NumberNet: Using Multi-Touch Technology to Support Within and Between-Group Mathematics Learning

Andrew Hatch, Steve Higgins, Andrew Joyce-Gibbons, Emma Mercier, Durham University Technology Enhanced Learning Lab, Leazes Road, Durham, DH1 1TA, UK.
Email: andrew.hatch; s.e.higgins; andrew.joyce-gibbons; emma.mercier@durham.ac.uk

Abstract: In this paper we present a new tool, NumberNet, designed to promote within and between group collaboration in a mathematics classroom. The activity builds on a standard individual mathematics activity (Explode-A-Number; Atkinson, 1992), to create a three-stage collaborative activity that promotes flexibility with numbers, operators and calculations. This tool uses a network of multi-touch tabletop computers in a classroom environment, taking students through a small group production activity, rotation of the activity between groups, and finally a sorting and structuring activity. Pilot results from 32 students indicate significant gains in the number of calculations that students produce from pre to post test. Further work and implications are described.

Introduction

Learning to use numbers in a flexible manner, and recognizing the breadth of operators that can be used and the different ways calculations can be constructed, is a key feature in mathematics education at the upper primary level in the United Kingdom. These skills establish a basis that prepares students for the more complex mathematical constructs and concepts that they will encounter during high school, such as algebra and geometry. However, while it is relatively easy to assess flexibility, designing curricula and activities to teach and foster numerical flexibility and adaptive expertise (Baroody & Dowker, 2003) is more challenging. In this paper, we describe a tool, NumberNet, that uses computer-supported collaborative learning activities to foster mathematical flexibility and reasoning through an interlinked series of small group and whole class activities. This tool builds on the standard classroom activity 'Explode-A-Number' (Atkinson, 1992), in which students are given a number and told to create as many calculations as possible that form that number (See figure 2 for an example of students work on a pre-test version of this task).

Although adult models of arithmetic calculation are relatively well known, the development of children's calculation skills is less well understood (for a recent overview see Kaufmann & Nuerk, 2005), and although the importance of developing flexibility through derived facts and mathematical reasoning is acknowledged, the relationships between mathematical concepts and processes is complex (Gray, 1994). It is generally agreed that derived fact and reasoning strategies are indicative of more successful mathematical learners (Baroody & Dowker, 2003) and, although teaching activities to develop these in computer-supported environments have not always been successful (Higgins, 2003; Sarama and Clemens, 2009) and the importance of experiences of teacher-supported mathematical activity in lessons is clearly essential (Ruthven, 1998)

Collaborative learning and problem solving activities have long been studied and recommended for teaching mathematics at many levels (see Esmonde, 2009; O'Donnell, 2006 for reviews). Research on collaborative learning indicates that the task-type and structure (e.g. Johnson & Johnson, 1991; Slavin, 1986), the quality of interaction, help seeking and problem-space creation can influence learning outcomes (Barron, 2003; Webb & Farivar, 1994; Roschelle, 1992). Bringing this research together, indicates the value of developing tools to support collaborative mathematics activities in the classroom, while paying close attention to how those activities are implemented and supported within the classroom. Adapting multi-touch technology for classroom use allows for the increased use of collaborative learning in the classroom.

Multi-touch technology provides a surface that can detect multiple touches, from one or more people, which creates opportunities for interaction between learners with multiple points of control. This is in contrast to single point of control computer technology that is more commonly used, and requires negotiation over control of the device (e.g. desktop or laptop computers that use a mouse or keyboard). In this study, we use a multi-touch enabled classroom (see Figure 1), which consists of four networked multi-touch student tables, an orchestration desk and a multi-touch interactive whiteboard. The student tables are designed to be sit-to-use, with space for between three and six students per table. The tables use a vision system that identifies up to thirty simultaneous touches, allowing all students to interact with the content through touch directly. The tables are networked, allowing for the teacher or student to 'slide' or send content from the orchestration desk to the student tables, or between the student tables. Finally, the orchestration desk can be used to move content from the student tables to the multi-touch interactive white-board. This means that teachers can project the content from a table to the whole class, using it to prompt a whole-class discussion about the methods or organization being used by an individual group. The teacher or students can interact directly with the projected content, explaining reasoning or pointing out difficulties to the whole class.

