Embodied Experiences within an Engineering Curriculum

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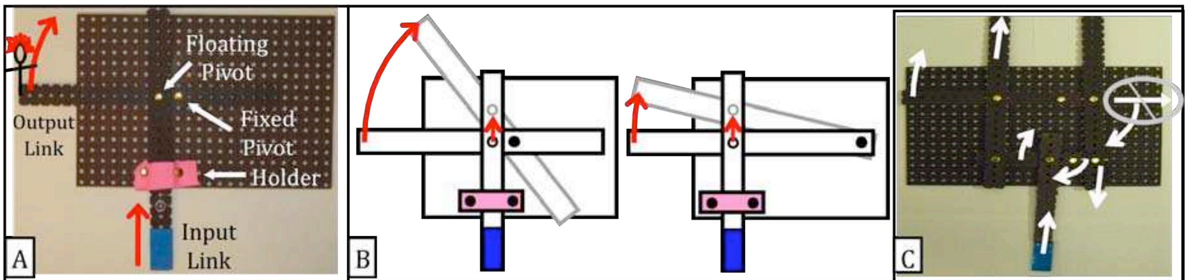
Abstract: Although simple mechanisms are commonplace, reasoning about how they work—mechanistic reasoning—is often challenging. To foster mechanistic reasoning, we engaged students in the third- and sixth-grades in the design of kinetic toys that consisted of systems of linked levers. To make the workings of these systems more visible, students participated in forms of activity that we conjectured would afford bodily experience of some of the properties of these mechanisms: constraint and rotary motion. Students progressively re-described and inscribed these embodied experiences as mathematical systems. We report a microgenetic study of one case study student, tracing how embodying and mathematizing motion supported the development of reasoning about how levered systems work.

Introduction

Reasoning about mechanisms (i.e., mechanistic reasoning) is a key to understanding the designed world. Yet, despite the ubiquity of machines in our culture, and concerted efforts to support mechanistic reasoning in schooling (e.g., simple machine curricula in elementary grades), most students continue to find this form of explanation challenging (Lehrer & Schauble, 1998; Metz, 1991). To support the growth of mechanistic reasoning, we engaged elementary students in the design and construction of kinetic toys. However, our approach to learning by design included efforts to embody and mathematize (Kline, 1980) mechanical systems, such as the simple levered machines depicted in Figure 1. Though mathematical ideas are an intricate part of explanations for mechanical systems, engineering curricula for children often neglect mathematics as a resource (Prevost & Nathan, 2009). Our guiding conjecture was that mathematical description of levered systems would help children reason causally about objects and relations within the system. Our approach was anchored in a body-syntonic (Papert, 1980) wherein initial mathematization emerged from bodily activity, which was then re-expressed with the operation of the simple machine. The instructional design was informed by a prior study of children's naïve reasoning about these simple machines, in which children predicted and explained the direction and amount of output motion.



Levered machines are constructed from pegboard and links. Floating pivots are formed by connecting two links and fixed pivots by connecting a link to the pegboard. Holders restricted a link's sideways motion. Outputs were marked with stick figurines.

Key: Floating Pivot ○ Fixed Pivot ● Amount of Input Push ↑

Contrasting machines A1 and A3 were used to elicit children's reasoning about relative amounts of motion for input and output. Given the same input push, the output for Machine A1 will move much further than that for Machine A3.

Arrows show motions that could be visualized when reasoning about motion of this machine. The circled and crossed out arrow shows impossible motion that preserves the static shape of the machine.

Figure 1. Pegboard machines explained by children.

