# Teachers' Conceptualizations of Computational and Mathematical Thinking

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**Abstract:** The importance of integrating computational thinking (CT) into existing school structures, like core content domains, has emerged from efforts to improve computer science education in the U.S. In the past, computer science has often been treated as an elective or enrichment activity, which limits students' exposure to foundational computing ideas, especially in underserved schools. However, given the ubiquity technology plays in our lives, it is imperative that all students have access to CT. Few studies have focused on how pre-service teachers (PSTs) learn about CT. Some researchers argue that CT integration into K-12 education belongs in teacher preparation programs and that teacher educators should develop courses aimed at supporting PSTs' understanding of CT in the context of schools. This paper explores the ways in which PSTs begin to understand CT and how they work to integrate CT into their core subject areas.

**Keywords:** Computational thinking, teacher education, mathematical thinking, video tagging

## Introduction

The ever-expanding use of computers and computing technology has led to calls for expanding computer science in U.S. classrooms and broadening the contexts in which computing is used and taught (Wilson et al., 2010; Wing, 2006). A key idea that emerged from these efforts is the importance of computational thinking (CT) in K-12 education. In the past, the ideas that constitute CT have been reserved for elective or enrichment activities, which limits students' CT exposure, especially in underserved schools. However, given the ubiquity technology plays in our lives, it is imperative that all students have access to CT. One way to address this is to integrate CT into existing school structures, like math and science curricula (Lee at al., 2014; Sengupta et al., 2013; Settle et al., 2012; Weintrop et al., 2016).

In order to support student learning of CT in math and science courses, an important intermediate question arises: how do math and science teachers learn about CT, conceptualize CT and work to implement it in the classroom? Given that CT now explicitly appears in current curriculum standards, including the Next Generation Science Standards (NGSS) (NGSS Lead States, 2013), it becomes even more crucial that core content area teachers learn how to support student learning of CT. Some researchers posit that CT learning for teachers should begin even before they enter the classroom, through teacher education programs (Yadav, Stephenson & Hong, 2017). However, little work has been done to determine how to best incorporate CT into education methods courses, how to determine pre-service teachers' (PSTs') incoming knowledge of CT or how PSTs best learn about CT. This study examines how PSTs working toward certification in middle grades math and science begin to learn about CT through a CT module in an interdisciplinary methods class. Specifically, we explore the following two research questions:

*What are math and science PSTs' early conceptions of CT as they are introduced to CT concepts?*

*How do math and science PSTs begin to conceptualize CT in relation to the core discipline they will teach?*

In this paper, we first discuss how CT has been introduced in the K-12 curriculum and what work has been done to explore CT instruction and the integration of CT into core content domains. We then discuss recent work that explores how teachers integrate CT into their core disciplines. Then we highlight an important question existing work leaves open- how do core content teachers begin to recognize the joint nature of CT and core content (in this case math and science)- and how do we begin to investigate this missing piece?

## Background

While not a new concept (see Papert 1980, 1996), Wing's (2006) piece on computational thinking reintroduced the idea that strategies from the field of computing, like abstraction (using symbolic representations), problem decomposition (breaking down complex problems into more manageable parts), algorithmic thinking (working in steps to complete a task) and automation (using computers to more efficiently solve problems) are broadly useful and should be a part of all students' K-12 education. Wing argued that computer science practices have applications to many disciplines and should be taught alongside traditional school subjects, like science and math. While a single unifying definition has yet to emerge, it is generally agreed that CT includes concepts such as variables, abstraction, conditional and iterative logic, and practices like debugging, problem decomposition, and generating and evaluating algorithms (Grover & Pea, 2013; NRC, 2010, Shute et al., 2017).

Much of the research related to K-12 education has focused on teaching students CT skills and concepts. Several studies have offered ways to define CT in the context of education and have suggested how CT can be integrated into a variety of traditional school subjects (Weintrop, et al., 2016; Barr & Stephenson, 2011). In practice with students, researchers have highlighted examples of how students engage with CT and how CT can be integrated in the K-12 curriculum through fields like robotics and game design (Lee, Martin, Apone, 2014; Lee, et al., 2011). In addition, several studies point out that infusing CT into regular math and science classrooms through computer simulation environments and programs, like Scratch, enhances learning (Calao, et al., 2015; Sengupta, et al., 2013).