Previous work on collaboration with multi-touch tables suggests that when multi-touch is compared to single-touch, the use of multi-touch results in more task focused conversation (Harris et al., 2009), more equitable participation of collaborators (Marshall et al., 2009) and more speedy conflict resolution (Hornecker, Marshall, Dalton, & Rogers, 2008). When compared to paper-based tasks, the use of multi-touch tables also appears to promote joint attention and the creation of a joint problem space (Higgins et al, under review). Levering this technology for whole-classroom learning provides the opportunity for increasing support of small-group learning, and better ways for the teacher to orchestrate the tasks and moves between individual, small group and whole class discussion (Dillenbourg & Jermann, 2010).

NumberNet was designed to use the affordances of multi-touch to help students become more flexible in their use of mathematics. The tool builds on the multi-touch features, to allow small groups of students to work towards a joint goal of creating calculations for a single target number, while retaining control of their own input devices, similar to numeric keypads. The tool also takes advantage of the networking aspects of this technology, allowing the teacher to move the target numbers between tables. In this way, students can learn from their immediate group, from the calculations passed to them from other groups, and through whole class discussion towards the end of the activity.

In this paper, we present the tool and data from a pilot study, to explore the value of this tool for increased flexibility generating arithmetic calculations in primary school children.

Figure 1. The Multi-touch Enabled Classroom.

Method

As this was a pilot study of the use of the new NumberNet tool, the study was designed to collect both pre and post data of math skills, and as a participant observation study to explore how students used the tool and any issues that arose in relation to the collaboration, math or technology.

Participants were 32, year six pupils (10-11 year olds; mean age 10.5 years; SD = 0.35), who attended two local public primary schools during the 2010-11 academic year. Eight male and eight female students were recruited from each school; in both schools, the students recruited were all part of one class group, and so were familiar with each other and used to working together.

Students were recruited during a visit to the school by members of the research team about a week before they visited the lab. The purpose of this visit was to explain the study and the multi-touch technology and lead the students through a number of activities to prepare them for the visit to the lab; the students also completed the pre-test (described below) during this visit to the school. All students in the class (approximately 25) participated in the introductory visit. Parental consent forms were distributed and the teacher selected 16 students to visit the lab from those who returned consent forms.

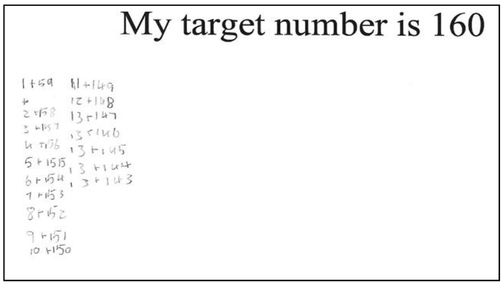
Students were brought to the lab in a group of 16, spending about 5 hours working on a variety of learning activities and games on the tables. These activities included a number of non-educational game-type activities to familiarize the students with the technology, two history activities and four mathematics mysteries. All of the math tasks were word-based problems; one was a logic problem, another required sorting number facts to find an answer, and two required calculations. These tasks were based of classroom based tasks, and were unlikely to influence the students' performance on the post-tests. The NumberNet activity was introduced after lunch, at which stage the students had been working on the tables for a number of hours, and were familiar with interaction and operation of the tables. After using NumberNet, the students piloted other new tasks for the non-educational aspects of this project, including a sorting task where they had to pass photographs between the tables, and a three-dimensional version of Tetris; after approximately an hour, the post-test was distributed.

The data described in this paper consists of comparison of the pre and post-test data, and an exploration of the process of one group. As this was a pilot study, each of the authors of this paper were present during this activity, acting as participant observers to evaluate the task; the groups at each table were each recorded by two cameras and a recording of the computer display was made. Participant observers were focused on helping the children complete the task while paying attention to any difficulty that they had with the task and the strategies that the group members used. During a de-briefing session after each day of data collection, the participant observers recounted what they had noticed, paying particular attention to processing of knowledge exchange, helping behaviour or on-task conversations within their group. The focus group was selected through the comments of the participant observers, where the sharing of a strategy had been noticed; the video of this group was reviewed in preparing the analysis for this paper. Further data from a larger study will be presented during the conference.

