What Makes Teaching Special?

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Abstract: The aim of this paper is to articulate some of our early attempts to understand how teaching interactions differ from everyday communicative interactions. In order to do this, we integrate theories from linguistics, particularly the branch of pragmatics, with work in math education and the learning sciences. Recognizing that communication is an inferential activity, we explore what makes teaching interactions a unique class of communication. Specifically, we suggest that individuals who take on a teaching role must deal with a prediction problem because knowledge is not equally shared between the communicators. Our analysis of classroom observations and tutoring interviews with third grade students learning single digit multiplication elucidate a few of the ways that the prediction problem is addressed in teaching activities.

Introduction

What makes teaching special in comparison to everyday communicative interactions? An underlying, though questionable, assumption commonly held in math education is that we must already know the answer to that question. That assumed understanding has been foundational in the design of curricula, pedagogical tools, and professional development interventions. How can we account for the activity in which these interventions are played out? Can we confidently say that we have a deep understanding of the factors that are involved in the moment-by-moment communicative interactions between a teacher and learner? We feel that it is imperative that as we continue to design and develop new teaching and learning interventions, we also work to develop a deeper understanding of what is involved in the communicative activity of teaching. This paper articulates our attempt to draw from multiple disciplinary backgrounds to answer the above question.

In keeping with the interdisciplinary nature of the learning sciences, we are integrating perspectives from multiple fields, including mathematics education, cognitive science, and linguistics. In particular, we draw from theories and models in pragmatics, the branch of linguistics that most deals with the interpretation processes and constraints involved in communication. As a practical matter, our work has proceeded both top-down and bottom-up. We have worked from our understanding of certain theories of pragmatics. In addition, we have worked through iterative analyses of a corpus of observations and interviews with elementary school mathematics students.

While some work has been done in education research that has merged linguistic theory with education, very little of it has dealt with pragmatics, and in particular, the pragmatics in teaching interactions. Often what we see in analysis of communication in the classroom is an explication of the discourse patterns that appear (Lemke, 1990) or how language functions in constituting the classroom culture (Lampert, 2001; Yackel & Cobb, 1996). Teaching studies that address communication tend to emphasize the particular moves that teachers make to promote a focus on student thinking (Carpenter & Fennema, 1992; Rittenhouse, 1998; Sherin, 2002; Williams & Baxter, 1996; Wood, 1994). Research that has addressed pragmatics has tended to emphasize the symbolic aspects of meaning (Pimm, 1987, 1995) or pragmatic obstacles that exist without shared experiences (Spanos, Rhodes, Dale, & Crandall, 1988).

In our own research, we start from the premise, drawn from pragmatics, that communication is strongly dependent on the inferences made by participants. One participant introduces an utterance. Then other participants infer the utterer's meaning, based on the utterance, as well as other knowledge and presumed constraints. Our focus is on how this process of inferring meaning works. This stands in contrast to typical analyses of classroom discourse patterns. These more typical analyses focus on the function that contributions play within a dialogue. For example, a contribution might function to pose a question, or to provide a response (Clark & Schaefer, 1989; Greeno & Engel, 1995; Lemke, 1990).

We are interested in communicative interactions in which one individual is taking on a teaching role. Typically, this would be a classroom teacher, but our intent is to speak to a whole class of communicative activities in which an individual is working to develop a content understanding in another individual. One marked difference between these teaching interactions and everyday communication is that the participating individuals may possess very different knowledge resources. Individuals who take on a teaching role are then posed with a challenge to do additional communicative work to coordinate the students understanding with their own. But rather than doing simply more communicative work, we posit that these individuals, whom we will for convenience refer to as teachers, do a different type of communicative work. To illustrate our view, we will first present a short excerpt from a classroom observation of third grade students, and use this as one vehicle for discussing our claims. We will proceed to introduce ideas from pragmatics that we have built upon in our own work. Finally, our discussion will explore further a couple of the communicative "tricks" that are used in these teaching interactions and relate them to diagrammatic-interpretive norms, a set of implicitly shared communicative resources that take on some of the inferential work in these interactions.

