Testing different models (fitting 2000 times)

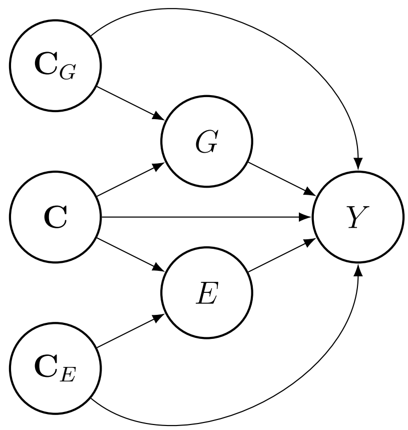


Figure 1: Directed Acyclic Graphs for causal interactions under two treatments G and E.

Let G and E denote the two binary treatments and Y be the binary outcome. Let $\mathbf{C}_G$, $\mathbf{C}_E$ be the set of covariates that confound G and E individually, and let $\mathbf{C}$ be the set of common confounders to both treatments.

Data generation

In this study, we will assume the outcome Y is binary and there are two treatments G and E. Let $\mathbf{C}_G = \{X_1, X_2\}$,$\mathbf{C}_E = \{X_3, X_4\}$ and $\mathbf{C} = \{X_5, X_6\}$ be the confounders of G, E and common confounder. Suppose $\mathbf{X} = \{X_1, \ldots, X_6\}$ is generated to follow a multivariate normal distribution such that $\mathbf{X} \sim \mathcal{N}_6(\mathbf{0}, \mathbf{I}_6)$. To generate the treatment assignment probabilities, we firstly generate probabilities for each treatment individually:

$$\pi_{G=1} = P(G = 1 \mid \mathbf{X}_G) = \frac{\exp\{f(\mathbf{X}_G)\}}{1 + \exp\{f(\mathbf{X}_G)\}}$$
$$\pi_{E=1} = P(E_i = 1 \mid \mathbf{X}_E) = \frac{\exp\{f(\mathbf{X}_E)\}}{1 + \exp\{f(\mathbf{X}_E)\}}$$

where $f(\mathbf{X}_G)$ and $f(\mathbf{X}_E)$ are the function of $\mathbf{X}_G$ and $\mathbf{X}_E$ such that $\mathbf{X}_G = \{X_1, X_2, X_5, X_6\}$ and $\mathbf{X}_E = \{X_3, X_4, X_5, X_6\}$. We now suppose $f(\mathbf{X}_G) = 0.3X_1 + 0.4X_2 + 0.5X_5 + 0.5X_6$ and $f(\mathbf{X}_E) = -0.4X_3 + 0.3X_4 + 0.5X_5 + 0.5X_6$. Note that here it is unnecessary to add the intercept as the expectation for each covariate is 0, making the marginal probabilities $\pi_{G=1}$ and $\pi_{E=1}$ are approximately 0.5. With the above setups, the probability of assignment for each combination of treatments can be computed as

$$\begin{aligned}
\theta_{00} &= p_{G=0,E=0} = (1 - \pi_{G=1})(1 - \pi_{E=1}) \\
\theta_{10} &= p_{G=1,E=0} = \pi_{G=1}(1 - \pi_{E=1}) \\
\theta_{01} &= p_{G=0,E=1} = (1 - \pi_{G=1})\pi_{E=1} \\
\theta_{11} &= p_{G=1,E=1} = \pi_{G=1}\pi_{E=1}
\end{aligned}$$

For each observation, the probability of receiving each pair of treatment follows a multinomial distribution such that $T_i \mid \mathbf{X} \sim \text{Multinomial}(1, \theta_{00}, \theta_{10}, \theta_{01}, \theta_{11})$. In addition, the outcome is generated as

$$\begin{aligned}
\Pr(Y = 1 \mid \mathbf{X}, G, E) = \text{logit}^{-1}(\beta_0 + \\
&0.4X_1 - 0.5X_2 + 0.7X_3 + 0.9X_4 - 0.2X_5 + 0.4X_6 + \\
&0.3G + 0.4E + 0.8GE)
\end{aligned}$$

The true value of RERI in terms of odds ratio can be computed as $\exp 0.3 + 0.4 + 0.8 - \exp 0.3 - \exp 0.4 + 1 = 2.64$

Using control only

$\beta_0 = -6$: **prevalence** ≈ 0.01

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	3.948(12.288)	4.326	0.968	1.362	52.67
CLOM(all)	4.017(9.252)	3.984	0.955	1.377	52.16
MSLOM(corresp.)	3.502(8.868)	3.078	0.946	0.916	35.42
DR	4.148(10.823)	2.440	0.734	1.508	57.12

