Students' Use of Multiple Strategies for Spatial Problem Solving

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Abstract: In scientific problem solving, spatial thinking is critical for reasoning about spatial relationships in three-dimensions and representing spatial information in diagrams. Despite the importance of spatial thinking, little is known about the underlying cognitive components of spatial thinking and the strategies that students employ to solve spatial problems. Namely, it is unclear whether students employ imagistic reasoning strategies while engaged in spatial thinking. In the present study, we investigate which strategies students use to solve spatial chemistry problems and the relationships between strategy choice, achievement, spatial ability and sex. The results indicate that students employ multiple strategies that include the use of diagrams and heuristics rather than merely relying on imagistic reasoning. Moreover we observed women to employ strategies differently than men after extended instruction in the domain.

Objectives & Theoretical Framework

A recent report from the National Research Council (2006) identifies spatial thinking as a critical component of scientific problem solving and reasoning and advocates for training spatial thinking in the science classroom. Such a call is consistent with the content of science instruction, which often requires students to reason about the three-dimensional relationships of objects and phenomena that are of interest to scientists. For example, chemistry students must learn about the three-dimensional structure of molecules, physics students must learn about the trajectory of projectiles and geology students must learn how geological structures transform over time. Given the prevalence of spatial thinking across the sciences, several researchers have suggested that student aptitude for spatial thinking, as measured by spatial ability psychometrics, predicts their success in science classrooms (Pallrand & Seeber, 1984; Wu & Shah, 2004) and careers (Shea, Lubinski, & Benbow, 2001). Indeed, a host of studies have shown positive correlations between visuo-spatial ability and achievement in several science domains (Carter, LaRussa, & Bodner, 1987; Hegarty & Sims, 1994; Keehner, Lippa, Montello, Tendick, & Hegarty, 2006). Consequently, these findings have led to claims that sex differences in spatial ability are responsible for sex differences in science achievement (cf. Fogg, 2005).

Despite the importance of visuo-spatial ability, questions remain about the cognitive components of spatial thinking. Typically, spatial thinking in science has referred to imagistic reasoning that includes mental imagery, mental rotation, spatial perspective taking and spatial visualization (Bodner & Guay, 1997). However, practicing scientists and novice students alike successfully solve spatial tasks through the use of external diagrams, models, and computer simulations that may or may not recruit these cognitive processes (Stieff, 2007; Stieff & Raje, 2010; Trafton, Trickett, & Mintz, 2005). Also, a variety of domain-specific analytic algorithms and heuristics have been reported that lead to solutions with little to no use of spatial information given in a spatial problem (Schwartz & Black, 1996; Stieff, 2007). The availability and utility of these alternative strategies raises several questions about the components of spatial thinking and their role in scientific problem solving at all levels.

The present paper aims to identify the underlying cognitive components that comprise spatial thinking in science. We address this aim with four questions: What strategies do problem solvers use to solve tasks that involve spatial thinking? Does strategy choice predict success on a variety of spatial tasks? Do spatial ability and sex predict strategy choice? How does instruction affect strategy choice? We address each of these questions by examining student problem solving in the domain of organic chemistry. Historically, this domain has privileged the role of visuo-spatial ability due to the content of organic chemistry which includes the analysis of three-dimensional relationships within and between molecular structures (Mathewson, 1999; Wu, Krajcik, & Soloway, 2001; Wu & Shah, 2004); yet, little is known about what strategies students employ when considering these relationships. Previously, Stieff and Raje (2010) have shown that expert chemists engage in spatial thinking using a variety of domain-specific diagrammatic and analytic strategies as opposed to mental imagery; however, strategy use among chemistry students remains unknown. Here, we build on the work of Stieff and Raje, by examining college students' choice of problem solving strategies for solving spatial organic chemistry problems to determine the extent to which chemistry students employ multiple strategies and how strategy choice changes with increasing domain knowledge.

