# How a Virtual Math Team Structured its Problem Solving

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**Abstract:** To develop a theory of small-group interaction in CSCL settings, we need an approach to analyzing the structure of computer-mediated discourse. Conversation Analysis examines informal face-to-face talk in terms of a fine structure of adjacency pairs, but needs to be adapted to online textual interaction and extended to analyze longer sequences built on adjacency pairs. This paper presents a case study of students solving a math problem in an online chat environment. It shows that their problem-solving discourse consists of a sequence of exchanges, each built on a base adjacency pair and each contributing a move in their collaborative problem-solving process.

## Structuring Group Cognition at Multiple Levels

A year ago in my opening keynote talk (Stahl, 2009a) at the International Conference of Computers in Education (ICCE 2009) in Hong Kong, I claimed that the discourse of group cognition (Stahl, 2006) has a hierarchical structure, typically including the following levels, as illustrated with a particular case study from the Virtual Math Teams (VMT) Project (Stahl, 2009c):

a.  **Group event**: E.g., Team B's participation in the VMT Spring Fest 2006.
b.  **Temporal session**: Session 4 of Team B on the afternoon of May 18, 2006.
c.  **Conversational topic**: Determining the number of sticks in a diamond pattern (lines 1734 to 1833 of the chat log of Session 4).
d.  **Discourse move**: A stage in the sequence of moves to accomplish discussing the conversational topic (e.g., lines 1767-1770—see Logs 1-10 below).
e.  **Adjacency pair**: The base interaction involving two or three utterances, which drives a discourse move (lines 1767 and 1769).
f.  **Textual utterance**: A text chat posting by an individual participant, which may contribute to an adjacency pair (line 1767).
g.  **Indexical reference**: An element of a textual utterance that points to a relevant resource. In VMT, actions and objects in the shared whiteboard are often referenced in the chat. Mathematical content and other resources from the joint problem space and from shared past experience are also brought into the discourse by explicit or implicit reference in a chat posting.

The multi-layered structure corresponds to the multiplicity of constraints imposed on small-group discourse—from the character of the life-world and of culture (which mediate macro-structure) to the semantic, syntactic and pragmatic rules of language (which govern the fine structure of utterances). A theory of group cognition must concern itself primarily with the analysis of mid-level phenomena—such as how small groups accomplish collaborative problem solving and other conversational topics.

The study of mid-level group-cognition phenomena is a realm of analysis that is currently underdeveloped in the research literature. For instance, many CSCL studies focus on coding individual (micro-level) utterances or assessing learning outcomes (macro-level), without analyzing the group processes (mid-level). Similarly, Conversation Analysis (CA) centers on micro-level adjacency pairs while socio-cultural Discourse Analysis is concerned with macro-level identity and power, without characterizing the interaction patterns that build such macro phenomena out of microelements. Understanding these mid-level phenomena is crucial to analyzing collaborative learning, for it is this level that largely mediates between the interpretations of individuals and the socio-cultural factors of communities.

The analysis in this paper illustrates the applicability of the notion of a 'long sequence' as vaguely suggested by both Sacks (1962/1995, II p. 354) and Schegloff (2007, pp. 12, 213). A longer sequence consists of a coherent series of shorter sequences built on adjacency pairs. This multi-layered sequential structure will be adapted in this paper from the informal face-to-face talk-in-interaction of CA to the essentially different, but analogous, context of groupware-supported communication and group cognition, such as the text chat of VMT. I will show how a small group of students collaborating online constructed a coherent long sequence, through which they solved the problem that they had posed for themselves. Methodologically, it is important to note that the definition of the long sequence—like that of the other levels of structure listed above—is oriented to by the discourse of the students and is not simply a construct of the researcher.

## An Analytic Method

Recently, I have been trying to apply the CA perspective and techniques in a systematic way to the analysis of VMT chat logs. Schegloff's (2007) book on *Sequence Organization in Interaction* represents the culmination of decades of CA analysis. As indicated by its subtitle, it provides a useful primer in CA. My goal is to transform CA to apply to online chat and to extend it to analyze the larger scale interactions of group cognition.

Schegloff's presentation makes clear the central role of the adjacency pair as the primary unit of sequence construction according to CA. An adjacency pair is composed of two conversational speaking turns by two different people, with an interactional order, such as a question followed by an answer to the question. The simple two-turn pair can be extended with secondary adjacency pairs that precede, are inserted between or follow up on the base pair, recursively. This yields "extensive stretches of talk which nonetheless must be understood as built on the armature of a single adjacency pair, and therefore needing to be understood as extensions of it" (p. 12).

