SPACE AND TIME IN CLASSROOM NETWORKS: MAPPING CONCEPTUAL DOMAINS IN MATHEMATICS THROUGH COLLECTIVE ACTIVITY STRUCTURES

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Abstract: This paper reports on a design-based research project in which we are developing collaborative activities for classroom networks. Research in generative design has explored classroom activities that analogize the many participants in a classroom with an often-infinite family of mathematical objects. By contrast, our project focuses on classroom activity structures that support interactions among two to four students, each aligned with different components of a shared mathematical object. In each case, these network-based relationships among students are intended to serve as resources to support learners' efforts to jointly navigate the conceptual territory delineated by their corresponding mathematical relationships in the space of the network. We compare the kinds of exploration of mathematical terrain respectively supported by whole-class and small group activities in classroom networks in order to examine the distinctive learning opportunities provided by each, and consider the potential of these different activity structures for providing mutually supportive instructional experiences.

Introduction

Simultaneously accounting for the cognitive and situative dimensions of learning remains an important and persistent challenge for contemporary mathematics education (Greeno, 1997; Sfard, 1998). Prior work along these lines highlights the importance of attending to intersections between discipline-specific content and practices in mathematics on the one hand, and social norms and interactions across different classroom activity structures on the other. For example, in a seminal account of the links between classroom interactions and mathematical activity, Yackel and Cobb (1996) observed that social structures, in the form of the collective, local norms for participating in mathematical discourse, serve as resources for structuring classroom mathematical practices. Following a similar insight in the reverse direction, research on generative design (e.g., Stroup, Ares and Hurford, 2005) has pointed to ways that mathematical structures can serve as resources for structuring social activity through a classroom network of handheld calculators. The intersection between individual students' engagement with mathematical objects in the private space of their respective devices and the collectively constructed artifacts visible in classroom networks provides rich possibilities for exploring this dynamic between conceptual and social.

In a similar vein, this paper reports on a design-based project in which we develop collaborative activities for classroom networks in order to investigate intersections between conceptual and social dimensions of mathematics learning. As in generative design, our own approach uses the mapping between student participants in a classroom group and mathematical objects in a shared virtual space to make important mathematical relationships and properties of these objects salient. Generative designs analogize the many participants in a whole-class activity with an often-infinite family of mathematical objects. For example, each student might be associated with a different function in a family of functions; a different point in a locus of points; or a different expression among a class of equivalent expressions (Stroup, Ares, Hurford & Lesh, 2007). By contrast, our project focuses on network designs and classroom activity structures that support collaborative interactions and investigations among groups of two to four students. These designs align each student participant in a small group with one of a small number of components of a single shared mathematical object, such as different representations of a common function (White, 2006).

In each case, these network-based relationships among students are intended to serve as resources to support learners' efforts to jointly navigate the conceptual territory delineated by their corresponding mathematical relationships in the space of the network. We see the forms of virtual exploration of interrelated graphical and symbolic terrain afforded by these designs as offering a visual example of Greeno's (1991) proposal that knowledge of a domain is analogous to familiarity with an environment. Our aim in this paper is to consider the ways these different activity structures for classroom networks (at whole-class and small group levels) might collectively map a corresponding conceptual territory in mathematics. In other words, we compare the kinds of exploration of mathematical terrain respectively supported by whole-class and small group activities in classroom networks in order to examine the distinctive learning opportunities provided by each, and to consider the potential of these different activity structures for providing mutually supportive instructional experiences.

Collective Activity in Classroom Networks

A rich body of prior research has explored ways of using of classroom networks to support whole-class interactions (Hegedus & Kaput, 2004; Stroup, Ares & Hurford, 2005). A major strand of this work emphasizes using the sociocultural diversity of the class group as an engine for exploring the possible variation among families and other collections of mathematical objects (Stroup, Ares, Hurford & Lesh, 2007). For instance, in studying the notion of algebraic equivalence, students might be asked to invent and contribute expressions equivalent to a given expression (e.g., 4x). The resulting collection of expressions that are the same as 4x indicates the breadth of the space of this *equivalence class* of expressions (Stroup, Ares & Hurford, 2005). Students make use of key algebraic skills as they work to populate this space, "unsimplifying" the given expression. Because of the diversity of ideas present in virtually any classroom-sized group of participants, the range of student contributions will reliably instantiate key concepts such as the distributive law and identity and inverse properties of multiplication and addition (see, e.g., Davis, 2007).

