Coordination and contextuality: Revealing the nature of emergent mathematical understanding by means of a clinical interview

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Abstract: Clinical interviews provide a rich source of data about the nature of student understanding. We use the "knowledge in pieces" (diSessa, 1993) epistemological framework to analyze a clinical interview of a sixth-grade student's knowledge of fraction equivalence. We focus on the coordination of his knowledge across representational contexts. Our analysis reveals a local understanding of "fraction as quantity" that we hypothesize could be leveraged in helping the student build a more coordinated understanding of fraction equivalence.

Introduction

In March of 2004, Deborah Loewenberg Ball conducted a public, ninety-minute clinical interview with one sixth-grade student, Brandon, at the first MSRI "Critical Issues in Mathematics Education" Workshop on Assessment. The purpose of the interview was to give a vivid demonstration of what clinical interviews reveal about student understanding. Over the course of 90 minutes, Ball and Brandon explored a variety of fraction concepts, using several different representational forms (e.g. area models, number lines, Hindu-Arabic notation, the physical activity of paper folding, and "real world" contexts). Schoenfeld's (2007) discussion of this interview illustrates the complex nature of what it means to learn and understand by highlighting various aspects of Brandon's emergent knowledge of fractions. In this poster, we develop a complementary line of analysis and explore what we can conclude about Brandon's understanding of fractions through the lens of a particular epistemological frame.

Theoretical perspective

The "knowledge in pieces" (KiP) epistemological framework starts from the assumption that it is productive to think about emergent knowledge of a domain as a complex system of diverse and loosely organized pieces (as opposed to an integrated, coherent system) (diSessa, 1993, 2004). Two central ideas of KiP are *coordination* and *contextuality*. One of the indications of deep conceptual understanding from this framework is "seeing" and coordinating relevant knowledge across a broad range of contexts. Furthermore, KiP highlights the productive role of prior knowledge in building deeper understandings of a domain.

Focus of inquiry

To illustrate the nature of our analysis, we share here a summary of one striking example of a "coordination issue" across three representational contexts in which he explores fraction equivalence: Hindu-arabic notation, area model, and number line. In order to get traction on why the lack of coordination across contexts is not salient to Brandon, we focus on the question of how Brandon attends to features of the representational forms and what this reveals about his understanding of fraction equivalence.

A summary of an exemplar preliminary analysis

Table 1: Contextuality and coordination in the case of Brandon's knowledge of fraction equivalence

Hindu-Arabic Notation Context	In the context of working with fraction notation, Ball asks Brandon if he can write another fraction for one-fourth. **Brandon writes down 2/8 as another fraction for 1/4** and explains "Umm, two-eights 'cause it takes four—it takes four two's to equal eight, so two would be 25 percent—or one-fourth."
Area Model Context	Later in the interview, Ball and Brandon are working together with area models. Ball asks Brandon to draw area models of 1/4 and 2/8. She then asks him which one he thinks is bigger. In this context, **Brandon answers that 1/4 is larger than 2/8** and reasons that it is the denominator alone that determines the size of a fraction: fourths are "bigger chunks" than eighths and so 1/4 is bigger than 2/8. This is despite having just constructed area models for 1/4 and 2/8 that suggest visually that 1/4 and 2/8 represent the same amount of shaded area. When Ball re-directs Brandon to the fraction notation context and his previous assertion, **Brandon sees no conflict with his previous assertion that 2/8 is another fraction for 1/4 and his new assertion that 1/4 is greater than 2/8.**
Number	After Ball and Brandon finish discussing area models, they turn to working on fraction

Line Context	equivalence using number lines. Brandon has labeled 1/4 and 2/8 at the same position on a number line. **In this context, Brandon again concludes that 1/4 is greater than 2/8** and reveals an understanding that is similar in nature to what he shows in the area model context. Specifically, he reasons that it is the length of the line segments that determine the size of the fraction: since fourths are longer segments than eighths, 1/4 is bigger than 2/8. This is despite having constructed a number line on which 1/4 and 2/8 label the same position that suggests visually that 1/4 and 2/8 represent the same distance from zero. When Ball re-directs Brandon to the location of 1/4 and 2/8 on the number line, **Brandon sees no conflict with the noticing that 1/4 and 2/8 label the same position on the number line juxtaposed with his statement that 1/4 is greater than 2/8.**

Discussion

In this poster, we used the "knowledge in pieces" epistemological perspective to frame the phenomenon of a student reasoning about fraction equivalence across a range of representational contexts. In the example discussed above, we see that Brandon has different ways of comparing fractions and determining "sameness" across the different representational contexts. When he works with area models and number lines, he cues up a rule involving the denominator in order to compare fractions. He can specify the logic of his rule locally in terms of features of the representational form at hand (i.e. chunks of pieces or lengths of segments).

We hypothesize that in order to understand equivalence of fractions, Brandon must first develop a coordinated understanding of fractions as quantities. That is, he must come to see a fraction as a relationship between the numerator and denominator that represents an amount (Mack, 1990). When reasoning about fraction equivalence, "quantity" is not salient to Brandon in his interpretations of area model and number line representations. Without recognizing fractions as quantities, it makes little sense to ask whether two fractions are equivalent or whether one fraction is bigger than another. We see resolving this issue as deeper than merely learning the conventions about how to properly express fractions as quantities in whatever representational system (number line, area model, etc) he is working with. We see the kernel of a "fraction as a quantity" understanding in his talk about the fraction notation 1/4 and 2/8 as both representing 25% of the whole. We hypothesize that this idea could be leveraged in helping Brandon build a more coordinated understanding of fraction equivalence.

Conclusions

The preliminary analysis discussed in this poster raises issues about how we interpret the nature of student reasoning revealed through clinical interviews. On one hand, probes across representational contexts can paint a complex picture of student understanding of mathematical ideas like equivalence of fractions. However, our analysis makes us aware that we cannot consider various representational forms as neutral windows into student thought about underlying mathematical ideas. Rather, expanding our focus to include the ways that students themselves understand the purpose of representation can help us give a more accurate characterization of how they understand underlying mathematical ideas.

References

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