These "extensive stretches of talk" are still focused on a single interaction of meaning making, and not a larger cognitive achievement like problem solving. However, both Sacks and Schegloff provide vague suggestions about the analysis of longer sequences. These suggestions have not been extensively developed within CA. This paper is an attempt to explore them in an online text-chat context.



Figure 1. VMT interface with stair-step pattern of horizontal and vertical sticks.

As I have frequently argued (e.g., Stahl, 2006; 2009c; Stahl, Koschmann & Suthers, 2006), I believe that adapting CA to computer-mediated communication offers the best prospects for analysis of interaction in groupware—i.e., for a theory of small groups appropriate to CSCL. I designed and directed the Virtual Math Teams (VMT) Project from 2003 to the present in order to produce a corpus of data that could be analyzed in as much detail as needed to determine the structure of group cognition, that is, of collaborative knowledge building through interaction at the group unit of analysis.

In looking at the VMT data corpus, the VMT research team has clearly seen the differences between online text chat and verbal conversation. The system of turn taking so important in CA (Sacks, Schegloff & Jefferson, 1974) does not apply in chat. Instead, chat participants engage in 'reading's work' (Zemel & Çakir, 2009), in which "readers connect objects through reading's work to create a 'thread of meaning' from the various postings available for inspection" (p. 274f). The first and second parts of an adjacency pair may no longer be literally temporally adjacent to each other, but they still occur as mutually relevant, anticipatory and responsive. The task of reading's work—for both participants and analysts—is to reconstruct the threading of the adjacency pair response structure (Stahl, 2009b).

We have tried to explore the larger sequential structure of problem-solving chat by using the CA notion of openings and closings (Schegloff & Sacks, 1973). VMT researchers looked at several math chats from 2004, which used a simple chat tool from AOL. We coded and statistically analyzed the fine-structure threading of adjacency pairs (Çakir, Xhafa & Zhou, 2009). In addition, we defined long sequences based on when opening and closing adjacency pairs achieved changes in topic (Zemel, Xhafa & Çakir, 2009). These long sequences were graphed to show their roles in constituting the chat sessions, but their internal sequential structures were not investigated.

My colleagues and I have subsequently conducted numerous case studies from the VMT corpus. We have been particularly drawn to the records of Team B and Team C in the VMT Spring Fest 2006. These were particularly rich sessions of online mathematical knowledge building because these teams of students met for over four hours together and engaged in detailed explorations of interesting mathematical phenomena. However, partially because of the richness of the interactions, it was often hard for analysts to determine a clear structure to the student interactions. Despite access to everything that the students knew about each other and about the group interaction, it proved hard to unambiguously specify the group-cognition processes at work (Medina, Suthers & Vatrapu, 2009; Stahl, 2009b; Stahl, Zemel & Koschmann, 2009).

Therefore, in the following case study, I have selected a segment of Team B's final session, in which the structure of the interaction seems to be clearer. The interaction is simpler than in earlier segments partially because two of the four people in the chat room leave. Thus, the response structure is more direct and less interrupted. In addition, the students have already been together for over four hours, so they know how to interact in the software environment and with each other. Furthermore, they set themselves a straightforward and well-understood mathematical task. The analysis of this relatively simple segment of VMT interaction can then provide a model for subsequently looking at other data and seeing if it may follow similar patterns.

## The Case Study

Three anonymous students (Aznx, Bwang, Quicksilver) from US high schools met online as Team B of the VMT Spring Fest 2006 contest to compete to be "the most collaborative virtual math team." They met for four
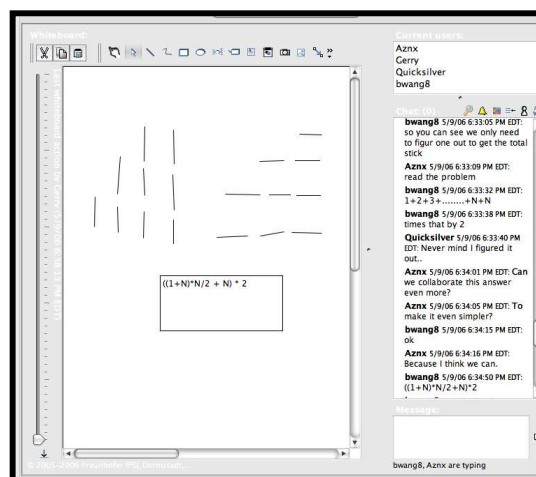
hour-long sessions during a two-week period in May 2006. A facilitator (Gerry) was present in the chat room to help with technical issues, but not to instruct in mathematics.