Thus, this strand of research taps into the protean sea of the classroom group, confident that it contains sufficient variability and diversity to represent the mathematical universe adequately for pedagogical purposes. The size of the classroom group appears as a critical asset in these classrooms, as it serves as the effective guarantee that the hypothesis of sufficient-diversity will hold. We interpret activities that rely on the diversity of student contributions in this way to be using the size and diversity of the classroom group to concretize the concept of one of several kinds of *infinity*. Indeed, the representative activities of this genre tend to focus on aggregate mathematical objects such as families of functions, loci of points, or instances of geometrical relationships—all consisting of an infinite set of constituent objects. In each case, a critical learning objective is the appreciation of the nature of this infinity and of structures within it. The design of the activities allows the student to grasp the infinite family, locus, or range of figures without losing contact with the specificity of the examples—her own and those of her peers—that serve to embody the infinite aggregate.

While these instances of infinity represent a rich and varied set of mathematical topics, they do not span the full array of ideas central to the discipline. Indeed, many core mathematical objects under study in the k-12 curriculum are composed of sets of just a few coordinated sub-objects: lines are uniquely determined by two points, quadratic expressions are comprised of three distinct monomial terms, equations compare two distinct algebraic expressions, functions are commonly represented through one of three modes—symbols, graphs or tables. Additionally, many of the conceptual challenges that face students as they grapple with these concepts consist of difficulties with the coordinated significance of these sub-objects: how to determine the slope of a line, to combine polynomial terms and simplify expressions, to solve equations, to interpret relations among representations.

With these concepts and challenges in mind, our own approach uses network links among student devices in order to align pairs or small groups of students with correspondingly small sets of mathematical objects. As in the case of analogizing the many students in a whole class to infinitely many related mathematical objects, these small group designs likewise seek to capitalize on network-defined social relationships to make corresponding mathematical relationships salient in learners' activity. To that end, our designs focus on collaborative learning tasks that are centered on collective mathematical objects that participants in a pair or small group must jointly manipulate through their networked devices. Such objects are collective to the extent that they or their attributes appear—and change—simultaneously on the devices of multiple students, and that they appear in a shared display as a consequence of contributions from multiple students. These designs map the students in a small group to sub-objects in a coordinated set: each learner manipulates a linked point to collectively form a curve in a Cartesian plane, or transforms a respective side of an algebraic equation, or combines different-ordered terms or enacts different binomial operations among polynomial expressions, or examines a different representation of the same function. Broadly, we aim to structure tasks around these collective mathematical objects in order to make the successful solving of problems dependent on contributions from and coordination between all participants in a small group. Below, we describe one such design in detail, and present an analysis of the kinds of student mathematical exploration supported by this environment.

The *Graphing in Groups* Design

The collaborative mathematics activities described in this paper were created using the NetLogo modeling environment (Wilensky, 1999) and HubNet architecture (Wilensky & Stroup, 1999) in concert with classroom sets of Texas Instruments graphing calculators connected through a TI-Navigator™ network. In the activity design called

Graphing in Groups, teams of two or more students share collective responsibility for jointly manipulating the graph of a function (for two students, a linear function; for three students, a quadratic). Each calculator displays a graphing window and allows the student to adjust the coordinates of a point graphed within that window using directional arrow keys on the calculator (Figure 1). The coordinate points inputted through the calculators of each member in a student pair, along with the line uniquely determined by those respective points, are displayed together in a single graphing window on the classroom server, creating a shared mathematical space for the members of a group (Figure 2). A grid composed of several groups' graphing windows and projected at the front of the classroom provides a collective display of all these small group-level graphs (Figure 3).

In the activities for the present study, each student's point was paired with that of a single partner so that their respective coordinates collectively defined a line. As students moved the individual point controlled by their respective graphing calculators, they could choose to "mark" the current coordinate location of that point by pressing a calculator key. Once both students in a pair marked points, the corresponding line appeared in their group's graphing window, along with an equation for that line in slope-intercept form (i.e., y=mx+b). Students could continue to move their respective points freely, and any time either student in a pair pressed the "mark" key again, the graph and equation automatically changed accordingly in the public display. In other words, the students controlled both an individual object, in the form of their respective points, and a collective object in the form of a line.

