Erasmus
School of
Economics

# MOOC Econometrics

## Training Exercise P.2

## Questions

1. For a standard normal distributed variable $y$, the following rules of thumb apply:

$$P(-1 \leq y \leq 1) \approx 0.68$$
$$P(-2 \leq y \leq 2) \approx 0.95.$$

   Transform these rules of thumb for a random variable $x \sim N(\mu, \sigma)$.

2. Let $z \sim \chi^2(n)$. Derive the variance of $z$.

3. Let $z_1$ and $z_2$ be two independent $\chi^2$ random variables with degrees of freedom $k_1$ and $k_2$. Show that $z_1 + z_2 \sim \chi^2(k_1 + k_2)$.

4. Let $x$ be an $(n \times 1)$ random vector that follows the multivariate normal distribution with mean vector $\mu$ and covariance matrix $\Sigma$. Let $L$ be an invertible matrix such that $LL' = \Sigma$.
   (a) What is the distribution of $y = L^{-1}(x - \mu)$.
   (b) Show that $z = (x - \mu)'\Sigma^{-1}(x - \mu)$ follows the $\chi^2(n)$ distribution.

5. Let $y$ be a $(n \times 1)$ random vector, $y \sim N(0, I)$. Let $z$ be a random variable, $z \sim \chi^2(k)$, independent from $y$. The variable $x$ is defined by $x = \frac{1}{\sqrt{z/k}} y$. Derive the distribution of $x'x/n$.