Children's naïve mechanistic reasoning

Children in our previous study (Bolger, Kobiela, Weinberg, & Lehrer, 2009) exhibited a wide range of ability in predicting and explaining machine motion. Mechanistic explanations existed, but more commonly children explained machine motion by describing pattern regularities that they noticed (ie they related a cause and effect without explaining the mechanism for the relationship) or they provided fragmented explanations with some of the elements necessary for a mechanistic explanation. Some aspects of levered machine's mechanisms seemed particularly problematic for most children. First, most did not seem to "see" rotation in ways consistent with the operation of the machines. For example, children's gestures often indicated impossible motions that preserved the shape of the machines (Figure 1C). Talk or gesture to indicate rotation of links was not common in most children's explanations. Among the 9 children interviewed, 3 never indicated rotation and 4 did so only in one or two instances. In 22 instances where children drew paths with their fingers to predict output motion that should have been arced, 17 drew straight lines. Even after children moved a machine, they infrequently described rotary paths, seemingly focusing on starting and ending points rather than intermediate motion, as described by Piaget, Inhelder & Szeminska (1960). Second, most children rarely if ever suggested that fixing a link to the board would constrain its motion (4 children never did so and 3 did so on one occasion). Out of 72 total machines explained, there were only 6 instances in which a child recognized *both* constraint of the fixed pivot and rotation, emphasizing the difficulty of *coordinating* these two ideas. Third, children also had difficulty reasoning about relative amounts of input and output motion (which is important to understanding the concept of leverage). Children rarely paid attention to how far links moved, even when a paired contrast was used to draw their attention to this feature.

Theoretical framework for designing instructional supports

In summary, students had difficulties with: 1) Recognizing an output link's rotation and causally ascribing that rotation to the constraint of the fixed pivot; and 2) Noticing and reasoning about the amount of motion traveled by an output link. In an effort to support these foundations of mechanistic reasoning we designed instruction to support development of reasoning about constraint, rotation and amount of relative motion of inputs and outputs.

Our instructional design is centered on embodied experiences that support student understandings of the mechanisms of the physical system and that provide a prospective pathway for mathematization of the system. Figure 2 suggests that mechanistic reasoning emerges from coordination of embodied experience and mathematization, a perspective consistent with Papert's (1980) conjecture about the role of embodied experience in mathematical development and the more recent work of Lakoff and Nunez (2001) and Barsalou (2008).

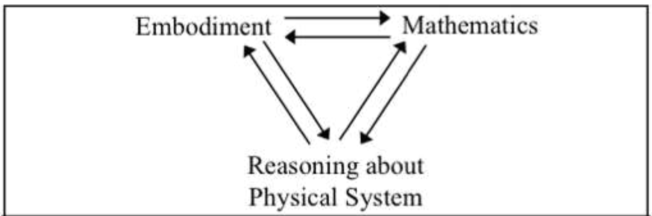


Figure 2. Supporting Learning.

Coordination between embodied experiences and mathematics is consistent with Freudenthal's (1973) notion of "progressive mathematization." Freudenthal states that to implement a program of progressive mathematization successfully, students have to use their experience (e.g., embodied experiences) and invent mathematics through well-developed instructional activities. We drew upon several decades of work leveraging experiences of walking (e.g., a path perspective) as grounds for developing a mathematics of space (e.g., Jordan & Henderson, 1995; Lehrer, Randle, & Sancilio, 1989). In this study, we capitalized on the students' embodied experiences through instruction that supported incorporation of experiential relations into more formal mathematics. These experiences allowed for students to take a path perspective of a moving part within the levered system. Further, we sought to help students integrate these mathematical ideas into their developing explanations for the function of the levered machines.

Disruption of "Straight": The Embodiment of Rotation

This section describes one embodied task designed to take advantage of the relations in our theoretical model (another task was also designed, but we do not describe it here). We unpack task features that could directly contribute to multi-modal conceptual development (Barsalou, 2008) and highlight the relevant mathematical features of the system as potentially experienced by a student. This task was framed around an apparent tension or disruption, thereby problematizing the construction of an explanation for the experienced phenomenon.

