Seeing What We Mean: Co-experiencing a Shared Virtual World

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Abstract: The ability of people to understand each other and to work together face-to-face is grounded in their sharing of our meaningful natural and cultural world. CSCL groups—such as virtual math teams—have to co-construct their shared world with extra effort. A case study of building shared understanding online illustrates these aspects: Asking each other questions is one common way of aligning perceptions. Literally looking at the same aspect of something as someone else helps us to see what each other means. The co-constructed shared world has social and temporal as well as objective dimensions. This virtual world grounds communicative, interpersonal, and task-related activities for online groups, making possible group cognition that exceeds the limits of the individual cognition of the group members.

The Shared World of Meaning

We all find others and ourselves within one world. We learn about and experience the many dimensions of this world together, as we mature as social beings. Infants learn to navigate physical nature in the arms of caregivers, toddlers acquire their mother tongue by speaking with others, adolescents are socialized into their cultures, and adults master the artifacts of the built environment designed by others. The world is rich with socially endowed meaning, and we perceive and experience it as immediately meaningful. Because we share the meaningful world, we can understand each other and can work together on concerns in common. Our activities around our common concerns provide a shared structuring of our world in terms of implicit goals, interpersonal relations, and temporal dimensions. These structural elements are reflected in our language: in references to artifacts, in social positioning, and in use of tenses. All of this is understood the same by us unproblematically based on our lived experience of the shared world. Of course there are occasional misunderstandings, particularly across community boundaries, but these are exceptions that prove the rule of shared understanding in general.

The "problem" of establishing intersubjectivity is a pseudo-problem in most cases. Human existence is fundamentally intersubjective from the start. We understand the world as a shared world and we even understand ourselves through the eyes of others and in comparison with others (Mead, 1934/1962). Rationalist philosophy—from Descartes to cognitive science—has made this into a problem by focusing on the mind of the individual as if it were isolated from the world and from other people. That raises the pseudo-problem of epistemology: how can the individual mind know about states of the world and about states of other minds? Rationalist philosophy culminated in an information-processing view of human cognition, modeled on computer architecture: understanding (as described by Dreyfus, 1992) is viewed as primarily consisting of a collection of mental representations (or propositions) of facts stored in a searchable memory.

Critiques of the rationalist approach (e.g., Dreyfus, 1992; Schön, 1983; Suchman, 1987; Winograd & Flores, 1986) have adopted a phenomenological (Heidegger, 1927; Husserl, 1936; Merleau-Ponty, 1945), hermeneutical (Gadamer, 1960/1988), or ethnomethodological (Garfinkel, 1967) approach, in which understanding is grounded in being-in-the-world-together, in cultural-historical traditions, and in tacit social practices. This led to post-cognitive theories, with a focus on artifacts, communities-of-practice, situated cognition, distributed cognition, group cognition, activity, and mediations by actor-networks. Human cognition is recognized to be a social product (Hegel, Marx, Vygotsky) of interaction among people, over time, within a shared world. Knowledge is no longer viewed as primarily mental representations of individuals, but includes tacit procedural knowledge (Polanyi, 1966), designed artifacts (Hutchins, 1996), physical representations (Latour, 1992), small-group processes (Stahl, 2006), embodied habits (Bourdieu, 1972/1995), linguistic meanings (Foucault, 2002), activity structures (Engeström, Miettinen & Punamäki, 1999), community practices (Lave, 1991), and social institutions (Giddens, 1984). The critique of human thought as purely mental and individual is now well established for embodied reality. But what happens in virtual worlds. Where the physical world no longer grounds action and reflection? That is the question for this paper.

Constructing a Shared Virtual World

However, the problem of shared understanding rises again—and this time legitimately—within the context of computer-supported collaborative learning (CSCL). That is because when students gather in a CSCL online environment, they enter a virtual world, which is distinct from the world of physical co-presence. They leave the world of nature, of physical embodiment, of face-to-face perception. They enter a world that they have not all grown into together. But this does not mean that "shared understanding" is just a matter of overlapping opinions of mental models for online groups either.

In the Virtual Math Teams (VMT) Project, we have been studying how students interact in a particular CSCL environment designed to support online discourse about mathematics. In this paper we will illustrate some of our findings about how interaction in the VMT environment addresses the challenge of constructing a shared virtual world, in which small groups of students can productively engage in collaborative mathematics.