Stephanie and the Ambiguous Cookies

We present here a short excerpt from a classroom observation to illustrate some of the interpretive ambiguity and online inferential activity in which participants are engaged. This excerpt comes from an enactment of Children's Math Worlds (CMW), an elementary reform mathematics curriculum designed at Northwestern University. All of the data we present here is drawn from three years of the CMW project, from 1998-2000, during which the project was engaged in the design and study of full-year curricula for third and fourth-grade mathematics.

In this excerpt, the teacher is facilitating a CMW lesson in which third grade students are learning about single-digit multiplication, and they are making use of a specific kind of drawn representations. Students were selected to come to the board and write down a multiplication number sentence, which they were free to choose, such as "2×3=6," and to make a drawing that could show this number sentence. One of the students who came to the board, Stephanie, produced the mathematical drawing shown in Figure 1.

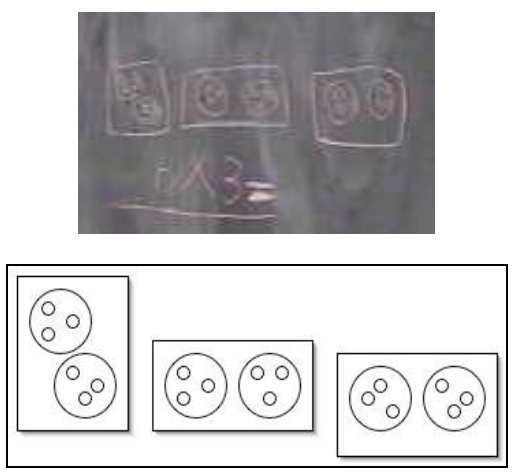

Figure 1. Stephanie's mathematical drawing of 6 × 3.

T: What's your multiplication sentence?
S: 6 times 3.
T: Ok, Stephanie, explain what you were doing.
S: I put 6 times 3 because there's umm, there's all these cookies together equals 6 and there's 3 boxes.

Stephanie touches the two leftmost large circles, motions over the entire drawing, then touches each of the rectangles as she moves from left to right.

T: Wow! Look at this. Stephanie, I was a little confused when I first looked at this. Third graders look up here because this is really interesting. I looked at this and Stephanie had 6 times 3. And I thought this kind of looks like 2 times 3 to me. But what she is saying is 2 times 3. <sic> What if these were plates and these were cookies on the plate and I wanted to know how many plates. What would my problem be if I wanted to know plates? Christina?
Teacher walks to chalkboard, briefly touches space next to drawing on the chalkboard with all fingers, then faces the class.
C: 2 times 3.
T: Right. If I wanted to know how many plates there were it would be 2 times 3, but what if I wanted to know how many *cookies* there were? There's 3 cookies on each plate and how many plates are there?
C: 6.
T: 6. So Stephanie was right.

What is immediately striking about this excerpt is the apparent ambiguity inherent in interpreting Stephanie's drawing. Given the abstract nature of her representation, it is reasonable that multiple interpretations could occur here. Stephanie's interpretation of her drawing appears to be inconsistent with the number sentence she selected. She states that the six in her number sentence refers to the total number of cookies, represented by the large circles, and the three is represented by the rectangles.

The teacher is working to assess Stephanie's understanding given the evidence that Stephanie presented in her above explanation. The teacher's interpretation is that the drawing represents 2 × 3 since she sees three equal groups, contained by the rectangles, with two large circles, representing plates, in each. This is consistent with her understanding of multiplication as repeated groups with an equal number of items. She articulates her interpretation of the drawing for the rest of the class and uses that as a platform to introduce alternative number sentences that the drawing could also represent. She eventually concludes that, under one interpretation, Stephanie's drawing could represent six times three. These differing interpretations are recorded in Table 1. We believe that, although this excerpt is very brief, there is nonetheless interesting interpretive and inferential work being done both by the teacher and the students. How do the teacher and the students make their respective attributions, both numerical and representational, to Stephanie's drawing? On what evidence do they base their understanding?