Study 1

In Study 1, we designed a strategy choice questionnaire that first asked students to solve 10 canonical organic chemistry assessment tasks. On each task, students were asked to indicate how they solved the problem using a list of known strategies applicable to the task. Previously, Stieff and Raje (2010) documented experts' use of specific imagistic and non-imagistic strategies for solving organic chemistry problems; the findings of that study were used to populate the list in the present work. The goal of Study 1 was to identify patterns of strategy use among students and any associations between strategy choice, achievement and sex.

Method

Thirty-nine college students (20 males, 19 females) who had completed 6 months of instruction in organic chemistry were asked to complete a chemistry strategy choice questionnaire. The strategy questionnaire consisted of 10 organic chemistry problems that asked participants (1) to identify spatial relationships between molecules or substituents within a molecule and (2) to consider spatial transformations of molecular diagrams. All chemistry problems were scored for correctness using a binary rubric (1 = correct, 0 = incorrect). Participants were also asked to report the strategy they used to solve each chemistry problem by selecting from a list of possible strategies applicable to each problem. Participants were allowed to choose more than one strategy and to write in their own strategy if they believed that none of the choices matched their strategy. Each list of strategies for individual problems was developed in an earlier protocol study conducted by Stieff and Raje (2010); each strategy was coded according to a priori categories of strategy type listed in Table 1. Briefly, categories included those strategies that relied more extensively on reasoning via mental imagery (Spatial-Imagistic), diagrams (Spatial-Diagrammatic), rules and heuristics that operated on spatial information (Spatial-Analytic) and rules and heuristics that operated on non-spatial information (Algorithmic). Participants could also indicate if they knew the answer to a problem (Recall) or if they randomly guessed (Guessing). We note that the three categories that include the *spatial* prefix involve the direct consideration of spatial information while the algorithmic category does not. In cases where participants wrote in their own strategies, two researchers independently coded the free responses according to the four categories in Table 1. Comparison of the two raters' codes indicated an inter-rater reliability score above 85%.

Table 1: Strategy Categories.

Strategy Type	Example Fixed-Choice Strategy Responses
Spatial-Imagistic	I tend to imagine the molecule in 3D and rotate it "in my head". I tend to imagine myself moving into the paper or around the molecule.
Spatial-Diagrammatic	I tend to first draw a basic skeletal structure and then make changes as I go. I tend to redraw the molecule using a different chemical representation to help me think about it.
Spatial-Analytic	I tend to assign R/S labels to each molecule.
Algorithmic	I just know that in stable molecules particular groups must be in a specific relationship. I tend to use a specific formula to calculate the number of stereoisomers.

Results & Discussion

Figure 1 summarizes the frequency of each strategy choice across the 10 tasks. Among the 418 strategies reported, participants selected Spatial-Analytic strategies most frequently (36%) followed by Spatial-Diagrammatic strategies (26%), Spatial-Imagistic strategies (22%) and finally Algorithmic strategies (16%). Figure 2 shows a detail of strategy frequencies by task. The distribution of strategies differed dramatically among the ten tasks, which suggests that students freely switched between the different types of strategies depending on each task. For example, the majority of reported strategies applied to Tasks 1, 5, 6, and 8 were Spatial-Analytic strategies, but Spatial-Imagistic strategies were reported more often on Tasks 9 and 10. The dataset was further analyzed to determine whether participants used primarily one strategy for each task or applied multiple strategies. In total, we were able to identify the strategy used by participants in 326 (83.5%) of the 390 cases of problem solving. The remaining 64 cases either lacked strategy choice information or were solved via Guessing or Recall. As Table 2 illustrates, 240 tasks (73.6%) were solved with only one type of strategy, 80 tasks (24.5%) were solved with two types of strategies, and 6 tasks (1.8%) were solved with three or more types of strategies. In cases where participants used only one strategy, Spatial-Analytic strategies were reported most frequently. Interestingly, in cases where participants selected two types of strategies, the majority of reported strategies involved the use of a Spatial-Diagrammatic strategy and one other type of strategy. Notably, we observed a negative correlation between the number of participants who successfully completed a problem and the number of participants who used two or more strategies, ($r(10) = -0.654$, $p = 0.040$), which suggests that students tend to apply multiple types of strategies as questions become more difficult.

