Math Engaged Problem Solving in Families

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Abstract: Research indicates that people engage in rich mathematical practices in everyday activities, yet little is known about school-aged children's mathematics learning within the family context. This paper reports results of an interview study with 20 families to understand contexts and activities that engage mathematics in the family setting. The results indicate that problem solving is frequent activity, and that mathematics is engaged in accomplishing problem solutions in a range of contexts or situations. We describe features of math engaged problem solving and describe how it is value driven. We see multiple kinds of math and multiple people drawn into problem solving, and we identify socially distributed mathematical practices. These findings implicate the family as an under-recognized, yet rich source of math teaching and learning.

Introduction

We investigate the diverse contexts and activities in which middle school age learners and their families engage in mathematical practices and problem solving. We describe several features of problem solving for the families we studied, characterizing the structure of their mathematical activities, and analyzing the social conditions and arrangements for their family-based mathematical practices. Several important features of problem solving around the home emerge: (1) the family is a setting giving rise to problems to be solved, which also generate mathematical activity; (2) values steer problem definition, identification, solutions and their judged adequacy; (3) different types of math and math at different levels come into play concurrently; and (4) it is critical to view mathematical thinking and learning as embedded in activity and in a social, historical, and distributed knowledge environment.

We first give some background on our approach and describe our research methods. Then we discuss each aspect of familial problem solving and present one example for each that is characteristic of the feature. We follow the descriptions with a summary and discussion of the significance of this characterization of problem solving with mathematics.

Background and Perspectives

Understanding mathematical problem solving in the complexity of family life activities is supported by certain orientations to theoretical and analytical approaches. Work on problem solving has had a long history, with significant roots in Gestalt psychological studies by Wertheimer (1945), Duncker (1945) and others, which had direct influences on the cognitive revolution in studying thinking (Newell, 1981; Simon, 1999). The mathematician George Polya in *How to Solve It* was another major contributor to conceptualizing problem solving processes in mathematics (1957). None of these approaches, however, sought to understand the situated nature of mathematical practices in relation to everyday life activities. In the 1970's, spurred by everyday cognition theoretical formulations from Neisser (1976), Bronfenbrenner (1979), and Cole (1971), an ecological orientation to looking at mathematics use *in situ* was developed. It was advanced throughout the 1980's by Lave (1988), Saxe (1988), Nunes et al. (1989), Scribner (1984), and others more recently (Goldman & Booker, 2009; Hoyles et al. 2001; Nasir, 2000; Satwicz & Stevens, 2008; Stevens et al., 2006). We know from such studies that people engage mathematically in the course of everyday activities from dieting to shopping to sports to the work of nursing and dairy order filling.

We take a view of problem solving that recognizes its complexity and finds problems being generated in many life situations. Problems and problem solving are part of the social fabric in peoples' lives, and they are distributed in time, place and across persons. They involve cognitive, social, cultural and physical means in their definition, generation and solutions, and a direct pursuit of both outside constraints and peoples' desires. We are especially interested in the problem-solving that people do during their family-based activities, how it both requires and generates mathematical practices, and how these mathematical problem-solving practices instantiate learning.

We see mathematics practice as ubiquitous in people's family-based activities. We also take a comprehensive view of learning that is based on activity in practice (Lave & Wenger 1991), and consider the family as a little examined yet significant cultural environment for activities and social relations that constitute a sizeable proportion of human experience and involve poorly understood forms of learning (Bronfenbrenner, 1979; Leichter, 1975). In relation to math, Ginsburg and his colleagues document how and what mathematics young children are "taught" at home (Ginsberg et al., 1999). Jackson and Remillard (2005) and Rogoff, Ellis, and Gardner (1984) show that the explicit teaching softens and changes once children enter school. The literature leaves us with a very obscured view of the significant role that families might play in mathematics teaching and learning, yet with the hopes that math engagements might be ubiquitous in familial circumstances and activity. Our work enters the family space to better understand the significance of the family as a setting for mathematics engagement, learning and the development of quantitative literacies (Steen, 2004).

Methods

The Family Math research project investigates the ways families with middle school aged children characterize their engagement in mathematical problem solving during the course of individual and family activities and events.[1] Our goal is to identify the social, cultural, and material contexts that are relevant to and create opportunities for mathematics learning and practice as well as to characterize similarities and differences in family math situations. We seek to identify the resources family members use for solving problems together, characterize the structure of these activities, describe the resources used in solving math-related problems, and analyze the social conditions and arrangements for family-based mathematics practices. Eventually, these understandings of mathematics of family life will help the field identify and design for points of synergy for strengthening the role of the family in mathematical teaching and learning and to build bridges between math in the home and math and school.