Table 1. Individual attributions of representational relations of situation to math drawing

	Stephanie's Interpretation	Teacher's and Christina's Interpretation
Small Circles	Chocolate Chips	Cookies
Large Circles	Cookies	Plates
Rectangles	Boxes	Groupings

Relevant Pragmatics

In this section, we will articulate a subset of assumptions in pragmatics from which we draw, both at the level of theory and at the level of posited models of communication. We feel that pragmatics is an appropriate area to use as a resource as it deals specifically with meaning and ambiguity reduction in communicative activity. The area of pragmatics has also shown significant success in increasing the visibility of the more subtle workings of interpretation. Pragmatics, as it has developed as an area, has also taken on a more cognitive focus, as it has extended its model of communication to account for individual interpretive processes.

The Code Model

According to Sperber and Wilson (1986), the dominant model of communication, traced as far back as Aristotle and throughout semiotics, had historically been the code model. The code model assumes that communication is a linear process in which a message starts at an information source and is then converted into a signal, or code. This signal then travels to the recipient, who uses her own decoding mechanism to extract the information in the signal. That information is then processed and stored by her, and then she can encode her own signal to transmit. The consequent foci in linguistic research had been centered on the purported codes that were transmitted, including how they were structured and what they meant. Research on syntax and semantics, for example, were primarily focused on the transmitted signal and the encoding/decoding processes.

An analogy can be drawn to a transmission model of teaching and learning. By substituting a teacher and a learner as the communicators in the situation above, it is simple to map teaching and learning to a code model. Knowledge is assumed to be encoded as signals that are then decoded and stored by the student. These signals take the form of lectures, textbooks, and other information-bearing tools. In this case, there is nothing remarkably special about the teacher except he has more information about a particular content area. As long as he can articulate the knowledge as words, he can teach. Teaching then becomes synonymous with extended telling, and then nothing is different about teaching interactions in comparison to everyday communication. Although this model is extremely simplistic, it appears to accurately characterize the model of the communicative activity of teaching that is frequently assumed. One major piece that appears to be missing is an account of how ambiguity or multiple interpretations can exist. Also missing are the specialized bodies of knowledge teachers must have in order to be effective (Ma, 1999; Shulman 1986).

The Inferential Model

Grice, a philosopher of language, led the way in his early work, toward an alternative model of communication exploring meaning (Grice, 1957, 1969). His work emphasized the observation that meaning must always be *inferred*, and that an understanding of intention by all participants, is essential. Grice is also credited with developing a set of communicative maxims for conversation (the maxims of quality, quantity, relation, and manner) as part of his cooperative principle (Grice, 1975). The maxims function as understood norms that do part of the communicative work in an interaction. They set expectations for likely and acceptable responses when knowledge is shared. The intersubjective understanding that these maxims are followed allows a communicator to constrain the likely inferences a listener makes about the speaker's intention and consequently constrain meaning. Violations of these maxims are also informative, as they provide additional evidence about the communicator's informative intent.

The inferential model of communication, suggested by Grice's work and later developed more fully by Sperber and Wilson (1986; Wilson & Sperber, in press) captures more of the ambiguity and online interpretation than the traditional code model. A communicator, rather than transmitting a signal, makes manifest evidence and assumptions in the mutual cognitive environment. In a teaching interaction, this mutual environment includes the physical setting, representations that have been constructed, previous utterances and gestures, shared presuppositions, and the shared bodies of cultural knowledge. Contributions to the mutual cognitive environment are made through ostensive communicative acts. The interpreter then constructs an understanding based on what new evidence or assumptions have been introduced in the mutual cognitive environment and formulates the most reasonable yet least cognitively demanding interpretation given the resources they have at their disposal in their cognitive environment. Sperber and Wilson use the term *relevance* to describe the degree to which an inferred understanding satisfies these conditions.

What we draw from this inferential model is that both a teacher and a learner are actively engaged in a continual process through which they are each contributing evidence and assumptions to a mutual cognitive environment, and they are also inferring each other's meaning. In standard inferential communication, each individual also shares the expectation that contributions to the mutual cognitive environment will bear a high degree of communicative relevance. However, this expectation may not be borne out in the case of teaching interactions.

Working to Constrain Responses

In teaching interactions, there is often a dramatic asymmetry in the knowledge possessed by teacher and learner. Specifically, the content understanding that a teacher and student would normally share in knowledge-symmetric situations is conspicuously absent. From the point of view of pragmatics, this asymmetry leads to some specific difficulties for the teacher. Most significantly, as compared to everyday interaction, the teacher faces much higher uncertainty of what the learner will construct as his interpretation of the teacher's utterance. It is the need to deal with this uncertainty that we believe makes teaching interactions qualitatively different from everyday communication; individuals who take on a teaching role must regularly leverage special communicative tricks to constrain the possible interpretations of their utterances.