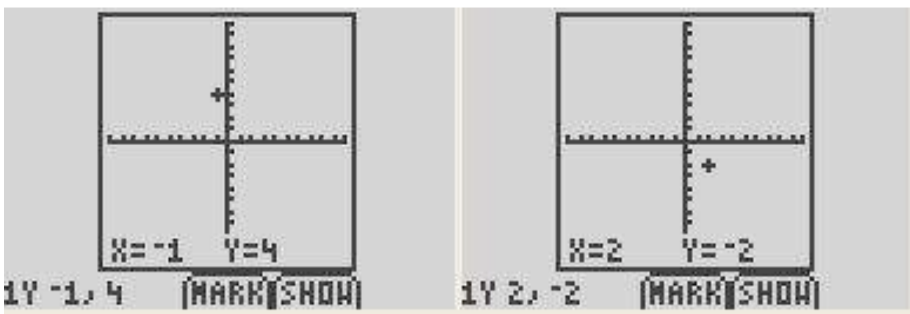

Figure 1. Two student calculator screens.

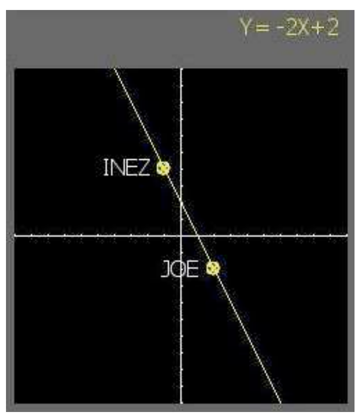

Figure 2. Two student points in a group-level graph.

Method

The *Graphing in Groups* design served as the basis for a three-week unit on linear graphing implemented in three ninth grade Algebra I classes, each taught by a different teacher. Over a series of several class sessions, students participated in a variety of *Graphing in Groups* activities intended to complement more conventional lessons on linear equations and their graphs. The topics addressed during this unit included slope, x- and y-intercepts, the point-slope and slope-intercept forms of linear equations, and parallel and perpendicular lines. Some of the *Graphing in Groups* activities conducted during the unit were teacher-led explorations and whole-class discussions of the relations among lines generated by different pairs. Other activities were problem-solving challenges assigned to student pairs. These problem-solving tasks generally involved asking student pairs to move their respective points so as to create a line with a certain characteristic or set of characteristics: a slope of three, a y-intercept of four, a negative slope, a line parallel to a teacher-generated line, etc. Such tasks were typically closed in the sense that each pair worked toward a single solution, but open with regard to the coordinates each student might assume and the process by which they identified that solution. In each case, the instructional focus was on framing these challenges in terms of pairs' collective lines, so that completing that task required students to consider that line and its equation in relation to the coordinates of their respective points.

