

## Testing different models for *strong* confounding effects (fitting 2000 times)

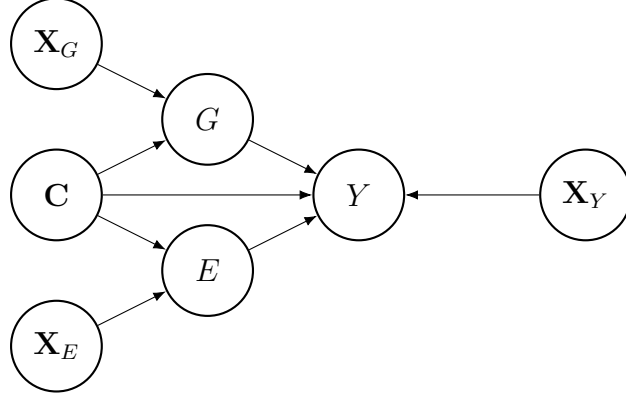


Figure 1: Directed Acyclic Graphs for causal interactions under two treatments  $G$  and  $E$ .

Let  $G$  and  $E$  denote the two binary treatments and  $Y$  be the binary outcome. Let  $\mathbf{X}_G$ ,  $\mathbf{X}_E$  be the set of instrumental variables for  $G$  and  $E$ , respectively. Let  $\mathbf{X}_Y$  be the outcome-only covariates. Let  $\mathbf{C}$  be the set of common confounders to both treatments.

### Data generation

In this study, we will assume the outcome  $Y$  is binary and there are two treatments  $G$  and  $E$ . Let  $\mathbf{X}_G = \{X_1, X_2\}$ ,  $\mathbf{X}_E = \{X_3, X_4\}$ ,  $\mathbf{X}_Y = \{X_5, X_6\}$  and  $\mathbf{C} = \{X_7, X_8\}$ . Suppose  $\mathbf{X} = \{X_1, \dots, X_8\}$  is generated to follow a multivariate normal distribution such that  $\mathbf{X} \sim \mathcal{N}_8(\mathbf{0}, \mathbf{I}_8)$ . To generate the treatment assignment probabilities, we firstly generate probabilities for each treatment individually:

$$\pi_{G=1} = P(G = 1 \mid \mathbf{Z}_G) = \frac{\exp\{f(\mathbf{Z}_G)\}}{1 + \exp\{f(\mathbf{Z}_G)\}}$$

$$\pi_{E=1} = P(E_i = 1 \mid \mathbf{Z}_E) = \frac{\exp\{f(\mathbf{Z}_E)\}}{1 + \exp\{f(\mathbf{Z}_E)\}}$$

where  $f(\mathbf{Z}_G)$  and  $f(\mathbf{Z}_E)$  are the function of  $\mathbf{Z}_G$  and  $\mathbf{Z}_E$  such that  $\mathbf{Z}_G = \{X_1, X_2, X_7, X_8\}$  and  $\mathbf{Z}_E = \{X_3, X_4, X_7, X_8\}$ . We now suppose

$$f(\mathbf{Z}_G) = 0.8X_1 + 0.5X_2 + 0.3X_7 + 0.5X_8$$

$$f(\mathbf{Z}_E) = -0.5X_3 + 0.2X_4 + 0.3X_7 + 0.5X_8$$

Note that here it is unnecessary to add the intercept as the expectation for each covariate is 0, making the marginal probabilities  $\pi_{G=1}$  and  $\pi_{E=1}$  are approximately 0.5. With the above setups, the probability of assignment for each combination of treatments can be computed as

$$\begin{aligned} \theta_{00} &= p_{G=0, E=0} = (1 - \pi_{G=1})(1 - \pi_{E=1}) \\ \theta_{10} &= p_{G=1, E=0} = \pi_{G=1}(1 - \pi_{E=1}) \\ \theta_{01} &= p_{G=0, E=1} = (1 - \pi_{G=1})\pi_{E=1} \\ \theta_{11} &= p_{G=1, E=1} = \pi_{G=1}\pi_{E=1} \end{aligned}$$

For each observation, the probability of receiving each pair of treatment follows a multinomial distribution such that  $T_i \mid \mathbf{X} \sim \text{Multinomial}(1, \theta_{00}, \theta_{10}, \theta_{01}, \theta_{11})$ , where  $T_i \in \{(0, 0), (1, 0), (0, 1), (1, 1)\}$ . In addition, denote  $\mathbf{Z}_Y = \{\mathbf{X}_Y, \mathbf{C}\} = \{X_5, X_6, X_7, X_8\}$ , the outcome is generated as

$$\begin{aligned} \Pr(Y = 1 \mid \mathbf{Z}_Y, G, E) &= \text{logit}^{-1}(\beta_0 + \\ &\quad 0.8X_5 + 0.5X_6 - 0.6X_7 + 0.4X_8 + \\ &\quad 0.3G + 0.4E + 0.8GE) \end{aligned}$$

The true value of conditional RERI in terms of odds ratio can be computed as  $\exp\{0.3 + 0.4 + 0.8\} - \exp\{0.3\} - \exp\{0.4\} + 1 = 2.64$ .

