Testing different models for *weak* confounding effects case 2 (fitting 2000 times)

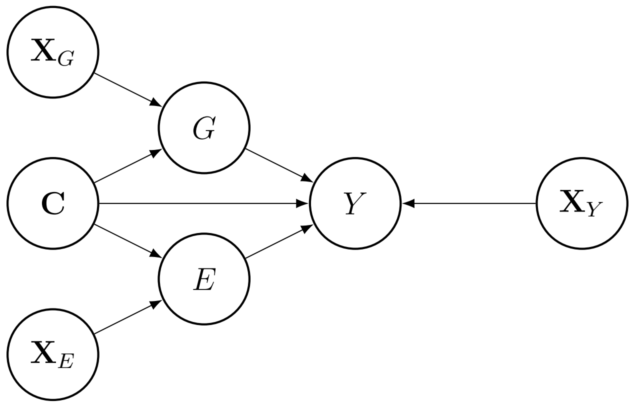


Figure 1: Directed Acyclic Graphs for causal interactions under two treatments G and E.

Let G and E denote the two binary treatments and Y be the binary outcome. Let $\mathbf{X}_G$, $\mathbf{X}_E$ be the set of instrumental variables for G and E, respectively. Let $\mathbf{X}_Y$ be the outcome-only covariates. Let $\mathbf{C}$ be the set of common confounders to both treatments.

Data generation

In this study, we will assume the outcome Y is binary and there are two treatments G and E. Let $\mathbf{X}_G = \{X_1, X_2\}$, $\mathbf{X}_E = \{X_3, X_4\}$, $\mathbf{X}_Y = \{X_5, X_6\}$ and $\mathbf{C} = \{X_7, X_8\}$. Suppose $\mathbf{X} = \{X_1, \ldots, X_8\}$ is generated to follow a multivariate normal distribution such that $\mathbf{X} \sim \mathcal{N}_8(\mathbf{0}, \mathbf{I}_8)$. To generate the treatment assignment probabilities, we firstly generate probabilities for each treatment individually:

$$\pi_{G=1} = P(G = 1 \mid \mathbf{Z}_G) = \frac{\exp\{f(\mathbf{Z}_G)\}}{1 + \exp\{f(\mathbf{Z}_G)\}}$$
$$\pi_{E=1} = P(E_i = 1 \mid \mathbf{Z}_E) = \frac{\exp\{f(\mathbf{Z}_E)\}}{1 + \exp\{f(\mathbf{Z}_E)\}}$$

where $f(\mathbf{Z}_G)$ and $f(\mathbf{Z}_E)$ are the function of $\mathbf{Z}_G$ and $\mathbf{Z}_E$ such that $\mathbf{Z}_G = \{X_1, X_2, X_7, X_8\}$ and $\mathbf{Z}_E = \{X_3, X_4, X_7, X_8\}$. We now suppose

$$f(\mathbf{Z}_G) = 0.8X_1 + 0.5X_2 + 0.3X_7 + 0.5X_8$$
$$f(\mathbf{X}_E) = -0.5X_3 + 0.2X_4 + 0.3X_7 + 0.5X_8$$

Note that here it is unnecessary to add the intercept as the expectation for each covariate is 0, making the marginal probabilities $\pi_{G=1}$ and $\pi_{E=1}$ are approximately 0.5. With the above setups, the probability of assignment for each combination of treatments can be computed as

$$\theta_{00} = p_{G=0,E=0} = (1 - \pi_{G=1})(1 - \pi_{E=1})$$
$$\theta_{10} = p_{G=1,E=0} = \pi_{G=1}(1 - \pi_{E=1})$$
$$\theta_{01} = p_{G=0,E=1} = (1 - \pi_{G=1})\pi_{E=1}$$
$$\theta_{11} = p_{G=1,E=1} = \pi_{G=1}\pi_{E=1}$$

For each observation, the probability of receiving each pair of treatment follows a multinomial distribution such that $T_i \mid \mathbf{X} \sim \text{Multinomial}(1, \theta_{00}, \theta_{10}, \theta_{01}, \theta_{11})$, where $T_i \in \{(0,0), (1,0), (0,1), (1,1)\}$. In addition, denote $\mathbf{Z}_Y = \{\mathbf{X}_Y, \mathbf{C}\} = \{X_5, X_6, X_7, X_8\}$, the outcome is generated as

$$\begin{aligned}\Pr(Y = 1 \mid \mathbf{Z}_Y, G, E) = \text{logit}^{-1}(\beta_0 + &\\ &0.8X_5 + 0.5X_6 - 0.05X_7 + 0.05X_8 \\ &+ 0.3G + 0.4E + 0.8GE)\end{aligned}$$

The true value of conditional RERI in terms of odds ratio can be computed as $\exp\{0.3 + 0.4 + 0.8\} - \exp\{0.3\} - \exp\{0.4\} + 1 = 2.64$.

