Coevolution of Symbolic and ConcreteDimensions of Understanding

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An important issue in early mathematics learning is the degree to which experiences with the physical world and experiences with signs influence developing understanding of mathematics concepts. The relationship between concrete and symbolic understanding has been investigated extensively (Bruner, Olver, & Greenfield, 1966; Piaget, 1966). Many researchers claim that concrete understanding develops prior to and underlies symbolic understanding (Lakoff & Núñez, 2000; Moses & Cobb, 2001; Resnick, 1992). Other researchers point out that symbolic or abstract understanding can precede qualitative understanding (Simons & Keil, 1995; Uttal, Scudder, & DeLoache, 1997). There is evidence that in general, as children develop expertise in a mathematical content area, they shift from using primarily concrete models and working with individual instances to a using numbers and developing general strategies for solving math problems (Carpenter, Fennema, Franke, Levi, & Empson, 1999). However, this trend should not be confused with the question of how understanding develops. To answer this question, researchers need to examine how concrete and symbolic resources each shape development (Vygotsky, 1978). Concrete resources are items like blocks, physically realistic drawings, and interactive virtual objects. Symbolic resources are items like written numbers, maps, and other notations. There is evidence that both signs and concrete objects can mediate learning (Fuson & Briars, 1990; Uttal, 2000; Wellman & Miller, 1986). At the same time, neither is sufficient for developing understanding (Resnick & Omanson, 1987; Uttal et al., 1997). A reasonable conjecture is that these resources mediate the development of understanding in a co-evolving fashion (Fuson, 1990; Rittle-Johnson, Siegler, & Alibali, 2001; Saxe, 1994).

I conducted two experiments that address the question of whether concrete and symbolic resources jointly participate in the development of the understanding that fractions are complex multiplicative quantities over time. To examine this question, the overall goal in these studies was to measure changes in how children used concrete and symbolic resources at a low level of granularity and to examine the development of the use of both kinds of resources over time. In these studies, fourth-grade students invented notations for fractional quantities displayed with a manipulative. Students solved problems that asked them to invent a way to use numbers to represent fractional amounts built with a manipulative called the Annaboard (Veit, June, 2002). I measured a features dimension and the numbers dimension. Development along the features dimension was growth in the number of relevant features of the concrete situation that students' invented notations included. Development along the numbers dimension was growth in the complexity and reasonableness of the signs the students used.

The results showed that students' notations included more features and employed more appropriate numbers and operators over time. Notably, neither aspect of the task dominated. Students' notations did not become complex on a features dimension and then improve on a numbers dimension. Neither did they follow the reverse pattern. Instead, these two dimensions improved together. This was true for early and late fourth-grade students. These results provide support for a co-evolution hypothesis that features and numbers information develop together.

From a developmental perspective, these results suggest that longitudinal trends do not necessarily explain how development happens. While there may be a development from a more perceptually grounded, case-specific type of understanding in various domain areas to a more general or principles-based type of understanding, this trend does not explain the mechanisms of that development. From the perspective of instructional design, these results suggest that while many instructional programs in elementary math begin with concrete resources and only later introduce symbolic resources, in a constructivist setting that is focused on meaning-making, introducing symbolic and concrete resources together and using them to productively interact can also be beneficial.

References

Please see the website version of this document for references (http://www.edb.utexas.edu/faculty/tmartin/papers.html).