Discovering Dependencies: A Case Study of Collaborative Dynamic Mathematics

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Abstract: The Virtual Math Teams (VMT) Project is exploring an approach to the teaching and learning of basic school geometry through a CSCL approach. As one phase of a design-based-research cycle of design/trial/analysis, two teams of three adults worked on a dynamic-geometry task in the VMT online environment. The case study reported here analyzed the progression of their computer-supported collaborative interaction, showing that each team combined in different ways (a) exploration of a complex geometric figure through dynamic *dragging* of points in the figure in a shared GeoGebra virtual workspace, (b) step-by-step *construction* of a similar figure and (c) discussion of the *dependencies* needed to replicate the behavior of the dynamic figure. The teams thereby achieved a group-cognitive result that most of the group members might not have been able to achieve on their own.

Based on a Vygotskian perspective, our CSCL approach to the teaching of geometry involves collaborative learning mediated by dynamic-geometry software—such as Geometer's Sketchpad or GeoGebra—and student discourse. During the past decade, we have developed the Virtual Math Teams (VMT) environment and have recently integrated a multi-user version of GeoGebra into it (Stahl, 2009; Stahl et al., 2010). Our environment and associated pedagogy focus on supporting collaboration and fostering significant mathematical discourse. In developing this system, we have tested our prototypes with various small groups of users. Recently, two small groups worked together on a problem based on the construction of inscribed equilateral triangles (see Figure 1).

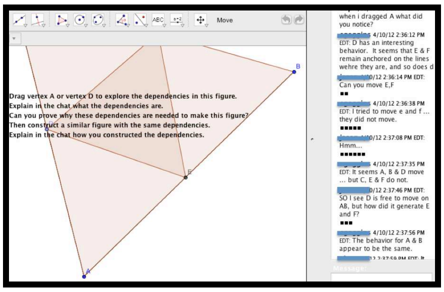

Figure 1. Discussion of the inscribed triangles problem.

The geometry problem is adapted to the VMT setting from (Öner, 2013). In her study, two co-located adults were videotaped working on one computer screen using Geometer's Sketchpad. We have "replicated" the study with teams of three adults working on separate computers with our multi-user version of GeoGebra in the VMT environment, allowing them to construct, drag, observe and chat about a shared construction. Öner chose this problem because it requires students to explore a dynamic-geometry figure to identify dependencies in it and then to construct a similar figure, building in such dependencies. We believe that the identification and construction of geometric dependencies is central to the mastery of dynamic geometry (Stahl, 2012b; 2013).

In this study, we analyzed the processes through which the two groups (A and B) identified and constructed the dependencies involved in an equilateral triangle inscribed in another equilateral triangle. We were able to replay the entire sessions of the groups in complete detail, observing all group interaction (text chat and dynamic-geometry actions) that group members observed—for logs and analysis, see (Stahl, 2013, Ch. 7).

Group A went through a collaborative process in which they explored the given figure by varying it visually through the procedure of *dragging* various points and noticing how the figure responded. Some points could move freely; they often caused the other points to readjust. Some points were constrained and could not be moved freely. The group then wondered about the constraints underlying the behavior. They conjectured that certain relationships were maintained by built-in dependencies. Without having figured out the constraints completely, they began trying to *construct* the figure as a way of exploring approaches experimentally using trial and error. Finally, the group figured out how to accomplish the construction of the inscribed equilateral triangles by defining the *dependencies* into their figure using the tools of GeoGebra.

Team B went through a similar process, with differences in the details of their observations and conjectures. Interestingly, Team B made conjectures leading to at least three different construction approaches. Like Group A, they initiated a collaborative process of exploring the given diagram visually with the help of *dragging* points. They developed conjectures about the constraints in the figure and about what dependencies would have to be built into a construction that replicated the inscribed equilateral triangles. They decided to explore trial *constructions* as a way of better understanding the problem and the issues that would arise in different approaches. Eventually, they pursued an approach involving *dependencies* among line segments in the three congruent smaller triangles.

Although both groups reached a similar conclusion, their paths were significantly different. First, they defined their problem differently (Zemel & Koschmann, 2013). Group A focused on listing the constraints that they noticed by dragging points and then on proving that the given triangles were in fact equilateral. Group B, in contrast, quickly realized that it would be difficult to construct triangle DEF to be both inscribed and equilateral, since these characteristics required quite different constraints, which would be hard to impose simultaneously. Whereas Group A coordinated its work so that the members followed a single path of exploration and conjecture, Group B's members each came up with different conjectures and even engaged in some divergent explorative construction on their own before sharing their findings. Despite these differences, both groups collaborated effectively. They listened attentively and responded to each other's comments. They solicited questions and agreement. They generally followed a shared group approach. Together, they reached an accepted conclusion to a difficult problem, which they might not all have been able to solve on their own, illustrating effective group cognition (Stahl, 2006).

The case study of Groups A and B illustrates the approach of collaborative dynamic geometry. The groups took advantage of the three central dimensions of dynamic geometry—dragging, construction and dependencies—to explore the intricacies of a geometric configuration and to reach—as a group—a deep understanding of the relationships within the configuration. They figured out how to construct the diagram and they understood why the construction would work as a result of dependencies that they designed into it.

The inscribed-triangle problem illustrates well the importance of dragging, constructing and dependencies in dynamic geometry. This argues against the current tendency in classroom usage of dynamic geometry software—and in the related research—to emphasize just the dragging. In our log analyses, we can observe clearly the role of all three aspects working together: in Öner's data of the dyad, in our case study of Group A and B as well as in subsequent logs of groups of math teachers collaborating on this problem.

In the Virtual Math Teams Project, we are currently refining the VMT software and developing curriculum (Stahl, 2012b) to guide the use of collaborative dynamic geometry in in-service-teacher professional development and high-school geometry (Stahl, 2012a). The curriculum centers on activities like the one in the case study. The curriculum is closely aligned to the new Common Core State Standards for basic geometry and their recommended mathematical practices (CCSSI, 2011). It covers the most important propositions of Euclid's *Elements*, translating them into research-based, contemporary approaches to geometry and mathematical discourse in a CSCL environment. We will continue to study the results of collaborative dynamic geometry through analysis of the discourse and geometric explorations (Stahl, 2012c). On the basis of a continuing series of trial studies like the one just reported, we feel that the approach of collaborative dynamic geometry can translate the geometry of Euclid into an effective tool of computer-supported collaborative learning.

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