Classroom Model, Model Classroom: Computer-Supported Methodology for Investigating Collaborative-Learning Pedagogy

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Abstract: We have been exploring the potential of agent-based modeling methodology for social-science research and, specifically, for illuminating theoretical complementarities of cognitive and socio-constructivist conceptualizations of learning (e.g., Abrahamson & Wilensky, 2005a). The current study advances our research by applying our methodology to pedagogy research: we investigate individual and social factors underlying outcomes of implementing collaborative-inquiry classroom practice. Using *bifocal modeling* (Blikstein & Wilensky, 2006a), we juxtapose agent-based simulations of collaborative problem solving with real-classroom data of students' collaboration in a demographically diverse middle-school mathematics classroom (Abrahamson & Wilensky, 2005b). We validate the computer model by comparing outcomes from running the simulation with outcomes of the real intervention. Findings are that collaboration pedagogy emphasizing group performance may forsake individual learning, because stable division-of-labor patterns emerge due to utilitarian preference of short-term production over long-term learning (Axelrod, 1997). The study may inform professional development and pedagogical policy (see interactive applet: http://ccl.northwestern.edu/research/conferences/CSCL2007/CSCL2007.html).

Background and Objective

We present a new methodology for developing and critiquing education theory, *agent-based modeling*. Agent-based modeling (hence ABM) has been increasingly used by natural scientists to study a wide range of phenomena such as the interactions of species in an ecosystem, the interactions of molecules in a chemical reaction, the percolation of oil through a substrate, and the food-gathering behavior of insects (e.g., Bonabeau, Dorigo, & Théraulaz. 1999; Wilensky & Reisman, 1998, 2006). Such phenomena, in which the elements within the system (molecules, or ants) have multiple behaviors and a large number of interaction patterns, have been termed *complex* and are collectively studied in a relatively young interdisciplinary field called *complex systems* or *complexity studies* (e.g., Holland, 1995). Typical of complex phenomena is that they lend themselves to two or more layers of description—e.g., collisions of particles in a gas chamber are the "micro" events, and pressure is the "macro" event— and the cumulative ('aggregate') patterns or behaviors at the macro level are not premeditated or directly actuated by any of the "lower-level" micro elements. For example, flocking birds do not intend to construct an arrow-shaped structure. Rather, each element ("agent") follows its "local" rules, and the overall pattern arises as epiphenomenal to these multiple local behaviors—the overall pattern *emerges*.

Specialized computer-based environments (Collier & Sallach, 2001; Langton & Burkhardt, 1997; Wilensky, 1999) have been developed as research tools for investigating complex phenomena (North et al., 2002; Wilensky, 2001; Wilensky & Reisman, 2006). The agents can be instantiated in the form of a computer program that specifies their rule-based behaviors. ABM is thus particularly powerful for studying complex phenomena, because once the modeler assigns agents their local rules, the modeler can set these virtual agents into motion and watch for any overall patterns that arise from the agents' interactions. E.g., the modeler might assign a group of virtual birds a set of rules and then watch their interactions to see whether typical flock structures emerge (Reynolds, 1987).

Whereas initially complex-systems methods and perspectives arose from the natural sciences, complexity, emergence, and micro- and macro levels of description of phenomena are all highly relevant to research in the *social* sciences. Indeed, the recent decades have seen a surge in social-science studies employing ABM (Axelrod, 1997; Diermeier, 2000; Epstein & Axtell, 1996). *Learning, too, we argue, can be construed as a complex phenomenon, and thus ABM is a potentially powerful research tool conducive to the investigation of patterns, including structures and rules, underlying the emergence of learning*. Specifically, we are proposing to use ABM in investigating the social dynamics underlying participation patterns observed when students interact around collaborative classroom assignments, such as construction projects. Thus, whereas our paper deals squarely with 'mice, minds, and

societies'—the theme of the CSCL 2007 conference—we are not so much dealing with computer-supported collaborative learning as such, as much as with computer-supported *inquiry into* collaborative learning (CSiiCL). Nevertheless, we hope to lay out agenda and methodology to facilitate synergies between the CSCL and complexity-studies communities—synergies that increase understanding, within education research, of mechanisms and practice pertaining to individual learning within social contexts. Thus, armed with computers as methodological tools for ultimately improving collaborative learning, this paper *is* about computer-supported collaborative learning.

We have been working with the *NetLogo* (Wilensky, 1999) multi-agent modeling-and-simulation environment. A vision of the NetLogo development effort is that building simulations will become common practice of natural/social-sciences scholars investigating complex phenomena: the scholars themselves—not hired programmers—build, run, and interpret the simulations (Tisue & Wilensky, 2004; Wilensky, 2003). The new lenses of ABM, we believe, will enable education researchers to explore, articulate, develop, and share an intuition we have struggled to study rigorously and express coherently: the intuition that individuals and communities are interdependent through myriad dynamic reciprocities (Cole & Wertsch, 1996; Greeno, 1998).

