A Design Framework for Teaching/Learning Activities Supporting Latino First Graders' Ten-Structured Thinking in Urban Classrooms

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Abstract: This paper presents a design framework for teaching/learning activities during year-long classroom teaching experiments that sought to support first graders' ten-structured thinking in two predominantly-Latino low-SES urban classrooms (one English-speaking and one Spanish-speaking). The design framework outlines (1) theoretical perspectives on the teaching/learning domain of multidigit concepts and multidigit addition and subtraction, (2) theoretical perspectives on teaching and learning processes, and (3) the pragmatic context of the research. The particular elements within the framework generalize to other design tasks to various extents. The teaching/learning activities were successful in enabling children in these two classes to achieve above grade level and to look more like East Asian than U.S. children in their multidigit concepts.

Children in the United States do considerably worse on many measures of mathematical thinking and performance than do children in China, Japan, and Korea [Fuson & Kwon 1992a, 1992b; Miller 1990, 1991; Miller & Stigler 1987; Miller & Zhu 1991; Song & Ginsburg 1987; Stigler, Lee, & Stephenson 1990; Stephenson & Stigler 1992]. Poor urban children, especially those who do not speak English or who have parents with low levels of education, perform even worse [Secada 1992]. This paper reports initial results from a project focused on designing teaching/learning activities for Latino-background urban children that are culturally appropriate but create learning experiences more like those received by East Asian children.

The focus is on the design framework for the teaching/learning activities, but summary achievement results are reported to indicate the effects of the activities. More details about the teaching/learning activities and the achievement data are in Fuson, Smith, and Lo Cicero [1996]. The mathematical focus was addition and subtraction of quantities expressible by 2-digit numbers and the conceptual relationships among number words, 2-digit numerals, and quantities (place-value concepts). Project staff teachers worked in one English-speaking and one Spanish-speaking first-grade classroom for a whole year.

The Design Framework

The design framework is given in Table 1. Its three major components are theoretical perspectives on the teaching/learning content domain, theoretical perspectives on teaching and learning, and attributes of the pragmatic context of the research. We view these as the miminum elements of any design framework. The particular elements within the framework generalize to other design tasks to various extents. The perspectives on the domain of teaching and learning (multidigit numbers) generalize the least, although aspects do generalize to any mathematical tasks using multidigit numbers and to decimals. The general frame within Figure 1 of triads of number words, written number marks, and quantity referents for these is also a general frame that can be used for any quantity domain (with appropriate specifics at each corner of the triad). The general categories within the theoretical perspectives on learning and teaching apply to any content domain, and many elements within these categories also generalize. Certain aspects within the pragmatic context also generalize to many school classrooms. For school-based projects (and probably also for most projects in non-laboratory settings), important attributes of this pragmatic context are often discovered during the course of the design and implementation process. The timing of these discoveries is vital, because they can occur after design decisions have been made. For technology designs, where there may be a long lag between design and actual implementation testing, failure to discover limiting aspects of the pragmatic context can be very costly in terms of a lack of fit of the designed technology and its pragmatic context of use.

The design process for our project had single or multiple recursive team cycles of design followed by mental implementation and then redesign. This was followed by implementation in one classroom, reflection and discussion, and often, redesign for the second classroom. Some elements of the framework were explicitly used from the beginning, some were used implicitly, and some emerged and/or were clarified during the year.

Theoretical Perspectives on the Domain of Teaching and Learning

A. Theory of multiunit conceptual structures: A connected web of unitary, tens-within-decades, and tens/ones triads (all paths in Figure 1) —> Need classroom supports for children's construction of all triads and connections among them —> Project classroom supports:

- a. Paths linking tens/ones quantities, number words, and number marks
  - i. Various introductory activities to see grouping in tens and relate groupings to number words and number marks (numerals)
  - ii. Ten-sticks and dots related to number words and number marks (numerals)
- b. Tens/ones words (53 said as five tens three ones)
- c. Ten-structured prerequisites for finger and mental single-digit methods around ten
- d. Special difficulties with teens words –> begin 2-digit web to 100 before extensive work with teens

B. Theory of Multiunit Addition and Subtraction

- a. Add/subtract like multiunits
- b. If necessary, make another ten (adddition)/open a ten to get more ones (subtraction)
- c. Single-digit add/subtract a given kind of multiunit

Theoretical Perspectives on Learning and Teaching Processes: A Constructivist View of Learning and a Vygotskiian View of Teaching

