

# Probabilistic Approach to Diffusion-mediated Surface Phenomena

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## Academic Training

- Undergraduate, Chemistry. 2015.9 - 2019.6  
*Xiamen University, Xiamen, China* **92.34/100.**
- Diplôme de l'ENS. 2019.9 - 2023.7  
*École Normale Supérieure, Paris, France*
- Research Assitant. 2020.9 - 2021.8  
*Hunan University, Changsha, China*
- Master 1, Chemistry. 2021.9 - 2022.7  
*École Normale Supérieure, Paris, France* **15.60/20.**
- Master 2, Physics, ICFP. 2022.9 - 2023.7  
*École Normale Supérieure, Paris, France* (1st semester) **13.70/20,**  
(2nd semester, without internship) **15.00/20.**

## Course - M1

- Theoretical Chemistry: Statistical Mechanics applied to Chemistry (Damien Laage & Guillaum Stirnemann, 4 ECTS) **16.00/20.**
- Bio-Inorganic Chemistry (Clotilde Policar, 4 ECTS) **15.90/20.**
- Inorganic Materials (Gilles Wallez, 4 ECTS) **15.25/20.**
- Biophysical Chemistry of the Living Matter (Ludovic Jullien, 4 ECTS) **14.00/20.**
- Nuclear Magnetic Resonance (Kong Ooi Tan, 4 ECTS) **14.00/20.**
- Heterogenous Catalysis (Clément Guibert, 2 ECTS) **13.80/20.**  
et al.
- Introduction of QFT (Adel Bilal & Guilhem Semerjian, 6 ECTS) **13.50/20.**
- Introduction of General Relativity (Marios Petropoulos, 6 ECTS) **13.40/20.**
- Research Internship M1S2 (Thomas Salez, 18 ECTS) **17.40/20.**  
*Brownian Motion near the Soft Surface* Feb.~Jul. 2022
- Research Internship L3S2 (Jéréemie Caillat, 6 ECTS) **15.00/20.**  
*Simulation of Vibrational ICD on Model Systems with Reduced Dimensions* Jun.~Jul. 2020

## Courses - M2S1

- Physics of Fluids and Nonlinear Physics  
(Arnaud Antkowiak & Camille Duprat, 6 ECTS) **14.86/20.**
- Statistical Field Theory and applications  
(Adam Nahum & Xiangyu Cao, 6 ECTS) **14.44/20.**
- Computational and Data-Driven Physics  
(Alberto Rosso & Rémi Monasson, 6 ECTS) **13.70/20.**
- Soft Matter Physics  
(Anke Lindner & Vincent Démery, 6 ECTS) **13.00/20.**
- Advanced Statistical Physics for soft matter  
(Christophe Texier & Jean-Noël Aqua, 6 ECTS) **12.50/20.**

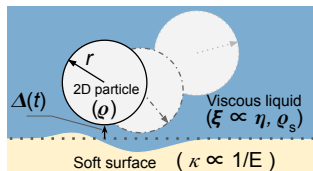
## Courses - M2S2

- Machine Learning (Marc Lelarge, 3 ECTS) **16.00/20.**
- Advanced Topics in Markov-chain Monte Carlo  
(Werner Krauth, 3 ECTS) **15.00/20.**
- Statistical Physics 2: Disordered Systems and Interdisciplinary Applications  
(Francesco Zamponi & Gregory Schehr, 3 ECTS) **15.00/20.**
- Conformal Field Theory  
(Benoît Estienne & Yacine IKHLEF, 3 ECTS) **14.00/20.**
- Research Internship M2S2 (Denis Grebenkov, 18 ECTS)  
*Probabilistic Approach to Diffusion-mediated Surface Phenomena* Apr.~Jul. 2023

# Internship - M1

## ElastoHydroDynamics interactions & Modified fluctuation-dissipation relation

Equations of motion (EOM) are non-linear coupled.



