

Problem 12 (The complex conjugate representation ¹)

Consider two N -dimensional representations ϕ_1, ϕ_2 of $SU(N)$ and $U(N)$, where $\phi_1(u) = u$, and $\phi_2(u) = u^*$, show that ϕ_1, ϕ_2 are

- (1) equivalent for $SU(2)$,
- (2) inequivalent for $U(2)$,
- (3) inequivalent for $SU(N)$ ($N > 2$).

Problem 13

- (1) Let H be a subgroup of G and $[G : H] = 2$. Show that $H \triangleleft G$.
- (2) Let $Z(G)$ be the center of group G . Show that if $G/Z(G)$ is cyclic, then G is abelian.

Problem 14

Show that

- (1) $SL(n, \kappa) \triangleleft GL(n, \kappa)$, and
- (2) $A_n \triangleleft S_n$.

Problem 15

For $g_1, g_2 \in G$, define the *group commutator*

$$[g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}.$$

The commutator subgroup $[G, G]$ is the subgroup generated by all the commutators. Show that

- (1) $[G, G] \triangleleft G$, and
- (2) If $H \triangleleft G$, then G/H is abelian if and only if $[G, G]$ is a subgroup of H .

¹p. 79 of [GM]