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.586$, $RERI_{RR} = 2.501$

$\beta_0 = -5.5$: **prevalence** ≈ 0.018

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.748(1.661)	1.533	0.964	0.274	11.08
CLOM(all)	3.126(2.415)	1.933	0.951	0.486	18.41
MSLOM(corresp.)	2.713(2.122)	1.767	0.948	0.239	9.66
DR	3.262(3.367)	1.054	0.693	0.622	23.56

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.474$, $RERI_{RR} = 2.348$

$\beta_0 = -5$: **prevalence** ≈ 0.03

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.484(1.134)	1.053	0.965	0.142	6.06
CLOM(all)	2.924(1.587)	1.364	0.951	0.284	10.76
MSLOM(corresp.)	2.443(1.289)	1.202	0.953	0.101	4.31
DR	2.985(1.759)	0.685	0.645	0.345	13.07

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.342$, $RERI_{RR} = 2.160$

$\beta_0 = -4.5$: **prevalence** ≈ 0.05

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.273(0.799)	0.773	0.957	0.074	3.37
CLOM(all)	2.810(1.104)	1.046	0.949	0.170	6.44
MSLOM(corresp.)	2.238(0.929)	0.883	0.951	0.039	1.77
DR	2.837(1.229)	0.512	0.624	0.197	7.46

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.199$, $RERI_{RR} = 1.946$

$\beta_0 = -4$**: prevalence** ≈ 0.08

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.096(0.611)	0.597	0.956	0.042	2.04
CLOM(all)	2.773(0.877)	0.859	0.955	0.133	5.04
MSLOM(corresp.)	2.098(0.706)	0.688	0.957	0.044	2.14
DR	2.792(0.969)	0.415	0.621	0.152	5.76
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.054$, $RERI_{RR} = 1.713$					

$\beta_0 = -3$**: prevalence** ≈ 0.18

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.807(0.397)	0.396	0.957	0.017	0.95
CLOM(all)	2.721(0.623)	0.635	0.956	0.081	3.07
MSLOM(corresp.)	1.869(0.469)	0.467	0.958	0.079	4.41
DR	2.728(0.675)	0.300	0.621	0.088	3.33
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.790$, $RERI_{RR} = 1.232$					

$\beta_0 = -2$**: prevalence** ≈ 0.4

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.616(0.306)	0.305	0.950	0.007	0.44
CLOM(all)	2.691(0.531)	0.529	0.949	0.051	1.93
MSLOM(corresp.)	1.734(0.376)	0.369	0.955	0.125	7.77
DR	2.704(0.585)	0.246	0.596	0.064	2.42
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.609$, $RERI_{RR} = 0.802$					

$\beta_0 = -1$**: prevalence** ≈ 0.6

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.546(0.287)	0.280	0.949	0.007	0.45
CLOM(all)	2.687(0.515)	0.500	0.954	0.047	1.78
MSLOM(corresp.)	1.698(0.355)	0.347	0.945	0.159	10.33
DR	2.696(0.562)	0.227	0.581	0.056	2.12
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.539$, $RERI_{RR} = 0.472$					

Using both control and case

$\beta_0 = -6$**: prevalence** ≈ 0.01

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	3.948(12.288)	4.326	0.968	1.362	52.67
CLOM(all)	4.017(9.252)	3.984	0.955	1.377	52.16
MSLOM(corresp.)	3.384(7.687)	3.046	0.945	0.798	30.86
DR	4.151(10.632)	2.412	0.737	1.511	57.23

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.586$, $RERI_{RR} = 2.501$

$\beta_0 = -5.5$**: prevalence** ≈ 0.018

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.748(1.661)	1.533	0.964	0.274	11.08
CLOM(all)	3.126(2.415)	1.933	0.951	0.486	18.41
MSLOM(corresp.)	2.638(2.079)	1.728	0.946	0.164	6.63
DR	3.263(3.183)	1.045	0.694	0.623	23.60

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.474$, $RERI_{RR} = 2.348$

$\beta_0 = -5$**: prevalence** ≈ 0.03

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.484(1.134)	1.053	0.965	0.142	6.06
CLOM(all)	2.924(1.587)	1.364	0.951	0.284	10.76
MSLOM(corresp.)	2.353(1.230)	1.163	0.945	0.011	0.47
DR	2.994(1.771)	0.689	0.646	0.354	13.41