$$\begin{aligned} \ddot{X}_G + \frac{2\varepsilon\xi}{3} \frac{\dot{X}_G}{\sqrt{\Delta}} + \frac{\kappa\varepsilon\xi}{6} \left[ \frac{19}{4} \frac{\dot{\Delta}\dot{X}_G}{\Delta^{7/2}} - \frac{\dot{\Delta}\dot{\Theta}}{\Delta^{7/2}} + \frac{1}{2} \frac{\ddot{\Theta} - \ddot{X}_G}{\Delta^{5/2}} \right] &= 0 \\ \ddot{\Delta} + \xi \frac{\dot{\Delta}}{\Delta^{3/2}} + \frac{\kappa\xi}{4} \left[ 21 \frac{\dot{\Delta}^2}{\Delta^{9/2}} - \frac{(\dot{\Theta} - \dot{X}_G)^2}{\Delta^{7/2}} - \frac{15}{2} \frac{\ddot{\Delta}}{\Delta^{7/2}} \right] &= 0 \\ \ddot{\Theta} + \frac{4\varepsilon\xi}{3} \frac{\dot{\Theta}}{\sqrt{\Delta}} + \frac{\kappa\varepsilon\xi}{3} \left[ \frac{19}{4} \frac{\dot{\Delta}\dot{\Theta}}{\Delta^{7/2}} - \frac{\dot{\Delta}\dot{X}_G}{\Delta^{7/2}} + \frac{1}{2} \frac{\ddot{X}_G - \ddot{\Theta}}{\Delta^{5/2}} \right] &= 0 \end{aligned}$$

T. Salez, and L. Mahadevan, *J. Fluid Mech.* **2015**, 779, 181-196

Add random force into EOM for modified fluctuation-dissipation relation.

$$\dot{v} + f(\Delta) v + \kappa g(\dot{v}, v, \Delta) = 0 \quad \rightarrow \quad \dot{v} = -\gamma_{\text{eff}} v + \delta F / M$$

$$v_i = v_{i0} + \kappa \cdot v_{i1} \rightarrow \langle v_{i0} v_{i1} \rangle(t, \kappa)$$

$$\gamma_i = \gamma_{i0}(\Delta) + \kappa \cdot \gamma_{i1}(\Delta) \rightarrow \text{noise correlator amplitude}$$

$$\delta F_i = \delta F_{i0} + \kappa \cdot \delta F_{i1} \rightarrow \langle \delta F_{i0} \delta F_{i1} \rangle(\kappa)$$

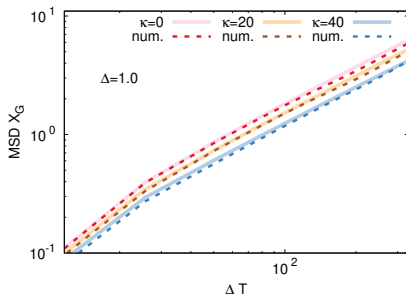
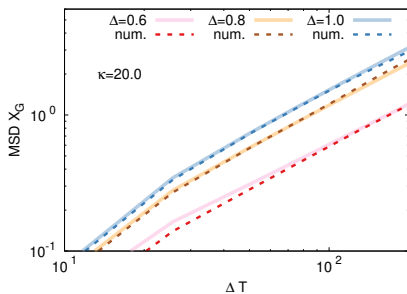
$$\langle \delta F_i(\tau_1) \delta F_i(\tau_2) \rangle \propto 2k_B T m_i \gamma_{i0} \delta(\tau_1 - \tau_2) \cdot \left[ 1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]$$

# Internship - M1

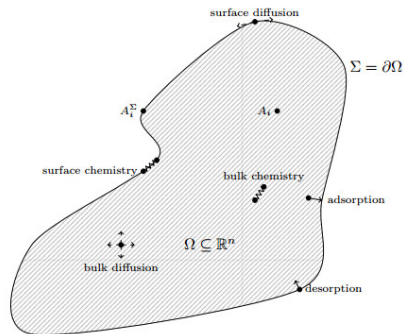
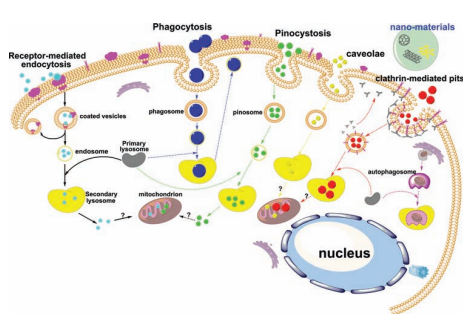
Numerical simulations with **fixed** height ( $\Delta$ ) - Effect of compliance ( $\kappa \neq 0$ )

