STA 5?2 HW9 Due 04/08/2020. Yi Mi. ym154.

1.

(a) Under H, $Y \sim N(0,1)$

the acc region is $A(0) = \{ Z : z_{\alpha(1-w)} < Z < z_{1-\alpha w} \}$

thus, type I error rate is

$$Pr(|\mu| \notin a \mid H)$$
$$= 1 - Pr(|\mu| \in a \mid H)$$
$$= 1 - Pr(z_{\alpha(1-w)} < Z < z_{1-\alpha w})$$
$$= 1 - [Pr(Z < z_{1-\alpha w}) - Pr(Z < z_{\alpha(1-w)})]$$
$$= 1 - [(1-\alpha w) - \alpha(1-w)]$$
$$= \alpha$$

(b) power is $Pr(Y \notin A_0 \mid \mu)$

$$= 1 - Pr(Y \in A_0 \mid \mu)$$
$$= 1 - Pr(z_{\alpha(1-w)} < Y < z_{1-\alpha w} \mid \mu)$$
$$= 1 - Pr(z_{\alpha(1-w)} - \mu < Y - \mu < z_{1-\alpha w} - \mu \mid \mu)$$

$\because Y \sim N(\mu, 1) \rightarrow Y - \mu \sim N(0,1)$

$$= 1 - [Pr(Y-\mu < z_{1-\alpha w} - \mu) - Pr(Y-\mu < z_{\alpha(1-w)} - \mu)]$$
$$= 1 - [\Phi(z_{1-\alpha w} - \mu) - \Phi(z_{\alpha(1-w)} - \mu)]$$

the plot is in appendix ($\alpha = 0.05$)

when I wanna make a hyp test with a symmetric double tails, I would use $w = 1/2$, otherwise I would use $1/4$

2.

(a) $F_\theta = \int_0^y p_\theta \, dy = \int_0^y e^{-y/\theta}/\theta \, dy = 1 - e^{-y/\theta}$

(b) $\Pr(\text{reject } H \mid H) = \Pr(y > b \mid H)$

$= 1 - \Pr(y \leq b \mid H)$

$= 1 - F_\theta(b \mid H)$

$= 1 - (1 - e^{-b/\theta})$

$= e^{-b/\theta}$

the prob of rejection $\leq \alpha$ for each $\theta < \theta_0$

$\Rightarrow \alpha \geq \sup_{\theta < \theta_0} \Pr(y > b \mid \theta) = \sup_{\theta < \theta_0} e^{-b/\theta} = e^{-b/\theta_0}$

$b \geq -\theta_0 \log \alpha$

Power $= e^{\theta_0/\theta \cdot \log \alpha}$

the plot is in appendix $(\alpha = 0.05,\ \theta_0 = 1)$

(c) $C(y) = -\frac{y}{\log \alpha}$

$\Pr(\theta \in (C(y), \infty) \mid \theta)$

$= \Pr(\theta > -\frac{y}{\log \alpha} \mid \theta)$

$= \Pr(y < -\theta \log \alpha \mid \theta)$

$= 1 - \alpha$

3.

(a) the usual two-sample t-statistic (with equal var) is

$$t(\underset{\sim}{y}) = \frac{|\bar{y}_A - \bar{y}_B|}{s\sqrt{1/n_A + 1/n_B}}$$

The p-value is the probability (under H_0) of observing a value of the test statistic the same as or more extreme than what was actually observed.

Under the continuous distribution, the p-value has a uniform(0,1) distribution.

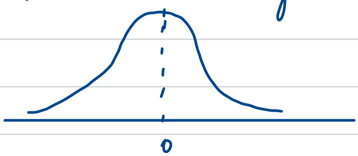
So the smallest p-value is 0.

(b) For permutation test,

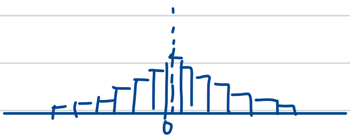
$$\text{p-value} = P_0(T > t_{obs}) = \frac{1}{N!}\sum_{j=1}^{N} I(T_j > t_{obs})$$

Because the tobs is one of the permutations statistics, it will be among those found within permutation distribution. Therefore, the smallest p-value under the most extreme situation is $1/N!$

(c) Normal-theory test



Randomization test



For normal theory-test, p-value = 0.0036.

