Problem 08

Construct a nontrivial homomorphism from the quaternion group to the Klein four group,

$$\phi : Q \to \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Show its kernel $\ker \phi$ and image $\mathrm{im}\phi$.

Problem 09

Show that the following diagram commutes if and only if $k_1 = k_2 \mod N$.

$$\begin{array}{ccc} \mathbb{Z}_N & \xrightarrow{m_{k_1}} & \mathbb{Z}_N \\ \downarrow \psi & & \downarrow \psi \\ \mu_N & \xrightarrow{p_{k_2}} & \mu_N \end{array}$$

Problem 10

Consider the linear action of $SU(2)$ on $\mathbb{C}^2$. Show that any linear equivariant map $T : \mathbb{C}^2 \to \mathbb{C}^2$ is of the form $T(\vec{z}) = \alpha\vec{z}$ for some $\alpha \in \mathbb{C}$.

Problem 11

What is the smallest symmetric group S_n that the dihedral group D_3 can be embedded? Construct the embedding and conclude that $D_3 \cong S_3$.

Problem 12

A permutation ϕ reverses the order of $\{1, 2, \ldots, n\}$ to $\{n, n-1, \ldots, 1\}$.

(1) Write down its cycle decomposition.

(2) Is it an even or odd permutation?

(3) Generate it using the generators $\sigma_i = (i\ i+1)$, where $1 \leq i < n$.