Street Mathematics and School Mathematics in a Video Game

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Abstract: Prior research has documented that school-age children tend to be more sensible and flexible in their strategies when thinking about quantitative relations in out-of-school settings. In classrooms children largely do math for math's sake. In other settings, when children engage in mathematical thinking, they often do so in service of purposeful actions situated in ongoing activities. This study rests on the premise that immersive video games can be designed to provide a context for purposeful mathematical thinking. This paper examines middle-school students' mathematical thinking in a video game designed to support applications of proportional reasoning. Data sources include screen-records of game play from two 6th-grade classrooms. Data analysis aims to identify and characterize the kinds of mathematical thinking students engaged in during game play. Findings suggest that in the same video game students' mathematical thinking varied, both within and between students, reflecting previously observed distinctions between street and school mathematics.

Background

This research involves designing an immersive video game to support purposeful mathematical thinking. In this background section, we briefly introduce 1) prior research on mathematical thinking across settings and 2) how immersive video games can support purposeful mathematical thinking.

Mathematical thinking across settings

Prior research in cultural psychology and in mathematics education has documented that people, including school-age children, engage in qualitatively different mathematical thinking when they are faced with similar mathematical problems in different settings, e.g., in a supermarket vs. in a test (Lave, 1988), in a market vs. in a classroom (Nunes, Schliemann, & Carraher, 1993). Studies that compared mathematical thinking in school vs. in everyday settings suggest that people are more sensible in their thinking and more flexible in their strategies when they reason about quantitative relations outside of school settings. For example, even though such mistake rarely occur in everyday situations, in school or school-like settings sometimes people will arrive at an amount of change to be returned that is greater than the amount of money given to the seller. This is because, in school,

> students commonly learn algorithms for manipulating numerical values without reference to physical quantities … By contrast, individuals in the workplace solve problems using mathematics as a tool to achieve practical goals that they keep in mind throughout the solution processes, while making continuous reference to the situation and the physical quantities involved (Carraher & Schliemann, 2002, p. 135).

School mathematics involves numerical manipulation and divorces it from meaning, while street mathematics is done in service of practical goals and preserves meaning.

Additionally, prior research has shown that school-age children are sensitive to variations in contexts even within the formal school settings, e.g., when similar tasks are posed in a conversation vs. on a worksheet (Cobb, 1987) or when similar tasks are posed in a mathematics classroom vs. in other classrooms, such as social studies (Säljö & Wyndhamn, 1993), language (Sfard, 2008) or religion (Dewolf, Van Dooren, & Verschaffel, 2011). Studies like these consistently show that in mathematics classrooms students often default to doing calculations without considering the meaning of the calculations and corresponding results. This widespread phenomenon is known as "suspension of sensemaking" (Schoenfeld, 1991) because it appears that students forego their sensemaking when they enter a mathematics classroom.

Overall, as Nunes et al. (1993) suggested in the title of their books, street mathematics and school mathematics are separate constructs. That is, again, when people do math in the context of use (street mathematics), they are more sensible and flexible in their thinking, compared to when they do math for math's sake (school mathematics). Educational designs for mathematics classrooms then need to account for the powerful normative forces of mathematics classrooms that often pressure students to do math for math's sake, as they do not reveal children at their intellectual best.

Immersive video games as contexts for mathematical reasoning

This research seeks to leverage new media technology (specifically immersive video games) to directly counter the trend that in mathematics classrooms students overwhelmingly engage in doing math for math's sake and rarely engage in putting math to use in service of purposeful actions. What is meant by immersive video games here is video games that integrate perceptual immersion (often by making use of a first-person perspective and a 3-D game world) and narrative immersion (often by developing an engaging storyline and dialogues). Video games afford opportunities for exploration and experimentation through iterative play. With their well-developed storylines and interactive game worlds, immersive video games in particular have the potential to provide a space for students to explore and deepen their understanding of mathematical concepts and procedures by using them to achieve desired game goals and observing the outcomes of their use of concepts and procedures (Gresalfi & Barab, 2011).