In the remaining sections of this paper, we: (1) introduce the case study—a collaborative construction project in a demographically diverse middle-school mathematics classroom studying combinatorial analysis (Abrahamson, Janusz, & Wilensky, 2006; Abrahamson & Wilensky, 2005b); (2) discuss the rationale, design, and implementation of a complexity-based analysis of the case-study's participation patterns, i.e., an agent-based model that purports to simulate this phenomenon; (3) introduce 'bifocal modeling' (Blikstein, Abrahamson, & Wilensky, 2006), a computer-assisted research technique for juxtaposing real and simulated data toward calibrating the simulation such that it emulates the real data—we demonstrate this juxtaposition by aligning participation patterns in our classroom data with simulated patterns emerging in the ABM; (4) report findings; and (5) offer concluding remarks on the implications of this study and the limitations of ABM and suggest directions for further research.

Case Study: Emergence of a Stratified Learning Zone in a Collaborative Project in a Demographically Diverse Mathematics Classroom

Complexity-studies methodology is particularly suitable for understanding student learning in pedagogical frameworks that support individual agency. When such classrooms engage in collaborative construction projects, participation patterns emerge, some that may be undesirable, from the educator's perspective. As we explain, below, these patterns emerge through iterative student-to-student negotiation of roles vis-à-vis students' skills and their interpretation of the overall classroom objectives. When these objectives are taken to be *production* rather than *learning*, inequitable participation patterns may emerge, because students are rewarded for their contribution to production rather than for their learning. The interactions of these two reward systems (the first, indexing students' contribution toward successful completion of a group project; the second, indexing students' own learning) is a complex system—ABM enables us to study the nature of students' iterative negotiations that give rise to the inequitable participation patterns. Thus, simulating participation patterns could provide designers and teachers valuable tools for running equitable classrooms. In particular, understanding the emergence of inequitable participation may help educators formulate responses that temper production to the benefit of learning. We now explain the case study that we investigated using ABM.

The Combinations tower: A Combinatorial-Analysis Collaborative Project

The current investigation uses data from a design-based research study of middle-school students' mathematical cognition pertaining to the topic of combinatorial analysis (Abrahamson, Janusz, & Wilensky, 2006; Abrahamson & Wilensky, 2005a). Central to the study was an implementation of a challenging classroom collaborative project—the construction of the *combinations tower*, the exhaustive sample space of a 3-by-3 grid of nine squares that can each be either green or blue (for a total of 512 distinct "9-blocks"). The classroom, working in groups, created all the 9-blocks and assembled them into a very tall "histogram." This histogram consisted of 10 columns running from "no-green" through "9-green" (the columns' heights were, respectively, 1, 9, 36, 84, 126, 126, 84, 36, 9, 1—coefficients in the binomial function $[a + b]^9$).

The Stratified Learning Zone: Group Dynamics From an Emergence Perspective

Data analysis revealed unanticipated participation patterns. Namely, individual students operating within groups assumed by-and-large restricted roles that we named: (a) "number crunchers"; (b) designers; (c) producers; (d) implementers; (e) checkers; and (f) assemblers; and in addition, some students operated between groups as (g)

ambassadors. We demonstrated the descending mathematical challenge of the *a*-through-*f* roles, e.g., the designers initiate combinatorial-analysis strategies, the implementers carry out these strategies, and the assemblers glue the 9-blocks onto a poster. We demonstrated that students' individual roles were related both to their mathematical achievement, as reported by the teacher, and their demographics. We argued that these roles were emergent and that they affected the students' learning opportunities and self image and that therefore it is important to understand how some students landed up on the lower rungs of the production line—how a stratified learning zone emerged.

A *stratified learning zone* is a design-engendered hierarchy of students' potential learning trajectories along problem-solving skill sets, each delimited in its conceptual scope, and all simultaneously occurring within a classroom. In comparison, the term *continuous learning zone* depicts a space wherein students can each embark from a core problem, sustain engagement in working on this problem, and build a set of skills wherein each accomplishment suggests, contextualizes, and supports the exploration and learning of the successive skill, so that a solution path is learned as a meaningful continuum.

Validation Through Feedback From the Students and Teacher

Based on interviews with the teacher and the students, we formulated the following agent-based explanation of the emergence of student task distribution. Students' roles emerged as a function of individual student interactions: Within a group, once a student realized that he had reached his limit in terms of mathematical problem solving as compared to another student within that group, the first student would often capitulate to his group-mate the task of pursuing that mathematical problem, and then she would take over, relegating to him a necessary task that was within his zone of achievement, thus freeing herself to focus on the problem he had abandoned. A network of symbiotic relationships crystallized as the more advanced students assumed leadership of their groups and as the emergent task specifications were articulated in terms of student roles and student-to-student and group-to-group negotiated partnerships. The likelihood of an individual student dominating another was affected by personality traits: of the mathematically advanced students, those who were less socially fluent preferred to work individually, whereas "bossier" students were more likely to assign tasks to other group members.

Exploration Vs. Exploitation: A Perennial Tradeoff of Collaborative Inquiry?

When a classroom that is engaged in collaborative project-based activity progresses towards successful completion of the project, could there be any justification to tamper with this progress? And yet, is a facilitator ethically permitted to sacrifice individual students' learning so as ensure the completion of the project? To address this design-and-facilitation dilemma, we will now turn to a complexity-studies perspective on organizations. One could arguably model the study classroom as an organization, a collective of individuals with some shared objective and a modus operandi for working towards this objective. There are no monetary stakes involved, but certain roles enable some students to gain knowledge capital, whereas other roles do not. Our motivation to model the classroom as an organization is that construction projects may tacitly import to the learning space ethics, ethos, and praxis of working spaces that may not be entirely beneficial for all students.