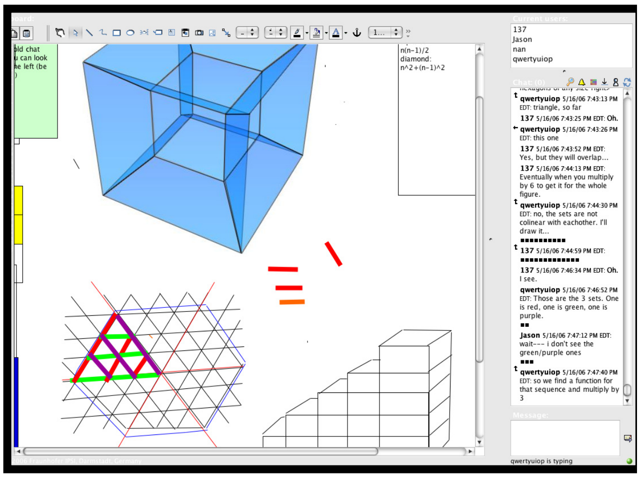

Figure 1. The VMT Environment during Session 3.

This paper will present a case study of Session 3 of Team C in the VMT Spring Fest 2006. Here, students aged 12-15 from different schools in the US met online for four hour-long sessions. Neither the students nor the researchers knew anything about the students other than their login user names and their behavior in the sessions. A researcher joined the students, but did not engage with them in the mathematics. Between sessions, the researchers posted feedback in the shared whiteboard of the environment. The VMT Project is described and discussed in (Stahl, 2009); its theoretical motivation is presented in (Stahl, 2006). The VMT environment is shown in Figure 1. The complete chat log of Session 3 of Team C is given in the Appendix of the online version of this paper (http://GerryStahl.net/pub/cscl2011.pdf) and a Replayer version can be obtained from the authors.

In the next sections, we illustrate the following aspects of building shared understanding: (a) Asking each other questions is one common way of resolving or avoiding troubles of understanding and aligning perceptions. (b) Literally looking at the same aspect of something as someone else helps us to see what each other means. (c) The co-constructed shared world has social and temporal as well as objective dimensions. (d) This world grounds communicative, interpersonal, and task-related activities for online groups.

Questioning to Share Understanding

We have analyzed how questions posed in the VMT environment often work to initiate interactions that resolve troubles of understanding and deepen shared understanding (Zhou, 2009; 2010; Zhou, Zemel & Stahl, 2008). This is in contrast to the rationalist assumption that questions are requests for propositional information. We will here review a number of questions from Session 3 of Group C and indicate how they lead to shared understanding. Unfortunately, due to space limitations, we will not be able to provide the full context for these questions or a detailed conversation analysis.

Log 1.

Chat Index	Time of Posting	Author	Content
685	19:06:34	qwertyuiop	has everyone read the green text box?
686	19:06:44	Jason	one sec
687	19:06:45	137	Yes...
688	19:07:01	Jason	alright im done

The question by Qwertyuiop in Log 1 serves a coordination function, making sure that all the students have read the feedback to Session 2 before any work begins in the new Session. This is an effort, taking the form of a question, to maintain a shared experience by having everyone take this first step together.

Log 2.

694	19:11:16	137	Great. Can anyone m ake a diagram of a bunch of triangles?
695	19:11:51	qwertyuiop	just a grid?
696	19:12:07	137	Yeah...
697	19:12:17	qwertyuiop	ok...

Log 2 is part of a complicated and subtle process of co-constructing shared understanding. It is analyzed in detail in (Çakır, Zemel & Stahl, 2009). The student named 137 has attempted to construct a grid of triangles in the whiteboard (similar to those in the lower left corner of Figure 1). He (or she) has failed (as expressed by the ironic "Great"), and has erased the attempt and solicited help by posing a question. Qwertyuiop requests clarification with another question and then proceeds to draw a grid of triangles by locating and then tweaking three series of parallel lines, following much the same procedures as 137 did. Qwertyuiop's understanding of 137's request is based not only on the "Yeah…" response to his/her "just a grid?" question, but also the detailed

Log 3.

698	19:14:09	nan	so what's up now? does everyone know what other people are doing?
699	19:14:25	137	Yes?
700	19:14:25	qwertyuiop	no-just making triangles
701	19:14:33	137	I think...
702	19:14:34	Jason	yeah
703	19:14:46	nan	good:-)
704	19:14:51	qwertyuiop	triangles are done

sequentially unfolding visual presentation of 137's failed drawing attempt.

