Peer scaffold in math problem solving

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Abstract: One of the most important issues that are dealt with in CSCL environments is self, and collaborative, regulated learning; independent of the support of teachers. In the first part of this paper, I will bring forward an innovative pedagogical approach for collaborative learning of math problem solving, accompanied by appropriate software (1) *Metafora*'s planning tool: a visual based planning and reflecting space for socio-meta-cognitive elicitation of collaborative learning processes, and, (2) *Geogebra*: a math application for the creation of dynamic Geometric figures in Cartesian domain. In the second part of this paper I will illustrate a learning scenario within the context of a collaborative math problem solving scenario. Then, I will highlight a behavior of collaborative learning, in which one team member, S1, makes progress with solving the problem, and goes back to help his peer, S2. S1 scaffolds his peer's work by (1) Reporting what he, S1, did on the shared planning-reflecting space (2) Monitoring his peer's error (3) Explicating this error to his peer (4) scaffolding his peer's construction of a Geogebra model, without giving him the whole answer. This observation serves as an important progress in the attempts of modern educators, and education design-researchers, to share some of the responsibility of the learning processes with students.

Supporting collaborative *Planning* & *Reflecting* in Math Problem Solving

Collaboration is considered as a central means for individual progress in modern society (Perret-Clermont, 2011, Wheelan, 1999). This serve as a good reason to progress *collaborative learning,* claiming that this mixture of individual and interactive activities can trigger learning mechanisms (Dillenbourg, 1999). In his attempt to illustrate a theoretical framework for such learning, Wegerif, (2006; 2011) explains that successful collaborative problem solving depends on the extent to which the solvers talk together and open up a reflective *Shared-Space*, which allows the emergence of ideas. When opening such a shared space, one's monitoring of his own cognition affects his peers' monitoring of their cognitions (Efklides, 2006).

But what would happen if the collaborative monitoring and regulation of learning is an *explicit* process that takes place as part of the collaborative solution? In the case of math problem solving this is a rather critical question. Heavy research strand (e.g.; Veenman and Spaans, 2005) shows the importance of metacognitive behaviors in the process of math problem solving, bringing forward the importance of being aware of the learning processes *before* (Weinberger, 2011; Rummel and Spada, 2005), *while* (Abdu and Schwarz, 2012; Schoenfeld, 1985) and/or *after* (Hamilton, Lesh, Lester & Yoon, 2007) performing it. My research team and I hypothesize that a tool that will afford the creation of a dialogic shared space between students can support emergence peer monitoring and regulation of collaborative learning process.

Metafora

The Metafora system is a software platform that encompasses a suite of tools used to support and encourage the development of "Learning to learn together" (L2L2) skills, through domain-specific activities in science and math. It is currently funded by the European Union. The idea is to have students collaboratively work in periods of 2-3 weeks, in order to give solution to a given challenge. During this solution process the students will plan, solve and reflect upon the learning and solving process in order L2L2. Their teacher and the software will scaffold their learning process. Students mutually engage in achieving the solution to a challenge through developing communication, strategic thinking and problem solving skills.

The System

The main tools that were developed and integrated into Metafora, for this end, include a virtual space for mapping of argumentative discussions (LASAD), a set of microworlds for simulating phenomena in science and math, and a planning tool. These tools are all interconnected, and monitored by an Analysis Component – an artificial intelligence component which is planned to take some moderating-load from the teacher. I will now elaborate about a couple of tools that were used in the current study: The planning/reflecting tool and Geogebra.

The *planning tool*: a shared space with which groups of students collaboratively, and autonomously (thus, in different computers), construct plans and reflections upon their work. This is being done by a creation of a constantly revised map; with the use of a set of icons we call "Visual Language Cards": a closed set of graphical ontology (See figure 1). This ontology is based on models of inquiry-based learning (e.g. Tamir, 2006), and of problem-solving (e.g. Polya, 1945). The ontology organizes the collaborative problem solving: Finding hypotheses, simulation, discussion, etc. The visual language also represents scientific/mathematical moves: understanding the problem, reflect, simulate, etc.