For randomization test, p-value $\to$ 0

the distribution of Randomization test would be more flat than that of normal-theory test, since in the formal one we have limited candidates values.

4.

(a) We know that $S^2 \stackrel{d}{=} \sigma^2 \chi^2/(n-1)$ with dif $= n-1$

thus $$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

let $$t(\underset{\sim}{y}) = \frac{(n-1)S^2}{\sigma_0^2}$$

and Acc region $= \{ y : \chi^2_{1-\alpha/2} < t(\underset{\sim}{y}) < \chi^2_{\alpha/2} \}$

this is a level-α hypothetis test for $H: \sigma^2 = \sigma_0^2$

(b) $$\chi^2_{1-\alpha/2} < \frac{(n-1)S^2}{\sigma_0^2} < \chi^2_{\alpha/2}$$

the $1-\alpha$ CI for σ^2 is $\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}} , \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}} \right)$

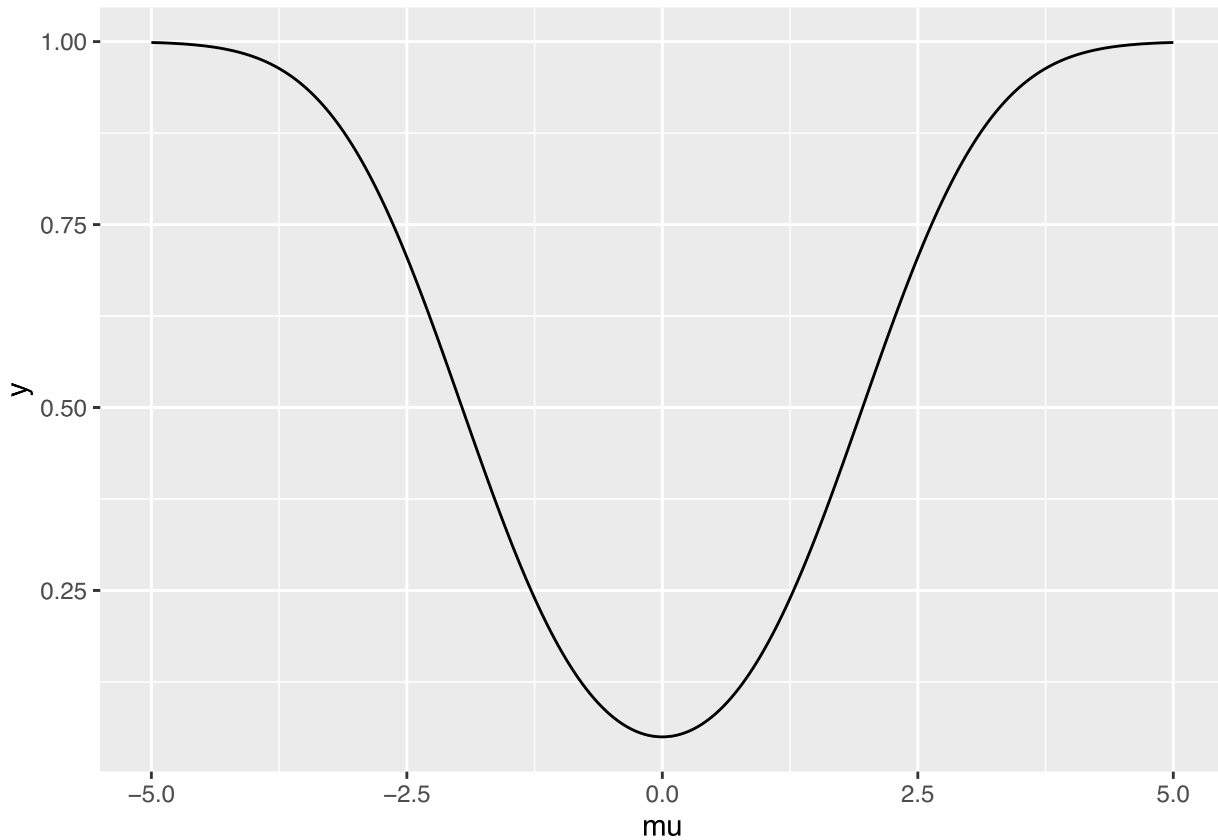
STA 532 Homework 9 - Appendix

Yi Mi

1.(b)

