Errata on Theorem 2 "False Data Inject Attacks in Control Systems"

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The proof can be divided into the following steps:

1. A matrix must be unstable. Suppose otherwise. Then we know there exists an l, such that $\|A^l\| = \alpha < 1$. As a result

$$\|\Delta e_{k+l}\| = \|A^l \Delta e_k - A^{l-1} K \Delta z_{k+1} - \cdots - K \Delta z_{k+l}\| \leq \alpha \|\Delta e_k\| + \|A^{l-1}K\| + \cdots + \|K\|.$$

   As a result, $\|\Delta e_k\|$ will be bounded, which contradicts with the fact that $\|\Delta e_k\| \to \infty$.

2. Given an unstable A, there exists an $l > n-1$, such that if $\|A^l p\| \geq \|p\|$, then $\lim_{k\to\infty} A^k p \neq 0$.

   Let us decompose A as

$$V \begin{bmatrix} A_u & 0 \\ 0 & A_s \end{bmatrix} V^{-1},$$

   where A_s is strictly stable and A_u contains all the unstable and critically stable eigenvalues. Define

$$\tilde{A} \triangleq V \begin{bmatrix} 0 & 0 \\ 0 & A_s \end{bmatrix} V^{-1}.$$

   Since $\tilde{A}$ is stable, there exists an $l > n-1$, such that $\|\tilde{A}^l\| < 1$.

   Now suppose that

$$V^{-1} p = \begin{bmatrix} p_u \\ p_s \end{bmatrix}.$$

   If $p_u = 0$, then

$$A^l p = V \begin{bmatrix} A_u^l p_u \\ A_s^l p_s \end{bmatrix} = V \begin{bmatrix} 0 \\ A_s^l p_s \end{bmatrix} = \tilde{A}^l p.$$

   As a result,

$$\|A^l p\| = \|\tilde{A}^l p\| \leq \|\tilde{A}^l\| \times \|p\| < \|p\|,$$

Thus, if $\|A^l p\| \geq \|p\|$, we can conclude that p_u cannot be zero, which further implies that

$$\lim_{k\to\infty} A^k p = V \begin{bmatrix} A_u^l p_u \\ 0 \end{bmatrix} \neq 0.$$

3. We can now conclude, using Lemma 2, that if $\|A^l p\| \geq \|p\|$, then there exists an unstable eigenvector v, such that

$$v \in \operatorname{span}\left\{p, Ap, \cdots, A^l p\right\}$$

4. The proof of Theorem 2 will remain almost the same. The difference is that we need to ensure $\{p_{j_k}\}$, ..., $\{p_{j_k-l}\}$ all converge, instead of $\{p_{j_k}\}$, ..., $\{p_{j_k-n+1}\}$ all converge (in the paragraph after Equation (31)).

   The next difference is the first paragraph of page 7. Instead of $\|Aq_1\| \geq \|q_1\|$, we have $\|A^l q_l\| \geq \|q_l\|$.