At home with mathematics: Meanings and uses among families

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Abstract: Everyday mathematics research has examined the activities of buyers (Lave, 1988) and sellers in markets (Nunes, Schliemann, & Carraher, 1993; Saxe, 1990); it has examined the practices of both high and lower status occupational work (e.g. Hall & Stevens, 1995; Scribner, 1985; Stevens & Hall, 1998), and it has examined the mathematics that is embedded in competitive play (Nasir, 2000) but the family as locus of mathematical activity is understudied. The three projects represented in this symposium focus on mathematical practices in the context of families. The first project takes an activity focus, examining mathematical practices in the context of a consequential context of family activity—money and its uses. The second project takes a topical focus, describing a wide range of practices in families that are seen as mathematical. The third paper focuses on a particular cultural group, attending to the meanings and uses of both informal and schooled mathematics in the family context. Together the papers provide a new perspective on qualities of mathematical practice outside of school and its actual, and potential, relations to school mathematics.

Money Matters: The Social and Material Organization of Consequential Financial Practices in Families

In the first project, we extend prior work that develops an approach through a series of ethnographic case studies of mathematical activity in and out of schools (Hall & Stevens, 1995; Stevens & Hall, 1998; Stevens, 1999; Stevens, 2000; Stevens, under review). It follows this prior line of research in focusing on how problems emerge in the context of daily activities and how they are resolved, often through quantitative practices not easily recognizable as "mathematics" (Stevens, 1999; Stevens, 2000; Stevens, under review). The activities that are our focus in this study—financial activities within families—were selected because these contexts of activity are recognizably *consequential* to the people we are studying. Activities that we documented range from routine bill-paying practices and monthly budgeting to major decisions including the selection of high-premium insurance policies, home buying, and saving for a child's college education.

This study builds upon prior everyday mathematics research for its theoretical framing but, by focusing on consequential contexts of quantitative practice, responds to perceived limitations in this line of work (see Palinscar, 1989) that suggest that by studying low-stakes problem-solving activities like supermarket best-buy choices (Lave, 1988) and weight-watchers measurements (delaRocha, 1986) we fail to see the full range of mathematical resources that people can and do use in everyday life. This was, of course, very much the theoretical point that studies like those by Lave were making—that mathematical practices in everyday contexts are fitted to endogenous criteria for success rather than an omni-relevant standard of mathematical problem solving dictated by disciplinary mathematics—but it leaves open the question of whether the mathematical practices in other more consequential everyday contexts would be appreciably different in response to different practical conditions. It also leaves open the question of whether more school-like forms and functions (Saxe, 1990) of mathematics will appear in these consequential situations of quantitative practice.

The study was conducted in the homes of eight families, each with a described intention to make a near-term consequential financial decision and with at least one resident high-school aged child. We purposively sampled for families with high-school aged children because we thought this would provide the most likely context among parents and children for informal teaching and learning around finances, since the young people are so close to legal adulthood and being "on their own." Families varied across socioeconomic levels, race, structure (single/two-parent households) and reported formal math abilities. Our data include videotaped observations of family financial activities over a 10-month period, documentation of resources and artifacts, two simulation tasks (cf. Scribner, 1985) and interviews with family members. In addition to seeking to understand the details of mathematical work in the context of family financial practices, the study also had the goal of exploring these financial practices as a context for informal, intergenerational learning between parents and children. The study was also intended to explore the resources that families, varying in SES, had and could access in solving the financial problems they posed for themselves or were posed for them by circumstance.

The first finding involves the ways the range of ways that informal learning was organized within financial practices among parents and children. One widely regarded model of informal learning is guided/intent participation (Rogoff, 2003). Though guided and intent participation differ as models and are understood to obtain under different practical and cultural conditions, both describes how young people are brought into practices thorough observation and guidance by knowledgeable others. These models of participation relate to broader notions of apprenticeship (Lave and Wenger, 1991) in which newcomers learn from oldtimers; though, typically this learning involves forms of participation that do not resemble those of Western schooling. In our study, we have found guided participation and apprenticeship-like conditions in our families, but we have also found other informal learning arrangements. The following are some of the different arrangements we have documented: in a couple cases young people are using their parents' practices reflexively as a resource for espousing alternative practices. They are in other words learning more from what they perceive as their parent's mistakes rather than being guided in their participation in shared practice as Rogoff describes. In other cases, guided participation and apprenticeship are not good descriptions, because parents deliberately occlude financial practices from children thus rendering them invisible; to use Hutchins' term, they are denied "horizons of observation" (1995) from which to occupy a position of peripheral participant (Lave & Wenger, 1991); in still other cases parents have opportunistically taken our study's presence in their lives as a pedagogical opportunity to teach their children about finances. For example, we provide $250 to families that they are asked to "invest" in some manner of their choosing, that we then document. In two higher SES families, this is considered a small enough sum that parents plan to have their children make the decision about the money's use as context for learning how to make good decisions with money. Taken together, we have a diversity of learning arrangements that challenge any single model of informal learning and in turn, we believe, the idea of singular models of learning. This study points us toward a view that learning may not be best understood in terms of any singular model or process—as typically pursued with psychology and cognitive science—but as something achieved by drawing from a repertoire of cultural practices for learning (e.g. proximal apprenticeship, coaching, intent participation, or direct instruction) that are fitted to current practical conditions and goals.

