SP500 Time-Series Forecasting
A Box-Jenkins and GARCH Approach

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- **Objective**: Forecast SP500 returns and volatility
- **Challenges**: Non-stationarity, volatility clustering, complex dependencies
- **Approach**:
  - Box-Jenkins (AR, MA, ARIMA) for returns
  - GARCH/ARCH for volatility
- **Applications**: Investment management, risk management, derivatives pricing

- **Period**: October 1, 2015 to October 30, 2025
- **Observations**: Over 2,000 daily observations
- **Target**: Daily log returns
- **Key Features**:
  - Stationary (d=0) - no differencing required
  - Fat-tailed distribution
  - Volatility clustering present
- **Variable Selection**: Elastic Net regularization selected key predictors

Methodology

Box-Jenkins Approach:

- Grid search: AR(p), MA(q), ARIMA($p, 0, q$)
- Order selection: AIC/BIC (in-sample)
- Model type selection: Out-of-sample RMSE
- Diagnostic checking: Ljung-Box, residual analysis

Volatility Models:

- GARCH(p, q) vs ARCH(q)
- Selection: BIC (captures volatility dynamics)
- Captures volatility clustering and persistence

Evaluation: Rolling-origin backtesting (80/20 split, hundreds of test folds)

Return Forecasting

Model	RMSE	Dir. Acc.
ARIMA(2,0,2)	**Lowest**	Lowest
AR(8)	Slightly higher	Higher
MA(2)	Slightly higher	**Highest**

Selected: ARIMA(2,0,2)

- Lowest RMSE (primary criterion)
- Magnitude accuracy > directional accuracy

Volatility Forecasting

Model	BIC
GARCH(1,1)	**Lowest**
ARCH(1)	Higher

Selected: GARCH(1,1)

- Lower BIC
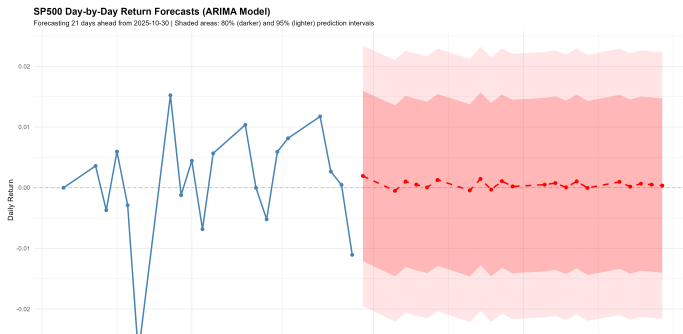- Time-varying volatility
- Captures clustering

Forecasts

Return Forecasts (21 days)

- Positive cumulative return
- Large majority positive days
- Bullish short-term outlook
- Wide prediction intervals

Volatility Forecasts

- Above historical mean
- Time-varying (increasing trend)
- Elevated risk ahead
- Risk management implications

SP500 Day-by-Day Return Forecasts (ARIMA Model)
Forecasting 21 days ahead from 2025-10-30 | Shaded areas: 80% (darker) and 95% (lighter) prediction intervals

Conclusions

Key Findings:

1. **ARIMA(2,0,2)** best for returns (lowest RMSE)
2. **GARCH(1,1)** best for volatility (realistic time-varying forecasts)
3. **Bullish outlook** but **increasing volatility** - balance opportunity with risk

Practical Implications:

- Investment: Maintain equity exposure, but account for uncertainty
- Risk: Reduce position sizes, increase hedging, adjust VaR upward

Note: Forecasts are probabilistic. Models assume stationarity and may break down during regime changes.

Thank you for your attention!