In the results reported here we present data based on two-hour interviews with 20 families providing narrative accounts of math in their daily lives. Narratives provide us with participants' accounts in their own words and allow for us to have records of how people relate stories about their math related activities, including family members self-identifying as African-American (N=5), Latino (N=23), Asian (N=7), Pacific Islander (N=8), Multi Racial (N=6) and white (N=25). The families also represent a spectrum of economic diversity, ranging from low income (children can receive free lunch) to upper-middle income families. The educational levels of the heads of families was also diverse, ranging from high school education through graduate school.

Data collection

We developed a semi-structured interview to have people discuss the activities they engage in as part of family life, work and school, allowing them to give us particularized versions of how they think about and accomplish their life tasks. The interview gave family members ample space for describing their experiences, while also collecting certain information from all families such as education levels, perceived comfort with mathematics, school attainment, number and description of math classes that parents (and students) have taken, and their attitudes towards mathematics in general and towards school math. The protocol included two segments to provide parity of data across all families. Each family member told a "Math in a Minute" story about an experience of any kind, positive or negative, alone or with others, that they had with mathematics. Each family also was asked to choose a cell phone plan by using brochures from four providers we provided that would meet the needs of the family. This task put the family into action and provided observable, interactive accounts of the family in a problem-solving situation. All interviews were videotaped and took approximately two hours.

Data Analyses

The analyses of the interviews proceeded through several stages and treatments. Three were particularly critical to the analyses of data for this paper: content logging, identification of talk and interactions involving problem solving, and development of a Math Inventory. These three data examinations eventually became the basis of a database on the intersection of problem solving and mathematics. All videotapes were first content logged to delineate events, turns in conversation topics, stories, events, and sub-topics of interest inside participants' accounts. Analyses of the segments resulted in 14 general categories, such as formalisms, gestures, props, efficiency, (math) avoidance, emotions, division of labor, and values. This paper emphasizes the categories of "problem-solving" and "type of mathematics" so we could better understand the occasions where family members depicted a situation *as creating a problem for them* and for which they engaged mathematics. These categorization processes resulted in a database comprising 315 stories—events that participants described where they were solving a problem using one or more types of mathematical practices.

We also developed a list of 20 categories for "types of mathematics used," establishing definitions with exemplars from our dataset for each. To consider overall patterns in our dataset of problems involving math, we examined the frequency with which the various types of math occurred within the various family activities.

Findings: Four Features of Math Engaged Problem Solving in the Family

Four aspects of math engaged problem solving are described through vignettes based on our families' accounts of their math engagements. We tell their stories and point out the ways in which they are representative and characteristic of patterns we saw in the larger data set.

I. Everyday problems lead the math

The most overarching feature of family problem solving is that problems come first, with math following. The complexity of family life and many interactions among family members and other institutions generate many contexts that require problem solving. The problems are complex (a need to renovate a bathroom), involve many steps over significant time periods (training for a race), and often are nested inside larger or connected problems (playing an extended game). Problems are defined and constrained by life imperatives (we must pay our credit card bills).

The knowledge and solutions derived from problem solving must align with situations and real constraints. People generally define their problems, and secondarily consider how to solve them, and then when to use math. Math rarely leads (except when family members directly want to learn or practice math). Single problems expand to fill the space, time and solution resources available. They may branch, with the resolution of one problem spawning new or next problems. People talked of the nesting and cascading of problems to be solved. We saw great examples of this in home improvement projects. People seem to meet up with problems once they start improvements, only to realize they must be solved in coordinated or timed ways. This is not to suggest that all problems are like runaway trains or that every problem facet being solved engages mathematics. It is to suggest that problems to be solved are complex ones that often require shifting attention, coordination of multiple nested activities, and unplanned-for triage processes.

When is mathematics brought to bear on problems in the rough and tumble of problem solving? Sometimes people are aware they are bringing math to the situation; other times they are engaging unawares of their mathematical practice. We had many instances in the database where we coded that the math was inferred—that people could not have accomplished what they told us they did without doing some mathematics. In this way, math could be *sub rosa*. The problems to be solved are often unique and situation dependent, people are generally required to evaluate their own solutions, and if correct, decide if the correct solution is really relevant or appropriate to the particularities of the situation. For example, people described both "doing the math" and "tossing it" when the idealized math did not fit the local situational constraints. We can see some of these characteristics of problems in a family's condo renovation project.