## Using control only

$\beta_0 = -6$ : prevalence  $\approx 0.01$

| Model  | $\beta_3$     | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|--|---------------|--------------------------|----------|-----------|---------|
| Default LOM  | 4.401(13.803) | 6.084                    | 0.958    | 1.834     | 71.45   |
| CLOM(X5X6)   | 4.546(12.550) | 6.078                    | 0.960    | 1.906     | 72.20   |
| CLOM(X7X8)   | 4.598(13.890) | 6.128                    | 0.948    | 1.958     | 74.17   |
| CLOM(X5678)  | 4.436(10.610) | 5.915                    | 0.956    | 1.796     | 68.03   |
| MSLOM(X7X8)  | 4.096(9.955)  | 4.583                    | 0.944    | 1.529     | 59.56   |
| MSLOM(X1278)   | 4.260(11.728) | 5.605                    | 0.936    | 1.693     | 65.95   |
| MSLOM(X3478)   | 4.287(11.649) | 4.787                    | 0.944    | 1.720     | 67.00   |
| MSLOM(X5678)   | 4.197(9.790)  | 5.023                    | 0.942    | 1.630     | 63.50   |
| MSLOM(X123478)   | 4.484(13.921) | 5.804                    | 0.940    | 1.917     | 74.68   |
| MSLOM(all)   | 4.644(13.849) | 7.204                    | 0.939    | 2.077     | 80.91   |
| DR   | 4.689(13.111) | 3.822                    | 0.769    | 2.049     | 77.61   |
| $RERI_{OR}^{true} = 2.64$ ; Empirical: $RERI_{OR} = 2.567$ , $RERI_{RR} = 2.501$ |               |                          |          |           |         |

$\beta_0 = -5.5$ : prevalence  $\approx 0.018$

| Model  | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|--|--------------|--------------------------|----------|-----------|---------|
| Default LOM  | 3.012(4.289) | 2.085                    | 0.949    | 0.519     | 20.82   |
| CLOM(X5X6)   | 3.291(7.281) | 2.258                    | 0.946    | 0.651     | 24.66   |
| CLOM(X7X8)   | 3.012(3.961) | 2.163                    | 0.935    | 0.372     | 14.09   |
| CLOM(X5678)  | 3.301(5.071) | 2.411                    | 0.953    | 0.661     | 25.04   |
| MSLOM(X7X8)  | 3.036(4.530) | 2.168                    | 0.946    | 0.543     | 21.78   |
| MSLOM(X1278)   | 3.144(5.241) | 2.533                    | 0.945    | 0.651     | 26.11   |
| MSLOM(X3478)   | 3.122(4.965) | 2.337                    | 0.944    | 0.629     | 25.23   |
| MSLOM(X5678)   | 3.110(4.384) | 2.192                    | 0.950    | 0.617     | 24.75   |
| MSLOM(X123478)   | 3.219(5.037) | 2.669                    | 0.944    | 0.726     | 29.12   |
| MSLOM(all)   | 3.300(4.955) | 2.717                    | 0.946    | 0.807     | 32.37   |
| DR   | 3.407(5.982) | 1.295                    | 0.709    | 0.767     | 29.05   |
| $RERI_{OR}^{true} = 2.64$ ; Empirical: $RERI_{OR} = 2.493$ , $RERI_{RR} = 2.391$ |              |                          |          |           |         |

$\beta_0 = -5$ : prevalence  $\approx 0.03$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.548(1.371) | 1.195                    | 0.948    | 0.148     | 6.17    |
| CLOM(X5X6)     | 2.763(1.503) | 1.289                    | 0.939    | 0.123     | 4.66    |
| CLOM(X7X8)     | 2.586(1.440) | 1.268                    | 0.918    | 0.054     | 2.05    |
| CLOM(X5678)    | 2.891(1.643) | 1.414                    | 0.950    | 0.251     | 9.51    |
| MSLOM(X7X8)    | 2.568(1.698) | 1.337                    | 0.947    | 0.168     | 7.00    |
| MSLOM(X1278)   | 2.628(2.097) | 1.573                    | 0.945    | 0.228     | 9.50    |
| MSLOM(X3478)   | 2.598(1.790) | 1.414                    | 0.950    | 0.198     | 8.25    |
| MSLOM(X5678)   | 2.669(1.740) | 1.383                    | 0.953    | 0.269     | 11.21   |
| MSLOM(X123478) | 2.680(2.330) | 1.676                    | 0.946    | 0.280     | 11.67   |
| MSLOM(all)     | 2.781(2.272) | 1.726                    | 0.952    | 0.381     | 15.88   |
| DR             | 2.946(2.095) | 0.717                    | 0.677    | 0.306     | 11.59   |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.400$ ,  $RERI_{RR} = 2.246$