C. Children as meaning makers

- a. Research says children initially need perceptual unit items for quantities
- b. Constructions with quantities take a long time before addition/subtraction methods can be abbreviated or internalized to mental or written numeric methods (no instamatic camera model of learning)
- c. Children choose their solution methods (they use paths of their choice)
- d. Use quantities so that they support reflection to move on to mental and numeric methods

D. Teaching as assisting all children to use cultural-historical semiotic domain tools meaningfully

- a. Necessary domain tools:
  - i. English/Spanish number words
  - ii. Written 2-digit marks (numerals)
- b. Pedagogical tools (potentially mathematical referrers and referents)
  - i. Tens/ones words (53 said as five tens three ones)
  - ii. Ten-sticks and dots
  - iii. Various tens/ones quantity activities (most of which did not meet pragmatic tests)
  - iv. Fingers for ten-structured single-digit addition and subtraction methods

E. Teaching as assisted performance of children with meanings-in-the-making

- a. Classroom learning experiences support children's learning of manageable multidigit addition/subtraction quantity methods
- b. Elicit, discuss, and demonstrate a range of solution methods
- c. Emphasize methods using tens and ones (decade count-by-tens, tens/ones count-the-tens)
- d. Assist children needing help with particular steps
  - i. Whole-class discussions focus on errors (why wrong? how to correct?)
  - ii. Assist individuals in class with missing or erroneous components
  - iii. Assist individuals outside of class if substantial chunks of web are missing
- e. Assist reflection so that children can use alternative methods
- f. Assist reflection so that children move on to mental and number methods

Attributes of the Pragmatic Context

F. Constraints of many urban schools

- a. Limits concerning pedagogical tools
  - i. Low cost in money and teacher time
  - ii. Not easily lost or taken
  - iii. Ease of management: distributing, keeping sets separate, objects dropping off desks, fiddling with rather than solving or listening

iv. Allow teacher to see children's solution methods after class because of large class sizes and frequent interruptions during class

b. Standardized tests affect curriculum topics covered

c. Individual differences in teacher emphasis and teacher style

G. Need to develop, try, and adapt learning activities and pedagogical tools

One class tried activities first (the project staff teacher was more experienced in the teaching domain); activities were modified as seemed necessary and were then tried in the second class (the Spanish-speaking class)