Recent research related to teacher learning of CT has centered primarily on in-service teachers' ability to integrate CT into existing school disciplines. Specifically, several studies show that elementary teachers can make use of their prior knowledge of curriculum and students to make connections between CT concepts and the math and science they teach (Rich, Yadav & Schwartz, 2019; Hestness, et al., 2018). With regard to the actual integration of CT into classes, Rich & Yadav (2019) found that elementary teachers' incorporation of CT into math and science lessons varied widely. A related line of work has investigated teachers' preconceptions of CT, finding that many teachers perceive CT as a difficult topic and are wary of claims about its seamless integration into their own disciplines (Cabrera, 2019).

Some researchers argue that CT integration into K-12 education belongs in teacher preparation programs and that teacher educators should develop educational methods courses aimed at supporting PSTs' understanding of CT in the context of schools (Yadav, Stephenson & Hong, 2017). In practice, researchers found that a short CT module in an educational technology class helped PSTs understand that they *can* teach CT across all disciplines and without the use of computers (Yadav, et al, 2011).

Fewer studies have focused on how PSTs *learn about* CT. When teachers have a more conceptual and multifaceted understanding of their discipline, like how to introduce the content to students (e.g., Schulman, 1986; Ball, 2000; Ball, Thames, Phelps, 2008), or aspects of the content that might be particularly difficult for students (Schulman, 1986), student learning is enhanced (e.g., Ergöneç, Neumann, & Fischer, 2014). In this exploratory study, we investigated PSTs' conceptualizations of CT as they were introduced to the concept and asked to consider its relationship to their core discipline, in this case mathematics or science.

## Methods

### Participants and context

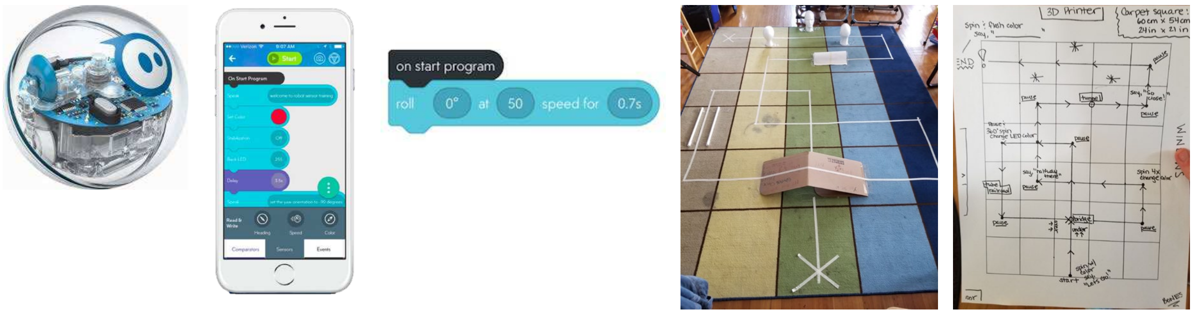
The study took place in an interdisciplinary teaching methods class at a large state university in the Mid-Atlantic United States. The class supports PSTs to use their knowledge of multiple subject areas to plan, integrate, implement, and assess curriculum and student learning. The course is required for PSTs in the middle grades (grades 4-9) math and science certification program. There were 16 PSTs in the course, 12 females and four males, all of whom were in their last semester of coursework before beginning their internship placements in local school districts.

The instructors (first and second authors) developed a three-class CT module aimed at introducing PSTs to key CT concepts and supporting PSTs to integrate these concepts into existing math or science curricula. The first class was online. PSTs read several practitioner-oriented articles that introduced CT terms (e.g. abstraction, problem decomposition) and gave examples for how CT can be integrated into various school subjects. In the second class, PSTs completed a small-group assignment in which they matched the CT practices from the articles to the Math Common Core State Standards (CCSS) and NGSS, which they displayed on whiteboards.

In the final class of the module, the instructors provided PSTs with an example of how CT can be integrated into math classes using a small robot called a Sphero (https://www.sphero.com). A Sphero is a spherical shaped robot that can be programmed using a mobile device (Figure 1). In order to explore PSTs' ideas about CT and math as enacted in the classroom, PSTs watched a video of 4$^{th}$ grade students completing a math task involving a Sphero. In prior work, video tagging tools have been used as a window into teachers' ideas about the nature of the domain (Walkoe, Sherin, & Elby, 2019) and student thinking in that domain (Walkoe, 2015). Using video tagging software (https://www.anotemos.com) (1), the PSTs were prompted to tag moments when they saw instances of mathematical thinking, instances of computational thinking or instances of mathematical thinking *and* CT happening simultaneously.