Axelrod and M. D. Cohen (1999) discuss *exploration versus exploitation*, a tradeoff inherent in *complex adaptive systems*, such as organizations. For instance, in allocating resources, an organization must determine which strategy will maximize its benefits—"mutating" to check for better fits with the changing environment or stagnating and cashing in on a proven model of success. Typically, "the testing of new types comes at some expense to realizing benefits of those already available" (Axelrod & M. D. Cohen, 1999, p. 44). We submit that a classroom can be seen as a complex adaptive system (Hurford, 2004), at least in terms of students' within-group free-range agency in problem solving and the interactions that shape these agencies. Initially, all students are explorative. Yet, once a functioning coordination scheme has evolved that is apparently well adapted to the environment, i.e. the classroom-as-a-whole is apparently progressing along a trajectory towards successfully completing a prescribed task and positive sanctioning is received from the forces that be (the facilitators), an implicit quietus is set on any further exploration, and the group achieves dynamic stability. From that point on, the individual cogs in the production mechanism hone their skills and produce (see Durkheim, 1947, for a social critique of the division of labor).

Finding: Some Answers, New Questions

When students are given the freedom to explore a problem collaboratively, both remarkable and undesirable group behaviors may emerge. It is not a zero-sum game—these "pros and woes" need not cancel each other out. An experienced and able teacher who anticipates this emergence and is sensitive to unforeseen behavior can steer this sensitive dependence so as to optimize student sharing and learning opportunities. The proposed

methodology introduced in this paper may provide education researchers, designers, and practitioners tools for understanding classroom dynamics such that they can identify points of leverage for working *with* students' natural behavioral inclinations to achieve equitable participation. The next section explores this possibility.

Implementing a Theoretical Model of the Case-Study Emergent Classroom Participation Pattern in the Form of "Runnable" Agent-Based Procedures

In this section, we demonstrate the applicability of ABM methodology for the investigation of pedagogical practice by explaining our design rationale for simulating the emergence of a stratified learning zone in a virtual classroom. Also, we demonstrate the iterative nature of this methodology by describing some of our key understandings, along the modeling process, that informed the improvement of the model. Whereas this paper is primarily *methodological*—we use particular research content so as to demonstrate an investigation technique—the reader may disagree with our *theoretical* model of the causes of stratification. We welcome such disagreement, because we regard it as manifesting a strength of the ABM methodology: scholars from across the disciplines, who may not share literature, constructs, or methodologies, can nevertheless critique each other's work pointedly—ABM is an interdisciplinary *lingua franca* (Abrahamson & Wilensky, 2005a). In fact, readers are welcome to download the model file and modify or replace the procedures so as to express their own hypotheses.

Rationale of the Stratified Learning Zone model: Selection of Key Parameters, Hypothesizing Behavior Rules, and Authoring the Rules Within the NetLogo Environment

Any model, regardless of the medium in which it is expressed, e.g., text, diagram, or agent-based model, is per force an attenuation of the "objective" reality. Initially, the modeler must use circumspection in answering the question, "What is the nature of the phenomenon we are attempting to model?" For example, we asked ourselves whether we are modeling: (a) a specific activity, i.e., "students collaborating on constructing the sample space of the binomial stochastic generator that has 9 variables each with the values "green" and "blue" that are glued onto purple construction paper"; or (b) "students collaborating on a task that demands a variety of roles that range by the content knowledge they foster." We chose the latter option. Next, in building an agent-based model, one defines the agents (e.g., students, teacher) and any other objects at play (e.g., portable artifacts), and assigns the agents properties evaluated as relevant to the phenomenon under investigation, including constants (e.g., gender) and variables (e.g., role in collaborative activity). The modeler's selection of these agents and properties is informed by a general rationale of the model, which the modeler articulates, e.g.:

- Classroom objectives are mandated by a curriculum
- A total of *n* individual students cluster in *m* groups of variable size; whom they group with is a mixture of student and teacher choice (teachers may opt to create either homogeneous or heterogeneous groups)
- Individuals are reinforced by their group-mates for contributing toward a group's objective, where 'contribution' is measured vis-à-vis the project specifications