H. The pragmatic sieve: the learning activity actually works in a whole classroom of children

Table 1: Design Framework for Multidigit Teaching/Learning Activities

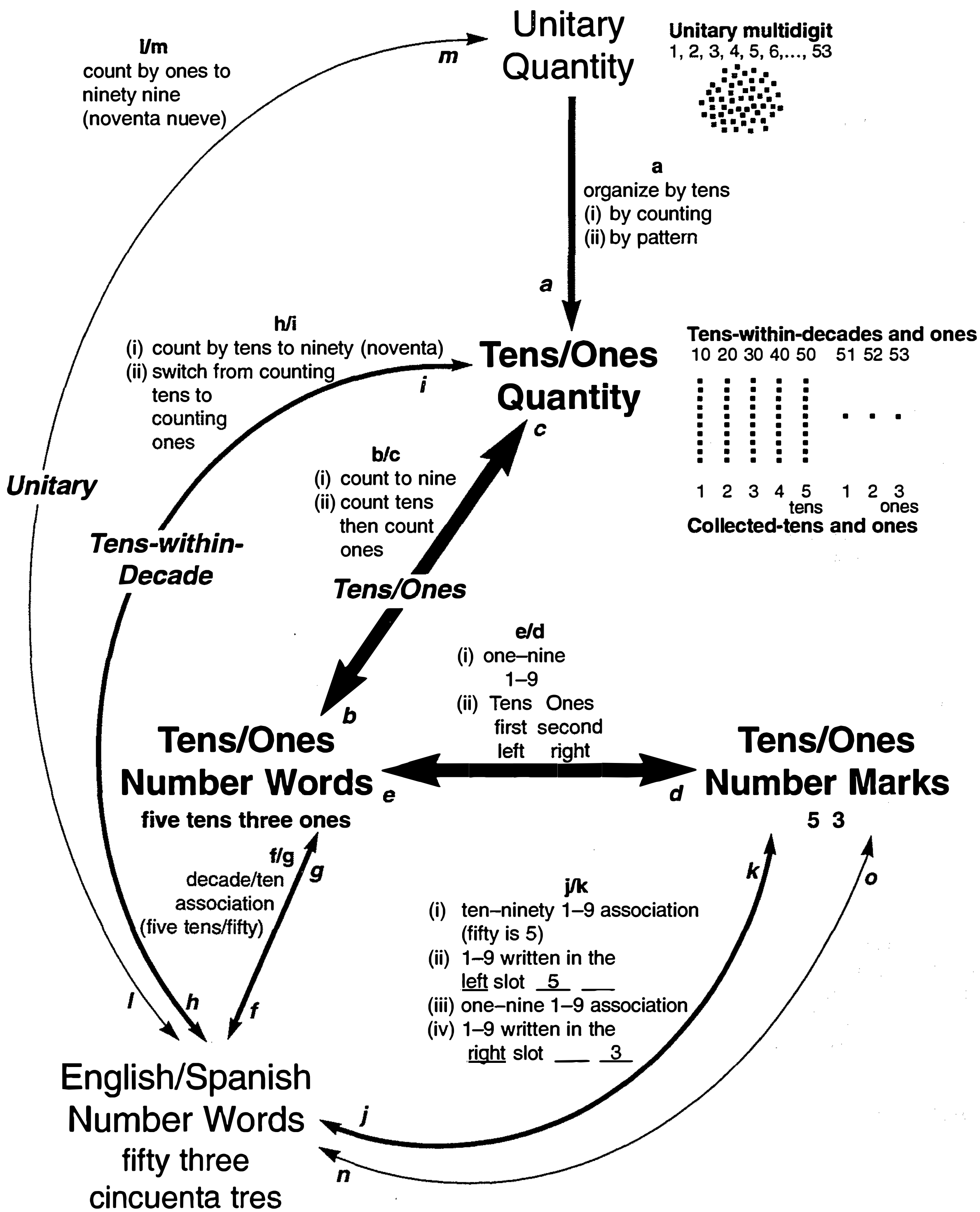
The project staff doing the design work were (1) a professor who was an experienced math teacher and had many years of experience doing basic and applied research on children's mathematical learning and on teaching based on such learning, (2) a post-doctoral fellow with a background in anthropology and considerable experience in designing and testing math learning activities for young children, and (3) a bilingual bicultural experienced language teacher. Input was sought from and given by the classroom teachers in whose classrooms we were working. The design framework was actually put into a table only when the results of the year were being written up. It now gives clarity to what was often an intuitive process with different perspectives by all participants. In retrospect, it seems that use of such a design framework would facilitate reflection and speed up the construction of shared meanings by participants as well as expose differences that would be helpful to discuss. Beginning any design project with an explicit framework, and revising it frequently throughout the design process, might stabilize and improve what is necessarily an emergent process.

Theoretical Perspectives on the Domain of Teaching and Learning: Multidigit Numbers

The theoretical work of this study was an extension of earlier theoretical work [Fuson 1990], instructional development work [Burghardt & Fuson 1996; Fuson & Briars 1990, Fuson, Fraivillig, & Burghardt 1992], and analytical and empirical work on linguistic and cultural supports for ten-structured thinking available for East Asian children [Fuson & Kwon 1991/92, 1992a, 1992b]. The extension and clarification of conceptual structures children construct for understanding multidigit numbers is summarized in Figure 1 and described in more detail in Fuson, Smith, and Lo Cicero [1996]. East Asian children need only construct the inner tens/ones triad (connected by the heavy lines) and the outer-most unitary triad (connected by the thinnest lines) because their number words are regular and name the tens for numbers between 10 and 100 (e.g., 53 is said "five ten three"). Children speaking European languages with a decade structure (e.g., twenty, thirty, forty, fifty, etc. instead of two tens, three tens, four tens, five tens, etc.) must construct the special middle tens-within-decade paths for the decade words in addition to the unitary and the tens/ones paths (meanings). Children can function meaningfully in ten-structured situations with only the decade or the tens/ones (thick line) paths, but a fully integrated and complete understanding of standard European number words and the number marks requires a flexible use of all paths in the web. Figure 1 indicates clearly that the learning and teaching tasks are much more complex in the U.S. than in East Asian countries, and that discourse within classrooms may get confused because children may have, or be using at that moment, different conceptions of multidigit contexts.