The four-minute clip that the PSTs tagged followed two 4th grade (ages nine and ten) students as they began to work on programming the Sphero to complete an obstacle course created by their teacher. The obstacle course was made out of cardboard, tape, and various other classroom supplies (Figure 1d). Students were not allowed to run the Sphero through the course before they fully programmed it. Instead, they used a to-scale map of the course as a reference for coding (Figure 1e). There was a grid on the map made up of rectangles that had a scale of 24 inches by 21 inches. The 4th graders had to figure out how to use the rectangle-grid to program the Sphero to complete the course in real life.

The 4th graders' overall goal in the video clip was to create a single Sphero command to make the Sphero travel the length of one square, which was 24 inches long. They could then repeatedly use this code as a *unit command* to align the Sphero's movements with the rectangle-grid on the obstacle course map. The video began with the 4th graders determining appropriate units to use (inches vs. centimeters). We call this event *unit conversion* (see Table 1). They then debated whether they should use the rectangles' area or length to create the unit command, which we call the *area vs. length* event. Using the map, they visually compared one of the rectangles' area to its length and they decided that the rectangle's length corresponded to the distance that the Sphero would have to travel in real life. The 4th graders then spent the bulk of the clip trying to get the Sphero to travel exactly 24 inches using the Sphero's `roll` command, which we call the *trial and error* event. The `roll` command has three inputs, the angle at which the Sphero will travel, the speed and the time (in seconds). The 4th graders began by setting a yardstick on the ground and marking out 24 inches. They then programmed the roll command to `roll  0° at 50 for.7s`, meaning that the Sphero rolled straight ahead at a speed of 50 for .7 seconds (Figure 1c). Running this command resulted in the Sphero rolling more than 24 inches, as measured by the yardstick. The 4th graders then iterated several times, changing the time while keeping the speed constant, until the Sphero rolled exactly 24 inches. The video clip ended with the *comparing to the map* event, in which the 4th graders returned to the map to count how many grid rectangles the Sphero would have to travel. The number of grid rectangles determined how many times they would have to repeat their unit command to make the Sphero travel through the obstacle course.




(a) (b) (c) (d) (e)

Figure 1. The Sphero robot (a), programming environment (b), a sample program (c), along with the obstacle course (d) and reference map (e) used in the video of the 4th grade task.

## Data collection

There were two primary sources for our data analysis. The first source was the whiteboard responses created by the teachers as they worked in small groups on the CT-standards matching assignment that was the focus of the second CT lesson. This allowed us to see PSTs' initial ideas about CT and their core discipline. There were four whiteboards in total, an example can be seen in Figure 2. In order to see how PSTs identified examples of CT and/or math thinking in situ, we saved the PSTs' video tags. We collected tags from 13 of the 16 PSTs (3 participants' responses were lost due to technology issues). PSTs had the options to tag moments of mathematical thinking, moments of computational thinking or moments of mathematical thinking and CT happening simultaneously. PSTs were also encouraged to comment on their tags.

## Data analysis

The categories in the Core Computational Thinking Concepts and Capabilities Framework (Barr & Stephenson, 2011) were used to categorize PSTs' responses for key components of CT in the small group assignment. The categories included data collection, data analysis, data representation, problem decomposition, abstraction, algorithms & procedures, automation, parallelization and simulation.

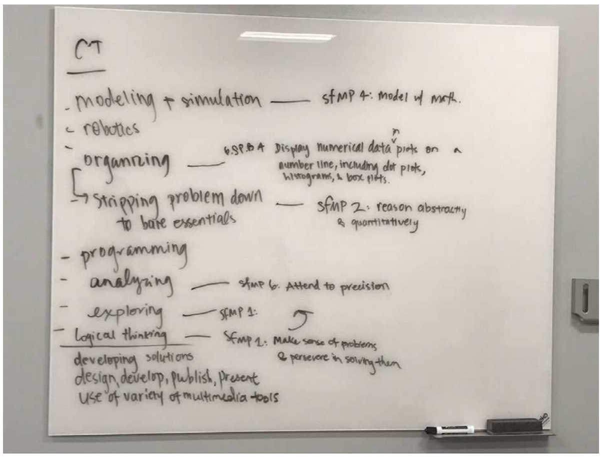



Figure 2. Example of the CT-standards matching activity.

