

Problem 16¹

Let $D \subset SU(2)$ be the subgroup of diagonal matrices. Note that $D \simeq U(1)$.

- (a) Compute explicitly its normalizer $N_{SU(2)}(D)$.
- (b) Compute the quotient group $N_{SU(2)}(D)/D$.
- (c) Show that conjugation by elements in $N_{SU(2)}(D)$ acts on elements of D by a permutation of the diagonal elements, and the permutation only depends on the projection to the quotient.
- (d) Show that there is no subgroup of $N_{SU(2)}(D)$ whose conjugation on D induces the permutation action.

Problem 17²

A group action of G on X can be viewed as a homomorphism $\phi: G \rightarrow S_X$

- (a) Show that the action is effective if and only if the homomorphism is injective.
- (b) Show that the subset of group elements that act ineffectively is a normal subgroup $H \triangleleft G$.
- (c) Show that there is an effective action of the quotient group G/H on X .

Problem 18³

Suppose G is finite and acts transitively on a finite set X with more than one point. Show that there is an element $g \in G$ with no fixed points on X .

¹p. 91 of [GM]

²p. 133 of [GM]

³p. 138 of [GM]