Theoretical Perspectives on Learning and Teaching

The perspective used by the research team can be summarized as a constructivist view of learning and a Vygotskiian view of teaching. Central attributes of our version of these perspectives are listed in the middle section of Table 1, but there is not space here to discuss them in detail. A brief discussion of children as meaning makers appears in Fuson, Fraivillig, and Burghardt [1992], and a paper explicating the perspectives in D and E is in preparation. The pragmatic context affected our search for pedagogical tools that would support meaning making for multidigit numbers. In earlier studies we had found that teachers can use base-ten blocks to help whole classrooms of children construct quantitative meanings for standard algorithms [Fuson & Briars 1990] and that small groups of children can use base-ten blocks to construct various meaningful methods of multidigit addition and subtraction if they link their actions on the blocks to their actions on the written numerals [Burghardt & Fuson 1996; Fuson, Fraivillig, & Burghardt 1992]. However, base-ten blocks are too expensive to have enough so that every child in the class can act on them. Therefore, our instructional



Unitary, Tens-within-Decades, and Collected-Tens Triads:
Quantity, Number-Mark, and Number-Word Relationships for Two-Digit Numbers

Figure 1

development focused heavily on devising pedagogical tools that would meet the pragmatic constraints outlined in the design framework.

The most powerful semiotic domain tool developed during the year was a drawn quantity referent we called "ten-sticks and dots." Ten-sticks were vertical sticks, each of which had a quantity of ten, and dots (or circles) had a quantity of one and were drawn in horizontal rows. The ten-sticks and dots met the pragmatic tests listed in Table 1 under Fa. They also facilitated linking of quantities and number marks because number marks could be written on or near the ten-sticks or dots, and they supported reflection by the child on their own constructions because the drawings were permanent. Linking is important in multidigit addition and subtraction because errors arise much more frequently if quantities and marks are not linked; errors can be eliminated by linking [Burghardt & Fuson 1996; Fuson 1986]. Written number marks do not take on quantitative meanings if they are not used to refer to multiunit quantities. With objects, the initial quantity is lost through the addition or subtraction operation of acting on the quantities. With the ten-sticks and dots, dots were enclosed to make another ten, or a ten-stick was opened up to make ten dots. Therefore all quantities were potentially there for reflection about the problem-solving process.