The matching standards were then divided between “practice standards” and “content standards.” Practice standards included the CCSS Standards for Mathematical Practice (SFMP) (CCSS, 2010) and the NGSS Science and Engineering Practices (SEP) (NGSS, 2013). Both sets of these standards are practices that apply to all grades and outline the types of actions that students should take when approaching any math problem or science question. Content standards include standards for specific math or science content that students should learn at a given grade level.

We began our analysis of the PSTs’ video tags by categorizing the events that PSTs identified as examples of CT and mathematical thinking in the video. Four PSTs’ tagging sessions were randomly selected and the time of each of their video tags was noted on a timeline. The tags timeline was then clustered and matched to the events taking place in the video in order to summarize the 4th graders’ actions that the PSTs tagged. Four events emerged (see Table 1). The events ranged from 7 seconds to 1 minute and 42 seconds. Once the tags were grouped based on event, they were divided into “instances of CT thinking,” “instances of mathematical thinking” and “instances of both CT and mathematical thinking,” based on the PSTs’ responses. Table 1 provides a summary of the PST’s tags.

Table 1: PST Video Tags

| **Event (time)** | **Instance of CT tag** | **Instance of math tag** | **Instance of both CT and math tag** |
|---|---|---|---|
| Unit conversion (0:00-0:07) | 0 | 3 | 0 |
| Area vs. length (0:08-0:38) | 0 | 10 | 2 |
| Trial and error (0:42-2:24) | 20 | 4 | 6 |
| Comparing to the map (2:31-3:02) | 3 | 8 | 8 |

## Findings

Our initial attempt to introduce PSTs to CT produced three important findings. From the analysis of PSTs’ mapping CT to standards, we found (1) PSTs are able to identify the overlap between key CT concepts and the disciplinary content they will teach. In analyzing the PSTs’ video tags, we found that (2) PSTs can identify CT in student thinking but, (3) when looking at student thinking, PSTs tend to foreground either CT or mathematical thinking even if both are present, or struggle to explain why both types of thinking are present. Below we further discuss each of these findings.

### PSTs were able to identify overlap between CT concepts and disciplinary standards

PSTs were able to link CT concepts to 18 different CCSS or NGSS standards. Interestingly, PSTs often looked for keyword matches between CT concepts and standards. For example, one group matched “analyzing data,” a CT concept, with the SEP “Analyzing and interpreting data” (NGSS, 2013). In other instances, PSTs looked for explicit connections between CT concepts and standards. For example, one group matched “automation” to the SFMP “Using appropriate tools strategically” (CCSS, 2010). PSTs also searched for explicit connections in the content standards. The PSTs who produced the board in Figure 2 saw the 6th grade math standard “Display numerical data in plots on a number line, including dot plots, histograms, and box plots*"* (CCSS, 2010) as related to the CT concept of organizing data. These connections generally seemed "surface-level" to us—an ill-defined working hypothesis we brought into our analysis of the other data stream.

## PSTs can identify CT in student thinking

PSTs were able to identify instances of students using CT in the video clip. In the *trial and error* video event, the 4th grade students used an iterative development approach to program the Sphero to travel exactly two feet. During this event, all 13 PSTs identified at least one moment when the 4th graders employed CT, resulting in 20 total tags (Table 1). Comments left by PSTs included "trial to see how far [the Sphero] needs to go (analysis)" and "exploration because they were testing things out."

While PSTs were able to identify CT in student thinking, they rarely identified both CT and math thinking in instances when both were present. Instead, PSTs often foregrounded either CT or math. Below, we focus on two of the events in Table 1, *area vs. length* and *trial and error*, as these events provide clear examples of PSTs' separation of mathematical thinking and CT.

In the *area vs. length* event, PSTs often foregrounded mathematical thinking rather than CT. The PSTs tagged the *area vs. length* event 12 times total, ten tags of which PSTs categorized as instances of mathematical thinking and two tags of which PSTs categorized as instances of both mathematical thinking and CT. Although PSTs mainly tagged this event as an instance of mathematical thinking, as researchers we classified much of the thinking throughout the event as students using both mathematical thinking and CT. For example, one moment showed both 4th graders looking at the map to decide whether to use area or length. Using his finger to point, one of the 4th graders started counting the rectangles that the Sphero would have to travel, while simultaneously saying, "24, 48…[referring to each rectangle's length]. " At this point, the other 4th grader said, "That's not the area." The first 4th grader responded, "I know, but it's the distance that we need to go." The second 4th grader responded, "Oh yeah, that's the length." This exchange shows how the 4th used their understanding of the differences between of area and length, which is mathematical thinking. However, in counting the rectangles while adding up inches, the first 4th grader also used abstraction, a CT skill, to translate between the map and the real-life situation of the Sphero's travel.

