Package ‘fdasrvf’

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Type Package

Title Elastic Functional Data Analysis

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Description Performs alignment, PCA, and modeling of multidimensional and unidimensional functions using the square-root velocity framework (Srivastava et al., 2011 <arXiv:1103.3817> and Tucker et al., 2014 <DOI:10.1016/j.csda.2012.12.001>). This framework allows for elastic analysis of functional data through phase and amplitude separation.

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LazyData TRUE

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URL https://github.com/jdtuck/fdasrvf_R

BugReports https://github.com/jdtuck/fdasrvf_R/issues

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R topics documented:

`align_fPCA` *Group-wise function alignment and PCA Extractions*

Description

This function aligns a collection of functions while extracting principal components.

Usage

```
align_fPCA(
  f,
  time,
  num_comp = 3L,
  showplot = TRUE,
  smooth_data = FALSE,
  sparam = 25L,
  parallel = FALSE,
  cores = NULL,
  max_iter = 51L,
```

```
  lambda = 0
)
```

Arguments

- `f` — A numeric matrix of shape $M \times N$ specifying a sample of N 1-dimensional curves observed on a grid of size M.
- `time` — A numeric vector of length M specifying the grid on which functions `f` have been evaluated.
- `num_comp` — An integer value specifying the number of principal components to extract. Defaults to `3L`.
- `showplot` — A boolean specifying whether to display plots along the way. Defaults to `TRUE`.
- `smooth_data` — A boolean specifying whether to smooth data using box filter. Defaults to `FALSE`.
- `sparam` — An integer value specifying the number of times to apply box filter. Defaults to `25L`. This argument is only used if `smooth_data == TRUE`.
- `parallel` — A boolean specifying whether computations should run in parallel. Defaults to `FALSE`.
- `cores` — An integer value specifying the number of cores to use for parallel computations. Defaults to `NULL` in which case it uses all available cores but one. This argument is only used when `parallel == TRUE`.
- `max_iter` — An integer value specifying the maximum number of iterations. Defaults to `51L`.
- `lambda` — A numeric value specifying the elasticity. Defaults to `0.0`.

Value

A list with the following components:

- `f0`: A numeric matrix of shape $M \times N$ storing the original functions;
- `fn`: A numeric matrix of the same shape as `f0` storing the aligned functions;
- `qn`: A numeric matrix of the same shape as `f0` storing the aligned SRSFs;
- `q0`: A numeric matrix of the same shape as `f0` storing the SRSFs of the original functions;
- `mqn`: A numeric vector of length M storing the mean SRSF;
- `gam`: A numeric matrix of the same shape as `f0` storing the estimated warping functions;
- `vfpca`: A list storing information about the vertical PCA with the following components:
  - `q_pca`: A numeric matrix of shape $(M + 1) \times 5 \times \mathrm{num\_comp}$ storing the first 3 principal directions in SRSF space; the first dimension is $M + 1$ because, in SRSF space, the original functions are represented by the SRSF and the initial value of the functions.
  - `f_pca`: A numeric matrix of shape $M \times 5 \times \mathrm{num\_comp}$ storing the first 3 principal directions in original space;
  - `latent`: A numeric vector of length $M + 1$ storing the singular values of the SVD decomposition in SRSF space;
  - `coef`: A numeric matrix of shape $N \times \mathrm{num\_comp}$ storing the scores of the N original functions on the first `num_comp` principal components;

  - U: A numeric matrix of shape $(M+1) \times (M+1)$ storing the eigenvectors associated with the SVD decomposition in SRSF space.
- Dx: A numeric vector of length max_iter storing the value of the cost function at each iteration.

References

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
## Not run:
  out <- align_fPCA(simu_data$f, simu_data$time)

## End(Not run)
```

---

beta *MPEG7 Curve Dataset*

Description

Contains the MPEG7 curve data set.

Usage

```
beta
```

Format

beta:
An array of shape $2 \times 100 \times 65 \times 20$ storing a sample of 20 curves from R to R^2 distributed in 65 different classes, evaluated on a grid of size 100.

---

bootTB *Tolerance Bound Calculation using Bootstrap Sampling*

Description

This function computes tolerance bounds for functional data containing phase and amplitude variation using bootstrap sampling

Usage

```
bootTB(f, time, a = 0.05, p = 0.99, B = 500, no = 5, Nsamp = 100, parallel = T)
```

Arguments

`f`	matrix of functions
`time`	vector describing time sampling
`a`	confidence level of tolerance bound (default = 0.05)
`p`	coverage level of tolerance bound (default = 0.99)
`B`	number of bootstrap samples (default = 500)
`no`	number of principal components (default = 5)
`Nsamp`	number of functions per bootstrap (default = 100)
`parallel`	enable parallel processing (default = T)

Value

Returns a list containing

`amp`	amplitude tolerance bounds
`ph`	phase tolerance bounds

References

J. D. Tucker, J. R. Lewis, C. King, and S. Kurtek, “A Geometric Approach for Computing Tolerance Bounds for Elastic Functional Data,” Journal of Applied Statistics, 10.1080/02664763.2019.1645818, 2019.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Jung, S. L. a. S. (2016). "Combined Analysis of Amplitude and Phase Variations in Functional Data." arXiv:1603.01775.

Examples

```
## Not run:
  out1 <- bootTB(simu_data$f, simu_data$time)

## End(Not run)
```

`boxplot.fdawarp` *Functional Boxplot*

Description

This function computes the required statistics for building up a boxplot of the aligned functional data. Since the process of alignment provides separation of phase and amplitude variability, the computed boxplot can focus either on amplitude variability or phase variability.

Usage

```
## S3 method for class 'fdawarp'
boxplot(
  x,
  variability_type = c("amplitude", "phase"),
  alpha = 0.05,
  range = 1,
  what = c("plot", "stats", "plot+stats"),
  ...
)

## S3 method for class 'ampbox'
boxplot(x, ...)

## S3 method for class 'phbox'
boxplot(x, ...)
```

Arguments

`x`	An object of class `fdawarp` typically produced by `time_warping()` or of class `ampbox` or `phbox` typically produced by `boxplot.fdawarp()`.
`variability_type`	A string specifying which kind of variability should be displayed in the boxplot. Choices are `"amplitude"` or `"phase"`. Defaults to `"amplitude"`.
`alpha`	A numeric value specifying the quantile value. Defaults to 0.05 which uses the 95% quantile.
`range`	A positive numeric value specifying how far the plot whiskers extend out from the box. The whiskers extend to the most extreme data point which is no more than `range` times the interquartile range from the box. Defaults to `1.0`.
`what`	A string specifying what the function should return. Choices are `"plot"`, `"stats"` or `"plot+stats"`. Defaults to `"plot"`.
`...`	Unused here.

Details

The function `boxplot.fdawarp()` returns optionally an object of class either `ampbox` if `variability_type = "amplitude"` or `phbox` if `variability_type = "phase"`. `S3` methods specialized for objects of these classes are provided as well to avoid re-computation of the boxplot statistics.

Value

If `what` contains `stats`, a list containing the computed statistics necessary for drawing the boxplot. Otherwise, the function simply draws the boxplot and no object is returned.

Examples

```
## Not run:
out <- time_warping(simu_data$f, simu_data$time)
```

```
boxplot(out, what = "stats")

## End(Not run)
```

---

calc_shape_dist *Elastic Shape Distance*

Description

Calculate elastic shape distance between two curves beta1 and beta2

Usage

```
calc_shape_dist(beta1, beta2, mode = "O", scale = F)
```

Arguments

beta1	array describing curve1 (n,T)
beta2	array describing curve
mode	Open ("O") or Closed ("C") curves
scale	Include scale (default =F)

Value

Returns a list containing

d	geodesic distance
dx	phase distance

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- calc_shape_dist(beta[, , 1, 1], beta[, , 1, 4])
```

curve_geodesic *Form geodesic between two curves*

Description

Form geodesic between two curves using Elastic Method

Usage

```
curve_geodesic(beta1, beta2, k = 5)
```

Arguments

beta1	array describing curve 1 (n,T)
beta2	array describing curve 2 (n,T)
k	number of curves along geodesic (default 5)

Value

a list containing

geod	curves along geodesic (n,T,k)
geod_q	srvf's along geodesic

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_geodesic(beta[, , 1, 1], beta[, , 1, 5])
```

curve_karcher_cov *Curve Karcher Covariance*

Description

Calculate Karcher Covariance of a set of curves

Usage

```
curve_karcher_cov(v, len = NA)
```

Arguments

`v`	array (n,T,N) for N number of shooting vectors
`len`	lengths of curves (default=NA)

Value

K covariance matrix

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_karcher_mean(beta[, , 1, 1:2], maxit = 2)
# note: use more shapes, small for speed
K <- curve_karcher_cov(out$v)
```

`curve_karcher_mean` *Karcher Mean of Curves*

Description

Calculates Karcher mean or median of a collection of curves using the elastic square-root velocity (srvf) framework.

