

# Teacher Groupwork Monitoring Routines and the Nature of Students' Conversation in Small Groups

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**Abstract:** Our goal in this study is to explore students' mathematical participation in collaborative groupwork with relation to teachers' instruction. By doing so, we address two challenges for research of teaching and learning: (1) the under-conceptualization of the teacher role during students' groupwork; and (2) the need to explicate the connection between teaching and learning. We suggest that, to connect student learning to teacher monitoring practices, it is valuable to link two different scales of analysis: (1) between-groups teacher groupwork monitoring routines; and (2) within-groups interactions with and without the teacher. We argue that such methodology affords a generative discussion of teachers' instructional practices and their disruption or reproduction of exclusionary aspects of students' conversations. To illustrate our approach, we look at two teachers. First, we describe and compare their groupwork monitoring routines. Then, we look at students' conversations within those classrooms and discuss connections between these conversations and instruction.

## Purpose

Groupwork is a promising teaching strategy to leverage students' ownership over mathematical ideas through talk and to promote inclusive classroom culture (Stein et al., 2008). However, while student collaborative work aims to disrupt narrow notions of math learning, these efforts often fall short and might actually reproduce exclusionary dynamics (Cohen & Lotan 2014), even among well-supported and equity-oriented teachers (Louie, 2017). While we know much about how to design collaborative work around groupworthy tasks (Lotan, 2003), the teachers' role during groupwork is understudied and ill-defined (Langer-Osuna, 2016). Even less clear is the connection between teacher facilitation and student learning during groupwork (Webb et al., 2009). The few scholars who study the teacher's role during groupwork offer different — and often contradictory — guidance for teachers' monitoring. On one hand, some scholars call for minimal interaction with groups. In the *Complex Instruction* model, Cohen and Lotan (2014) call to intervene only when a group is "hopelessly off task." Similarly, in the *5 Practices* model, Stein and colleagues (2008) state that teachers' main (although not only) role during monitoring is to listen to students' conversations in preparation for the whole class discussion. On the other hand, others suggest that teachers should get involved in students' conversations, probing their thinking and modeling dialogue (Webb et al., 2009). Given these contradictions and our experience as teacher educators, we acknowledge that the field is still learning how to support math teachers' shift toward instruction that centers students' sensemaking, particularly during groupwork facilitation. At the crux of this issue, we need to better understand how teachers' monitoring moves support students' productive mathematical talk. Toward this end, we examine two classrooms in the following manner: We first look at the two teachers' *monitoring routines*, the patterned ways they monitor groupwork and interact with students (Ehrenfeld & Horn, in press). Then, we look at *students' participation* in those groups during the lesson. Finally, we explore the relation between the two. Our goal in this paper is both methodological and empirical. That is, we explore students' mathematical participation in collaborative groupwork with relation to teachers' interventions and illustrate a method for doing so. We thus ask three related questions. First, *how do teachers' monitor groupwork?* Second, *how do students participate in the groups?* And finally, *how are the two related to each other?*

## Theoretical perspectives: Situative view on teaching and learning

In line with sociocultural and situative perspectives on teaching and learning (Greeno, 1998), we see mathematical competencies as aspects of participation in certain activities (Gresalfi et al., 2009). This means that we focus on processes of discursive *positioning* and consequently of identity development (Gresalfi & Hand, 2019) and how they enable or inhibit students' rich math learning experiences (Louie, 2017). We use the term *status* (Cohen & Lotan, 2014; Horn, 2012) to indicate students' positioning (either by their peers or by their teacher) as having more or less academic and social authority as mathematicians and group members. While abstract frameworks of monitoring practices can be useful heuristics to support students' collaboration, teachers always exercise pedagogical judgment in their own teaching situations (Horn, 2020). With this perspective in mind, we

conceptualize teacher groupwork monitoring (Ehrenfeld & Horn, in press) as a *situated interactional routine*. This framing captures tensions between classroom structures and teacher agency: while individual teachers monitor groupwork in patterned ways that we can document and study, participants also have agency to disrupt these patterns as they interact with one another, and teachers’ practice their judgment about groupwork as it unfolds. We can not stress enough that we do not see teacher monitoring routines as teaching “styles” or as teachers’ “property.” For the last 3 years of our research project, we have repeatedly seen the same teachers facilitate groupwork differently for different classes, topics, and tasks.