In their first session, they solved a given problem, finding a mathematical formula for the growth pattern of the number of squares and the number of sticks making up a stair-step arrangement of squares. They determined the number of sticks by drawing just the horizontal sticks together and then just the vertical ones (see Figure 1). They noticed that both the horizontals and the verticals formed the same pattern of **1 + 2 + 3 + … + n + n** sticks at the **n**th stage of the growth pattern. They then applied the well-known Gaussian formula for the sum of consecutive integers, added the extra **n**, and multiplied by **2** to account for both the horizontal and vertical sets of sticks.

In the second session, they explored problems that they came up with themselves, related to the stair-step problem, including 3-D pyramids. Here they ran into problems drawing and analyzing 3-D structures. However, they managed to approach the problem from a number of perspectives, including decomposing the structure into horizontal and vertical sticks. In the third session, Team B was attracted to a diamond-shaped variation of the stair-step figure, as explored by Team C in the Spring Fest. They tried to understand how the other team had derived its solution. They counted the number of squares by simplifying the problem through filling in the four corners surrounding the diamond to make a large square; the corners turned out to follow the stair-step pattern from their original problem. In the fourth session, they discovered that the other team's formula for the number of sticks was wrong. In the following, we join them an hour and 17 minutes into the fourth session, when one of the students as well as the facilitator had to leave.

## Problem-Solving Moves

In this section of the paper, the interaction is analyzed as a sequence of moves in the problem-solving interaction between Bwang and Aznx, the two remaining students. Each move is seen to include a base adjacency pair (in **bold** face), which provides the central interaction of the move and accomplishes the focal problem-solving activity. The captions of log segments indicate the aim of the move, according to the analysis.

In line 1734 (see Log 1), Bwang states that the team is close to being able to solve the problem of the number of sticks in the **n**th stage of the diamond pattern, suggesting that they might stay and finish it up. Note that this is the end of the last of the scheduled four sessions for the contest, despite some arrangements underway to allow the team to continue to meet. Aznx responds in line 1736, indicating—and implicitly endorsing the suggestion—that the team could indeed continue to work on the current topic. This opens the topic for the group. Quicksilver apologetically stresses that he must leave immediately. He just wants to know the location of the new chat room that the facilitator is setting up for the team to continue its math explorations on a future date. The facilitator supplies this information and everyone says goodbye to Quicksilver.

Aznx expresses uncertainty about how to proceed now that Quicksilver has gone and the facilitator has arranged things for the future. Line 1749 (see Log 2) questions whether he and Bwang need to go as well. Bwang then reiterates his suggestion that they could stay and finish solving the problem. He argues that it should not take much longer. Bwang directly asks Aznx if he wants to solve the problem now. Aznx agrees by responding to Bwang's question in the affirmative. This effects a decision by the pair of students to start working on the problem right away. Bwang continues to argue for starting on the problem now—posting line 1754 just 3 seconds after Aznx' agreement, probably just sending what he had already typed before reading Aznx' response. Bwang then notes the response.

Log 1. Open a Topic.

| LINE | TIME | AUTHOR | TEXT OF CHAT POSTING |
|------|------|--------|----------------------|
| **1734** | **08.17.20** | **bwang8** | **i think we are very close to solving the problem here** |
| 1735 | 08.17.35 | Quicksilver | Oh great...I have to leave |
| **1736** | **08.17.39** | **Aznx** | **We can solve on that topic.** |
| 1737 | 08.17.42 | Quicksilver | Sorry guys |
| 1738 | 08.17.45 | bwang8 | oh |
| 1739 | 08.17.46 | Aznx | It shouldn't take much time. |
| 1740 | 08.17.47 | bwang8 | ok |
| 1741 | 08.17.50 | Aznx | k, bye Quicksilver |
| 1742 | 08.17.52 | Quicksilver | Just tell me the name of the room |
| 1743 | 08.17.52 | bwang8 | bye |
| 1744 | 08.18.14 | Gerry | The new room is in the lobby under Open Rooms |
| 1745 | 08.18.44 | Gerry | It is under The Grid World. It has your names on it |
| 1746 | 08.18.49 | Quicksilver | [leaves the room] |
| 1747 | 08.19.00 | Aznx | Alright found it. |
| 1748 | 08.19.04 | Aznx | Thanks. |

