Concrete vs. Abstract Problem Formats: A *Dis*advantage of Prior Knowledge

Andrew F. Heckler, Department of Physics, Ohio State University Columbus, OH 43210
Email: heckler.6@osu.edu

Abstract: Three experiments examine the effects of varying the relative concreteness of physics word problems on student performance. Previous studies have found that concrete representations benefit performance for relatively simple problems, whereas abstract representations are beneficial for more complex problems. These findings are replicated in a physics context. More importantly, a significant disadvantage for concrete representations is identified for some questions. When a problem potentially elicits prior knowledge that is contrary to scientific knowledge, e.g. a scientific "misconception", it is found that the concrete representation invokes incorrect answers more frequently than abstract representations. In addition, an interaction is found between the final course grade of the student and the abstract vs. concrete problem format, with higher grade students performing disproportionally better on abstract problems. This is consistent with previous findings in reasoning and an explanation is provided that involves the cuing of familiar, automatic knowledge and skills vs. explicit deliberate processes.

Introduction

One truism in education research is that student responses to a particular question depend on the how the question is represented. This idea can be applied in a productive way for the topic of problem solving: while it is clear that student performance on problem solving depends on the representation (or context) of the problem, one can gain a better understanding of the nature of student problem solving by determining *which* dimensions of representational variation matter and *how* the variations affects performance (e.g., Kotovsky, Hayes, & Simon, 1985; Collins & Ferguson, 1993, Zhang, 1997).

This paper builds on a series of previous studies that examine the effects of varying the dimension of concrete vs. abstract problem representations on student performance. These studies indicate that there is often a tradeoff between concrete and abstract representations. In particular, in cases where the problem complexity is substantial, abstract problem representations have been found to be beneficial for problem performance (Koedinger, Alibali, & Nathan, 2008), and have been found to facilitate transfer in problem solving performance (Goldstone & Son, 2005; Kaminsky, Sloutsky & Heckler, 2008).

In contrast, concrete representations can be beneficial because, for example, previous knowledge can facilitate situated reasoning that allows for the use of familiar, intuitive strategies practiced frequently (Kintsch & Greeno, 1985; Carraher, Carraher, & Schliemann, 1987; Nathan et al., 1992; Pollard & Evans, 1987) and provides redundancy that is helpful for avoiding and detecting errors (e.g. Koedinger, Nathan, 2004; Koedinger, Alibali, & Nathan, 2008).

We consider two points regarding abstract and concrete problem representations. First, while grounded or concrete problem representations may produce problem-solving benefits via the cuing of helpful prior knowledge of a specific situation, it may also be the case that the grounded representation can cue knowledge that drives the solver toward an *incorrect* solution path. Potential examples of this are the well known findings of patterned and "incorrect" student answering to simple conceptual questions about physical phenomena (e.g., Pfundt & Duit, 2000). Some have interpreted these findings as evidence of coherent "scientific misconceptions", while others have explained it in terms of the cuing of "knowledge in pieces" or "resources" (diSessa, 1988; Hammer, 2000). In either case, the patterned incorrect answers are a result of prior knowledge of some type.

The second point to consider is that individual differences can elucidate important insights into the nature of the effect of representational variation. For example, in reasoning studies involving the Wason card task, Stanovich and West (1998) provide evidence that both high and low SAT (test score) participants can easily solve the task in some concrete, familiar representations, but in the generic form the task is more difficult and is solved much more frequently by high SAT participants. They argue that concrete, familiar tasks tend to invoke practiced and automatic knowledge that is independent of ability, and abstract tasks require higher order, more deliberate thinking skills.

Therefore in this paper we address two questions. First, if prior knowledge can in some cases include "incorrect" scientific knowledge, then in these cases will concrete, familiar representations tend to cue this incorrect knowledge more frequently than abstract and less familiar representations? Put more plainly, will students display misconceptions more frequently in concrete problems than abstract problems? Second, will there be a significant interaction between ability level of the student and the representational format? A disproportionate increase in performance for high ability on abstract questions students would lend more support for the idea that concrete representations cue familiar and automatic solution methods that are common to all students.

