

# Doing Their Own Math: Computer Support of Discursive Approaches to "Real" Math Problems

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**Abstract:** Traditionally, there has been no obvious role for the computer in an elementary-level classroom oriented toward "talking about math" as a way of deepening children's understanding of mathematical problem situations. We suggest that an appropriate computer learning environment can serve as a useful tool to both complement and be enhanced by discursive approaches to mathematics. In this paper we describe software, some of which is currently being tested in classrooms, which allows students to create and share word problems motivated by classroom discussion of an arithmetically relevant situation, and to dynamically explore links between linguistic and other symbolic representations of their problems and those of other children. In conjunction with the discursive approach, these facilities support pedagogical methods such as the integration of writing, science and mathematics; roles for students as knowledge contributors rather than as passive recipients; and a motivated use of the computer for understanding mathematical problem relationships.

## 1. Introduction

"Sorry, the kids can't talk now--they're doing their math." But is it *their* math? Traditionally, young students have "done their math problems" by reading arithmetic word problems constructed by educators and trying, usually in isolation, to come up with an answer, often by guesswork manipulation of the numbers given and with little attention to common sense or to the meaning constructed by the sentences of the problem [Verschaffel & DeCorte 1996].

Some approaches in elementary mathematics education, however, have increasingly recognized the mediating role of natural language in expressing everyday problem situations as a potential focus for both research on children's mathematical problem solving and new classroom approaches to teaching "mathematical thinking." The NCTM Curriculum Standards [NCTM 1989] emphasize the pedagogical importance of representing and reading mathematics; both the Curriculum Standards and the Professional Standards [NCTM 1991] focus on encouraging children to write mathematics and particularly to *talk about* mathematical aspects of situations.

Consistent with this research, a growing number of classrooms are engaging in group discussion of mathematical problems. Informal group discourse about mathematics is an important step in increasing children's

conceptual understanding. Yet in certain respects this approach is incomplete with respect to the above goals. *By itself* group discussion may not help those who do not actively participate and who do not feel that they have a role in solving the problem; it does not relate natural language to other modalities, symbolic systems or representations [see Janvier 1987] which might play a particular role in some children's understanding; it does not allow most children in any given discussion to individually initiate problem conceptualizations; a child's particular level or difficulty may not be addressed by the discussion.

A more individual-oriented approach has been offered commercially through software which has entertaining situation pictures and includes quantitative mappings, but does not relate to any difficulties which might be particular to an individual child at a certain stage, does not actively involve the child in his own learning, does not relate to a situation in which the child is currently involved, and is not social in nature. An exception to some of these criteria is provided by the Word Problem Processors (WPP) software [Pogrow 1994], which enables children of fourth-grade level and above to become actively involved by writing their own word problems with feedback from the WPP, presented as a "space creature." Like other software, however, it does not focus at all on the role of significant types of linguistic constructs, and is more oriented toward game-playing and competition than real-world problems.

Is either the discursive *or* the typical software approach sufficient to help the student conceptualize the problem in natural, mathematical or other symbolic language, thereby promoting precise reasoning and perhaps communication skills? We would claim that while discourse about mathematical situations is an excellent partial approach, a different and expanded role for learning software would do much to enhance and complement the discourse approach, while providing a means of relating situations, quantitative concepts and language.

To start determining what such a computer learning environment should do, we pose the question, why is it that children can understand e.g. 'a piece of pie,' and even '3 pieces of pie with ice cream on top,' but can't/won't relate these to other pieces of pie mentioned in the problem, even though they know [Cummins 1991] part-part-whole relationships? With the original linguistic statement of an arithmetic problem situation as the focus of our attention, we will suggest that several key competences underlie the desired understanding and that these might be strengthened by a computer learning environment interacting with the context of a natural classroom situation. In the following discussion, a sketch of how such a software design would fit into this approach is preceded by a general and then a more specific consideration of aspects of language about mathematics that require extension of children's thinking.

## 2. Difficulties in Language Processing

Research in cognitive science and mathematics education has emphasized that understanding arithmetic word problems involves a complex interaction of text comprehension and mathematical processes. This research includes results on semantic types of problems and children's logico-mathematical competencies [Riley & Greeno 1988], the locus of low-achieving students' eye-fixations [Hegarty, Mayer & Green 1992] and the developmental sequence of central numerical structures [Okamoto 1992]. Lacking in the above work, however, is an account of how the student's ability to arrive at mathematical connections might depend on identifiable ways in which natural language expresses or facilitates such connections. Results concerning the effects of small changes in problem wording [Cummins 1991, Davis-Dorsey, Ross & Morrison 1991, Staub & Reusser 1995, DeCorte, Verschaffel & DeWin 1985 and others] and the role of integrated propositions in memory [Trabasso & Sperry 1985] provide evidence that problem difficulty is determined by text comprehension factors as well as by semantic structure and the development of central numerical structures. The information processing and learning assumptions in this research reflect the idea that different kinds of natural language used to convey a problem situation have different effects on the ability to conceptualize the problem in terms of correct mathematical relationships.