Usage

```
curve_karcher_mean(
  beta,
  mode = "O",
  rotated = T,
  scale = F,
  lambda = 0,
  maxit = 20,
  ms = "mean"
)
```

Arguments

`beta`	array (n,T,N) for N number of curves
`mode`	Open ("O") or Closed ("C") curves
`rotated`	Optimize over rotation (default = T)
`scale`	Include scale (default = F)

lambda	A numeric value specifying the elasticity. Defaults to 0.0.
maxit	maximum number of iterations
ms	string defining whether the Karcher mean ("mean") or Karcher median ("median") is returned (default = "mean")

Value

Returns a list containing

mu	mean srvf
beta	centered data
betamean	mean or median curve
type	string indicating whether mean or median is returned
v	shooting vectors
q	array of srvfs
gam	array of warping functions
cent	centers of original curves
len	length of curves
len_q	length of srvfs
mean_scale	mean length
mean_scale_q	mean length srvf
E	energy
qun	cost function

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_karcher_mean(beta[, , 1, 1:2], maxit = 2)
# note: use more shapes, small for speed
```

curve_pair_align *Pairwise align two curves*

Description

This function aligns to curves using Elastic Framework

Usage

```
curve_pair_align(beta1, beta2, mode = "O")
```

Arguments

beta1	array describing curve 1 (n,T)
beta2	array describing curve 2 (n,T)
mode	Open ("O") or Closed ("C") curves

Value

a list containing

beta2n	aligned curve 2 to 1
q2n	aligned srvf 2 to 1
gam	warping function
q1	srvf of curve 1
beta1	centered curve 1
beta2	centered curve 2

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_pair_align(beta[, , 1, 1], beta[, , 1, 5])
```

curve_principal_directions
Curve PCA

Description

Calculate principal directions of a set of curves

Usage

```
curve_principal_directions(v, K, mu, len = NA, no = 3, N = 5, mode = "O")
```

Arguments

v	array (n,T,N1) of shooting vectors
K	array (n*T*,*n*T) covariance matrix
mu	array (n,T) of mean srvf
len	length of original curves (default NA)
no	number of components
N	number of samples on each side of mean
mode	Open ("O") or Closed ("C") curves

Value

Returns a list containing

s	singular values
U	singular vectors
coef	principal coefficients
pd	principal directions

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_karcher_mean(beta[, , 1, 1:2], maxit = 2)
# note: use more shapes, small for speed
K <- curve_karcher_cov(out$v)
out <- curve_principal_directions(out$v, K, out$mu)
```

curve_srvf_align *Align Curves*

Description

Aligns a collection of curves using the elastic square-root velocity (srvf) framework.

Usage

```
curve_srvf_align(
  beta,
  mode = "O",
  rotated = T,
  scale = F,
  lambda = 0,
  maxit = 20,
  ms = "mean"
)
```

Arguments

`beta`	array (n,T,N) for N number of curves
`mode`	Open ("O") or Closed ("C") curves
`rotated`	Optimize over rotation (default = T)
`scale`	Include scale (default = F)
`lambda`	A numeric value specifying the elasticity. Defaults to `0.0`.
`maxit`	maximum number of iterations
`ms`	string defining whether the Karcher mean ("mean") or Karcher median ("median") is returned (default = "mean")

Value

Returns a list containing

`betan`	aligned curves
`qn`	aligned srvfs
`betamean`	mean curve
`q_mu`	mean SRVFs

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
data("mpeg7")
# note: use more shapes and iterations, small for speed
out = curve_srvf_align(beta[,,1,1:2],maxit=2)
```

---

curve_to_q *Convert to SRVF space*

Description

This function converts curves to SRVF

Usage

```
curve_to_q(beta)
```

Arguments

beta	array describing curve (n,T)

Value

q array describing srvf

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
q <- curve_to_q(beta[, , 1, 1])$q
```

---

elastic.depth *Calculates elastic depth*

Description

This functions calculates the elastic depth between set of functions

Usage

```
elastic.depth(f, time, lambda = 0, pen = "roughness", parallel = FALSE)
```

Arguments

f	matrix of N function of M time points (MxN)
time	sample points of functions
lambda	controls amount of warping (default = 0)
pen	alignment penalty (default="roughness") options are second derivative ("roughness"), geodesic distance from id ("geodesic"), and norm from id ("norm")
parallel	run computation in parallel (default = T)

Value

Returns a list containing

amp	amplitude depth
phase	phase depth

References

T. Harris, J. D. Tucker, B. Li, and L. Shand, "Elastic depths for detecting shape anomalies in functional data," Technometrics, 10.1080/00401706.2020.1811156, 2020.

Examples

```
depths <- elastic.depth(simu_data$f[, 1:4], simu_data$time)
```

---

elastic.distance *Calculates two elastic distance*

Description

This functions calculates the distances between functions, D_y and D_x, where function 1 is aligned to function 2

Usage

```
elastic.distance(f1, f2, time, lambda = 0, pen = "roughness")
```

Arguments

f1	sample function 1
f2	sample function 2
time	sample points of functions
lambda	controls amount of warping (default = 0)
pen	alignment penalty (default="roughness") options are second derivative ("roughness"), geodesic distance from id ("geodesic"), and norm from id ("norm")

Value

Returns a list containing

Dy	amplitude distance
Dx	phase distance

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
distances <- elastic.distance(
  f1 = simu_data$f[, 1],
  f2 = simu_data$f[, 2],
  time = simu_data$time
)
```

`elastic.logistic` *Elastic Logistic Regression*

Description

This function identifies a logistic regression model with phase-variability using elastic methods

Usage

```
elastic.logistic(
  f,
  y,
  time,
  B = NULL,
  df = 20,
  max_itr = 20,
  smooth_data = FALSE,
  sparam = 25,
  parallel = FALSE,
  cores = 2
)
```

Arguments

f	matrix (N x M) of M functions with N samples
y	vector of size M labels (1/-1)
time	vector of size N describing the sample points
B	matrix defining basis functions (default = NULL)
df	scalar controlling degrees of freedom if B=NULL (default=20)
max_itr	scalar number of iterations (default=20)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
parallel	enable parallel mode using `foreach()` and `doParallel` package
cores	set number of cores to use with `doParallel` (default = 2)

Value

Returns a list containing

alpha	model intercept
beta	regressor function
fn	aligned functions - matrix (N x M) of M functions with N samples
qn	aligned srvfs - similar structure to fn
gamma	warping functions - similar structure to fn
q	original srvf - similar structure to fn
B	basis matrix
b	basis coefficients
Loss	logistic loss
type	model type ('logistic')

References

Tucker, J. D., Wu, W., Srivastava, A., Elastic Functional Logistic Regression with Application to Physiological Signal Classification, Electronic Journal of Statistics (2014), submitted.

elastic.lpcr.regression
Elastic logistic Principal Component Regression

Description

This function identifies a logistic regression model with phase-variability using elastic pca

Usage

```
elastic.lpcr.regression(
  f,
  y,
  time,
  pca.method = "combined",
  no = 5,
  smooth_data = FALSE,
  sparam = 25
)
```

Arguments

f	matrix (N x M) of M functions with N samples
y	vector of size M labels
time	vector of size N describing the sample points
pca.method	string specifying pca method (options = "combined", "vert", or "horiz", default = "combined")
no	scalar specify number of principal components (default=5)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)

Value

Returns a lpcr object containing

alpha	model intercept
b	regressor vector
y	label vector
warp_data	fdawarp object of aligned data
pca	pca object of principal components
Loss	logistic loss
pca.method	string specifying pca method used

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, "Elastic Functional Principal Component Regression," Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

`elastic.mlogistic` *Elastic Multinomial Logistic Regression*

Description

This function identifies a multinomial logistic regression model with phase-variability using elastic methods

Usage

```
elastic.mlogistic(
  f,
  y,
  time,
  B = NULL,
  df = 20,
  max_itr = 20,
  smooth_data = FALSE,
  sparam = 25,
  parallel = FALSE,
  cores = 2
)
```

Arguments

f	matrix (N x M) of M functions with N samples
y	vector of size M labels (1,2,...,m) for m classes
time	vector of size N describing the sample points
B	matrix defining basis functions (default = NULL)
df	scalar controlling degrees of freedom if B=NULL (default=20)
max_itr	scalar number of iterations (default=20)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
parallel	enable parallel mode using `foreach()` and `doParallel` package
cores	set number of cores to use with `doParallel` (default = 2)

Value

Returns a list containing

alpha	model intercept
beta	regressor function
fn	aligned functions - matrix (N x M) of M functions with N samples
qn	aligned srvfs - similar structure to fn
gamma	warping functions - similar structure to fn
q	original srvf - similar structure to fn
B	basis matrix
b	basis coefficients
Loss	logistic loss
type	model type ('mlogistic')

References

Tucker, J. D., Wu, W., Srivastava, A., Elastic Functional Logistic Regression with Application to Physiological Signal Classification, Electronic Journal of Statistics (2014), submitted.