Experiment 1

The goal of the first experiment is to replicate previous studies which demonstrate that concrete, more grounded problems are more successfully solved than more abstract, generic problems if the problems are sufficiently simple and familiar (e.g., Koedinger et al., 2004; 2008). A simple physics problem is posed in two different contexts: abstract and generic vs. concrete and familiar. In addition, the problem is constructed to include a physical situation which is known to elicit misconceptions.

Participants

The participants were 170 university students enrolled in the first quarter of a calculus-based introductory physics course at a large research university. They completed this assignment for participation credit as part of their total course grade. Participants were randomly placed in one of two conditions, the abstract (n=92) and concrete (n=78) contexts conditions.

Materials, Design, and Procedure

Participants were given one of the two physics problems shown in Table 1. They were given 15-20 minutes to complete the task at the beginning of a physics lab class in a proctored environment, and were given credit of participation. The problems are simple mechanics problems that would be typically found in the back of a textbook or on an exam, and are given routinely in homework assignments. The problem is typically solved by determining the two unknowns in the problem (friction and acceleration) via solutions to (a minimum of) two simple equations, one involving the definition of friction force ($F_{friction} = \mu F_{\text{normal}} = \mu mg$) and Newton's second law ($F_{\text{net}} = F_{applied} - F_{friction} = ma$).

While it may not be obvious at first glance, even to the expert, the physical situation described by the problem is somewhat counterintuitive. In particular, the net force resulting from the two horizontal forces on the object (applied force and friction force) is in the direction opposite the motion. That is, the applied force is less than the friction force. Nonetheless, the object can still be moving, just slowing down. It is well-known that many students have difficulty with the idea that the net force can be opposite the direction of motion (Viennot, 1979; Clement, 1982).

Table 1. Question Set 1: isomorphic abstract and concrete versions of a force-motion show-work problem.

Abstract version	Concrete version
A force F is exerted on an object in the positive direction. The object is sliding on a surface in the positive direction toward a mark on the surface. At the very moment the object passes the mark, the force F is 80N. The mass of the object is 32Kg and the coefficient of friction between the object and the surface is μ_k=0.3. What is the magnitude and direction of the acceleration of the object at the moment it passes the mark?	In front of a house, a boy is pulling on a large toy rocking-horse in the direction towards the driveway. The rocking-horse is on the sidewalk sliding toward the driveway. At the very moment the rocking-horse crosses onto the driveway, the boy is pulling it with a force of 80N. The mass of the large toy rocking-horse is 32Kg and the coefficient of friction between the toy and the concrete is μ_k=0.3. What is the magnitude and direction of the acceleration of the toy rocking-horse at the moment it crosses onto the driveway?

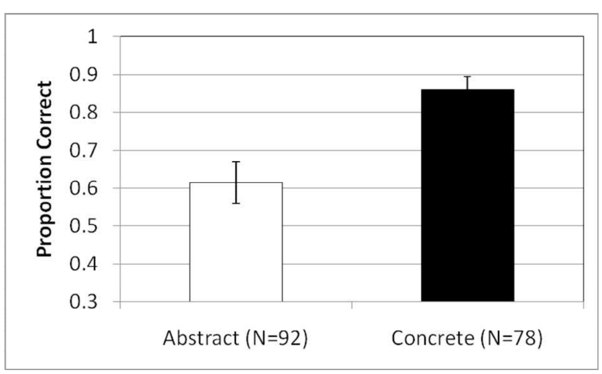

Figure 1. Student performance on Question 1, separated by condition. Error bars are 1 SEM.