$\beta_0 = -4.5$ : prevalence  $\approx 0.05$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.362(0.880) | 0.859                    | 0.962    | 0.075     | 3.28    |
| CLOM(X5X6)     | 2.628(0.970) | 0.948                    | 0.939    | 0.012     | 0.45    |
| CLOM(X7X8)     | 2.422(0.946) | 0.923                    | 0.902    | 0.218     | 8.26    |
| CLOM(X5678)    | 2.791(1.088) | 1.059                    | 0.956    | 0.151     | 5.72    |
| MSLOM(X7X8)    | 2.378(0.993) | 0.961                    | 0.951    | 0.091     | 3.98    |
| MSLOM(X1278)   | 2.420(1.262) | 1.132                    | 0.947    | 0.133     | 5.82    |
| MSLOM(X3478)   | 2.398(1.052) | 1.016                    | 0.954    | 0.111     | 4.85    |
| MSLOM(X5678)   | 2.510(1.058) | 1.009                    | 0.962    | 0.223     | 9.75    |
| MSLOM(X123478) | 2.445(1.333) | 1.198                    | 0.947    | 0.158     | 6.91    |
| MSLOM(all)     | 2.582(1.417) | 1.259                    | 0.954    | 0.295     | 12.90   |
| DR             | 2.820(1.184) | 0.520                    | 0.647    | 0.180     | 6.82    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.287$ ,  $RERI_{RR} = 2.063$

$\beta_0 = -4$ : prevalence  $\approx 0.08$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.182(0.661) | 0.650                    | 0.957    | 0.019     | 0.88    |
| CLOM(X5X6)     | 2.508(0.744) | 0.738                    | 0.931    | 0.132     | 5.00    |
| CLOM(X7X8)     | 2.277(0.724) | 0.709                    | 0.867    | 0.363     | 13.75   |
| CLOM(X5678)    | 2.719(0.854) | 0.841                    | 0.949    | 0.079     | 2.99    |
| MSLOM(X7X8)    | 2.213(0.748) | 0.730                    | 0.954    | 0.050     | 2.31    |
| MSLOM(X1278)   | 2.235(0.897) | 0.861                    | 0.947    | 0.072     | 3.33    |
| MSLOM(X3478)   | 2.231(0.790) | 0.772                    | 0.958    | 0.068     | 3.14    |
| MSLOM(X5678)   | 2.378(0.810) | 0.779                    | 0.968    | 0.215     | 9.94    |
| MSLOM(X123478) | 2.251(0.945) | 0.909                    | 0.952    | 0.088     | 4.07    |
| MSLOM(all)     | 2.420(1.016) | 0.971                    | 0.964    | 0.257     | 11.88   |
| DR             | 2.732(0.910) | 0.406                    | 0.637    | 0.092     | 3.48    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.163$ ,  $RERI_{RR} = 1.849$

$\beta_0 = -3$ : prevalence  $\approx 0.18$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.915(0.415) | 0.420                    | 0.959    | 0.000     | 0.00    |
| CLOM(X5X6)     | 2.367(0.500) | 0.507                    | 0.889    | 0.273     | 10.34   |
| CLOM(X7X8)     | 2.082(0.470) | 0.473                    | 0.705    | 0.558     | 21.14   |
| CLOM(X5678)    | 2.683(0.602) | 0.604                    | 0.956    | 0.043     | 1.63    |
| MSLOM(X7X8)    | 1.986(0.490) | 0.482                    | 0.953    | 0.071     | 3.71    |
| MSLOM(X1278)   | 2.005(0.597) | 0.572                    | 0.955    | 0.090     | 4.70    |
| MSLOM(X3478)   | 1.995(0.522) | 0.510                    | 0.958    | 0.080     | 4.18    |
| MSLOM(X5678)   | 2.245(0.566) | 0.538                    | 0.945    | 0.330     | 17.23   |
| MSLOM(X123478) | 2.011(0.626) | 0.602                    | 0.960    | 0.096     | 5.01    |
| MSLOM(all)     | 2.275(0.716) | 0.674                    | 0.960    | 0.360     | 18.80   |
| DR             | 2.690(0.653) | 0.285                    | 0.636    | 0.050     | 1.89    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.915$ ,  $RERI_{RR} = 1.362$