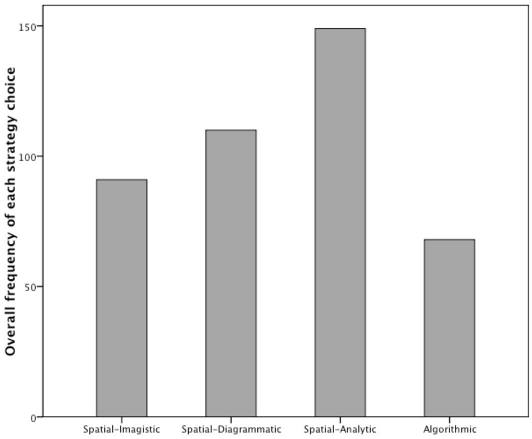

Figure 1. Overall frequency of strategies reported by category.

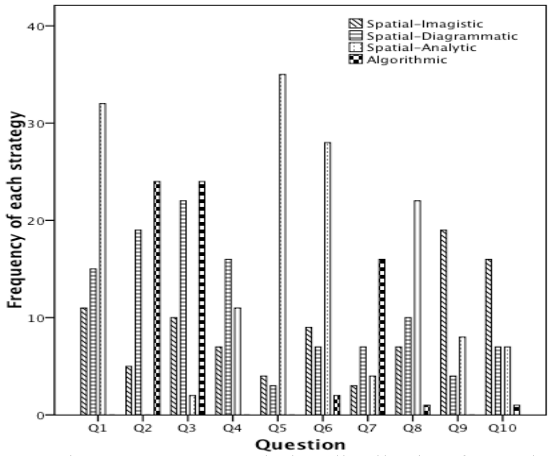

Figure 2. Strategy choice distribution for each task.

Table 2: Numbers and types of strategy used for each task.

No. reported strategies used	Strategy type	Frequency	Total
1	Spatial-Imagistic	49	240 (73.6%)
	Spatial-Diagrammatic	47	
	Spatial-Analytic	107	
	Algorithmic	37	
2	SI+SD	16	80 (24.5%)
	SI+SA	15	
	SD+SA	21	
	SD+AL	21	
3 or more	-	-	6 (1.8%)
Total			324

Note. Dashes indicate no further analysis was conducted. SI=spatial-imagistic, SD=spatial-diagrammatic, SA=spatial-analytic, AL=algorithmic. 64 tasks coded as recall/guessing/unknown are not included.

The relationship between correctness and type of strategy used was tested using a Pearson's χ^2 test for 2 (use of each strategy) x 2 (correctness) contingency table. Using an alpha level of 0.05, no association between success and strategy use was found, indicating that strategy choice does not have an impact on whether a participant answer a task correctly. Sex differences in problem-solving success and strategy choice were tested using an independent two-sample t-test. The mean total correctness score of male participants ($M = 4.25$, $SD = 1.45$) was not found to differ from the mean total correctness score of female participants ($M = 4.15$, $SD = 1.12$), $t(37) = 0.22$, $p = 0.41$. Likewise, strategy choice did not differ significantly between female and male participants, as illustrated in Figure 3. Men and women displayed similar patterns of strategy choice: in order of reported strategy use, both groups employed Spatial-Analytic, Spatial-Diagrammatic, Spatial-Imagistic and Algorithmic strategies. In order to examine the relationship between sex and strategy choice, strategy scores of participants were calculated by counting the numbers of each strategy used across the ten survey items. t-tests to compare Spatial-Imagistic, Spatial-Diagrammatic, Spatial-Analytic and Algorithmic strategy scores between male and female were not found to be statistically significant at an alpha level of 0.05.

