

# Errata on Theorem 2 in “False Data Inject Attacks in Control Systems”

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September 17, 2017

The proof can be divided into the following steps:

1.  $A$  matrix must be unstable. Suppose otherwise. Then we know there exists an  $l$ , such that  $\|A^l\| = \alpha < 1$ . As a result

$$\|\Delta e_{k+l}\| = \|A^l \Delta e_k - A^{l-1} K \Delta z_{k+1} - \cdots - K \Delta z_{k+l}\| \leq \alpha \|\Delta e_k\| + \|A^{l-1} K\| + \cdots + \|K\|.$$

As a result,  $\|\Delta e_k\|$  will be bounded, which contradicts with the fact that  $\|\Delta e_k\| \rightarrow \infty$ .

2. Given an unstable  $A$ , there exists an  $l > n - 1$ , such that if  $\|A^l p\| \geq \|p\|$ , then  $\lim_{k \rightarrow \infty} A^k p \neq 0$ .

Let us decompose  $A$  as

$$V \begin{bmatrix} A_u & 0 \\ 0 & A_s \end{bmatrix} V^{-1},$$

where  $A_s$  is strictly stable and  $A_u$  contains all the unstable and critically stable eigenvalues. Define

$$\tilde{A} \triangleq V \begin{bmatrix} 0 & 0 \\ 0 & A_s \end{bmatrix} V^{-1}.$$

Since  $\tilde{A}$  is stable, there exists an  $l > n - 1$ , such that  $\|\tilde{A}^l\| < 1$ .

Now suppose that

$$V^{-1} p = \begin{bmatrix} p_u \\ p_s \end{bmatrix}.$$

If  $p_u = 0$ , then

$$A^l p = V \begin{bmatrix} A_u^l p_u \\ A_s^l p_s \end{bmatrix} = V \begin{bmatrix} 0 \\ A_s^l p_s \end{bmatrix} = \tilde{A}^l p.$$

As a result,

$$\|A^l p\| = \|\tilde{A}^l p\| \leq \|\tilde{A}^l\| \times \|p\| < \|p\|,$$

Thus, if  $\|A^l p\| \geq \|p\|$ , we can conclude that  $p_u$  cannot be zero, which further implies that

$$\lim_{k \rightarrow \infty} A^k p = V \begin{bmatrix} A_u^l p_u \\ 0 \end{bmatrix} \neq 0.$$

3. We can now conclude, using Lemma 2, that if  $\|A^l p\| \geq \|p\|$ , then there exists an unstable eigenvector  $v$ , such that

$$v \in \text{span} \{p, Ap, \dots, A^l p\}$$

4. The proof of Theorem 2 will remain almost the same. The difference is that we need to ensure  $\{p_{j_k}\}, \dots, \{p_{j_k-l}\}$  all converge, instead of  $\{p_{j_k}\}, \dots, \{p_{j_k-n+1}\}$  all converge (in the paragraph after Equation (31)).

The next difference is the first paragraph of page 7. Instead of  $\|Aq_1\| \geq \|q_1\|$ , we have  $\|A^l q_l\| \geq \|q_l\|$ .