$\beta_0 = -2$ : prevalence  $\approx 0.4$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.730(0.325) | 0.317                    | 0.950    | 0.003     | 0.17    |
| CLOM(X5X6)     | 2.277(0.418) | 0.405                    | 0.810    | 0.363     | 13.75   |
| CLOM(X7X8)     | 1.949(0.377) | 0.367                    | 0.503    | 0.691     | 26.17   |
| CLOM(X5678)    | 2.678(0.515) | 0.497                    | 0.948    | 0.038     | 1.44    |
| MSLOM(X7X8)    | 1.838(0.397) | 0.377                    | 0.944    | 0.111     | 6.43    |
| MSLOM(X1278)   | 1.848(0.476) | 0.448                    | 0.953    | 0.121     | 7.01    |
| MSLOM(X3478)   | 1.839(0.418) | 0.399                    | 0.947    | 0.112     | 6.49    |
| MSLOM(X5678)   | 2.206(0.498) | 0.442                    | 0.835    | 0.479     | 27.74   |
| MSLOM(X123478) | 1.850(0.498) | 0.473                    | 0.954    | 0.123     | 7.12    |
| MSLOM(all)     | 2.224(0.621) | 0.559                    | 0.898    | 0.497     | 28.78   |
| DR             | 2.684(0.563) | 0.228                    | 0.594    | 0.044     | 1.67    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.727$ ,  $RERI_{RR} = 0.888$

$\beta_0 = -1$ : prevalence  $\approx 0.6$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.655(0.286) | 0.289                    | 0.954    | 0.005     | 0.30    |
| CLOM(X5X6)     | 2.233(0.373) | 0.377                    | 0.766    | 0.407     | 15.42   |
| CLOM(X7X8)     | 1.892(0.331) | 0.337                    | 0.397    | 0.748     | 28.33   |
| CLOM(X5678)    | 2.673(0.461) | 0.467                    | 0.953    | 0.033     | 1.25    |
| MSLOM(X7X8)    | 1.802(0.366) | 0.356                    | 0.948    | 0.152     | 9.21    |
| MSLOM(X1278)   | 1.803(0.451) | 0.427                    | 0.952    | 0.153     | 9.27    |
| MSLOM(X3478)   | 1.805(0.392) | 0.379                    | 0.942    | 0.155     | 9.39    |
| MSLOM(X5678)   | 2.276(0.484) | 0.436                    | 0.745    | 0.626     | 37.94   |
| MSLOM(X123478) | 1.806(0.477) | 0.453                    | 0.946    | 0.156     | 9.45    |
| MSLOM(all)     | 2.288(0.633) | 0.559                    | 0.835    | 0.638     | 38.67   |
| DR             | 2.687(0.521) | 0.205                    | 0.562    | 0.047     | 1.78    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.650$ ,  $RERI_{RR} = 0.511$

## Using both control and case

$\beta_0 = -6$ : prevalence  $\approx 0.01$

| Model  | $\beta_3$     | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|--|---------------|--------------------------|----------|-----------|---------|
| Default LOM  | 4.401(13.803) | 6.084                    | 0.958    | 1.834     | 71.45   |
| CLOM(X5X6)   | 4.546(12.550) | 6.078                    | 0.960    | 1.906     | 72.20   |
| CLOM(X7X8)   | 4.598(13.890) | 6.128                    | 0.948    | 1.958     | 74.17   |
| CLOM(X5678)  | 4.436(10.610) | 5.915                    | 0.956    | 1.796     | 68.03   |
| MSLOM(X7X8)  | 4.062(9.855)  | 5.017                    | 0.943    | 1.495     | 58.24   |
| MSLOM(X1278)   | 4.237(11.653) | 6.688                    | 0.936    | 1.670     | 65.06   |
| MSLOM(X3478)   | 4.234(11.277) | 5.244                    | 0.943    | 1.667     | 64.94   |
| MSLOM(X5678)   | 4.204(11.796) | 4.866                    | 0.942    | 1.637     | 63.77   |
| MSLOM(X123478)   | 4.293(12.278) | 9.428                    | 0.940    | 1.726     | 67.24   |
| MSLOM(all)   | 4.421(14.160) | 11.028                   | 0.935    | 1.854     | 72.22   |
| DR   | 4.843(13.981) | 3.942                    | 0.773    | 2.203     | 83.45   |
| $RERI_{OR}^{true} = 2.64$ ; Empirical: $RERI_{OR} = 2.567$ , $RERI_{RR} = 2.501$ |               |                          |          |           |         |