Log 2. Decide to Start.

| 1749 | 08.19.12 | Aznx | I guess we should leave then. |
|------|----------|------|-------------------------------|
| **1750** | **08.19.34** | **bwang8** | **well do you want to solve the problem** |
| 1751 | 08.19.36 | bwang8 | i mean |
| 1752 | 08.19.39 | bwang8 | we are close |
| **1753** | **08.19.48** | **Aznx** | **Alright.** |
| 1754 | 08.19.51 | bwang8 | i don't want to wait til tomorrow |
| 1755 | 08.19.53 | bwang8 | ok |

Log 3. Pick an Approach.

| 1756 | 08.19.55 | Aznx | How do you want to approach it? |
|---|---|---|---|
| 1757 | 08.20.14 | bwang8 | 1st level have 1*4 |
| 1758 | 08.20.20 | Gerry | You can put something on the wiki to summarize what you found today |
| 1759 | 08.20.29 | bwang8 | 2st level have (1+3)*4 |
| 1760 | 08.20.32 | Aznx | bwang you put it. |
| 1761 | 08.20.35 | Aznx | for the wiki |
| 1762 | 08.20.37 | bwang8 | ok |
| 1763 | 08.20.42 | Aznx | we actually did quite a lot today |
| 1764 | 08.20.53 | bwang8 | 3rd level have (1+3+5)*4 |
| 1765 | 08.21.05 | bwang8 | 4th level have (1+3+5+7)*4 |
| 1766 | 08.21.10 | Gerry | This is a nice way to solve it |

Log 4. Identify the Pattern.

| 1767 | 08.21.12 | Aznx | So it's a pattern of +2s? |
|---|---|---|---|
| 1768 | 08.21.15 | Aznx | Ah ha! |
| 1769 | 08.21.15 | bwang8 | yes |
| 1770 | 08.21.20 | Aznx | There's the pattern! |

Log 5. Seek the Equation.

| 1771 | 08.21.39 | bwang8 | now we have to find a equation that describe that pattern |
|---|---|---|---|
| 1772 | 08.21.49 | Aznx | Hold on. |
| 1773 | 08.21.51 | Aznx | I know it. |
| 1774 | 08.21.57 | bwang8 | what is it |
| 1775 | 08.21.58 | Aznx | But I'm trying to remember it. =P |
| 1776 | 08.22.04 | Aznx | and explain it as well. |
| 1777 | 08.22.17 | Aznx | try and think of it |
| 1778 | 08.22.53 | Gerry | Maybe Quicksilver can come back here tomorrow or next week to finish it with you |
| 1779 | 08.23.01 | Gerry | I have to go now |
| 1780 | 08.23.05 | Gerry | Bye! |
| 1781 | 08.23.06 | bwang8 | ok |
| 1782 | 08.23.07 | bwang8 | bye |
| 1783 | 08.23.23 | Gerry | [leaves the room] |
| 1784 | 08.23.29 | bwang8 | ok |
| 1785 | 08.23.32 | bwang8 | so |
| 1786 | 08.23.37 | bwang8 | i think it is this |
| 1787 | 08.23.53 | Aznx | ok |
| 1788 | 08.23.55 | Aznx | i found it |
| 1789 | 08.24.00 | Aznx | n^2 |
| 1790 | 08.24.01 | bwang8 | (2*n)*n/2 |
| 1791 | 08.24.09 | Aznx | or (n/2)^2 |

Once a decision has been made to solve the problem, the question of how to approach the problem is raised in line 1756. Bwang immediately lays out his approach in lines 1757, 1759, 1764 and 1765 of log 3. The approach is the same as they used in the first session: visualize just the vertical or just the horizontal sticks. The two sets follow the same pattern. In fact, the diamond is also symmetric left/right and top/bottom, so the vertical sticks can be divided left/right into two identical sets, which can then be divided top/bottom. This produces four sets of sticks, each having rows of **1, 3, 5, 7, …** sticks, up to **(2n-1)** for the **n**[th] stage of the diamond pattern. Interspersed with this defining of the approach is a parting reminder from the facilitator, before he logs out, to summarize the team's work on the Spring Fest wiki for other teams to view.