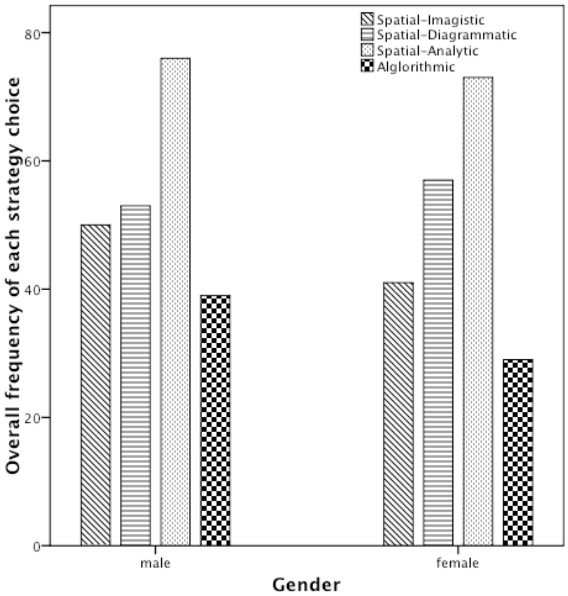

Figure 3. Overall frequency of strategy choice by males and females.

Study 2

In Study 2, we adapted the strategy choice questionnaire for group administration via a remote personal response system (i.e., "clickers") in an organic chemistry classroom during instruction. Although Study 1 established that students primarily made use of spatial-analytic strategies for solving organic chemistry tasks, the participants in that studied had completed several months of instruction in the domain. Thus, Study 1 yielded no information about how student strategy choice changes with instruction. Therefore, we conducted Study 2 to determine whether students employed spatial-analytic strategies in the context of an organic chemistry course and whether students employed the same strategies uniformly over the course of instruction.

Method

103 undergraduate students enrolled (sex was reported for 90 students: 33 males and 57 females) in a 6-week intensive organic chemistry course were assigned unique personal response devices to respond to adapted strategy choice questions administered during the course. Over the duration of the course, students were asked 10 unique organic chemistry questions and related strategy choices. Questions were administered approximately once each week of instruction. During the final meeting of the course, students were asked 8 organic chemistry questions and related strategy choices that included 6 of the 10 questions administered during earlier sessions of the course. All questions were presented on large LCD televisions at the front of the classroom and students answered questions by clicking a multiple-choice answer on their assigned device. The scoring rubric and strategy categories from Study 1 were used to analyze student responses. Notably, the adapted questions in Study 2 did not contain algorithmic strategies as the course instructor deemed that the strategy survey questions that included algorithms were beyond the scope of her course. In addition, unlike the strategy survey questionnaire, students were not able to choose more than one strategy per problem because the classroom clicker system could not capture multiple answers per student for a given question. Students were able to report their own strategies after each class if they employed a strategy not presented in the provided options.

Among the 103 students, 91 students volunteered to complete a spatial ability battery that included the Vandenberg Mental Rotation Test (Vandenberg & Kuse, 1978) and Guay's Visualization of Views (McDaniel & Guay, 1976). Descriptive statistics of strategy choice were generated for each task and strategy use on both administrations of the 6 questions was compared. Unlike Study 1, group administration of the questions permitted students to interact and discuss their responses prior to inputting an answer on their clicker devices and the course instructor assigned these questions for course credit; therefore, the independence of student answers to chemistry problems could not be guaranteed and reports of student achievement were not considered valid for analysis. In contrast, because students did not receive credit for strategy responses and the instructor emphasized that there was no correct answer to these questions, we considered student responses to these questions valid for analysis.

Results & Discussion

The distribution of strategy choices at each administration time point in the classroom is presented in Table 3. As indicated, the students reported that they employed Spatial-Imagistic strategies more than any other strategies both during and after instruction. Excluding recall, guess, and unreported strategies, Spatial-Imagistic strategies were most frequently reported by students (947 times, 64.95%), followed by Spatial-Diagrammatic strategies (397 times, 27.23%) and Spatial-Analytic strategies (114 times, 7.82%). Although Spatial-Imagistic strategies dominated both during and after organic chemistry instruction, comparison between the two occasions suggests that fewer Spatial-Imagistic strategies were employed after instruction while Spatial-Diagrammatic and Spatial-Analytic strategies were reported more frequently.