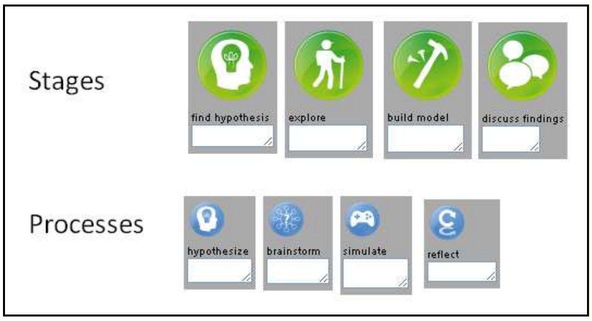

Figure 1. Examples of two visual language elements: Stages of the problem solving and processes undertaken during these stages.

The second tool that was used in this study is *Geogebra:* A math application that affords the creation of dynamic Geometric figures in a cartesian domain. Geogebra serves as a rich ground for learning math (e.g. Gergelitsove and Holan, 2012; Stahl, 2009), and we therefore decided to facilitate it within the overall Metafora system.

The Pedagogical Approach

The main idea of our approach is that bringing students to perform explicit discourse upon the meaning of actions/cards, within the context of the solution process of a specific challenge, elicits the learning of communication, metacognitive and problem solving skills (Rogoff, 1990). We bring students to engage with computer supported collaborative problem solving in mathematics, while explicating their learning processes with the help of the planning tool.

At first we had a rather rigid idea of a tool that affords students with planning their work ahead, and to some extent commit to that plan. After some observations upon students' work with the planning tool we realized two things: First, students were often reluctant to plan in advance, before they make sense of the problem, or, "Understand the problem". Second, we found out (Abdu and Schwarz, 2012) that teachers that use the planning tool, first needed to make sense of the problem and then they were able to reflect upon their solution to that point. When they reached the "current" point they were able to plan ahead their collaborative solution. These two complementary findings led us to the understanding that asking students to plan ahead their collaborative solution should be done mainly after they made sense of the challenge. Therefore, the planning tool becomes also a planning tool, in which the solvers create a model of their own learning process (Hamilton, Lester, Lesh, & Yoon, 2006).

Other observations made by the Metafora pedagogical team led us to identify four key skills that are necessary for any process in which students are learning together, and on a higher level, L2L2. These skills are: *Distributed leadership*, *Mutual engagement, Peer assessment*, and *Group reflection on the learning process.* In the conclusions I will show the emergence of these skills. In particular, I will show a behavior that was identified as *Peer Scaffold* by our research team, as a result of the existence of these skills.

The *City* challenge

The class, the course

Sixteen 8th grade male math-competent students from a religious school, all from mid-high class families in Jerusalem, participated in this study**.** The students met once a week in a computer class and participated in 8 month course of computer-supported collaborative math problem solving. The teacher in charge was an experienced math teacher, and teachers' tutor, that gave the course as a part of her master's thesis program, at the Hebrew University of Jerusalem.

We created a course with fourteen units that appear as a succession of activities. We adopted the approach of a design research (Cobb, 2001; Collins, Joseph & Bielaczyc, 2004) in which the learning environment is assessed and refined throughout a course.

The course had three phases. **Collaborative learning establishment:** Two double lessons in which the students took part in paper and pencil collaborative problem solving, involving teacher's orchestration. The purpose of this stage was to practice the students with group work and attracting them into the course. **Learning heuristics, strategies, and how to use the computerized tools**: Eight double lessons, in which groups of students solved 1-3 problems in every lesson. Each problem focused on a specific heuristic or strategy that was learned in the context of one or more problems. Students also learned to use different computerized tools: The Planning tool, Geogebra, and other micro-worlds. The teacher focused on the following heuristics and strategies: Planning, Reflecting, "Thinking outside the box", abduction (backward strategies), introducing proper notations, Creating a model, Allocating tasks, Generalizing, Checking a simpler case, Hypothesizing, Checking hypothesis, Trial and error and looking for patterns. **Solving challenges***:* Fifteen weeks in which, groups of students were given challenges with longer time frames, in order to collaboratively solve relatively complex problems.