---

elastic.mlpcr.regression

Elastic Multinomial logistic Principal Component Regression

Description

This function identifies a multinomial logistic regression model with phase-variability using elastic pca

Usage

```
elastic.mlpcr.regression(
  f,
  y,
  time,
  pca.method = "combined",
  no = 5,
  smooth_data = FALSE,
  sparam = 25
)
```

Arguments

`f`	matrix (N x M) of M functions with N samples
`y`	vector of size M labels
`time`	vector of size N describing the sample points
`pca.method`	string specifying pca method (options = "combined", "vert", or "horiz", default = "combined")
`no`	scalar specify number of principal components (default=5)
`smooth_data`	smooth data using box filter (default = F)
`sparam`	number of times to apply box filter (default = 25)

Value

Returns a mlpcr object containing

`alpha`	model intercept
`b`	regressor vector
`y`	label vector
`Y`	Coded labels
`warp_data`	fdawarp object of aligned data
`pca`	pca object of principal components
`Loss`	logistic loss
`pca.method`	string specifying pca method used

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, "Elastic Functional Principal Component Regression," Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

`elastic.pcr.regression` *Elastic Linear Principal Component Regression*

Description

This function identifies a regression model with phase-variability using elastic pca

Usage

```
elastic.pcr.regression(
  f,
  y,
  time,
  pca.method = "combined",
  no = 5,
  smooth_data = FALSE,
  sparam = 25,
  parallel = F,
  C = NULL
)
```

Arguments

f	matrix (N x M) of M functions with N samples
y	vector of size M responses
time	vector of size N describing the sample points
pca.method	string specifying pca method (options = "combined", "vert", or "horiz", default = "combined")
no	scalar specify number of principal components (default = 5)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
parallel	run in parallel (default = F)
C	scale balance parameter for combined method (default = NULL)

Value

Returns a pcr object containing

alpha	model intercept
b	regressor vector
y	response vector
warp_data	fdawarp object of aligned data
pca	pca object of principal components
SSE	sum of squared errors
pca.method	string specifying pca method used

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, “Elastic Functional Principal Component Regression,” Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

`elastic.prediction` *Elastic Prediction from Regression Models*

Description

This function performs prediction from an elastic regression model with phase-variability

Usage

```
elastic.prediction(f, time, model, y = NULL, smooth_data = FALSE, sparam = 25)
```

Arguments

f	matrix (N x M) of M functions with N samples
time	vector of size N describing the sample points
model	list describing model from elastic regression methods
y	responses of test matrix f (default=NULL)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)

Value

Returns a list containing

y_pred	predicted values of f or probabilities depending on model
SSE	sum of squared errors if linear
y_labels	labels if logistic model
PC	probability of classification if logistic

References

Tucker, J. D., Wu, W., Srivastava, A., Elastic Functional Logistic Regression with Application to Physiological Signal Classification, Electronic Journal of Statistics (2014), submitted.

elastic.regression *Elastic Linear Regression*

Description

This function identifies a regression model with phase-variability using elastic methods

Usage

```
elastic.regression(
  f,
  y,
  time,
  B = NULL,
  lam = 0,
  df = 20,
  max_itr = 20,
  smooth_data = FALSE,
  sparam = 25,
  parallel = FALSE,
  cores = 2
)
```

Arguments

f	matrix (N x M) of M functions with N samples
y	vector of size M responses
time	vector of size N describing the sample points
B	matrix defining basis functions (default = NULL)
lam	scalar regularization parameter (default=0)
df	scalar controlling degrees of freedom if B=NULL (default=20)
max_itr	scalar number of iterations (default=20)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
parallel	enable parallel mode using `foreach()` and `doParallel` package
cores	set number of cores to use with `doParallel` (default = 2)

Value

Returns a list containing

alpha	model intercept
beta	regressor function
fn	aligned functions - matrix (N x M) of M functions with N samples

qn	aligned srvfs - similar structure to fn
gamma	warping functions - similar structure to fn
q	original srvf - similar structure to fn
B	basis matrix
b	basis coefficients
SSE	sum of squared errors
type	model type ('linear')

References

Tucker, J. D., Wu, W., Srivastava, A., Elastic Functional Logistic Regression with Application to Physiological Signal Classification, Electronic Journal of Statistics (2014), submitted.

`elastic_amp_change_ff` *Elastic Amplitude Changepoint Detection*

Description

This function identifies a amplitude changepoint using a fully functional approach

Usage

```
elastic_amp_change_ff(
  f,
  time,
  d = 1000,
  h = 0,
  smooth_data = FALSE,
  sparam = 25,
  showplot = TRUE
)
```

Arguments

f	matrix (N x M) of M functions with N samples
time	vector of size N describing the sample points
d	number of monte carlo iterations of Brownian Bridge (default = 1000)
h	window selection of long range covariance function (default = 0)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
showplot	show results plots (default = T)

Value

Returns a list object containing

pvalue	p value
change	indice of changepoint
DataBefore	functions before changepoint
DataAfter	functions after changepoint
MeanBefore	mean function before changepoint
MeanAfter	mean function after changepoint
WarpingBefore	warping functions before changepoint
WarpingAfter	warping functions after changepoint
WarpingMeanBefore	mean warping function before changepoint
WarpingMeanAfter	mean warping function after changepoint
change_fun	amplitude change function
Sn	test statistic values
mu	mean srsfs
mu_f	mean functions

References

J. D. Tucker and D. Yarger, “Elastic Functional Changepoint Detection of Climate Impacts from Localized Sources”, Envirometrics, 10.1002/env.2826, 2023.

---

elastic_change_fpca *Elastic Changepoint Detection*

Description

This function identifies changepoints using a functional PCA

Usage

```
elastic_change_fpca(
  f,
  time,
  pca.method = "combined",
  pc = 0.95,
  d = 1000,
  n_pcs = 5,
  smooth_data = FALSE,
  sparam = 25,
  showplot = TRUE
)
```

Arguments

f	matrix (N x M) of M functions with N samples
time	vector of size N describing the sample points
pca.method	string specifying pca method (options = "combined", "vert", or "horiz", default = "combined")
pc	percentage of cummulation explained variance (default = 0.95)
d	number of monte carlo iterations of Brownian Bridge (default = 1000)
n_pcs	scalar specify number of principal components (default = 5)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
showplot	show results plots (default = T)

Value

Returns a list object containing

pvalue	p value
change	indice of changepoint
DataBefore	functions before changepoint
DataAfter	functions after changepoint
MeanBefore	mean function before changepoint
MeanAfter	mean function after changepoint
WarpingBefore	warping functions before changepoint
WarpingAfter	warping functions after changepoint
WarpingMeanBefore	mean warping function before changepoint
WarpingMeanAfter	mean warping function after changepoint
change_fun	amplitude change function
Sn	test statistic values

References

J. D. Tucker and D. Yarger, “Elastic Functional Changepoint Detection of Climate Impacts from Localized Sources”, Envirometrics, 10.1002/env.2826, 2023.

`elastic_ph_change_ff` *Elastic Phase Changepoint Detection*

Description

This function identifies a phase changepoint using a fully functional approach

Usage

```
elastic_ph_change_ff(
  f,
  time,
  d = 1000,
  h = 0,
  smooth_data = FALSE,
  sparam = 25,
  showplot = TRUE
)
```

Arguments

`f`	matrix (N x M) of M functions with N samples
`time`	vector of size N describing the sample points
`d`	number of monte carlo iterations of Brownian Bridge (default = 1000)
`h`	window selection of long range covariance function (default = 0)
`smooth_data`	smooth data using box filter (default = F)
`sparam`	number of times to apply box filter (default = 25)
`showplot`	show results plots (default = T)

Value

Returns a list object containing

`pvalue`	p value
`change`	indice of changepoint
`DataBefore`	functions before changepoint
`DataAfter`	functions after changepoint
`MeanBefore`	mean function before changepoint
`MeanAfter`	mean function after changepoint
`WarpingBefore`	warping functions before changepoint
`WarpingAfter`	warping functions after changepoint
`WarpingMeanBefore`	mean warping function before changepoint

WarpingMeanAfter
: mean warping function after changepoint

change_fun
: amplitude change function

Sn
: test statistic values

mu
: mean shooting vectors

References

J. D. Tucker and D. Yarger, “Elastic Functional Changepoint Detection of Climate Impacts from Localized Sources”, Envirometrics, 10.1002/env.2826, 2023.

---

fdasrvf *Elastic Functional Data Analysis*

Description

A library for functional data analysis using the square root velocity framework which performs pair-wise and group-wise alignment as well as modeling using functional component analysis.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using Fisher-Rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

J. D. Tucker, W. Wu, and A. Srivastava, Phase-amplitude separation of proteomics data using extended Fisher-Rao metric, Electronic Journal of Statistics, Vol 8, no. 2. pp 1724-1733, 2014.

J. D. Tucker, W. Wu, and A. Srivastava, “Analysis of signals under compositional noise with applications to SONAR data," IEEE Journal of Oceanic Engineering, Vol 29, no. 2. pp 318-330, Apr 2014.

Tucker, J. D. 2014, Functional Component Analysis and Regression using Elastic Methods. Ph.D. Thesis, Florida State University.

Robinson, D. T. 2012, Function Data Analysis and Partial Shape Matching in the Square Root Velocity Framework. Ph.D. Thesis, Florida State University.

Huang, W. 2014, Optimization Algorithms on Riemannian Manifolds with Applications. Ph.D. Thesis, Florida State University.

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

W. Xie, S. Kurtek, K. Bharath, and Y. Sun, A geometric approach to visualization of variability in functional data, Journal of American Statistical Association 112 (2017), pp. 979-993.

Lu, Y., R. Herbei, and S. Kurtek, 2017: Bayesian registration of functions with a Gaussian process prior. Journal of Computational and Graphical Statistics, 26, no. 4, 894–904.

Lee, S. and S. Jung, 2017: Combined analysis of amplitude and phase variations in functional data. arXiv:1603.01775, 1–21.

J. D. Tucker, J. R. Lewis, and A. Srivastava, “Elastic Functional Principal Component Regression,” Statistical Analysis and Data Mining, vol. 12, no. 2, pp. 101-115, 2019.

J. D. Tucker, J. R. Lewis, C. King, and S. Kurtek, “A Geometric Approach for Computing Tolerance Bounds for Elastic Functional Data,” Journal of Applied Statistics, 10.1080/02664763.2019.1645818, 2019.

T. Harris, J. D. Tucker, B. Li, and L. Shand, "Elastic depths for detecting shape anomalies in functional data," Technometrics, 10.1080/00401706.2020.1811156, 2020.

