A Two-stage Angiographic Parameter Calibration Method for Coronary Artery 3D Reconstruction

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***Abstract*—Three-dimensional(3D) reconstruction technology of coronary artery allows doctors observe more intuitively thus compensating the limitations of X-ray coronary angiography. However, directly using the original parameters to reconstruct 3D vessels can lead to substantial errors due to factors such as surgical table movement, heart beating, and inaccuracies in clinical calibration. Therefore, calibration of imaging system parameters is crucial for accurate 3D coronary artery reconstruction. This paper proposes a novel method for reconstructing 3D coronary artery from two uncalibrated angiographic views. It thoroughly analyzes the projection principle of X-ray coronary angiography and performs coarse calibration using coronary tree topology constraints and epipolar constraints. Moreover, in addition to the traditional reprojection error, this method introduces vascular diameter error for fine calibration. Experimental results demonstrate that this method achieves more accurate reconstruction results in a shorter time, with a reprojection intersection over union (IoU) ratio of 0.872, thereby confirming the effectiveness of this approach.**

Keywords—calibration,3D reconstruction, coronary artery tree, medical image

I. INTRODUCTION

Coronary artery disease represents a significant threat to human health. X-ray coronary angiography, widely acknowledged as the 'gold standard' for diagnosing cardiovascular conditions, has found widespread application in clinical settings [1].However, X-ray coronary angiography is fundamentally limited in some aspects. For example, the projection view lacks depth information, and issues such as vascular overlap and artifacts compromise the physician's diagnosis [2]. 3D reconstruction of the coronary artery offers a comprehensive representation of vascular structure, facilitating quantification of vascular stenosis and other lesion parameters. This approach yields more precise diagnostic outcomes, holding significant clinical relevance.

3D reconstruction of the coronary artery is achieved by two angiography images of same vascular segment from different angles. At present, the single-plane angiography system is commonly used in hospitals. Data collection involves generating image sequences from various perspectives and time points during the rotation of the C-arm. However, due to the movement of the platform, heart beat, equipment records and other reasons, parameter calibration is often inaccurate. F. Cheriet discovered that errors in the geometric parameters of the angiography system and inaccuracies in two-dimensional (2D) image processing significantly impact the accuracy of 3D reconstruction of coronary artery. To enhance the accuracy of 3D reconstruction, calibration of the geometric parameters of the angiography system is essential. [3].

G.Kristiansen has proposed a complex and time-consuming calibration method, but it is difficult to be applied in clinical practice [4]. Based on the deformable model, A.B.Merle constantly corrects the coronary skeleton through iteration, but ignores the impact of mismatch [5]. S-Y.Chen proposed a calibration method based on prior knowledge, but it is necessary to obtain the angle and ray source distance information between two projection views [6]. E.Montin uses the optimal fitness function of genetic algorithm to optimize, but this method has a large error when the number of views is small [7]. J.X.Huang takes the back-projection error as the optimization objective function, and considers the direction vector of the back-projection point, but the angle calculation is large, the calculation is coupled, and it is not easy to converge [8]. J.Yang and Z.Y.Fu perform nonlinear optimization based on the LM algorithm, but they are calibrated after the reconstruction is completed, which introduces unnecessary errors. If the initial projection parameter error is large, it will take a lot of time and fall into the local optimum [9][10].

Given the challenges observed in the aforementioned 3D reconstruction methods, this paper introduces a novel approach: a two-stage angiographic parameters calibration-based method for reconstructing accurate 3D coronary tree from two uncalibrated angiographic views. This study has two main contributions : 1) First, the coarse calibration is performed by topological constraints and epipolar constraints, and the vascular reconstruction is completed by using intermediate parameters, and then the fine calibration of parameters is performed by reprojection error. 2) Taking into account the diameter error during the reprojection process, a novel optimization objective function is formulated, and the adaptive simulated annealing algorithm is employed to address this issue.

The remainder of this paper is structured as follows: The second section provides a detailed introduction to the proposed method outlined in this paper. Following that, the third section elucidates the experimental design and outcomes. Subsequently, the last section presents the conclusions drawn from the study.