This study

Working from a theoretical frame that commits to describing thinking and reasoning as embedded in contexts of culture and activity, in this paper we present an analysis of middle-school students' mathematical thinking during video game play and how their mathematical thinking is shaped by the broader scope of activity (Lave, 1988; Scribner, 1997). Specifically, the following questions are addressed:

- When did mathematical thinking happen during game play? What kinds of mathematical thinking?
- What purposes did students' mathematical thinking serve?
- Did the kinds of mathematical thinking that happened in the game correlate with parts of the game they happened in, and how?

Previous studies that have examined how settings and contexts influence mathematical thinking often took the form of quasi-experiments, but here, we have an opportunity to examine how variations in context within the same classrooms influenced mathematical thinking both within and across pairs of students. This paper contributes additional evidence of street mathematics and school mathematics as separate constructs, and how they interacted with the problem-solving task posed to students, in the familiar setting of a mathematics classroom, but in a novel activity (to this setting) of video game play. Next, we describe the video game and the problem-solving task posed to students in the game.

The video game: The Adventure in Boone's Meadow

The video game, *The Adventure in Boone's Meadow*, is a video-game rendition of a video-based lesson from the Jasper Series (Bransford, Zech, Schwartz, Barron, & Vye, 2000). In the game, students play the role of a wildlife rescue assistant (Figure 1) and are tasked with transporting injured animals from a nature reserve to an animal clinic.

Figure 1. A screenshot of a typical dialogue in the game.

To construct an animal rescue plan, students have to decide between different vehicles and different routes, taking constraints such as time travelled and gas used into account. While there are multiple sensible ways to conceive of and represent the quantitative relations in this situation, here we describe the particular situational constraints imposed by the game. Most importantly, in order to arrive at the correct numerical answers as programmed into the game, students have to assume the following: 1) the plane travels in a straight line (such that the distance travelled by the plane is the same as the distance between the start and end points), and 2) the plane travels at its maximum speed at all times. The game also specifies a solution path that students have to follow by requiring that students construct their rescue plans on an interactive form called the Route-Planning Tool (Figure 2). When students create a complete trip, they are able to proceed to “run route plan.” Here, students partly maneuver an airplane in an interactive animation and partly watch a non-interactive animation of a plane moving in space. They may succeed in bringing the animal back to the clinic, or they may fail the mission, most likely by running out of gas. It is important to note that the accuracy of the numerical answers is not a prerequisite for proceeding to “run route plan.”