The embodied task (illustrated in Figure 3A) was designed to address two qualities of the mechanisms: 1) fixing a link in one place generates a circular path and 2) the length of this circular path is related to the distance from the pivot to the output. We first engaged students in a conception of "straight" from a path perspective as no turns—a constant heading while walking. We then paired students - one held one end of the rope and remained in place (acting as the fixed pivot) while the other held the other end of the rope and attempted to walk in a straight path perpendicular to the rope's orientation (See Figure 3A). The student walking literally experienced the effect of constraint on path, while the student in the "center" experienced the force required to accelerate (constantly changing direction) the walker. The task was experienced twice with different lengths of rope. The constraint disrupted straightness and produced a circular (i.e., rotary) path, turning a constant amount as the student continued to walk at about the same rate. Ideally, this disrupted straight path, and its description as a mathematical object, could serve as a resource for explaining the operation of a system of levers.

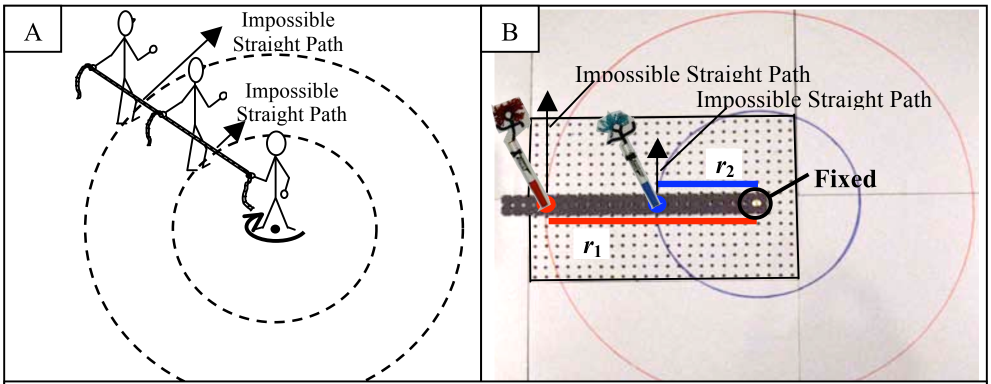


Figure 3. Disrupting "Straight" through Embodied Circular Walks with Rope.

Table 1 elaborates the task's embodied experiences, the mathematics involved in those experiences, and their relations to the physical levered system. During the embodied task, the student walking experienced the disrupted straight path: as he tried to walk straight, the constraint of the rope forced him to turn. Later when the student would be asked to reason about actual levered machines, the system's constraints would be made real by the experience of feeling the rope's pull. To help bridge between the embodied experiences and the physical system "little men" figurines were placed on the links to resemble walkers within the system (Figure 3B). The student who stood in the center, holding the rope, experienced standing in place, feeling "stuck," while pulling the other student toward a circular path. For her, the role of the fixed pivot was magnified as important in resisting the other student's pull to move straight, a resource as she later reasoned about the role of the fixed pivot in the physical system. The different perspectives taken by the two students could influence how the experience served as a resource for reasoning about the physical system. The perspective of the first student was that of a circular path, seeing himself move *around* the student holding the other end of the rope. As he walked, he may have also felt his body constantly turning. When the rope was shortened, this turning accelerated and he may have felt himself going around faster (that is greater angular rotation in the same amount of time) because of the decreased path length. The perspective of the second student did not include a circular walk, but seeing the walker move around her, furthered her experience as "center." When the rope was shortened, the she may have seen the other student moving around her at a faster rate. We conjecture these perspectives are significant in helping students mentally animate the operation of the pegboard machine.

