a)

As given by the question:

$b_R = (X_1'X_1)^{-1}X_1'y$ (1)

$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$ (2)

replacing y in (1) with (2) results:

$b_R = \left(X_1'X_1\right)^{-1}X_1'(X_1\beta_1 + X_2\beta_2 + \varepsilon) = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\beta_2 + (X_1'X_1)^{-1}X_1'\varepsilon$

as $(X_1'X_1)^{-1}X_1'X_2 = P$, and $E\left(\left(X_1'X_1\right)^{-1}X_1'\varepsilon\right) = \left(X_1'X_1\right)^{-1}X_1'E(\varepsilon)$, given the assumption that $E(\varepsilon) = 0$, we get $E(b_R) = \beta_1 + P\beta_2$

b)

$Var(b_R) = Var(\left(X_1'X_1\right)^{-1}X_1'\varepsilon)$

let $(X_1'X_1)^{-1}X_1' = A$

$Var(b_R) = A\,Var(\varepsilon)A' = A\,\sigma^2 IA' = AA'\sigma^2$

$AA' = (X_1'X_1)^{-1}X_1'X_1(X_1'X_1)^{-1} = (X_1'X_1)^{-1}$, therefore,

$Var(b_R) = \sigma^2(X_1'X_1)^{-1}$

c)

$y = X_1b_1 + X_2b_2 + e$ (3)

replace y in (1) with (3), we get

$b_R = \left(X_1'X_1\right)^{-1}X_1'(X_1b_1 + X_2b_2 + e) = b_1 + Pb_2 + (X_1'X_1)^{-1}X_1'e$

compare model $y = Xb + e_f$ with model $y = X_1b_1 + X_2b_2 + e$, it can be seen that

$e_f = e$

therefore, $X'e_f = 0 \Rightarrow X'e = 0 \Rightarrow X_1'e = 0$

Because $X'e_f = X'(y - Xb) = X'y - X'X\left(X'X\right)^{-1}X'y = 0$

Therefore, $(X_1'X_1)^{-1}X_1'e = 0$ and $b_R = b_1 + Pb_2$

d)

$P = (X_1'X_1)^{-1}X_1'X_2$, note that for model $y = Xb + e$, we estimate β as $b = (X'X)^{-1}X'y$. It can be seen that if we replace X with X_1, y with X_2, for i_{th} column in P, it is the same as the coefficients by regressing the i_{th} column in X_2, i.e., i_{th} column in P can be obtained by regressing $X_1^{'}(\text{constant, Female})$ on i_{th} column in X_2 respectively (Age, Educ, Parttime).

e)

According to d), we get

$$P = \begin{pmatrix} 40.05 & 2.26 & 0.20 \\ -0.11 & -0.49 & 0.25 \end{pmatrix}$$

f)

$P = \begin{pmatrix} 40.05 & 2.26 & 0.20 \\ -0.11 & -0.49 & 0.25 \end{pmatrix}$, $b_2 = \begin{pmatrix} 0.031 \\ 0.233 \\ -0.365 \end{pmatrix}$, $b_1 = \begin{pmatrix} 3.05 \\ -0.04 \end{pmatrix}$ therefore

$$Pb_2 = \begin{pmatrix} 1.70 \\ -0.21 \end{pmatrix}, b_1 + Pb_2 = \begin{pmatrix} 1.70 + 3.05 \\ -0.21 - 0.04 \end{pmatrix} = \begin{pmatrix} 4.75 \\ -0.25 \end{pmatrix}$$

which is almost identical to $b_R = \begin{pmatrix} 4.73 \\ -0.25 \end{pmatrix}$ regarding the numerical values.