## Data and methods

### Research context and focal classrooms

This analysis comes from a larger research-practice partnership with a professional development organization (PDO) for experienced math teachers. To support teachers in making sense of instruction, we have developed a video-based formative feedback (VFF) cycle (Horn, 2020). In this analysis, we focus on data collected during classroom observations. We used two cameras and four audio recorders: Camera 1 captured the whole class with a focus on the teacher’s movements. Camera 2, a point-of-view camera, captured what teachers saw as they moved through the classroom and interacted with students. Audio recorders captured student conversations from four teacher-selected groups. In addition to these recordings, our data from lessons include artifacts, photos of whiteboards and student work, and conversations with the focal teachers before and after instruction.

For this analysis, we selected lessons from two teachers, Brad and Veronica, as distinct but generatively comparable cases of monitoring (Yin, 2013). Specifically, Brad and Veronica share many salient similarities as teachers, yet used very different monitoring routines in the focal lessons. They both teach in secondary public urban schools in the western U.S. They are both White, though their classrooms and schools have a majority of Black and Latinx students. They have both taught for over five years, share a commitment to promoting equitable participation in groupwork; indeed, they reminded students “don’t leave any of your classmates behind, work together, be collaborative” (Brad) and to agree on equitable rules for participation (Veronica). Despite these similarities, Brad and Veronica took different approaches to monitoring groupwork, as we report below. Thus, they offer productive contrasting cases to begin an inquiry into the interplay between teachers’ monitoring practices and students’ participation during groupwork.

During Brad’s Algebra I class, students worked on a task of graphing quadratic functions. To be more “groupworthy” (Lotan, 2003), Brad designed these tasks to incorporate new challenges (for them) such as finding roots with decimal x-values and graphing the function beyond a given grid. During Veronica’s 7th-grade lesson, students discussed various claims (e.g., “The sum of three consecutive numbers is divisible by 3”) to discuss if they were *sometimes*, *always*, or *never* true. Both classes included approximately 30 students working in groups of three or four. Because we want to connect student social and academic talk with teacher monitoring, we purposefully selected groups in each class with the most salient status issues, indicated by asymmetrical participation patterns, Brad’s Group 5 and Veronica’s Group 7.

### Phase 1 of data analysis: Teacher groupwork monitoring routines

We analyzed Brad’s and Veronica’s monitoring routines according to the teacher monitoring framework from Ehrenfeld & Horn (in press). We summarize the framework and each of its moves in Table 1.

**Table 1: Interpretive questions for the 4 monitoring moves and the overarching participation pattern**

Move	Interpretive questions	Examples
Initiation	How do teachers approach groups and initiate conversations?	<ul style="list-style-type: none"> <li>• T is walking in a fixed pattern between groups</li> <li>• T/Ss are initiating conversations</li> </ul>
Entry	What do teachers first say to the group as they approach?	<ul style="list-style-type: none"> <li>• T asks the group what they talk about</li> <li>• T redirects Ss conversation to a new topic</li> </ul>
Focus	What is the focus of the teacher’s interaction with the group?	<ul style="list-style-type: none"> <li>• T is probing students math thinking</li> <li>• T discusses task directions</li> </ul>
Exit	How do teachers exit the conversation?	<ul style="list-style-type: none"> <li>• T gives directive of “next steps to do”</li> <li>• T suggests “things to think about”</li> </ul>
Participation pattern	Do all students in the group participate in the conversation?	<ul style="list-style-type: none"> <li>• T is discussing with whole group</li> <li>• T is discussing with subgroup</li> </ul>

Teacher monitoring routines give a high-level overview of classroom dynamics. However, being teacher-centered, they do not tell us how students interact, what is happening in the moment-to-moment level of the teacher-student interactions, or the effect of the interaction. For these reasons, phase 2 focuses on a student-centered micro-level of analysis.

## Phase 2 of data analysis: Groups' conversations (with and without the teacher)

To explore the nature of students' conversation in the microgenetic level, we established a coding system around four interpretive questions (see Table 2). To investigate these questions, we used interaction analysis (Jordan & Henderson, 1995) and constant comparison (Strauss & Corbin, 1998) to shape our codes. Our team met regularly and maintained a "coding dilemmas" document, memos about group's dynamics, and a codebook with definitions and examples. The codes were refined in peer debriefing with the larger research team and the project's advisory board to ensure the ecological validity (Lincoln & Guba, 1985). Drawing on our memos and codes, we then characterized the interactional structure of each group, with and without the teacher.