For the stratified learning zone (SLZ) model, we chose a puzzle task (see Figure 1, below). This linear puzzle consists of set of pieces that need to be concatenated according to a logical sequence. Necessary activities within this task are retrieving pieces (simplest task), connecting pieces (most demanding task), and verifying (intermediate demand). Thus, the roles that students might specialize in are piece-retrievers, piece-connectors, and puzzle-verifiers. Puzzle pieces are scattered all over the classroom. Retrievers wander around and, when they find a piece that they evaluate as useful (it may in fact be incorrect), they go back to their group's table, deliver the piece to the connector and then return to retrieve more puzzle pieces. Upon receiving a piece, the piece-connector evaluates its fit to the puzzle in its current state. If the piece is not suitable, the piece-connector orders the piece-retriever to drop the piece somewhere else and bring a new one. If the piece is suitable, the piece-connector takes it and tries to add it to the puzzle. Once the puzzle is completed, the puzzle-verifiers check it. If one piece is out of place, the group has to re-assemble parts of the puzzle. For each task, students increase their skill (faster and/or more accurate). Overall group performance is evaluated by the correctness of the puzzles and time-to-completion. Our independent variables are: (a) pedagogical style (with or without mandated role rotation); (b) students' initial skill level for each task and distribution of skill levels within student; and (c) task difficulty. Note: As a measure of achieving initial "reliability"—evaluating whether the model rationale indeed expresses what it purports to express—the modelers first worked individually and only then shared notes. We could thus partially validate our conjectures through inter-modeler triangulation.
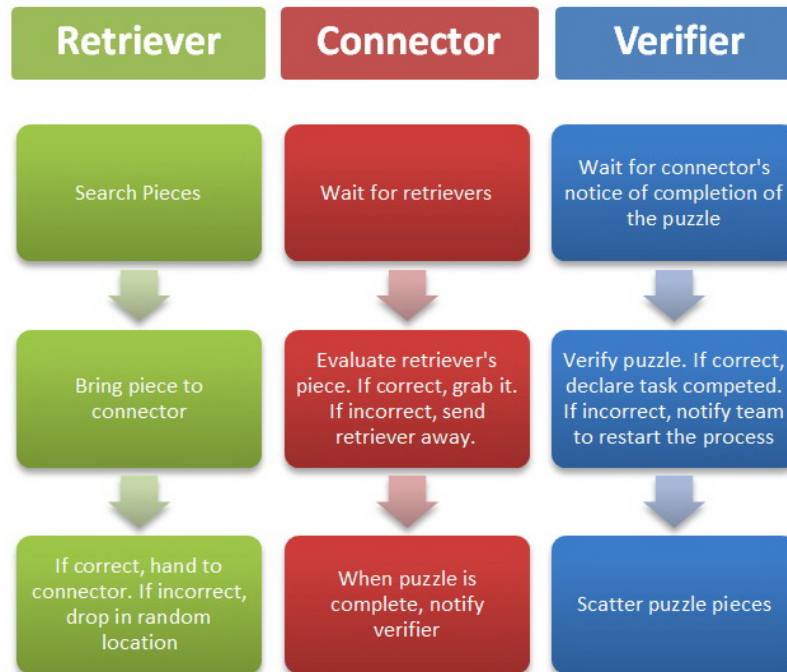
Figure 1. Design rationale for the Stratified Learning Zone agent-based model.

Once the model rationale has been articulated, as above, the modeler couches the agents' rules of interacting with each other and the environment in IF–THEN couplets and packages each topical set of rules in a procedure. These procedures express the researcher's conceptual model. For example, the following procedure (simplified for rhetorical clarity) is for the retriever–connector interaction, and delineates the agent's commands—retrievers gather pieces, connectors receive and evaluate pieces, and retrievers drop unfitting pieces.

```
to retrieve pieces
     ask retrievers
       [
        find-piece-around
        create-link-with chosen-piece
        go-back-to-group

        if any? connectors around me
           [
            if (chosen-piece fits puzzle)
                 [
                  unlink-piece-from-retriever
                  deliver-piece-to-connector
                  update-skill-retriever
                  update-skill-connector
                 ]
           if (chosen-piece does-not-fit puzzle)
                 [
                  go-far-and-drop-piece
                  update-skill-retriever
                  update-skill-connector
                 ]
     ]]
end
```

See http://ccl.northwestern.edu/research/conferences/CSCL2007/CSCL2007.html for an interactive applet of the NetLogo simulation.

Bifocal Modeling: Juxtaposing Real and Simulated Data as a Research Methodology for Iteratively Improving and Evaluating a Conceptual Model

In creating the SLZ model, we worked with videotaped data from the original study. To facilitate the modeling and to iteratively evaluate its "curve fit" to the classroom data, we employed bifocal modeling. We now introduce this methodology and then demonstrate its application to our case study.
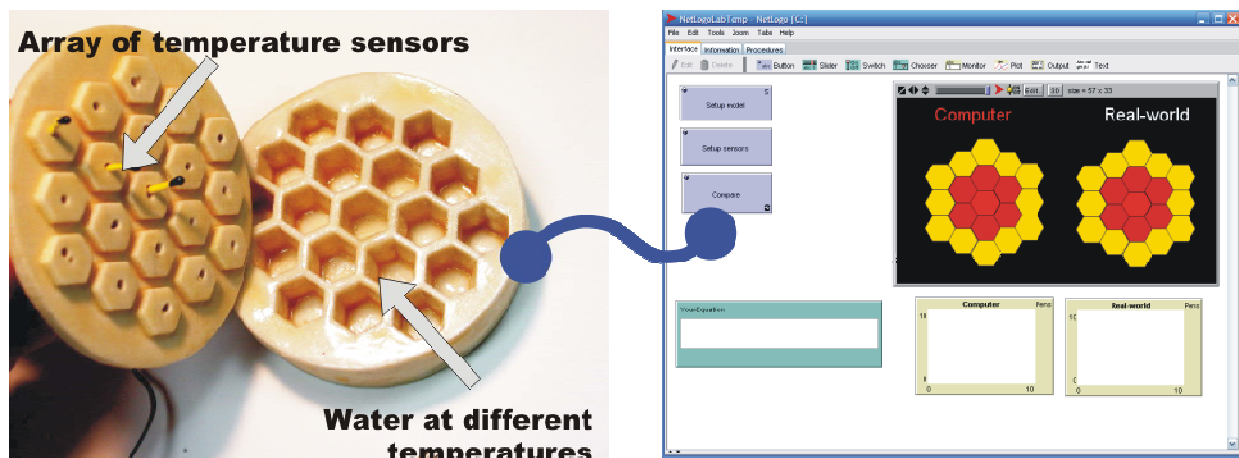
Introduction of bifocal modeling