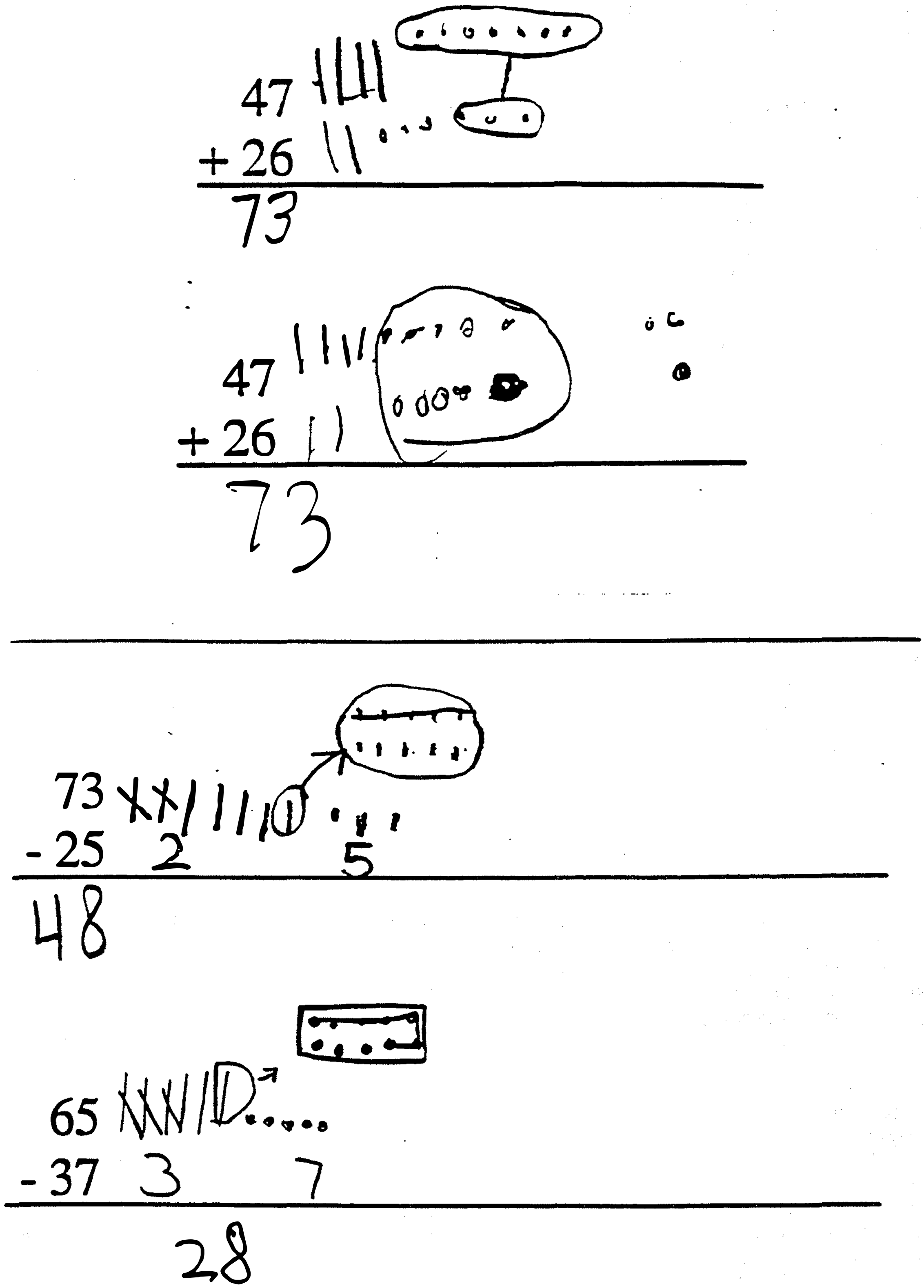
Ten-sticks and dots were first developed out of unitary quantities (dots) grouped in tens. Children therefore could think of these quantities using any of the conceptual structures in Figure 1 (unitary, decade, or tens/ones). To make a record of 2-digit quantities the class was accumulating, children counted by ones while they made columns of ten dots and then a horizontal row of dots for the ones (often with a space between the first five dots and the last four dots to facilitate "seeing" these larger numbers as patterns of $5 + x$). To check their quantity, children could count then count the columns of ten dots by ones, by tens (10, 20, 30, 40), or as tens (1, 2, 3, 4 tens). These different ways to count were all modelled by the teacher and by children. When children confidently could make such drawings, the columns of ten dots were connected by a line drawn through them as the counting by tens or of tens was done; some children had already spontaneously begun to do this. Eventually only the vertical stick was drawn to show a ten. These activities occupied part of the class period for about two weeks.

The ten-sticks and dots facilitated meaningful addition and subtraction of multidigit quantities because they supported all three aspects of these operations (see B in Table 1, taken from Fuson, 1990). In classroom instructional conversations, the children and the project teacher discussed various addition (and, later, subtraction) methods using the ten-sticks and dots. Dots were combined to make another ten when possible, and a ten-stick was opened (its ten dots were drawn within an ellipse) when necessary in subtraction [see Figure 2]. Addition and subtraction took place within word problem contexts, for example as buying/selling doughnuts packed in boxes of ten, so boxes were packed or opened as necessary. Children could use either a decade or a collected ten conception in adding or subtracting: They counted the "stick and dot" ("las barras y los puntos") quantities by tens and ones (10, 20, 30, 40, 41, 42, 43) or as tens and as ones (1, 2, 3, 4 tens and 1, 2, 3 ones).

The Pragmatic Context

The major pragmatic constraints on pedagogical semiotic tools are given as Fa in Table 1; they were discussed above with respects to the ten-sticks and dots. The attributes A through G concern the cycle of learning/teaching activity development throughout the whole year. H, the pragmatic sieve, was an immediate test of every teaching/learning activity. It was more difficult to anticipate and repair problems in this general "Does it work?" aspect than in the others. We usually could determine low cost in money and teacher time, and we then could reject invented options that did not meet these criteria. We frequently could anticipate management problems and modify to reduce or eliminate them. But the pragmatic seive involved complexities of making an activity that would be interesting to children, would be at the right level of difficulty for many, and could fit within a few core classroom activity structures that could be learned and routinized. We did not always anticipate all of the necessary aspects, so learning activities frequently needed to be adapted.

