CHARACTERIZING THE NATURE OF DISCOURSE IN MATHEMATICS CLASSROOMS

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Abstract: A key idea in mathematics education reform today is the need to support discourse in classrooms. Despite the importance of this goal however, research has shown that realizing this vision of discourse can be hard to achieve. To address this issue, I introduce a framework that can be used to study and implement discourse in mathematics classrooms. Data come from two second-grade classrooms in which instruction was based on a reform-based curriculum. The study examines the character of the discourse that occurred in these two classrooms. I first identify six distinct discourse structures that were used as building blocks for talking about students' solutions to mathematical problems. I also identify three broad patterns in the ways these structures were used to define interactions in the classrooms. These patterns vary in the level of complexity from getting discourse started to increasingly complex patterns where students were engaging in mathematically complex discussions.

Introduction

One of the key ideas in mathematics education reform today is the need to support discourse in classrooms (NCTM 1991, 2000). This emphasis is motivated by a desire to change the way in which mathematics teaching and learning is envisioned and implemented in classrooms. Work in the area of situated cognition argues that learning is an interactive process where students learn by interacting and participating in practices with others in social and cultural contexts (Brown, Collins, & Duguid, 1989; Cobb & Yackel, 1996; Cobb, Wood, Yackel, & McNeal, 1992; Greeno, 1991; Lave & Wenger, 1991; Vygotsky, 1978). This is in contrast with the traditional teacher-dominated view of instruction. In traditional classrooms the teacher is seen as the authority and he or she evaluates students to see how well they have acquired knowledge (Sinclair & Courthard, 1975). In this model of classroom instruction and learning, students passively receive knowledge from the teacher and answer directives from the teacher.

As researchers and educators begin to use sociocultural perspectives to consider teaching and learning, an emphasis is instead placed on establishing classroom environments in which students are engaged in meaningful mathematical activities. In particular students are expected to have opportunities to talk about mathematics and to share their ideas with their teachers as well as with others in the community (Ball, 1993; Davis, Mahir & Noddings, 1990; Dorfler, 2000; Greeno, 1991; Hufferd-Ackles, Fuson & Sherin, 2004; Lampert, 1990; NCTM, 2000). In this context, students explain and clarify their thinking and respond to others' questions about their ideas. Students and the teacher talk about the mathematics as well as about executing the procedures and algorithms.

Despite the importance of this goal, research has shown that realizing this vision of discourse in classrooms can be hard to achieve (Bauersfeld, 1995; Gallimore & Tharp, 1990; Sherin, 2002). First, it can be hard to get discourse started in the classroom. For many teachers this involves a significant rethinking of their goals for instruction. And even in cases where teachers begin to make changes in their instruction to include aspects of mathematics reform (for example focusing on understanding versus executing algorithms), they may tend to follow traditional patterns of discourse (Leinhardt, 1993; Spillane & Zeuli, 1999). In this way some long-established norms of school may dominate the patterns of discourse, resulting in an emphasis on right answers and a reliance on teachers' knowledge (Ball, 1991, 1993).

Second, discourse is hard to establish because of what is asked of the students. Students may be unsure of what is expected and talking about mathematics may not come naturally to them (Cobb, Wood & Yackel, 1994; NCTM, 2000). Teachers have to model, guide and help them learn how to do so. Finally, even when reform-oriented discourse is established, it can be difficult to maintain over time. Furthermore, it can be quite challenging to move beyond an initial state in which discussion is centered on explaining one's work, to one in which the discourse is consistently around significant mathematical ideas (Sherin, 2002).

This study contributes to these issues by introducing a framework for exploring how students and teachers talk about mathematical problems and solutions. I also apply this framework to two second-grade classrooms. The classrooms were using a new reform-based curriculum that emphasizes the establishment of productive discourse among the teacher and students. In particular, the curriculum encourages students to make visual representations of their individual solution methods. The class then discusses these different methods. This study examines the discourse that occurs as the class discusses these different methods.

Specifically this study asks the question: What discourse structures are used by elementary students and teachers to talk about students' solutions to mathematical tasks? By discourse structures I am referring to units or components that can be used to talk about the mathematics. Further I ask the question: How are these structures used together to form patterns of classroom discourse?