Our second finding involves the *resources* that family members *assemble* and *coordinate* to solve problems they encounter in these financial situations. Building upon an analysis of quantitative practice in a high status workplace environment (Stevens, 1999; under review), we are characterizing problems that are resolved through *routines* and those that are resolved with *innovations* (cf. Schwartz, Bransford, & Sears, in press). Innovations appear to be born of new and often urgent forms of trouble in family circumstances or from routines within which family members recognize "chronic snags" (delaRocha, 1986). Our key finding is that in comparison to the resources that are required for school mathematics problems, the assembled resources used to solve problems in these domestic situations are *radically heterogeneous*. An example of such a problem resolution, that we characterize as an innovation, was initiated by a bit of trouble in one of our low SES families. The family was suddenly faced with a problem, because the teenage son in the family wrecked his car. Rather than take the full pay out amount for the insurance, the single mother in the family collaborated with her son to take $400 less than the maximum payout while keeping the totaled car. The mother and son then decided to cannibalize the car for parts that the son would then sell through an online vendor or, in the case of aftermarket parts like an alarm and radio, reuse in his next car. They made this decision with the expectation of making or saving more than the $400 they gave up by keeping the totaled car rather than relinquishing it. These funds, along with the insurance money, would be put toward the purchase of a newer and more expensive car than the one he crashed. As we followed the course of this problem's resolution, we found that the family did indeed make a profit of $200 as well as recouping the cost of the alarm and car stereo. In addition however, there were a number of emergent learning opportunities that were presented and taken within the family. First, since the son did not know enough to disassemble the car, he took the opportunity to ask friends who had this knowledge to help him to do so, which they did. In so doing, he learned a great deal about the inner workings of a car and how to disassemble one. The mother reported learning a great deal about this as well. The cannibalization approach also provided the mother with a temporary pool of liquid funds to pay off some expensive debts, in effect using the insurance payout as interest free loan until her son has completed his work and was ready to buy the replacement car of his choice, a Stealth. The story has more elements but for purposes of our argument, this portion of this story involves at least the following heterogeneous resources that were innovatively assembled: knowledge of options available under an insurance policy, a network of friends and their manual skills, basic calculations, and a local online marketplace for the sale of used merchandise called Craig's List.

Both of our findings urge us toward for more complex models of learning and problem solving and, more generally, toward a rethinking of received goals in the cognate disciplines that directly inform the learning sciences—psychology and cognitive science—of singular models of phenomena like learning and problem solving. In turn, these findings pose interesting questions about and suggest design implications for the conduct of school mathematics education.

Problem Emergence, Problem Solving, and Mathematics in Family Life

The second project has investigated the diverse contexts and activities in which middle school age learners and their families engage in mathematical practices and problem solving. We describe the resources family members use for solving problems, characterize the structure of their mathematical activities, and analyze the social conditions and arrangements for family-based mathematics practices.

This draws its results from interviews and observations of 13 families who reside in the San Francisco Bay Area representing economic, racial and ethnic diversity. We used a semi-structured interview protocol organized around mathematically relevant contexts identified in prior studies, including: financial and time budgeting, hobbies and crafts, home repairs and home chores for kids, purchasing of larger items (such as phone plans, cars, homes, or major appliances), local and long-distance travel, family entertainment activities, and cooking (Goldman, 2005). This focus allowed interviews to center on activities that most families engage in, while allowing families to give us their particularized versions of how they accomplish each life task (including technology use and systems for representing mathematical relationships). The interview also included a simulation task in which families chose a cell phone plan to meet the family's needs.

Analysis of data is qualitative, and includes individual's descriptions, family discourse and interaction. Our approach is to understand how the problems arising in family-based, math-relevant settings generate new knowledge and make certain kinds of thinking and action adaptive. Data analyses show that mathematical problem solving in the family looks quite different than school-based problem solving. School-based math problems vary in their realism, but the particulars of school problems are most often vehicles for math content. In contrast, the mathematics involved in tackling problems in families generally remains subordinate to the problem itself, and as such, family math can easily remain invisible.