The mathematization of the embodied experience is intended to help highlight relevant features of the experience and the relations between those features that can then be used to reason about the physical system. The stationary student may be thought of as a center of a circle, where the circle is created by the person walking along the constrained path caused by the fixed length of the rope. The rope gives significance to the (otherwise invisible) constant radius of the circle. In this sense, the mathematical features of the experience map directly onto the physical system: the person in center of the circle becomes the fixed pivot, the walking student becomes an output on a link traveling in a circular path. Additionally, the mathematical features can be used to help explain the "unable to walk straight" phenomenon (the path *must* be circular *because* there is a constant radius, - r_1 and r_2 in Figure 3B). Because a radius exists between the fixed pivot and the output, this mathematical relationship causally connects the constraint of the fixed pivot with the rotary motion of the link. The shorter rope may be then thought of as a shorter radius (r_2). In this sense, the path walked is shorter because the circle created is smaller. Mathematically, this occurs because of the proportional relation between the radius and the circumference. Later, when students are asked to reason about the *amount* of motion of an output as it relates to the distance between output and fixed pivot (i.e., radius), they can take the perspective of walking a smaller or larger circle using the longer or shorter rope.

Table 1: Embodied Experiences, Mathematics and the Physical System Related to the Rope Walk.

Embodied Experience	Mathematics	Physical System
A ***person*** holds one end of a ***rope***. Another ***person*** holds the other end.	A point (***center***) is drawn with a line (***radius***) (r_1) extending out from it. Another ***point*** is drawn at the other end.	A ***link*** is attached to a pegboard at the end with a ***fixed pivot***. The other end is marked as the ***output*** (with a figurine).
One person tries to walk straight while the other rotates in place. His straight path is quickly disrupted by the constraining force of the rope held by the other person.	One end of the radius cannot be swept in a straight path without moving *both* ends of the line.	The output cannot be moved in a straight line without removing the fixed pivot.
The person walking is forced by the rope onto a ***path going around the person*** rotating in place. The walker is always the ***same distance*** from the center person.	A ***circle*** is created by sweeping the radius (***constant length***) ***around the center point***.	The output ***rotates around the fixed pivot***. At any point in the rotation, the output always remains the ***same distance*** from the fixed pivot.
The rope is now exchanged for a ***shorter rope*** and the person walking again attempts to walk straight.	The new, ***shorter, radius*** (r_2) is created. Again, the radius cannot be swept in a straight path without moving both ends of the line.	The output (figurine) is now moved in ***closer to the fixed pivot***. Again, the output cannot be moved in a straight line without removing the fixed pivot.
The walker is again forced onto a path going around the person rotating in place. This time, the path is ***shorter*** and the walker goes around ***faster***.	A new ***smaller circle*** (with smaller circumference) is created when the shorter radius is swept around the center point.	The output again rotates around the fixed pivot. This time, the path looks ***smaller***.

Methods

A microgenetic method was employed, featuring one child and one teacher, lasting an average of approximately 7.2 hours over 8 days. The rope embodiment activity, described above, took place on the first day of instruction. Around the fourth day of instruction, most students participated in a second embodiment activity designed to highlight relative input and output motions. Most instruction was centered around simple design challenges (for example, 'Can you make something that moves using these 3 links?' or 'Can you change this machine so that the output moves in the opposite direction?'). Students also designed and constructed a MechAnimation (a decorated toy driven by levers). Participants (n=11, 5 male) (5 third graders, 6 sixth graders) attended two urban schools in the southeastern United States. Sixty-percent of students at the middle school and ninety-percent at the elementary qualified for free or reduced lunch. The children were ethnically diverse and represented a wide spectrum of achievement in school. To assess gains in predicting and explaining machine motion, children were interviewed before and after instruction.

We chose to carefully study the learning of one child, Sarah. We chose Sarah primarily because of her willingness to express her thinking to us. Though she was a fairly successful student in school, she did not demonstrate a strong naïve ability to predict or explain levered machines in her pre-interview. Analysis of Sarah's learning included coding of her interviews (using the framework from our first study), group video noticings (Jordan & Henderson, 1995) of each day of instruction, and microgenetic analysis of the transcript and video to trace prospective relations between embodiment experiences and other instructional activities.

One Student's Use of Embodied Activity in Reasoning about Pegboard Machines

We follow Sarah as she engaged in the embodiment task, represented her experience through drawing, and considered the operation of a single link connected to a pegboard. We highlight ways that the experience seemed to be a resource for reasoning about difficult concepts—that is, how constraint of the fixed pivot necessitates rotary motion, mental animation of the system and relations influencing amount of lever motion.

