The Corona Theorem

These are my notes for my talk in the winter semester of 2024, in the analysis seminar at the Technical University of Vienna.

We will denote the space of all complex-valued, bounded, analytic functions on the unit disk $\mathbb{D}$ as H^∞. The space of all multiplicative, bounded, linear functionals on H^∞ not identically zero is denoted $\Delta(H^\infty)$ and is called the *Gelfand space* of H^∞. We endow this space with the trace of the weak-$*$ topology on the dual $(H^\infty)'$ and refer to this as the *Gelfand topology*. For each $z \in \mathbb{D}$ we consider the point-evaluation functional

$$\pi_z : H^\infty \to \mathbb{C},\ f \mapsto f(z)$$

and the set

$$\Delta_0 \coloneqq \{\pi_z : z \in \mathbb{D}\}.$$

Clearly $\Delta_0 \subset \Delta(H^\infty)$. The *corona* is defined as the complement of closure of Δ_0 in the Gelfand topology. The corona theorem now states:

Theorem 1 (L. Carleson)**.** The corona is empty. In other words, Δ_0 is dense in $\Delta(H^\infty)$.