II. METHODS

A. Mathematical Principles of 3D Reconstruction

1) Topological structure of 2D coronary tree: The feature points of the 2D coronary tree include bifurcation points, crossing points and endpoints. Each vessels can be regarded as a sequence of two feature points as endpoints and represented by a series of connected points. Among them, the vascular segments distinguished by the intersection point overlap because of the projection, and in fact the vascular segments do not intersect in the three-dimensional space.

In this paper, the binary tree is used to describe the topological structure of the two-dimensional coronary artery centerline, and the feature points on the centerline are used as the nodes of the binary tree. As shown in Fig. 1, to unequivocally ascertain the topological characteristics of the binary tree, nodes may be assigned weights to differentiate their left and right subtrees. The weights are calculated as follows :

$$w = d \times 100 + n \tag{1}$$

Where w denotes the weight of the node, d denotes the depth of a node in a binary tree, and n denotes the number of nodes contained in a subtree of a node.

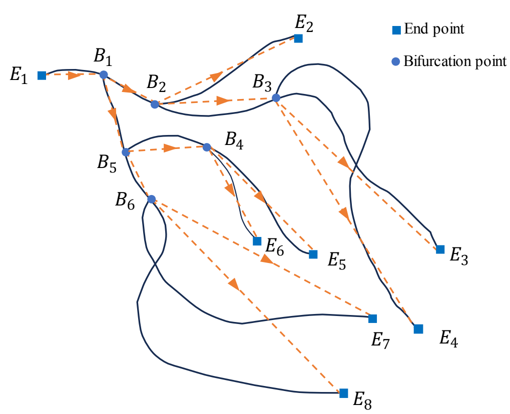

Fig. 1. Node selection of binary tree.

Due to the limited number of feature points contained in the centerline $(n < 100)$, both d and n can be calculated from the weight of the node. It is stipulated that the two child nodes of the same parent node have smaller weights as the left node. Moreover, if there is only one child node, it is also considered the left node. These rules ensures the unique binary tree description of the 2D coronary tree.

2) Reconstruction of 3D points: The X-ray coronary angiography system operates on the principle of perspective projection, where objects in the X-ray image are magnified and projected onto the image plane, with X-rays typically considered as the ideal light source. Fig. 2 illustrates the perspective projection model of the angiography system. Among them, $OXYZ$ is the world coordinate system, which is established through the patient 's lying platform. $S_iX_iY_iZ_i$ is the local coordinate system, which is established by X-ray source S_i in different directions. $U_iO_iV_i$ is the image coordinate system of the imaging plane corresponding to the projection direction. α_i, β_i represent the projection angle of the ray emitted from S, SID_i represent the distance from the ray source S_i to the image plane $U_iO_iV_i$, SOD_i represent the distance from the ray source S_i to the target, specifically the origin O of the world system ($i = 1,2$).

A point $\boldsymbol{P}$ in the 3D coronary artery is projected onto the image plane $U_iO_iV_i$ due to the ray emitted by the ray source S_i. The projection process can be described by the following coordinate transformation :

$$\mathbf{P}(x, y, z) \rightarrow \mathbf{P}_i(x_i, y_i, z_i) \rightarrow \mathbf{P}_i^K(u_i, v_i) \tag{2}$$

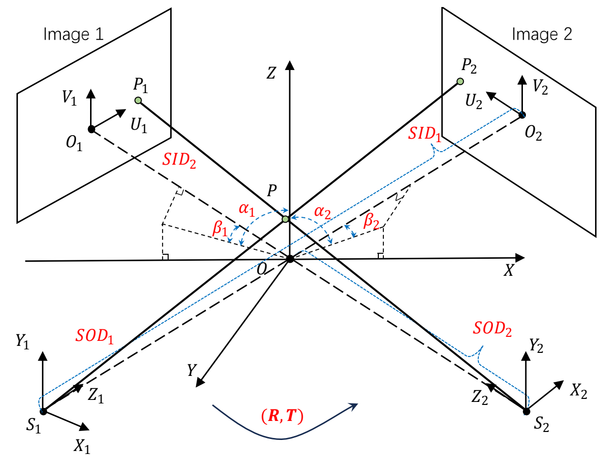

Fig. 2. The mathematical model of imaging system.