Table 2: Interpretive questions and codes for the themes for understanding the nature of students' conversations

Phase	Interpretive questions	Code examples
Segments	<i>How was the conversation segmented by topic and structure?</i> (Jordan & Henderson, 1995)	<ul style="list-style-type: none"> <li>• Problem number</li> <li>• Conversation number</li> </ul>
Sensemaking	<i>What is the nature of math sensemaking throughout the conversation?</i> (Stein et al., 2008)	<ul style="list-style-type: none"> <li>• Social talk</li> <li>• Math talk {sub-codes: Results ('what'), Procedural ('how'), Conceptual ('why')}</li> </ul>
Positioning	<i>Who is positioned as a help seeker and who is positioned as a resource for mathematical knowledge?</i> (Horn, 2012)	<ul style="list-style-type: none"> <li>• Help seeking with peer/T</li> <li>• Being a resource for peer/T questions</li> </ul>
Identity	<i>What are additional indicators for the nature of students academic and social status throughout the conversation?</i> (Engle et al., 2014)	<ul style="list-style-type: none"> <li>• Directive (not math) talk</li> <li>• Direct identity labelling</li> <li>• Checking in / offering help</li> </ul>

The micro-analysis of group conversations revealed how students interact with each other and the teacher, and how status was constructed in these interactions. However, it does not show the interactions within the larger context of the lesson and teachers' decision-making. For this reason, Phase 3 joins the two levels of analysis.

## Phase 3 of data analysis: Linking monitoring and groups' conversations

Phase 3 is where we connect the top-level account and micro-level interactions by locating groups' conversations (with and without the teacher) in the larger context of the lesson. Analytically, building on Phases 1 and 2 we asked: (1) are the interactions we looked at a part of a larger pattern of groupwork monitoring?; (2) to what extent are the interactions typical of the classroom dynamics?; and (3) what exogenous (i.e., analysts' perspective) instructional dilemmas do we see emerging across the two scales?

## Findings

### Part 1: Brad's and Veronica's groupwork monitoring routines

We represent Brad's and Veronica's monitoring through what we call *monitoring traces* (Figures 1, 2, 3). The horizontal axis represents time and the vertical axis represents student groups with numbers. On the horizontal axis, -1 represents quiet circulation (QC), moments when the teachers observed students without directly interacting with a group. Each rectangle represents an interaction between the teacher and a group, with horizontal lengths showing the interactions' durations. Brad had 11 interactions with an average duration of 1:39 minutes. Veronica had 26 interactions with an average duration of 56 seconds. Initiation routines (Figure 1) highlight the contrast between teacher- and student-initiated interactions.

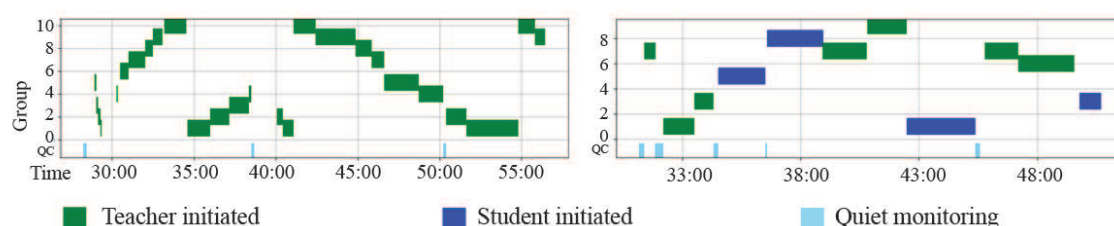


Figure 1. Monitoring traces of Veronica's (left) and Brad's (right) initiation routines.

Looking at Veronica's *initiation* routines (Figure 1), we can see that: (1) Veronica initiated all of her conversations with students; and (2) Veronica moved in a fixed pattern between the groups. Both of these routines are different for Brad, who practiced a mixed pattern of either approaching groups or being called upon. Initiation routines such as Veronica's secure a relatively equal teacher time allocation between the groups. For example, Brad did not talk with groups 2 and 4 at all. However, equality is different than equity, and teachers might have good reasons not to intervene (e.g. students are doing well).

When it comes to *entry* and *focus* routines (Table 3), Brad and Veronica both started most of their conversations by listening or asking groups to summarize where they were. Then, they both mostly focused on probing and hinting (*focus* is not a mutually exclusive move and can be multiple coded). Nine of Brad's 11 interactions focused on probing students' thinking, and 9 of his 11 interactions focused on scaffolding them. This pattern of probing and then hinting resulted in him *exiting* conversations with closed endings, where he told students what was correct or gave them firm directions on what to do next (See Figure 2). For Veronica, 15 of her 26 conversations focused on probing students, and 8 of her 26 interactions focused on scaffolding their work, which resulted in her *exiting* conversations with a mixed pattern of closed and open moves (See Figure 2). In the latter, she left students with something to think about, a subtle hint, encouragement, or new challenges.