Using control only

$\beta_0 = -6$: **prevalence** ≈ 0.01

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	5.813(17.581)	12.156	0.947	3.194	121.95
CLOM(X5X6)	5.962(18.369)	11.893	0.949	3.322	125.83
CLOM(X7X8)	6.077(18.766)	12.142	0.940	3.437	130.19
CLOM(X5678)	6.318(18.787)	12.742	0.943	3.678	139.32
MSLOM(X7X8)	6.053(18.607)	15.203	0.921	3.434	131.12
MSLOM(X1278)	7.078(24.627)	45.142	0.921	4.459	170.26
MSLOM(X3478)	5.993(17.724)	16.303	0.924	3.374	128.83
MSLOM(X5678)	6.362(20.548)	16.659	0.924	3.743	142.92
MSLOM(X123478)	6.864(28.239)	19.945	0.919	4.245	162.08
MSLOM(all)	6.655(21.117)	21.543	0.922	4.036	154.10
DR	6.580(21.808)	7.957	0.761	3.940	149.24

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.619$, $RERI_{RR} = 2.565$

$\beta_0 = -5.5$: **prevalence** ≈ 0.018

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	3.740(10.368)	4.380	0.951	1.162	45.07
CLOM(X5X6)	3.764(8.338)	3.376	0.950	1.124	42.58
CLOM(X7X8)	3.588(7.549)	3.537	0.941	0.948	35.91
CLOM(X5678)	3.791(7.462)	3.679	0.950	1.151	43.60
MSLOM(X7X8)	3.553(8.347)	3.313	0.940	0.975	37.82
MSLOM(X1278)	3.713(8.258)	3.613	0.934	1.135	44.03
MSLOM(X3478)	3.622(7.148)	3.280	0.943	1.044	40.50
MSLOM(X5678)	3.737(7.672)	3.282	0.941	1.159	44.96
MSLOM(X123478)	3.833(10.037)	3.880	0.939	1.255	48.68
MSLOM(all)	3.832(8.389)	3.902	0.941	1.254	48.64
DR	3.951(9.007)	2.225	0.731	1.311	49.66

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.578$, $RERI_{RR} = 2.493$

$\beta_0 = -5$**: prevalence** ≈ 0.03

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.787(2.340)	1.510	0.949	0.265	10.51
CLOM(X5X6)	3.069(3.733)	1.633	0.953	0.429	16.25
CLOM(X7X8)	2.784(2.686)	1.600	0.929	0.144	5.45
CLOM(X5678)	3.006(2.671)	1.711	0.951	0.366	13.86
MSLOM(X7X8)	2.817(2.706)	1.656	0.938	0.295	11.70
MSLOM(X1278)	2.895(3.159)	1.951	0.939	0.373	14.79
MSLOM(X3478)	2.838(3.057)	1.761	0.941	0.316	12.53
MSLOM(X5678)	2.922(2.922)	1.712	0.947	0.400	15.86
MSLOM(X123478)	2.921(3.271)	2.059	0.942	0.399	15.82
MSLOM(all)	3.042(3.690)	2.127	0.946	0.520	20.62
DR	3.065(3.024)	0.884	0.685	0.425	16.10

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.522$, $RERI_{RR} = 2.389$

$\beta_0 = -4.5$**: prevalence** ≈ 0.05

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.567(1.049)	1.005	0.958	0.118	4.82
CLOM(X5X6)	2.848(1.170)	1.110	0.964	0.208	7.88
CLOM(X7X8)	2.552(1.092)	1.049	0.930	0.088	3.33
CLOM(X5678)	2.836(1.219)	1.162	0.956	0.196	7.42
MSLOM(X7X8)	2.568(1.199)	1.116	0.952	0.119	4.86
MSLOM(X1278)	2.608(1.462)	1.301	0.944	0.159	6.49
MSLOM(X3478)	2.593(1.267)	1.181	0.949	0.144	5.88
MSLOM(X5678)	2.702(1.269)	1.169	0.959	0.253	10.33
MSLOM(X123478)	2.640(1.566)	1.381	0.942	0.191	7.80
MSLOM(all)	2.782(1.665)	1.450	0.945	0.333	13.60
DR	2.865(1.341)	0.574	0.667	0.225	8.52