Prior research on classroom discourse has examined strategies that teachers can use to facilitate discourse (Ball, 1993; Fraivillig, Murphy & Fuson, 1999; Hufferd-Ackles, 2000; Kazemi, 1999). These address specific instructional techniques that seem to support the development of classroom discourse. For example, revoicing or restating a student's idea helps other students to follow along (Michaels & O'Connor, 1996). Hufferd-Ackles (2000) describes specific types of probing questions that are critical to the establishment of productive discourse.

Other research on classroom discourse has examined discourse interaction patterns in classrooms. For example, Mehan (1979) found a reproducible pattern of Initiation, Response, and Evaluation in interactions between teachers and students. The teacher initiates the interaction by asking a question. There is a response, followed by an evaluation of the response. Other researchers have described a process of funneling as a way in which teachers can get students to focus in or a few ideas for the class to discuss (Sherin, 2002; Wood, 1997). This study considers both of these perspectives and attempts to describe specific discourse strategies as well as discourse patterns that are comprised of these strategies.

This study contributes to current research in several ways. First, the framework helps us to characterize the nature of classroom discourse in mathematics beyond what is currently known. Specifically this research identifies particular discourse structures that are used as students explain, ask questions, and communicate with one another about their solutions to mathematical problems and situations. Explaining one's thinking and being able to participate in productive discussions of mathematical ideas are important learning goals of mathematics education reform (Hiebert et al., 1997; Lampert, 1990; NCTM, 2000). By detailing the discourse in two classrooms, this study enhances our knowledge of what discourse in mathematics can look like. Furthermore the development and classification of discourse structures increases our understanding of the different ways in which students can be encouraged to talk about mathematical problems and situations.

Second, the framework is useful because it offers directions for teachers who want to pursue such discourse in their own classrooms. In other words, the identified structures can be thought of as particular strategies that teachers can attempt to implement in their classrooms. Further, the results of this research provide valuable ideas that can be used to inform professional development around creating and maintaining discourse in mathematics classrooms.

Methodology and Setting

Context

The research reported in this study is based on observations in two mathematics classrooms in an urban elementary (K-8) school. The research is based on principles of a case-study approach, which allows us to isolate and study in an in-depth manner the issues within a specific context. This is particularly appropriate given the complexity of studying classroom interactions (Cohen, 1990; Fraivillig, 1995; Spillane, 1998; Yin, 1994). The two classrooms were second-grade classrooms where the teachers were in their first year of implementing the Children's Math World curriculum (herein referred to as CMW). The CMW (Fuson et.al, 2000) classroom is seen as one where there is a collaborative effort towards learning and where learning occurs through interactions with others. The curriculum includes many features that support these goals, three of which are particularly relevant for this study.

First, the CMW curriculum places a strong emphasis on students creating visual representations of their solutions to the problem situations. These in turn help them to make sense of a problem situation, as well as to facilitate their thinking and explanations. A second aspect of the curriculum that is relevant is the focus on having students use different methods for solving problems. Finally, another aspect of the curriculum that pertains to this study is a focus on encouraging discourse among students and the teacher. One of the ways in which a culture of collaboration and discourse is encouraged is through what is called "math talk". Math talk involves the discussion of ideas and problem-solving strategies amongst the community after students have drawn models to solve and represent problem situations. This study seeks to unpack the nature of this math talk as the students engage in discussion specifically of different solution methods. In particular it examines the kinds of discourse structures that are used in these discussions. Further, it examines the kinds of patterns of discourse that are built with these structures.

Data collection

During the school year 1999-2000 the mathematics classes of the two classrooms were observed and videotaped. The classes were observed (on average) every week from September through May. In addition, field notes were collected during each observation. Observation notes contained detailed information concerning (1) general lesson procedures, (2) students' engagement and participation in the activities, (3) key events that occurred and (4) how the teacher and students communicated about mathematics ideas. Notes on general lesson procedures, students' engagement and participation and events provided contextual information for the examination of the interactions around students' drawings.

A variety of types of interactions around students' representations of their solutions were noted, those involving (a) the teacher and individual students as she moved through the classroom, (b) the teacher and all students, and (c) students and other students. Particular attention was paid to the discussions at the board because a significant portion of the lessons comprised this type of interaction. The displays of the students' representations of their solution methods that appeared on the board were recorded both by video and within the field notes.