In Log 3, the moderator, Nan, asks a question to make visible in the chat what members of the group are doing. Qwertyuiop is busy constructing the requested grid in the whiteboard and the others are presumably watching that drawing activity and waiting for its conclusion. The students do not seem to feel that there is a problem in their understanding of each other's activities. However, due to the nature of the virtual environment—in which the attentiveness of participants is only visible through their chat and drawing actions—Nan cannot know if everyone is engaged during this period of chat inaction. Her question makes visible to her and to the students the fact that everyone is still engaged. The questioning may come as a minor interference in their group interaction, since Nan's questioning positions her as someone outside the group ("everyone"), exerting authority by asking for an accounting, although it is intended to increase group shared understanding ("everyone know what other people are doing").

See What I Mean

Studies of the use of interactive whiteboards in face-to-face classrooms have shown that they can open up a "shared dynamic dialogical space" (Kershner et al., 2010) as a focal point for collective reasoning and co-construction of knowledge. Similarly, in architectural design studios, presentation technologies mediate shared ways of seeing from different perspectives (Lymer, Ivarsson & Lindwall, 2009) in order to establish shared understanding among design students, their peers, and their critics. Clearly, a physical whiteboard that people can gather around and gesture toward while discussing and interpreting visual and symbolic representations is different from a virtual shared whiteboard in an environment like VMT.

We have analyzed in some detail the intimate coordination of visual, narrative and symbolic activity involving the shared whiteboard in VMT sessions (Çakir, 2009; Çakır, Stahl & Zemel, 2010; Çakır, Zemel & Stahl, 2009). Here, we want to bring out the importance of literally looking at some mathematical object together in order to share the visual experience and to relate to—intend or "be at"—the object together. People often use the expression "I do not see what you mean" in the metaphorical sense of not understanding what someone else is saying. In this case study, we often encounter the expression used literally for not being able to visually perceive a graphical object, at least not being able to see it in the way that the speaker apparently sees it.

While empiricist philosophy refers to people taking in uninterpreted sense data much like arrays of computer pixels, post-cognitive philosophy emphasizes the phenomenon of "seeing as." Wittgenstein notes that one sees a wire-frame drawing of a cube not as a set of lines, but as a cube oriented either one way or another (Wittgenstein, 1953, sec. 177). For Heidegger, seeing things as already meaningful is not the result of cognitive interpretation, but the precondition of being able to explicate that meaning further in interpretation (Heidegger, 1927/1996, pp. 139f). For collaborative interpretation and mathematical deduction, it is clearly important that the participants see the visual mathematical objects as the same, in the same way. This seems to be an issue repeatedly in the online session we are analyzing as well.

137 proposes a mathematical task for the group in line 705 of Log 4. This is the first time that the term, "hexagonal array," has been used. Coined in this posting, the term will become a mathematical object for the group as the discourse continues. However, at this point, it is problematic for both Qwertyuiop and Jason. In line 706,

Log 4.

705	19:15:08	137	So do you want to first calculate the number of triangles in a hexagonal array?
706	19:15:45	qwertyuiop	What's the shape of the array? a hexagon?
707	19:16:02	137	Ya.
708	19:16:15	qwertyuiop	ok...
709	19:16:41	Jason	wait-- can someone highlight the hexagonal array on the diagram? i don't really see what you mean...
710	19:17:30	Jason	hmm.. okay
711	19:17:43	qwertyuiop	oops
712	19:17:44	Jason	so it has at least 6 triangles?
713	19:17:58	Jason	in this, for instance

Log 5.

714	19:18:53	137	How do you color lines?
715	19:19:06	Jason	there's a little paintbrush icon up at the top
716	19:19:12	Jason	it's the fifth one from the right
717	19:19:20	137	Thanks.
718	19:19:21	Jason	there ya go :-)
719	19:19:48	137	Er... That hexagon.

Log 6.

720	19:20:02	Jason	so... should we try to find a formula i guess
721	19:20:22	Jason	input: side length; output: # triangles
722	19:20:39	qwertyuiop	It might be easier to see it as the 6 smaller triangles.
723	19:20:48	137	Like this?
724	19:21:02	qwertyuiop	yes
725	19:21:03	Jason	yup
726	19:21:29	qwertyuiop	side length is the same...
727	19:22:06	Jason	yeah
728	19:22:13	Jason	so it'll just be x6 for # triangles in the hexagon
729	19:22:19	137	Each one has 1+3+5 triangles.