Aznx has previously been oriented toward finding patterns of growth in the mathematical objects the group has been exploring. Often, someone will create a graphical representation of the object in such a way that it makes the pattern visible. Aznx will then formulate a textual description of the pattern. Then the group will work on a symbolic representation to capture the pattern in a mathematical formula. (See (Çakır, Zemel & Stahl, 2009) for an analysis of the intertwining of graphical/visual, textual/narrative and symbolic/mathematical modes of interaction within the work of Team C.) In log 4, line 1767, Aznx describes the pattern as involving adding numbers that successively increase by **2**. The number of sticks in a given stage of the diamond shape is a sum of numbers that start at **1** and increase successively by **2**. When going from one stage to the next, one simply adds another number to this sum that is **2** more than the highest previous one. Aznx presented his description as a question and Bwang affirmed it at the same time as Aznx posted line 1768. Aznx then emphasized that they had identified the pattern

In log 5, Bwang indicates that the next step in their work is to "find an equation that describes the pattern." Aznx asks Bwang to let him state the equation, implicitly agreeing that this is the next step by trying to produce the equation. In line 1774, Bwang asks Aznx to state the equation and Aznx expresses difficulty in formulating an adequate and accountable answer. After a half minute of silence with still no formulation from Aznx, the facilitator suggests that Aznx and Bwang might want to wait until a future time when the whole group can work together to finish the problem. The facilitator then says goodbye and leaves the room. After more than a minute since Aznx posted anything, Bwang starts to preface the presentation of his own formulation. Eventually, Aznx joins back in. Simultaneously, Aznx and Bwang post their formulae. For Aznx, it is either $n^2$ or $(n/2)^2$. For Bwang, it is $2n(n/2)$. Aznx has not given any indication of how he got his proposed formulae. Bwang's formula suggests the use of Gauss' summation, which the students have used repeatedly in the past. According to this summation of an arithmetic sequence of integers, the result is the sum of the first and last member of the sequence times half the number of members. For a sequence of **n** members, **1 + 3 + 5 + … + (2n-1)**, the sum would be **[1 + (2n-1)]*(n/2)**. Adding the **1** and the **-1**, yields Bwang's

Log 6. Negotiate the Solution.

| 1792 | 08.24.14 | Aznx | I'm simplifying |
|------|----------|------|-----------------|
| 1793 | 08.24.30 | Aznx | if u simplify urs |
| **1794** | **08.24.35** | **Aznx** | **its n^2** |
| 1795 | 08.24.59 | Aznx | bwang |
| 1796 | 08.25.01 | Aznx | you there? |
| **1797** | **08.25.03** | **bwang8** | **so that's wrong** |
| 1798 | 08.25.07 | bwang8 | yeah |
| 1799 | 08.25.08 | bwang8 | i am here |

Log 7. Check Cases.

| 1800 | 08.25.11 | Aznx | so |
|------|----------|------|-----------------|
| 1801 | 08.25.13 | Aznx | the formula |
| **1802** | **08.25.22** | **Aznx** | **would be 4n^2?** |
| 1803 | 08.25.28 | bwang8 | let's check |
| 1804 | 08.25.55 | bwang8 | Yes |
| **1805** | **08.26.00** | **bwang8** | **it actually is** |
| 1806 | 08.26.02 | Aznx | So we got it! |

Log 8. Celebrate the Solution.

| 1807 | 08.26.02 | bwang8 | omg |
|------|----------|------|-----------------|
| 1808 | 08.26.04 | Aznx | yay! |
| **1809** | **08.26.08** | **bwang8** | **i think we got it!!!!!!!!!!!!!** |
| **1810** | **08.26.12** | **Aznx** | **WE DID IT!!!!!!** |
| 1811 | 08.26.12 | bwang8 | and it is so simple |
| 1812 | 08.26.14 | Aznx | YAY!!!! |
| 1813 | 08.26.16 | Aznx | i know |
| 1814 | 08.26.17 | bwang8 | lol |
| 1815 | 08.26.18 | Aznx | lol |

Log 9. Present a Formal Solution.