$\beta_0 = -5.5$ : prevalence  $\approx 0.018$

| Model  | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|--|--------------|--------------------------|----------|-----------|---------|
| Default LOM  | 3.012(4.289) | 2.085                    | 0.949    | 0.519     | 20.82   |
| CLOM(X5X6)   | 3.291(7.281) | 2.258                    | 0.946    | 0.651     | 24.66   |
| CLOM(X7X8)   | 3.012(3.961) | 2.163                    | 0.935    | 0.372     | 14.09   |
| CLOM(X5678)  | 3.301(5.071) | 2.411                    | 0.953    | 0.661     | 25.04   |
| MSLOM(X7X8)  | 3.002(4.774) | 2.160                    | 0.946    | 0.509     | 20.42   |
| MSLOM(X1278)   | 3.110(5.442) | 2.481                    | 0.943    | 0.617     | 24.75   |
| MSLOM(X3478)   | 3.052(4.413) | 2.278                    | 0.943    | 0.559     | 22.42   |
| MSLOM(X5678)   | 2.966(4.011) | 2.135                    | 0.947    | 0.473     | 18.97   |
| MSLOM(X123478)   | 3.197(5.542) | 2.645                    | 0.942    | 0.704     | 28.24   |
| MSLOM(all)   | 3.158(4.855) | 2.621                    | 0.942    | 0.665     | 26.67   |
| DR   | 3.428(5.941) | 1.329                    | 0.712    | 0.788     | 29.85   |
| $RERI_{OR}^{true} = 2.64$ ; Empirical: $RERI_{OR} = 2.493$ , $RERI_{RR} = 2.391$ |              |                          |          |           |         |

$\beta_0 = -5$ : prevalence  $\approx 0.03$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.548(1.371) | 1.195                    | 0.948    | 0.148     | 6.17    |
| CLOM(X5X6)     | 2.763(1.503) | 1.289                    | 0.939    | 0.123     | 4.66    |
| CLOM(X7X8)     | 2.586(1.440) | 1.268                    | 0.918    | 0.054     | 2.05    |
| CLOM(X5678)    | 2.891(1.643) | 1.414                    | 0.950    | 0.251     | 9.51    |
| MSLOM(X7X8)    | 2.509(1.639) | 1.307                    | 0.945    | 0.109     | 4.54    |
| MSLOM(X1278)   | 2.569(2.045) | 1.537                    | 0.942    | 0.169     | 7.04    |
| MSLOM(X3478)   | 2.541(1.818) | 1.387                    | 0.947    | 0.141     | 5.88    |
| MSLOM(X5678)   | 2.507(1.643) | 1.309                    | 0.943    | 0.107     | 4.46    |
| MSLOM(X123478) | 2.615(2.187) | 1.630                    | 0.944    | 0.215     | 8.96    |
| MSLOM(all)     | 2.613(2.276) | 1.636                    | 0.944    | 0.213     | 8.88    |
| DR             | 2.952(2.166) | 0.723                    | 0.683    | 0.312     | 11.82   |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.400$ ,  $RERI_{RR} = 2.246$

$\beta_0 = -4.5$ : prevalence  $\approx 0.05$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.362(0.880) | 0.859                    | 0.962    | 0.075     | 3.28    |
| CLOM(X5X6)     | 2.628(0.970) | 0.948                    | 0.939    | 0.012     | 0.45    |
| CLOM(X7X8)     | 2.422(0.946) | 0.923                    | 0.902    | 0.218     | 8.26    |
| CLOM(X5678)    | 2.791(1.088) | 1.059                    | 0.956    | 0.151     | 5.72    |
| MSLOM(X7X8)    | 2.307(0.952) | 0.933                    | 0.945    | 0.020     | 0.87    |
| MSLOM(X1278)   | 2.345(1.206) | 1.097                    | 0.945    | 0.058     | 2.54    |
| MSLOM(X3478)   | 2.325(1.007) | 0.985                    | 0.950    | 0.038     | 1.66    |
| MSLOM(X5678)   | 2.303(0.939) | 0.933                    | 0.947    | 0.016     | 0.70    |
| MSLOM(X123478) | 2.369(1.273) | 1.158                    | 0.941    | 0.082     | 3.59    |
| MSLOM(all)     | 2.364(1.258) | 1.157                    | 0.941    | 0.077     | 3.37    |
| DR             | 2.824(1.184) | 0.525                    | 0.650    | 0.184     | 6.97    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.287$ ,  $RERI_{RR} = 2.063$



$\beta_0 = -4$ : prevalence  $\approx 0.08$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 2.182(0.661) | 0.650                    | 0.957    | 0.019     | 0.88    |
| CLOM(X5X6)     | 2.508(0.744) | 0.738                    | 0.931    | 0.132     | 5.00    |
| CLOM(X7X8)     | 2.277(0.724) | 0.709                    | 0.867    | 0.363     | 13.75   |
| CLOM(X5678)    | 2.719(0.854) | 0.841                    | 0.949    | 0.079     | 2.99    |
| MSLOM(X7X8)    | 2.124(0.707) | 0.702                    | 0.948    | 0.039     | 1.80    |
| MSLOM(X1278)   | 2.143(0.846) | 0.825                    | 0.940    | 0.020     | 0.92    |
| MSLOM(X3478)   | 2.141(0.746) | 0.741                    | 0.949    | 0.022     | 1.02    |
| MSLOM(X5678)   | 2.121(0.694) | 0.702                    | 0.953    | 0.042     | 1.94    |
| MSLOM(X123478) | 2.158(0.890) | 0.869                    | 0.946    | 0.005     | 0.23    |
| MSLOM(all)     | 2.155(0.876) | 0.868                    | 0.945    | 0.008     | 0.37    |
| DR             | 2.735(0.908) | 0.411                    | 0.645    | 0.095     | 3.60    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 2.163$ ,  $RERI_{RR} = 1.849$