Qwertyuiop poses a question for clarification and receives an affirmative, but minimal response. Jason, unsatisfied with the response, escalates the clarification request by asking for help in seeing the diagram in the whiteboard *as* an "hexagonal array," so he can see it *as* 137 sees it. Between Jason's request in line 709 and acceptance in line 710, Qwertyuiop and 137 work together to add lines outlining a large hexagon in the triangular array. Demonstrating his ability to now see hexagons, Jason thereupon proceeds with the mathematical work, which he had halted in the beginning of line 709 in order to keep the group aligned. Jason tentatively proposes that every hexagon "has at least 6 triangles" and he makes this visible to everyone by pointing to an illustrative small hexagon from the chat posting, using the VMT graphical pointing tool.

In Log 5, 137 asks the group to share its knowledge about how to color lines in the VMT whiteboard. Jason gives instructions for 137 to visually locate the appropriate icon in the VMT interface. Demonstrating this new knowledge, 137 changes the colors of the six lines outlining the large hexagon, from black to blue, making the outline stand out visually (see Figure 1). 137 thereby finally clarifies how to look at the array of lines *as* a large hexagon, a task that is more difficult than looking at the small hexagon that Jason pointed to. In this excerpt, the group shares their working knowledge of their virtual world (the software functionality embedded in it), incidentally to carrying out their task-oriented discourse within that world.

In Log 6, Jason proposes a specific mathematical task for the group to undertake, producing a formula for the number of triangles in an hexagonal array of any given side length. (As we shall see below, the group uses the term "side length" as the measure of a geometric pattern at stage n.) Qwertyuiop responds to this proposal with the suggestion to "see" the hexagon (of any size) as a configuration of six triangular areas. (To see what Qwertyuiop is suggesting, look at Figure 1; one of the six triangular areas of the large hexagonal array has its "sticks" colored with thick lines. Looking at this one triangular area, you can see in rows successively further from the center of the hexagon a sequence of one small triangle, then three triangles, then five triangles.)

In line 723, 137 seeks confirmation that he is sharing Qwertyuiop's understanding of the suggestion. After posting, "Like this?" with a reference back to Qwertyuiop's line 722, 137 draws three red lines through the center of the large hexagon, dividing it visually into six triangular areas. Upon seeing the hexagon divided up by 137's lines, Qwertyuiop and Jason both confirm the shared understanding. Now that they are confident that they are all seeing the mathematical situation the same, namely *as* a set of six triangular sub-objects, the group can continue its mathematical work. Jason draws the consequence from Qwertyuiop's suggestion that the formula for the number of small triangles in a hexagon will simply be six times the number in one of the triangular areas of that hexagon, thereby subdividing the problem. 137 then notes that each of those triangular areas has 1+3+5 small triangles, at least for the example hexagonal array that they are looking at. The fact that the three members of the group take turns making the consecutive steps of the mathematical deduction is significant; it demonstrates that they share a common understanding of the deduction and are building their shared knowledge collaboratively.

The observation, "Each one has 1+3+5 triangles," is a key move in deducing the sought equation. Note that 137 did not simply say that each triangular area had nine small triangles. The posting used the symbolic visual representation, "1+3+5." This shows a pattern of the addition of consecutive odd numbers, starting with 1. This pattern is visible in the posting. It indicates that 137 is seeing the nine triangles *as* a pattern of consecutive odd numbers—and thereby suggests that the reader also see the nine triangles *as* such a pattern. This is largely a visual accomplishment of the human visual system. People automatically see collections of small numbers of objects as sets of that specific size (Lakoff & Núñez, 2000). For somewhat larger sets, young children readily learn to count the number of objects. The team has constructed a graphical representation in which all the members of the team can immediately see features of their mathematical object that are helpful to their mathematical task. The team is collaborating within a shared virtual world in which they have co-constructed visual, narrative, and symbolic objects in the chat and whiteboard areas. The team has achieved this shared vision by enacting practices specific to math as a profession for shaping witnessed events, such as invoking math terms and drawing each others' attention to relevant objects in the scene (Goodwin, 1994). They have learned and taught each other how to work, discuss, and perceive as a group in this shared virtual world.

