Given the true data-generation model

$$\begin{aligned}\operatorname{logit}(p(Y=1 \mid G, E, \mathbf{X})) &= \beta_0 + \boldsymbol{\gamma}^\top \mathbf{x} + 0.3g + 0.4e + 0.8ge \\ &= \eta_{ge}(\mathbf{x})\end{aligned}$$

The true (marginal) RERI_{OR} are different with empirical ones as shown as following

True (Marginal) RERI_{OR} from the data generation process

Denote $p_{ge}(x) = p(Y=1|G=g, E=e, X=x)$, OR_{11} can be expressed as

$$\begin{aligned}\mathrm{OR}_{11} &= \frac{p_{11}(x)/(1-p_{11}(x))}{p_{00}(x)/(1-p_{00}(x))} \\ &= \frac{\exp\{\eta_{11}(\mathbf{x})\}}{\exp\{\eta_{00}(\mathbf{x})\}} \\ &= e^{0.3+0.4+0.8}\frac{\exp\{\eta_{00}(\mathbf{x})\}}{\exp\{\eta_{00}(\mathbf{x})\}} \\ &= e^{0.3+0.4+0.8}\end{aligned}$$

Similarly, we can have

$$\mathrm{OR}_{10} = e^{0.3}$$

$$\mathrm{OR}_{01} = e^{0.4}$$

Hence the (marginal) RERI_{OR} can be expressed as

$$\mathrm{RERI}_{OR} = e^{0.3+0.4+0.8} - e^{0.3} - e^{0.4} + 1 \approx 2.64$$

Empirical (crude) risks and odds from a simulation of 1 million

Define the marginal risk as

$$p^c_{ge} = P(Y=1|G=g, E=e)$$

which can be expressed as

$$\begin{aligned}p^c_{ge} &= \int_{\mathcal{X}} P(Y=1, X=x|G,E)dx \\ &= \int_{\mathcal{X}} P(Y=1|G,E,X=x)\, f_{X|G,E}(x)\, dx \\ &= \mathbb{E}_{X|G,E}[p(Y=1|G,E,X)] \\ &= \mathbb{E}_{X|G,E}\left[\frac{\exp\{\eta_{ge}(X)\}}{1+\exp\{\eta_{ge}(X)\}}\right]\end{aligned}$$

Consequently

$$\begin{aligned}\mathrm{odds}_{ge} &= \frac{p^c_{ge}}{1-p^c_{ge}} \\ &= \frac{\mathbb{E}_{X|ge}\left[\frac{\exp\{\eta_{ge}(X)\}}{1+\exp\{\eta_{ge}(X)\}}\right]}{\mathbb{E}_{X|ge}\left[\frac{1}{1+\exp\{\eta_{ge}(X)\}}\right]}\end{aligned}$$

For example, for $\mathrm{OR}_{11} = \mathrm{odds}_{11}/\mathrm{odds}_{00}$,

$$
\begin{aligned}
\mathrm{odds}_{11} &= \frac{\mathbb{E}_{X|11}\left[\frac{\exp\{\eta_{11}(X)\}}{1+\exp\{\eta_{11}(X)\}}\right]}{\mathbb{E}_{X|11}\left[\frac{1}{1+\exp\{\eta_{11}(X)\}}\right]} \\
&= e^{0.3+0.4+0.8}\frac{\mathbb{E}_{X|11}\left[\frac{\exp\{\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}{1+\exp\{1.5+\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}\right]}{\mathbb{E}_{X|11}\left[\frac{1}{1+\exp\{1.5+\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}\right]} \\
\mathrm{odds}_{00} &= \frac{\mathbb{E}_{X|00}\left[\frac{\exp\{\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}{1+\exp\{\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}\right]}{\mathbb{E}_{X|00}\left[\frac{1}{1+\exp\{\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}\right]}
\end{aligned}
$$

Clearly $\mathrm{odds}_{11}/\mathrm{odds}_{00} \neq \exp\{0.3+0.4+0.8\}$, and thence the empirical estimated $\widehat{RERI}_{OR}$ is not equal to 2.64 even in 1 million simulation.

However, when the outcome is getting rarer, then $odds = \frac{p^c_{ge}}{1-p^c_{ge}} \approx p^c_{ge}$. Therefore,

$$
\begin{aligned}
\mathrm{odds}_{11} &\approx \mathbb{E}_{X|11}\left[\frac{\exp\{\eta_{11}(X)\}}{1+\exp\{\eta_{11}(X)\}}\right] \\
&= e^{0.3+0.4+0.8}\mathbb{E}_{X|11}\left[\frac{\exp\{\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}{1+\exp\{1.5+\beta_0+\boldsymbol{\gamma}^\top\mathbf{x}\}}\right] \\
\mathrm{odds}_{00} &\approx \mathbb{E}_{X|00}\left[\frac{\exp\{\eta_{00}(X)\}}{1+\exp\{\eta_{00}(X)\}}\right]
\end{aligned}
$$