Sarah's Embodied Experience

As Sarah performed the embodiment task, described earlier, the teacher took hold of one end of the rope and asked Sarah to hold another part of the rope several feet away. The teacher then asked Sarah to "try walking in a straight line that way" (pointing to indicate direction) while keeping the rope taut. Sarah immediately sensed difficulty, at first hesitating ("but uh") and then asked whether she should "turn." The teacher repeated the request to "try to walk in a straight line." As Sarah walked, she described feeling constrained and then *having* to turn. Note that italics in transcript represent ideas that we wish to emphasize; student's or teacher's emphasis is denoted by all caps.

> S: The um-*I can't move any more*…Like the rope can't (?) you *have to turn*.
> T: And then if you kept on just moving (Sarah continues the rotary walk)…what would start to happen?
> S: I would go in like (gestures in a circle with her arm) a *circle*.

Feeling the force of the rope constraining her, Sarah acknowledged that her path was circular. By taking this perspective, she seemed to experience a causal relation between the constraint of the rope and the rotary path. The teacher then highlighted the constant rope length of the walk. She asked Sarah to approximate the amount of rope between them, first standing at one position ("a fourth of the rope") and then standing at another position ("*still* a fourth"). The teacher asked about *all* possible positions: "And then so *anywhere* we turn?" Sarah completed the generalization by concluding that "it's *still* a fourth," By highlighting this feature of their experience, the teacher attempted to help Sarah relate the circular path of her walk to the unchanging distance between her and the teacher.

Finally, Sarah was asked to walk the circular path again, but with a shorter rope length. This time, Sarah was not directed to walk straight but instead immediately asked to "turn." While walking, Sarah initially observed that the path was again a circle and when the teacher asked her for more details, she noticed that "it'd be smaller." The teacher again scaffolded a generalization of the experience, this time emphasizing the relation between rope length and circle size. Moving her hand along the rope, she asked, "so when this becomes smaller?" Sarah completed the generalization, "the circle gets smaller." Thus, through the contrasting walks, a sense of magnitude as "smaller circle" emerged, as well as a relation between the rope length and the distance of the circular path.

Inscribing the Embodied Experience: Moving towards Mathematization

Immediately following the embodied activity, the teacher requested that the student draw two pictures to show the two different rotations: with the long and the short rope. Her drawing (see top of Figure 4A) showed two circles, one smaller than the other, each with a line extending from it with a point at each end. As Sarah began to explain the drawing, it was evident that she had mathematized all of the relevant *components* of the experience (that is, the people as points, the rope as a line and the walk as a circle), but not all of the *relations* between them. Although her drawing accurately depicted relative differences in circle size and radius (smaller rope, smaller circle), it did not represent radius as moving *around* a center and the circular path as emerging from that interaction. However, Sarah realized this problem when attempting to reenact the experience within the drawing.

> S: I drew me (points to top dot in her drawing) and then you (points to bottom dot in her drawing) and then (moves finger back to top dot) I was holding the rope and (moves finger back to the bottom dot) then *when*, no but that's wrong.

Sarah's inability to animate her drawing motivated a new one (see bottom of Figure 4A). This time, she animated the static representation as she drew and explained. As the static components of the drawing were made dynamic, relations among them emerged. While explaining her drawing, she not only imparted agency within it, but also re-enacted the experience (via gesture) through it: "This one (points to the dot inside the rightmost circle) would be you. And me (points to the dot on the circle) holding the rope *going around* (moving her finger around the circle)."

Sarah's first drawing suggests that while she related "rope length" to circle circumference, this length had not yet been fully mapped to "radius". As she reasoned about the embodied experience and the representation together, her mathematization of the system showed structural relations of the circle—the rope became radius and the teacher (precursor for the fixed pivot) moved *inside* the circle. Her final drawing also symbolized three ideas that became important later in the session – fixed pivot as the center of the circle, the constant radius of the circle, and the relationship between radius and circumference.