Dimensions of a Virtual World

There has not been much written about the constitution of the intersubjective world as the background of shared understanding, particularly in the CSCL online context. There has not been much written about the constitution of the intersubjective world as the background of shared understanding, particularly in the CSCL online context. This is largely the result of the dominance of the cognitive perspective, which is primarily concerned with mental models and representations of the world; this rationalist view reduces the shared world to possible similarities of individual mental representations. Within the VMT Project, we have analyzed the dimensions of domain content, social interaction, and temporal sequencing in the co-construction of a virtual math team's world or joint problem space (Sarmiento & Stahl, 2008; Sarmiento-Klapper, 2009a; Sarmiento-Klapper, 2009b). In this work, we have found the following conceptualizations to be suggestive: the joint problem space (Teasley & Roschelle, 1993) and the indexical ground of reference of domain content (Hanks, 1992); the social positioning of team members in discourse (Harré & Gillet, 1999) and their self-coordination (Barron, 2000); and the temporal sequentiality of discourse (Schegloff, 1977) and the bridging of temporal discontinuities.

In previous sessions, the group has tried to derive formulae for the number of two-dimensional objects (small squares or small triangles) in a growing pattern of these objects, as well as the number of one-dimensional sides, edges or "sticks" needed to construct these objects. A major concern in counting the number of sides is the issue of "overlap." In a stair-step two-dimensional pattern (like the 2-D version of the stair-step pyramid in the lower right section of Figure 1), one cannot simply multiply the number of squares by 4 to get the number of sides because many of the sides are common to two squares. In Session 1, Team C had seen that in moving from one stage to the next of the stair-step pattern most new squares only required two new sides.

In Log 7, Qwertyuiop moves on from the derivation of the number of triangles to that of the number of sides. He "bridges" back to the group's earlier in-sight that the addition of "each polygon corresponds to [an additional] 2 sides." In bridging to past sessions, we found, it is necessary to re-situate a previous idea in the current context. In line 731, Qwertyuiop is reporting that for their hexagon formula, such situating does not work—i.e., that the current problem cannot be solved with the same method as the previous problems. The group then returns to the formula for the number of triangles and efficiently solves it by summing the sequence of consecutive odd numbers using Gauss' technique—the sum of n consecutive odd integers is n(2n/2)—which they had used in previous sessions.

Log 7.

731	19:22:29	qwertyuiop	the "each polygon corrisponds to 2 sides" thing we did last time doesn't work for triangles
732	19:23:17	137	It equals 1+3+...+(n+n-1) because of the "rows"?
733	19:24:00	qwertyuiop	yes- 1st row is 1, 2nd row is 3...
734	19:24:49	137	And there are n terms so... n(2n/2)
735	19:25:07	137	or n^2
736	19:25:17	Jason	yeah
737	19:25:21	Jason	then multiply by 6
738	19:25:31	137	To get 6n^2

In Log 8, Qwertyuiop makes a particularly complicated proposal, based on a way of viewing the sides in the large hexagon drawing. He tries to describe his view in chat, talking about sets of collinear sides. Jason does not respond to this proposal and 137 draws some lines to see if he is visualizing what Qwertyuiop has proposed, but he has not. Qwertyuiop has to spend a lot of time drawing a color-coded analysis of the sides as he sees them. He has decomposed the set of sides of one triangular area into three subsets, going in the three directions of the array's original parallel lines. He can then see that each of these subsets consists of 1+2+3 sides. There are 3 subsets in each of the 6 triangular areas. Based on this and generalizing to a growing hexagonal array, which will have sums of consecutive integers in each subset, the team can derive a formula using past techniques. At some point, they will have to subtract a small number of sides that overlap between adjacent triangular areas. Qwertyuiop has proposed a decomposition of the hexagonal array into symmetric sets, whose constituent parts are easily visible. Thus, his approach bridges back to previous group practices, which are part of the shared world of the group—see the analysis of a similar accomplishment by Group B in (Medina, Suthers & Vatrapu, 2009). The hexagonal pattern, which Team C came up with on its own, turns out to be considerably more difficult to decompose into simple patterns that the original problem given in Session 1. It strained the shared understanding of the group, requiring the use of all the major analytic tools they had co-constructed (decomposing, color-coding, visually identifying sub-patterns, summing series, eliminating overlaps, etc.).

Log 8.