$\beta_0 = -3$ : prevalence  $\approx 0.18$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.915(0.415) | 0.420                    | 0.959    | 0.000     | 0.00    |
| CLOM(X5X6)     | 2.367(0.500) | 0.507                    | 0.889    | 0.273     | 10.34   |
| CLOM(X7X8)     | 2.082(0.470) | 0.473                    | 0.705    | 0.558     | 21.14   |
| CLOM(X5678)    | 2.683(0.602) | 0.604                    | 0.956    | 0.043     | 1.63    |
| MSLOM(X7X8)    | 1.860(0.445) | 0.453                    | 0.946    | 0.055     | 2.87    |
| MSLOM(X1278)   | 1.875(0.538) | 0.531                    | 0.942    | 0.040     | 2.09    |
| MSLOM(X3478)   | 1.868(0.474) | 0.477                    | 0.949    | 0.047     | 2.45    |
| MSLOM(X5678)   | 1.857(0.434) | 0.452                    | 0.952    | 0.058     | 3.03    |
| MSLOM(X123478) | 1.881(0.564) | 0.557                    | 0.948    | 0.034     | 1.78    |
| MSLOM(all)     | 1.877(0.553) | 0.557                    | 0.951    | 0.038     | 1.98    |
| DR             | 2.691(0.648) | 0.294                    | 0.648    | 0.051     | 1.93    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.915$ ,  $RERI_{RR} = 1.362$

$\beta_0 = -2$ : prevalence  $\approx 0.4$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.730(0.325) | 0.317                    | 0.950    | 0.003     | 0.17    |
| CLOM(X5X6)     | 2.277(0.418) | 0.405                    | 0.810    | 0.363     | 13.75   |
| CLOM(X7X8)     | 1.949(0.377) | 0.367                    | 0.503    | 0.691     | 26.17   |
| CLOM(X5678)    | 2.678(0.515) | 0.497                    | 0.948    | 0.038     | 1.44    |
| MSLOM(X7X8)    | 1.677(0.346) | 0.344                    | 0.939    | 0.050     | 2.90    |
| MSLOM(X1278)   | 1.684(0.409) | 0.403                    | 0.947    | 0.043     | 2.49    |
| MSLOM(X3478)   | 1.677(0.363) | 0.362                    | 0.944    | 0.050     | 2.90    |
| MSLOM(X5678)   | 1.676(0.336) | 0.344                    | 0.948    | 0.051     | 2.95    |
| MSLOM(X123478) | 1.684(0.426) | 0.422                    | 0.949    | 0.043     | 2.49    |
| MSLOM(all)     | 1.682(0.418) | 0.423                    | 0.953    | 0.045     | 2.61    |
| DR             | 2.684(0.554) | 0.243                    | 0.638    | 0.044     | 1.67    |

$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.727$ ,  $RERI_{RR} = 0.888$

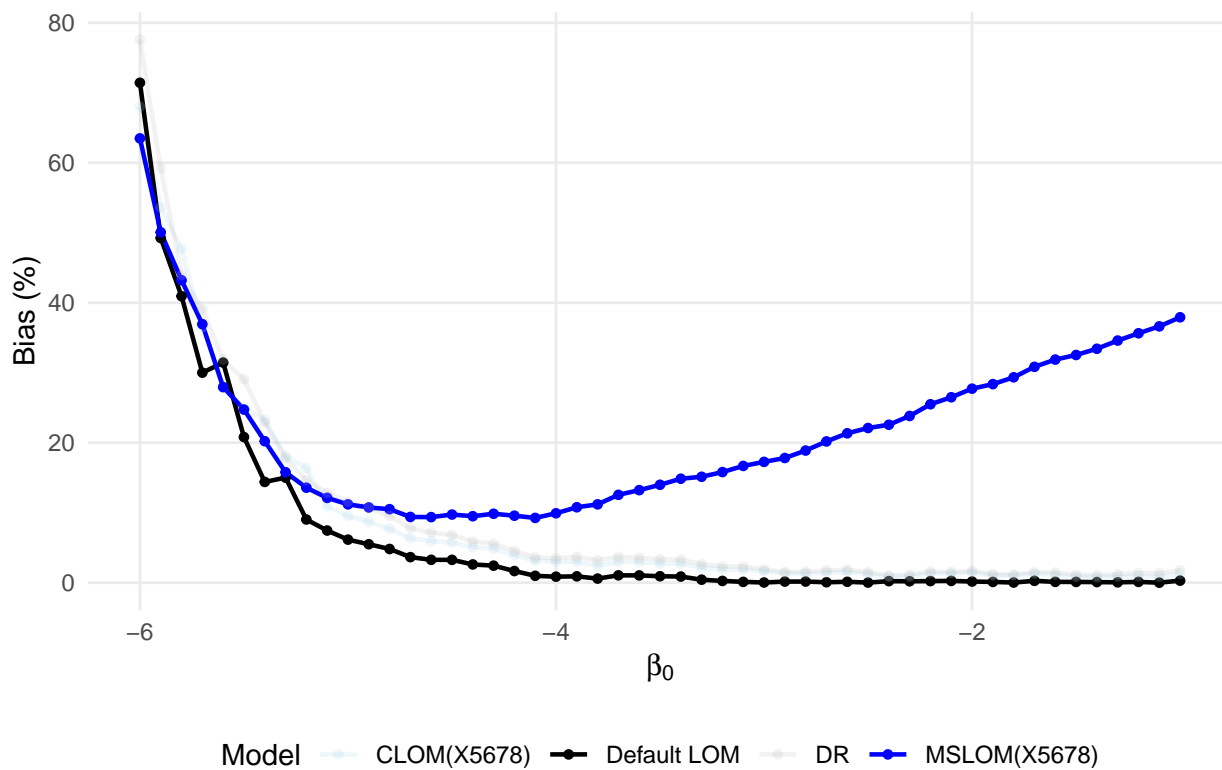
$\beta_0 = -1$ : prevalence  $\approx 0.6$