Data Analysis

This study seeks to understand the nature of discourse in a mathematics classroom by looking closely at interactions around students' displays of their solutions. The research reported is largely qualitative in nature, describing aspects of the communication as students and the teacher discussed the different solution methods made by students to represent and solve problems. The observation field notes along with the videotapes of these observations form the corpus of the data for this study. All of the videotapes were fully transcribed. Data analysis was guided by a combination of fine-grained analysis of the video (Schoenfeld, Smith & Arcavi, 1993) and discourse analysis techniques (Erikson, 1986). The analysis focused on studying in detail one portion of the daily lesson – the discussions of the different solution methods at the board. There were three stages of analysis. These are described below.

Stage 1

In this initial stage, all of the 45 lessons were examined and the notion of 'display events' was defined. A display event is a lesson segment in which a set of one of more solution methods related to a particular problem is explored. I called these "display events" because the focus of discussion was the drawings and representations students displayed on the board. The basis for these drawings or representations could be a known representation (e.g. the number line), a representation introduced by the curriculum, or one invented by students. The display events in all 45 lessons were identified. Two researchers performed this analysis independently in order to ensure the reliability of the process and agreement was over 95%.

Stage 2

The goal of this stage was to identify and characterize the different kinds of talk that occurred in the classroom within the display events. To explore this, five lessons were chosen, spaced over the year, for closer examination. The display events that had been identified within these lessons were examined with particular attention to examining what occurred as the teacher and students discussed the different solutions to mathematical problems. Here I looked at various factors including (1) who was speaking, (2) what was being said, (3) what was the purpose of what was said, (4) how was this related to the initial problem, (5) what mathematical concept was being addressed, (6) how many representations were considered, and (7) were there any changes made to the representations during the discussion. The display events were then divided into chunks of activities, according to when the nature of the talk changed. From this chunking, patterns and categories emerged. In particular I identified six different discourse structures. These structures will be discussed in the next section.

Stage 3

With these discourse structures in mind, I proceeded to the next stage of analysis. Here ten lessons were selected from each of the two teachers - making a total of twenty lessons chosen for this stage of analysis. The lessons chosen were spread throughout the year, while ensuring that they covered varied topics as well. All of the display events in each of the twenty lessons were coded using the six discourse structures identified earlier. Another trained researcher using the same categories independently examined the lessons to ensure inter-coder reliability. Agreement was over 95%.

Having identified and coded these structures I also looked at how these structures were used together. Here I looked at the sequence of structures in these lessons – the order and the frequency of the use of these structures. Next I explored what recurrent patterns of interactions occurred around the use of these discourse structures. Specifically, snapshots of the twenty lessons were examined with attention to (a) the order in which the discourse structures were used and (b) the frequency of re-occurring sequences of structures. Again through an iterative process three distinct patterns were identified and coded for in the snapshots of the twenty lessons. Another trained researcher independently coded the lesson snapshots for the three patterns. Agreement was over 98%.

Once these patterns had been identified in the twenty lessons, the corresponding notes and videotapes were again examined. This was done in order to better understand the nature of the different interaction patterns and fine-grained differences within each pattern. In summary, this stage of analysis focused on trying to understand at a deeper level than the previous stage, the kinds of interaction patterns that were happening as the community talked about students' solutions to mathematical tasks. The results of this stage (Stage 3) of the analysis form the central results of the study and are described in the Results section that follows.

Results and Discussion

The results from this study are organized in two parts. The first part identifies discourse structures that were used in these classrooms as the students and teachers discussed student solutions to mathematical tasks. The second part describes recurrent patterns in the ways that these six structures were used to promote discourse and math talk in the community. In the next few paragraphs I briefly outline the discourse structures and the interaction patterns. The paper will discuss these in detail including providing examples from the data.

Discourse in the classroom community

Building blocks

There were six distinct discourse structures that formed the building blocks for interaction in the classroom around students' representations of their solutions. The structures that emerged were

- Answer and partial explanation [1a]: Different solution(s) for the problem are produced or displayed on the board. Within the representation there is an answer and a partial solution path that is available to the audience. The explanation and answer that can be inferred is non-verbal.
- Explicit explanation [1b]: Explicit explanations are provided, by the maker or others, of the problem solved. This explanation is the verbal extension of the previous structure. The explanation may be about the elements in the representation, how the representation was created, or about the strategy or method used to solve the problem being represented.
- Extension [2a]: A discussion of mathematical ideas. There is no longer just an explanation of the representation, but rather statements that go beyond the explanation and extend the problem situation or solution. Types of extensions include discussing the conceptual meaning of solution procedure, or discussing other mathematical ideas.
- Comparison [2b]: A comparison is made among the different solution methods displayed on the board. The comparison may be about the features of the different representations, about the strategies used in the different solutions, or about the answers obtained.