Pre and Post Test

Students were given a pre-test during the initial classroom visit, and a post-test about an hour after they used the NumberNet tool in the lab. The test consisted of a single sheet of paper, with space for the student's name, and the statement: My target number is x, with x being replaced with one of up to ten possible numbers (six at pre-test and four at post-test). Different target numbers were used for the pre and post-tests, and no two children at the same table received the same target numbers.

The tests and pencils were distributed to each student, and they were asked to put their names on the top of the page. There were then given brief instructions (all students were familiar with the task from prior use in their mathematics class), and told they had two minutes to write as many calculations as possible. After two minutes, the students were asked to put down their pencils and their tests were collected by members of the research team.



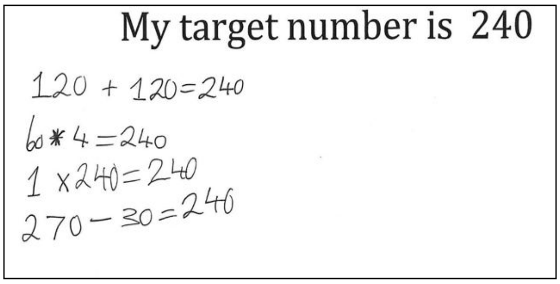


Figure 2. Examples of the Pre and Post-test (Standard Explode-A-Number Activity).

NumberNet

The NumberNet activity is based on a standard classroom activity known as 'Explode a Number' (e.g. Atkinson, 1992). This task is commonly used in upper primary school (9-11 year olds) in the mental/oral arithmetic section of the mathematics lesson. Students are given a number and told to create as many alternative calculations as possible, which give that total, usually within a certain time limit. The task can be varied depending on the students' ability, areas in which they need practice and the learning objectives for the activity by specifying which operations can be used to achieve the answer. Over the course of time, students are expected to identify a variety of strategies for this task, including using simple addition and subtraction patterns (e.g. 499+1=500; 498+2=500; 497+3=500 etc), simple multiplication or division patterns (e.g. 50*10=500; 5*100=500; 0.5*1000=500; 5000/10=500 etc), multiplication or division by one (e.g. 500/1 = 500; 500*1= 500), commutative statements (e.g. 200+300=500; 300+200=500; 250*2 =500; 2*250=500). The overall goal is for students to be able to identify the different uses of operations and develop a flexible understanding of how numbers and operators interact.

While this task provides an easy way of assessing a child's current understanding of numbers and operators, and can be used over time as a practice activity, it is explicitly an individual activity. While some teachers attempt various whole-class extension activities after students have written their calculations, it remains difficult to adapt this from a practice or assessment activity into one that provides opportunities to learn to develop a wider range of strategies.

The NumberNet tool takes the principle of the task, but by making it a collaborative task, and allowing for an extension sorting activity, provides a way for students to learn from each other during production of calculations, and for the teacher to structure and scaffold their learning in the conversations that follow. The three phases of NumberNet take the students through small group, between group and whole class interactions, allowing them to learn from their group members as well as other teams and from the classroom discourse.

Phase 1

In the first phase of NumberNet a target number is assigned to each table (a different number for each table) and each student at the table has access to a multi-touch numeric keypad on the surface of the table (see Figure 2) that is designated for their use. The teacher can choose when to make the target numbers appear, and how long the students are allowed to spend creating calculations for that number. Each student can work alone, or can discuss with their peers at the table how to address this task or particular calculations within the task.

The numeric keypads allow the students to use the digits 0-9, the four primary operators (addition, subtraction, multiplication and division) and brackets (parentheses). While the teacher controls allow for the removal of any digit or operator on the numeric keypads (to make the task more difficult or encourage certain students to focus on a particular operator, factors etc), these features were not used during the experiment described here: all students were able to use all digits and operators. When the student has created a calculation, they use the send button to move the calculation from the number pad and onto the open space on the table.