The local coordinate system SID_i can be seen as the world coordinate system $OXYZ$ clockwise α_i around the Y axis, then rotates β_i around the X axis, and finally translates SOD_i along the Z axis. The coordinate transformation relationship can be described as follows :

$$\mathbf{P} = \mathbf{R}_y^{-1}(\alpha_i) \cdot \mathbf{R}_x^{-1}(\beta_i) \cdot (\mathbf{P}_i - \mathbf{T}_i) \tag{3}$$

Where $\boldsymbol{R}_x(\beta_i)$, $\boldsymbol{R}_y(\alpha_i)$ are rotation matrices and $\boldsymbol{T}_i$ is translation matrix $(i = 1,2)$.

Through (3), the coordinate transformation relationship between the two local coordinate systems $S_1X_1Y_1Z_1$ and $S_2X_2Y_2Z_2$ can be further obtained, which can be described as follows :

$$\mathbf{P}_2 = \mathbf{R} \cdot (\mathbf{P}_1 - \mathbf{T}) \tag{4}$$

Where $\boldsymbol{R} = \boldsymbol{R}_x(\beta_2) \cdot \boldsymbol{R}_y(\alpha_2) \cdot \boldsymbol{R}_y^{-1}(\alpha_1) \cdot \boldsymbol{R}_x^{-1}(\beta_1)$ and $\boldsymbol{T} = -\boldsymbol{R}^{-1} \cdot \boldsymbol{T}_2 + \boldsymbol{T}_1$. $\boldsymbol{R}, \boldsymbol{T}$ represent the rotation and translation matrix, which are determined entirely by the angiography parameters α_i， β_i， $SOD_i (i = 1,2)$.

The coordinate transformation from the $S_iX_iY_iZ_i$ to the $U_iO_iV_i$ is obtained by the single point perspective projection relationship, which can be calculated as follows :

$$[x_i, y_i, z_i]^T = z_i \cdot [\xi_i, \eta_i, 1]^T \tag{5}$$

Where $\xi_i = u_i/SID_i = x_i/z_i, \eta_i = v_i/SID_i = y_i/z_i$ $(i = 1,2)$.

When two angiography views are available, the conversion relationship between the two local coordinate systems can be obtained from $\boldsymbol{R}, \boldsymbol{T}$, thereby establishing the correspondence between the coordinates of the same point $\boldsymbol{P}$ in image coordinate system of different angiography images. This relationship can be described as follows :

$$U_1O_1V_1 \xleftrightarrow{SID_1} S_1X_1Y_1Z_1 \xleftrightarrow{\mathbf{RT}} S_2X_2Y_2Z_2 \xleftrightarrow{SID_2} U_2O_2V_2 \tag{6}$$

Combining (4) with (5) and eliminating z_2, we can obtain that $\boldsymbol{P}_1(x_1 y_1 z_1)$ as follows :

$$\begin{bmatrix} 1 & 0 & -\xi_1 \\ 0 & 1 & -\eta_1 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vec{a}\cdot\vec{t} \\ \vec{b}\cdot\vec{t} \end{bmatrix} \tag{7}$$

Where $\vec{a} = [r_{11} - r_{31}\xi_2, r_{12} - r_{32}\xi_2, r_{13} - r_{33}\xi_2]$, $\vec{b} = [r_{21} - r_{31}\eta_2, r_{22} - r_{32}\eta_2, r_{23} - r_{33}\eta_2]$, $r_{ij}(i,j = 1,2,3)$ denotes the componet of $\boldsymbol{R}$.

Equation (7) is abbreviated as $\boldsymbol{A}\cdot\boldsymbol{P} = \boldsymbol{b}$. For this overdetermined system of equations, the optimal coordinate position of the spatial point can be obtained using the least squares solution. According to the normal equation, it can be calculated as follows :

$$\mathbf{P}_1 = (\mathbf{A}^T\cdot\mathbf{A})^{-1}\cdot\mathbf{A}^T\cdot\mathbf{b} \tag{8}$$

Finally, the 3D points in the world coordinate system are transformed from (3), thus achieving the reconstruction of 3D points.

