Exploring Transgressive Actions in Mathematics: Situated Perspectives on Undergraduate Mathematics Learning

Cihan Can, Kathleen M Clark, and Mark Watford
ccan@fsu.edu, kclark@fsu.edu, jwatfor@fsu.edu
Florida State University

Abstract: We explore the role of using Primary Source Projects (PSPs) in undergraduate mathematics instruction on students' mathematical growth from the perspective of situated theory of learning. Using PSPs in teaching creates opportunities for student learning, as well as challenges. To make sense of their challenges, we refer to Kozielecki's (1986) transgressions framework. In our study, we demonstrate empirical justification of a relation between a learning experience with PSPs and students' mathematical growth.

Introduction

In his lament, Lockhart (2008) raised a provocative question that might offend mathematics educators: "Why don't we want our children to learn mathematics? Is it that we don't trust them, that we think it's too hard?" (p. 6). He argues that there is a fundamental problem in mathematics education and it basically originates from how we define mathematics. In his view, mathematics is an art similar to music and painting. In this regard, he contends that we need not teach, but let students experience the art of mathematics. The best way to teach art, he believes, is not to teach but let learners do art.

From our perspective, Lockhart's (2008) view on learning mathematics resonates with the theory of situated learning. According to Lave and Wenger (1991), learning is about newcomers' or novices' increased access to expert performance, and not simply the acquisition of knowledge and facts of a particular discipline. Based on a situated perspective on learning, learners need to "assemble a general idea of what constitutes the practice of the community" (Lave & Wenger, 1991, p. 95) as a result of their learning experience.

In our research we seek to understand the psychology of students who had a mathematics learning experience that we identify as a viable pedagogical practice for increasing students' access to the community of practice of mathematicians. This form of mathematics instruction is based on using curriculum materials that were developed to promote exploration and discovery of mathematical ideas by making use of primary historical sources. These materials are called Primary Source Project (PSPs) and include (1) excerpts from one or several primary historical sources, (2) student tasks designed to illuminate the mathematical concepts that form the focus of the sources, (3) secondary narrative by the project author to replace or support the related content of undergraduate mathematics curriculum.

Notably, PSP-guided instruction creates an unfamiliar context in terms of learning for students compared to a learning experience dominated by textbooks. Clearly, the level of unfamiliarity depends on the classroom implementation of the PSPs, but the content and its presentation are relatively unique in the PSPs and different than standard textbooks. In the study presented here, we adopt Kozielecki's (1986) transgressive theory to make sense of students' challenges with a PSPs-guided learning experience and contribution of these challenging experiences to their mathematical growth from the perspective of the theory of situated learning.

Transgressive Theory

The term transgression may often be considered with a negative connotation, such as violating the norms of a standard practice, but we use the term as behavior of purposeful action that leads to an outcome which exceeds the boundaries of an individual's past achievements. These actions differ from their counterpart of protective actions which seek to maintain the status quo of the subject ((Kozielecki, 1987). Expansive transgressive behavior is of interest to our research because it involves the acquisition and assimilation of existing material. We wish to identify symbolic expansion, in which a student widens the knowledge of the world or of the self. Pieronkiewicz (2015) couples Kozielecki's traditional definition of transgression with the proviso that the referenced barriers "preclude one's mathematical growth and development" (p. 1263). We therefore seek to bridge the theoretical work of Pieronkiewicz and Kozielecki by exploring transgressive actions in mathematics while making a case for students' mathematical growth from the perspective a situated theory of learning.

Methods

The data used in this study were collected by the evaluation-with-research working group of a National Science Foundation-funded project aimed at improving undergraduate mathematics education via developing, testing, and

disseminating PSPs. One goal of the project is to explore students' learning experiences with PSPs in a holistic way.

For our analysis of students' transgressions, we used a subset of student data collected in classrooms where instructors used PSPs at some point in their teaching. For this analysis we used student data from four Abstract Algebra (or similarly titled) courses implemented in Spring 2018 (Instructor A, 14 students; Instructor B, 15 students) and Fall 2018 (Instructor C, 12 students; Instructor D, 13 students).

We conducted our analysis of students' transgressive actions at the individual level. We profiled each student with his or her classifications of transgressive or protective actions. We also looked for student responses that indicate an identity-changing or -learning process from the perspective of situated-theory of learning.

Results

In our analysis, we wanted to document the ways that students perceived their learning experience as a barrier for their learning of mathematics, while considering our hypothesis of such an unfamiliar learning experience. This way, we can argue for students' crossing their boundaries purposefully, thus making a case for growth.

The most prevalent student view that we identified was related to students' difficulty with the primary source excerpts, although the resulting emotional expressions were revealed in many different forms. In such cases, students' discomfort and frustration with reading unfamiliar and dense texts, compared to what they are used to reading in textbooks, are indications of boundaries that they were expected to cross as part of a learning experience with PSPs. Some students even decided not to continue studying mathematics professionally in their future career as a result of gaining a better sense of what it means to study at the expert level. In these cases, frustrated with not staying within the boundaries of status quo (e.g., using a textbook), we believe that students' actions are transgressive for the purposes of studying what will make them happier in the future.

As for the main goal in this paper, searching for transgressive actions and potential access to a community of practice, or expert performance, associated with these actions, we identified that a majority of students' responses (we were not interested in quantification at that point) regarding their PSP experience indicate transgressive actions. Respectively, we identified the dominant characteristic of these transgressions as resulting in positive affective experiences, but with several different flavors: feeling of accomplishment after having to read dense text and depth of study to understand complex mathematical ideas, being able to see connections between mathematical concepts proposed by expert mathematicians, and learning how mathematical ideas were developed, to list a few. In almost all of these instances, students' expressions about their experience included explicit references to what we identify as indications of their increased participation within a community of practice, and thus identity-changing or -shaping processes from the perspective of situated theory of learning. Due to space considerations, we share the following student response (Student A_2) to illuminate our claim about the relation between transgression and mathematical growth from the perspective of situated learning.

> When going through each "task" I get this small sense of accomplishment by solving proofs and making conjectures, giving a sense [of] discovery and fulfillment. Notation is the roughest part of trying to learn from old manuscripts, though many issues that may be had during the reading can be mitigated by the lecture or group work the following day.

Student A_2 is explicit in stating how notation was a boundary in learning, but also states clearly the influence of crossing that boundary as a sense of accomplishment and fulfillment. As a response to another item in the survey, the student further notes how such an experience was helpful in terms of getting an "insight into what the mathematicians were thinking and how they went about creating something completely new," thus shedding light on the relation between transgression and mathematical growth.

References

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