Figure 2. Bifocal modeling: A 'linked' (hybrid) system for real-time physical/virtual investigating of heat transfer.

Bifocal modeling (Blikstein & Wilensky, 2006a; Blikstein, Abrahamson, & Wilensky, 2006) combines two ostensibly disparate research practices that are in fact methodologically complementary: agent-based modeling and enhanced visualization. Side by side on a split computer screen (see Figure 2, above, on the right) run the real and the virtual: (a) a movie or graphical representation of a phenomenon under inquiry (whether directly captured by cameras/sensors or enhanced through micro/macro or slow/fast-motion treatment), e.g., crystallization, heat transfer, or clinical interviews with preschoolers engaged in mathematical inquiry (see Figure 2, the physical model on the left, with sensor connecting into the computer and visualized in the 'Real-World' image); and (b) a multi-agent model simulating the same phenomenon in the form of a procedurally expressed and "runnable" scientific model, e.g., myriad interacting avatars representing elements in the conjectured process of annealing or cognitive elements of students' conceptual construction in classroom argumentation. Because the computer models have been carefully constructed to imitate the phenomenon's visual language, the bifocal methodology minimizes interpretive challenges typical of multi-media research. That is, the *seen* and the *hypothesized* are displayed such that their perceptual differences are backgrounded and, therefore, their procedural differences are more likely to be revealed. By thus utilizing the power of computation and representation, bifocal modeling constitutes a multi-disciplinary research tool that offloads aspects of both the interpretive and menial burden of scientific practice, freeing cognitive, discursive, and material resources that can thus be allocated toward validation of the hypotheses. The adaptable quality of the NetLogo multi-agent modeling-and-simulation environment enables users to keep calibrating their proceduralized hypotheses until their visualization reaches compelling micro/macro similarity to the real-data, such that there are grounds to assume that the proceduralized model indeed emulates this phenomenon.

A Bifocal Model of Emergent Collaboration Practices in a Mathematics Classroom

Figure 3, below, shows three examples of real-data (on the right) and simulated data (on the left). Note that our choice to model a generic collaborative activity, rather than modeling the precise activity, makes for surface differences between the real and the simulated data. The comparison is thus analogical: the real sample space corresponds with the linear puzzle, and eight roles in the original data have been simplified to three. In introducing such simplification, one must adopt a skeptical stance, because in any act of modeling lies the inherent possibility that some critical aspect of a situation has been overlooked. And yet, this challenge of modeling is certainly not unique to agent-based modeling but is typical of any scientific endeavor in which a researcher seeks to articulate patterns and mechanisms underlying phenomena under inquiry.

Onset of group work: before the assignment of individual roles emerge.

Student (standing) whose role is to help other groups improve on a specific skill.

