Perspectives and problem solving in an algebra classroom

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Abstract: Perspective theory, an extension and generalization of schema theory, adapts the notion of perspective from psycholinguistics to capture the cognitive and interactional creativity in reasoning and problem solving. This poster applies perspective theory to two problem solving episodes of classroom mathematical discourse. The episodes illustrate the negotiation of differing perspectives, participants' commitment to perspective and demonstrate how perspectival constructions can be interactively achieved. Characteristics of task and positioning may encourage participation in exploratory perspectival construction.

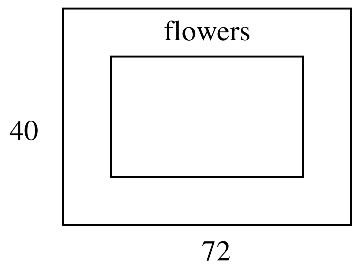
In our analyses of classroom mathematical discourse, our research group has encountered many instances of cognitive and interactional creativity that cannot be understood by appealing to schema theory. In order to characterize these episodes of learning, we have identified a need for a generalization of schema theory that incorporates the concept of entities and relations but that also includes a point of view, or perspectival, construct. See Greeno & MacWhinney (these proceedings) for a discussion of perspective theory. In problem solving discussions, the participants are often involved in considering alternatives and engaging in effortful interactions in order to reach mutual understanding. These interactions can be characterized as instances of exploratory construction, in which at least one of the participants needs to discover and arrange a perspectival construction, rather than fitting entities into a prefabricated arrangement based on a schema.

In support of a perspective theory of understanding, this poster presents episodes from a progressive eighth grade algebra classroom. In one episode, students negotiate alternative perspectives on constructing (linear) functions to describe pattern configurations. The discussion reflects each participant's commitment to her own perspective and the role validity plays in the resolution of multiple perspectival views. In the second episode involving an algebraic word problem, the teacher and two students interactively construct a perspective that is different than the schema-based perspective that the teacher initially planned to develop. Both episodes provide evidence for the existence of perspectives in the context of problem solving and allow us to distinguish between perspectives that are schema-based and perspectives that are not schema-based but have schema-based components. In addition, both episodes provide an account of the way in which perspectival flow and shift result in conceptual schema alignment and the discourse surrounding constructive listening. We conclude that perspectival alignment is a critical component of mathematical learning that can occur within a single representational system.

Exploring Patterns

As a homework assignment, students completed two complementary worksheets entitled "Exploring Patterns" and "Graphing Functions." "Exploring Patterns" consisted of six pattern configurations of either toothpicks or tiles in which the first three 'steps' were depicted. Students were instructed to draw steps 4 and 5 for each configuration, answer how many elements would be in step 10, and describe how the pattern was changing or growing. Each pattern configuration corresponded to a function for which students were to construct a T-table for values 0 through 5, 10, 17, and 'n' and draw a corresponding graph. Students were organized in small groups during the homework check to compare and discuss their solutions. In the group of students (J, D, and G) we observed, the answer to the question of how the pattern is changing or growing revealed three differing perspectives, each setting up relations between the mathematical entities from a point of view. J focused on the mathematical relationship between step numbers and reasoned (incorrectly since the linear relationships had a constant term) that step 10 would have twice as many elements as step 5. J maintained this perspective throughout the discussion despite the presentation (and validation) of the other group members' perspectives. D focused on the relationship between step numbers and number of toothpicks, identifying a relationship that worked for each step/toothpick configuration (e.g. $2n + 1$, where n denotes step number). In contrast to these schema-based perspectives, G focused on the number of elements in step 1 and the relationship between successive steps, namely how many elements were added each time (e.g. $3 + (n - 1)*2$). G and D constructively listened to one another during the discussion and established that both of these perspectives were valid.

The Garden Lap Problem



Sally's backyard is 40 feet wide by 72 feet long, and it is structured so that a central rectangle of grass is surrounded by an even border of flowers. The area of the border (flowers) is 5/12 of the area of the whole garden. If Sally wants to be sure to walk her dog for .25 mile, how many laps will she have to take around the grass?

Figure 1. The Garden lap problem

On the first day of the garden lap problem (see Figure 1) discussion, students calculated numerical values for the area of the whole garden (2880 ft^2), the border (1200 ft^2), and the central rectangle (1680 ft^2) and guessed values for the dimensions of the central rectangle that would satisfy the two solution constraints. On the second day of discussion, the goal of the class was to construct an algebraic solution to the problem. This was judged to be a difficult task for the students, and the teacher (T) attempted to communicate her perspective to the class. After reproducing the problem diagram with numerical area values on the board, T labeled the left and top border widths with question marks (?'s) and, surmising that she "gave it away" with this cue, asked the class "What would my variable be?" Note that assigning a variable, say w, for the border width would result in the quadratic equation (72 – 2w)(40 - 2w) = 1680, the solution of which could then be subtracted from the dimensions of the whole garden to produce the dimensions of the central rectangle, and subsequently the central rectangle perimeter needed to answer the question of the requisite number of laps. The students, however, did not respond to T's hinted suggestion of identifying a variable with the border width; they did not share the schema and, despite hints, were unable to 'view the problem' as T had with a focus on the border width. Instead, when the question of variable was again raised in conjunction with the question "What am I trying to figure out here?," one of the students (G) introduced an alternative perspective by drawing attention to "the two lengths of the inside square" and suggesting that these could be given labels, x and y. Because this was a different perspective than T had entertained and because G was only able to present disjoint pieces of the corresponding solution, arriving at mutual understanding required considerable effort and negotiation. It was at this juncture that T, G, and another student (H) embarked on an extended exploratory construction that included explaining entities (G: "So, if you do 40 minus x divided by 2, you'll get the two widths, I think."), questioning referents (T: "Can you point out where the 72 minus y is going to be that question mark?"), and suggesting relations (H: "I mean, you could make another equation, x times y equals 1680."). The result was a publicly available (via the white board) system of two equations in two unknowns [x times y = 1680 and (72-y)/2 = (40-x)/2], the solution of which would yield numerical values for the dimensions of the central rectangle, its perimeter, and consequently the requisite number of laps. Following this exploratory construction, T acknowledged to the class that "this is a completely different way than I looked at it" and instructed the class to finish solving the problem from this perspective.

Conclusions

We show how students exercise conceptual agency by presenting, constructing, and negotiating alternative perspectives in the classroom and how novel understanding resulted from conceptual alignment.

References

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Acknowledgments

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