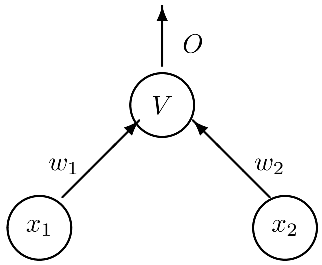
Homework-2

Question 1



	x_1	x_2	y
e_1	1	0	1
e_2	1	1	1
e_3	2	0	0
e_4	2	1	0

You are given a perceptron implemented with a sigmoid, with $\beta = 1$. There are NO bias connections. The initial values of the weights are $w_1 = 0$, $w_2 = 1$.

Part 1

Give explicit expressions to the way the weights change if the network is given the example e_3. Use $\epsilon = 0.1$. You may use temporary variables in your answer, but make sure that they are all specified in terms of the given values. You may use the notation $S(.)$ instead of explicitly computing sigmoid values.

Part 2

a. If you could choose ϵ to be as small as you like, and run back propagation as many epochs as you like with the four examples e_1, e_2, e_3, e_4, do you expect the computed output of the perceptron to be within 0.001 of the desired output (the value of y) for all four examples?

Answer: YES / NO / IMPOSSIBLE-TO-TELL

b. Will your answer to a. remain the same if bias connection is added (everything else stays as in a.)?

Answer:

- YES. My answer is exactly the same as in Part a.
- NO. My answer changes. With this new condition
  - my new answer to a. is YES.
  - my new answer to a. is NO.
  - my new answer to a. is IMPOSSIBLE-TO-TELL

Question 2

Part I

We would like to use the perceptron with a sigmoid unit to determine weights w_1, w_2, w_3 which are to be used to compute O, an approximation of the desired response y, according to:

$$\begin{aligned} h &= w_1\phi_1 + w_2\phi_2 + w_3\phi_3 \\ O &= \frac{1}{1+\exp\{-2\beta h\}} \end{aligned}$$

with

$$\phi_1 = x_1, \quad \phi_2 = x_2, \quad \phi_3 = 1.$$

The 4 training examples are given by:

	x_1	x_2	y
e_1	0	0	0
e_2	0	1	1
e_3	1	0	1
e_4	1	1	0

a. With $\beta = 1, \epsilon = 0.5$, show that when the algorithm is given e_1 the values of w_1, w_2, w_3 change according to:

$$\begin{aligned} O &\leftarrow \frac{1}{1+\exp\{-2w_3\}} \\ \delta &\leftarrow O^2(O-1) \\ w_1 &\leftarrow w_1 \\ w_2 &\leftarrow w_2 \\ w_3 &\leftarrow w_3 + \frac{\delta}{2} \end{aligned}$$

b. Give explicit formulas to how w_1, w_2, w_3 change when the algorithm is given e_2.

c. Give explicit formulas to how w_1, w_2, w_3 change when the algorithm is given e_3.

d. Give explicit formulas to how w_1, w_2, w_3 change when the algorithm is given e_4.

Part II

Write a program that implements these equations. Start with $w_1 = w_2 = w_3 = 0$ and compute their value after (a) 1 training epoch, (b) 2 epochs, (c) 100 epochs, (d) 1000 epochs, (e) 2000 epochs. (An epoch is a pass through the entire training set which in our case is e_1, e_2, e_3, e_4.)

Part III

The results obtained in Part II are used to classify examples by labeling a pair (x_1, x_2) of real numbers as a positive example when $O > 0.5$, and as a negative example when $O \leq 0.5$.

a. for the values of w_1, w_2, w_3 that were obtained in (a) show in a diagram of the x_1, x_2 plane what area is labeled as positive by shadowing that area.

b. Same as (a) for the values of w_1, w_2, w_3 that were obtained in (b).

c. Same as (a) for the values of w_1, w_2, w_3 that were obtained in (c).

d. Same as (a) for the values of w_1, w_2, w_3 that were obtained in (d).

e. Same as (a) for the values of w_1, w_2, w_3 that were obtained in (e).