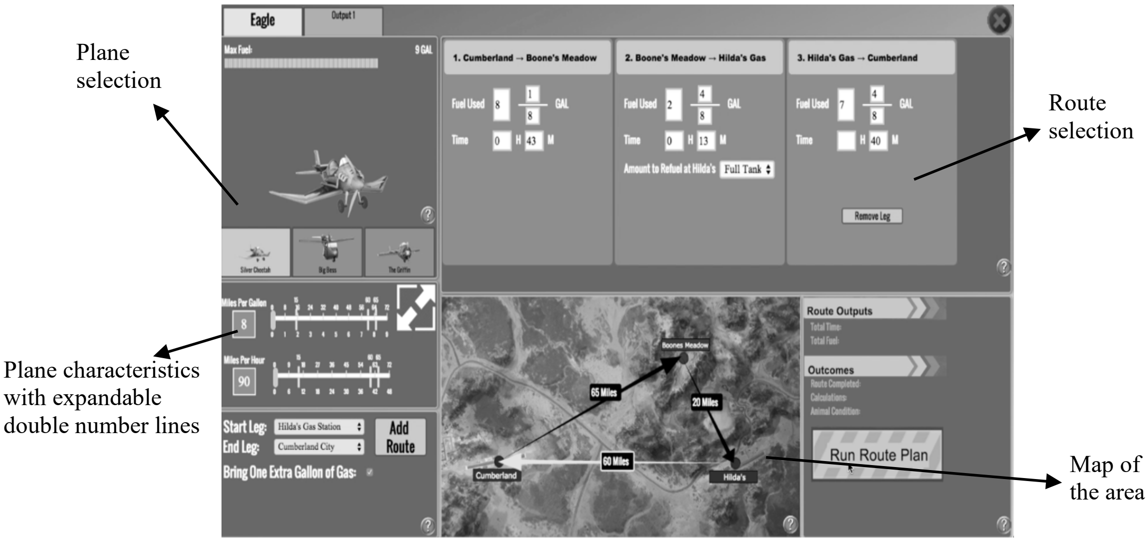

Figure 2. A screenshot of the interactive form where students construct their animal rescue plan. In the game, this form is referred to as the Route-Planning Tool.

Methods

The data in the current analysis includes screencaptures of game play collected from two self-contained sixth-grade classrooms in an urban city in the United States. These classrooms were chosen from the corpus because of completeness of data, i.e., more than 90% of students consented to participate in the study, and all who consented were screencaptured. Students in both classrooms played the video game in pairs for about 4 hours over 3-4 days. Overall, 23 pairs were included in the analysis, 12 from one classroom and 11 from the other. (One pair from each classroom was excluded from the analysis because of data loss due to equipment failure.)

Playing Boone’s Meadow includes some time that looks very much like commercial video games, for example when students explore the game world (exploring and engaging in activities on the streets of Cumberland City, the virtual setting where the animal clinic is). It also includes experiences that look much more like school, such as when they have to answer specific mathematical questions. Figure 3 shows the contrast between more “street-like” and more “school-like” parts of the game. Previous analyses clarified that at least for some students, these different experiences felt and asked for very different kinds of activity. Thus, our analysis in this paper sets out to systematically explore how students used mathematical ideas both when they engaged with the “street” aspects of the game and when they engaged with the “school” aspects of the game. To explore this question, we reviewed the screen-records and marked episodes of mathematical thinking in different parts of the game. The “school” parts of the game were well defined, since screens that request numerical inputs appeared at specific points in the game as programmed. Identifying episodes in the “street” parts involved noting when students mentioned quantities (e.g., “speed” or “fast”, “distance” or “far”) and quantitative relations (for example, “it uses more gas”). We characterized these episodes using Thompson’s distinction between quantitative and numerical operations (Thompson, 1994). Next, we reviewed episodes of mathematical thinking and described what purposes the mathematical thinking served.

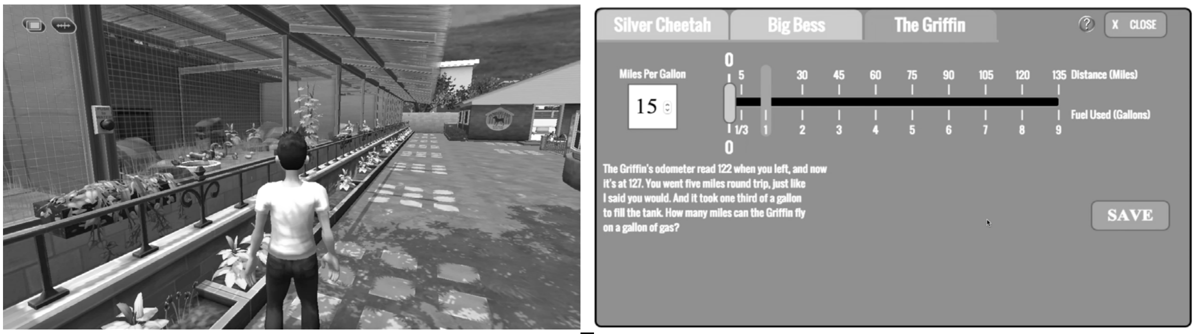

Figure 3. Screenshots of examples of the "street" and "school" parts of the video game

Findings

Mathematical thinking mostly happened in the "school" parts of the game, specifically around the interactive form that students had to use to construct their rescue plans. There, mathematical thinking largely entailed calculating gas used and time travelled for each leg of the trip, as those were prompted for by the form. Mathematical thinking rarely happened in the "street" parts of the game. In the few cases where it happened, students' mathematical thinking had the characteristics of street mathematics—reasoning with physical quantities (rather than disconnected numerical values), and continuously referring to the problem situation.