The keypads are set to identify duplicate calculations (e.g. calculations that are already on the table, regardless of who created them), turning them red and not allowing the student to slide it into the middle of the table. Incorrect calculations and invalid expressions are permitted at this stage, although are flagged in the teacher's control panel.

Once a calculation is added to the table, any student at that table can pull it onto their number pad and edit it, allowing for corrections or discussion within the group. The teacher's control records which number pads create each calculation, and whether the calculations are new or edits to existing calculations. This provides a way for the teacher to monitor the task from a distance during the task, or, what is more likely, examine the activity afterwards, or over time. This information can be used to identify students who appear to be stuck on a particular method, students who alter calculations during the tasks, and students who do not get the calculations correct. Over time, this can provide the teacher with a method of assessing the students' performance, learning and areas of difficulty and indicate where prompting exploration of additional strategies would be useful.

Throughout the task, the total number of correct calculations on each table is displayed in the corner of the screen. This feature can be turned on or off at any time during the activity, depending on whether the teacher is interested in encouraging between-group competition. For the pilot study described here, this feature was enabled, allowing the students to see which table was creating the most calculations.

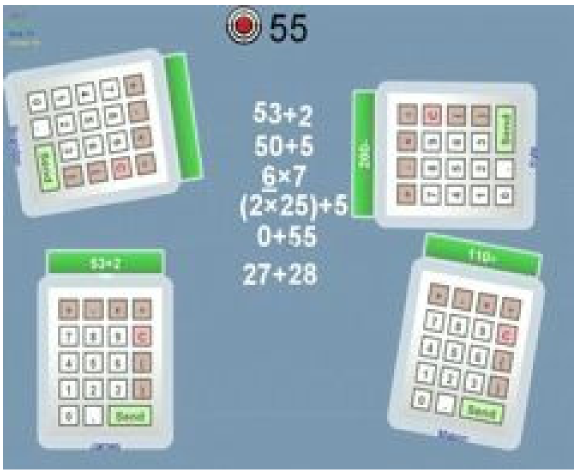


Figure 3. Phase One of NumberNet, during Creation of Calculations.

Phase 2

After a length of time determined by the teacher (either before or during the task), the second phase of NumberNet begins. The number pads disappear, the correct calculations cluster near the top of the table, while the incorrect calculations remain in the center of the table; the teacher can allow the group to examine their incorrect calculations before moving onto the next part of the task. The correct calculations and the target number are then rotated to the next table, so that each group receives the last group's number and their calculations. They are given a few minutes to explore the calculations that have been sent to their table, and then the number pads appear so they can add more calculations. The length of time before the number pads appear can be altered, and increases for each rotation, as the students have increasing numbers of calculations to explore. As before, the tool identifies duplications, so the new group cannot replicate calculations created by the previous group.

Phase 3
Once the target numbers have been rotated around all the tables (four tables in our case), the teacher can activate the final phase of the activity. In this phase, the groups get their original target numbers back, with all the calculations that have been created by their class. The number pads disappear and the group's task is to link the calculations together in a way that makes sense to them (see Figure 3). This can then be projected by the teacher, so that the whole class can discuss the reasons for linking and what the similarities or differences are between calculations. There are many possible ways to create structures for the calculations (e.g. similar patterns, the same operators, etc), the important feature is that the students can explain why they linked particular calculations, and, over time, understand the range of possibilities.

Calculations are linked together by placing a finger on each one and bringing them together; when the two calculations touch, a link is formed. Links can be broken by pulling the calculations apart. Groups can enable or disable a dynamic automatic layout of the resulting graphs of expressions. The layout algorithm used is a simple force-directed graph layout that attempts to minimize edge crossings.

Due to technical issues, not every group in our sample was able to use Phase Three, and the teacher noted that additional work will be necessary to develop pedagogical supports for the classroom discussion of this activity; however changes to the technology have resulted in increased stability for this phase, and the data presented at the conference will include use of Phase Three.