In one family, mom Yali and daughters Vivian and Renee are in the process of renovating a condo that Yali recently bought. Vivian is in law school and Yali recently became a teacher after a mid-career change. They want to save money on the renovations and Vivian likes carpentry, so she is managing the renovations and doing some of the work. Vivian describes how there are so many decisions to make—moldings, the placement and the size of walls, cabinets, which way to place the tub, where to put the sink? Closets, should there be one or two? Vivian discusses how they conceived of the renovation.

> *Interviewer*: Did you use any tools or ways to visualize to make these decisions? How did you actually decide where to put things?
>
> *Vivian*: It was a combination of just paper, like drawing a two dimensional…this is a square that is the kitchen. Where are the things going to be? And then actually showing up on site. I had to go down there a number of times, which was the hardest part, because, you know, we're trying to figure out, okay, we have this space between the tub and the toilet and how much space do you need for your knees? So we would just take the tape measure, and we're like okay, we want the [height of the] bench by the dining table. And we're like, well, how high is the bench by the dining table? There are these various measurements. So we literally took the tape measure and measured our knees from the floor sitting on [the bench]. So then you want to design, okay, what are our options? And you draw those on paper and then you actually show up and then you sit on the toilet. How many inches do I need, if the tub comes to here will I be able to sit down? <Yali laughs> We will, but if we sell the house to someone who's taller, will they? So it was kind of a back and forth.
>
> *Interviewer*: So you used your body for some of that?
>
> *Yali and Vivian*: Oh Yeah…

> *Interviewer*: …And you drew plans. When you drew the plans did you draw them to scale? Or did you just free form draw them?
>
> *Vivian*: I did both. Some things, well I'd mostly draw the free form just to give to the contractor an idea, okay this is what I decided. The tub is going to go here and I'd draw the inches...But I actually got on my computer and found some grid to work with to actually get more of the proportions. I didn't have a ruler in my whole room. So I couldn't even just measure a piece of paper and draw inches myself. I had to do it all online.

The family enjoys that they measured with their bodies, and upon Vivian's telling of the story, they all move around and giggle. Vivian's story on managing renovations reveals how informed critical decision-making is to their home improvement project. Vivian uses math in this work. She draws plans free form, but also draws them to scale in order to communicate with the contractor. The math involves knowing the dimensions of the large fixtures and pieces and the dimensions of the spacing they are being placed in. Drawing the scale version of the plans involves assigning scale units and engaging in proportional reasoning and problem solving. If a bathtub is 60 X 30 inches, how big is that on this particular graph paper? Still, even scaled plans might not be enough, and the women use their bodies to give a reality check to the calculations to make sure what is accurate on the 2D scaled paper plan will also work in the 3D world of real people moving around in the spaces. In the renovation project, daughter Vivian knows not only how to make and use scale models, but also what their limitations are, and when to switch into reality mode. This back and forth cycling of model to reality is interesting, and typical in this type of situation.

The renovation also included a new kitchen and new closets in the entranceway. There was a lot of figuring and fitting going on. To measure for the stone countertop, Vivian entices thirteen year old Renee into realizing she could use the Pythagorean theorem to complete the measurements for the counter in the stovetop corner. There was also clever financial planning around emerging problems as they were discovered, from having to convert from metric to standard measurements to figure out if the cabinets would fit before ordering, to balancing loans for the construction costs against 0% financing on appliances and fixtures, with the projection of a well-paying job when law school was over in a year. The problem space was huge, cascading and needing attention over an extended period of time. The math identified in the renovation story was extensive as well and included arithmetic, comparing magnitudes, geometry, measurement, optimization, unit conversion; proportional reasoning, decimals, unit conversion, formula, measurement, estimation, data representation, and interest rates.

II. Values steer the problem definition

Why engage all of this problem solving? Our analyses revealed that exploring values answered that question. Why bother to take a student loan to help finance your mom's condo renovation and juggle a huge amount of debt for several years? Values also answer the *how* question. Vivian could hire someone to design the new kitchen and bathroom, or she could figure it herself, especially since she had lots of ideas and she thought she knew enough to get started. Values drive some of the decision-making around problem solving, and in volatile emotional moments, steer people into certain practices and solutions (instead of arguing, let's check the figures). Peoples' preferences and their values actually are determinant in whether or not a problem will even be engaged (Pea & Martin, 2010).

