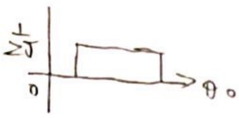
STA 601 HW10

Yi Mi

ym154

1. Hoff 10.1

① $\theta_0, \theta_1 > \delta$

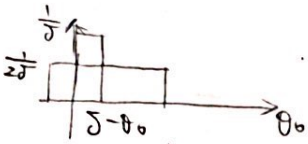


$J(\theta_1|\theta_0) = \frac{1}{2\delta}$

$J(\theta_0|\theta_1) = \frac{1}{2\delta}$

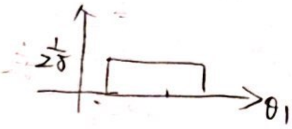
$\Rightarrow J(\theta_1|\theta_0) = J(\theta_0|\theta_1)$

② $\theta_0 \leq \delta < \theta_1$



$\theta_1 > \delta > \delta - \theta_0$

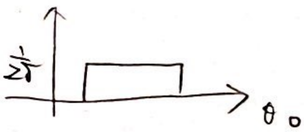
$J(\theta_1|\theta_0) = \frac{1}{2\delta}$



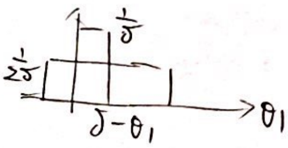
$J(\theta_0|\theta_1) = \frac{1}{2\delta}$

$\Rightarrow J(\theta_1|\theta_0) = J(\theta_0|\theta_1)$

③ $\theta_1 \leq \delta < \theta_0$



$J(\theta_1|\theta_0) = \frac{1}{2\delta}$

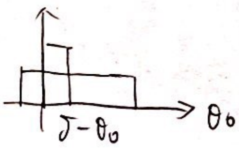


$\theta_0 > \delta > \delta - \theta_1$

$J(\theta_0|\theta_1) = \frac{1}{2\delta}$

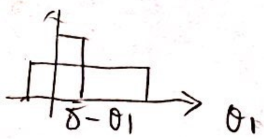
$\Rightarrow J(\theta_1|\theta_0) = J(\theta_0|\theta_1)$

④ $\theta_1, \theta_0 < \delta$ and $\theta_1 + \theta_0 \leq \delta$



$\theta_1 \leq \delta - \theta_0$

$J(\theta_1|\theta_0) = \frac{1}{\delta}$

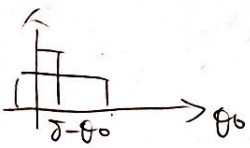


$\theta_0 \leq \delta - \theta_1$

$J(\theta_0|\theta_1) = \frac{1}{\delta}$

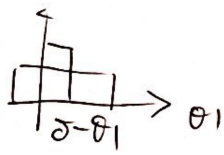
$\Rightarrow J(\theta_1|\theta_0) = J(\theta_0|\theta_1)$

⑤ $\theta_1, \theta_0 < \delta$ and $\theta_1 + \theta_0 > \delta$



$\theta_1 > \delta - \theta_0$

$J(\theta_1|\theta_0) = \frac{1}{2\delta}$



$\theta_0 > \delta - \theta_1$

$J(\theta_0|\theta_1) = \frac{1}{2\delta}$

$\Rightarrow J(\theta_1|\theta_0) = J(\theta_0|\theta_1)$

Therefore, the proposal distribution is symmetric.

STA 601 HW10
Yi Mi
ym154

2. Consider the following sampling model,

$$y_1 \cdots y_n | \theta \sim p(y | \theta_1, \theta_2)$$

and prior,

$$\theta_1 \sim g(\theta_1), \quad \theta_2 \sim h(\theta_2)$$

where,

$$\theta_1, \theta_2 \in R$$

Write down the acceptance probability for MH.

① Based on Full conditional: $J(\theta_1^* | \theta_1^{(s)}, \theta_2^{(s)}) = p(\theta_1 | y_1 \cdots y_n, \theta_2^{(s)})$

$$r = \min\left(1, \frac{p(\theta_1^* | y, \theta_2^{(s)})}{p(\theta_1^{(s)} | y, \theta_2^{(s)})} \times \frac{J(\theta_1^{(s)} | \theta_1^*, \theta_2^{(s)})}{J(\theta_1^* | \theta_1^{(s)}, \theta_2^{(s)})}\right)$$

$$= \min\left(1, \frac{p(\theta_1^* | y, \theta_2^{(s)})}{p(\theta_1^{(s)} | y, \theta_2^{(s)})} \times \frac{p(\theta_1^{(s)} | y, \theta_2^{(s)})}{p(\theta_1^* | y, \theta_2^{(s)})}\right)$$

$$= \min(1, 1)$$

$$= 1$$

② Based on Prior: $J(\theta_1^* | \theta_1^{(s)}, \theta_2^{(s)}) = g(\theta_1)$

$$r = \min\left(1, \frac{p(\theta_1^* | y, \theta_2^{(s)})}{p(\theta_1^{(s)} | y, \theta_2^{(s)})} \times \frac{J(\theta_1^{(s)} | \theta_1^*, \theta_2^{(s)})}{J(\theta_1^* | \theta_1^{(s)}, \theta_2^{(s)})}\right)$$

$$= \min\left(1, \frac{p(y | \theta_1^*, \theta_2^{(s)})\, g(\theta_1^*)}{p(y | \theta_1^{(s)}, \theta_2^{(s)})\, g(\theta_1^{(s)})} \times \frac{g(\theta_1^{(s)})}{g(\theta_1^*)}\right)$$

$$= \min\left(1, \frac{p(y | \theta_1^*, \theta_2^{(s)})}{p(y | \theta_1^{(s)}, \theta_2^{(s)})}\right)$$

$$= \min\left(1, \frac{\prod_{i=1}^n p(y_i | \theta_1^*, \theta_2^{(s)})}{\prod_{i=1}^n p(y_i | \theta_1^{(s)}, \theta_2^{(s)})}\right)$$

③ Based on Random walk: $J(\theta_1^* | \theta_1^{(s)}, \theta_2^{(s)}) = \text{Normal}(\theta_1^{(s)}, \delta^2)$

$$r = \min\left(1, \frac{p(y | \theta_1^*, \theta_2^{(s)})\, g(\theta_1^*)}{p(y | \theta_1^{(s)}, \theta_2^{(s)})\, g(\theta_1^{(s)})} \times \frac{\frac{1}{\sqrt{2\pi\delta^2}} \exp(-\frac{1}{2}(\theta_1^{(s)} - \theta_1^*)^2)}{\frac{1}{\sqrt{2\pi\delta^2}} \exp(-\frac{1}{2}(\theta_1^* - \theta_1^{(s)})^2)}\right)$$

$$= \min\left(1, \frac{p(y | \theta_1^*, \theta_2^{(s)})\, g(\theta_1^*)}{p(y | \theta_1^{(s)}, \theta_2^{(s)})\, g(\theta_1^{(s)})}\right)$$

$$= \min\left(1, \frac{g(\theta_1^*) \prod_{i=1}^n p(y_i | \theta_1^*, \theta_2^{(s)})}{g(\theta_1^{(s)}) \prod_{i=1}^n p(y_i | \theta_1^{(s)}, \theta_2^{(s)})}\right)$$