The PSTs who tagged the *area vs. length* event as only mathematical thinking largely focused on the word "area." One of the PSTs wrote, "trying to decide whether or not they need to find the area-- other student disagrees," while another wrote, "They were explaining how to measure on the grid and find the area." Another PST recognized CT in the event, but only recognized it as mathematical thinking. She wrote, "Student wants to find the area so that they can determine how many blocks they have to cover. But then later they determine they just can add the inches of the [rectangle]." In this comment, the PST identifies the 4th graders' use of abstraction when she writes about the 4th graders determining how many blocks they have to cover. Yet, she labels this moment only as an instance of mathematical thinking.

At times, the PSTs switched the type of thinking they emphasized and foregrounded CT over mathematical thinking. Returning to the *trial and error* event, PSTs tagged this interval 30 times. Of those tags, PSTs labeled 20 of them as instances of CT, four of them as instances of math thinking and six of them as instances of both math thinking and CT happening simultaneously. Despite PSTs generally labeling this event as an example of CT, in our own analysis we identified that the 4th graders employed both CT and mathematical thinking throughout. In addition to the iterative development approach used in this event as described above, the 4th graders used their understanding of covariation and held speed constant, while only manipulating time, to make the Sphero travel 24 inches, which is an instance of mathematical thinking. The 4th graders also exhibited knowledge of ordering decimals and precision, which are both mathematical thinking skills. While iterating to get the Sphero to travel exactly two feet, the 4th graders try .5 seconds, which makes the Sphero travel just under two feet. At this point, one of the students says, "Try .573." The student knows that adding three thousandths of a second will cause the Sphero to travel slightly farther than the previous trial. .573 works and the Sphero travels exactly 2 feet. Consequently, one of the 4th graders says, "Right on the dot," indicating that he has an attention to precision.

The PSTs who labeled the event as only CT had varying reasons for their tags. Some PSTs focused only on the trial and error aspect of the video event, as can be seen in the quotes at the beginning of this section. Other PSTs referenced mathematical thinking, but also recognized such thinking as CT. For example, one PST who tagged the event as an instance of CT commented, "make sure that the measurement was exact," referring to the 4th graders' effort at precision. Another PST wrote, "Modifying speed to determine perfect speed." While she mistakenly referred to the 4th graders' modification as changing speed instead of time, this PST noticed that the students manipulated one variable in an effort to make the Sphero travel two feet. However, the PST considered the 4th graders' thinking an instance of only CT.

The final event in the video, *comparing to the map,* provided another example of PSTs foregrounding mathematical thinking over CT. However, it also showed that when PSTs actually identified instances of both CT and mathematical thinking occurring together, they struggled to articulate why that was the case. In this event, the 4th graders returned to examining the map to determine how many times they would have to repeat their unit command to get the Sphero to travel the desired distance through the obstacle course. One of the 4th graders used his finger to count the number of grid-rectangles that correspond to the number of 24-inch increments that the Sphero would have to move. He said, "We need to go 1, 2, 3, … 4 of those [rectangles]." The other student responded, "We've done one of them so we need to do two more," meaning that their unit command already covered the distance of one rectangle. The students briefly debated where the Sphero would actually be located on the map after running one unit command, and eventually agreed that they had to go two and a half additional rectangles. In determining how to travel half of a rectangle, one of the students said, "So, we'll just cut the speed in half." Like in the *area vs. length* event, the 4th graders use abstraction as they switch between charting their progress on the map and coding the Sphero to move in real life. They also employ modular solutions and reusing their code. In addition, the end of the clip shows the students using proportional reasoning as they determine how to make the Sphero move the equivalent of a half of a rectangle. As a result, we categorized this moment as an instance of both CT and mathematical thinking. Like in previous events, in this event some PSTs separated instances of mathematical thinking and CT, with more PSTs recognizing mathematical thinking. Of the 19 times this event was tagged, eight of the tags were for mathematical thinking only. PSTs' comments for the mathematical thinking tags were also similar to previous events in that some PSTs focused on mathematical language, like "cut in half," while other PSTs identified instances of CT as mathematical thinking. For example, one PST wrote, "Expanding their model for how far they actually need to go."

