Erasmus School of Economics

MOOC Econometrics

Training Exercise 6.2

Questions

If y_t is a stationary process with mean μ, then the k-th order autocovariance is defined as $\gamma_k = E((y_t - \mu)(y_{t-k} - \mu))$. In particular, the variance is $\gamma_0 = E(y_t - \mu)^2$. The k-th order autocorrelation is defined as $\rho_k = \text{cov}(y_t, y_{t-k})/\text{var}(y_t)$. In this exercise, you are asked to derive the autocorrelations of the AR(1) model

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t$$

In your derivations, you may use that $E(\varepsilon_t) = 0$ and that ε_t is uncorrelated with the values of y_s for all $s < t$.

(a) Show that the mean of the AR(1) model is equal to $\mu = \alpha/(1 - \beta)$.

(b) Define $z_t = y_t - \mu$. Show that $z_t = \beta z_{t-1} + \varepsilon_t$ and that $\text{var}(z_t) = \sigma^2/(1 - \beta^2)$.

(c) Use the idea of part (b) to show that the autocorrelations of y_t are equal to $\rho_k = \beta^k$.

(d) Argue that stationarity requires that $-1 < \beta < 1$.

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