Results and Discussion

As indicated in Figure 1, students were significantly more successful in obtaining a correct solution in the concrete condition (86%) than the abstract condition (62%), $\chi^2(1) = 13.2$, $p < .0001$, effect size $d = .6$. These results are consistent with previous findings that concrete, grounded context results in better student performance compared to the abstract context Further analysis of the solutions supported a previously described explanation for this difference (e.g. Koedinger et al. 2004), namely that the abstract condition inhibits the use of familiar knowledge that may help in understanding and solving the problem. In the abstract condition, the most common errors involved issues of not properly understanding the physical situation described in the problem text. For example in the abstract condition a 10/92 students confused (i.e., switched) the values of the two of the forces provided in the problem (the applied force and the weight, via mass). In addition, 5/92 students in the abstract condition ignored friction. In contrast, none of the students in the concrete condition made these errors; the most common error in the concrete condition was a simple algebraic error. Therefore, these results suggest that students in the abstract condition were less able to understand the physical details of the problem, and this lack of physical understanding could in turn inhibit them in their solution to the problem.

However, responses from a small number of students suggest that in some cases the concrete context can interfere with success in solving the problem as well. In particular, the common conceptual error that an object must be moving in the direction of the net applied force was made by a small number of students (4/78) in the concrete condition. For example, one of these students wrote, "Because net force is less than zero, static friction was not overcome and acceleration is zero". In contrast, *none* (0/92) of the students in the abstract condition made this error. However, the elicitation of this misconception may have been minimized in the particular concrete context given, as the friction force (large toy on concrete) is fairly salient compared to the force in the direction of motion (via a boy). This may partially explain why only a small number of students displayed the misconception.

Experiment 2

Experiment 1 replicated the finding that the concrete condition provides benefits to problem solving. However, there was also an indication that more concrete contexts can cue misconceptions more frequently. The purpose of Experiment 2 is to determine more directly whether a concrete context cues a misconception more frequently than an abstract context. We elicit the same misconception as in Experiment 1, namely the student belief that the direction of the net force on an object and its velocity must be in the same direction.

Participants

Two hundred and fifty-two participants were enrolled in the first quarter of a calculus-based introductory physics series at a large research university. They completed this assignment for participation credit as part of their total course grade.

Table 2. Question Set 2: isomorphic abstract and concrete versions of a force-motion conceptual question.

Abstract version	Concrete version
At a particular instant of time, there are several forces acting on an object in both the positive and negative direction, but the forces in the negative direction (to the left) are greater. Which statement best describes the motion of the object at this instant? a) it is moving to the right b) it is moving to the left c) it is not moving d) both a and b are possible e) both a and c are possible f) a, b, and c are possible	A force sensor is attached inside a soccer ball that is used during a match. The force sensor measures the forces acting on the ball. At a randomly chosen instant during the game, the sensor detects that there is only one horizontal force on the ball, and that force is directed toward the home team goal. Which statement best describes the motion of the ball at this instant? a) the ball is moving toward the home team goal b) the ball is moving away from the home team goal c) the ball is not moving d) both a and b are possible e) both a and c are possible f) a, b, and c are possible

Note: For both questions, answer *f* is the correct answer.

Materials design and Procedure

Experiment 2 is a within-subject design. Participants completed the multiple choice questions shown in Table 2 in a quiet, proctored room. These two questions were part of a larger set of multiple choice questions and the design was counter-balanced for question order (no reliable differences in order were found). The validity and reliability of the multiple choice questions was verified through extensive testing and interviews (Rosenblatt, Sayre & Heckler, 2008). These two questions are designed to elicit student beliefs about the relationship between the direction of net force on an object and the direction of its velocity[1].

The final course grades of the students were also collected. The student were then categorized as the High grade students (n =123) and Low grade students (n = 119), indicating whether they were in the top half or bottom half of the class as measured by the final course grade.