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.449$, $RERI_{RR} = 2.247$

$\beta_0 = -4$**: prevalence** ≈ 0.08

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.408(0.753)	0.748	0.955	0.049	2.08
CLOM(X5X6)	2.768(0.861)	0.854	0.958	0.128	4.85
CLOM(X7X8)	2.382(0.782)	0.778	0.898	0.258	9.77
CLOM(X5678)	2.740(0.895)	0.891	0.955	0.100	3.79
MSLOM(X7X8)	2.387(0.840)	0.826	0.948	0.028	1.19
MSLOM(X1278)	2.420(1.040)	0.969	0.939	0.061	2.59
MSLOM(X3478)	2.408(0.893)	0.873	0.946	0.049	2.08
MSLOM(X5678)	2.561(0.910)	0.881	0.960	0.202	8.56
MSLOM(X123478)	2.443(1.115)	1.025	0.940	0.084	3.56
MSLOM(all)	2.624(1.215)	1.096	0.955	0.265	11.23
DR	2.758(0.970)	0.433	0.650	0.118	4.47

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.359$, $RERI_{RR} = 2.062$

$\beta_0 = -3$**: prevalence** ≈ 0.18

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.164(0.476)	0.470	0.963	0.016	0.74
CLOM(X5X6)	2.728(0.593)	0.586	0.961	0.088	3.33
CLOM(X7X8)	2.137(0.499)	0.487	0.742	0.503	19.05
CLOM(X5678)	2.693(0.618)	0.609	0.956	0.053	2.01
MSLOM(X7X8)	2.133(0.527)	0.517	0.951	0.015	0.70
MSLOM(X1278)	2.155(0.642)	0.611	0.943	0.007	0.33
MSLOM(X3478)	2.144(0.569)	0.546	0.946	0.004	0.19
MSLOM(X5678)	2.421(0.608)	0.581	0.952	0.273	12.71
MSLOM(X123478)	2.165(0.683)	0.645	0.940	0.017	0.79
MSLOM(all)	2.459(0.779)	0.726	0.959	0.311	14.48
DR	2.694(0.660)	0.289	0.624	0.054	2.05

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.148$, $RERI_{RR} = 1.576$

$\beta_0 = -2$**: prevalence** ≈ 0.4

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.973(0.353)	0.346	0.951	0.010	0.51
CLOM(X5X6)	2.717(0.482)	0.468	0.952	0.077	2.92
CLOM(X7X8)	1.948(0.365)	0.357	0.476	0.692	26.21
CLOM(X5678)	2.679(0.500)	0.484	0.947	0.039	1.48
MSLOM(X7X8)	1.950(0.388)	0.386	0.947	0.013	0.66
MSLOM(X1278)	1.958(0.466)	0.460	0.945	0.005	0.25
MSLOM(X3478)	1.954(0.406)	0.409	0.949	0.009	0.46
MSLOM(X5678)	2.374(0.504)	0.461	0.892	0.411	20.94
MSLOM(X123478)	1.959(0.485)	0.485	0.945	0.004	0.20
MSLOM(all)	2.391(0.627)	0.583	0.932	0.428	21.80
DR	2.690(0.542)	0.224	0.576	0.050	1.89

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.963$, $RERI_{RR} = 1.032$

$\beta_0 = -1$**: prevalence** ≈ 0.6

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.880(0.309)	0.312	0.954	0.001	0.05
CLOM(X5X6)	2.700(0.435)	0.437	0.954	0.060	2.27
CLOM(X7X8)	1.857(0.318)	0.320	0.319	0.783	29.66
CLOM(X5678)	2.663(0.447)	0.450	0.952	0.023	0.87
MSLOM(X7X8)	1.864(0.356)	0.356	0.942	0.015	0.80
MSLOM(X1278)	1.870(0.429)	0.428	0.945	0.009	0.48
MSLOM(X3478)	1.874(0.381)	0.379	0.944	0.005	0.27
MSLOM(X5678)	2.393(0.492)	0.445	0.829	0.514	27.35
MSLOM(X123478)	1.879(0.456)	0.453	0.947	0.000	0.00
MSLOM(all)	2.421(0.624)	0.571	0.885	0.542	28.85
DR	2.676(0.501)	0.198	0.576	0.036	1.36