J. D. Tucker and D. Yarger, “Elastic Functional Changepoint Detection of Climate Impacts from Localized Sources”, Envirometrics, 10.1002/env.2826, 2023.

`function_group_warp_bayes`

Bayesian Group Warping

Description

This function aligns a set of functions using Bayesian SRSF framework

Usage

```
function_group_warp_bayes(
  f,
  time,
  iter = 50000,
  powera = 1,
  times = 5,
  tau = ceiling(times * 0.04),
  gp = seq(dim(f)[2]),
  showplot = TRUE
)
```

Arguments

`f`	matrix (N x M) of M functions with N samples
`time`	sample points of functions
`iter`	number of iterations (default = 150000)
`powera`	Dirichlet prior parameter (default 1)
`times`	factor of length of subsample points to look at (default = 5)

tau	standard deviation of Normal prior for increment (default ceil(times*.4))
gp	number of colors in plots (defaults seq(dim(f)[2]))
showplot	shows plots of functions (default = T)

Value

Returns a list containing

f0	original functions
f_q	f aligned quotient space
gam_q	warping functions quotient space
f_a	f aligned ambient space
gam_a	warping ambient space
qmn	mean srsf

References

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

Examples

```
## Not run:
  out <- function_group_warp_bayes(simu_data$f, simu_data$time)

## End(Not run)
```

---

function_mean_bayes   *Bayesian Karcher Mean Calculation*

Description

This function calculates karcher mean of functions using Bayesian method

Usage

```
function_mean_bayes(f, time, times = 5, group = 1:dim(f)[2], showplot = TRUE)
```

Arguments

f	matrix (N x M) of M functions with N samples
time	sample points of functions
times	factor of length of subsample points to look at (default = 5)
group	(defaults 1:dim(f)[2])
showplot	shows plots of functions (default = T)

Value

Returns a list containing

distfamily	dist matrix
match.matrix	matrix of warping functions
position	position
mu_5	function mean
rtmatrix	rtmatrix
sumdist	sumdist
qt.fitted	aligned srsf functions
estimator	estimator
estimator2	estimator2
regfuncs	registered functions

References

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

Examples

```
## Not run:
  out <- function_mean_bayes(simu_data$f, simu_data$time)

## End(Not run)
```

`f_to_srvf` *Transformation to SRSF Space*

Description

This function transforms curves from their original functional space to the SRVF space.

Usage

```
f_to_srvf(f, time, multidimensional = FALSE)
```

Arguments

f Either a numeric vector of a numeric matrix or a numeric array specifying the functions that need to be transformed.

- If a vector, it must be of shape M and it is interpreted as a single 1-dimensional curve observed on a grid of size M.

- If a matrix and `multidimensional == FALSE`, it must be of shape $M \times N$. In this case, it is interpreted as a sample of N curves observed on a grid of size M, unless $M = 1$ in which case it is interpreted as a single 1-dimensional curve observed on a grid of size M.
- If a matrix and `multidimensional == TRUE`,it must be of shape $L \times M$ and it is interpreted as a single L-dimensional curve observed on a grid of size M.
- If a 3D array, it must be of shape $L \times M \times N$ and it is interpreted as a sample of N L-dimensional curves observed on a grid of size M.

`time` A numeric vector of length M specifying the grid on which the curves are evaluated.

`multidimensional` A boolean specifying if the curves are multi-dimensional. This is useful when `f` is provided as a matrix to determine whether it is a single multi-dimensional curve or a collection of uni-dimensional curves. Defaults to `FALSE`.

Value

A numeric array of the same shape as the input array `f` storing the SRSFs of the original curves.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using Fisher-Rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
q <- f_to_srvf(simu_data$f, simu_data$time)
```

---

gam_to_v *map warping function to tangent space at identity*

Description

map warping function to tangent space at identity

Usage

```
gam_to_v(gam, smooth = TRUE)
```

Arguments

`gam` Either a numeric vector of a numeric matrix or a numeric array specifying the warping functions

`smooth` Apply smoothing before gradient

Value

A numeric array of the same shape as the input array gamma storing the shooting vectors of gamma obtained via finite differences.

---

gauss_model *Gaussian model of functional data*

Description

This function models the functional data using a Gaussian model extracted from the principal components of the srvfs

Usage

```
gauss_model(warp_data, n = 1, sort_samples = FALSE)
```

Arguments

warp_data	fdawarp object from time_warping of aligned data
n	number of random samples (n = 1)
sort_samples	sort samples (default = F)

Value

Returns a fdawarp object containing

fs	random aligned samples
gams	random warping function samples
ft	random function samples

References

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
out1 <- gauss_model(simu_warp, n = 10)
```

gradient *Gradient using finite differences*

Description

This function computes the gradient of `f` using finite differences.

Usage

```
gradient(f, binsize, multidimensional = FALSE)
```

Arguments

`f` Either a numeric vector of a numeric matrix or a numeric array specifying the curve(s) that need to be differentiated.

- If a vector, it must be of shape M and it is interpreted as a single 1-dimensional curve observed on a grid of size M.
- If a matrix and `multidimensional == FALSE`, it must be of shape $M \times N$. In this case, it is interpreted as a sample of N curves observed on a grid of size M, unless $M = 1$ in which case it is interpreted as a single 1-dimensional curve observed on a grid of size M.
- If a matrix and `multidimensional == TRUE`,it must be of shape $L \times M$ and it is interpreted as a single L-dimensional curve observed on a grid of size M.
- If a 3D array, it must be of shape $L \times M \times N$ and it is interpreted as a sample of N L-dimensional curves observed on a grid of size M.

`binsize` A numeric value specifying the size of the bins for computing finite differences.

`multidimensional`
A boolean specifying if the curves are multi-dimensional. This is useful when `f` is provided as a matrix to determine whether it is a single multi-dimensional curve or a collection of uni-dimensional curves. Defaults to `FALSE`.

Value

A numeric array of the same shape as the input array `f` storing the gradient of `f` obtained via finite differences.

Examples

```
out <- gradient(simu_data$f[, 1], mean(diff(simu_data$time)))
```

growth_vel — *Berkeley Growth Velocity Dataset*

Description

Combination of both boys and girls growth velocity from the Berkley dataset.

Usage

```
growth_vel
```

Format

growth_vel:

A list with two components:

- `f`: A numeric matrix of shape 69×93 storing a sample of size $N = 93$ of curves evaluated on a grid of size $M = 69$.
- `time`: A numeric vector of size $M = 69$ storing the grid on which the curves `f` have been evaluated.

horizFPCA — *Horizontal Functional Principal Component Analysis*

Description

This function calculates vertical functional principal component analysis on aligned data

Usage

```
horizFPCA(warp_data, no, ci = c(-1, 0, 1), showplot = TRUE)
```

Arguments

`warp_data`	fdawarp object from time_warping of aligned data
`no`	number of principal components to extract
`ci`	geodesic standard deviations (default = c(-1,0,1))
`showplot`	show plots of principal directions (default = T)

Value

Returns a hfpca object containing

gam_pca	warping functions principal directions
psi_pca	srvf principal directions
latent	latent values
U	eigenvectors
vec	shooting vectors
mu	Karcher Mean

References

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
hfpca <- horizFPCA(simu_warp, no = 3)
```

---

im *Example Image Data set*

Description

Contains two simulated images for registration.

Usage

```
im
```

Format

`im`:

A list with two components:

- `I1`: A numeric matrix of shape 64×64 storing the 1st image;
- `I2`: A numeric matrix of shape 64×64 storing the 2nd image.

invertGamma *Invert Warping Function*

Description

This function calculates the inverse of gamma

Usage

```
invertGamma(gam)
```

Arguments

gam	vector of N samples

Value

Returns gamI inverted vector

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
out <- invertGamma(simu_warp$warping_functions[, 1])
```

inv_exp_map *map square root of warping function to tangent space*

Description

map square root of warping function to tangent space

Usage

```
inv_exp_map(Psi, psi)
```

Arguments

Psi	vector describing psi function at center of tangent space
psi	vector describing psi function to map to tangent space

Value

A numeric array of the same length as the input array psi storing the shooting vector of psi

---

jointFPCA *Joint Vertical and Horizontal Functional Principal Component Analysis*

Description

This function calculates amplitude and phase joint functional principal component analysis on aligned data

Usage

```
jointFPCA(
  warp_data,
  no,
  id = round(length(warp_data$time)/2),
  C = NULL,
  ci = c(-1, 0, 1),
  showplot = T
)
```

Arguments

warp_data	fdawarp object from time_warping of aligned data
no	number of principal components to extract
id	integration point for f0 (default = midpoint)
C	balance value (default = NULL)
ci	geodesic standard deviations (default = c(-1,0,1))
showplot	show plots of principal directions (default = T)

Value

Returns a list containing

q_pca	srvf principal directions
f_pca	f principal directions
latent	latent values
coef	coefficients
U	eigenvectors
mu_psi	mean psi function
mu_g	mean g function
id	point use for f(0)
C	optimized phase amplitude ratio

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Jung, S. L. a. S. (2016). "Combined Analysis of Amplitude and Phase Variations in Functional Data." arXiv:1603.01775.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
jfpca <- jointFPCA(simu_warp, no = 3)
```

---

joint_gauss_model — *Gaussian model of functional data using joint Model*

Description

This function models the functional data using a Gaussian model extracted from the principal components of the srvfs using the joint model

Usage