To foster the acculturation to problem solving and collaboration we adapted well known activities to problem solving challenges that have multiple solutions and/or multiple paths to a solution. Part of the content of the problems is directly related to school curriculum. Some other challenges were open-ended, thus affording the elaboration and the application of strategies to solve the challenge (Wee & Looi, 2009). One of them will be brought here: the "City" challenge.

The course was subdivided to fourteen Learning units with different durations: from 45 (one lesson) to 180 minutes (3 double lessons). However, the scenario of a challenge was quite stable: At the beginning the teacher presented a challenge to the class. The students then initiated their work through explorations. In some key milestones of their solution process we asked them to use the planning tool. And at least the couple that we filmed complied and even seldom used it when not asked. Three kinds of reflections were implemented: (1) *within group reflection* that was done throughout the process, when needed (2) a whole class reflection, with the teacher as a leader that gave an overview of different solutions and solution paths. (3) a reflection that was done by a group of students, upon their collaborative problem solving and solution processes, in front of the whole class.

Design Principles of the challenge

The *City challenge*, posed and supported by the teacher, is a relatively complex problem in math that was designed to be solved over three double lessons. In this challenge, students need to find a point that is equidistant from 7 general points in a 2D space. Math fan readers are invited to take a break and come up with the answer for this geometrical place. To the ones that are not big fans of math, I will tell this- there isn't such a point except for one particular case: When the 7 points are located on one circle. However, the ideas that can develop from such inquiry are vast, if done properly. In order to solve such a challenge, students need to find a simpler case of the problem (See table 1) and gradually find the solution to more complex cases. Through such a solution of a challenge, the students need to apply concepts such as "median", "medians' intersecting point" and "a perpendicular bisector". Most of these concepts were a part of the "ordinary" curriculum for these students and some –such as the perpendicular bisector- were rather new. Geogebra serves here as a facilitator for the construction of these shapes and by that- supports the meaning making of these concepts (Stahl, 2009).

A challenge that spans over three double lessons and solved by groups of competent students needs to be hard enough for them, so they will not be able to answer it immediately, but it should be within grasp and attractive enough from their point of view. Thus, they will not give up easily. We carefully planned our scaffold for the solution, while letting the students the freedom to explore directions to the solution and construct their own Geogebra models and planning-reflecting maps. One of the ways to achieve these goals is to plant a *Cognitive conflict*. At the end of step 2 (See table 1), students reach a conclusion about the equilateral triangle: The equidistant point is in the meeting point of the angles bisectors'/medians meeting point/heights. Obviously, in the case of an equilateral triangle it does not matter, since the three loci are at the same point. In preliminary observations it seems that students come to this conclusion rather fast. However, when they tried this solution in the case of a general triangle (step three, see table 1) they found out that this does not work.

Solving the City challenge

In the city challenge students are asked to place an energy system in an imaginary city, in the center of seven other institutions. Since the challenge was too complicated to be solved in one step, the teacher guided her students to solve it in four steps as can be seen in table 1 below.

Table 1: the four steps of the City challenge:

Step One: Place the energy center in the center of seven institutions, all of which are important to the city and all are packed with people. What is the conclusion of the conference committee? **Step two:** Simpler case- 3 institutions located in the structure of an equilateral triangle. Where to place the point? **Step Three:** Three institutions located in the structure of any triangle. Where to place the point? Try to formulate a final conclusion. Is there an equally distant point to the three vertices of a triangle? If so where it is located? **Step four:** Give a general answer: Where should we put the energy center?

We follow two students S1 and S2 while they solve step three - locating the energy center in the middle of a general triangle. This is the second lesson out of three that were dedicated for this challenge. The two came to a solid conclusion in the previous lesson, that the point that is equidistant from the three vertices of an equilateral triangle is the medians' intersection. When they moved to step three, they tried a couple of conjectures. First, they checked the medians' intersection and found out that it is not the desired point. Second, S1 found the solution for the case of a right angled triangle: The equidistant point is the midpoint of its hypotenuse (Thus segments x, y and z are equal in figure 2). However, this solution was only constructed by S1, while S2 is only watching him.