742	19:25:48	qwertyuiop	an idea: Find the number of a certain set of colinear sides (there are 3 sets) and multiply the result by 3
746	19:26:36	137	As in those?
747	19:27:05	qwertyuiop	no-in one triangle. I'll draw it...
748	19:28:10	qwertyuiop	those
749	19:28:28	qwertyuiop	find those, and then multiply by 3
750	19:28:50	137	The rows?
751	19:30:01	qwertyuiop	The green lines are all colinear. There are 3 identical sets of colinear lines in that triangle. Find the number of sides in one set, then multiply by 3 for all the other sets.
752	19:30:23	137	Ah. I see.

In Log 9, the group work is interrupted by an interesting case of bridging across teams. At the end of

Log 9.

804	19:48:49	nan	(we got a question for you from another team, which was posted in the lobby:
805	19:48:53	nan	Quicksilver 7:44:50 PM EDT: Hey anyone from team c, our team needs to know what n was in your equations last week
806	19:49:04	Jason	oh
807	19:49:15	137	The length of a side.
808	19:49:16	qwertyuiop	was n side length?
809	19:49:33	Jason	are you talking about the original problem with the squares
810	19:49:48	137	I think nan is.
811	19:49:58	qwertyuiop	i think it's squares and diamonds

each session, the teams had posted their findings to a wiki shared by all the participants in the VMT Spring Fest 2006. During their Session 3, Team B had looked at Team C's work on a pattern they had invented: a diamond variation on the stair-step pattern. In their wiki posting, Team C had used their term, "side length." Because members of Team B did not share Teams C's understanding of this term, they were confused by the equation and discussion that Team C posted to the wiki. Team B's question sought to establish shared understanding across the teams, to build a community-wide shared world. As it turned out, Team C had never completed work on the formula for the number of sides in a diamond pattern and Team B eventually discovered and reported the error in Team C's wiki posting, demonstrating the importance of community-wide shared understanding.

Grounding Group Cognition

CSCL is about meaning making (Stahl, Koschmann & Suthers, 2006). At its theoretical core are questions about how students collaborating online co-construct and understand meaning. In this paper, we conceptualize this issue in terms of online groups, such as virtual math teams, building a shared meaningful world in which to view and work on mathematical objects.

Log 10 illustrates a limit of shared understanding, closely related to the notion of a "zone of proximal development" (Vygotsky, 1930/1978, pp. 84-91). The original stair-step pattern consisted of one-dimensional sides and two-dimensional squares. In their Session 2, Team C had generalized this pattern into a three-dimensional pyramid consisting of cubes. Now Qwertyuiop proposes to further generalize into a mathematical fourth dimension and derive formulae for patterns of one, two, three, and four-dimensional objects. He had previously imported a representation of a four-dimensional hyper-cube (see the upper area of Figure 1) into the whiteboard for everyone to see.

Log 10.

20:12:22	qwertyuiop	what about the hypercube?
20:12:33	137	Er...
20:12:39	137	That thing confuses me.
20:13:00	137	The blue diagram, right?
20:13:13	qwertyuiop	can you imagine extending it it 4 dimensions, and a square extends into a grid?
20:13:17	qwertyuiop	yes
20:13:30	137	I didn't get that?
20:13:32	qwertyuiop	I'm having trouble doing that.
20:13:45	qwertyuiop	didn't get this?
20:13:50	137	Ya.
20:15:02	qwertyuiop	If you have a square, it extends to make a grid that fills a plane. A cube fills a space. A simaller pattern of hypercubes fills a "hyperspace".
20:15:19	137	The heck?
20:15:29	137	That's kinda confusing.
20:15:43	qwertyuiop	So, how many planes in a hyper cube latice of space n?
20:16:05	137	Er...
20:16:07	qwertyuiop	instead of "how many lines in a grid of length n"
20:16:17	qwertyuiop	does that make any sense?
20:16:30	137	No. No offense, of course.

At this point late in Session 3, Jason had left the VMT environment. Qwertyuiop was unable to guide 137 to see the drawing in the whiteboard as a four-dimensional object. Apparently, Qwertyuiop had been exposed to the mathematical idea of a fourth dimension and was eager to explore it. However, 137 had not been so exposed. They did not share the necessary background for working on Qwertyuiop's proposal. This shows that tasks for student groups, even tasks they set for themselves, need to be within a shared group zone of proximal development. The stair-step problem was in their zone—whether or not they could solve it themselves individually, they were able to solve it collectively, with enough shared understanding that they could successfully work together. Their three-dimensional pyramid turned out to be quite difficult for them to visualize in a shared way. Their diamond pattern seemed to be easy for them, although they forgot to work on some of it and posted an erroneous formula. The hexagonal array required them to develop their skills in a number of areas, but they solved it nicely. However, the hyper-cube exceeded at least 137's ability (or desire) to participate.