We have found that mathematical problem solving in the family, while less explicit than school math, is complex and varied, and while occasionally based in routines, commonly requires adaptive flexibility due to specifics of the situation. Unlike many school-based problems, math in family life does not come prepackaged with normative solution paths. When problems emerge or re-occur, family members must decide how to deal with them. These decisions involve consideration of, at a minimum, the availability of resources such as materials, money, space and time), historical approaches to the problem within the family, the distribution of expertise within the family, who is available to work on solutions (Pea 1993), the timetable at hand, and the potential risks and benefits at stake. The complex web of contingencies that is considered in even ordinary family problem solving contexts greatly exceeds what would be expected in most school settings. The mathematical content put to use in families is wide ranging as well, including fractions, decimals and percents; ratios and proportions (direct and indirect); measurement and conversion; probability and odds; basic geometry; charts and graphs; statistics (such as averages), and statistical comparisons. Problems may take several steps to solve and, most often, more than one type of math is brought to bear on the problem. Math in the context of family life is often co-occurring with solving compelling life issues, and it is the family issues to which people resonate. Problem solving in the family can approximate the ways adults problem solve problems with math in the workplace. Such work skills and practices include logical reasoning, categorization, approximation and estimation (Steen, 2004), which all appear frequently in family problem solving. While these skills are often identified with work, our research finds that people apply these mathematical skills in the contexts of family life.

We present examples from the data, discuss types of problems, their characteristics, and ways people define problems and accomplish solutions. We discuss implications of our research for understanding how learning at home provides unique opportunities and how those opportunities correspond to, or contradict learning in other settings.

Supporting Math Achievement at Home: Practices that Matter for the School Math Achievement of African-American Students

The third project explores the relation between home mathematics practices, sense of mathematical identity, and mathematics achievement among high and low- achieving African American students in math. This project speaks to a long history of disparities by race in mathematics achievement (such that African-American students routinely perform less well than other groups) are widely cited and pervasive (Oakes, 1990; Secada, 1995; Walker & McCoy, 1997). One approach to addressing such inequities is to better understand differences between more and less successful African American students. In this project, we take such an approach to understanding the kinds of home and out-of-school experiences that seem to support the math achievement of African American students. This is critical to understanding the role home practices might play in the math achievement of African American students, as we know very little about how African American students interact with mathematics in practices outside of school (Nasir, 2000), nor do we know much about how African American families may be supporting (or failing to support) the mathematics learning of their high school students.

In this project, we focus on differences in students' experiences with mathematics outside of school and in support for mathematics at home. Data are drawn from a study of African-American students' experiences with mathematics at school and at home. 12 high and 12 low achieving African-American students were interviewed about the type and extent of their experiences with informal mathematics at home and in out-of-school contexts, and about the type and availability of support for school mathematics at home, as well as about their sense of themselves as math learners during school and home math activities. The interview also included questions on students' sense of their identities and learning in the math classes, and their sense of themselves as African-American. The sample was half male and half female and all were 10^{th} and 11^{th} graders. Interviews ranged from one hour to one and a half hours.

Findings reveal several interesting differences in out-of-school mathematics practices and home math of high- and low-achieving African American students. These include differences in: 1) level of direct family support, 2) participation in activities that seem to mediate academic identity in general, and 3) differences in how students "see" the math in settings outside of school, and 4) differences in how students' view themselves as competent math students (including how students draw upon conceptions of "the kind of math person" other families members are).

Differences in the direct level of family support were not surprising. While all students reported that their families checked their report card grades and wanted them to do well in mathematics, the families of higher achieving students more often expected higher grades in mathematics (and in general). Further, higher achieving students were more likely to have family members who helped them with their math homework, and with selecting mathematics courses. Clearly, this direct support (and cultural capital around which courses to take) influenced students' math achievement in very straightforward ways.

Rather than directly affecting students' ability to engage in mathematics classes, participation in other out-of-school activities seemed to mediate students' academic identities more generally. Specifically, higher achieving students were more likely to participate in organized sports and church. Student reports suggest that both of these settings seem to offer students positive messages about their ability to be strong students, and they also seem to encourage continued and full participation in school and school activities.

The third difference we describe is that higher-achieving students were more likely to “see” mathematics in practices outside of school. This was interesting because low- and high- achieving students did *not* seem to differ in their participation in the types or numbers of settings outside of school where they engaged with mathematics. The difference, then, was in students’ ability to see these activities as being mathematically-relevant. For instance, while almost all of the students mentioned shopping as an activity that they were involved with, lower-achieving students were less likely to see it as an activity that might involve math, while higher-achieving students tended to describe the way they used math in shopping activities. For instance, one higher-achieving student said, “ So if I buy something that’s like 80 dollars save 25% off, that means it’s like 60 dollars.”

Finally, higher-achieving students were more likely to see themselves as competent in math and were more likely to be comfortable with problems that they couldn’t figure out—that is, facing a difficult problem did not trigger for them a sense that they were “not good” at math, as it did for lower-achieving students. Hence, they were more willing to persist in the face of difficult problems.

These differences between low- and high- achieving African American students in mathematics are an important start to initial efforts to describe the types of home and out-of-school practices that seem to be related to math achievement for African American students. The relative influence of direct factors like level of homework support and guidance with course selection, as well as more indirect factors like supporting engagement in school generally, and seeing oneself as a “math person” are interesting to consider, though further work will be needed to understand the nuances of how much, and how each of these kinds of influences may matter.

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