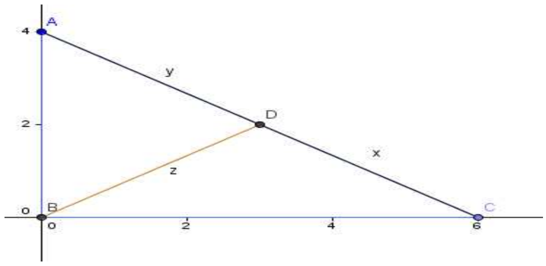

Figure 2: The point that is equidistant from the three vertices of a right angled triangle is the midpoint of the hypotenuse (point D)

In the beginning of the second lesson, the students were asked to reflect upon their work, and plan their solution to the challenge with the planning tool, before they continue to solve the challenge.

Peer scaffold: construction of a Geogebra model for the case of right angled triangle

Between various instances in which we observed peer-scaffold behaviors, the most rigorous episode is the current, in which we show that although S1 and S2 established that their next step will be- checking the intersection of medians; when we looked at their next actions we observed that while S1 regarded it as a reflective move, S2 constructed an intersection of the medians, with Geogebra. This is despite the fact that both constructed this model in the previous lesson, and found out that this point is not equidistant to all vertices. Later, S1 progresses to reflecting upon his solution for the right-angled triangle, and only when he finishes, he scaffolds S2's construction of building such a solution with Geogebra.

The Episode

It starts as S2 inspects the case of the medians' intersection, with the help of Geogebra; while S1 avoided S2's work and reports on the success he had in the previous lesson: Discovering that the equidistant point in the case of a right angled triangle is the midpoint of its hypotenuse. We join them as S2 measures the lengths from the vertices to the medians meeting points in his Geogebra model and S1 looks at the planning tool.

1. S1: *"Let's reflect again on the process…OK, S2, now I am going to write the story of a lifetime".* S1 writes in the "reflect on process" card: "With pure genius of his honorable S1 …"
2. S2 [Refers to his measurements]: *"And as we thought: a mistake"*
3. S1 [continues to write in the card]: "we decided that we will (We=I will) solve for a right angled triangle. I tried to check the median to the hypotenuse, and I made it"
4. S2: *"S1, there are medians, bisectors what else are there? Amm…perpendicular… how do you create a perpendicular? …Amm, I don't think it's going to work…"*
5. S1 [Keeps writing]"… (After whole 80 minutes!)":

After he is done creating a median's intersection with Geogebra, S2 is ready to work on the case of a right angled triangle that was reported by his friend in the planning-reflecting map. S1 is in a different place: Since he constructed the triangle on the first lesson, he just reported in Planning-reflecting tol that "we decided that we will (We=I will) solve for a right angled triangle. I tried to check the median to the hypotenuse, and I made it". S1 knows the solution for a right angled triangle, and he made it clear so everybody (The teacher, the video camera) will know. Now, S1 is willing to scaffold S2's process of building a right angled triangle, with GeoGebra. First, S2 makes an attempt to create a right angled triangle, but the right angle is not accurate, since S2 did not define it properly (See figure 3).

6. S2: "*So, in the case of right angled triangle, what is the answer, remind me?"*
7. S1: "*You insult me, didn't you hear my answer?* [Points to the map] *You can read it here. "*
8. S2:" *I don't feel like reading*
9. S1:" *Read! You do it all the time, you can do it now…"*
10. S2: "*Ah, it is the median, no?"*
11. S1 [looks at the camera]: "*Shush! Don't tell everyone"*
12. S2: *"Which one?"*
13. S1: *"Of the hypotenuse"*
14. S2: *Of the hypotenuse…*

Now S2 creates a hypotenuse to the right angled triangle he constructed while S1 looks at his work (Figure 3)

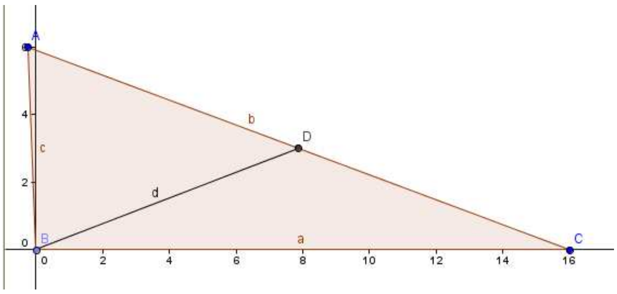

Figure 3: S2's building of a median to the "hypotenuse" of his erroneous right triangle model

