Erasmus School of Economics

MOOC Econometrics

Test Exercise 5

Notes:

- See website for how to submit your answers and how feedback is organized

Goals and skills being used:

- Get experience with the mathematical structure of the logit model
- Get experience with the interpretation of parameters of the logit model

Questions

Consider again the application in lecture 5.5, where we have analyzed response to a direct mailing using the following logit specification

$$\Pr[\text{resp}_i = 1] = \frac{\exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}$$

for $i = 1, \ldots, 925$. The maximum likelihood estimates of the parameters are given by

Variable	Coefficient	Std. Error	t-value	p-value
Intercept	-2.488	0.890	-2.796	0.005
Male	0.954	0.158	6.029	0.000
Active	0.914	0.185	4.945	0.000
Age	0.070	0.036	1.964	0.050
$(\text{Age}/10)^2$	-0.069	0.034	-2.015	0.044

(a) Show that

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{age}_i} + \frac{\partial \Pr[\text{resp}_i = 0]}{\partial \text{age}_i} = 0.$$

(b) Assume that you recode the dependent variable as follows: $\text{resp}_i^{\text{new}} = -\text{resp}_i + 1$. Hence, positive response is now defined to be equal to zero and negative response to be equal to 1. Use the odds ratio to show that this transformation implies that the sign of all parameters change.

(c) Consider again the odds ratio positive response versus negative response:

$$\frac{\Pr[\text{resp}_i = 1]}{\Pr[\text{resp}_i = 0]} = \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2).$$

During lecture 5.5 you have seen that this odds ratio obtains its maximum value for age equal to 50 years for males as well as females. Suppose now that you want to extend the logit model and allow that this age value is possibly different for males than for females. Discuss how you can extend the logit specification.