Reports of strategy use on the six questions appearing both during and after instruction were examined further to clarify changes in strategy use after instruction. As indicated in Table 4, after instruction the average number of Spatial-Imagistic strategies across all tasks reported decreased ($t(102) = -3.98$, $p < .001$), and the average number of Alternative strategies increased ($t(102) = 4.95$, $p < .001$). Figure 4 illustrates the frequency of reported strategies for each of the six questions at each presentation. Examination of these items indicates that students do indeed employ Spatial-Imagistic strategies less frequently after instruction. Interestingly, distributions of strategy choice after instruction varied across the six question items. For questions 1 and 6, reports of using Spatial-Analytic strategies rose dramatically, while reports of using Spatial-Diagrammatic strategies rose relatively higher on questions 2 and 4. In contrast, no noticeable difference in the relative use of each strategy type was seen on questions 3 and 5. Examination of these six items revealed that students not only adopted strategies alternative to Spatial-Imagistic Strategies after instruction, but the choice of strategy after instruction was related to the task itself.

Table 3: Frequency of strategy use.

	No. strategy choice		
Types of strategy	During instruction [a]	After instruction [a]	Total
Spatial-Imagistic	596 (73.22%)	351 (54.50%)	947 (64.95%)
Spatial-Diagrammatic	172 (21.13%)	225 (34.94%)	397 (27.23%)
Spatial-Analytic	46 (5.65%)	68 (10.56%)	114 (7.82%)
Total	814 (100 %)	644 (100%)	1458 (100%)

[a] 10 question items were administered during instruction and 8 items were administered after instruction.

Table 4: Mean number of Spatial-Imagistic and Alternative strategies reported during and after instruction.

	During the instruction		After the instruction	
Types of strategy	*M*	*SD*	*M*	*SD*
Spatial-Imagistic strategies	3.56	1.48	2.63	1.91**
Alternative strategies	0.99	1.09	1.83	1.79**

Note. Scores for each category range from 0-6 excluding recall and guessing strategies.
** $p < 0.001$

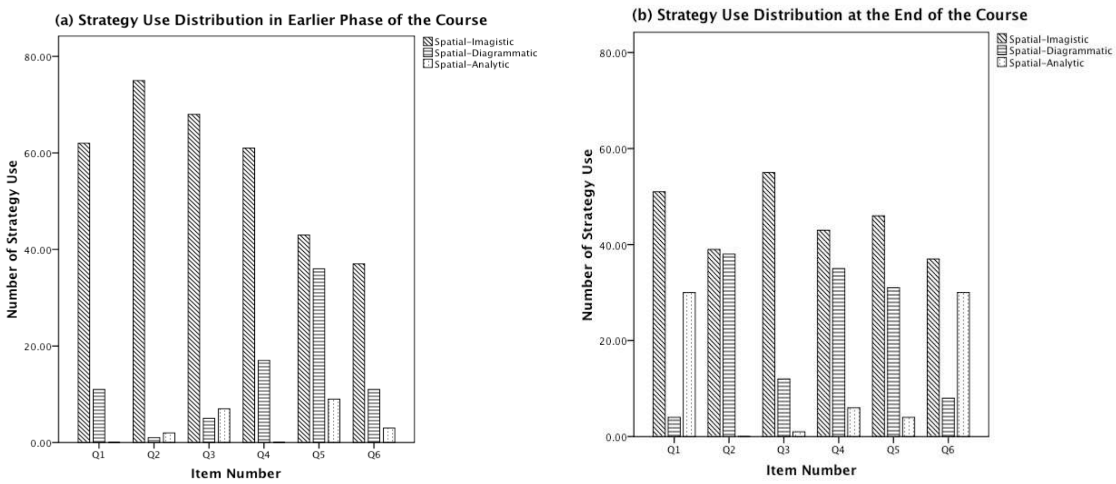

Figure 4. Frequency of strategy use reported by students (a) during and (b) after instruction.