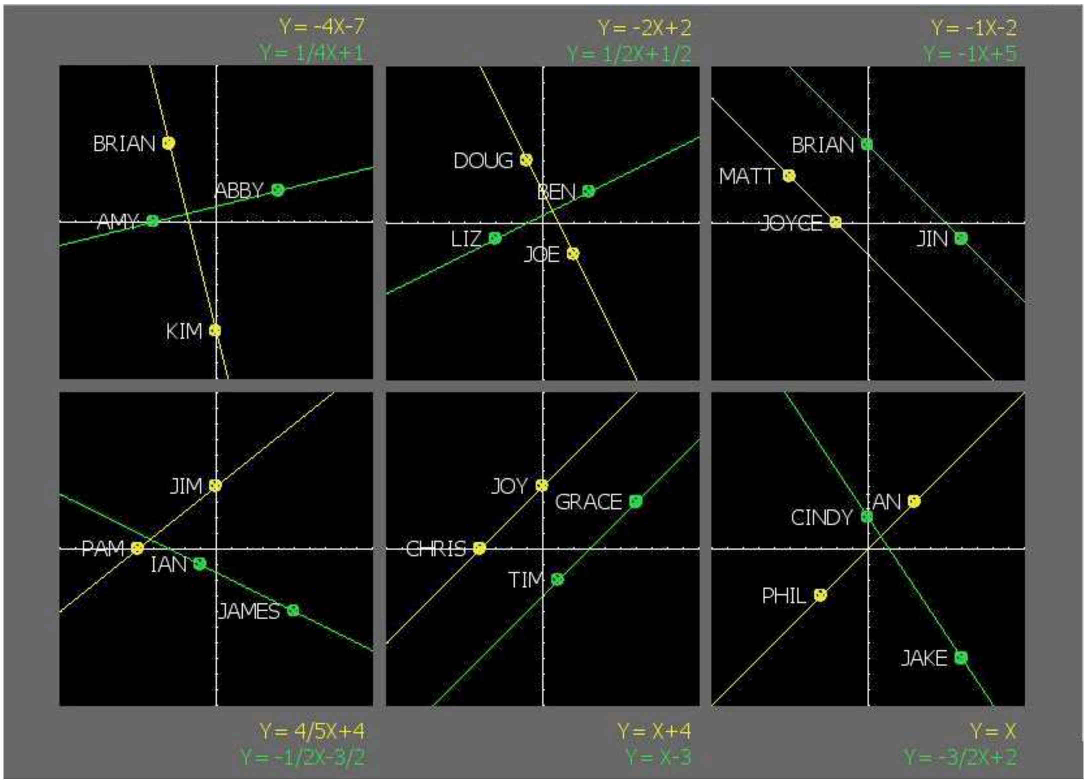

Figure 3. *Graphing in Groups* whole-class display.

Students were assigned to pairs by the teacher, and remained in the same pair throughout the unit. Two to three student pairs in each class were selected, on the basis of informed consent, as focus groups and videotaped during all collaborative activities throughout the unit. An additional camera captured the projected server display at the front of the room, as well as whole-class discussions and other instructional interactions. Server logs captured coordinates entered by the students as they manipulated their respective points to complete the collaborative tasks. Analysis of these data focused on the ways students individually and collectively interpreted the mathematical objects (i.e. points, lines and equations) and on their relations in this environment as they developed approaches to solving problem-solving tasks. Video segments of six student pairs working on between 13 and 19 tasks each were transcribed and examined alongside time-synched video of each group's shared graphing window from the server display. Students typically worked three to four minutes before completing or stopping each task; the longest sessions ran approximately 10 minutes and the shortest lasted about 30 seconds. Each transcript was then annotated with detailed descriptions of the ways students moved and marked their points. The first author and another researcher developed two different coding schemes for analyzing these annotated transcripts for each task episode: one focused on problem-solving strategies and the other on interactions between students.

Below, we analyze an episode from the video data in order to illustrate the ways learners engaged with mathematical objects and relationships in the context of this network-supported activity for student pairs. In particular, we examine the ways students explored and interpreted the relations between their respective points and a collective line, and between the graph and the equation of that line, over the course of a problem-solving task. This episode is typical of the broader data set relative to the analytic coding scheme; both the problem-solving strategies employed and kinds of student interactions occurred frequently in other tasks and among other pairs. We have selected it for presentation in the current paper because it provides a clear example of students' explorations of this mathematical domain as they undertook these problem-solving tasks, and because it features a task reminiscent of the "make expressions that are the same as 4x" example of generative whole-group activity in classroom networks.

Analysis

This episode finds two students, Priyana and Kate, in the midst of efforts to locate their respective points at coordinates that would collectively form a line with equation y=4x-1. Just prior to the excerpt below, the students had worked for a few minutes by moving each of their respective points before realizing that they could establish a line with the correct y-intercept by having Kate position her point at -1 on the y-axis. In the moments that followed, Kate kept her point stationary at the y-intercept, watching and offering commentary and suggestions as Priyana moved her own point in a series of attempts to construct a line with slope 4. To do so, she began taking a series of paired horizontal and vertical unitary steps, marking and examining the results of each move:

1. [Kate has just marked at (0,-1) to make y=(5/3)x-1. From (3,4), Priyana moves up and right to (4,5), marks to form y=(3/2)x-1.]
2. Kate: That slope has to be the same, so…what was the slope before, a 4?
3. Priyana: [Moves up and right again to (5,6)] A negative 4. [Presses mark to form y=(7/5)x-1]. I mean a 4. Ok…[moves up and right again to (6,7)]
4. Kate: Ok, so…
5. Priyana: [Presses mark at (6,7) to form y=(4/3)x-1. Moves right and up to (7,8) and presses mark to form y=(9/7)x-1. Moves up and right again to (8,9), where the label of her point partially covers the pair's equation]
6. Kate: [Trying to read the obscured equation] What does it say? [Priyana now marks at (8,9) to form y=(5/4)x-1] Can you go down more?
7. Priyana: Yeah, maybe I should go down. [Moves down five units and left five to (3,4), then back one unit right to (4,4), presses mark. Equation does not change. Looks down at her calculator and then up at the screen again, moves down and left to (3,3) and marks to form y=(4/3)x-1. Moves down and left again to (2,2) and marks to form y=(3/2)x-1. Moves down and left again to (1,1) and marks to form y=2x-1.] Oh, there it is.

