Problem 25 (From the lecture)

Verify the following statements from the lecture:

- (1) The actions of S_n and a group G on $V^{\otimes n}$ as defined in the lecture commute.
- (2) The tensor coefficients in the irrep corresponding to the Young tableau

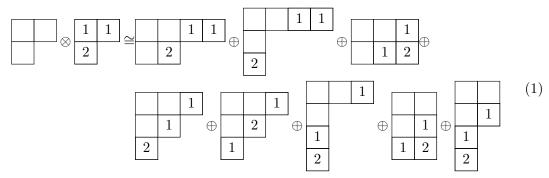
satisfy $a_{ijk} + a_{jki} + a_{kij} = 0$ and $a_{ijk} = -a_{jik}$.

(Optional) Problem 26 (Irreps of SU(N))

As discussed in the lecture, the Schur-Weyl duality dictates that distinct irreps of SU(N) are given by different Young diagrams with at most N rows. Using the Young diagrams, the decomposition of tensor product of irreps $T_1 \otimes T_2$ can be computed in a pictorial way as described below:

- (1) assign distinct labels to boxes in each row of T_2 , e.g. $\frac{1}{2}$
- (2) attach boxes of T_2 row by row to T_1 in all possible ways that result in another semistandard Young tableau.
- (3) after attaching each row, one should keep only those tableaux that when reading from right to left while going from top to bottom, the sequence of numbers attached should contain at least as many 1's as 2's, 2's as 3', etc at any point.

Below is an example:



Note that the tableau $\frac{1}{1}$ is not allowed because it violates condition (3).

(1) Perform the decomposition for the following tensor product for SU(3):



(2) The dimension of the irrep is given by $D = \prod_b \frac{(N+d(b))}{h(b)}$, where the multiplication goes over all boxes b in the diagram. h(b) is the hook length of box b, and d(b) defined in the following example diagram

0	1	2	3
-1	0	1	2
-2	-1	0	1

Check that the dimensions match in the decomposition you just performed. For more details consult textbooks, e.g. Jones, *Groups*, representations and physics, 2nd Ed., IoP 1998.