Most of these researchers, however, stop short of defining how natural language is somehow "translated" into correct mathematical relationships or into a solution, choosing instead to start their studies with propositional or idealized models of the problem. Addressing this gap has been a research goal of LeBlanc's cognitive process model of arithmetic word problem solution (LeBlanc & Weber-Russell, in press), which parses (EDUCE) and performs text integration (SELAH) on such problems. (EDUCE [Burns 1993] is based on

conceptual analysis of natural language [Schank & Riesbeck 1981], extended for mathematically relevant language.) These processes have been monitored through predictor variables including MEMORY (the average number of "conceptual units" in memory at the end of each sentence) and INFERENCES (the average number of times per sentence that i) significant relationships facilitating text integration or ii) arithmetic actions are implied rather than made explicit in the text). Regression analyses show the MEMORY + INFERENCES variables *together* (i.e., holding conceptual units in working memory while making inferences) as accounting for approximately 60% of the variance when correlated with empirical evidence of solution difficulty (grades K-3) for 18 benchmark problems.

The results suggest that while young students are able to handle the memory load of sentence comprehension and are capable of making inferences if the memory load is not high (neither of these variables proving to be significant by themselves), the task of making such inferences while retaining conceptual input presents a particular difficulty for some students. The implication of taking observations about language to this level of detail is not just that students fail to make conceptual connections leading to a correct problem structure, but that the focusing power needed to retain previous information while attempting such connections, for one reason or another, does not appear to be there. We suggest that an approach to strengthening students' focusing abilities and thus making them more active learners should offer reinforcement of interrepresentational mapping, encourage precise conceptualization of the problem, and be something in which students would be *motivated* to participate.

### 3. Language-related Dimensions of Word Problem Understanding

Software which reinforces conceptual mappings should be based on a theory of what transitions the student needs to accomplish. The above research on arithmetic word problem solution has usefully considered linguistic aspects of such problems in terms of their "semantic structure" (Change, Combine, Compare, Equalize). However, this somewhat general characterization does not tell us what concepts, analyses, or types of wordings make such problems difficult or easy. It is therefore not evident how learning environment components could address observed difficulties in a specifically targeted way. In an effort to come closer to the "elementary competences" involved in word problem understanding, we propose taking the semantic structures apart, giving dimensions in terms of the following task components:

- (1) An entire problem structure, not just a simple assertion, must be understood as presented. A complication is an "unknown" referent which cannot be processed immediately, but must be retained while the rest of the problem is read or processed. In *nonquantitative* domains, this task may be relatively easy for most young students. However, in combination with (2), i.e., for quantitative problems, a perhaps unfamiliar mode of reading is required.
- (2) Words and phrases must be understood in their quantitative rather than in their ordinary linguistic sense. Mathematically relevant language is *embedded* in the natural language, and must be abstracted from the linguistic statements into (ultimately) a numeric domain. Young children may have difficulty recognizing certain phrases as "mathematically significant"; they may recognize that a quantity is involved in any individual assertion in the problem, but not focus on it as if it "made a difference" (pun intended).

As in (1), in isolation this task seems doable by most young students; they understand 'three pieces of pie.' In combination with (1), however, i.e., in the context of a word problem, extra effort is required. For the "expert" solver, 'three' triggers the quantitative "mode." This mode extends 'three pieces of pie' to a quantity (3) which will relate to other quantities in the problem. (See also [Nesher 1982] on the understanding of number-words vs. numbers.) The mathematical mode of reading requires both abstraction and active focusing. Focusing is even more difficult in the case of an unknown quantity, such as in 'some pieces of pie,' since children, using linguistic senses familiar to them, may understand the meaning of 'some' simply as an adjective [Cummins, Kintsch, Reusser & Weimer 1988], rather than retain it in memory as an unspecified *quantity*. Similarly, we would point out that the relational word 'of' can be understood conventionally ('a piece of pie') or quantitatively as "subtracted from," that is, as

'1 of ? pieces,' requiring an extension to the numeric domain.[1]

As an example of the difficulty presented by the concurrence of problem processing and quantitative abstraction, consider:

John has 3 million dollars.