This segment illustrates the kind of incremental movements in a graphical space that students often used in order to investigate variations in a corresponding algebraic expression in this environment. Priyana began with five successive moves up one unit and over one (Figure 4a), marking and observing the resulting slope each time (lines 1-6). She used these paired unitary vertical and horizontal steps—one up and one over—to make incremental adjustments to the slope of the line, watching the resulting changes in the collective display and reacting to each with soft vocalizations (e.g. "Ok" in line 3) or pursed lips before adjusting again. In each case, she used minor adjustments to the graph as a means of gradually adjusting the value of the linear coefficient in the equation. Importantly, though, she did not simply make the smallest possible adjustment in *her position*—one unit up *or* one over. Rather, taking an incremental step in each dimension before marking to form a new line allowed her to make the smallest possible adjustment to *their line*.

The next short series of movements further highlights the students' careful efforts to alternately maintain or only minimally rotate the current line. As Priyana's point moved toward the upper right corner of the graphing window, the label on her point actually obscured part of the pair's equation (line 5). Kate, who was closely watching each move, asked Priyana to "go down more" so that she could read the equation clearly (line 6). Priyana, who was quickly running out of screen room, agreed that "maybe (she) should go down" (line 7). She retraced her steps in reverse, moving down five units and left five and then briefly pausing as though to consider returning to (3,4), the point at which she had started the sequence of moves that opened this episode. Instead, she moved one unit back to the right to position herself on the same line before marking at (4,4), and so leaving the current line unchanged. Again, these moves appeared carefully selected to make only minimal changes to the current line—to seek out the correct parameter through incremental adjustments. Priyana thus abandoned her initial sequence of similar moves only when display limitations interfered, and worked carefully to minimize changes in the line even as she adopted a new line of exploration. Continuing to display the same preference for relatively small variations in slope, Priyana began a new series of paired unitary horizontal and vertical steps, this time down and to the left from (4,4) (Figure 4b). Three such steps established a slope of two, at which point she noted the appearance of an integer slope value (line 7). At this point, Kate and Priyana briefly disagreed about how to proceed:

8. Kate: Go up like… go up a little bit. [Ignoring Kate's instruction, Priyana moves down to x-axis, left to origin, and up to (0,2), all without marking] It's supposed to be 4.
9. Priyana: [Continues circling her point around her last mark—two units right to (2,2), then back to x-axis, then left toward origin] Yeah, maybe if I keep going down, like this… [briefly brings her point to rest on the origin, as if to mark there, one unit left and one down from her last mark, but then moves down again to sit on top of

Kate's point at (0,-1) and presses mark so that the pair's line disappears]. That's not right. [moves back up to (1,1) and marks to again form y=2x-1].

10. Kate: [As Priyana moves up and over one to (2,2), but does not mark, and instead moves back to (1,2)] Go up, like, little by little.
11. Priyana: [marks at (1,2), forming y=3x-1]. There you go. Oh, hold on, I'll just do one more. [Moves up to (1,3)]
12. Kate: [reading the y-intercept from the equation] That's a negative 1 though.
13. Priyana: [Marks at (1,3) to form y=4x-1] There you go.

This excerpt highlights an important contrast between Priyana's continuing incremental exploration and Kate's recognition of a more direct path toward the desired line. Reacting to the newly established slope of two, Kate prompted Priyana to "go up a little bit" (line 8). Ignoring the instruction, Priyana wavered, appearing to consider movement in all possible directions as she traced a square path around her last mark at (1,1) (Figure 4c). Wondering aloud, "maybe if I keep going down, like this," she briefly came to a stop at the origin and directly above Kate's point, where a mark would have continued her previous pattern of paired unitary steps down and left, but also formed a line with undefined slope (line 9). While Kate now recognized that moving Priyana's point up would bring about the desired increase in the value of the slope, Priyana had to this point continued relying on incremental steps in the graphical space in order to feel her way about in search of a solution. However, the fact that she chose not to mark here at the origin indicates that she recognized she was reaching the limits of this approach. Indeed, as she instead moved down to mark on top of Kate's point and briefly erase the line, she immediately realized that her new location was "not right" and returned to her previous mark at (1,1) (line 9). Kate again urged her to "go up…little by little" (line 10). Finally acquiescing, Priyana marked one unit up at (1,2). When the slope changed from two to three, Priyana clearly recognized that they were now moving toward a line with the correct slope, and that she needed to "just do one more" similar step, quickly marking at (3,1) to form the target slope and line (Figure 4d).