- Conjecture [3a]: A hypothesis that extends the problem situation in a more generalized direction. Generalized positions that deal with the underlying mathematical concept are addressed. It is sometimes implicitly formulated and needs teacher support to be a conjecture. The purpose is to construct an understanding of a general concept.
- Justification [3b]: An explanation or justification is given for the conjecture that was offered. By working through the conjectures, an understanding of the general concept is constructed.

Each of the structures represents a discourse activity that was performed using the solutions represented and displayed in the classroom. The structures vary in the level of mathematical reasoning required from (a) answering and explaining the problem in the first two structures (Answer and Partial explanation and Explicit explanation); (b) going beyond the problem situation in the Extension and Comparison structures; and (c) generalizing in the Conjecture and Justification structures. Further, structures 1a-b and 3a-b can be thought of as being in parallel. In structures 1a and 3a, there is a drawing or a statement made without an explanation. Structures 1b and 3b provide the justification for the drawing (structure 1a) or statement (structure 3a). The structures will be discussed in detail in the paper.

To further understand the ways in which these structures were used in these classrooms, the analysis also looked for patterns of interactions formed around the use of these structures. As a result three recurrent specific types of interactions around students' solutions to mathematical tasks were identified. One pattern that was used was a *Getting Started* pattern. This basic interaction pattern serves to initiate discourse around a representation. This pattern begins when a display of a solution is presented to the class. Discourse then moves to giving an explicit explanation. This is followed by an extension activity based on the representation. These interaction sequences are usually brief and often multiple instances of problem sets are explored. Following the Getting Started pattern, the discourse may continue in one of the following two ways. In some cases the discourse continues as the class compares the different solution methods represented on the board. This is called *Investigating by Comparing*. Here the comparison structure is used along with the other structures. In other cases, the class engages in what I call *Going Deeper*. This involves an extended discussion of mathematically significant ideas. Such interaction sequences tend to be longer and fewer instances of problem sets are explored as students engage in discussions relating to a greater number of issues related to one set of solutions. In the paper I relate these patterns specifically to the discourse patterns identified earlier. Detailed examples of these patterns from the data will also be discussed.

Implications and future work

This study provided a description of the different ways in which students in these classrooms talked about their solutions and representations of mathematics problems. The study identified six structures of various levels of complexity that were used to define discourse around these represented solutions. The study also identified three recurrent patterns in the ways these structures were used. The identification and description of the different structures and patterns is useful for a few reasons.

First, the definition and identification of the structures and patterns helps to further characterize what mathematics reform looks like in practice. In particular, the six discourse structures define individual components of mathematics talk. Exploring the different patterns is also an important step in moving towards a better understanding of classroom discourse. We see that the discourse structures are not just isolated units – but can be coordinated and integrated into these patterns. Second, each of the structures addresses a different purpose that teachers and students have in mind while talking about the mathematical problem situation. The structures define different types of discourse, each with it own learning goals. It is important at this time to clarify the kind of learning goals that this research addresses. One measure of assessing learning by students is through performance measures on standardized tests. The research in this study does not address this aspect of testing learning. Rather, this research addresses current mathematics learning goals by investigating other measures of understanding. For instance, there is a non-verbal aspect of understanding where a representation is made to solve a given problem. Another measure consists of the verbal discussion and explanation of one's own or others' represented solutions. The six discourse structures are thus verbal and non-verbal indications of mathematical understanding.

From this viewpoint then, the identification and characterization of the discourse in these classrooms addresses mathematics reform efforts that are tied to engagement in mathematical activity through participation in discourse. Work in the area of situated cognition argues that learning is an interactive process and students learn by participating in practices with others in social and cultural contexts (Brown, Collins, & Duguid, 1989; Cobb & Yackel, 1996). Thus researchers began to emphasize the importance of participating in meaningful mathematical activities where students do not just learn how to execute the algorithms. Instead researchers emphasize that students should be able to hypothesize, try things out, execute mathematical procedures, communicate and defend results and reflect on the methods selected and the results generated (Cobb, & Bauersfeld, 1995; Davis, Maher, Noddings, 1990; NCTM, 2000). The six discourse structures address these different aspects of engagement in mathematical activity and learning.