Table 3: Brad's & Veronica's focus routines

	Codes	Brad	Veronica
Math	Probing	9 (82%)	15 (58%)
	Results centered	0 (0%)	0 (0%)
	Hinting	9 (82%)	8 (31%)
Not math	Participation centered	2 (18%)	3 (12%)
	Task centered	2 (18%)	0 (0%)
	Technical issues or brief comment	0 (0%)	7 (27%)

\* Out of a total of 11 interactions (Brad) and 26 interaction (Veronica). Codes sum up to more than 100% since focus routines could be multiply coded.

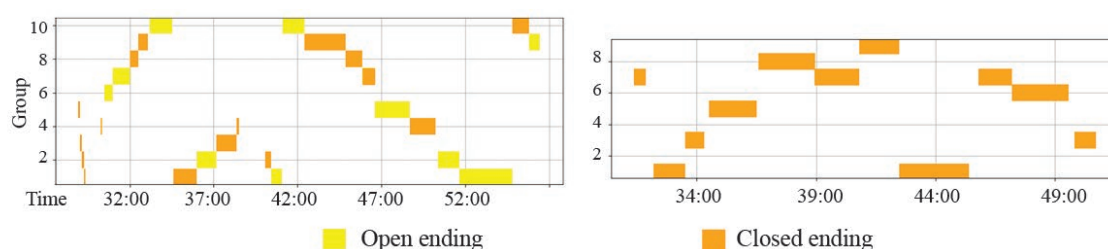


Figure 2. Monitoring traces of Veronica's (left) and Brad's (right) exit routines.

Participation patterns refers to the extent to which everyone in the group is included. We observed an inclusive conversational formation based on talk, participants' eye gaze and body orientation, which means it does not require all participants to speak. Generally speaking, both teachers were mindful about interacting with groups as a whole. However, even within *teacher-group* conversations, the teachers' interactions may play into status dynamics, which can be seen at the micro-interactional level of the conversations in the following section.

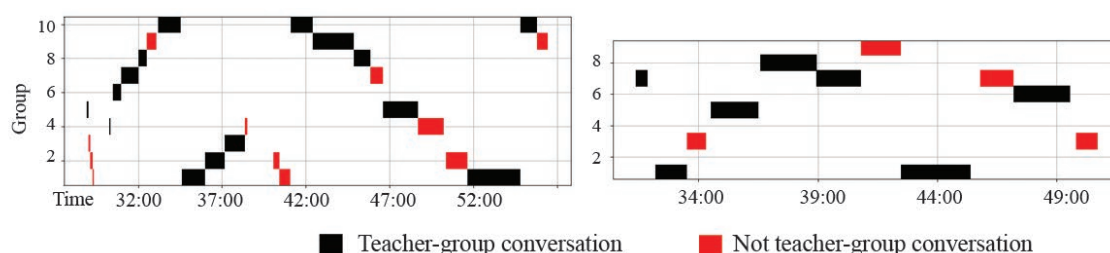


Figure 3. Monitoring traces of Veronica's (left) and Brad's (right) participation routines.

## Part 2: The nature of groups' conversations

Through reading the literature and our professional experience, we recognize there are a multitude of ways for groups to have "status issues." Thus, discerning group dynamics and responding to them is an essential (and complex) part of monitoring. In this section, we describe students' participation in Brad's Group 5 and Veronica's Group 7 (see Figures 1-3). These groups had notably asymmetrical participation patterns (Horn, 2012; Sung, 2018) constructed through group interactions with and without the teacher.

### Positioning and identity in Brad's Group 5

Group 5 was a relatively quiet group of three students: Mike, Mira, and June. As soon as the three got the task, June started to report her answers by telling her groupmates what to write:

- June: Alright so for the first one, write concave up since the leading term is positive. And the y intercept is at 5. And then for the second one write concave up, and x- I mean y intercept at -6. That's like the most you can do.
- Mira: Okay wait, you said concave up.
- June: Yeah.
- Mira: At -6 for the second one.
- June: Mhm (affirmative).