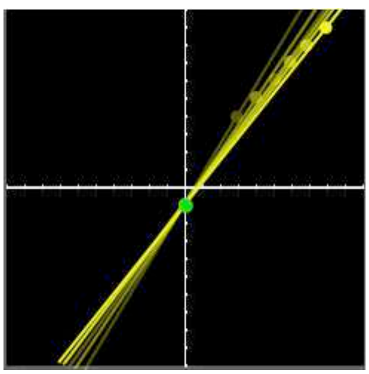
Figure 4a.

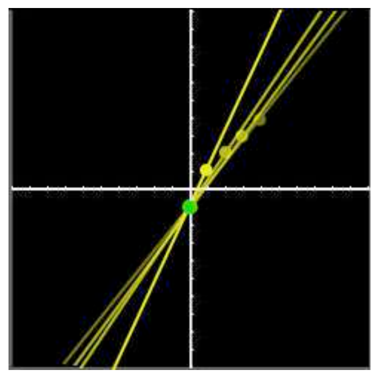
Figure 4b.

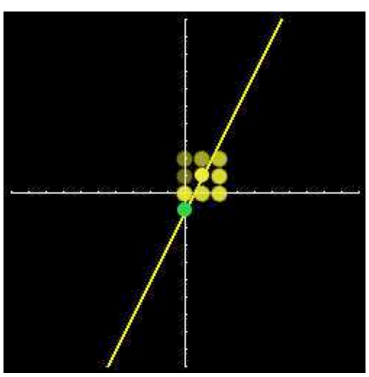
Figure 4c.

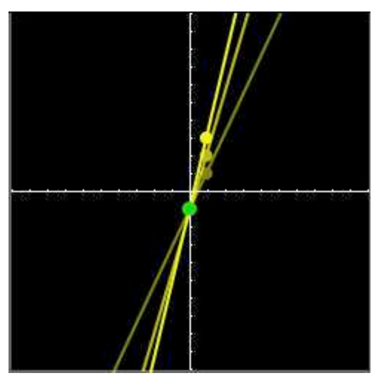
Figure 4d.

Throughout this sequence, Priyana moved from incrementally adjusting the slope downward and away from the correct value using paired unitary horizontal and vertical steps, to adjusting the slope in the same manner but upward and toward the correct value, to increasing the slope from two to the target value of four by making unitary vertical steps upward. In the first two instances, neither she nor Kate appeared to recognize the relationship between the steepness of the line and the value of the slope. In the absence of a clear target toward which to adjust the line, Priyana focused on making small and systematic changes away from the current state, first in one direction and then in another, until she established a parameter value she and Kate were better able to interpret. That moment arrived when the slope went from a series of fractions to an integer; at that point Kate urged her to "go up a little bit" because "it's supposed to be four" (line 8), clearly associating upward vertical change in Priyana's point location with increases in the slope. By contrast, Priyana initially appeared to interpret the achievement of an integer close to the target as an indication of the fruitfulness of her unitary moves down and left. When applying this interpretation proved unsuccessful, she instead followed Kate's advice, and appeared to quickly come to the same insight about the relations between her vertical moves and the slope of the line.

Priyana's transition from movements that indicate some confusion about the relationship between the graphical and the symbolic to those that reflect a clearer and more correct interpretation of that relationship suggest ways that successive, iterative movements in this space serve as resources for making meaning about slope. Both students appeared to be coming to clearer understanding of the relations between graphical movements and linear equations through real-time incremental adjustments over the course of the episode. Indeed, this episode in particular and our analysis of students' strategic engagement with these tasks generally suggests that patterned, iterative sequences of incremental movements like Priyana's here are particularly important resources for students' efforts to navigate this mathematical terrain. Though these movements are sometimes exploratory and ad hoc, much of the time they indicate students' clear expectation that these particular changes will bring about particular results. For example, students often articulated specific directions ("go up, like, little by little," line 10) to one another or narrated their own moves ("I'll just do one more," line 11), suggesting that they were beginning to anticipate certain outcomes from these moves.