One of the foci of our work has been to begin to catalog these "tricks" and to understand how they work. A full catalog and description of the range of tricks we have seen is beyond the scope of this paper. Instead, we will overview two of the basic tricks that have been observed, both in a third-grade classroom setting and in individual tutoring interviews with the same set of third grade students. The interviews that we discuss here were structured around a set of challenging multiplication and division word problems. Initially, a student was encouraged to attempt to solve each problem without assistance from the interview. But then, if a student struggled, the interviewer progressively increased the amount of assistance provided and took on a teaching role.

Enriching Contributions to Capitalize on Shared Knowledge

We emphasized above that teaching, like all communication, is strongly dependent on inferences made by participants. For the student, these inferences take off from the communicative contributions of the other participants, especially the teacher. Thus, one technique that a teacher can use is simply to add additional contributions to the mutual cognitive environment. According to this strategy, when communication becomes problematic, a teacher simply increases the number and content of the utterances that the student has as a base for inferences. This is especially effective when the utterances are chosen so that the inferences required by the student draw on knowledge that is shared by the teacher and student.

In the case of the interactions around elementary mathematics that we analyzed, this strategy took a specific form. In elementary mathematics, students are asked to solve or model word problems that are extremely concise in description. A teacher can choose to add to this description, by introducing real world knowledge that is not given in the problem, but that leverages established connections a student may have with objects or situations in the world. Doing so helps to elicit a particular understanding in the child of the activity or situation described in the word problem.

A simple example can be found in the cookie episode described above. The teacher introduces the idea of a plate being a part of the picture and uses that added detail to make sense of the drawing and communicate that understanding to the class. By introducing the plates as being in the drawing and having three cookies on each of them, the teacher is evoking a specific interpretation for other students to make, thus dealing with the prediction problem inherent in the interaction. This then gives her and the students an agreed upon referent with which to work and shapes the spoken responses and interpretations that will follow. What is noteworthy, in this simple example, is that the teacher has manipulated the space of likely interpretations for the students through her use of everyday knowledge. We would like to mention, though, that this only works as long as the plates and cookies are consistent with the body of cultural and experiential knowledge that the students have, which typically is passively assessed in such a way that if no objections or inconsistencies are expressed, then mutual understanding of the referent is, albeit sometimes incorrectly, assumed.

Similarly, this works on a smaller scale. Take, for example, one of the interview questions posed to the same group of students: If Susan can make three cups of coffee in five minutes, how many can she make in 30 minutes? If a student is appearing to have difficulty with this question, then the interviewer can choose to elaborate and enrich the mutual cognitive environment in a multitude of ways. For example, the interviewer can give the characters in the word problem a specific purpose, or describe other items that could be used in the situation. If the repeating five minute increments of the coffee problem are completely opaque to the student, then the interviewer can elaborate it by situating the problem as having a character who places the three cups of coffee that take five minutes to make on a serving tray. The question then changes to how many cups of coffee will be on the serving tray after 30 minutes, which may be more easily represented in a student's drawing. In order for this to work effectively, the student must be able to easily manifest knowledge of the objects and situations being described as usable resources in their individual cognitive environment.

We believe that the use of these small pieces of real world knowledge is not a trivial matter. It has a significant influence on how interpretation happens and ambiguity is reduced, and it fits very well in a pragmatics-oriented analysis of the teaching interaction. An effective teacher will have knowledge of the real world experience that a student can easily access as a resource.

Intermediate Value Assessment

Another way that a teacher can deal with the difficult problem of predicting students' inferences, is to initiate active cycles of assessment of a learner's understanding, using the evidence and assumptions that a student introduces in the mutual cognitive environment. Thus many of the communicative tricks that we have observed have to do with this active assessment. Here we illustrate this with a brief discussion of one of those assessment tricks.