When families spoke about their problem solving strategies and gave examples of the kinds of problems that arose in their lives, family members consistently talked about what they valued. Families often had to consider the factors that mattered when making decisions. Was getting the "best" deal most important? Did social relationships play a part? Was time an important consideration? Was it important to make decisions by taking into consideration *all* of the relevant information? Each family's answers to these questions varied, sometimes even from one problem to the next. The answers to these questions often organized the problems to be solved by defining the constraints, the resources, and the criteria by which the outcomes would be judged. Values helped families determine *how* to solve problems once they were identified as such. When one mother wanted a savings plan to pay down credit card debt, she chose a socially regulated solution that traded the "best" financial deal for the power of social commitment and cultural participation.

One family's approach to purchases illuminates not only how values enter into the mathematics of their decision-making, but in this case, the creation of a new kind of value exchange unit (product goods vs. days mother has to work away from home). The dad, Jim, is an analyst and the mom, Liz is a part-time financial consultant. Liz works part-time because she tries to stay at home with the children as much as she can. She does take consulting jobs to keep up with expenses, but she tries to optimize her time at home with what she can earn through consulting gigs. When we asked how they handle the purchase of big-ticket items, they responded by going straight to the importance of their values. Jim said that he and Liz had been thinking about "bigger ticket item" purchases:

> We have got into an interesting concept about what the value of bigger ticket items is to us personally, and value is a very personal thing. Since my wife is a consultant and works on billable hours, we have gotten into this thing of looking at something, and if it is a big ticket item, thinking about how many hours it costs for us to get it. Whatever it is: I have to work three days, or it will cost me three weeks of effort if I want to do this, whatever the big ticket item is. And we sit back and we say, you know: is this really worth three weeks? No. Is this worth three days? No. So it depends – and sometimes – is this worth three days worth of effort? Yes. It's an interesting way of thinking of things, and trying to assess the value, which we do on bigger ticket items… We oftentimes say, you know, it's not worth that to me. I am not going to work five days to get this thing. It's just not worth it to me. It is an interesting value concept that just the two of us think about.

The family actually pits Liz's time at home against their wants, knowing they can have one or the other. Liz's home time usually wins, and the family goes without. This is especially so if the purchase was a flat screen TV. However other kinds of big ticket items, such as paying to offset the tuition for their middle school aged daughter's private school, is considered worth the consulting hours. Jim and Liz went on to tell us that even their children have come to use a similar approach in their own thinking. It started as the children were getting to the ages where they wanted to purchase souvenirs when the family went on a vacation. They decided to give their children $10.00 and let their children budget their own purchases, forcing them to ask the question of whether what they would purchase was worth it.

> Jim: But it has translated down a bit with the kids and the money that we give them on vacation, and we…they have ten dollars, and then they go through this value exercise to say 'do I really want that candy bar for two dollars or not?' And they are starting to make value judgments themselves on what is important to them, and whether or not they are going to spend the money on a particular item, or save it.

What such a structuring of the values undergirding decision making for bigger ticket purchase items exemplifies is that how a family weighs, or creates equivalences between, different categories of resource (incremental earnings, big ticket purchases, family time), affects the decisions that they make in how resources are allocated. In turn, features of the ideology, as well as of the mathematical practices, can serve as a model that the children emulate in their own age-relevant versions of the adult problems. For example, the math involved in Jim and Liz's purchase to consulting hours trade off includes unit conversion, arithmetic, and proportional reasoning. As the parents discuss family purchases, the children are exposed to mathematical reasoning involving these concepts, as well as the value considerations their parents consider important.

III. Different maths engaged in service of problem solving

Two findings relating to the math family members engaged while problem solving are of special interest. The first is that a number of problems in our database indicate that people engaged multiple maths on the way to problem resolution, even when we exclude simple arithmetic or computation from the data set. The second is that, even for the same problem, family members approach problems with different ways of bringing math to the solutions. There are several conditions for multiple maths to co-occur. People in the family sometimes approached the same problem in different ways. In some of the cases, they knew the way of another family member, but chose to use their own. In a few cases, this led to circulation of a problem from one person to the other, and in these instances, we found family member playing to perceived expertise of others.

Engaging multiple kinds of math to problem solving is a very interesting and positive aspect of family mathematical practice, and we see it in stark contrast to how problem solving is taught in school math. In school, the approach to math most preferred is a development-sequential one. Math in school tends to concentrate on learning and practicing one kind of math at a time, and this is generally the case, even when problems are of an applied nature. In school, the applied problem is often designed to practice a singular math concept; at home, the problem is generally designed to accomplish a goal that is desired and sought, often with the engagement of a variety of math practices for accomplishment. The family offers children the opportunity to develop mathematical practice under conditions that matter to them.