```
joint_gauss_model(warp_data, n = 1, no = 5)
```

Arguments

`warp_data`	fdawarp object from time_warping of aligned data
`n`	number of random samples (n = 1)
`no`	number of principal components (n=4)

Value

Returns a fdawarp object containing

`fs`	random aligned samples
`gams`	random warping function samples
`ft`	random function samples
`qs`	random srvf samples

References

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Jung, S. L. a. S. (2016). "Combined Analysis of Amplitude and Phase Variations in Functional Data." arXiv:1603.01775.

Examples

```
out1 <- joint_gauss_model(simu_warp, n = 10)
```

---

kmeans_align *K-Means Clustering and Alignment*

Description

This function clusters functions and aligns using the elastic square-root slope function (SRSF) framework.

Usage

```
kmeans_align(
  f,
  time,
  K = 1L,
  seeds = NULL,
  centroid_type = c("mean", "medoid"),
  nonempty = 0L,
  lambda = 0,
  showplot = FALSE,
  smooth_data = FALSE,
  sparam = 25L,
  parallel = FALSE,
  alignment = TRUE,
  omethod = c("DP", "DP2", "RBFGS"),
  max_iter = 50L,
  thresh = 0.01,
  use_verbose = FALSE
)
```

Arguments

`f` Either a numeric matrix or a numeric 3D array specifying the functions that need to be jointly clustered and aligned.

- If a matrix, it must be of shape $M \times N$. In this case, it is interpreted as a sample of N curves observed on a grid of size M.
- If a 3D array, it must be of shape $L \times M \times N$ and it is interpreted as a sample of N L-dimensional curves observed on a grid of size M.

`time` A numeric vector of length M specifying the grid on which the curves are evaluated.

`K` An integer value specifying the number of clusters. Defaults to `1L`.

`seeds` An integer vector of length `K` specifying the indices of the curves in `f` which will be chosen as initial centroids. Defaults to `NULL` in which case such indices are randomly chosen.

centroid_type	A string specifying the type of centroid to compute. Choices are "mean" or "medoid". Defaults to "mean".
nonempty	An integer value specifying the minimum number of curves per cluster during the assignment step. Set it to a positive value to avoid the problem of empty clusters. Defaults to 0L.
lambda	A numeric value specifying the elasticity. Defaults to 0.0.
showplot	A boolean specifying whether to show plots. Defaults to FALSE.
smooth_data	A boolean specifying whether to smooth data using a box filter. Defaults to FALSE.
sparam	An integer value specifying the number of box filters applied. Defaults to 25L.
parallel	A boolean specifying whether parallel mode (using foreach::foreach() and the **doParallel** package) should be activated. Defaults to FALSE.
alignment	A boolean specifying whether to perform alignment. Defaults to TRUE.
omethod	A string specifying which method should be used to solve the optimization problem that provides estimated warping functions. Choices are "DP" or "RBFGS". Defaults to "DP".
max_iter	An integer value specifying the maximum number of iterations. Defaults to 50L.
thresh	A numeric value specifying a threshold on the cost function below which convergence is assumed. Defaults to 0.01.
use_verbose	A boolean specifying whether to display information about the calculations in the console. Defaults to FALSE.

Value

An object of class fdakma which is a list containing:

- f0: the original functions;
- q0: the original SRSFs;
- fn: the aligned functions as matrices or a 3D arrays of the same shape than f0 by clusters in a list;
- qn: the aligned SRSFs as matrices or a 3D arrays of the same shape than f0 separated in clusters in a list;
- labels: the cluster memberships as an integer vector;
- templates: the centroids in the original functional space;
- templates.q: the centroids in SRSF space;
- distances_to_center: A numeric vector storing the distances of each observed curve to its center;
- gam: the warping functions as matrices or a 3D arrays of the same shape than f0 by clusters in a list;
- qun: cost function value.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using Fisher-Rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Sangalli, L. M., et al. (2010). "k-mean alignment for curve clustering." Computational Statistics & Data Analysis 54(5): 1219-1233.

Examples

```
## Not run:
  out <- kmeans_align(growth_vel$f, growth_vel$time, K = 2)

## End(Not run)
```

`LongRunCovMatrix` *Long Run Covariance Matrix Estimation for Multivariate Time Series*

Description

This function estimates the long run covariance matrix of a given multivariate data sample.

Usage

```
LongRunCovMatrix(mdobj, h = 0, kern_type = "bartlett")
```

Arguments

`mdobj`	A multivariate data object
`h`	The bandwidth parameter. It is strictly non-zero. Choosing the bandwidth parameter to be zero is identical to estimating covariance matrix assuming iid data.
`kern_type`	Kernel function to be used for the estimation of the long run covariance matrix. The choices are `c("BT", "PR", "SP", "FT")` which are respectively, bartlett, parzen, simple and flat-top kernels. By default the function uses a `"barlett"` kernel.

Value

Returns long run covariance matrix

---

multiple_align_functions
Group-wise function alignment to specified mean

Description

This function aligns a collection of functions using the elastic square-root slope (srsf) framework.

Usage

```
multiple_align_functions(
  f,
  time,
  mu,
  lambda = 0,
  pen = "roughness",
  showplot = TRUE,
  smooth_data = FALSE,
  sparam = 25,
  parallel = FALSE,
  omethod = "DP",
  MaxItr = 20,
  iter = 2000
)
```

Arguments

f	matrix (N x M) of M functions with N samples
time	vector of size N describing the sample points
mu	vector of size N that f is aligned to
lambda	controls the elasticity (default = 0)
pen	alignment penalty (default="roughness") options are second derivative ("roughness"), geodesic distance from id ("geodesic"), and norm from id ("norm")
showplot	shows plots of functions (default = T)
smooth_data	smooth data using box filter (default = F)
sparam	number of times to apply box filter (default = 25)
parallel	enable parallel mode using foreach() and doParallel package (default=F)
omethod	optimization method (DP,DP2,RBFGS,dBayes,expBayes)
MaxItr	maximum number of iterations
iter	bayesian number of mcmc samples (default 2000)

Value

Returns a fdawarp object containing

`f0`	original functions
`fn`	aligned functions - matrix (N x M) of M functions with N samples
`qn`	aligned SRSFs - similar structure to fn
`q0`	original SRSF - similar structure to fn
`fmean`	function mean or median - vector of length N
`mqn`	SRSF mean or median - vector of length N
`gam`	warping functions - similar structure to fn
`orig.var`	Original Variance of Functions
`amp.var`	Amplitude Variance
`phase.var`	Phase Variance
`qun`	Cost Function Value

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

`optimum.reparam` *Align two functions*

Description

This function aligns the SRSFs of two functions defined on an interval $[t_{\min}, t_{\max}]$ using dynamic programming or RBFGS

Usage

```
optimum.reparam(
  Q1,
  T1,
  Q2,
  T2,
  lambda = 0,
  pen = "roughness",
  method = c("DP", "DPo", "SIMUL", "RBFGS"),
  f1o = 0,
  f2o = 0,
  nbhd_dim = 7
)
```

Arguments

- `Q1` — A numeric matrix of shape `n_points x n_dimensions` specifying the SRSF of the 1st `n_dimensions`-dimensional function evaluated on a grid of size `n_points` of its univariate domain.
- `T1` — A numeric vector of size `n_points` specifying the grid on which the 1st SRSF is evaluated.
- `Q2` — A numeric matrix of shape `n_points x n_dimensions` specifying the SRSF of the 2nd `n_dimensions`-dimensional function evaluated on a grid of size `n_points` of its univariate domain.
- `T2` — A numeric vector of size `n_points` specifying the grid on which the 1st SRSF is evaluated.
- `lambda` — A numeric value specifying the amount of warping. Defaults to `0.0`.
- `pen` — alignment penalty (default="roughness") options are second derivative ("roughness"), geodesic distance from id ("geodesic"), and norm from id ("l2gam"), srvf norm from id ("l2psi")
- `method` — A string specifying the optimization method. Choices are `"DP"`, `"DPo"`, `"SIMUL"`, or `"RBFGS"`. Defaults to `"DP"`.
- `f1o` — A numeric vector of size `n_dimensions` specifying the value of the 1st function at $t = t_{\min}$. Defaults to `rep(0, n_dimensions)`.
- `f2o` — A numeric vector of size `n_dimensions` specifying the value of the 2nd function at $t = t_{\min}$. Defaults to `rep(0, n_dimensions)`.
- `nbhd_dim` — size of the grid (default = 7)

Value

A numeric vector of size `n_points` storing discrete evaluations of the estimated boundary-preserving warping diffeomorphism on the initial grid.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using Fisher-Rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
q <- f_to_srvf(simu_data$f, simu_data$time)
gam <- optimum.reparam(q[, 1], simu_data$time, q[, 2], simu_data$time)
```

outlier.detection *Outlier Detection*

Description

This function calculates outlier's using geodesic distances of the SRVFs from the median

Usage

```
outlier.detection(q, time, mq, k = 1.5)
```

Arguments

q	matrix (N x M) of M SRVF functions with N samples
time	vector of size N describing the sample points
mq	median calculated using `time_warping()`
k	cutoff threshold (default = 1.5)

Value

q_outlier	outlier functions

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
q_outlier <- outlier.detection(
  q = toy_warp$q0,
  time = toy_data$time,
  mq = toy_warp$mqn,
  k = .1
)
```

`pair_align_functions` *Align two functions*

Description

This function aligns two functions using SRSF framework. It will align f2 to f1

Usage

