Erasmus School of Economics

MOOC Econometrics

Training Exercise 3.4

Questions

(a) Consider the Chow break test, with

$$\begin{aligned} y_1 &= X_1\beta_1 + \varepsilon_1, \\ y_2 &= X_2\beta_2 + \varepsilon_2, \end{aligned}$$

which can be combined to give

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}.$$

The Chow break test has H_0: $\beta_1 = \beta_2$, such that

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \beta + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix},$$

which is tested against the above unrestricted set-up.

Prove that the vector of residuals from the unrestricted model $e_U = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ and thus $e_U'e_U = e_1'e_1 + e_2'e_2 \equiv S_1 + S_2$. Show that this is equivalent to calculating the sum of squared residuals for a regression for only the first sample, thus a regression of y_1 on X_1, plus the sum of squared residuals for a regression for only the second sample, thus a regression of y_2 on X_2. Hint: As an intermediate step, show that $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} (X_1'X_1)^{-1}X_1'y_1 \\ (X_2'X_2)^{-1}X_2'y_2 \end{pmatrix}$. You may use that $\begin{pmatrix} X_1'X_1 & 0 \\ 0 & X_2'X_2 \end{pmatrix}^{-1} = \begin{pmatrix} (X_1'X_1)^{-1} & 0 \\ 0 & (X_2'X_2)^{-1} \end{pmatrix}$.

(b) Now consider the Chow forecast test. First write the Chow forecast model

$$y_i = x_i'\beta + \sum_{j=n_1+1}^{n_1+n_2} \gamma_j D_{ji} + \varepsilon_i$$

in matrix form similar to the matrix form given for the Chow break test of question (a). Then provide the F-test for γ in this form and show that it is equivalent to the expression $F = \frac{(S_0-S_1)/n_2}{S_1/(n_1-k)}$ of the lecture.