3) Conjugate point pairs matching: In order to reconstruct the 3D model of the coronary artery, it is necessary to match the corresponding feature points from two angiographic views. The epipolar constraint is a common method for conjugate point pairs matching automatically. As illustrated in Fig. 3, the epipolar plane is fomed by spatial point $\boldsymbol{P}$ and the X-ray sources S_1, S_2. And two epipolar lines L_1 and L_2 are the intersection line of the epipolar plane and two image planes respectively. The epipolar constraint dictates that the projections $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ of the spatial point $\boldsymbol{P}$ on the two image planes must lie on the epipolar lines L_1 and L_2 [5].

By substituting (5) into (4) and eliminating z_1, the equation of L_2 can be obtained, which has the following form :

$$\xi_2(m_3n_2 - m_2n_3) + \eta_2(m_1n_3 - m_3n_1) + (m_2n_1 - m_1n_2) = 0 \tag{9}$$

Where $m_i = r_{i1}\xi_1 + r_{i2}\eta_1 + r_{i3}$, $n_i = r_{i1}t_1 + r_{i2}t_1 + r_{i3}t_3$, t_i is the component of the translation matrix $\mathrm{T}(i = 1,2,3)$.

According to (9), L_2 on the image plane 2 is determined, and the conjugate point $\boldsymbol{P}_2$ of $\boldsymbol{P}_1$ can be obtained by solving the intersection point of L_2 and vascular segment, so as to realize the automatic matching of conjugate point pairs.

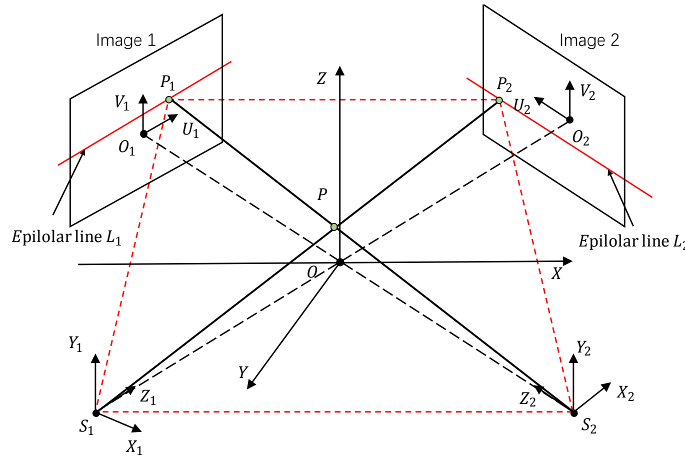

Fig. 3. Schematic diagram of *the epipolar constraint*.

B. Two-stage parameter calibration

1) Coarse calibration

Due to the error of the clinically calibrated imaging geometric parameters, the matching based on the epipolar constraint may not be accurate, which further enlarges the error of the optimal approximation solution of the spatial point, making the subsequent optimization take more time and even fall into the local optimum. Therefore, this paper proposes a two-stage imaging parameter calibration method. Before the point pair matching, the original imaging parameters are coarsely calibrated to quickly obtain the intermediate parameters, and more accurate matching point pairs are obtained according to the intermediate parameters.

Since the feature points of the coronary tree are the necessary invariants in the projection images from different perspectives [11], and the feature points can be matched by the topological constraints described above, the feature points can be used as coarse calibration samples. As illustrated in Fig. 6(b), the corresponding point in the other image plane is not located on the epipolar line, and this matching error can be calculated as the distance of point to epipolar line :

$$d_2 = \frac{\left|\xi_2(m_3n_2 - m_2n_3) + \eta_2(m_1n_3 - m_3n_1) + (m_2n_1 - m_1n_2)\right|}{\sqrt{(m_3n_2 - m_2n_3)^2 + (m_1n_3 - m_3n_1)^2}} \tag{10}$$

Let $\boldsymbol{VAR} = [\alpha_i, \beta_i, SOD_i, SID_i](i = 1,2)$. Therefore, the objective function in the coarse calibration phase is defined as follows :

$$\arg\min F_1(\mathbf{VAR}) = \sum_{j=1}^{N} {d_{2j}}^2 \tag{11}$$

The initial imaging parameters are coarsely calibrated by minimizing the objective function F_1, so that the distance of the sample point to corresponding epipolar line is minimized, and the matching accuracy of the conjugate point pair is improved.