Students in this group worked quietly and mostly individually. In our analysis, the code *social talk* was not used much, and in a 21-minute episode of groupwork, we noted six segments of "quiet" lasting from 25 to 80 seconds. When students spoke, it was usually *procedural* math talk as in the example above, without any instances of *conceptual* math talk. Group 5's teacher-like procedural talk was the main student positioning mechanism. For instance, when Mira needed help, she asked and received it from June. We coded 13 instances of *help seeking*, of which 11 were Mira's and two were June's. In one situation, June asked for help from Mike, and in another from Brad, the teacher. In accordance, we coded 12 instances of June's *help giving* and one of Mike's. Both Mira and June engaged in *explicit identity talk*, including when Mira tells June "you're smart" and when she calls herself "stupid" and "bad at math" in the following, somewhat heartbreaking, dialogue:

- Mira: I'm still confused about something. I'm stupid.
- June: Mhm.
- Mira: I'm just gonna use the quadratic method. So like when you do the thing do you? Do you distribute or do you?
- June: Okay so you always do this number first so you get negative four and then you multiply negative four times negative six. You don't put negative four times negative no, first you do this.
- Mira: Mhm.
- June: What's negative four times negative six?
- Mira: Negative four times negative six, that's-
- June: You can say it!
- Mira: I'm bad, I'm bad at math.

Overall, Group 5's form of collaboration focused on June's willingness to procedurally support Mira, which constantly positioned her as mathematically smart and a resource for math learning. In contrast, Mira, and to some extent Mike, were positioned as help seekers and "bad at math."

#### Relation between students' conversation in Group 5 and Brad's intervention

Brad visited the group once during the class, at June's request. June asked him if the x-value of roots can be a decimal. The interaction was relatively long (2 min), but it focused on a conversation between June and Brad:

- June: I have a question, for, let's say I wanna do quadratic formula for it?  
Brad: Mhm (affirmative)  
June: And then I solve it.  
Brad: Mhm (affirmative)  
June: Does it always work? Is it always like a whole number?  
Brad: And what exactly are you saying 'solved for'?

At this point, Brad asked June to elaborate on her question (a "probing" *focus* move) and their conversation went on with only some involvement by Mira and no talk by Mike (although Brad did call his name). Then Brad substantiated the idea that not every result of the quadratic formula (i.e. every root) would always "run beautifully" on whole numbers and that it is possible to get a decimal (a hint *focus* move and closed *exit* move). At this point, when Brad left, the group went back to individual work for almost 2 minutes until Mira sought more help ("What are you doing for the second one? You did the quadratic formula?").

June, seen as a resource by her peers, did not reciprocate this view. Rather, when in need of help— an opportunity to balance the interactional pattern — June called the teacher. The teacher solved her problem, which stood at odds with her needing her peers. To be clear, we do not argue that the status issues in the group were the direct result of Brad's intervention. Instead, we claim that such interventions (especially if part of a larger pattern) did not disrupt status issues, but rather reproduced them.

#### Positioning and identity in Veronica's Group 7

Veronica's Group 7 consisted of Sam, Steve, and Abby, who engaged in substantial conceptual conversation as they grappled with the task's statements. Their body orientations often went into an exclusionary conversational formation, where Steve was in the middle holding the whiteboard marker, Abby was to the right, and Sam to the far left, sometimes not even in front of the writing space. This physical orientation resembled their overall status dynamics, both mathematical and social. Without Veronica's presence, this group developed asymmetrical dynamics, with Steve taking on an authority role and directing the group to begin:

- Steve: So, let's start with this one. If you add two numbers the result is zero. So, we have to prove this wrong.  
Sam: Yeah.  
Steve: Who would like to go first? No? Okay, I'll go first. Can I grab my purple.... This is not purple. This is brown. Okay, so this says that if I add two numbers the result is zero. But what if I—what if those numbers were let's say two, so two plus two would be four, which is higher—which is greater than zero.

Steve's dialogue with his peers is consistent with this example throughout the groupwork period (20 instances of *directive talk*) in that he maintains social authority and directs his peers to participate in the task, often speaking before they have a chance to engage:

- Steve: That is a good statement. Sam, do you have one for this statement, as well?  
Sam: Um... No.  
Steve: Okay, let me help you with that. So, let's say maybe a very high number like 100. Negative 100 plus 100 equals zero. So, I think that with three statements that prove this true [Abby giggles], that the answer to this statement is that it is always true. Do you guys agree?  
Abby: I agree.

Sam: I agree.