| **1816** | **08.26.34** | **Aznx** | **So you're putting it in the wiki, right?** |
|------|----------|------|-----------------|
| **1817** | **08.26.37** | **bwang8** | **yes** |
| 1818 | 08.26.41 | Aznx | Alright then. |
| 1819 | 08.26.43 | bwang8 | ok |
| 1820 | 08.26.53 | Aznx | Give an email to Gery, telling him that we got it. =) |
| 1821 | 08.26.57 | bwang8 | ok |
| 1822 | 08.26.59 | Aznx | I meant Gerry |
| 1823 | 08.27.04 | bwang8 | are you going to do it |
| 1824 | 08.27.07 | bwang8 | or am i |
| 1825 | 08.27.12 | Aznx | You do it. |
| 1826 | 08.27.14 | bwang8 | ok |
| 1827 | 08.27.19 | Aznx | Tell him that we both dervied n^2 |
| 1828 | 08.27.29 | Aznx | And then we saw that pattern |
| 1829 | 08.27.37 | Aznx | and we got the formula |

Log 10. Close the Topic.

| 1830 | 08.27.44 | Aznx | when should we meet again? |
|------|----------|------|-----------------|
| 1831 | 08.27.49 | Aznx | hat's your email? |
| **1832** | **08.27.52** | **Aznx** | **we should keep in touch** |
| **1833** | **08.27.57** | **bwang8** | **yeah** |

formula, $2n(n/2)$. Note that the $n^{th}$ odd integer can be represented by $(2n-1)$. It is likely that Aznx used a similar method, working on his own during his prolonged silence, but got confused about the result when he simplified his expression. As Aznx shows next, Aznx's first answer is equivalent to Bwang's answer, once Aznx simplifies it. His second answer is related to part of Bwang's unsimplified answer.

Aznx simplifies Bwang's formula: $2n(n/2) = n^2$ in line 1794 of log 6. This is the same as one of Aznx' proposed formulae. When Bwang does not respond to this posting, Aznx wonders if Bwang is still present online. Bwang was apparently already typing "so that is wrong" when be received Aznx' question concerning his presence. This message in effect confirmed that Aznx' second formula, $(n/2)^2$, is wrong and his first one, which agrees with Bwang's, is correct. The selection of the solution is thereby negotiated.

Going along with this in line 1802 of log 7, Aznx then multiplies their agreed upon formula by **4** because there were 4 sets of horizontal or vertical sticks, each numbering **1 + 3 + …**. Aznx poses his message as a question, soliciting confirmation from Bwang. By offering this next step in the symbolic representation, Aznx demonstrates that he understands where Bwang's formula came from and he understands the larger strategy of approaching the problem that Bwang had proposed. In other words, Aznx demonstrates a level of mathematical competence and of shared understanding that he did not always display in the previous sessions. Before being ready to answer whether $4n^2$ is actually the correct formula for the number of sticks, Bwang suggests that they first check if the formula works by testing it for a number of values of **n** and counting the sticks in drawings of diamonds at the corresponding $n^{th}$ stage. A half-minute later, Bwang concludes that the formula does check out. He therefore answers Aznx' question with confidence, perhaps mixed with a touch of surprise. Aznx concludes that they got the solution for the number of sticks in the diamond pattern—a problem that Team C had posed for itself, but for which they had derived the wrong formula, without, however, realizing it. Team B had been shocked earlier to discover that the formula they had been struggling to understand from Team C had been wrong; that it did not check out for any values of **n**.

Their surprise and excitement is almost uncontrollable. They use every chat technique they know to express their joy in log 8. Their postings intertwine like a frenzied dance.

Once the mathematical exploration is done, it is time to write up a report of ones findings. Professional mathematicians would do this in the form of a proof. Bwang agrees in log 9 to post a narrative of their solution to the Spring Fest wiki.

Finally, in log 10, Aznx and Bwang wrap up the conversational topic by exchanging email

addresses and agreeing to meet again online with Quicksilver and pursue further mathematical adventures together.

## The Sequence of Pairs

Within each of the preceding log segments we have identified a base adjacency pair by means of which the work of a specific move in the problem-solving effort of the small group is interactively accomplished. In most cases, a question is posed and a response is then given to it. As Schegloff (2007) argues, an adjacency pair is itself a sequence. It embodies a temporal structure, with the first element of the pair projecting the opportunity and expectation of a response in the interactional immediate future. The second element constitutes an uptake of a first element that it implicitly references as in the interactional immediate past (Suthers et al., 2010). In engaging in the exchange of an adjacency pair, the participants in the interaction effectively co-construct an elementary temporal structure within which future and past events can be located and referenced.