If someone has millions of dollars, he is rich.

Is John rich?

If this problem had proceeded quantitatively after the first statement, e.g., 'He lost 2 million dollars in the stock market,' a child who did not carry through quantitative expectations while processing the problem structure might be focused on nonmathematical (irrelevant) aspects of the problem, with possible implications for ease of problem solution. This potential difference in the mode of reading a problem points out the importance of a motivating purpose for creating the problem.

- (3) Higher-order thinking is required for e.g. Compare problems, which involve relations *between* entities as opposed to entities themselves. This ability requires a problem solver at the least to retain more than one entity in memory at a time, with roles assigned to those entities. A particular difficulty is introduced by quantification of relations. While many children can understand the ordinary sense of 'greater than' or 'less than,' they may not understand 'How many more...' The identification of this dimension helps to account for the relative ease of solution of Kintsch's (nonrelational) situation models [Kintsch 1989] as opposed to relationally worded problems.

A possible generalization across these three dimensions is that the quantification and retention of nonconcrete concepts (unknowns and relations) present a particular difficulty, since, as in LeBlanc's results above, it involves concurrent processes--a task we believe can be addressed by a learning context which encourages active focusing.

#### 4. Word Problem Learning Environments as a Classroom Tool

We propose an integrated pedagogical approach which a) balances exploration and initiative on one hand and guidance and reinforcement of conceptually relevant mappings on the other; and b) is motivated by classroom discourse about and experience with "naturally" arising situations. Consistent with the previous section [Language-Related Dimensions of Word Problem Understanding], the guidance component should aim to:

- (1) help young students to conceptualize problem structures including unknown quantities;
- (2) extend students' existing knowledge to analogous situations and numeric representations;
- (3) foster the understanding and conceptualization of relational language.

Our own interactive pilot implementations [LeBlanc 1992] have started by using the output of EDUCE/SELAH to dynamically link (in the sense advocated in [Kaput 1992]) natural-language assertions to progressively more abstract representations (icons, dots, numbers). For example, our "Language Link" tool allows students to see immediately how language (e.g., 'have more than') is changed when they alter certain iconic representations. Planned versions include the converse of this process, additional symbolic modes (such as number lines) and a dynamic linkage between mathematically significant prepositions and corresponding actions on icons. These features should make conceptual components of the problem solving process explicit and should extend students' awareness of the power of mathematics to account for many analogous situations [see Thompson 1989]. However, as emphasized above, there must be an active focus on these conceptual steps and patterns if students are to effectively "construct" [Cobb 1994] mathematical meaning from these

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[1] That children may be ready to make such extensions (see [Vosniadou 1995] for emphasis on the developmental role of analogical reasoning) is suggested by the many extensions (though unconscious) they have previously learned through the assimilation of language which is actually metaphoric [Brugman & Lakoff 1988, Russell 1986]. For example, from the presumably literal 'piece of pie,' a (nonquantitative) extension of the sense of 'of' can be made to expressions such as 'color of the ball.'

associations. By itself, this approach may still not motivate children (especially those jaded by years of passive learning and entertainment). While students can express a preference of learning style through choice of conceptual modes (numeric, iconic, spatial, linguistic) offered by such a learning environment, this approach does not really address the "initiative" requirement stated above. The following scenario, in the spirit of "anchored instruction" [Barron & Kantor 1993], reflects a far more motivating environment where the learning is self-directed, goal-oriented and situated.

The class, with the teacher, looks at a real-world problem (e.g., weights of different types of whales) and talks about it until some degree of understanding is involved, specifically the inherent relationships that exist in the situation (e.g., differences between male and female weights). Then students are encouraged to get the computer to help them finish solving the problem (or a variation of the problem), and to see what the computer does with the problem as *created by the students in their own words*. This step will in most cases be preceded by a phase where students write about the situation, the relationships of interest and possibly an open question. This includes conferral with the teacher or possibly with other students. Writing is thereby used to facilitate "ownership" of mathematical language. A necessary side effect of this write-rewrite cycle is to present the computer with something intelligible and spelled correctly. This "linguistic conceptualization," especially the expression of problem relationships, is consistent with the "mathematical writing" approach advocated by [Winograd & Higgins 1995] and represents a "role reversal" with respect to the traditional situation in which the student is given a word problem to solve. In submitting the problem to a computer supplied with the type of software outlined above, students would see the abstractions the computer goes through, providing a link between the experienced situation and the quantitative characterization. Students could share their problems with other students, who would also have the option of exploring the problem interactively. Comparing similarities or differences between their problems should also further their sense of how problem parts relate to the whole picture.