2) 3D diameter of vessels: After coarse calibration, the intermediate projection parameters are used to match the remaining points on the vessels segment, and the 3D spatial coordinates of the sampling points in $OXYZ$ are calculated from (8). Next, the 3D skeleton can be fitted by B-spline curve. Finally, the 3D of the vessels needs to be calculated directly to complete the surface reconstruction of the vessels. According to the generalized cylinder [12], the cross-section of coronary artery vessels can be approximately regarded as circular. Many scholars have used the projection information provided by a single angiography view to calculate the three-dimensional diameter [13][14]. However, because the projection image plane is not always perpendicular to the normal line of the vascular cross section, information from a single view is incomplete. More generally, the vascular projection is illustrated in Fig. 4.

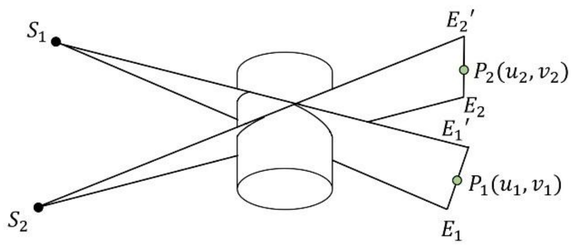

Fig. 4. General situation of vascular projection at two angles.

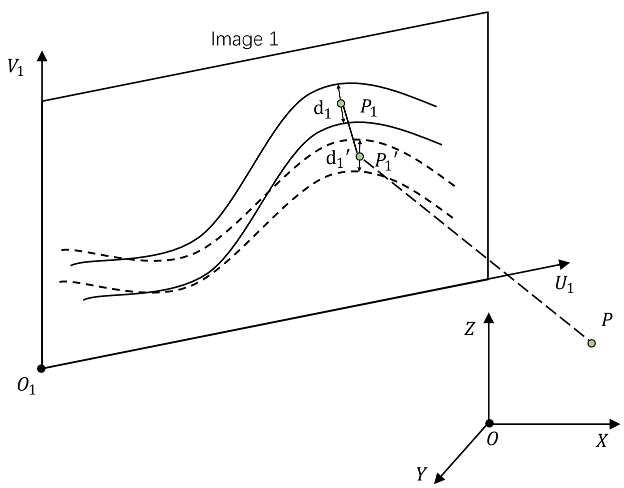

Fig. 5. Reprojection distance error and diameter error.

The 2D diameter of the vessels can be obtained by measuring the length of the line segment that the vertical line passing through the point is cut out by two sides of the contour. E and E' denote the intersection point with the two sides of the contour, then EE' is the two-dimensional diameter at this point. For the vascular skeleton point P_j, the 3D diameter in the two projection directions can be solved based on projection relationships. The diameter in this direction is calculated as follows :

$$d_j^i = E_j^i E_j^i{}' \frac{\left\| [x_j^i, y_j^i, z_j{}^i] \right\|_2}{\left\| [u_j^i, v_j^i, SID_i] \right\|_2} \tag{12}$$

Since the distance of the ray source and the target is much larger than the diameter of the vessels, the 3D diameter at the P_j point can be taken as the average diameter of the two projection directions :

$$d_j = (d_j^1 + d_j^2)/2 \tag{13}$$

Where $i = 1,2, j = 1,2, \cdots, N$.

3) Fine calibration: Reprojection is a common method to evaluate the accuracy of 3D reconstruction [15]. As depicted in Fig. 5, the reconstruction point $\boldsymbol{P}$ of the conjugate point pair $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ is projected back to the two image planes to obtain the projection points $\boldsymbol{P}_1{}'$ and $\boldsymbol{P}_2{}'$, respectively. $\boldsymbol{P}_i$ and $\boldsymbol{P}_i{}'$ will not completely coincide because of the error, and the error can be evaluated by the sum of the lengths of the line segments $\boldsymbol{P}_i\boldsymbol{P}_i{}'(i = 1,2)$, as follows :

$$\varepsilon_1(\mathbf{P}) = \left\| \overrightarrow{\mathbf{P}_1\mathbf{P}_1{}'} \right\|_2 + \left\| \overrightarrow{\mathbf{P}_2\mathbf{P}_2{}'} \right\|_2 \tag{14}$$