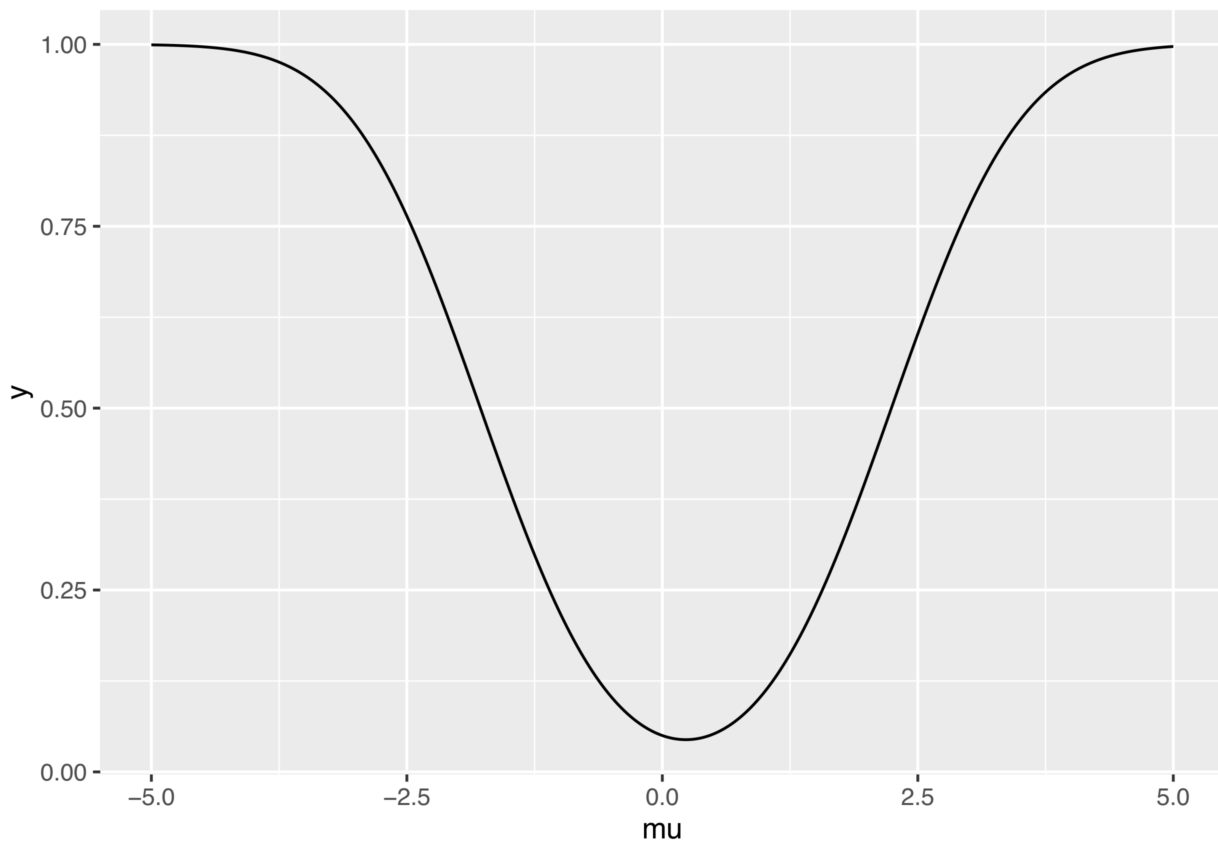
Let $\alpha = 0.05$. *And* $w = 1/2$.

```
mu = seq(-5, 5, 0.01)
#fa = qnorm(1 - a/2)*sqrt(a)
w = 0.5
y = 1-(pnorm(qnorm(1-0.05*w)-mu) - pnorm(qnorm(0.05 - 0.05*w)-mu))
df = data.frame(y,mu)
ggplot(df, aes(x=mu,y=y))+geom_line()
```



Let $\alpha = 0.05$. *And* $w = 1/4$.