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.879$, $RERI_{RR} = 0.573$

Using both control and case

$\beta_0 = -6$**: prevalence** ≈ 0.01

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	5.865(17.729)	12.204	0.947	3.246	123.94
CLOM(X5X6)	6.000(18.443)	11.927	0.949	3.360	127.27
CLOM(X7X8)	6.091(18.770)	12.149	0.940	3.451	130.72
CLOM(X5678)	6.316(18.783)	12.742	0.943	3.676	139.24
MSLOM(X7X8)	5.859(17.718)	11.317	0.923	3.240	123.71
MSLOM(X1278)	6.625(21.074)	33.662	0.918	4.006	152.96
MSLOM(X3478)	5.895(18.659)	30.086	0.923	3.276	125.09
MSLOM(X5678)	5.756(17.263)	8.831	0.923	3.137	119.78
MSLOM(X123478)	6.103(19.122)	15.832	0.922	3.484	133.03
MSLOM(all)	6.055(19.504)	14.509	0.923	3.436	131.20
DR	6.430(21.428)	7.788	0.766	3.790	143.56

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.619$, $RERI_{RR} = 2.565$

$\beta_0 = -5.5$**: prevalence** ≈ 0.018

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	3.740(10.368)	4.380	0.951	1.162	45.07
CLOM(X5X6)	3.764(8.338)	3.376	0.950	1.124	42.58
CLOM(X7X8)	3.588(7.549)	3.537	0.941	0.948	35.91
CLOM(X5678)	3.791(7.462)	3.679	0.950	1.151	43.60
MSLOM(X7X8)	3.681(8.792)	3.186	0.939	1.103	42.79
MSLOM(X1278)	3.691(8.296)	3.628	0.934	1.113	43.17
MSLOM(X3478)	3.650(7.804)	3.347	0.943	1.072	41.58
MSLOM(X5678)	3.637(7.786)	4.683	0.937	1.059	41.08
MSLOM(X123478)	3.748(8.214)	3.806	0.939	1.170	45.38
MSLOM(all)	3.751(8.272)	3.848	0.940	1.173	45.50
DR	3.961(9.109)	2.255	0.734	1.321	50.04

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.578$, $RERI_{RR} = 2.493$

$\beta_0 = -5$**: prevalence** ≈ 0.03

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.787(2.340)	1.510	0.949	0.265	10.51
CLOM(X5X6)	3.069(3.733)	1.633	0.953	0.429	16.25
CLOM(X7X8)	2.784(2.686)	1.600	0.929	0.144	5.45
CLOM(X5678)	3.006(2.671)	1.711	0.951	0.366	13.86
MSLOM(X7X8)	2.818(2.772)	1.657	0.938	0.296	11.74
MSLOM(X1278)	2.891(3.045)	1.939	0.939	0.369	14.63
MSLOM(X3478)	2.854(3.422)	1.750	0.939	0.332	13.16
MSLOM(X5678)	2.811(2.626)	1.652	0.941	0.289	11.46
MSLOM(X123478)	2.919(3.294)	2.055	0.942	0.397	15.74
MSLOM(all)	2.920(3.348)	2.044	0.943	0.398	15.78
DR	3.091(3.820)	0.931	0.688	0.451	17.08

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.522$, $RERI_{RR} = 2.389$

$\beta_0 = -4.5$**: prevalence** ≈ 0.05

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.567(1.049)	1.005	0.958	0.118	4.82
CLOM(X5X6)	2.848(1.170)	1.110	0.964	0.208	7.88
CLOM(X7X8)	2.552(1.092)	1.049	0.930	0.088	3.33
CLOM(X5678)	2.836(1.219)	1.162	0.956	0.196	7.42
MSLOM(X7X8)	2.567(1.197)	1.114	0.953	0.118	4.82
MSLOM(X1278)	2.606(1.453)	1.295	0.942	0.157	6.41
MSLOM(X3478)	2.591(1.262)	1.177	0.950	0.142	5.80
MSLOM(X5678)	2.563(1.177)	1.113	0.953	0.114	4.65
MSLOM(X123478)	2.637(1.551)	1.371	0.940	0.188	7.68
MSLOM(all)	2.633(1.533)	1.371	0.941	0.184	7.51
DR	2.869(1.343)	0.578	0.669	0.229	8.67