In many methods, ε_1 is optimized as a reprojection error, and only the position information of the sampling points on the coronary artery skeleton is used, while the influence of geometric projection parameter error on the three-dimensional diameter of the vessels in the surface reconstruction process is ignored. In practical clinical applications, in order to assist doctors to observe, the reconstruction of three-dimensional coronary tree must also consider the diameter information at the spatial point. Therefore, the vascular diameter after reprojection is also used as an error evaluation index in this paper, which is defined as follows :

$$\varepsilon_2(\mathbf{P}) = \left| d_1 - d_1{}' \right| + \left| d_2 - d_2{}' \right| \tag{15}$$

Where d_i is the diameter measured at $\boldsymbol{P}_i$ in the original projection view, and $d_i{}'$ is the diameter at $\boldsymbol{P}_i{}'$ after reprojection $(i = 1,2)$, which can be calculated according to (12).

Therefore, the reprojection error at point $\boldsymbol{P}$ is defined as $\varepsilon = \varepsilon_1 + \varepsilon_2$. The objective function in the fine calibration phase is defined as follows :

$$\arg\min F_2(\mathbf{VAR}) = \sum_{j=1}^{N} \left(\left\| \overrightarrow{\mathbf{P}_j^1\mathbf{P}_j^1{}'} \right\|_2 + \left\| \overrightarrow{\mathbf{P}_j^2\mathbf{P}_j^2{}'} \right\|_2 + \left| d_j^1 - d_j^1{}' \right| + \left| d_j^2 - d_j^2{}' \right| \right) \tag{16}$$

Equation (11) and (16) are nonlinear least squares problems. The commonly used methods are Gradient descent method, Levenberg-Marquardt (LM) method and adaptive simulated annealing (ASA) method. ASA has the following advantages : 1) It can offer an effective approximate solution algorithm for problems with NP complexity ; 2) It overcomes the issue of the optimization process being prone to getting trapped in local minima. ; 3) It overcomes the dependency on initial values [16]. Therefore, this paper uses the adaptive simulated annealing algorithm to perform two-stage calibration of the angiographic geometric parameters.

III. Experiment

The experimental data in this paper are LAO/CRAN and RAO/CRAN perspective imaging images from the same patient, and the initial parameters are read from the device. Before the

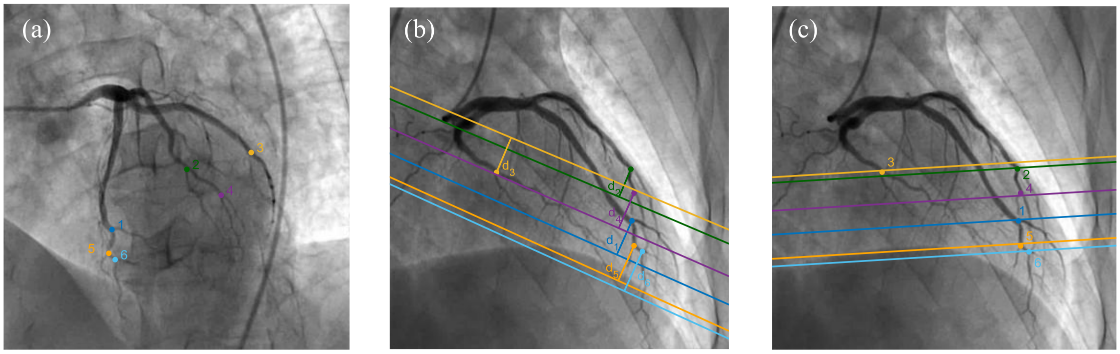

Fig. 6.(a) Selected feature points. (b) Uncalibrated epipolar line. (c) Coarse calibrated epipolar line

3D reconstruction experiment, the original angiography image needs to be preprocessed, including vessels segmentation, two-dimensional centerline extraction, and feature point recognition [17]. In the experiment, the feature points in the two contrast views were first selected for rough calibration. As in [3], 6 pairs of feature points were selected as samples, and the corresponding points were represented by the same number as the matching relationship, as shown in Fig. 6(a). Then the 3D coronary tree is reconstructed according to the intermediate parameters obtained by the coarse calibration. Finally, the fine calibration is performed to minimize the reprojection distance error and diameter error. The experiment was completed on a PC with a 3.3GHz AMD Ryzen 9 CPU and 16G Memory and programmed in Matlab R2023b.

