

Understanding Teachers' Conceptual and Pedagogical knowledge of Probability

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Recent reform movement in mathematics education has witnessed an increasing emphasis on “Teaching for Understanding” (NCTM, 2000). Teaching for understanding involves teaching practices that are complex and demanding, especially when the subject of teaching is a set of advanced and sophisticated mathematical ideas. It requires that teachers not only have coherent understanding of these ideas, but also know how one might get to it, i.e., knowing ways of supporting students to reach these understanding (Thompson, 2002). However, research on statistics education has attended to neither teachers’ understanding of probability, nor their thinking on how to teach (Garfield & Ben-Zvi, 2003). Against this background, we undertook a teaching experiment with 8 high school statistics teachers in the format of a professional development workshop. The goal of this study was to develop a conceptual framework for understanding teachers’ conceptual and pedagogical understanding of probability.

We designed the workshop with the purpose of provoking the teachers to reflect and rethink their instructional goals, objectives, and practices in teaching probability. We conducted the study using a combination of design experiment (Gravemeijer, 1994) and multi-tiered constructivist teaching experiment methodologies (Lesh & Kelly, 2000). A multi-tiered teaching experiment highlights the aspects of investigating teachers’ conceptual, psychological, and pedagogical understanding using what one knows about students’ learning experiences. Video data and students’ work from two high school teaching experiments (Saldanha & Thompson, 2002) were employed as points of discussion with the teachers to support our attempts to reveal their understanding of ideas, objectives, and practices in teaching probability and statistics. The teaching experiment had 8 day-long workshop sessions. All sessions were recorded with two video cameras. Each teacher was interviewed three times. Data included video recordings and copies of teachers’ work, workshop notes, and observer field notes.

Our analysis indicated that teachers' understandings of probability covered a broad spectrum: 1) thinking that probability is about predicting the state of a specific completed (or to-be-completed) event about which one does not know the actual result, 2) thinking that probability is about selecting one outcome from a set of possible outcomes, and 3) thinking that probability is about imagining a collective of results generated by a single random process. Only two teachers began the workshop with a stochastic understanding of probability. Three teachers masked a non-stochastic understanding of probability with their sophisticated knowledge of rules for computing probabilities. Even when some teachers did conceive a situation stochastically, they did not have in mind that an event's probability is determined by the underlying stochastic processes one imagines consciously or presumes tacitly. Teachers believed that a probabilistic situation should not be subject to multiple interpretations. When it is, they believed simulation could decide which interpretation is correct, without realizing that a simulation is designed according to one's interpretation of the situation. With respect to teachers’ pedagogical understanding, we found several instances in which teachers’ lack of conceptual understanding impeded their pedagogical considerations. For example, all teachers were unaware that even when they designed instruction to “teach for understanding”, they were actually teaching prescribed ways of doing mathematics that was insensitive to alternative ways of thinking.

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