In talk-in-interaction, as analyzed by CA, the immediacy of response is intimately related to the turn-taking structure of vocal conversation (Sacks, Schegloff & Jefferson, 1974). As discussed above, the completion of the adjacency pair can be postponed by insertion sequences, such as repairs of misunderstandings or clarification exchanges. The base adjacency pair can also be preceded by introductory exchanges, such as announcements of what is coming, or succeeded by follow-up exchanges or confirmations.

In chat-in-interaction, as seen in the preceding log extracts, adjacency pairs can in addition be delayed by a more complicated response structure, in which multiple participants can be typing simultaneously and postings do not always directly follow the message to which they are responding. Thus, in Log 1, Quicksilver or Gerry can be initiating other topics in the midst of an interaction between Aznx and Bwang. Also, Aznx and Bwang can be typing to each other simultaneously as in Log 6, particularly if there has been an extended period of inactivity. This often makes textual chat harder to follow and to analyze than verbal conversation.

Nevertheless, it is generally possible to identify base adjacency pairs carrying the discourse along. In the previous section, we identified ten such pairs. The discourse moves in the log segments (each including one of these base adjacency pairs) formed a problem-solving sequence:

- Log 1. Open the topic
- Log 2. Decide to start
- Log 3. Pick an approach
- Log 4. Identify the pattern
- Log 5. Seek the equation
- Log 6. Negotiate the solution
- Log 7. Check cases
- Log 8. Celebrate the solution
- Log 9. Present a formal solution
- Log 10. Close the topic

The integrity of each of the ten moves is constructed by the discourse of the participants. Each move contains its single adjacency pair, which drives the interaction. In addition, there may be several utterances of secondary structural importance, which introduce, interrupt or extend the base pair; there may also be some peripheral utterances by other participants.

## The Group Perspective

The analysis of this paper is an attempt to make explicit the structure of adjacency pairs and a problem-solving longer sequence that is experienced by the participants and is implicit in the formulation of their contributions to the discourse. This is in contrast to analytic approaches that to some degree impose a set of coding categories based on the analyst's research interests or on an a priori theoretical framework.

Lines 1795 and 1796, for instance, show the power *for the participants* of the adjacency pairings. Here, Aznx has addressed a mathematical proposal to Bwang: "**If you simplify yours** [expression]**, it is n$^2$.**" After 24 seconds of inaction, Aznx cannot understand why Bwang has not replied, expressing agreement or disagreement with the first part of the proposal, for which Aznx expects a response. Because it is not a preferred move at this point for Aznx to reprimand Bwang for not responding, Aznx inquires if Bwang has disappeared, perhaps due to a technical software problem, which would not be anyone's fault. Two seconds later, we see that Bwang was typing a more involved response that implicitly accepted Aznx' proposal. Bwang then immediately explicitly accepts the proposal in line 1798, allowing Aznx to continue with the start of a new move with line 1802. Here we see Aznx and Bwang clearly orienting to the adjacency-pair structure of their discourse, in terms of their expectations and responses.

Aznx and Bwang co-constructed the longer (ten-move) problem-solving sequence by engaging in the successive exchange of adjacency pairs. Sometimes one of the students would initiate a pair, sometimes the other. As soon as they completed one pair, they would start the next. This longer sequence also has a temporal structure. It is grounded in their *present* situation, trying to find a formula for the number of sticks in the diamond figure. It makes considerable use of resources from their shared (co-experienced) *past* during the previous four hours of online sessions. It is strongly driven forward into the *future* by the practices they have learned for engaging in problem solving, culminating teleologically in the presentation of a solution.

The problem-solving sequence analyzed in this paper—covering 100 lines of chat during 10 minutes—is not selected arbitrarily or imposed in accordance with criteria external to the interaction, but is grounded in

the discourse as structured by the participants. The excerpted sequence is defined as a coherent conversational topic *by the discourse* of Aznx and Bwang. They open this topic with their interaction in Log 1 and they close it with the discourse move in Log 10 (Schegloff & Sacks, 1973).