Notwithstanding Steve's directive authority, indicative of his high social status, the dynamic of the overall group was more complex. Abby seemed to be consistently positioned as a resource for mathematical knowledge (11 instances of *help-giving*). Sam, on the other hand, was generally excluded, verbally engaging with his peers a few times, often only when prompted to do so by Steve. While Steve's explicit inclusion of Sam might seem positive and equitable, we see something more subtle at play. Namely, Sam was occasionally given access to *interactional space* (i.e. to talk) without being given, or better, without having, meaningful access to the *conversational floor* (Engle et al., 2014) to influence the mathematical conversation. This dynamic became apparent through our coding: First, Steve's directive inclusion of Sam never resulted in Sam's positioning as a resource for mathematical information (no instances of *help giving* in contrast with Steve's six and Abby's 11). Second, Sam's one spontaneous utterance was a mathematical question answered incorrectly by Steve, leaving Sam to answer his own question. From Sam's responses to Veronica and his correction of Steve after asking him a question, it is evident that Sam had the mathematical knowledge necessary to participate in the conceptual discussion mandated by the task but was prevented from sharing it. Our perspective on status relations in this group echoes Goffman's (1983) discussion about preferred treatment by individuals who can choose whether to include another or not as a mechanism that maintains power hierarchy and may result in one being "patronized and disparaged." Group 5's conversations without the teacher illustrated that Steve and Abby held strong perceived status with Steve exercising his power constructed a pattern of marginalization.

#### Relation between students' conversation in Group 7 and Veronica's interventions

Veronica's interactions with Group 5 revealed her attempts to balance participation, even though these status interventions were not substantial enough to last after she leaves the group.

She approached Group 7 twice, once at the very beginning of the groupwork period and again towards the end (see Figure 1). When she arrived at the group, she asked general check-in questions to find out where the group stands. The rest of her questions, however, addressed particular students, especially Sam and Abby, suggesting that she may intentionally addressed the group's status imbalance. Even when she explicitly directed a question to Abby or Sam, Steve sometimes answered, further asserting his access to the group's interactional space. Veronica asked the students eight mathematical questions, and the respondents were relatively equal, with Abby answering three questions, Sam answering two, and Steve answering three.

Veronica's interaction shifted who spoke, with increased participation from Sam and Abby, suggesting that her strategy of explicitly addressing questions to specific group members successfully disrupted the inequitable status dynamic present when students worked without her support. However, after Veronica left the group, the hierarchical relationship among students seemed to return, indicating that her intervention did not create a lasting disruption in the group's interactional patterns.

### Part 3: Connecting teacher monitoring routines to groups' conversations

For Brad, the micro-analysis of his interaction with Group 5 revealed that he answered June's question by talking mostly with her. In doing so, he indirectly contributed to the asymmetrical status hierarchy in the group, where Mira sought help from June, and June sought help from Brad. His monitoring routines revealed that his interactional *focus* centered probing and hinting, resulting in closed *exit* moves. By consistently positioning himself (and taking up students' positioning of him) as a resource for mathematical information, this might work against building a classroom culture where students like June and Mira see their peers as resources. For Veronica, the micro-analysis of her interactions with Group 7 showed how she only temporarily disrupted the asymmetrical interactional pattern. From a monitoring routines perspective, Veronica (1) spent very short amounts of time quietly monitoring the classroom; (2) walked in a fixed pattern, initiating all her conversations with groups; and (3) talked with groups about math much more than she did about participation norms. Given the subtle interactional dynamics of Group 7, coordinating the monitoring analysis with the micro-analysis suggests new instructional possibilities for Veronica with regards to (1) normalizing walking between the groups quietly and listening to their conversations since recognizing group dynamics and responding to them is an essential part of monitoring; and (2) explicitly discussing participation norms with the groups.

## Discussion

Methodologically, we argued and illustrated that the microgenetic level of Brad's and Veronica's interactions with the groups is useful to learn about the teachers' role during groupwork, but it is also insufficient. The additional analysis of the teachers' monitoring routines provided essential additional information to glean the patterns that framed the micro-analyses. This lens is central in our ongoing effort to conceptualize the teacher's role during

students' collaborative work. Empirically, we argue that the two different scales of analysis — whole lesson and micro-analysis of interaction — offer a generative approach for discussing instructional dilemmas during groupwork, a promising step towards connecting small group instruction and student learnings. We hope that such efforts will allow us to better support teachers in facilitating deliberate, productive, and inclusive mathematical discourse in their own teaching contexts.

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