Reasoning about Rotation around a Pivot: Mapping to the Physical System

Within the pegboard system, Sarah's notion of "center" began to take on functional significance. After inscribing the embodied experience, the teacher gave Sarah a pegboard, a link, and a brad fastener and asked her to attach the link to the pegboard any way she wanted. Sarah placed the pivot at the very end of the link, a structure resembling the teacher standing and holding the end of the rope. Interestingly, when asked to describe her machine, Sarah spontaneously mapped the link's movement to this experience: "it [the link] can move around but, yeah like it can move around like rotating *how we did*." Moreover, as she continued describing the machine, she noted new relations among the parts of the system. We suspect that the literal resemblance of the machine's structure to the embodied experience perhaps afforded a "bird's eye" perspective of her previous experience that made these relations salient.

> S: …it *can only rotate* in like this spot (points to the fixed pivot) and no other (waves hand over the mechanism) spot…Like the starting point (touches fixed pivot) you can only start here (touches pivot again) and not start (points to the left of the fixed pivot) somewhere else because this (points to the fixed pivot multiple times) is *stuck* to THAT (points to pivot again) place.

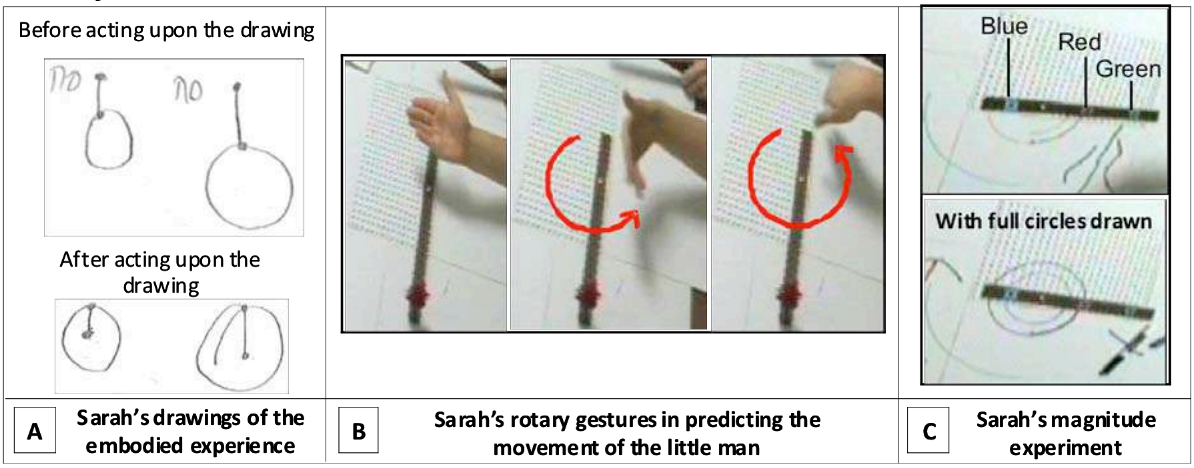


Figure 4. Sarah's use of her embodied experience.

Sarah's notion of "stuckness" attributed to the fixed pivot illustrates an important notion of constraint: that it is unable to move. Furthermore, by describing the rotation as only occurring "in like this spot," she indicated some sense of the source of rotation originating from the fixed pivot.

These ideas of coordinating the rotation of the link to the constraint of the fixed pivot resurfaced again a little later in the session. The teacher had moved the fixed pivot in towards the center of the link and had placed a little man figurine on the left end. She asked Sarah to predict the motion of the little man supposing they were to push the link, pointing to a particular place to the right of the fixed pivot. Sarah correctly predicted the direction of the man, invoking her knowledge of the link's rotary motion. She explained that, "this [the link] will *spin*," gesturing with her hand in a circular motion (Figure 4B). During her pre-interview, Sarah didn't talk about turning or spinning and didn't make rotary hand gestures (with the exception of her noticing the rotary motion of a round-shaped piece of pegboard), suggesting a different way of thinking that may have been supported by the tasks that proceeded this episode.