Associations between spatial ability and strategy choices were analyzed via ANOVA. The 91 students who completed the spatial ability psychometrics were categorized into three groups based on their performance on the Mental Rotation Test (MRT) and Visualization of Views Test (VoV): High (N=31, M=51.94, SD=13.02 for MRT and M=17.74, SD=4.56 for VoV), Medium (N=30, M=34.20, SD=10.39 for MRT and M=8.71, SD=4.44 for VoV), and Low (N=30, M=15.40, SD=12.08 for MRT and M=4.21, SD=3.58 for VoV). Table 5 illustrates the results from the ANOVA. On the first presentation of each strategy question, the use of Alternative strategies did not vary with spatial ability (F(2, 88)=0.96, ns) at an alpha level of 0.05. After instruction, however, we observed a trend in the data that indicated students in the lower ability group employed Alternative strategies more frequently than higher spatial ability students (F(2, 88)=3.10, p=0.05). Associations between each student's strategy choice and spatial ability were analyzed via a Multivariate Analysis of Variance (MANOVA) test of Alternative strategy scores with within-subjects effect of administration time (i.e., during and after the instruction) and between-subjects effect of spatial ability group. The analysis failed to show a significant interaction between student strategy choice after instruction and spatial ability, *Wilk's λ*= 0.968, F (2, 88)=1.43, ns. Thus, spatial ability was not found to predict the use of any particular strategy after instruction.

Table 5: Comparison of Alternative scores during and after the instruction in three spatial ability groups

	High		Medium		Low		
Occasions of the task	*M*	*SD*	*M*	*SD*	*M*	*SD*	*F* (2,88)
During the instruction	0.90	1.07	1.30	1.08	1.06	1.20	0.96
After the instruction	1.35	1.56	2.43	1.94	2.17	1.80	3.10

Finally, relationships among sex, spatial ability and strategy choice were investigated. Using an alpha level of 0.05, males were found to outperform females on the Mental Rotation Test (M = 45.63, SD = 16.07 for male, M = 27.62, SD = 17.63 for female, t(83) < 0.001) and on the Visualization of Views (M = 13.42, SD = 8.27 for male, M = 8.66, SD = 5.94 for female, t(83) = 0.003). During instruction, males and females did not differ in use of Spatial-Imagistic strategies (M = 3.81, SD = 1.36 for male, M = 3.32, SD = 1.40 for female, ns) or the use of Alternative strategies (M = 1.00, SD = 0.87 for male, M = 1.07, SD = 1.10 for female, ns). After instruction, however, females were observed to use Alternative strategies more frequently than males (M = 1.24, SD = 1.56 for male, M = 2.47, SD = 1.79 for female, t(88) = 0.002); however, the difference between male and female use of Spatial-Imagistic strategies was marginal (M = 3.21, SD = 1.92 for male, M = 2.53, SD = 1.83 for female, t(88) = 0.096). Repeated measures analysis of Alternative strategy scores involving within-subjects effect of administration time (i.e., during and after instruction) and between-subjects effect of sex resulted in significant interaction between sex and occasion of the tasks (MANOVA, *Wilk's λ* = 0.892, F(1, 88) = 21.31, p = 0.002).

Conclusions & Implications

The above results offer some tentative answers to the questions we posed initially. First, the findings clearly illustrate that students employ a variety of strategies to solve tasks that involve spatial thinking. In Study 1, we observed students to rely more consistently on Spatial-Analytic and Spatial-Diagrammatic strategies as opposed to Spatial-Imagistic strategies, as typically believed. Likewise, in Study 2, we observed students to employ Spatial-Imagistic strategies preferentially during instruction, yet adopt more alternative strategies by the end of the course. Moreover, we also observed students to fluidly switch between different types of strategies between tasks. The findings of the present work suggest that students choose task-dependent strategies in a manner similar to expert chemists and apply multiple strategies on problems of increased difficulty. These results indicate that students are aware of the availability of diverse strategies and are willing to employ alternative strategies. In other words, students are not limited to reasoning about spatial information in molecular structures via imagistic reasoning, but can reason about spatial information with a variety of strategies.