Students whose emergent role involves conducting a mathematically trivial skill
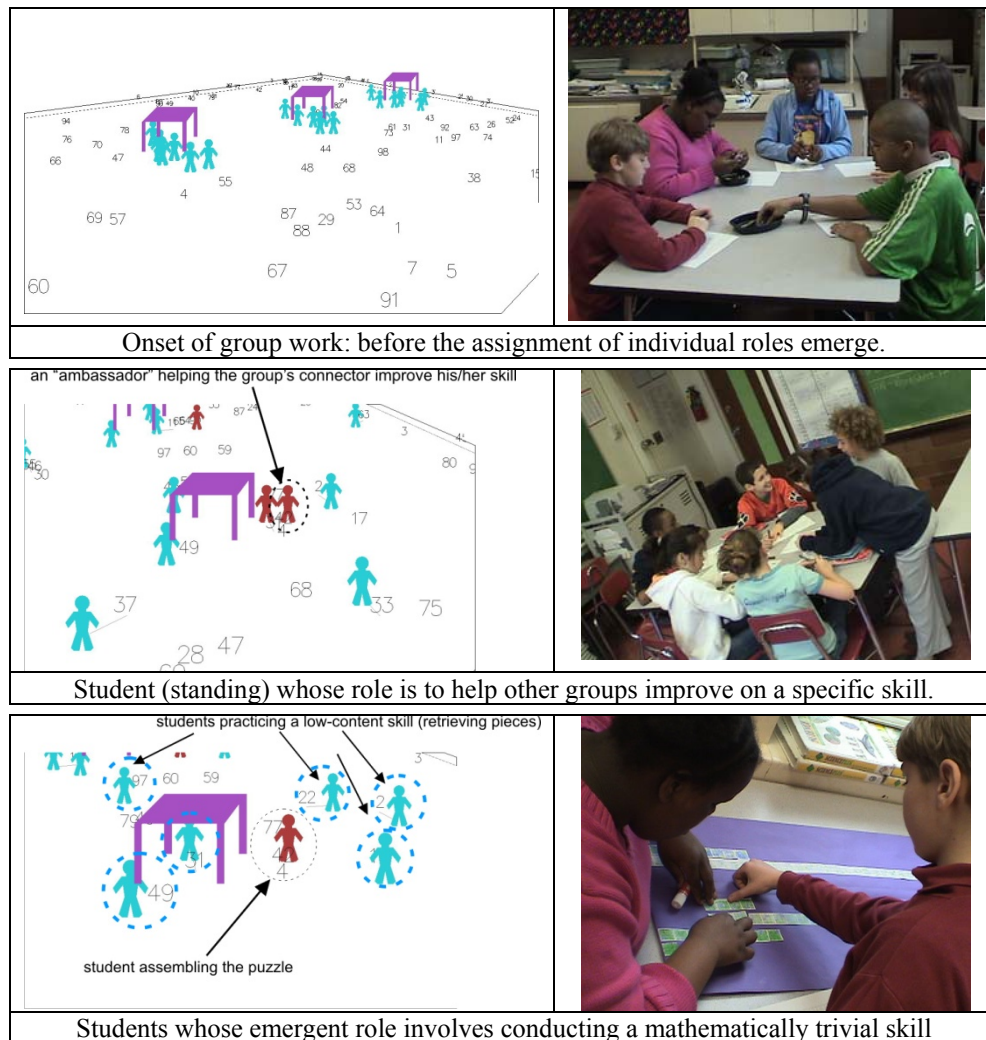
Figure 3. Bifocal modeling of collaborative learning: Three samples of paired states in student collaborative practice—computer simulation (left) and classroom data (right).

Findings From the Agent-Based Inquiry Into the SLZ Participation Pattern

We succeeded in simulating the emergence of a stratified learning zone. Furthermore, the model plausibly demonstrates relations between pedagogical practice and student learning, as follows:

a) When student–agents are reinforced for group production rather than individual learning, students become entrenched within skills reflecting their initial within-student skill-level distribution and increase their personal level in those skills.

b) However, when role rotation is mandated, *production slows down yet more learning occurs, per student, in both levels (high- and low-level skills).*

c) A careful analysis of the impact of each task performance on group performance is necessary to build a causal explanation of our numerical results. Indeed, different tasks have diverse impact on overall group performance. Increasing a low-level task skill (i.e., increasing the number of puzzle pieces a retriever–student can bring to the group per time tick) appears to improve group performance linearly (see red line in Figure 4, next page). On the other hand, increasing the high-level task skill (i.e., increasing the probability of the connector–student choosing a correct piece for the puzzle) effects a non-linear trend (see the blue line, in Figure 4).
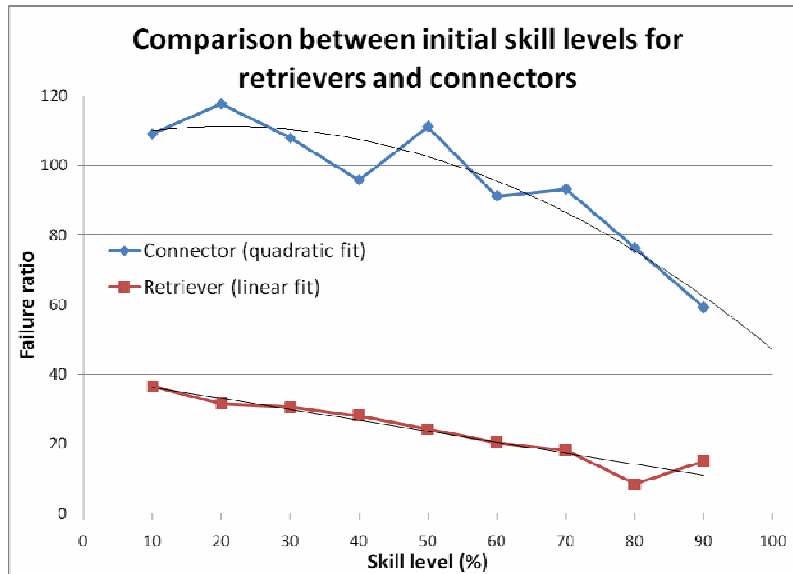
Figure 4. Comparison between initial skill levels for retrievers and connectors. Each data point represents the average of 20 runs of the model with the same initial parameters.