In this paper, I have shown how the group constructs its mid-level problem-solving structure through a longer sequence built on micro-level adjacency pairs and contributing significantly to their macro-level collaborative learning, knowledge building and group cognition. Reviewing the hierarchy of levels introduced previously, it is now clear that each level can be analyzed as *oriented to* by the group discourse and the contributions to it by individual members:

a.   **Group event**: The group members log in to the event and comment explicitly on its goals, characteristics and duration as an event in which they are participating.
b.   **Temporal session**: Participants start each session with greetings and end with good byes.
c.   **Conversational topic**: The group explicitly opened and closed the topic analyzed in this case study.
d.   **Discourse move**: Each move was executed with a single adjacency pair.
e.   **Adjacency pair**: An adjacency pair includes an elicitation  and a response by each participant.
f.   **Textual utterance**: A text chat posting is defined by a participant actively typing and sending a message.
g.   **Indexical reference**: A reference is made by a word choice or graphical action by a participant.

This case study provides an unusually clear and simple example of problem solving in a virtual math team. In earlier sessions, the students encountered many difficulties, although they also achieved a variety of successes and learned much about both collaboration and mathematics. At the beginning of their first session, they did not know how to behave together and showed rather poor collaboration skills. Bwang said very little in English, often simply producing drawings or mathematical expressions. Aznx, at the other extreme, tried hard to engage the others, but seemed to have a weak mathematical understanding of what the others were discussing. At various points in the sessions, misunderstandings caused major detours and breakdowns in the group work. Moreover, from an analyst's perspective the interaction was often almost impossible to parse (Stahl, 2009b). By contrast, in the final conversational topic, which is here reviewed, the interaction is focused on two participants; they work well together; they seem to follow each other well; and their work goes quite smoothly. The structure of the interaction is also relatively easy to follow.

It seems that Aznx and Bwang have substantially increased their skills in online collaborative mathematics. The level of their excitement—especially in the segment of Log 8—shows they are highly motivated. Log 10 indicates that they would like to continue this kind of experience in the future.

## The Structure of Group Cognition

The analysis of the case study in this paper provides a first analysis of the long-sequence-of-moves structure of collaborative mathematical problem solving in a virtual math team. This is a paradigmatic example of group cognition. The small group—here reduced to a dyad—solves a math problem whose solution had until then eluded them (and had escaped Team C as well).

The students accomplish the problem solving by successively completing a sequence of ten moves. Each of the moves seems almost trivial, but each takes place through an interaction that involves both students in its achievement. The moves are commonplace, taken-for-granted practices of mathematical problem solving. They are familiar from individual and classroom problem solving in algebra classrooms. They have also been encountered repeatedly by Team B in their previous four hours of collaborative problem solving (Medina, Suthers & Vatrapu, 2009). It is this sequence of moves that accomplishes the problem solving. The sequence has an inner logic, with each move requiring the previous moves to have already been successfully completed and each move preparing the way for the following ones.

The common assumption about mathematical problem solving is that math facts and manipulations are what are most important. In our analysis of problem solving in a group context, math content is simply, unproblematically included in individual postings. In fact, more often than not, it is implicitly used and understood "between the lines" of the text chat. Of course, this is only possible because the group had already co-constructed a 'joint problem space' (Kershner et al., 2010; Medina, Suthers & Vatrapu, 2009; Sarmiento & Stahl, 2008; Teasley & Roschelle, 1993) that included this math content as already meaningful for the group. Rather, the important aspects of discourse engaged in collaborative math problem solving are matters of coordination, communication, explanation, decision making and perspective shifting (e.g., moving between visual, verbal and symbolic modes (Çakır, Zemel & Stahl, 2009)). To some extent, these are interactional moves required by most group activities; to some extent, these are adapted to the nature of mathematical discourse.

In conclusion, the group-cognitive achievement of the solution to the group's final problem was accomplished by a sequence of moves. Each move was mundane when considered by itself. The moves and their sequencing were common practices of mathematical problem solving. Each move was interactively achieved through the exchange of base adjacency pairs situated in the ongoing discourse. The problem solving was an act of group cognition structured as a sequence of these interactive moves.

Perhaps this case study can serve as an unusually clear and simple model of the structure of group cognition in mathematical problem solving by a virtual math team. It shows the group cognition taking place through the co-construction of a temporal sequence of problem-solving moves in the group discourse. While the fine structure adheres to the adjacency-pair system of interactional exchange, the larger problem-solving structure builds on these elements through a sequence defined by the topical moves of mathematical deduction.

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