Technical Details
Each student table ran an instance of the NumberNet student software which was built on our software framework which was written to support distributed applications for the classroom. This framework uses Java with OpenGL bindings through the jME2 scene graph library. Our framework specifically provides components for touch interaction, content management and teacher control over the network. Inter-table messaging is achieved with opens standards messaging systems (XMPP). The teacher's software for NumberNet was built using Multiplicity with a Swing GUI.

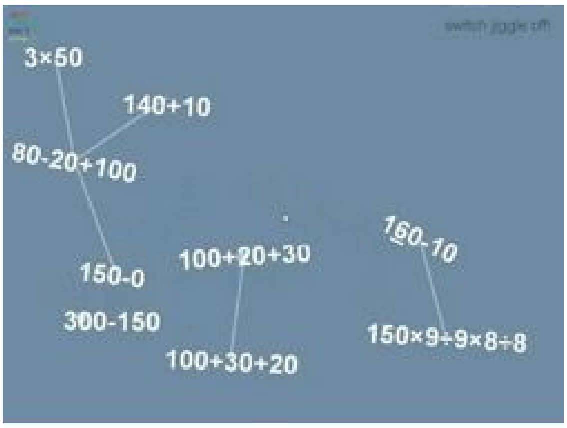


Figure 3. Phase Three of NumberNet: Sorting the Calculations.

Results

Outcome Measures
The data from the pre and post-test were collected and analyzed to explore whether there were differences in number of calculations created, range of operators used and maximum number of operators use in a single calculation. Paired sample t-tests were used to determine whether these scores differed significantly from pre to post test.

Results indicated that the difference in number of calculations created from pre to post test was statistically significant, $t(31) = -3.63$, $p < .001$. The differences in range of operators was not significant, $t(31) = .55$, $p=.59$. The difference in maximum number of operators used was not significant, $t(31)=-1.53$, $p = .14$. Descriptive statistics are displayed in table 1.

Table 1: Descriptive statistics for number of calculations, range of operators and maximum number of operators.

	Pre-test		Post-test	
	Mean	SD	Mean	SD
Number of Calculations	5.91	3.49	8.59	4.63
Range of Operators	2.5	1.14	2.41	.84
Max number of operators	1.38	.94	1.56	1.13

Within-group Learning

While our data indicated an increased number of calculations created, but not an increase in the maximum number of operators used, or an increase in the range of operators used, through exploration of one group's process, we can see some changes to their calculations during the task, shedding light on the mechanisms through which NumberNet might support the developmental of mathematical understanding.

The focus group is made up of two boys, Jack and Thomas, and two girls, Olivia and Amy. This group represent a wide range of mathematical skills, ranging from Jack, who produced two correct (and one incorrect) calculation at pre-test to Thomas who produced seven correct calculations at pre-test. The group also included Olivia, who only produced three calculations at pre-test, although one of these was a 4-operator calculation, one of the longest calculations seen in the entire data-set and the only multi-operator produced at pre-test by the members of this group. At pre-test, Amy produced five correct and one incorrect calculation. The calculations created at pre and post tests (as well as the target numbers) are shown in table 2.

There were two primary changes to the group's post-test data that can be tracked back to the group's process. The first is the introduction of multi-operator expressions. During the course of the activity, Olivia created a number of multi-operator calculations; members of her group were then seen to also create multi-operator expression, possibly as a reaction to seeing her use this form of calculation (usually just two operators, without accurate use of brackets/parentheses). At the post-test, we see Amy produce a multi-operator calculation (6*8*10), suggesting the possibility that she had picked this up from observing Olivia's behavior during the activity.

The second change that we saw was in Jack's calculations from pre to post test. At pre-test, Jack created two correct and one incorrect calculation. His pre-test calculations contained one correct subtraction and one correct addition, at post-test this increased to three correct additions, two correct subtractions and a multiplication calculation (all single operator expressions). Three of these are also small manipulations around his target number (e.g. 499+1=500; 500-0=500). As this type of pattern was created by other members of his group, it suggests that he may have noticed that he could create these simple calculations, and used this strategy as a way to create more calculations at post-test.