```
pair_align_functions(
  f1,
  f2,
  time,
  lambda = 0,
  pen = "roughness",
  method = "DP",
  w = 0.01,
  iter = 2000
)
```

Arguments

`f1`	function 1
`f2`	function 2
`time`	sample points of functions
`lambda`	controls amount of warping (default = 0)
`pen`	alignment penalty (default="roughness") options are second derivative ("roughness"), geodesic distance from id ("geodesic"), and norm from id ("norm")
`method`	controls which optimization method (default="DP") options are Dynamic Programming ("DP"), Coordinate Descent ("DP2"), Riemannian BFGS ("RBFGS"), Simultaneous Alignment ("SIMUL"), Dirichlet Bayesian ("dBayes"), and Expo-Map Bayesian ("expBayes")
`w`	controls LRBFGS (default = 0.01)
`iter`	number of mcmc iterations for mcmc method (default 2000)

Value

Returns a list containing

`f2tilde`	aligned f2
`gam`	warping function

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

Lu, Y., Herbei, R., and Kurtek, S. (2017). Bayesian registration of functions with a Gaussian process prior. Journal of Computational and Graphical Statistics, DOI: 10.1080/10618600.2017.1336444.

Examples

```
out <- pair_align_functions(
  f1 = simu_data$f[, 1],
  f2 = simu_data$f[, 2],
  time = simu_data$time
)
```

---

pair_align_functions_bayes

Align two functions

Description

This function aligns two functions using Bayesian SRSF framework. It will align f2 to f1

Usage

```
pair_align_functions_bayes(
  f1,
  f2,
  timet,
  iter = 15000,
  times = 5,
  tau = ceiling(times * 0.4),
  powera = 1,
  showplot = TRUE,
  extrainfo = FALSE
)
```

Arguments

`f1`	function 1
`f2`	function 2
`timet`	sample points of functions

iter	number of iterations (default = 15000)
times	factor of length of subsample points to look at (default = 5)
tau	standard deviation of Normal prior for increment (default ceil(times*.4))
powera	Dirichlet prior parameter (default 1)
showplot	shows plots of functions (default = T)
extrainfo	T/F whether additional information is returned

Value

Returns a list containing

f1	function 1
f2_q	registered function using quotient space
gam_q	warping function quotient space
f2_a	registered function using ambient space
q2_a	warping function ambient space
match_collect	posterior samples from warping function (returned if extrainfo=TRUE)
dist_collect	posterior samples from the distances (returned if extrainfo=TRUE)
kappa_collect	posterior samples from kappa (returned if extrainfo=TRUE)
log_collect	log-likelihood of each sample (returned if extrainfo=TRUE)
pct_accept	vector of acceptance ratios for the warping function (returned if extrainfo=TRUE)

References

Cheng, W., Dryden, I. L., and Huang, X. (2016). Bayesian registration of functions and curves. Bayesian Analysis, 11(2), 447-475.

Examples

```
out <- pair_align_functions_bayes(
  f1 = simu_data$f[, 1],
  f2 = simu_data$f[, 2],
  timet = simu_data$time
)
```

pair_align_functions_expomap
Align two functions using geometric properties of warping functions

Description

This function aligns two functions using Bayesian framework. It will align f2 to f1. It is based on mapping warping functions to a hypersphere, and a subsequent exponential mapping to a tangent space. In the tangent space, the Z-mixture pCN algorithm is used to explore both local and global structure in the posterior distribution.

Usage

```
pair_align_functions_expomap(
  f1,
  f2,
  timet,
  iter = 20000,
  burnin = min(5000, iter/2),
  alpha0 = 0.1,
  beta0 = 0.1,
  zpcn = list(betas = c(0.5, 0.05, 0.005, 1e-04), probs = c(0.1, 0.1, 0.7, 0.1)),
  propvar = 1,
  init.coef = rep(0, 2 * 10),
  npoints = 200,
  extrainfo = FALSE
)
```

Arguments

`f1`	observed data, numeric vector
`f2`	observed data, numeric vector
`timet`	sample points of functions
`iter`	length of the chain
`burnin`	number of burnin MCMC iterations
`alpha0, beta0`	IG parameters for the prior of sigma1
`zpcn`	list of mixture coefficients and prior probabilities for Z-mixture pCN algorithm of the form list(betas, probs), where betas and probs are numeric vectors of equal length
`propvar`	variance of proposal distribution
`init.coef`	initial coefficients of warping function in exponential map; length must be even
`npoints`	number of sample points to use during alignment
`extrainfo`	T/F whether additional information is returned

Details

The Z-mixture pCN algorithm uses a mixture distribution for the proposal distribution, controlled by input parameter zpcn. The zpcn$betas must be between 0 and 1, and are the coefficients of the mixture components, with larger coefficients corresponding to larger shifts in parameter space. The zpcn$probs give the probability of each shift size.

Value

Returns a list containing

`f2_warped`	f2 aligned to f1
`gamma`	Posterior mean gamma function
`g.coef`	matrix with iter columns, posterior draws of g.coef
`psi`	Posterior mean psi function
`sigma1`	numeric vector of length iter, posterior draws of sigma1
`accept`	Boolean acceptance for each sample (if extrainfo=TRUE)
`betas.ind`	Index of zpcn mixture component for each sample (if extrainfo=TRUE)
`logl`	numeric vector of length iter, posterior loglikelihood (if extrainfo=TRUE)
`gamma_mat`	Matrix of all posterior draws of gamma (if extrainfo=TRUE)
`gamma_q025`	Lower 0.025 quantile of gamma (if extrainfo=TRUE)
`gamma_q975`	Upper 0.975 quantile of gamma (if extrainfo=TRUE)
`sigma_eff_size`	Effective sample size of sigma (if extrainfo=TRUE)
`psi_eff_size`	Vector of effective sample sizes of psi (if extrainfo=TRUE)
`xdist`	Vector of posterior draws from xdist between registered functions (if extrainfo=TRUE)
`ydist`	Vector of posterior draws from ydist between registered functions (if extrainfo=TRUE)

References

Lu, Y., Herbei, R., and Kurtek, S. (2017). Bayesian registration of functions with a Gaussian process prior. Journal of Computational and Graphical Statistics, DOI: 10.1080/10618600.2017.1336444.

Examples

```
## Not run:
  # This is an MCMC algorithm and takes a long time to run
  myzpcn <- list(
    betas = c(0.1, 0.01, 0.005, 0.0001),
    probs = c(0.2, 0.2, 0.4, 0.2)
  )
  out <- pair_align_functions_expomap(
    f1 = simu_data$f[, 1],
    f2 = simu_data$f[, 2],
    timet = simu_data$time,
    zpcn = myzpcn,
    extrainfo = TRUE
  )
```

```
  # overall acceptance ratio
  mean(out$accept)
  # acceptance ratio by zpcn coefficient
  with(out, tapply(accept, myzpcn$betas[betas.ind], mean))

## End(Not run)
```

---

pair_align_image *Pairwise align two images This function aligns to images using the q-map framework*

Description

Pairwise align two images This function aligns to images using the q-map framework

Usage

```
pair_align_image(
  I1,
  I2,
  M = 5,
  ortho = TRUE,
  basis_type = "t",
  resizei = FALSE,
  N = 64,
  stepsize = 1e-05,
  itermax = 1000
)
```

Arguments

I1	reference image
I2	image to warp
M	number of basis elements (default=5)
ortho	orthonormalize basis (default=TRUE)
basis_type	("t","s","i","o"; default="t")
resizei	resize image (default=TRUE)
N	size of resized image (default=64)
stepsize	gradient stepsize (default=1e-5)
itermax	maximum number of iterations (default=1000)

Value

Returns a list containing

Inew	aligned I2
gam	warping function

References

Q. Xie, S. Kurtek, E. Klassen, G. E. Christensen and A. Srivastava. Metric-based pairwise and multiple image registration. IEEE European Conference on Computer Vision (ECCV), September, 2014

Examples

```
## Not run:
  # This is a gradient descent algorithm and takes a long time to run
  out <- pair_align_image(im$I1, im$I2)

## End(Not run)
```

---

pcaTB *Tolerance Bound Calculation using Elastic Functional PCA*

Description

This function computes tolerance bounds for functional data containing phase and amplitude variation using principal component analysis

Usage

```
pcaTB(f, time, m = 4, B = 1e+05, a = 0.05, p = 0.99)
```

Arguments

f	matrix of functions
time	vector describing time sampling
m	number of principal components (default = 4)
B	number of monte carlo iterations
a	confidence level of tolerance bound (default = 0.05)
p	coverage level of tolerance bound (default = 0.99)

Value

Returns a list containing

pca	pca output
tol	tolerance factor

References

J. D. Tucker, J. R. Lewis, C. King, and S. Kurtek, “A Geometric Approach for Computing Tolerance Bounds for Elastic Functional Data,” Journal of Applied Statistics, 10.1080/02664763.2019.1645818, 2019.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Jung, S. L. a. S. (2016). "Combined Analysis of Amplitude and Phase Variations in Functional Data." arXiv:1603.01775.

Examples

```
## Not run:
  out1 <- pcaTB(simu_data$f, simu_data$time)

## End(Not run)
```

`predict.lpcr` *Elastic Prediction for functional logistic PCR Model*

Description

This function performs prediction from an elastic logistic fPCR regression model with phase-variability

Usage

```
## S3 method for class 'lpcr'
predict(object, newdata = NULL, y = NULL, ...)
```

Arguments

`object`	Object of class inheriting from "elastic.pcr.regression"
`newdata`	An optional matrix in which to look for variables with which to predict. If omitted, the fitted values are used.
`y`	An optional vector of labels to calculate PC. If omitted, PC is NULL
`...`	additional arguments affecting the predictions produced

Value

Returns a list containing

`y_pred`	predicted probabilities of the class of newdata
`y_labels`	class labels of newdata
`PC`	probability of classification

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, “Elastic Functional Principal Component Regression,” Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

---

predict.mlpcr *Elastic Prediction for functional multinomial logistic PCR Model*

Description

This function performs prediction from an elastic multinomial logistic fPCR regression model with phase-variability

Usage