15. S2: *"Does it work?"*
16. S1 [Leans towards S2's computer]: *"You see?! But the problem is…how did you build this? S2, I am sorry that I need to break the news, but this is not a right angled triangle."*
17. S2 [points with the mouse cursor on the shape]: *"Look"*
18. S1 [Points with his finger to the values of the segments, in the variables section of GeoGebra]: "*Look here… It is not right angled……"* [Goes back to his computer and grabs the mouse]
19. S2: *"Why…so how am I creating a right angled [triangle]?"*
20. S1: *"You build a right angled triangle."*
21. S2: *"And how can I know it is a right angled triangle?"*
22. S1: "*You build it on a tangent and then build [segments] to sectors"*
23. S2: "*But this is what I am trying to say"*
24. S1: "*And build another point"*
25. S2: "*But it looks like a right angled"*

26. S1: *"...it looks like it, but it is not...Do you want me to show you how to do it? Happily...move!"*[Grabs the computer mouse] *"This is a much easier method."*
27. S2: "*Erase everything"*
28. S1 erases the Geogebra screen.
29. S1: " *OK, now we take random points, pay attention* [puts a point in (0,0)]"
30. S2: "*No, don't put it there…"*
31. S1: *"Zero, Zero"*
32. S2: "*mmm…"*
33. S1 [Puts a point in (0,4), puts a point in (6,0)]: *"Zero four"*
34. S2: "*Is this the only way?"*
35. S1 [Gives S2 the computer mouse]:*"No, but it worked, because I simply built it on the vertices of the…X and Y"*
36. S2 Receives the computer mouse from S1 and creates segments AB, BC and AC.
37. S1: *"Now you got it!"*
38. S2 [Builds a median to section BC]: "*OK, now...you looked at the line to the hypotenuse ...And now lines"* [Looks at the values of the segments: x=y=z in figure 2]
39. S1: *"See?!"*
40. S2: *"Now what?"*
41. S1 [Points to the screen]: *Now you can check and see that AD BD and DC are equal.*
42. S2: *OK*

Now S2 sees that the midpoint of the hypotenuse in a right angled triangle is equidistant from the three vertices. Later, he will also explain this point to his teacher.

Discussion

The pedagogical setting, instructions and the affordance of the software led S1 and S2 to an interesting situation. In the beginning of the episode, the two worked in parallel- while S2 revisited the work that was done, S1 reports about this work in the planning tool, this presumably gives heads up to S2. In addition, the rather immodest choice of words "His honorable S1" (Lines 1, 3 and 5) implies that S1 wanted to report on the stages taken to that point, in order to talk about his achievement. But he had S2 as his peer, and S2 wanted to make sure he understands what happened in the last session, before they move forward. A possible explanation for that is that they wanted to be aligned with each other since the challenge was given to groups, rather than individuals. S2 asks for help from his peer. This ignites a process in which S1 scaffolds S2's progress.

Although I mentioned the term "scaffold", earlier in this paper, I was yet to define it. I will now do so, and the definition will stay fresh in the reader's mind in the next paragraph. Scaffolding is a kind of mediation in teaching, first termed by J.S. Bruner (Puntambekar & Hubscher, 2005). The main idea is that through *Scaffolding* a caregiver gradually transfers the responsibility of learning through a learner's ZPD. He does not give him the answer to the problem nor lets him figure it out all by himself. We identify four main behaviors that are associated with scaffolding: 1. Modeling- the caregiver shows the child how he performs an assignment. 2. Ongoing diagnosis- The caregiver monitors and regulates the child's actions. 3. Calibrated support- Achieved by recruiting the child's interest, reducing the degrees of freedom by simplifying the task, maintaining direction, highlighting the critical task features, controlling frustration, and demonstrating ideal solution paths, and, 4. Fading out- gradual transfer of responsibilities to the learning- from the caregiver to the child (Puntambekar & Hubscher, 2005).