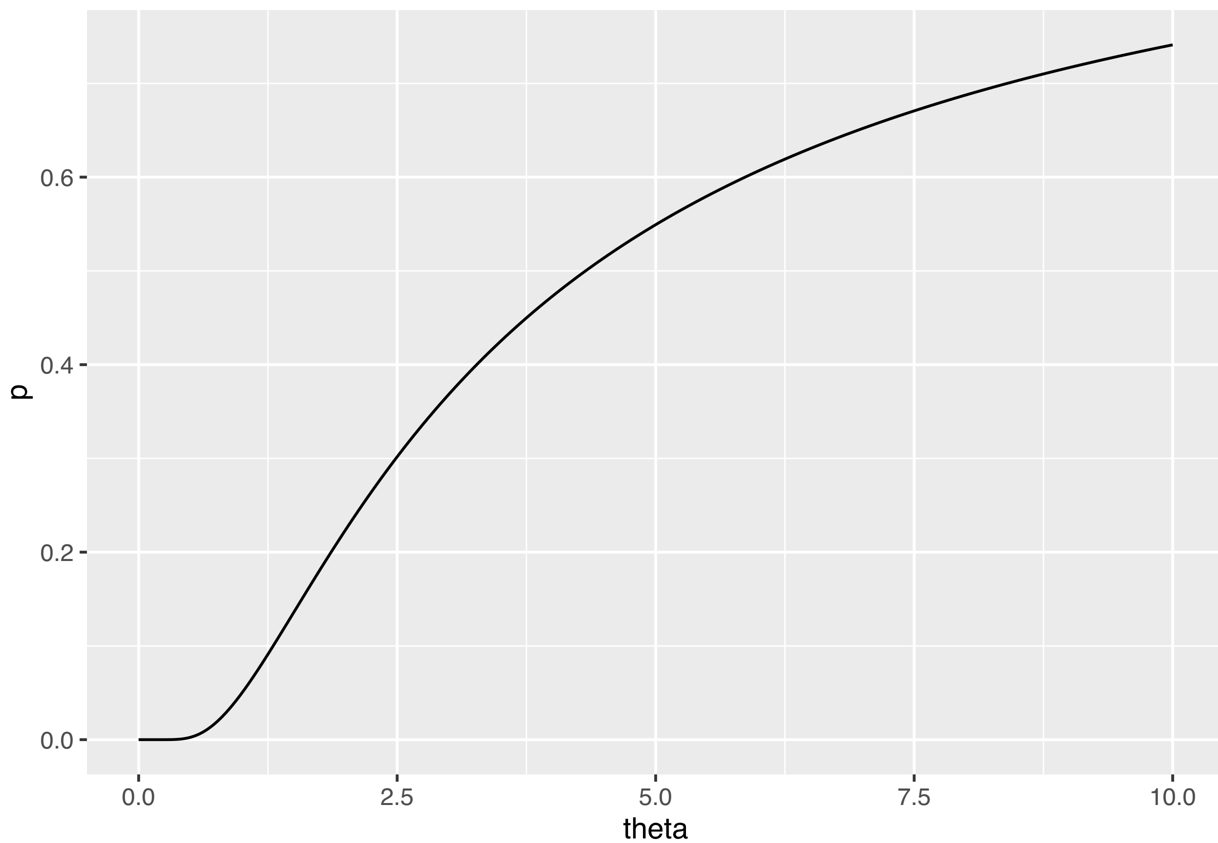
```
mu = seq(-5, 5, 0.01)
#fa = qnorm(1 - a/2)*sqrt(a)
w = 0.25
y = 1-(pnorm(qnorm(1-0.05*w)-mu) - pnorm(qnorm(0.05 - 0.05*w)-mu))
df = data.frame(y,mu)
ggplot(df, aes(x=mu,y=y))+geom_line()
```



2.(b)

Let $\alpha = 0.05$ *and* $\theta_0 = 1$.

```
theta = seq(0, 10, 0.01)
p = exp(log(0.05)/theta)
df = data.frame(p,theta)
ggplot(df, aes(x=theta,y=p))+geom_line()
```


0.6
0.4
0.2
0.0
p
0.0
2.5
5.0
7.5
10.0
theta