Of the three events described in this section, *comparing the map* had the most tags of both CT and mathematical thinking happening simultaneously, with eight. However, although PSTs identified both CT and mathematical thinking, they seemed unsure as to why both types of thinking occurred. This uncertainty can be seen through the lack of comments left by PSTs. Of the eight tags, only two had corresponding comments. One of those comments was quoting the 4th graders, which did not provide any insight into the PST's thinking. The other comment was, "applying analysis of distance to start thinking about further programming," which shows that the PST saw a relationship between the 4th graders' distance calculations and their translation back to the map. The other six tags had no comments, possibly indicating that, while PSTs thought they saw an instance of CT and mathematical thinking together, they could not explain why or how the two types of thinking worked in tandem. This further highlights the challenges associated with supporting PSTs in attending to students engaging in reasoning at the intersection of two conceptual areas. These results, broadly speaking, align with our tentative conclusion from the CT-Standards linking activity that the PSTs were more attuned to superficial, keyword-based connections between CT and mathematics than to connections based on the underlying cognition or practices involved, connections that may have led to the PSTs' "seeing" more co-occurrences of CT and math than was the case.

## Discussion and conclusion

The purpose of this study is to further our understanding of PSTs' early conceptions of CT and how they begin to think about infusing CT into school disciplines like math and science. Our analysis reveals that PSTs can find areas of overlap between CT and disciplinary content. In addition, PSTs were able to identify instances of CT when watching students engage in mathematical and CT tasks. However, when presented with moments when students were engaged in both CT and mathematical thinking, PSTs often identified only one type of thinking, or struggled to communicate why both types of thinking were present. This is important because if we want to integrate CT with other content domains, it is essential that teachers can identify when and how CT supports other domains.

The keyword-based connections PSTs made between CT and math and science curricula, along with PSTs' difficulty identifying instances of mathematical and computational thinking co-occurring, is in line with PSTs' status as novice learners of CT. In both cases, the PSTs related CT to math and science in superficial ways, if at all. These matches did not invite meaningful infusion of CT into math and science curricula because they do not show how CT could support math or science learning in specific ways, like how iterative development supported student learning of covariation and precision in the video tagging example above. In contrast, experts in a particular subject can recognize more meaningful connections and their strategic consequences (Bransford, Brown, Cocking, 2000). Given that computing can serve as a powerful context for disciplinary exploration, future work should explore how to continue to develop PSTs' CT knowledge so that they move toward expertise in recognizing the "deep structure" of CT skills and how those skills can support math and science learning.

This work also serves as another example of the pedagogical value of video for helping prepare PSTs. There is an established body of literature around teachers' use of video as a way to notice and analyze student thinking (Sherin & van Es, 2009; van Es & Sherin, 2010). Other researchers have called for video clips that show CT integrated with school disciplines as a means to engage teachers in how to implement CT in their classrooms (Yadav, Krist, Good, & Caeli, 2018). Given the multifaceted nature of CT and its constituent skills, video is a particularly generative medium for exploration and instruction. The ability to watch and re-watch videos of CT and disciplinary knowledge co-occurring can help refine PSTs' noticing of these moments.

As teacher educators, we think it is important and potentially impactful to integrate CT into methods classes. This work shows the potential that methods classes hold for supporting PSTs in their understanding of how CT fits into existing school disciplines. Building on Yadav, et al. (2011), our work shows that PSTs not only understand the value in infusing CT in subjects like math and science, but they can also begin the work of identifying where CT fits into their future classrooms. Moving forward, we have several questions guiding future research. In our study, identifying overlap between CT and CCSS and NGSS did not cue PSTs to look for math and CT intersections in student thinking. Consequently, how do we demonstrate the joint nature of CT and school subjects in ways that support PSTs in identifying this mutual relationship? Is it better to focus on CT concepts and definitions, or provide examples of the overlap? In addition, once PSTs can identify the joint nature of CT and school disciplines, how do we support them as they begin to infuse CT into their own lessons and classes? Answers to these questions will become increasingly important as we strive to equip teachers to prepare students to navigate the increasingly technological world that awaits them.

## Endnotes

(1) The Anotemos software used in this paper has been developed at the GRIP Lab, University of Michigan with partial support from NSF grant DRL- 1316241. Opinions expressed in this paper do not necessarily represent the views of the National Science Foundation or the University of Michigan.

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