Analogous situations/problems could then be brought in. Either the teacher introduces another real-world problem which the students "discover" can be stated in similar terms (or dissimilar terms, giving another way to solve the problem), or the teacher asks for different themes using the same structure, or the student chooses a topic. In whatever case, a desirable effect is that students, having once followed dynamically displayed conceptual steps for their *own* problems, as described above, may be motivated to engage in a similar process for these *other* problems.

Up until recently, a bottleneck in this kind of approach has been a lack of software which processes arithmetic word problems from their original statements. The Word Problem Processors of [Pogrow 1994] "understand" students' word problems sufficiently to allow, for example, feedback by the computer as to comprehensibility of the problem. However, writing creativity is constrained by prompting for expected sentence parts. The reading/solution capabilities of EDUCE/SELAH, on the other hand, parse natural language statements without such constraints, and are able to handle syntactic and semantic variations which influence problem difficulty. The (planned) integration of EDUCE/SELAH with a recently created program, *eXpress Math* [LeBlanc & Natola 1995], which supports students in writing their own word problems, will allow a reasonable freedom of style in writing about problems which can then be translated to various conceptual depictions. The use of *eXpress Math* allows the teacher to: integrate writing, science and mathematics; facilitate ownership of mathematical language by encouraging students to highlight quantitative relationships in situations independent of arithmetic; view students as knowledge-contributors rather than passive recipients; and encourage the construction of problems at unexpected levels of difficulty [see Fuson 1994]. *eXpress Math* (beta version) has been distributed to over 50 requesting sites and is currently operational as a pilot project in a number of classrooms nationwide. The integration of *eXpress Math* into the proposed classroom scenario can be described as follows:

- (1) Classroom *discussion* of problem situation
- (2) Linguistic *conceptualization* of problem with support of *eXpress Math*:

The teacher decides *how* to constrain the problem, e.g., chooses 2 or 3 relational words (from a long list) that must be included, for example: 'more than' and 'altogether.' Different combinations of words will make problems vary widely in level of difficulty, where the intent is to (a) force students to really understand the language and (b) encourage them to write (at least) two-step problems, perhaps even problems for grades above their own. The teacher can also constrain the topic, chosen from a long list (e.g., entomology, oceanography). The student writes the problem, clue, and solution "by hand," reviews it with the teacher (rewrite cycle), selects constraining words (if any), and enters the problem into *eXpress*

*Math* [see Fig.1].

- (3) *Exploration* and manipulation of problem solution processes in alternative symbolic modes through EDUCE/SELAH and interactive graphics tools.
- (4) *Sharing* of both printed and dynamically representable versions of children's problems.

Currently, *eXpress Math* only prints the problem to share with others. Problems can be shared "horizontally" among students in the same class/grade or "vertically" between students in upper or lower levels. Work in progress will allow students to submit their problems to a World Wide Web server. Teachers/students could search the database and download word problems (eventually those written in other languages). For example, they could search for "all problems containing 'more than' && 'altogether' && grade < 5." Web access to the interactive graphics tools is also planned.

The screenshot displays the eXpress Math web interface. At the top left, a form contains the student's information: Name: Jenn, School: Beechwood Knoll Elementary School, City: Quincy, State: MA, and Grade: 4. To the right, a 'Sample Topics' list includes Astronomy, Botany, Entomology (Insects), Geology, Oceanography (highlighted), and Whales. Below the student information, an 'About the Author' section features a message: 'I like math. Mr. Natola is a nice teacher. He makes it fun.' and a navigation bar with icons for Home, Author, Clue, and Solution. The main content area is titled 'Oceanography' and lists terms: coral reef, current, density, diatom, dissolved oxygen, echolocation (highlighted), and ecosystem. Below this is a 'Dolphin Weigh' problem: 'A dolphin weighs 300 pounds. His sister weighs 367 pounds and his brother weighs 524 pounds. How much do they weigh altogether?' and 'How much more does the brother weigh than the sister?'. At the bottom right, there are buttons for 'Add Word to List' and 'Delete'.

Figure 1: *eXpress Math* screens for an actual student-created problem

## 5. Conclusion

In the described approach to learning about word problems, mathematically relevant real-world problems discussed in class motivate students' own construction of word problems in interaction with teachers and with a computer learning environment. The computer graphics make explicit the multi-modal conceptual components of the problem portion of the student's written input, allowing the discovery of analogies between conceptually or mathematically similar problems. Central to this environment are the children's own conceptualization and creation of word problems, their derivation from a real-world context, their conversion to various symbolic modes and their sharing with peers--an integrated means of promoting a situation-related sense of mathematics in young children.

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