Discussion

In the episode above, moving points in a graphical display allowed students to explore the mathematical space of linear functions with rational-number slopes. By systematically and incrementally varying coordinates, Priyana and Kate were able to feel their way about in the unfamiliar territory of fractionally-sloped lines, and to make use of a more familiar landmark when they encountered an integer value for slope. These kinds of exploratory movement typical of students' engagement with the *Graphing in Groups* interface illustrate the ways that developing strategies for moving points and manipulating functions in these classroom network activities might be understood as learning to navigate a conceptual environment. In this sense, we see students' joint exploration of virtual mathematical terrain as a productive alternative interpretation of the physical metaphor for a conceptual domain proposed by Greeno (1991).

From the standpoint of this analogy to conceptual environments, whole- and small-group activity structures may form complementary dimensions of a complex mathematical domain. The episode above suggests an illustrative contrast between the whole-class activity to "provide different expressions that are equivalent to 4x" and the *Graphing in Groups* challenge to "make a line with a slope of 4" from the raw materials of the two students' respective coordinate locations. In the whole-class activity, the equivalent expressions are manifested as objects that fill out a conceptual *space*; in the *Graphing in Groups* activity, the small group's solution is experienced as a process *in time* through a sequence of individual explorations and collaborative interactions. In *Graphing in Groups*, student movements of their individual points generate fluctuations and modifications of the collective graph, as the group moves incrementally towards a goal. Each student moves freely, but those movements shape and constrain the emergent activity of the group as teams explore the impact of incremental changes in their constituent sub-objects upon the nature and attributes of the collective, composite object. That collective activity is characterized not so much by attention to the family of objects that fill out a function space but rather by the manipulation over time of a single function through small perturbations—the exploration of *infinitesimal modification* rather than the unfolding of an infinite parameterization.

In generative designs for whole-group classroom network activity structures, diversity and individual expression in the classroom group serve as surrogates for infinity. In contrast, in the approach for small groups presented above, distinctive student actions take the form of infinitesimal variations in the group's collective object. Because students are identified with complex or compound mathematical objects, with each group member given the role of a component sub-object, these variations demonstrate the dependence of the collective object on the characteristics of its components. Moreover, while the whole-class activity highlights the infinite variety of expressions equivalent to 4x, the pairs task emphasizes infinitesimal variations among linear terms increasingly close to 4x. In these ways, we see designs for whole- and small-group activity as respectively and jointly mapping spatial and temporal dimensions in a conceptual environment central to the broader curricular landscape of secondary mathematics.

Conclusion

The relationship between the conceptual dimensions spanned by these small-group and (generative) whole-group activity structures suggests corresponding ways that they might be combined to form a complementary instructional sequence. While the analytic focus of this paper and the early phases of this design-based research project have been on mathematics learning opportunities afforded by small-group interactions, the *Graphing in Groups* collective display (Figure 3) also explicitly supports an additional layer of artifacts and instructional possibilities oriented toward the larger classroom group. For example, in a different *Graphing in Groups* task from the one depicted here, each pair of students creates a line with the same slope. While the activity at the level of each pair might unfold along the lines of the episode presented in this paper, the collection of groups in the class would typically together produce several representative members of a family of parallel lines. When the small-group phase of the activity is complete, each of the groups has produced a distinct solution to the challenge, and the aggregate begins to fill out the mathematical space of possibility defined by that challenge. This sequence aims to use the classroom network to integrate two levels of classroom activity, one in which the small group focus is on understanding the interrelations of points and lines through strategies of infinitesimal variation, and another in which the classroom group as a whole can see the infinite variety of possible lines that meet the activity's challenge criterion. Our current classroom research focus is on jointly examining these small-and whole-group levels of mathematical activity and instructional interaction in this hybrid environment. In these ways, we see small-group designs of the kind outlined in this paper as expanding the map of the social and mathematical terrain in classroom networks charted by prior research, and broadening the pedagogical modes across which those networks might support mathematics teaching and learning. Collectively, these whole and small-group activity structures highlight the rich potential of the social space of networks for mapping conceptual spaces in mathematics.

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