For one middle school boy, Ravi, a pre-algebra student, opportunities to apply his new problem-solving skills abound at home. His mom talked about how he constantly enjoyed trying to calculate the best deal. The family told the story of Ravi's part in helping them decide what car to purchase. The family was looking at various Toyota models, and they were considering a number of factors in making the decision. Ravi compared the options and saw gas mileage as the deciding factor:

> *Ravi*: Yeah, uh, so we were considering between a Toyota Corolla, a Toyota Camry, and a Prius. And I said that we should buy a Prius because my mom drives a lot of miles [Mom &

> Son laugh]. And so, uh, with a Prius, uh, you get, uh, I believe you get some tax benefits, car, auto insurance [Mom & Sister smile].
>
> *Dad*: [returns, sits down and pats son on the back while smiling and laughing] You don't need to know that.
>
> *Ravi*: … and, a Prius, um, is better for the environment also.

While Mom and Dad would also take the tax benefits into account, they knew Ravi was not yet ready to weigh those benefits. Still, he was intent on making a case for the mileage in terms of environmental effects along with cost savings at the pump and at income tax time. He had figured the differences and savings for his parents and argued for the ways the pump and environmental impact savings would even out the extra purchase price. His interest in optimizing was balanced by his knowledge of his mom's commute and a general appreciation in the family for thoughtful, mathematical arguments. Ravi demonstrated how he put what he learned in school to work for him on the Prius problem:

> *Ravi*: Yeah, like um, usually in the car, like uh, for the Prius, like miles per gallon. When you see 50 miles per gallon, that means if you drive, you will use 1 tank of gas if you drive 500 miles. That means, if you fill up your tank you can drive from here to LA in that tank.

In Ravi's example, he set up a problem and argumentation for his choice that would put the mileage benefits of the Prius into perspective. He knew he could make it from Palo Alto to LA on a 10-gallon tank. It is important to recognize what drives Ravi to set up a proportional calculation. He wants to find the best of three car options for his family's needs. His interest in optimizing gas mileage to get his family the best financial deal while making a responsible environmental decision works nicely with his penchant for math problem solving generally. The car purchase context sets up the opportunity for Ravi to draw on several types of math problem solving at once. The car purchase demonstrates the way Ravi is able to draw on his school knowledge inside a relevant family context. He tackles everything from arithmetic and measurement to optimization and proportional reasoning in order to support his argument.

IV. Mathematical thinking as activity in distributed knowledge environments

The stories that families told us indicated that math engaged problem solving was most often a social affair. This was so even when we asked an individual for a story. Children often mentioned how siblings, parents, grandparents, or friends were part of their math stories. Adult problem solving often involved others, even if they did not always involve children. Social, historical and cultural practices were deeply embedded in problem solving. Playing math games with a grandfather, working on puzzles as a family, inventing fantasy businesses for play, and saving money and managing debt were all deeply social and distributed across persons, time and setting. When math was involved, it became entwined with the persons in activity. Math engaged events, like other learning situations, were embedded and distributed among people, artifacts, activities and settings (Gonzales et al., 2001; Pea, 1993). Individual accounts of how the math practice was distributed differed, yet we found only few instances in family-based activities where math engagements were isolated or individual.

Mathematics during a road trip plays out in one family for whom mental math is key, particularly for the father, Steve. Throughout our interview, Steve's use of mental math to figure out problems is a common theme recognized by the rest of the family and becomes a source of amusement. For example, Steve describes doing "nutrition math" around each meal to stay on the *Zone* diet and also decrease the amount of cholesterol he and his wife consume. Such activity involves calculating protein grams, different forms of carbohydrates, and fat per meal, as well as the amount of cholesterol and total number of calories. According to Steve, "you almost need a computer to do the nutrition analysis of each meal at this point. So, I have this much saturated fat, so if I use olive oil instead of this oil…a lot of [calculations like] that." However, rather than using a computer to keep track of the calculations, Steve says he adds up the different quantities in his head as he builds a menu. His comment - "I do it in my head" - causes laughter from his wife, Sharron, and daughter, Elise. Sharron jokingly asks us: "Do you see a theme here?" - referring to the father's focus throughout the interview on doing calculations in his head. Sharron and Elise also figure cholesterol, but with different methods—one cooks with no fat, since "zero times zero is zero", and the other computes cholesterol with paper calculations.