After testing Sarah's prediction, the teacher asked her how she had known that the path would be curved. In her explanation, Sarah again invoked the importance of the fixed pivot's immobility, but this time indirectly referencing the experience of trying to walk straight: "…it's not going up, it's just … on one brad and when you hit it (gestures pushing link), it will go, that way (gestures rotation) cause it (points near the fixed pivot) *can't always go straight*. Yeah cause it can't move up. It's *in its place*." Here, unlike in her earlier talk, the rotation of the link and the "stuckness" of the fixed pivot are more than simply related. Rather, Sarah now attributes the rotation to the constraint of the fixed pivot, giving it functional significance.

Using Mathematics of the Circle to Reason about Magnitude of Link Motion

The final task undertaken on the first day of instruction was to focus on the relationship between two distances - the distance from a point on the link to the fixed pivot (radius) and the distance traveled by that point (arc length or circumference). This relationship forms the precursor for understanding the relationship between the magnitude of input and output motions. For this task, the teacher stipulated that the link would be rotated ¼ turn, marking the starting and stopping points. She then asked Sarah to choose 3 places on the link to put colored stickers to correspond with where the "little men" would be placed (Figure 4C, top image).When asked to predict how each sticker would move, Sarah understood that all stickers would rotate, but their paths would have different lengths: "I think that. They will move different-No they won't move differently but they're um (points to the board and then moves finger in a circle) lengths I think will be different." When the teacher talked further to Sarah about her prediction, Sarah spontaneously referred to her embodied experience "So they'll like (gestures finger in circles again) turn the same way but like our *rope* was different lengths and their shape is going to be smaller or bigger." Hence, Sarah seemed to use her experiences to reason that the rope length (radius) would affect the size of path walked (circumference) in this simple physical system. However, her specific mapping of the "rope length" onto the link was less clear as she went on to incorrectly predict the relative distances that each sticker would travel.

Sarah was guided by her teacher to find a generalization that would allow her to accurately predict the relative amounts of motion for points along the link, but she eventually came to an impasse. She explained her first prediction (blue>green>red) this way: "So yeah like the outer it is the bigger it's gonna be so this (points to blue sticker) is outer". Here, Sarah would be correct, if the link rotated around its center rather than the fixed pivot. After measuring the relative amounts traveled by the stickers (green>red>blue), Sarah created a new incorrect explanation to fit the observations at hand, "I think that the closer it is to (points to green sticker) this end, it will go the most and the farthest it is to that end (points to the right end of the link), it will go the least." When the teacher offered a counter example to disprove this new theory (the distance traveled by a point at the far left end is greater than the distance traveled by the blue sticker), Sarah was unable to pose a new explanation. At this point, it seemed that the fixed pivot was invisible when Sarah reasoned about these relative distances.

Finally, the teacher helped Sarah draw the full circles traveled by the stickers and by the point at the far left end of the link (Figure 4C, bottom image). This far left point, which began as an unexplainable counter example, was repurposed to provoke an explanation that involved the radius and the fixed pivot as center of the circle:

> T: So why do you think the [point at the left end] and the red went the same? Are on the same *circle*?...
> S: (gasps) Oh:: Maybe because like uh this one is (points to the far left end) one two three four (counting the holes) five six and that one's (points to the red sticker) one two three four five six. It's the same distance close to the brad.