| Model          | $\beta_3$    | $\widehat{SE}_{\beta_3}$ | coverage | Abs. Bias | Bias(%) |
|----------------|--------------|--------------------------|----------|-----------|---------|
| Default LOM    | 1.655(0.286) | 0.289                    | 0.954    | 0.005     | 0.30    |
| CLOM(X5X6)     | 2.233(0.373) | 0.377                    | 0.766    | 0.407     | 15.42   |
| CLOM(X7X8)     | 1.892(0.331) | 0.337                    | 0.397    | 0.748     | 28.33   |
| CLOM(X5678)    | 2.673(0.461) | 0.467                    | 0.953    | 0.033     | 1.25    |
| MSLOM(X7X8)    | 1.611(0.307) | 0.316                    | 0.952    | 0.039     | 2.36    |
| MSLOM(X1278)   | 1.606(0.367) | 0.369                    | 0.947    | 0.044     | 2.67    |
| MSLOM(X3478)   | 1.610(0.324) | 0.332                    | 0.952    | 0.040     | 2.42    |
| MSLOM(X5678)   | 1.609(0.296) | 0.316                    | 0.961    | 0.041     | 2.48    |
| MSLOM(X123478) | 1.605(0.384) | 0.387                    | 0.947    | 0.045     | 2.73    |
| MSLOM(all)     | 1.604(0.375) | 0.387                    | 0.955    | 0.046     | 2.79    |
| DR             | 2.683(0.504) | 0.229                    | 0.637    | 0.043     | 1.63    |

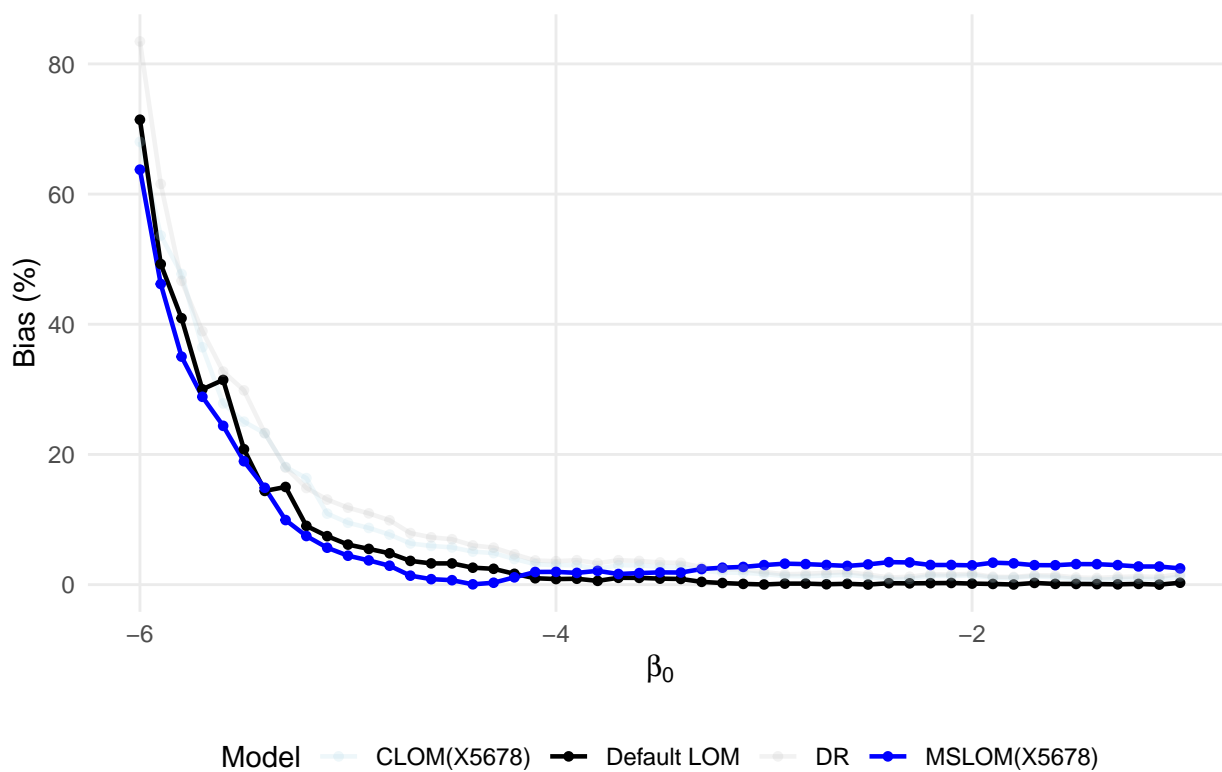
$RERI_{OR}^{true} = 2.64$ ; Empirical:  $RERI_{OR} = 1.650$ ,  $RERI_{RR} = 0.511$