This assessment trick, which we refer to as *intermediate value assessment*, allows the teacher or interviewer to constrain the space of likely predictable responses. If the learner presents a value that is correct, it provides more evidence to suggest a mutually coordinated understanding. Alternately, if the learner presents a value that is incorrect or inconsistent with the desired model, the teacher still has the benefit of new evidence about the learner's understanding. If the learner's response is consistent with a reasonable 'bug' in their thinking, then the teacher or interviewer can begin to infer the possible origins of the bug. When the value is incorrect, and there is no indication of its origin, the teacher or interviewer has strong evidence for an uncoordinated understanding and the need to use a different trick and re-evaluate her understanding of the learner's understanding.

In the case of the coffee problem discussed above, we observed several instances of intermediate value assessment. Often, as the interviewers adopted a teaching role, they would work with the student to re-represent the problem using their computational activity as a representation for the situation described in the word problem. For example, the interviewer would work with the student to decompose the problem into five-minute increments. The students would then figure out how many cups of coffee were made in the first five minutes, then proceed to compute how many cups were made in the next five minutes, and so on. Each step in the computation in part served as a representation of a five-minute time step in a model of the word problem. At each time step, an intermediate value was produced at the request of the interviewer, such as how many cups of coffee were made after 10 minutes and how were made after 15 minutes. These acts served as overt assessments to evaluate the student's understanding of the steps in the representational activity. In this trick, the interviewer is making certain conditions manifest. In the coffee problem, the state of ten minutes is one such condition.

The teacher made use of intermediate value assessment in the cookie episode described above, though in the context of the classroom and much less formally. When the teacher asks the class how one would figure out the number of plates represented by Stephanie's drawing assuming that part of the drawing included cookies on plates, she is checking for an explanation that would serve as an intermediate value in the computation process. An affirmative answer provides evidence that another student is making the same interpretation as the teacher and is therefore likely to generate the appropriate model for computing a solution. In the cookie situation, Christina provides such a response (2 times 3) and the teacher can assume that she is being understood and can continue her work to elicit a desired understanding.

In our proposed inferential framework of teaching interactions, the trick of intermediate value assessment is more than simply incidental. While these interactions often have the structure of inquiry-response-evaluation, the pragmatic aspects of these interactions suggest that a certain amount of important and useful evidence is being gathered for the teacher through the posed constraints. We believe that this is particular to situations where significant guidance is needed for the student to get started in order to work toward a solution. In a more general case, consider the situation where a student is asked to answer a question for which there are just a few possible response choices. If the student gets it right, the teacher has limited evidence from which to infer the student's understanding. On the other hand, if there are an abundance of possible choices and the student selects the correct one, then the teacher has much stronger evidence that the student had constructed the right answer. When math questions are constructed properly such that the potential misinterpretations of a student's understanding are minimized, then a teacher can easily generate a question where there are a large number of possible answers. The key is for the teacher to construct the intermediate value assessment in such a way that it is extremely unlikely that a student could give the correct answer accidentally, and without a reasonable understanding. For example, if the teacher constructs an intermediate value assessment so that the answer should be 58, and the student gives that, then in most cases she can be nearly positive that the student's interpretation is correct.

Diagrammatic-Interpretive Norms

We now turn our discussion to what must also exist beyond just teaching tricks in order for shared diagrammatic interpretation to happen. Given the absence of any ostensive gestures by the teacher in the above cookie excerpt, how can Christina successfully interpret what the teacher means when she is referencing Stephanie's math drawing? We believe that for the teacher and students in Stephanie's cookie problem to be able to interpret each other, a certain set of shared general assumptions must exist in the backdrop. In part, we were inspired by Grice's communicative maxims and how he claimed they function to facilitate conversation. Specifically, we recognized that in these classroom interactions and in interviews, these Gricean-like rules or assumptions were taking on a portion of the communicative work, and thus we extracted what we call *diagrammatic-interpretive norms*. These norms constrain likely interpretations of mathematical drawings whenever they are used in a communicative interaction. We will briefly describe three of these norms.