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.449$, $RERI_{RR} = 2.247$

$\beta_0 = -4$**: prevalence** ≈ 0.08

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.408(0.753)	0.748	0.955	0.049	2.08
CLOM(X5X6)	2.768(0.861)	0.854	0.958	0.128	4.85
CLOM(X7X8)	2.382(0.782)	0.778	0.898	0.258	9.77
CLOM(X5678)	2.740(0.895)	0.891	0.955	0.100	3.79
MSLOM(X7X8)	2.386(0.838)	0.824	0.948	0.027	1.14
MSLOM(X1278)	2.417(1.030)	0.962	0.938	0.058	2.46
MSLOM(X3478)	2.406(0.889)	0.869	0.946	0.047	1.99
MSLOM(X5678)	2.382(0.819)	0.823	0.951	0.023	0.97
MSLOM(X123478)	2.439(1.101)	1.016	0.941	0.080	3.39
MSLOM(all)	2.435(1.087)	1.016	0.942	0.076	3.22
DR	2.761(0.971)	0.438	0.658	0.121	4.58

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.359$, $RERI_{RR} = 2.062$

$\beta_0 = -3$**: prevalence** ≈ 0.18

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	2.164(0.476)	0.470	0.963	0.016	0.74
CLOM(X5X6)	2.728(0.593)	0.586	0.961	0.088	3.33
CLOM(X7X8)	2.137(0.499)	0.487	0.742	0.503	19.05
CLOM(X5678)	2.693(0.618)	0.609	0.956	0.053	2.01
MSLOM(X7X8)	2.131(0.524)	0.514	0.951	0.017	0.79
MSLOM(X1278)	2.151(0.632)	0.603	0.943	0.003	0.14
MSLOM(X3478)	2.141(0.564)	0.542	0.948	0.007	0.33
MSLOM(X5678)	2.127(0.509)	0.514	0.953	0.021	0.98
MSLOM(X123478)	2.160(0.670)	0.634	0.939	0.012	0.56
MSLOM(all)	2.156(0.651)	0.633	0.946	0.008	0.37
DR	2.697(0.658)	0.296	0.638	0.057	2.16

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 2.148$, $RERI_{RR} = 1.576$

$\beta_0 = -2$**: prevalence** ≈ 0.4

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.973(0.353)	0.346	0.951	0.010	0.51
CLOM(X5X6)	2.717(0.482)	0.468	0.952	0.077	2.92
CLOM(X7X8)	1.948(0.365)	0.357	0.476	0.692	26.21
CLOM(X5678)	2.679(0.500)	0.484	0.947	0.039	1.48
MSLOM(X7X8)	1.946(0.382)	0.381	0.947	0.017	0.87
MSLOM(X1278)	1.950(0.450)	0.447	0.947	0.013	0.66
MSLOM(X3478)	1.948(0.398)	0.401	0.948	0.015	0.76
MSLOM(X5678)	1.944(0.372)	0.381	0.953	0.019	0.97
MSLOM(X123478)	1.950(0.467)	0.469	0.944	0.013	0.66
MSLOM(all)	1.949(0.460)	0.469	0.952	0.014	0.71
DR	2.689(0.535)	0.236	0.604	0.049	1.86

Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.963$, $RERI_{RR} = 1.032$

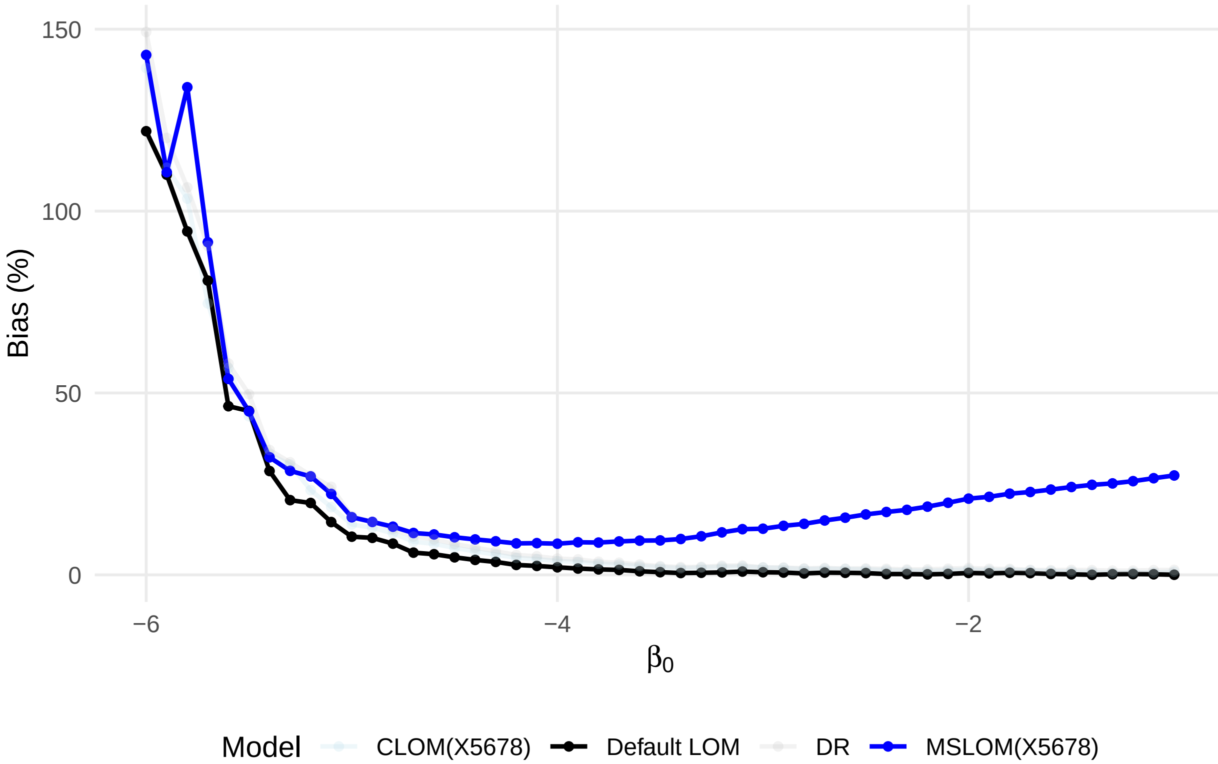
$\beta_0 = -1$**: prevalence** ≈ 0.6

Model	β_3	$\widehat{SE}_{\beta_3}$	coverage	Abs. Bias	Bias(%)
Default LOM	1.880(0.309)	0.312	0.954	0.001	0.05
CLOM(X5X6)	2.700(0.435)	0.437	0.954	0.060	2.27
CLOM(X7X8)	1.857(0.318)	0.320	0.319	0.783	29.66
CLOM(X5678)	2.663(0.447)	0.450	0.952	0.023	0.87