Interaction between different effects of low- and high-level tasks might indicate that the multidimensional combinatorial space of all possible skills levels and types is not a linear *n*-dimensional surface, but might contain discontinuities and multiple local minima and maxima. Specialized functionality of the agent-based environment (*BehaviorSpace*, Tisue & Wilensky, 2004), combined with the bifocaling technique, enable researchers to sketch the topography of this territory, locate minima and maxima, and connect them to real classroom scenarios. For example, the non-linear behavior of connector skills, concurrent with the linear behavior of retriever skills, describe a "hilly" 3-D surface (see Figure 5, below). In some region, slight improvement in connecting skills may significantly impact group performance. Within the same region, improvement in low-level skills renders negligible impact on group performance. This could correspond to a classroom scenario in which one heterogeneous group of students suddenly improves its performance, while other groups, perhaps more homogeneous, still struggle to solve the task at hand. The gain in performance could be attributed to a single student who advanced on a high-level task. Group mates may not have learned the new skill at all, but the group performance, as observed by the teacher, would have improved greatly.
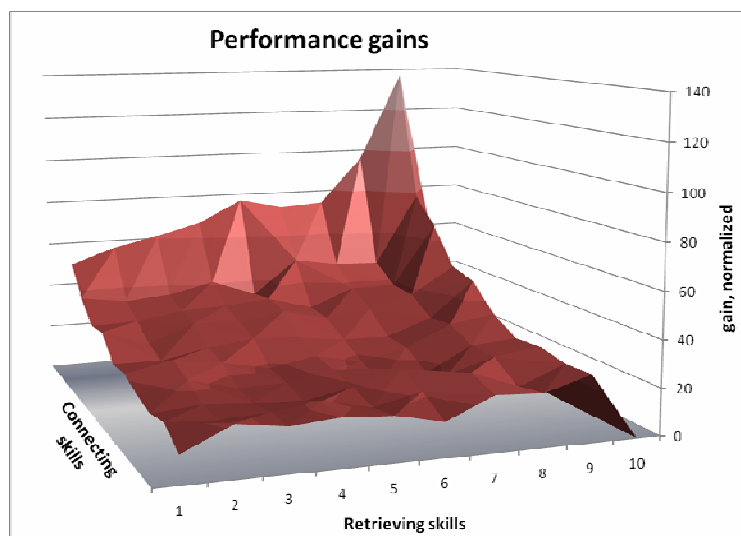

Figure 5. A multivariable experiment, in which both connecting and retrieving skills varied from 1 to 10 (10 runs per data point, for a total of 1000 runs). While still in exploratory stages, this kind of visualization can reveal local minima and maxima, as well as different patterns of performance gains for the combinatorial space of skills.

We are improving the model so as to further examine relations between pedagogy and equity. We are particularly interested to simulate pedagogical practices (e.g., Aaronson, Blaney, Srephan, Sikes, & Snapp, 1978; E. G. Cohen, 1986) so as to understand their underlying mechanisms and gauge their potential.

Conclusions

We have presented a computer-based methodology for conducting research into collaborative learning. To demonstrate the methodology, we described the design and implementation of an agent-based model for studying the emergence of inequitable participation patterns observed in a middle-school implementation of a collaborative-inquiry activity. Based on the functional resemblance of the simulated and real behaviors, on interviews with the teacher and students, and on inter-modeler reliability, we conclude that the model constitutes a viable, if not complete, explanation for the emergence of the observed patterns. We submit that our case study is sufficiently generic so as to shed light on behavior patterns observed in a range of classroom interactions around collaborative projects. We end this paper with remarks on the applicability and limitations of ABM methodology.

Applicability of Agent-Based Modeling to Educational Research

The current study extends our previous modeling-based inquiry into the dynamics of individual learning in social contexts. The inquiry encompasses different "levels" of learning: individual cognition is viewed as arising from distributed dynamics of cognitive elements, and group learning is viewed as arising from distributed dynamics of human elements. An ambitious goal of this work is to combine these two levels of modeling. Such cognitive–social modeling would advance the cause of an integrated learning-sciences theory (Cole & Wertsch, 1996).

Strengths and Limitations

Papert (1980) has demonstrated the intrinsic power of modeling—by virtue of creating a procedure-based model of a phenomenon under inquiry, one has occasion to hone and critique one's own conceptual model of the phenomenon. The application of agent-based modeling to research in the social sciences is still young, and the specific application of this mode of inquiry in the learning sciences is only nascent. Yet we believe in the potential of this methodology. The caveat of proceduralization that agent-based methodology imposes on the researcher can be a humbling experience, because it imperviously demands such clarity and precision as are quite uncommon to the social sciences. Yet once the model appears to plausibly simulate a complex system, it enables powerful exploration. In particular for the social science, where human-subjects issues naturally limit the scope of research, agent-based modeling provides opportunities to investigate the 'what ifs' that are key to fostering change. This study has illuminated for us regions of potentiality that are exciting in their prospects. Of course, in order to validate our findings from the virtual medium, we will have eventually to return to the source phenomenon, where we will evaluate whether the classroom model indeed helps us create a model classroom.

Regardless of the medium in which it is implemented, any model is still just a model—it is an embodiment of a system of conjectures held together through theoretical plausibility, sensed coherence, and empirical support. One must be especially wary in the case of models embodied in dynamic visual media, because the superficial features might divert the researcher from attending to the deep mechanisms ('seduction of sim,' Starr, 1994). It might look and smell like a duck but still be a rabbit. Also, our source study was not designed with a focus on participation patterns within groups, so our video data do not track any one particular group over the entire duration of the implementation, but rather sample from the groups. We can only conjecture as to the group dynamics that transpired in that classroom during the data gaps. In future studies, we will work with data that cover the entire evolution of group dynamics. Finally, student interactions are vastly richer than we have portrayed in our case-study model. A possibly provocative statement is that any theoretical lacuna detected in the current model could be covered by further procedures, such that one could plausibly build agent-based models that capture, if not exhaust, the wealth of learning-sciences theory.