One such norm deals with the interpretation of *identical elements*. In a mathematical drawing that is intended to have abstract representations, figures that are roughly identical are considered to represent instances of the same thing within a particular problem. This is particularly useful for a communicator who does not necessarily need to go through a process of enumerating every instance of a particular element. Instead, they can refer to one or a few of the identical elements as representatives of the element set and still elicit the correct inference. For example, Stephanie, when first explaining "there are 6 cookies altogether", points at each of the two left-most large circles and then motions her hand over the rest of the entire drawing. What she is communicating here, with the aid of the identical elements norm, is that elements that are identical to the two circles to which she had pointed first were meant to be included in the set of cookies to which she is referring.

A second norm is that of *numerical exhaustiveness.* An attribution made to a drawing should be such that the number of instances of a particular identical element corresponds with the number in the situation that the drawing is intended to represent. For instance, in the above excerpt, the teacher says:

> T: Right. If I wanted to know how many plates there were it would be 2 times 3, but what if I wanted to know how many *cookies* there were? There's 3 cookies on each plate and how many plates are there?

Interpreting what the teacher is referring to as plates and cookies without the aid of specific highlighting gestures is not a trivial matter. The student, Christina, is able to use the information from the teacher's utterance to shape her interpretation. Knowing that a shared interpretation requires there to be three cookies represented in the drawing, Christina's inference is constrained by the need to attribute the label of cookies to a drawn element of which there are three instances. Following from numerical exhaustiveness, there are two elements that can represent the number 3: the rectangles and the small circles. The added information that cookies must be represented on a plate further constrains the interpretation that the small circles must be cookies and the large circles must be plates.

Another norm that appears to exist is that of *global exhaustiveness*. Every element in the drawing should map to a part of the situation being represented diagrammatically. Misuse of this norm tends to be very informative, as it reflects a learner's misunderstanding of parts of the situation that need to be represented. For example, in Stephanie's number sentence there are only two quantities that appear: 6 and 3. But her mathematical drawing has three entities: small circles, large circles, and rectangles. Most immediate interpretations of the diagram in terms of the number sentence must violate the global exhaustiveness norm, since an element of the diagram will not be used, and thus interpretive ambiguity ensues.

At first glance, it could appear that these norms simply reflect a consistency with standards for representation in the sense described by Palmer (1977). However, we believe there is a subtle difference. These norms are less a property of the drawn representation itself and more a property of the communicative activity that involves the interpretation of the drawing. Evidence that is introduced in the mutual cognitive environment drives the understanding of what the representation represents, and we have witnessed in other classroom observations and in interviews that, within seconds, new evidence introduced in the mutual cognitive environment does indeed dramatically change what in the drawing are considered to be representational units and what those units represent. We believe that this dynamic interpretation of a representation is part of the nature of an inferential model of teaching that draws from multiple cognitive environments.

The utility of these norms in teaching interactions is realized as they are automatically invoked through teaching tricks. Adherence to them is meant to reduce ambiguity and deal with the prediction problem by constraining the possible responses. Violations, though rare, produce evidence of a communicative breakdown and consequently a need for repair. None of this is accidental. In a sense, these representations are introduced precisely because they create a circumstance in which there is a set of norms of this sort, and they can be leveraged to constrain interpretation of the utterances of all participants. We would like to point out that these norms typically work in concert with each other, rather than in isolation. Items that are identical elements are the same items that must align with numerical exhaustiveness. It is very likely that other norms exist beyond the ones described here and that norms are specific to particular communicative media.

Conclusion

The incorporation of pragmatic models and ideas has the potential to be very informative to math education and the learning sciences. In our own work, we have suggested that the communicative activity involved in math teaching interactions is inferential, rather than code-like, in nature. These particular interactions are different from standard communicative activity because of the knowledge asymmetry that exists between the teacher and student in the situation. That knowledge asymmetry introduces a prediction problem for the individual who takes on a teaching role, in which the explanation that a student has for particular problem or task can be constructed in unanticipated ways that are different from a desired understanding. Teachers can use a variety of tricks to deal with the prediction problem that they face, and by reducing ambiguity and the space of likely relevant responses, they are able to better manage a focus on student thinking. We have discussed a couple of those tricks here and some of the norms that do a portion of the cognitive work involved in inferring meaning. As we continue this work, we are examining other higher-level tricks that teachers use to constrain interpretation and are further exploring situations in which diagrammatic-interpretive norms are being violated. Still, much more research remains to be done to understand the inferential processes that are unique to teaching.

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