A. *Qualitative analysis*

Fig. 6(b) draws the epipolar line calculated by the feature points in the LAO view. It can be seen that there is a significant error in the matching of point pairs using uncalibrated initial parameters, and the epipolar line can correctly match the conjugate points after coarse calibration, as shown in Fig. 6(c). As shown in Fig. 7(a1)(a2), the reconstructed vesselss using intermediate parameters exhibit poor anastomosis with the original angiography vesselss. The structure after fine calibration is shown in Fig. 7 (b1)(b2). It is evident that the optimization effect is very pronounced.

B. *Quantitative analysis*

To quantitatively analyze the accuracy of 3D reconstruction, taking the IoU as the evaluation index, and the calculation formula is :

$$IoU = \frac{TP}{TP + FN + FP} \tag{17}$$

In the formula, TP is the number of pixels where the reprojection vessels cover the angiographic vessels, FP is the number of pixels where the vessels re-projection falls outside the angiographic vessels, and FN is the number of pixels in the angiographic vessels that are not covered by the reprojection vessels. The value range of IoU is [0, 1], where a larger value indicates better anastomosis between the reconstructed vessels and the original angiography vessels.

To further analyze the optimization effect of the proposed method, the error statistcs before and after calibration are shown in Table I. And it is compared with the related methods, as shown in Table II. The proposed method demonstrates significant improvements in accuracy and efficiency, as evidenced by the experimental results, thereby verifying the reliability of the approach.

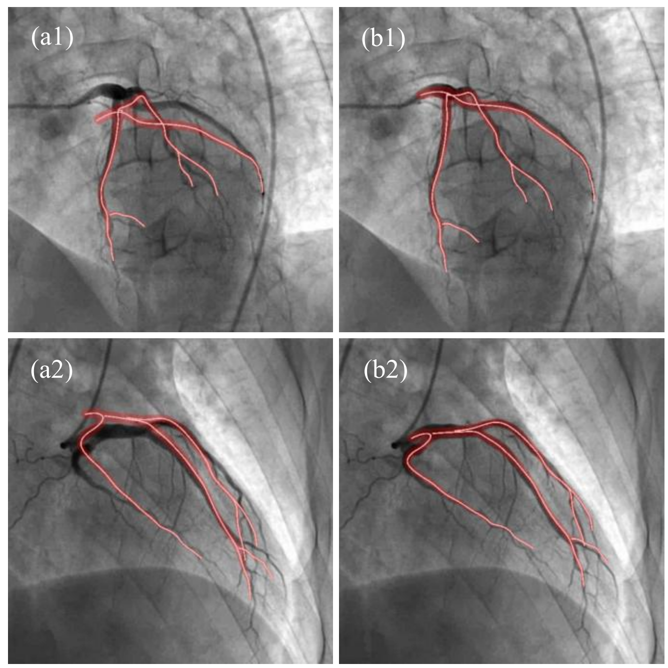

Fig. 7. Reprojection results before and after fine calibration.

TABLE I. ERROR STATISTCS

Methods	*Maximum (mm)*	*Minimum (mm)*	*Mean (mm)*	*RMS (mm)*
Uncalibrated	19.2365	0.2362	8.4766	8.3547
Proposed	0.8026	0.0003	0.1376	0.2452

TABLE II. IOU AND TIME CONSUMING OF DIFFERENT METHODS

Methods	*IoU*	*Time cost(s)*
Uncalibrated	0.534	
J.Yang et al.[9]	0.718	18813.9
Z.Y.Fu et al.[10]	0.854	7771.5
Proposed	0.872	6870.8

a. IoU is the average of two images.

IV. CONCLUSIONS

In this paper, a novel method for 3D reconstruction of coronary artery based on two-stage angiographic geometric parameter calibration is proposed. Specifically, the method coarsely calibrates the geometric parameters based on the topological constraints and epipolar constraints of the coronary tree, and uses the intermediate parameters to complete the conjugate point pair matching and three-dimensional coordinate solution. Then, a vascular diameter projection model is established, and a new optimization function is devised considering the reprojection distance error and diameter error. Finally, ASA is used to complete the fine calibration of geometric parameters. Experimental results demonstrate the proposed method exhibits good stability and computational efficiency, significantly enhancing the accuracy of reconstructing the 3D coronary artery from two uncalibrated angiographic views.