```
## S3 method for class 'mlpcr'
predict(object, newdata = NULL, y = NULL, ...)
```

Arguments

`object`	Object of class inheriting from "elastic.pcr.regression"
`newdata`	An optional matrix in which to look for variables with which to predict. If omitted, the fitted values are used.
`y`	An optional vector of labels to calculate PC. If omitted, PC is NULL
`...`	additional arguments affecting the predictions produced

Value

Returns a list containing

`y_pred`	predicted probabilities of the class of newdata
`y_labels`	class labels of newdata
`PC`	probability of classification per class
`PC.comb`	total probability of classification

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, “Elastic Functional Principal Component Regression,” Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

predict.pcr *Elastic Prediction for functional PCR Model*

Description

This function performs prediction from an elastic pcr regression model with phase-variability

Usage

```
## S3 method for class 'pcr'
predict(object, newdata = NULL, y = NULL, ...)
```

Arguments

`object`	Object of class inheriting from "elastic.pcr.regression"
`newdata`	An optional matrix in which to look for variables with which to predict. If omitted, the fitted values are used.
`y`	An optional vector of responses to calculate SSE. If omitted, SSE is NULL
`...`	additional arguments affecting the predictions produced

Value

Returns a list containing

`y_pred`	predicted values of newdata
`SSE`	sum of squared errors

References

J. D. Tucker, J. R. Lewis, and A. Srivastava, "Elastic Functional Principal Component Regression," Statistical Analysis and Data Mining, 10.1002/sam.11399, 2018.

q_to_curve *Convert to curve space*

Description

This function converts SRVFs to curves

Usage

```
q_to_curve(q, scale = 1)
```

Arguments

`q`	array describing SRVF (n,T)
`scale`	scale of original beta (default 1)

Value

beta array describing curve

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
q <- curve_to_q(beta[, , 1, 1])$q
beta1 <- q_to_curve(q)
```

`reparam_curve` *Align two curves*

Description

This function aligns two SRVF functions using Dynamic Programming

Usage

```
reparam_curve(
  beta1,
  beta2,
  lambda = 0,
  method = "DP",
  w = 0.01,
  rotated = T,
  isclosed = F,
  mode = "O"
)
```

Arguments

`beta1`	array defining curve 1
`beta2`	array defining curve 1
`lambda`	controls amount of warping (default = 0)
`method`	controls which optimization method (default="DP") options are Dynamic Programming ("DP")

w	controls LRBFGS (default = 0.01)
rotated	boolean if rotation is desired
isclosed	boolean if curve is closed
mode	Open ("O") or Closed ("C") curves

Value

return a List containing

gam	warping function
R	rotation matrix
tau	seed point

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
gam <- reparam_curve(beta[, , 1, 1], beta[, , 1, 5])$gam
```

---

reparam_image *Find optimum reparameterization between two images*

Description

Finds the optimal warping function between two images using the elastic framework

Usage

```
reparam_image(It, Im, gam, b, stepsize = 1e-05, itermax = 1000, lmark = FALSE)
```

Arguments

It	template image matrix
Im	test image matrix
gam	initial warping array
b	basis matrix
stepsize	gradient stepsize (default=1e-5)
itermax	maximum number of iterations (default=1000)
lmark	use landmarks (default=FALSE)

Value

Returns a list containing

gamnew	final warping
Inew	aligned image
H	energy
stepsize	final stepsize

References

Q. Xie, S. Kurtek, E. Klassen, G. E. Christensen and A. Srivastava. Metric-based pairwise and multiple image registration. IEEE European Conference on Computer Vision (ECCV), September, 2014

---

resamplecurve *Resample Curve*

Description

This function resamples a curve to a number of points

Usage

```
resamplecurve(x, N = 100, mode = "O")
```

Arguments

x	matrix defining curve (n,T)
N	Number of samples to re-sample curve, N usually is > T
mode	Open ("O") or Closed ("C") curves

Value

xn matrix defining resampled curve

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
xn <- resamplecurve(beta[, , 1, 1], 200)
```

rgam *Random Warping*

Description

Generates random warping functions

Usage

```
rgam(N, sigma, num)
```

Arguments

N	length of warping function
sigma	variance of warping functions
num	number of warping functions

Value

gam warping functions

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
gam = rgam(N=101, sigma=.01, num=35)
```

sample_shapes *Sample shapes from model*

Description

Sample shapes from model

Usage

```
sample_shapes(mu, K, mode = "O", no = 3, numSamp = 10)
```

Arguments

mu	array (n,T) of mean srvf
K	array (2T,2T) covariance matrix
mode	Open ("O") or Closed ("C") curves
no	number of principal components
numSamp	number of samples

Value

samples list of sample curves

References

Srivastava, A., Klassen, E., Joshi, S., Jermyn, I., (2011). Shape analysis of elastic curves in euclidean spaces. Pattern Analysis and Machine Intelligence, IEEE Transactions on 33 (7), 1415-1428.

Examples

```
out <- curve_karcher_mean(beta[, , 1, 1:2], maxit = 2)
# note: use more shapes, small for speed
K <- curve_karcher_cov(out$v)
samples <- sample_shapes(out$mu, K)
```

---

simu_data *Simulated two Gaussian Dataset*

Description

A functional dataset where the individual functions are given by: $y_i(t) = z_{i,1}e^{-(t-1.5)^2/2} + z_{i,2}e^{-(t+1.5)^2/2}$, $t \in [-3,3]$, $i = 1,2,\ldots,21$, where $z_{i,1}$ and $z_{i,2}$ are *i.i.d.* normal with mean one and standard deviation 0.25. Each of these functions is then warped according to: $\gamma_i(t) = 6(\frac{e^{a_i(t+3)/6}-1}{e^{a_i}-1}) - 3$ if $a_i \neq 0$, otherwise $\gamma_i = \gamma_{id}$ ($gamma_{id}(t) = t$) is the identity warping). The variables are as follows: f containing the 21 functions of 101 samples and time which describes the sampling.

Usage

```
simu_data
```

Format

`simu_data`:

A list with 2 components:

- `f`: A numeric matrix of shape 101×21 storing a sample of size $N = 21$ of curves evaluated on a grid of size $M = 101$.
- `time`: A numeric vector of size $M = 101$ storing the grid on which the curves `f` have been evaluated.

`simu_warp` *Aligned Simulated two Gaussian Dataset*

Description

A functional dataset where the individual functions are given by: $y_i(t) = z_{i,1}e^{-(t-1.5)^2/2} + z_{i,2}e^{-(t+1.5)^2/2}$, $t \in [-3, 3]$, $i = 1, 2, \ldots, 21$, where $z_{i,1}$ and $z_{i,2}$ are *i.i.d.* normal with mean one and standard deviation 0.25. Each of these functions is then warped according to: $\gamma_i(t) = 6(\frac{e^{a_i(t+3)/6}-1}{e^{a_i}-1}) - 3$ if $a_i \neq 0$, otherwise $\gamma_i = \gamma_{id}$ ($gamma_{id}(t) = t$) is the identity warping). The variables are as follows: f containing the 21 functions of 101 samples and time which describes the sampling which has been aligned.

Usage

```
simu_warp
```

Format

`simu_warp`:

A list which contains the output of the `time_warping()` function applied on the data set `simu_data`.

`simu_warp_median` *Aligned Simulated two Gaussian Dataset using Median*

Description

A functional dataset where the individual functions are given by: $y_i(t) = z_{i,1}e^{-(t-1.5)^2/2} + z_{i,2}e^{-(t+1.5)^2/2}$, $t \in [-3, 3]$, $i = 1, 2, \ldots, 21$, where $z_{i,1}$ and $z_{i,2}$ are *i.i.d.* normal with mean one and standard deviation 0.25. Each of these functions is then warped according to: $\gamma_i(t) = 6(\frac{e^{a_i(t+3)/6}-1}{e^{a_i}-1}) - 3$ if $a_i \neq 0$, otherwise $\gamma_i = \gamma_{id}$ ($gamma_{id}(t) = t$) is the identity warping). The variables are as follows: f containing the 21 functions of 101 samples and time which describes the sampling which has been aligned.

Usage

```
simu_warp_median
```

Format

`simu_warp_median`:

A list which contains the output of the `time_warping()` function finding the median applied on the data set `simu_data`.

`smooth.data` *Smooth Functions*

Description

This function smooths functions using standard box filter

Usage

```
smooth.data(f, sparam)
```

Arguments

`f`	matrix (N x M) of M functions with N samples
`sparam`	number of times to run box filter

Value

fo smoothed functions

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
fo <- smooth.data(simu_data$f, 25)
```

SqrtMean *SRVF transform of warping functions*

Description

This function calculates the srvf of warping functions with corresponding shooting vectors and finds the mean

Usage

```
SqrtMean(gam)
```

Arguments

gam	matrix (N x M) of M warping functions with N samples

Value

Returns a list containing

mu	Karcher mean psi function
gam_mu	Karcher mean warping function
psi	srvf of warping functions
vec	shooting vectors

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
out <- SqrtMean(simu_warp$warping_functions)
```

SqrtMeanInverse — *SRVF transform of warping functions*

Description

This function calculates the srvf of warping functions with corresponding shooting vectors and finds the inverse of mean

Usage