This work has implications for teacher education in important ways. Establishing and maintaining patterns of discourse is not easy and places significant demands on teachers. This work considers the abstract notion of 'mathematical discourse' and breaks it down into concrete discourse structures. These structures are at a grain size that is useful for teachers. By identifying these structures and then combining them to form patterns of discourse, this work is an important step in moving teachers towards a better understanding of discourse in mathematics classrooms. The units are a useful step in helping teachers make sense of how to foster discourse in classrooms by giving them components that are easier to understand. The structures that were identified could allow teachers to identify different ways in which students might be encouraged to engage in mathematical discourse. Relating the talk of students in their classrooms to these structures could contribute to teachers' understanding of how to foster discourse.

Furthermore, this work provides us with a model of classroom discourse by breaking it down into components. Teachers can use these models to study their own classrooms and find ways of implementing discourse. By exploring the different ways in which the structures can be used to facilitate and form discourse patterns, the work offers a new approach to the design of mathematical discourse communities.

Discussion of the Framework developed

In particular, the study contributes to this effort through the development and identification of a framework that can be used to study and develop discourse. The framework that was developed has important implications for mathematics research. First the framework addresses both individual discourse strategies as well as how they interact. The first part of the framework identified six distinct discourse structures that could be used to characterize the discussions that students and the teacher had about students' solutions to mathematical tasks. The list of structures defined offers a manageable way in which to characterize these discussions. There are enough distinct structures to capture the different kinds of talk. At the same time, the list is not too long. In the second part of the framework, I showed how these different structures could be used to form discourse patterns for interacting in classrooms about students' solutions to problems.

Second, the framework describes trajectories – at two levels. First there is a continuum along which the structures fall. There are three levels at which these structures fall. At the first level, the two structures (Answer and partial explanation, and explicit explanation) address answering and explaining the problem. At the next level, we have the extension and comparison structures that address going beyond the initial problem. Finally at the third level the conjecture and justification structures address generalizing. Thus there is a continuum in the levels at which the structures fall, with each level addressing increasingly complex reasoning. In addition, there is a continuum along which the discourse patterns fall. The Getting Started pattern is a way of getting classroom discourse started. It can be seen as a bridge and jumping off point for other interactions. It is a good starting step and is a basic interaction pattern that is not very difficult to achieve. It has easily reproducible bits or sections and has a flow that is natural. You begin with the display of the solution, explain what you did and then explore some mathematical concept embedded in the display. Once this pattern is achieved, by adding the comparison structure the Investigating by comparing pattern can by built. Further, the Going Deeper pattern uses the same basic structure – but encourages a deeper and longer sequence of interactions.

Lastly the framework offers a modular approach to building discourse. The framework first identified specific discourse structures that could be used to talk about mathematics solutions. These can then be used to form patterns of discourse. So the structures provide the building blocks or components that can be combined in different ways to form discourse patterns (modules).

In conclusion, the classrooms engaged in significant mathematical discourse. More importantly, the classrooms moved along a continuum as they engaged in discourse around students' solutions to mathematical tasks. Students encouraged one another and formed a community where students learned from one another. The discussions and exchanges were mostly amongst students, with the teacher playing the role of facilitator of the discussions. Besides, the classroom did not just use one kind of structure or pattern of interaction for discourse. The discussions were diverse and addressed important mathematical issues as well.

There are a few points worth noting here. Firstly these classrooms were not 'special' classrooms, serving a gifted population. The classrooms were in an urban school district serving low-income students many of who were speaking English as a second language. Secondly, the students were second-graders and to see the level of discourse and the kinds of discussions and talk they were having is encouraging and noteworthy. Furthermore, the teachers in these classrooms were not veteran teachers. Both were in their fourth year of teaching and in their first year of implementing a reform based mathematics curriculum. In summary, it is encouraging to see the kind of engagement and discourse that these students engaged in, where the discourse covered mathematically significant issues. This study shows that such a result is achievable and thus provides encouragement for our reform efforts.

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