The results also indicate that strategy choice does not predict success on spatial tasks in chemistry. The findings in Study 1 suggest that students reach equivalent levels of achievement regardless of whether they employ strategies that involve reasoning via mental imagery or alternative strategies. Equally important, we did not observe significant differences in achievement between men and women on chemistry tasks. Despite these findings, we did observe that multiple strategies were applied on tasks that the majority of students failed to solve. This finding is consistent with the literature on flexible strategy choice that reports individuals employ multiple strategies on tasks of increased difficulty (cf. Siegler, 1996). The use of multiple strategies, however, did not lead to increased success on such tasks. Thus, it did not appear that the application of one or more strategy types (e.g., Spatial-Imagistic, Spatial-Analytic, Spatial-Diagrammatic, Algorithmic) predicts achievement. That is, each strategy is equally likely to result in success or failure on a given task.

Study 2 permitted us to examine the relationship between strategy choice and instruction in the context of an organic chemistry classroom. The results of that study clearly illustrate that instruction has a direct effect on strategy choice. In the beginning of the course, we observed students rely primarily on Spatial-Imagistic strategies to solve spatial tasks; by the end of the course, we observed a sharp increase in the use of strategies alternative to Spatial-Imagistic strategies. Interestingly, the participants in Study 2 reported greater use of Spatial-Imagistic strategies at the end of instruction while the participants in Study 1 reported greater use of Spatial-Analytic strategies. We believe the reason for this discrepancy is due to two important differences between the participants in each study. First, the students received instruction over different time periods. The students in Study 1 completed ~20 weeks of instruction during course of an academic year; however, the students in Study 2 learned less material in a 6 week summer course. It is possible that the longer duration of study in Study 1 resulted in better apprehension of and preference for alternative strategies. Similarly, the instructors for each course reported notable differences in their own emphasis on strategy use. The instructor in Study 1 reported she was 'bad at visualization' and emphasized diagrammatic and algorithmic heuristics, but the instructor in Study 2 reported she attempted to teach as many strategies as possible for the benefit of the students. Thus, instructional differences may have resulted in the observed differences in strategy preference. Nevertheless, although students in Study 2 reported using Spatial-Imagistic strategies as their primary strategy, the increased use of domain-specific alternative strategies suggests that as expertise develops, students may rely less on imagistic reasoning and more on heuristics to solve spatial tasks.

Study 2 also permitted us to examine the relationship between spatial ability, sex and strategy choice in the classroom. Although the results of that study do not indicate a direct relationship between spatial ability, sex and strategy choice, they do suggest a potential interaction may exist. First, our findings clearly show that over the course of instruction women reported a significant increase in the use of alternative strategies compared to men. Second, our findings tentatively suggest that low spatial students may preferentially switch from Spatial-Imagistic strategies to alternative strategies after instruction; high spatial students do appear to rely on Spatial-Imagistic strategies throughout instruction. Thus, the data suggests that low-spatial females preferentially switch to alternative strategies. Two major limitations of the Study limit the validity of these findings. First, our analysis relies solely on students' strategy reports on 6 questions. The results of Study 1 indicate that several strategies are task-specific and our reliance on so few tasks casts doubt on the interpretation of these findings. Second, students were asked to respond to the clicker questions in Study 2 under classroom time constraints and they were also permitted to collaborate on their responses. Thus, there was an increased risk in Study 2 of failing to detect changes in strategy choice and individual differences in spatial ability. Nevertheless, we believe the trends in the data suggest a potential interaction between spatial ability, sex and strategy choice does exist and warrants further investigation.