MSLOM(X7X8)	1.856(0.345)	0.346	0.943	0.023	1.22
MSLOM(X1278)	1.857(0.405)	0.405	0.945	0.022	1.17
MSLOM(X3478)	1.862(0.364)	0.364	0.942	0.017	0.90
MSLOM(X5678)	1.854(0.332)	0.346	0.951	0.025	1.33
MSLOM(X123478)	1.861(0.424)	0.425	0.945	0.018	0.96
MSLOM(all)	1.860(0.414)	0.425	0.948	0.019	1.01
DR	2.674(0.487)	0.221	0.639	0.034	1.29

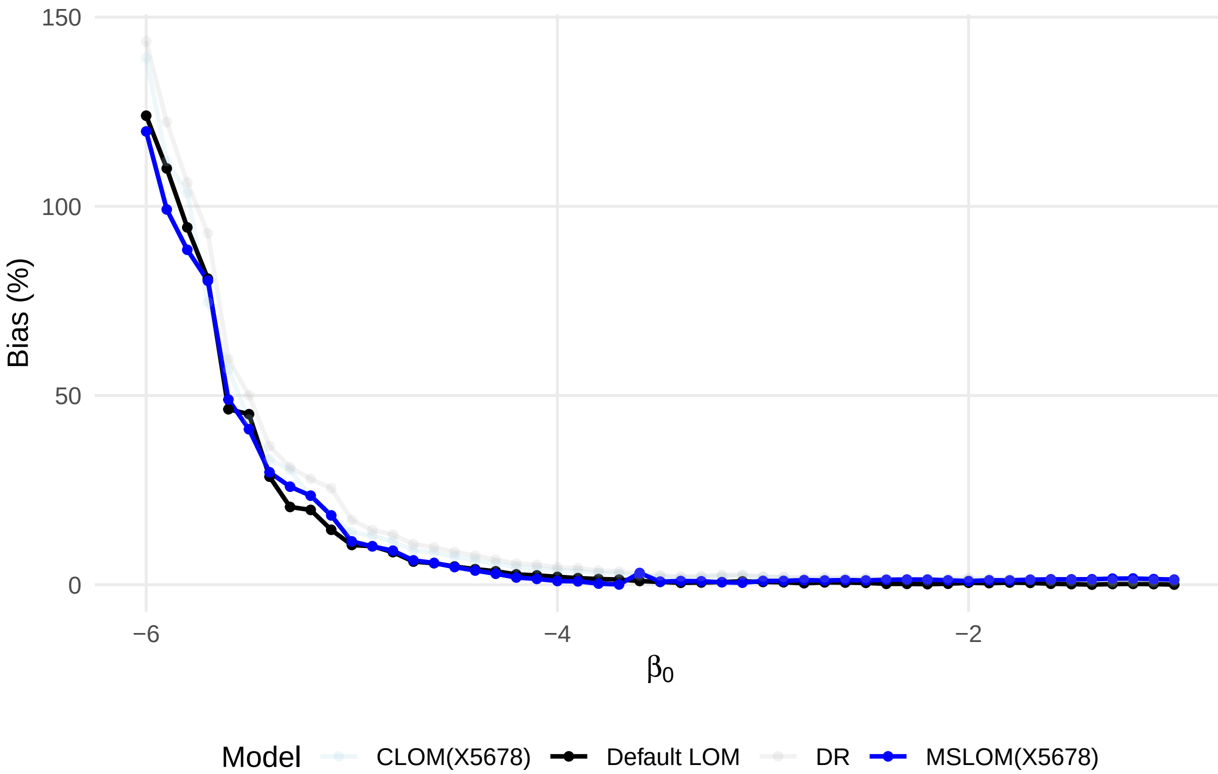
Conditional: $RERI_{OR} = 2.64$; Marginal: $RERI_{OR} = 1.879$, $RERI_{RR} = 0.573$

