The Effects of Base Ratio and Conceptual Structure on Accuracy in Multiplicative Situations

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Abstract: We examined the effects of base ratio and conceptual structure on accuracy in multiplicative situations. Forty-eight 3rd and 5th graders solved 36 problems that varied in base ratio and conceptual structure. ANOVA results indicated that conceptual structure made a greater impact on accuracy than did the size of the base ratio. The effect of structural constraint on accuracy supports the call for deeper conceptual instruction and learning in mathematics education.

Introduction

Research on children's arithmetic understanding has concentrated on addition, rather than multiplication. Three common constructs of the investigations in both additive and multiplicative research are strategy (Sherin & Fuson, 2005), problem size, and problem structure (Vergnaud, 1983). Historically, these constructs have been studied independently of one another. This paper reports on one part of a larger effort to build a more integrated understanding of relationships among strategy, problem size, problem structure, and tool use. We examine the link between the effects of base ratio and conceptual structure on 3rd and 5th graders' accuracy in multiplicative situations.

Method

Third-grade (n=24) and fifth-grade (n=24) public school students in a large urban district were individually asked six sets of six questions. Accuracy (accurate/inaccurate) was measured as a function of grade, base ratio, and conceptual structure. Base ratio was systematically varied across the six sets, with each set using different ratios, specifically, 1:6, 1:5, 1:4, 1:2, 1:3, and 1:9. The six questions in each set tapped four conceptual structures based on Vergnaud's (1983) theory of conceptual fields (see Figure 1). The first three questions were consistent with structure *a* but varied in number magnitude (i.e., 5, 8/9, and 10). The other three questions mapped on to structures *b*, *c*, and *d*, respectively. The six questions were presented in a fixed order hypothesized to reflect increasing question difficulty.

M_1	M_2
1	a
n	?

(a)

M_3	M_2
1	a
n	?

(b)

M_1	M_2	M_3
c	a	1
?	b	n

(c)

M_1	M_2
1	a
?	n

(d)

Figure 1. Conceptual structures for the six question types in the structured interview.

For each set, students first answered three simple proportion problems (Q1-Q3) followed by another simple proportion problem in which the dimensions changed (Q4), which in turn was succeeded by a concatenation of two simple proportions problem (esp., quotitive division). The structural variance of these five questions, coupled with the invariant question order lead to the following predicted order of accuracy:

$$Q1 \approx Q2 \approx Q3 > Q4 > Q5 \quad (1)$$

Because Q1 and Q3 had number magnitudes of 5 and 10, we wanted to evaluate the relationship between these two questions. Additionally, a sixth question, Q6, was asked to examine the children's use of pragmatic knowledge to solve simple quotitive division problems that involved a remainder.

Results

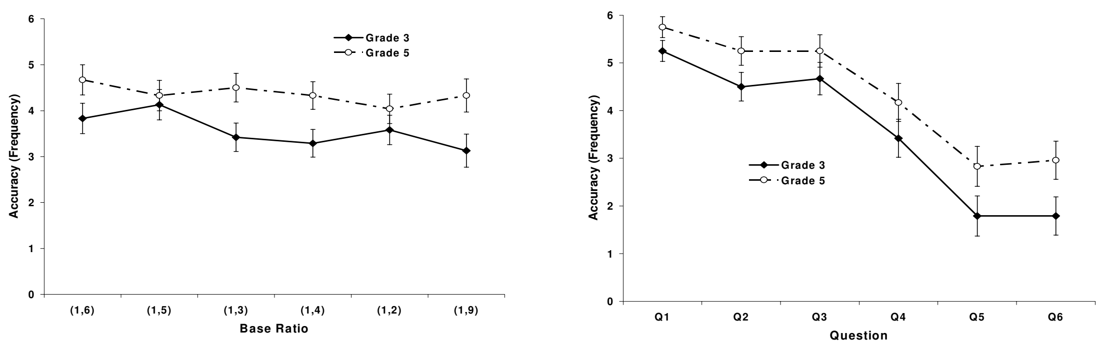
Grade x Base Ratio

Figure 2a illustrates mean accuracy as a function of grade and base ratio. Employing a 2 (grade) x 6 (base ratio) mixed model ANOVA, with grade (3, 5) as the between-groups variable and base ratio (1:6, 1:5, 1:4, 1:2, 1:3, 1:9) as the within-groups variable, we found main effects for grade (F (1, 46) = 4.46, p = .04, η_p^2 = .09) and base ratio (F (5, 230) = 2.50, p = .03, η_p^2 = .05) on accuracy. The interaction was not significant. *Post hoc* comparisons of base ratio means using the Fisher-Hayter procedure indicated that despite a main effect for ratio, no significant difference was detected between the largest (1:6; M = 4.25, SD = 1.64) and (1:9; M = 3.73, SD = 1.85) smallest group means.

Grade x Question

Mean accuracy as a function of grade and question is presented in Figure 2b. A 2 (grade) x 6 (structure) mixed design ANOVA was performed on accuracy with grade (3, 5) as the between-groups factor and conceptual structure of the questions (Q1, Q2, Q3, Q4, Q5, Q6) as the within-groups factor. Not surprisingly, the analysis showed a main effect for grade (F (1, 46) = 4.37, p = .04, η_p^2 = .09) and also question (F (5, 230) = 61.66, p < .001, η_p^2 = .57) and no grade x question interaction.

Figure 2b shows that simple proportion questions (Q1-Q3) were about equal in accuracy, Q4 was more difficult, and Q5 and Q6, the two quotitive division questions, were least accurate. A planned comparison using customized contrast coefficients aligned with the prediction in Equation 1 showed general trend agreement, F_{fit} (4, 184) = 100.90, p < .001, η_p^2 = 25. This finding was validated by a confirmatory test of model-to-data fit, ($F_{failure}$ (1, 138) = 1.50, *ns*), indicating that nothing other than structure contributed to the differences in the means. An additional planned comparison revealed a significant mean difference between Q1 (M = 5.50, SD = 1.09) and Q3 (M = 4.96, SD = 1.66) on accuracy, F (1, 46) = 8.78, p < .01, η_p^2 = .16. The results show that the conceptual structure of the question tempered solution accuracy: Those involving quotitive division (Q5 & Q6) were more difficult than simple proportion questions (Q1-Q3). Simple proportion problems were more difficult when the dimensions changed (Q4).



(a) (b)

Figure 2. Accuracy as a function of base ratio (a) and question (b).

Discussion

This study's findings support NCTM's (2000) recommendation for deeper conceptual learning and instruction of mathematics concepts. Our data do not support the notion that small problems are solved more accurately than large problems. Actually, our results showing no differences due to base ratio suggest that the dimensional and relational structure of multiplicative concepts impose greater constraints on response accuracy than problem size does for 3rd and 5th grade students. This is substantiated by differences in the effect sizes of the tested models and the finding that difference in mean accuracy between Q1 and Q3 were not large enough to invalidate the test of the model to the data even though that difference was a function of number magnitude (i.e., 5 vs. 10). Finally, the accuracy means of the division questions were nearly identical, [Q5 (M = 2.31, SD = 2.10) and Q6 (M = 2.38, SD = 2.04)]. This finding counters the idea that division-with-remainder is more difficult than simple division. Future research will evaluate the relation between conceptual structure and students' strategies, reasoning, students' tool (graph, table) use in learning, and teachers' use of tools in ininstruction.

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