References

Aaronson, E., Blaney, N., Srephan, C., Sikes, J., & Snapp, M. (1978). *The jigsaw classroom*. Beverly Hills, CA: Sage.

Abrahamson, D., Janusz, R., & Wilensky, U. (2006). There once was a 9-block...—A middle-school design for probability and statistics [Electronic Version]. *Journal of Statistics Education*, *14*(1) from http://www.amstat.org/publications/jse/v14n1/abrahamson.html.

Abrahamson, D., & Wilensky, U. (2005a). *Piaget? Vygotsky? I'm game!: Agent-based modeling for psychology research.* Paper presented at the annual meeting of the Jean Piaget Society, Vancouver, Canada.

Abrahamson, D., & Wilensky, U. (2005b). *The stratified learning zone: Examining collaborative-learning design in demographically-diverse mathematics classrooms.* Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Axelrod, R. M. (1997). *The complexity of cooperation: Agent-based models of competition and collaboration.* Princeton, NJ: Princeton University Press.

Axelrod, R. M., & Cohen, M. D. (1999). *Harnessing complexity: Organization implications of a scientific frontier.* New York: Free Press.

Blikstein, P., & Wilensky, U. (2006a). The missing link: A case study of sensing-and-modeling toolkits for constructionist scientific investigation. In *Proceedings of the 6th IEEE International Conference on Advanced Learning Technologies (ICALT 2006)*, Kerkrade, The Netherlands, 980-982.

Blikstein, P., Abrahamson, D., & Wilensky, U. (2006b). *Minsky, mind, and models: Juxtaposing agent-based computer simulations and clinical-interview data as a methodology for investigating cognitive-developmental theory.* Paper presented at the annual meeting of the Jean Piaget Society, Baltimore, MD.

Blikstein, P., & Wilensky, U. (2006). *Learning about learning: Using multi-agent computer simulation to investigate human cognition.* Paper presented at the International Conference of Complex Science, Boston.

Bonabeau, E., Dorigo, M., Théraulaz, G. (1999). *Swarm intelligence: From natural to artificial systems.* London: Oxford University Press.

Cohen, E. G. (1986). *Designing groupwork: Strategies for the heterogeneous classroom.* New York: Teachers College Press.

Cole, M., & Wertsch, J. V. (1996). Beyond the individual-social antinomy in discussions of Piaget and Vygotsky. *Human Development, 39*(5), 250-256.

Collier, N., & Sallach, D. (2001). *Repast.* Chicago: University of Chicago. http://repast.sourceforge.net

Diermeier, D. w. A. M. (2000). Government turnover in parliamentary democracies. *Journal of Economic Theory, 94*, 46-79.

Durkheim, E. (1947). *The division of labor in society* (G. Simpson, Trans.). New York: The Free Press. (Original work published 1893).

Epstein, J., & Axtell, R. (1996). *Growing artificial societies: Social science from the bottom up.* Washington: Brookings Institution Press.

Greeno, J. G. (1998). The situativity of knowing, learning, and research. *American Psychologist, 53*(1), 5-26.

Holland, J. H. (1995). *Hidden order: How adaptation builds complexity.* Reading, MA: Helix Books.

Hurford, A. (2004). *A dynamical systems model of student learning and an example from a HubNet Participation Simulation.* Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.

Langton, C., & Burkhardt, G. (1997). *Swarm.* Santa Fe: Santa Fe Institute. http://www.swarm.org/release.html.

North, M., Macal, C., Cirillo, R., Conzelman, G., Koritarov, V., Thimmapuram, P., et al. (2002). *Modeling of electricity markets.* Paper presented at the Agents 2002, Chicago, IL.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* NY: Basic Books.

Reynolds, C. W. (1987) Flocks, herds, and schools: A distributed behavioral model. *Computer Graphics, 21*(4) (SIGGRAPH '87 Conference Proceedings), 25-34

Starr, P. (1994). Seductions of Sim. *The American Prospect, 17*, 19-29.

Tisue, S., & Wilensky, U. (2004). *NetLogo: Design and implementation of a multi-agent modeling environment.* . Paper presented at the Agent 2004, Chicago.

Wilensky, U. (1999). *NetLogo.* Northwestern University, Evanston, IL: The Center for Connected Learning and Computer-Based Modeling http://ccl.northwestern.edu/netlogo/.

Wilensky, U. (2001). Modeling nature's emergent phenomena with multi-agent modeling languages. In *Proceedings of EuroLogo 2001.* Linz, Austria.

Wilensky, U. (2003). Statistical mechanics for secondary school: The GasLab modeling toolkit. *International Journal of Computers for Mathematical Learning, 8*(1, special issue on agent-based modeling), 1-41.

Wilensky, U., & Reisman, K. (1998). *Learning biology through constructing and testing computational theories - An embodied modeling approach.* Paper presented at the Second International Conference on Complex Systems, Nashua, NH.

Wilensky, U., & Reisman, K. (2006). Thinking like a wolf, a sheep or a firefly: Learning biology through constructing and testing computational theories—an embodied modeling approach. *Cognition & Instruction, 24*(2), 171-209.