The focus on mental math is not only important to Steve, but it is also passed down to Elise, with the involvement of the whole family. Elise describes a game she played in the car on a trip with her parents:

> *Elise*: We were coming home across the Golden Gate Bridge, and I wanted to see if I could hold my breath across it. Our guidebook said it was 2 miles long, and at 45 miles/hour, so we had to figure out how long I had to hold my breath, and it ended up being 3 minutes. And that's a long time, so… I didn't hold it for the whole entire time.
>
> *Sharron* (chimes in laughing): We drove faster!

> *Steve* adds: We realized we'd have to change some of the physics in order for this to work, since 3 minutes was too long. So we drove faster and shortened the span to just the part that was kind of out there. She figured out the stuff while we were driving. That was her project to do the math.

This game was initiated by Elise, as holding her breath across a bridge was something she had done on a fieldtrip with her friends. However, it was taken up by the whole family, and became a problem to solve by the family unit. According to Sharron, for Elise to figure out whether she would be able to hold her breath "was a good exercise that took part of the time between Santa Rosa and the Golden Gate Bridge." Once she figured out that she would not be able to hold her breath long enough, the family unit took it on to figure out how to help her succeed, both by changing the constraints of the problem, the factors involved, and what would count as a solution. As a family, they decided that she should not try to hold her breath for the entire bridge, but rather concentrate on a shorter span of the bridge that was more manageable. The father also changed the speed of the car, speeding across a section of the bridge to make it easier for his daughter to succeed. This episode was recounted by the family with enthusiasm, with all members actively chiming in to the description. What started out as a game for the daughter became a challenge for the family to solve.

This family focus on mental math is not new for Elise, and she gets practice in many situations with her parents' help: "There are a lot of times in cars when dad has a problemish thing, like if we're going this fast, how long will it take us to get somewhere." Steve adds, "That's right. And you have to do it in your head. Which is hard, isn't it? But really good practice." At the store, Elise's parents often ask her to help with the "shopping math," asking her to do price comparisons to figure out which is the best value, as well as figuring out the nutritional analysis of foods based on the serving size they actually eat. In talking about shopping math, Steve says:

> They try to teach rounding in the schools obviously, but when [Elise] gets a problem that we would just instinctively round off, because we think, 'I'm never going to calculate 16.3 divided by 8.7' or something, she's instinctively trying, you can see, especially - that's why it's good to try to do stuff in your head – try[ing] in her head to do 16.3 divided by 8.7 (Steve gestures as though writing out a long division problem.) And it's like, 'ok, this is a good time for rounding'. And she's like 'Oh, well then, 16 divided by 8. Now it's easy'. But it's not an instant conclusion that you leap to.

Sharron adds: "probably more than anything, that ability to get common number sense for everyday use for quick, (snaps) quick answers just seems to be really key for her, and she's getting there." Her family is explicit about using rounding as a strategy, as Elise recognizes. She says about Steve: "You do that a lot. I have trouble with a math problem. And then he says, well what if the numbers are this and this, could you figure it out then? Then, it's oh, that's how you do it."

Summary

Four features of mathematically engaged problem solving were found across our interview data: Mathematics was engaged as people solved problems in their lives; problems that received attention did so because people valued the end goals related to the problem solving; multiple maths were brought to bear on problems; and, math was embedded in the socially distributed learning opportunities. Mathematics followed as people attended to other goals. These features varied across families, situations, and events, to the extent that family position, histories, styles and cultural practices had a lot to do with how and when problems and math were engaged.

These results have several implications. First, rich problem solving in the family is not appreciated, and family mathematics even less so. If mathematical engagements are recognized, they are usually considered to be of less consequence than the more formal math structures taught in schools. Second, the results of our analysis indicate that family mathematics practices are socially embedded in some of the families' most important and valued activities. Family members engage with math because they need to or want to. They support each other through reliance on, and knowledge of, their distributed expertise. Third, children are often witnesses and participants in these vital family problem-solving contexts, and the mathematics engaged is definitely providing learning opportunities—about math and life. Fourth, the mathematics covered has alignment with school math objectives through pre-algebra—another reason why the learning opportunities available in families are rich ones.

Endnotes

(1) The Family Math Study is conducted as part of the LIFE Center (NSF REC 0354453). Eventually, its results will also inform a range of studies about learning in both formal and informal learning environments (for more about the LIFE Center, http://www.life-slc.org).

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