## Graphs

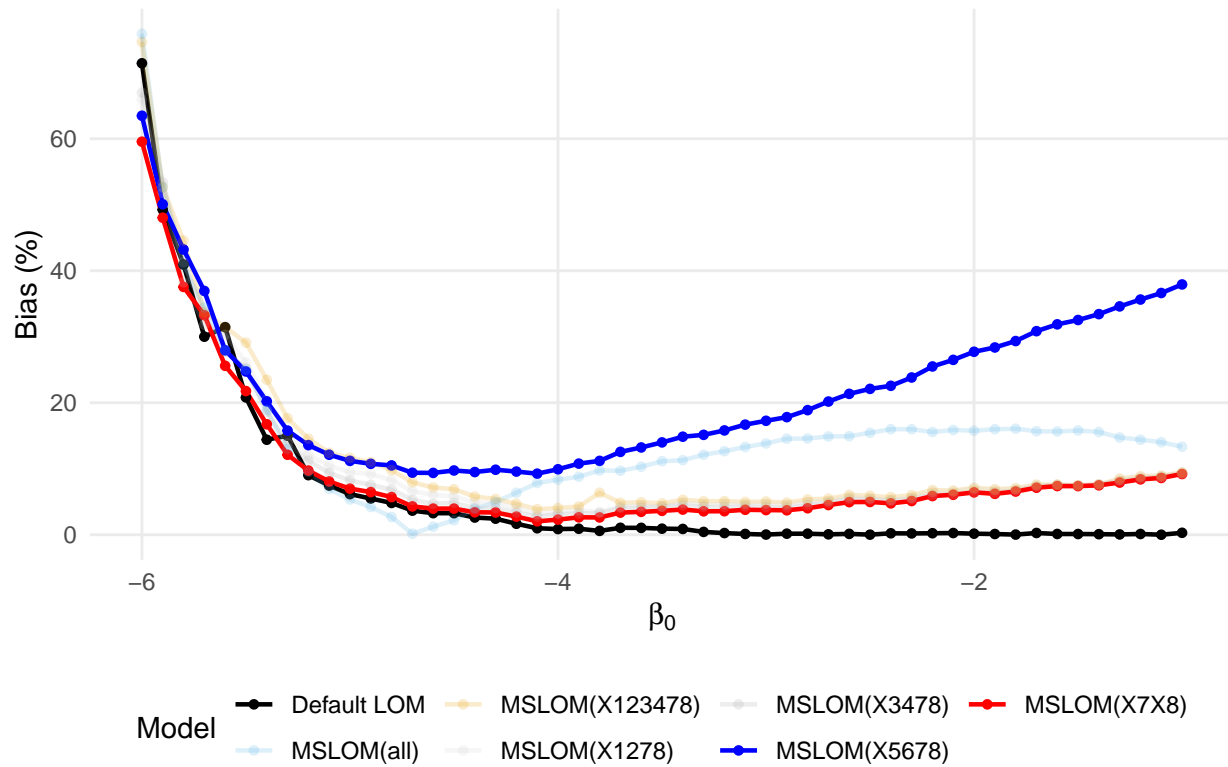
Bias of  $RER_{I_{OR}}$  across models, with weights using control data only



Bias of  $RER_{I_{OR}}$  across models, with weights using both data



Bias of  $RER_{OR}$  across MSLOM models, with weights using control data only



Bias of  $RER_{OR}$  across MSLOM models, with weights using both data