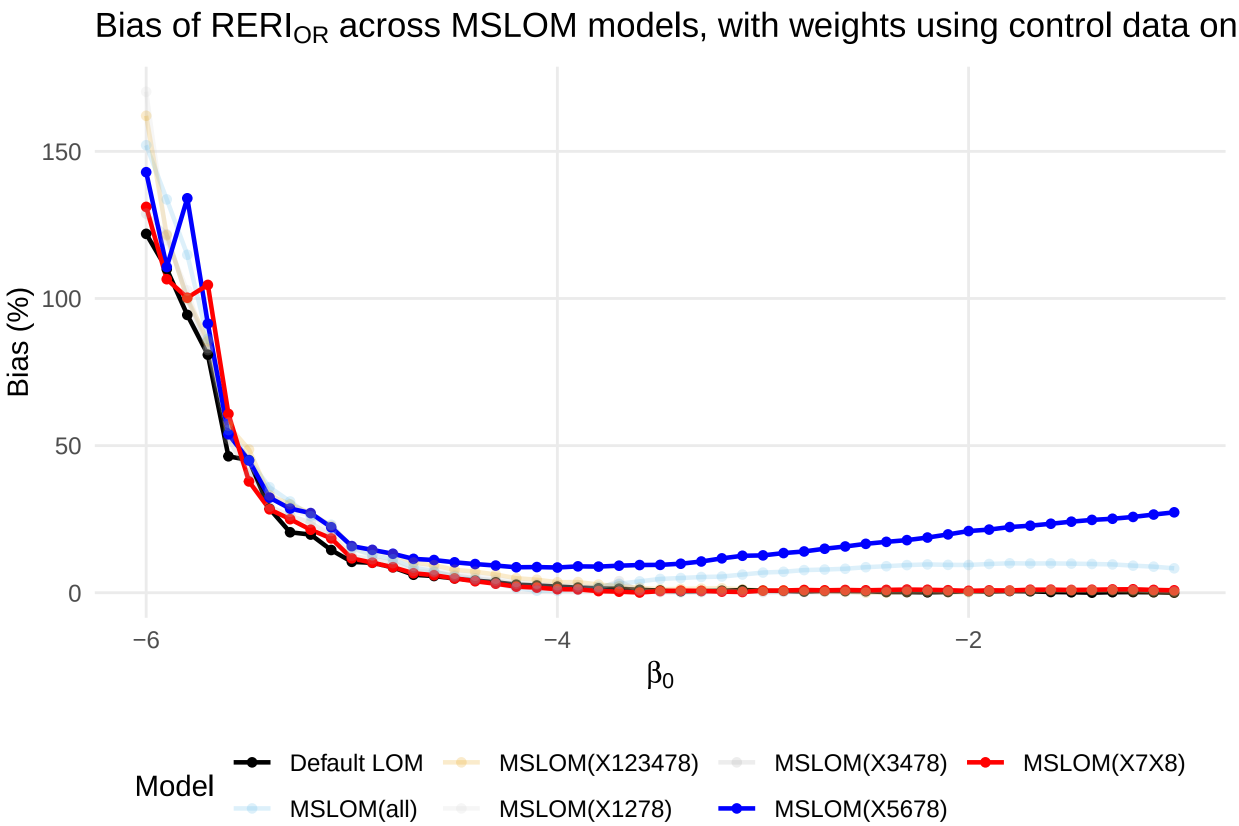
Graphs

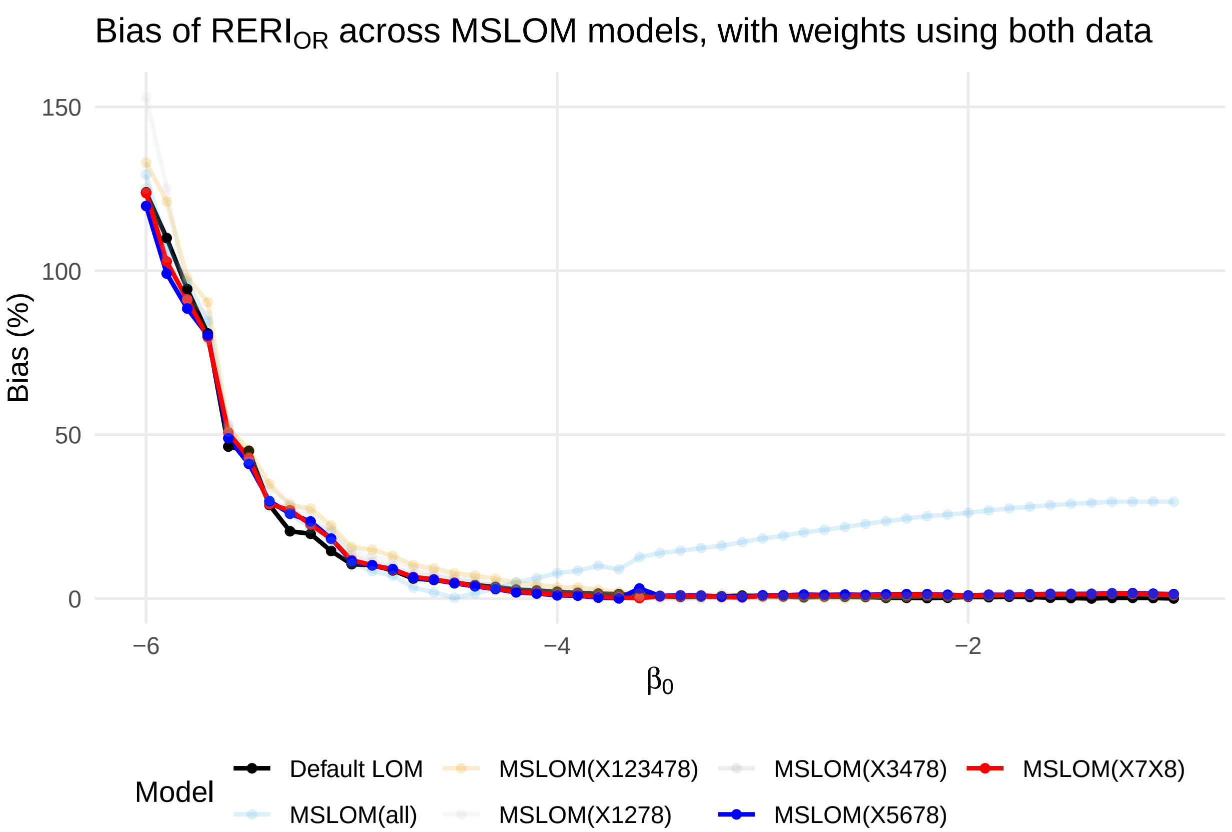
Bias of RERI_{OR} across models, with weights using control data only



Bias of RERI_{OR} across models, with weights using both data




Bias of RERIOR across MSLOM models, with weights using control data onl
Bias (%)
150
100
50
0
−6
−4
−2
β0
Model
Default LOM
MSLOM(X123478)
MSLOM(X3478)
MSLOM(X7X8)
MSLOM(all)
MSLOM(X1278)
MSLOM(X5678)


Bias of RERIOR across MSLOM models, with weights using both data
Bias (%)
150
100
50
0
−6
−4
−2
β0
Model
Default LOM
MSLOM(X123478)
MSLOM(X3478)
MSLOM(X7X8)
MSLOM(all)
MSLOM(X1278)
MSLOM(X5678)