Taken together, the results of the present studies indicate that spatial thinking in advanced scientific problem solving, specifically organic chemistry, involves a range of strategies that vary significantly in the extent to which they rely on imagistic reasoning. Of particulate note, our findings suggest that students approach the study of organic chemistry using mental rotation and other spatial-imagistic strategies to reason about molecular structures, but quickly adopt a variety of algorithms and heuristics after instruction. This behavior leads us to question the utility of instructional methods that emphasize the exclusive focus on training students to use imagistic strategies (e.g., by improving students' visuo-spatial ability, Ferk, Vrtacnik, Blejec, & Gril, 2003). Rather, we suggest instead that students may benefit most from instruction that teaches the applicability of multiple strategies, as in Study 2. Moreover, the present study did not identify significant correlations between sex and chemistry problem solving success. This result contradicts previous claims that men outperform women in science due to their aptitude for spatial reasoning (Fogg, 2005). Rather, our findings suggest that female students apply the same strategies as male students with equal levels of success in chemistry and that they are likely to switch to alternative strategies when necessary in a course.

References

Bodner, G. M., & Guay, R. B. (1997). The Purdue visualization of rotations test. *The Chemical Educator, 2*(4), 1-18.

Carter, C. S., LaRussa, M. A., & Bodner, G. M. (1987). A study of two measures of spatial ability as predictors of success in different levels of general chemistry. *Journal of Research in Science Teaching, 24*(7), 645-657.

Ferk, V., Vrtacnik, M., Blejec, A., & Gril, A. (2003). Students' understanding of molecular structure representations. *International Journal of Science Education, 25*(10), 1227-1245.

Fogg, P. (2005, January 28). Harvard's president wonders aloud about women in science and math. *The Chronicle of Higher Education*, p. A12.

Hegarty, M., & Sims, V. K. (1994). Individual differences in mental animation during mechanical reasoning. *Memory & Cognition, 22*(4), 411-430.

Keehner, M., Lippa, Y., Montello, D. R., Tendick, F., & Hegarty, M. (2006). Learning a spatial skill for surgery: How the contributions of abilities change with practice. *Applied Cognitive Psychology, 20*, 487-503.

Mathewson, J. H. (1999). Visual-spatial thinking: An aspect of science overlooked by educators. *Science Education, 83*, 33-54.

National Research Council (2006). *Learning to think spatially*. Washington, D.C.: The National Academy Press.

McDaniel, E. D., & Guay, R. B. (1976). *Spatial ability, mathematics achievement, and the sexes*. Paper presented at the Annual Meeting of American Educational Research Association, San Francisco, CA.

Pallrand, G. J., & Seeber, F. (1984). Spatial ability and achievement in introductory physics. *Journal of Research in Science Teaching, 21*(5), 501-516.

Schwartz, D. L., & Black, J. B. (1996). Shuttling between depictive models and abstract rules: Induction and fallback. *Cognitive Science, 20*, 457-497.

Shea, D. L., Lubinski, D., & Benbow, C. P. (2001). Importance of assessing spatial ability in intellectually talented young adolescents: A 20-year longitudinal study. *Journal of Educational Psychology, 93*(3), 604-614.

Siegler, R. S. (1996). *Emerging minds*. New York: Oxford University Press.

Stieff, M. (2007). Mental rotation and diagrammatic reasoning in science. *Learning and Instruction, 17*, 219-234.

Stieff, M. & Raje, S. (2010). Expertise and imagistic reasoning in chemistry. *Spatial Cognition & Computation*, *10*(1), 53-81.

Trafton, J. G., Trickett, S. B., & Mintz, F. E. (2005). Connecting internal and external representations: Spatial transformations of scientific visualizations. *Foundations of Science, 10*, 89-106.

Vandenberg, S. G., & Kuse, A. R. (1978). Mental rotation: Group test of three-dimensional spatial visualization. *Perceptual and Motor Skills*, 47, 599-604.

Wu, H.-K., Krajcik, J. S., & Soloway, E. (2001). Promoting conceptual understanding of chemical representations: Students' use of a visualization tool in the classroom. *Journal of Research in Science Teaching, 38*(7), 821-842.

Wu, H.-K., & Shah, P. (2004). Exploring visuospatial thinking in chemistry learning. *Science Education, 88*(3), 465-492.

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