Mathematical thinking in the "school" parts of the game

When working on the interactive form they had to use, most students did calculations, and most of the time they did them accurately. However, students did not use those calculations in service of purposeful actions, e.g., to determine whether the plane would have enough gas. Table 1 provides an overview of students' mathematical thinking in this part of the game. Across the two classrooms, the total number of calculations that students could have done while working on this form was 116 calculations (the number of calculations differed depending on the route chosen). Students attempted to do 97 (83%) of those calculations. Their calculations were accurate on 73% of those attempted. However, only 8 (8%) of those attempted (regardless of accuracy) were used in service of decision making.

Table 1: Numbers of total, attempted, accurate and connected calculations carried out by students

	Calculations			
	Total	Attempted (% of total)	Accurate (% of those attempted)	Connected (% of those attempted)
Class 1	60	50 (83%)	32 (64%)	4 (8%)
Class 2	56	47 (84%)	37 (79%)	4 (8%)
Total	116	97 (83%)	69 (73%)	8 (8%)

It was not apparent why students did calculations and then failed to use them to reason about what would happen. Our interpretation was that students did these calculations because they thought they had to fill in the "worksheet." It was unfortunate that this part of the game had the appearance of a worksheet. Most worksheets only require students to fill in the blanks with short answers, so unsurprisingly, over time, students come to take their tasks with worksheets to be filling in the blanks (Scherr & Hammer, 2009). In Class 1, the game in fact was not programmed to stop students from moving on if the form was left blank. Yet students in that classroom still filled in the blanks at the same rate as did those in the other classroom, and similarly rarely used the calculations they had done. A few incidental cases supported this interpretation that students thought of these calculations in this "worksheet" as their tickets to flying. First, one pair of students in Class 1 accidentally proceeded to flying without having filled in any blanks. Having found out they were allowed to do so, they never filled out those blanks in subsequent plays. Another incidental case came from a classroom not included in this particular analysis, but which is relevant here. One student had an older brother who played the game the year before and was told by the brother that one could play this game without having done the calculations; as he played the game, this student entered random inputs into the form, quoting his brother to his partner. These cases showed that, when students learned they did not have to do the calculations, they did not do them.

In sum, in the "school" parts of the game, students perfunctorily did calculations but did not use them for other purposes beyond producing numerical answers, possibly as "tickets" to the next stage in the game. This was consistent with the well-documented observation that in school students overwhelmingly do math for math's sake. In the "school" parts of this game, students did school mathematics.

Mathematics in the "street" parts of the game

Mathematical thinking rarely happened in the "street" parts of the game. In the rare cases it happened, students' mathematical thinking was different from that in the "school" parts of the game. When using calculations, students used different, simpler strategies (than the one imposed by the structure of the interactive form). Their calculations were done with a clear focus on determining whether the plane would have enough gas.