REFERENCES

[1] SY Chen, KR Hoffmann and JD Carroll et al, Computer Assisted Coronary Intervention: 3D Reconstrution and Determination of Optimal Views. IEEE Computers in Cardiology,vol.23,pp.117-120,1996.

[2] Çimen, Serkan, et al. "Reconstruction of coronary arteries from X-ray angiography: A review." Medical image analysis 32 (2016): 46-68.

[3] Cheriet, F., and J. Meunier. "Self-calibration of a biplane X-ray imaging system for an optimal three dimensional reconstruction."Computerized medical imaging and graphics 23.3 (1999): 133-141.

[4] Kristiansen G, Wunderlich W, Fischer F et al, Accuracy and precision of the analytic calibration method in quantitative coronary arteriography, IEEE computers in Cardiogy, vol.23,pp.401-404,1996.

[5] A. B. Merle, G. Finet, J. Lienard and I. E. Magnin, "3D reconstruction of the deformable coronary tree skeleton from two X-ray angiographic views," Computers in Cardiology 1998. Vol. 25 (Cat. No.98CH36292), Cleveland, OH, USA, 1998, pp. 757-760.

[6] S-Y Chen, C. E. Metz, K. R. Hoffmann and J. D. Carroll, "Improved determination of biplane imaging geometry and 3D coronary arterial tree from two views," Computers in Cardiology 1994, Bethesda, MD, USA, 1994, pp. 653-656.

[7] E.Montin et al., "A method for coronary bifurcation centerline reconstruction from angiographic images based on focalization optimization," 2016 38th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), Orlando, FL, USA, 2016, pp. 4165-4168.

[8] J.X.Huang, D.Y.Yu , X.D.Chen, et al. Optimization of transformation for 3D re construction of coronary arterial tree.Proceedings of SPIE, the International Society for Optical Engineering.2004,vol.5444:P547-552.

[9] Yang, Jian, et al. "Novel approach for 3-D reconstruction of coronary arteries from two uncalibrated angiographic images." Ieee transactions on image processing 18.7 (2009): 1563-1572.

[10] Z.Y.Fu, Z.Fu, et al., "Optimization For 3D Reconstruction Of Coronary Artery Tree By Two-stage Levenberg-Marquardt Algorithm," 2021 27th International Conference on Mechatronics and Machine Vision in Practice (M2VIP), Shanghai, China, 2021, pp. 84-89.

[11] A. Iskurt, Y. Becerikli and K. Mahmutyazicioglu, "A fast and automatic calibration of the projectory images for 3D reconstruction of the branchy structures," 2013 47th Annual Conference on Information Sciences and Systems (CISS), Baltimore, MD, USA, 2013, pp. 1-6.

[12] A. Wahle, W. Ernst, M. Ignace et al .Assessment of difuse coronary artery disease by quantitative analysis of coronary morphology based upon 3-D reconstruction from biplane angiograms,IEEE Transactions on Medical Imaging,1995,14 (2): 203-241.

[13] C.V. Bourantas, I. C. Kourtis, M. E. Plissiti, D. I. Fotiadis, C. S. Katsouras, M. I. Papafaklis, and L. K. Michalis, "A method for 3D reconstruction of coronary arteries using biplane angiography and intravascular ultrasound images," Comput. Med. Imag. Graph., vol. 29, pp. 597–606, Dec. 2005.

[14] C. Blondel, R. Vaillant, F. Devernay, G. Malandain, and N. Ayache, "Automatic trinocular 3D reconstruction of coronary artery centerlines from rotational X-ray angiography," in Proc. Computer Assisted Radiology and Surgery, 2002, pp. 1073–1078.

[15] Klein, J. Larry, et al. "A quantitative evaluation of the three dimensional reconstruction of patients' coronary arteries." The International Journal of Cardiac Imaging 14.2 (1998): 75-87.

[16] Ingber L. Adaptive simulated annealing (ASA):lessons learned [J]. Journal of Control and Cybernetics, 1996, 25(1): 33-54.

[17] Sui, Chenxin, et al. "A Novel Method for Vessel Segmentation and Automatic Diagnosis of Vascular Stenosis." 2019 IEEE International Conference on Robotics and Biomimetics (ROBIO). IEEE, 2019.