Modeling the solution S1 gave to the case of an equilateral triangle was done by him in the previous lesson, and now he refers to S2 with the planning and reflecting tool as their tool for communication (line 9). It is safe to assume that S1 does not want to give S2 the solution "right away", since he wants S2 to read the "conclusions" that he wrote into the "reflect on process" card (lines 7 and 9). When S2 does not want to use this line of communication and takes a guess (line 10), S1 completes the insight, orally, (line 13). This, leads S1 to choose a different path, as he starts **monitoring** S2's work (Line 16) and commenting on it: "*…how did you build this?"* providing him with feedback "*S2, I am sorry that I need to break the news, but this is not a right angled triangle."* Moreover, S1 refers S2 to evidences they could both see - the values of the segments, in the variables section of GeoGebra- (line 18) "*Look here… It is not right angled…"* When S2 asks him*" ... how am I* *creating a right angled [triangle]?*" S1 applies **calibrated support** as he reduces the degrees of freedom with the creation of a scaffold with Geogebra, while verbalizing his actions (lines 29 to 33). He puts three vertices of a right angled triangle are in points (0, 0), (6,0) and (4,0), but does not complete the model. Now the three vertices lie on the axes X and Y (see figure 2) and S2 can build the three segments (line 36). Then, based on a prompt by S1, S2 accurately checks if the point that is equidistant from the three vertices of a right angled triangle is the midpoint of the hypotenuse. S1 encourages his friend when he sees that S2 created a right angled triangle based on his scaffold, by saying "now you got it" when S2 creates the right angled triangle (line 37).

So, what are the characteristics of this particular interaction between S1 and S2 that allowed such an interaction to take place? I claim that lots of it has to do with the emergence of L2L2 behaviors, that progress this collaborative learning scenario: (1) We observed a *leadership move,* when S1 monitors S2's faulty construction of right angled triangle (line 16) and goes to sit next to his computer (Line 18), and then S2 asks for his help (line 19): *"...so how am I creating a right angled [triangle]?"* (2) The two are *Mutually engaged* although they are still quite far from the solution*,* (3) We can see *peer assessment* when, for example, S1 monitors S2's faulty construction. However, in this particular episode we observed that (4) *Group reflection* was shown only from S1's side, when he implicitly complains about his partner, as he writes in the "reflect on process" card: "we decided that we will *(We=I will)* solve for a right angled triangle, (line 3).

Conclusions

In this study I bring an example for collaborative learning between two students, over a challenging math problem. The learning environment and tools afford their collaborative problem solving, dynamic simulations of their mental models and discuss their collaborative work on a shared discussion space. This is being done with the aid of L2L2 skills. Through peer assessment that was done by one student, after his reflection upon his own solution, an error was discovered. This led him to scaffold his peer's creation of a model of a right angled triangle with a median to it hypotenuse, in Geogebra. He scaffolds his peer's work by (1) Reporting what he did in the previous lesson (2) Monitoring his peer's error (3) Explicating this error to his peer (4) scaffolding his peer's construction of his Geogebra model. In this spirit, as S1 supports S2's work, he even gives S2 some moral support by stating his success. However, this is also a socio-metacognitive move that is being done by S1.

The idea of having peers learning together and supporting each other's learning is very appealing for educators: Students that do not give to each other only the bottom line, but guide their peers throughout the learning. We can see how such environment gives some responsibilities of the teacher in the hands of one of the students. For this end, I offer two operational outcomes that can be derived from this instance. The first stems from the assumption that the act of reflection prompted the two towards a process in which they needed to level. This leads me to look up for more peer-regulating learning phenomena that emerge as a result of the use of the planning tool. The second comes as support to the importance of developing appropriate affordances for reflective moves, upon learning scenarios (Suthers, 2003).

There are several limitations, though, to this paper. It presents a study in progress, but it is far from sufficient. The described scenario is an example taken from over 25 hours of learning and instruction in an environment that elicits many interesting learning scenarios, such as peer scaffold. But as this learning episode stands on its own, questions of validity and reliability should come up. The reliability question was addressed in two meetings that were taken in which this episode was presented to my co-researchers in Metafora project, which maintained the same opinion as mine. Addressing the validity question is more complex, and I will have to identify more instances that will help me to define this kind of phenomena in a wide perspective, and eventually help in implementing it as a part of my coding system.

Endnotes

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