```
SqrtMeanInverse(gam)
```

Arguments

gam	matrix (N x M) of M warping functions with N samples

Value

gamI inverse of Karcher mean warping function

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
gamI <- SqrtMeanInverse(simu_warp$warping_functions)
```

SqrtMedian — *SRVF transform of warping functions*

Description

This function calculates the srvf of warping functions with corresponding shooting vectors and finds the median

Usage

```
SqrtMedian(gam)
```

Arguments

gam	matrix (N x M) of M warping functions with N samples

Value

Returns a list containing

`median`	Karcher median psi function
`gam_median`	Karcher mean warping function
`psi`	srvf of warping functions
`vec`	shooting vectors

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
out <- SqrtMedian(simu_warp_median$warping_functions)
```

`srvf_to_f` *Transformation from SRSF Space*

Description

This function transforms SRVFs back to the original functional space.

Usage

```
srvf_to_f(q, time, f0 = 0, multidimensional = FALSE)
```

Arguments

`q` Either a numeric vector of a numeric matrix or a numeric array specifying the SRSFs that need to be transformed.

- If a vector, it must be of shape M and it is interpreted as a single 1-dimensional curve observed on a grid of size M.
- If a matrix and `multidimensional == FALSE`, it must be of shape $M \times N$. In this case, it is interpreted as a sample of N curves observed on a grid of size M, unless $M = 1$ in which case it is interpreted as a single 1-dimensional curve observed on a grid of size M.
- If a matrix and `multidimensional == TRUE`,it must be of shape $L \times M$ and it is interpreted as a single L-dimensional curve observed on a grid of size M.
- If a 3D array, it must be of shape $L \times M \times N$ and it is interpreted as a sample of N L-dimensional curves observed on a grid of size M.

`time` A numeric vector of length M specifying the grid on which SRSFs are evaluated.

`f0` Either a numeric value or a numeric vector of or a numeric matrix specifying the initial value of the curves in the original functional space. It must be:

- a value if `q` represents a single 1-dimensional SRSF.
- a vector of length L if `q` represents a single L-dimensional SRSF.
- a vector of length N if `q` represents a sample of N 1-dimensional SRSFs.
- a matrix of shape $L \times M$ if `q` represents a sample of N L-dimensional SRSFs.

`multidimensional` A boolean specifying if the curves are multi-dimensional. This is useful when `q` is provided as a matrix to determine whether it is a single multi-dimensional curve or a collection of uni-dimensional curves. Defaults to `FALSE`.

Value

A numeric array of the same shape as the input `q` storing the transformation of the SRSFs `q` back to the original functional space.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using amplitude and phase separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
q <- f_to_srvf(simu_data$f, simu_data$time)
f <- srvf_to_f(q, simu_data$time, simu_data$f[1, ])
```

`time_warping` *Alignment of univariate functional data*

Description

This function aligns a collection of 1-dimensional curves that are observed on the same grid.

Usage

```
time_warping(
  f,
  time,
  lambda = 0,
  penalty_method = c("roughness", "geodesic", "norm"),
  centroid_type = c("mean", "median"),
  center_warpings = TRUE,
```

```
  smooth_data = FALSE,
  sparam = 25L,
  parallel = FALSE,
  optim_method = c("DP", "DPo", "DP2", "RBFGS"),
  max_iter = 20L
)
```

Arguments

`f`	A numeric matrix of shape $M \times N$ specifying a sample of N curves observed on a grid of size M.
`time`	A numeric vector of length M specifying the common grid on which all curves `f` have been observed.
`lambda`	A numeric value specifying the elasticity. Defaults to `0.0`.
`penalty_method`	A string specifying the penalty term used in the formulation of the cost function to minimize for alignment. Choices are `"roughness"` which uses the norm of the second derivative, `"geodesic"` which uses the geodesic distance to the identity and `"norm"` which uses the Euclidean distance to the identity. Defaults to `"roughness"`.
`centroid_type`	A string specifying the type of centroid to align to. Choices are `"mean"` or `"median"`. Defaults to `"mean"`.
`center_warpings`	A boolean specifying whether to center the estimated warping functions. Defaults to `TRUE`.
`smooth_data`	A boolean specifying whether to smooth curves using a box filter. Defaults to `FALSE`.
`sparam`	An integer value specifying the number of times to apply the box filter. Defaults to `25L`. This is used only when `smooth_data = TRUE`.
`parallel`	A boolean specifying whether to run calculations in parallel. Defaults to `FALSE`.
`optim_method`	A string specifying the algorithm used for optimization. Choices are `"DP"`, `"DPo"`, and `"RBFGS"`. Defaults to `"DP"`.
`max_iter`	An integer value specifying the maximum number of iterations. Defaults to `20L`.

Value

An object of class `fdawarp` which is a list with the following components:

- `time`: a numeric vector of length M storing the original grid;
- `f0`: a numeric matrix of shape $M \times N$ storing the original sample of N functions observed on a grid of size M;
- `q0`: a numeric matrix of the same shape as `f0` storing the original SRSFs;
- `fn`: a numeric matrix of the same shape as `f0` storing the aligned functions;
- `qn`: a numeric matrix of the same shape as `f0` storing the aligned SRSFs;
- `fmean`: a numeric vector of length M storing the mean or median curve;
- `mqn`: a numeric vector of length M storing the mean or median SRSF;

- `warping_functions`: a numeric matrix of the same shape as `f0` storing the estimated warping functions;
- `original_variance`: a numeric value storing the variance of the original sample;
- `amplitude_variance`: a numeric value storing the variance in amplitude of the aligned sample;
- `phase_variance`: a numeric value storing the variance in phase of the aligned sample;
- `qun`: a numeric vector of maximum length `max_iter + 2` storing the values of the cost function after each iteration;
- `lambda`: the input parameter `lambda` which specifies the elasticity;
- `centroid_type`: the input centroid type;
- `optim_method`: the input optimization method;
- `inverse_average_warping_function`: the inverse of the mean estimated warping function;
- `rsamps`: TO DO.

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using Fisher-Rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative models for functional data using phase and amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
## Not run:
  out <- time_warping(simu_data$f, simu_data$time)

## End(Not run)
```

`toy_data` *Distributed Gaussian Peak Dataset*

Description

A functional dataset where the individual functions are given by a Gaussian peak with locations along the x-axis. The variables are as follows: f containing the 29 functions of 101 samples and time which describes the sampling.

Usage

```
toy_data
```

Format

`toy_data`:

A list with two components:

- `f`: A numeric matrix of shape 101×29 storing a sample of size $N = 29$ of curves evaluated on a grid of size $M = 101$.
- `time`: A numeric vector of size $M = 101$ storing the grid on which the curves `f` have been evaluated.

`toy_warp` *Aligned Distributed Gaussian Peak Dataset*

Description

A functional dataset where the individual functions are given by a Gaussian peak with locations along the x-axis. The variables are as follows: f containing the 29 functions of 101 samples and time which describes the sampling which as been aligned.

Usage

```
toy_warp
```

Format

`toy_warp`:

A list which contains the output of the `time_warping()` function applied on the data set `toy_data`.

`vertFPCA` *Vertical Functional Principal Component Analysis*

Description

This function calculates vertical functional principal component analysis on aligned data

Usage

```
vertFPCA(
  warp_data,
  no,
  id = round(length(warp_data$time)/2),
  ci = c(-1, 0, 1),
  showplot = TRUE
)
```

Arguments

warp_data	fdawarp object from time_warping of aligned data
no	number of principal components to extract
id	point to use for f(0) (default = midpoint)
ci	geodesic standard deviations (default = c(-1,0,1))
showplot	show plots of principal directions (default = T)

Value

Returns a vfpca object containing

q_pca	srvf principal directions
f_pca	f principal directions
latent	latent values
coef	coefficients
U	eigenvectors
id	point used for f(0)

References

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
vfpca <- vertFPCA(simu_warp, no = 3)
```

---

v_to_gam *map shooting vector to warping function at identity*

Description

map shooting vector to warping function at identity

Usage

```
v_to_gam(v)
```

Arguments

v	Either a numeric vector of a numeric matrix or a numeric array specifying the shooting vectors

Value

A numeric array of the same shape as the input array v storing the shooting vectors of v obtained via finite differences.

warp_f_gamma *Warp Function*

Description

This function warps function f by γ

Usage

```
warp_f_gamma(f, time, gamma, spl.int = FALSE)
```

Arguments

f	vector function
time	time
gamma	vector warping function
spl.int	use spline interpolation (default F)

Value

fnew warped function

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
fnew <- warp_f_gamma(
  f = simu_data$f[, 1],
  time = simu_data$time,
  gamma = seq(0, 1, length.out = 101)
)
```

`warp_q_gamma` *Warp SRSF*

Description

This function warps srsf q by γ

Usage

```
warp_q_gamma(q, time, gamma, spl.int = FALSE)
```

Arguments

`q`	vector
`time`	time
`gamma`	vector warping function
`spl.int`	use spline interpolation (default F)

Value

qnew warped function

References

Srivastava, A., Wu, W., Kurtek, S., Klassen, E., Marron, J. S., May 2011. Registration of functional data using fisher-rao metric, arXiv:1103.3817v2.

Tucker, J. D., Wu, W., Srivastava, A., Generative Models for Function Data using Phase and Amplitude Separation, Computational Statistics and Data Analysis (2012), 10.1016/j.csda.2012.12.001.

Examples

```
q <- f_to_srvf(simu_data$f, simu_data$time)
qnew <- warp_q_gamma(q[, 1], simu_data$time, seq(0, 1, length.out = 101))
```

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