The relevant mathematical relationship seemed to appear when the circles were drawn. This teacher move served a dual purpose – drawing the visual focus to the fixed pivot as center and bringing the task closer to the embodied experience. In the preceding section, we saw that Sarah seemed to draw from her embodied experience as she reasoned about the direction of turn for this same machine. However, when she moved to a slightly more complex task, seemingly small details (i.e. locating the pivot away from the end of the link and drawing arcs instead of complete circles) may have been enough to disrupt a literal mapping to the embodied experience. This speaks to the importance of teacher sensitivity about appropriate scaffolds that will allow for best use of embodied experiences. Interestingly, on later days of instruction, Sarah spontaneously invoked the embodied experience when asked to reason again about the radius-circumference relationship, suggesting that the experience was a useful support beyond its immediate context. For example, in the third day of instruction, when Sarah was asked again to explain the same phenomenon (also with a single link mechanism), she provided the following explanation.

> T: Why is it when it's really far it's making a big circle, like what MAKES the circle?
> S: Cause us like when we were doing the *rope*. There is only like, pretend this is me (points to green sticker) and that was you (points to pivot). There was only like this much *distance* between us.
> T: You can only like (turns her body in constrained manner)
> S: Yeah, like move that much.

Sarah's difficulty with reasoning about magnitude of link motion prompted the development of another embodied experience used later in instruction.

Conclusion

We designed embodied experiences (Papert, 1980; Abelson & diSessa, 1980) to support "mathematization" (Kline, 1980) of mechanical systems, namely those containing simple levered machines. Drawing from our previous studies, we targeted particular naïve student conceptual difficulties that were seen in most students, even when working with the machines. These were: viewing rotary motion in the machines, attending to the amount of output link motion, and reasoning about the constraint resultant from fixing a link to the pegboard. Embodied experiences served as a resource for progressive mathematization of the system (Kline, 1980; Freudenthal, 1973), aided by strategic teaching supports, such as: asking the student to represent the literal embodied experience on paper, asking the student to reason about a physical system that closely resembled the embodied experience and frequently revisiting how relevant features of the embodied experience mapped to the physical system.

With our case study student, Sarah, the embodied rope experience and subsequent teaching supports seemed to help her reason about each of our target concepts. Though Sarah did not address the rotation of links in her pre-interview, this idea flowed readily from the embodiment experience. Within the experience, she noticed herself going “around,” an idea later resurfacing in her drawings, talk and gesture. Though this experience helped Sarah readily see circular paths, she required additional supports to see other mathematical features of circles. Reasoning around her representation seemed to help Sarah map the “rope” to radius. As Sarah revised her drawing, she appeared to reenact the experience, imparting agency and replaying motion through gesture. Further, the relationship between radius and circumference, a necessary precursor to understanding the *amount* of output motion, was not obvious to Sarah. By helping Sarah work with the physical system (and adjusting it to visually cue the embodied experience), the teacher was able to guide her to discover this relationship and map it in a lasting way to the embodied experience.

Sarah’s process of reasoning about how the fixed pivot served to constrain motion was refined throughout the first day of instruction. During the embodied experience, she talked about how her walk was constrained to a circular path, but the source of this constraint was not evident. As she refined her representation, the fixed pivot (teacher) was moved *inside* the circle. When she built the simple physical system and mapped it back to the embodied rope experience, she noted for the first time that the fixed pivot was “stuck” and assigned it significance as the source of rotation. When directed to reason about the path of a “little man” on the link she causally connected his circular motion with the fixed pivot as constraint. However, as instruction continued Sarah had difficulty locating the fixed pivot as the circles’ center as well as maintaining a notion of fixed pivot as the source of the disruption of straight. We conjecture that Sarah’s lack of experience acting as the fixed pivot in the embodied experience may have made her less aware of its role as a constraint.

Our results suggest that integrating mathematics into an engineering curriculum can support students as they begin to construct explanations for simple machines. However, many questions remain as to how best integrate mathematics in ways that will be accessible and meaningful to students and that will best support mechanistic reasoning. Current research is focused on answering these questions through further microgenetic analysis as well as analysis of data from intact classrooms. Classroom instruction used similar embodied activities, but was able to capitalize on the variety of student ideas. Comparison of ideas was used to motivate student reasoning.

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