In this paper, we describe only one of these rare cases, chosen as an illustrative case here because students in this pair engaged in mathematical thinking in both "street" and "school" parts of the game, hence illustrating the difference between kinds of mathematical thinking in different parts of the game within students. These students were given pseudonyms Boston and Adam. When Boston and Adam were exploring the game world, they learned from the veterinarian that an endangered eagle had just been shot down and needed to be picked up immediately. Boston and Adam halted the conversation with the doctor, pulled up the map of the game world (accessible anytime in the game), and started devising a rescue plan:

Boston: So, the fuel capacity is 12 gallons, so it can go 7 times 12, what's 7 times 12?
Adam: 7 times 12, 84, I think.
Boston: So, it can go 84 miles, on an entire tank.
Adam: But then we have to think about going back, too. So, we can just go like that and come back to Hilda's. Yeah 75, and then we can go to Hilda's and then come back.
Boston: Are you sure? Oh yeah, we just have to gas.
Adam: We use, it's 60 here, so we would have 25 left, 15 to Hilda's, and 10 gallons left, then we have enough just to get back.
Boston: So Silver Cheetah.

Their mathematical thinking here was put in service of considering "would this work." Their strategy was arguably less cumbersome than that imposed by the interactive form. Given the layout of the form, to work with it, students would have to calculate both fuel used and time travelled for each leg, then add them up to find total time travelled and fuel used. This often involved doing multiple divisions (i.e., gas used = distance ÷ gas mileage, time travelled = distance ÷ speed). On the other hand, Boston and Adam started by figuring out how far this plane could fly with a full tank of gas (Lines 1-3: 84 miles = 7 mpg × 12 gallons). Then, they subtracted the distance along a path from the that distance. This meant that they only had to do multiplication, involving quantities of different units, only once. The rest of their calculations involved additive comparisons within the same quantity (distance in miles). Most importantly, their calculations were continuously aimed at considering "would this work." This was notably different from when most students did calculations on the interactive "worksheet," since they almost never connected their calculations to considering whether their plan would work.

After they had decided on the plane and the route, they picked up the conversation with the doctor, and continued to the "school" part as the game was programmed. Given their spontaneous solution with sensible mathematical thinking earlier, we had evidence that these two students understood well the goal they tried to accomplish in the game along with quantitative relations relevant in this situation. However, when Boston and Adam opened the interactive form, they did not engage in mathematical thinking that was sensible as they did earlier. In marked contrast to what had happened prior to opening the form, in filling in blanks, Boston and Aaron made no connections either to their earlier thinking or to the broader narrative of the game. Indeed, when calculating the gas for each leg, they mistakenly changed the miles-per-gallon that they were working with, sometimes confusing it with another plane. These were not arithmetic mistakes, and in fact, all their calculations were accurate given the values they were using. Instead, these mistakes suggest a disconnect between the calculations and the situation.

Boston and Adam engaged in qualitatively different mathematical thinking. When they were exploring and engaging in activities in the streets of Cumberland City, their problem solving reflects characteristics of street mathematics. When they worked on the form with an appearance of a worksheet requiring calculations, they engaged in numerical manipulations unconnected to physical quantities, characteristic of school mathematics.

Summary

This research attempted to leverage immersive videogame technology to support purposeful mathematical thinking. It did so by developing an interactive 3-D game world, populated with lively characters and accompanied by an engaging storyline that motivates the relevance of mathematical ideas (ratio and rate) in achieving desired game outcomes. Findings in this study showed that, contrary to the design's intention, the game rarely supported students to engage in purposeful mathematical thinking. Students often engaged in numerical manipulations divorced from physical quantities, characteristic of school mathematics, rather than reasoning with quantities in order to accomplish practical goals, characteristic of street mathematics. This behavior appeared to be influenced by the structure for solution paths imposed by the interactive form designed to scaffold students' mathematical reasoning. In a few rare cases, when students reasoned mathematically about what to do in the game without this form in front of them, their mathematical thinking served specific practical purposes and connected to the broader narrative of the game. The division between mathematical thinking in the "street" and "school" parts of the game reflected the well-documented division between street mathematics and school mathematics. The current design of this game did not succeed in countering the normative forces to do math for math's sake in mathematics classrooms. Thus, it reinscribed the division between street and school mathematics onto this newer form of educational technology, and fell short of supporting students to make connections between everyday and school forms of mathematical thinking.

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