Rationalist philosophy reduces the complexity of social human existence to a logical, immaterial mind that thinks about things by representing them internally. It confuses the mind with the brain and conflates the two. It assumes that someone thinking about a hexagon or working on a math problem involving a hexagon must primarily be representing the hexagon in some kind of mental model. But one of the major discoveries of phenomenology (Husserl, 1936/1989) was that intentionality is always the intentionality of some object and that cognition takes place as a "being-with" that object, not as a mental act of some transcendental ego. As an example, we have seen that the members of Team C are focused on the graphical image of the hexagon in their virtual world on their computer screens. They reference this image and transform it with additional lines, colors, and pointers. They chat about this image, not about some personal mental representations. They work to get each other to see that image in the same way that they see it. This "seeing" is to be taken quite literally. Their eyes directly perceive the image. They perceive the image in a particular way (which may change and which they may have to learn to see). "Seeing" is not a metaphor to describe some kind of subjective mental process that is inaccessible to others, but a form of contact with the object in the world. We may say that shared understanding is a matter of the group members being-there-together at the graphical image in the whiteboard.

Being-there-together is a possible mode of existence of the online group. The "there" where they are is a multi-dimensional virtual world. This world was partially already there when they first logged in. It included the computer hardware and software. It included the VMT Spring Fest as an organized social institution. As they started to interact, the students fleshed out the world, building social relationships, enacting the available technology, interpreting the task instructions, and proposing steps to take together. Over time, they constructed a rich world, furnished with mathematical objects largely of their own making and supporting group practices that they had introduced individually but which they had experienced as a group.

Being-there-together in their virtual world with their shared understanding of many of this world's features, the group was able to accomplish mathematical feats that none of them could have done alone. Each individual in the group shared an understanding of their group work at least enough to make productive contributions that reflected a grasp of what the group was doing. Their group accomplishments were achieved through group processes of visualization, discourse, and deduction. They were accomplishments of group cognition, which does not refer to anything mystical, but to the achievements of group interaction. The group cognition was possible because of, and only on the basis of, the shared understanding of the common virtual world. Shared understanding is not a matter of similar mental models, but of experiencing a shared world.

Of course, there are limits to group cognition, just as there are limits to individual cognition. We saw that Team C could not understand Qwertyuiop's ideas about the fourth dimension. Without shared understanding about this, the group could not engage in discourse on that topic. Group cognition can exceed the limits of the individual cognition of the group members, but only by a certain amount. The individuals must be able to stretch their own existing understanding under the guidance of their peers, with the aid of physical representations, tools, concepts, scaffolds, and similar artifacts, whose use is within their grasp—within their zone of proximal development (Vygotsky, 1930/1978). We have seen that Team C was able to solve a complex mathematical problem that they set for themselves involving a hexagonal array by building up gradually, systematically, and in close coordination a meaningful virtual world.

An analysis of the log of the interaction in our case study has demonstrated much about the team's group cognition. Their group work proceeded by contributions from different individuals, with everyone contributing in important ways. Their questions showed that their individual cognition was initially inadequate to many steps in the work; but their questions also served to expand the shared understanding and to ensure that each member shared an understanding of each step. Because, the students demonstrated an understanding of the group work through their successive contributions, we can see not only that individual learning took place, but we can analyze the interactional processes through which it took place through detailed analysis of their chat and drawing actions.

As Vygotsky argued, not only does group cognition lead individual cognition by several years, but individual cognition itself develops originally as a spin-off of group cognition. Individuals can learn on their own, but the cognitive and practical skills that they use to do so are generally learned through interaction with others and in small groups. This is a powerful argument for the use of CSCL in education. It is incumbent upon CSCL research to further analyze the processes by which this takes place in the co-construction of shared understanding within co-experienced virtual worlds. As we have seen, participants in CSCL virtual environments co-construct worlds to ground their interactions. These virtual worlds exploit meaning-making, perceptual and referential practices learned in the physical social world.

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