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.342$, $RERI_{RR} = 2.160$

$\beta_0 = -4.5$**: prevalence** ≈ 0.05

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.273(0.799)	0.773	0.957	0.074	3.37
CLOM(all)	2.810(1.104)	1.046	0.949	0.170	6.44
MSLOM(corresp.)	2.130(0.875)	0.846	0.942	0.069	3.14
DR	2.846(1.233)	0.517	0.625	0.206	7.80

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.199$, $RERI_{RR} = 1.946$

$\beta_0 = -4$**: prevalence** ≈ 0.08

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.096(0.611)	0.597	0.956	0.042	2.04
CLOM(all)	2.773(0.877)	0.859	0.955	0.133	5.04
MSLOM(corresp.)	1.967(0.651)	0.652	0.945	0.087	4.24
DR	2.800(0.970)	0.421	0.627	0.160	6.06

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.054$, $RERI_{RR} = 1.713$

$\beta_0 = -3$**: prevalence** ≈ 0.18

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.807(0.397)	0.396	0.957	0.017	0.95
CLOM(all)	2.721(0.623)	0.635	0.956	0.081	3.07
MSLOM(corresp.)	1.694(0.420)	0.432	0.946	0.096	5.36
DR	2.735(0.677)	0.310	0.626	0.095	3.60

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.790$, $RERI_{RR} = 1.232$

$\beta_0 = -2$**: prevalence** ≈ 0.4

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.616(0.306)	0.305	0.950	0.007	0.44
CLOM(all)	2.691(0.531)	0.529	0.949	0.051	1.93
MSLOM(corresp.)	1.524(0.326)	0.334	0.942	0.085	5.28
DR	2.712(0.586)	0.259	0.622	0.072	2.73

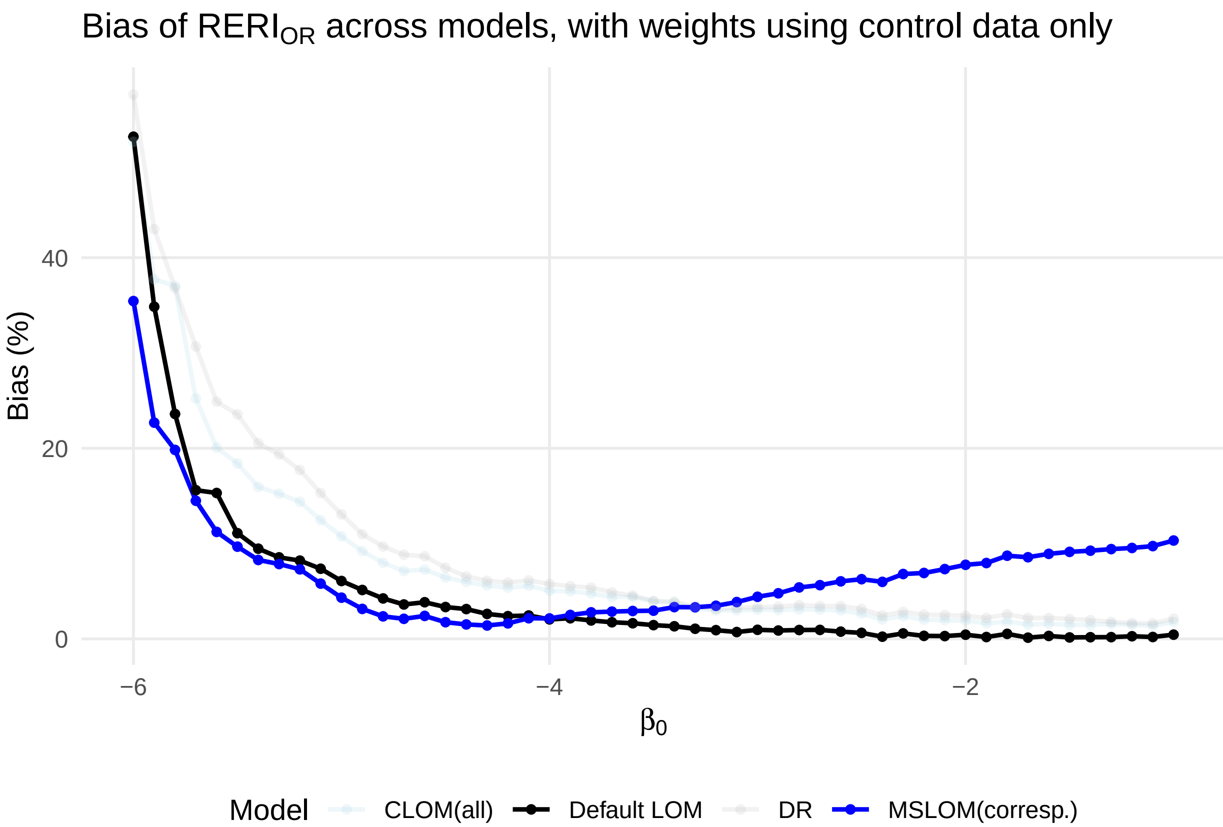
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.609$, $RERI_{RR} = 0.802$