Diffusion coefficient depends on vertical positions and soft wall modulus.

$$D(\kappa, \Delta) = D(0, \Delta) \left[ 1 - \kappa \cdot \frac{\gamma_{i1}(\Delta)}{\gamma_{i0}(\Delta)} \right]$$



# Internship - M2



**How does the complicated confining media affect the diffusion?**

B. Augner, and D. Bothe. arXiv:1911.13030 (2019).

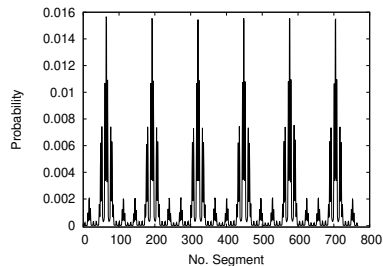
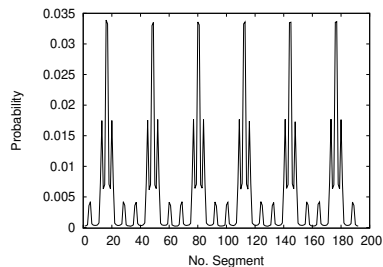
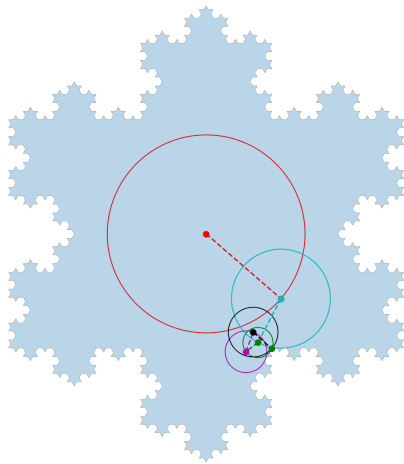
F. Zhao, et al. Small, 7(10), 1322-1337 (2011)



# Internship - M2

## Geometry-adapted fast random walk

Find minimal radius & Diffuse uniformly

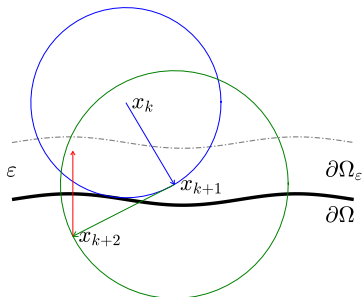


# Thesis Project

$$\vec{x}_{k+1} = \vec{x}_k + \rho(\cos \theta, \sin \theta_k)$$

$$\tau = \frac{\delta^2}{4D} \quad t_{k+1} = t_k + \tau$$

$$\ell_{t_{k+1}} = \ell_{t_k} + \sqrt{\frac{\pi}{2} D \tau}$$

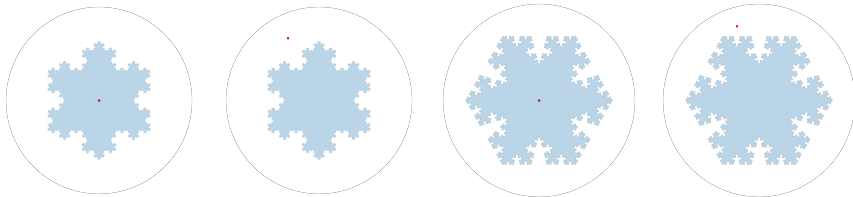


- **Local Time  $\ell$ :** How much time would one particle move near the complex surface?
- “Target-finding” diffusion near the surface;
- Collective effect of multiple independently diffusing particles;

Y. Zhou, et al. Comm. Math. Sci., 15, 237-259 (2017).

Y. Lanoiselée, et al. Nature Commun. 9, 4398 (2018).

# Programme



- Efficient numerical methods towards different cases;
- Diffusion on the surface & Local residence time;
- Reversible reactions controlled by diffusion on the surface;
- From 2D model to 3D model;

**Thanks for your attention!**