Table 2: Calculations created by each of the focus group students at pre and post (not all are correct).

Jack		Thomas		Olivia		Amy	
Pre (320)	Post (500)	Pre (320)	Post (160)	Pre (240)	Post (235)	Pre (180)	Post (480)
311+9	1+499	3200/10	1600-100	200+40	200+35	60*3	470+10
10-330	503-3	200+120	80*2	100+140	200+30+5	1800/10	479+1
20+300	10+490	20+300	16*10	50+50+50 +50+40	100+100+35	179+1	6*8*10
	300+200	310+10	100+60		50+50+50+50 +30+5	380-200	400+80
	100*5	1+319	17*10-10		100+100+20+ 10+4+11	174+6	500-20
	500-0	320+0	40+120		200+19+11+3 +2		438-3
		2+318	1+159				
			20+140				
			10+150				

Discussion

The results of our pilot study exploring the value of NumberNet for increasing mathematical flexibility and developing adaptive expertise in upper primary students indicated that there was a significant increase in the number of calculations created by students from pre to post test. This result suggests that from a single use of

NumberNet, improvements in flexibility with mathematical concepts were evident. Exploration of the process also indicated that these improvements may have been due to the collaborative nature of the activity.

While the results from the pilot study presented in this paper indicate the possibilities of NumberNet, other explanations include the chances that these occurred due to the opportunity of practicing creating calculations, rather than any direct effect of the tool. Further studies comparing students who used NumberNet with students who practiced the paper-based version of the activity are necessary to determine whether this was in fact the case. Further studies will also explore the role of the third phase of NumberNet in more detail to understand how the process of sorting or creating structure of calculations may foster numerical flexibility.

Additionally, the data presented in this study came from two pilot tests of the tool; each test was orchestrated by a different teacher. While both teachers have extensive in-classroom experience using the Explode-A-Number activity in a standard classroom and experience using the multi-touch enabled classroom; the NumberNet activity was new to them. They both expressed the need to develop the pedagogical features of this activity in more detail, helping the teacher to understand how best to manage and orchestrate the activity and scaffold students' conversations in the final stages of the task to take the most advantage of the tool. Future development will include the pedagogical supporting documents and further testing with a range of teachers. Finally, due to technical issues, the third phase of the activity could not be completed by every group; we anticipate that this phase will lead to more complex conversation around the activity, and that greater changes in flexibility and the range of calculations would be seen in groups who had the opportunity to fully engage with this part of the tool.

NumberNet was designed to take advantage of the affordances of a multi-touch classroom. in this way, the tool is an example of CSCL embedded in the classroom, which supports small group, intra-group and whole class learning and facilitates the movement between these phases for the teacher. It also provides evidence for the way in which integrating technology into the classroom can be used to introduce new types of interactions between students, and engagement with content and ideas. By leveraging the affordances of the technology to promote sharing of strategies in tasks similar to Explode-A-Number, students have the opportunity to learn from each other and recognize different ways of approaching a similar task.

Learning to be flexible users of knowledge has been described as adaptive expertise (e.g. Hatano & Inagaki, 1986), and recognized as an important skill to develop for future learning and transfer. To develop adaptive expertise, Schwartz, Bransford & Sears (2005), suggest that patterns of learning should balance the development of efficiency using strategies, with experiences of innovation, keeping learning with the 'Optimal Adaptability Corridor'. Evidence from prior research indicates that when students learn mathematics in ways focus on the development of efficiency, deep understanding of the constructs can be limited, while students who learn using a more innovative approach took longer to master the fundamental ideas but understood the concepts and were better prepared to transfer their knowledge into a new situation (Martin & Schwartz, 2005). In NumberNet, students can move between using their pre-existing efficient strategies for creating calculations and innovating new strategies through observation or collaboration with their peers during the initial stages of the task. The third phase requires all students to innovate to structure the calculations that were created. The teacher can guide this activity in a number of ways, and provide support for the whole class through projection of each group's calculation maps. In future work, the use of NumberNet over time will be explored, to determine whether it promotes the development of flexibility and adaptive expertise in primary mathematics.

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