There also was interaction between the pragmatic seive and the different paths within the web. Part of the adaptation process involved trying to achieve a balance among opportunities for children to work on various paths and learn necessary knowledge for these. Another part of the balancing act was maintaining enough continuity of activity structures and mathematical content so that children could function profitably and could learn over sessions, but enough variation so that the teacher and children did not become bored. Furthermore, there were individual differences in the need for the predictable old and the strange new. We found that our first graders, and our first-grade teachers, felt comfortable with the many days of sustained experience in each of several different types of activities it took to build up multiunit concepts.



2-Digit Addition and Subtraction Using Ten-Sticks and Dots

Figure 2

Results of the Teaching/Learning Activities

The ten-sticks and dots allowed children to use different solution methods, some that required decade conceptions and some that required tens/ones conceptions. Some children in each class used each of these kinds of conceptions. However, most children in the Spanish-speaking class used decade conceptions, and more that half of the children in the English-speaking class used tens/ones conceptions. These emphases resulted from different emphases by the teachers. By the end of the year, almost all of the first graders could accurately use ten-stick and dot drawings to add 2-digit numbers that require trading (regrouping), most of the English-speaking first graders could use such drawings to subtract 2-digit numbers with trading (regrouping), and most of the Spanish-speaking children could use them to do 3-digit adding without trading (the latter two different foci were necessitated by differences in the standardized tests used in these classes).

End-of-the-year interview tasks on Figure 1 knowledge paths indicated that all children in both classes constructed all of the knowledge paths for the inner tens/ones triad. All but three children could do the middle decade triad tasks. Children in the Spanish-speaking class tended to do these directly by using paths h/i and j/k, and children in the English-speaking class tended to use the tens/ones triad paths b/c and d/e and the path f/g connecting tens/ones number words to English/Spanish number words.

On various tasks assessing children's conceptual structures for 2-digit numbers, first graders from both classes looked more like East Asian children in other studies, who give answers reflecting ten-structured concepts of numbers, than like first graders in the United States, who predominantly respond unitarily. On some tasks, the first graders did better than second and third graders from their school before this intervention and than suburban U.S. children from higher grades reported in other studies.

We are not aware of comparable data to assess the relative competence of our children on multidigit addition tasks. Stigler, Lee, and Stevenson [1990] reported that Japanese, Taiwanese, and U.S. first graders were 29%, 25%, and 13% correct on a Combine word problem adding 26 + 19. Hiebert and Wearne [1992] reported that 35% of their conceptually instructed first graders and 25% of the traditionally instructed first graders were correct on a Change Add To: unknown Result problem with trades. Our children were given a word problem with base-ten blocks (89% correct) and a 2-digit addition problem in numerals, for which most children used ten-sticks and dots (90% correct). Thus, our children's performance on multidigit addition problems with trading considerably exceeded that of children in other studies.

Conclusion

It did prove to be possible to support most children's construction of most elements of the web of relations in Figure 1. Most children demonstrated most relations for all three conceptions: the tens and ones conception, the sequence-tens decade and ones conception, and the unitary multidigit conception. Furthermore, on a range of unfamiliar tasks, many children showed a robust preference for ten-structured conceptions, looking like children in China, Japan, and Korea rather than like age-mates in the United States, or like children in higher grades in the United States. Many children also were able to carry out ten-structured solutions to 2-digit addition problems (also subtraction, for the English-speaking class). This is considerably above what first graders in the United States ordinarily have an opportunity to learn, because such problems with trades are usually not included in first-grade textbooks [Fuson 1992]. Performance was considerably above that reported for U.S. children receiving traditional and nontraditional instruction and was above that reported for Japanese and Taiwanese first graders.

The work on 2-digit addition (and 2-digit subtraction and 3-digit addition with no trades) was conceptualized partly as work on that domain but also as an opportunity for children to continue to construct and to use in more complex activities many relations from the web. Therefore, the teaching criteria during these activities were different from those usually used with the simple topics in the first-grade curriculum, where high levels of success are expected of most children. The 2-digit activities were viewed as involving an apprenticeship in complex activities during which children might make many partial errors that gradually would come to be corrected as they had opportunities to construct more adequate understandings. In our view, the whole web of knowledge European children must construct is so complex, and so many different orders of construction are possible, that its full construction and use is best conceptualized as a 2-year task requiring much of first and second grade. However, unlike current practice, much more of this web can be constructed in first grade if teaching/learning activities stemming from the design framework given in Table 1 are used. The tasks for second grade should then focus on consolidation and automatization of the web and on moving from using drawn concrete quantity referents to using written and mental numerical methods that have those quantity meanings.

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