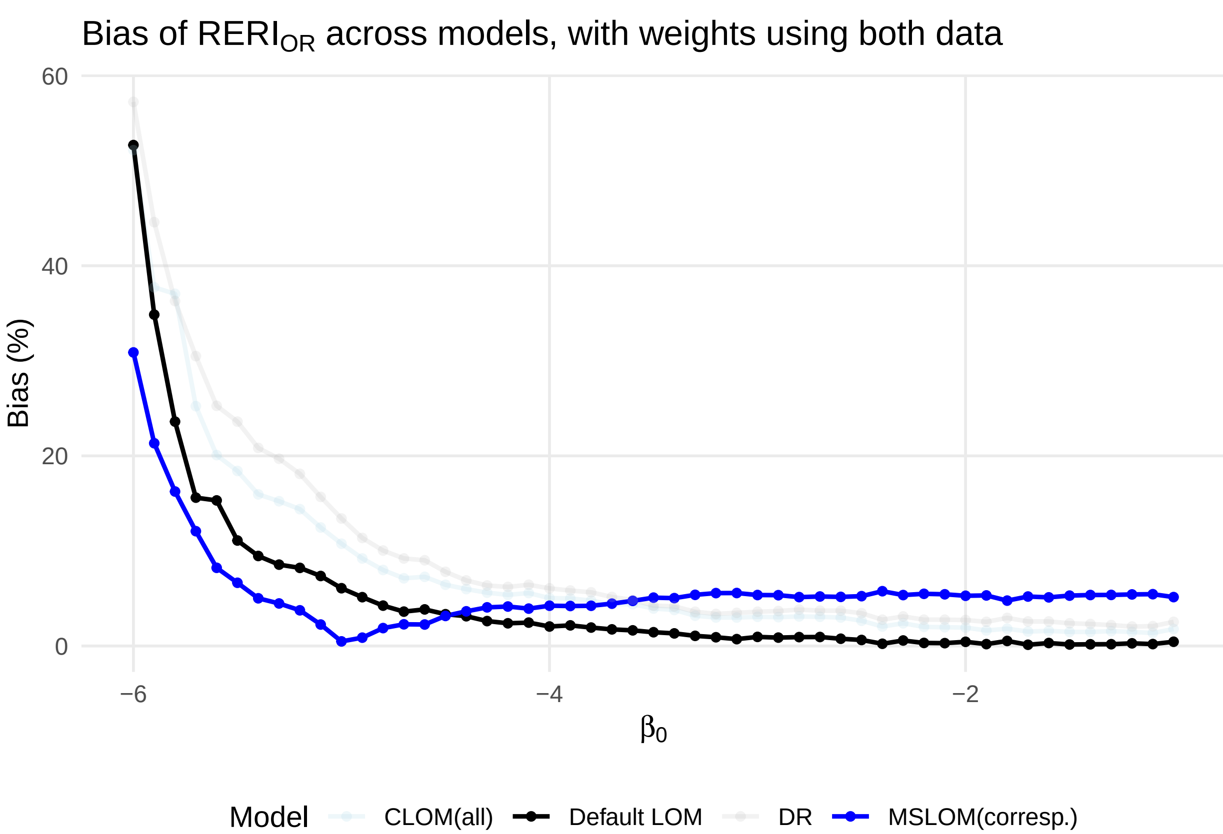
$\beta_0 = -1$**: prevalence** ≈ 0.6

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.546(0.287)	0.280	0.949	0.007	0.45
CLOM(all)	2.687(0.515)	0.500	0.954	0.047	1.78
MSLOM(corresp.)	1.460(0.307)	0.308	0.936	0.079	5.13
DR	2.707(0.568)	0.247	0.617	0.067	2.54

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.539$, $RERI_{RR} = 0.472$

Graphs


Bias of RERI$_{OR}$ across models, with weights using control data only
Bias (%)
40
20
0
−6
−4
−2
β_0
Model
CLOM(all)
Default LOM
DR
MSLOM(corresp.)


Bias of RERI$_{OR}$ across models, with weights using both data
Bias (%)
60
40
20
0
−6
−4
−2
β_0
Model
CLOM(all)
Default LOM
DR
MSLOM(corresp.)