Results and Discussion

While both questions were difficult overall, students answered the Abstract question correctly more often (20%) than the Concrete question (9%), McNemar's test $p < .001$, effect size $d = 0.32$. Figure 2 shows the proportion of students choosing the correct answer choice separated out by High and Low Grade students. Inspection of Figure 2 reveals that High and Low grade students perform similarly (poorly) on the Concrete question and the High Grade students perform significantly better on the Abstract question than the low grade student. A repeated measures analysis, with question type as the repeated measure and grade rank as the between student factor, reveals significant main effects of question type ($F(1) = 18.4, p < .001$), and rank ($F(1) = 4.5, p = .035$), and there is a reliable interaction between question type and grade rank, $F(1) = 8.0, p = .005$. This interaction is mainly due to the High Grade students choosing the correct answer more often for the abstract question (26%) than for the concrete question (10%), McNemar's test, $p < .001$. There is no significant difference in choosing the correct response between the concrete (12%) and abstract (8%) question for Low grade students, McNemar's test $p = .5$.

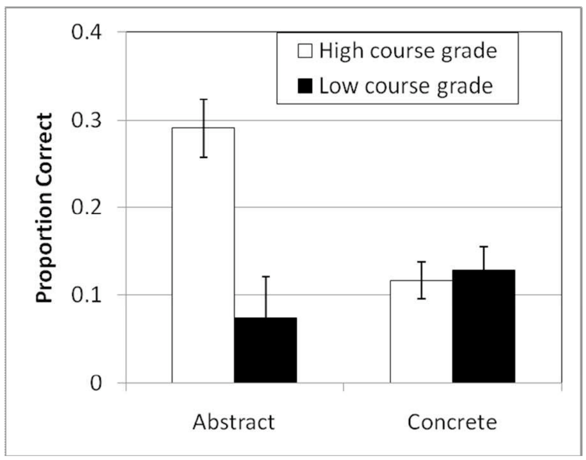

Figure 2: Student performance on Question set 2, separated by High course grade students (n = 123) and Low course grade students (n =119). Error bars are 1 SEM.

Experiment 3

The purpose of this experiment is two-fold, first it is to replicate the finding that while concrete context help with simple problems, more complicated problems may have increased performance in more abstract problem representations (Koedinger et al., 2008). Therefore, we pose a slightly more complicated problem (involving three rather than two equations and unknowns). The second purpose is somewhat exploratory. Experiments 1 and 2 indicate that everyday knowledge may in some cases hinder problem solving because it may misguide the solver to an incorrect solution path. In Experiment 3, we pose a simple kinematics problem that lends itself to using everyday knowledge, but may also present cues in the concrete context that lead to incorrect assumptions.

Participants

The participants were 68 university students enrolled in the first quarter of a calculus-based introductory physics course at a large research university. They completed this assignment for participation credit as part of their total course grade. Participants were randomly placed in one of two conditions, the abstract (n=32) and concrete (n=36) contexts conditions.

Materials design and Procedure

Participants were given one of the two physics problems shown in Table 3. They were given 15-20 minutes to complete the task in a quiet, proctored room. The problems are simple kinematics problems that would be typically found in the back of a textbook or on an exam, and are given routinely in homework assignments. The problems are typically solved by determining three unknowns, the distance that each person travelled and the acceleration of one of them. This involved using three equations: the total distance $d_{tot} = d_1 + d_2$, the kinematic equation for the constant velocity person $d_1 = v_1 t$, and the kinematic equation for the constant acceleration person $d_2 = a_2t^2/2$. Therefore this problem is slightly more complicated than the problem in Experiment 1, which only needed two equations and two unknowns.

Table 3. Question Set 3: isomorphic abstract and concrete versions of a kinematics show-work problem.

Abstract version	Concrete version
Two objects are 60 kilometers apart and move toward each other such that they will meet at a point in between the original starting positions. The first object travels in the positive direction toward the meeting point at a constant speed of 18 meters/second. The second object begins at zero speed and constantly accelerates in the negative direction toward the meeting point. If both objects meet in exactly 30 minutes, what was the acceleration of the second object?	Two friends, Kevin and Claire, live 60 kilometers apart and decide to meet in a town called "Tristville" in between them. Claire drives eastward toward Tristville at a constant speed of 18 meters/second. Kevin begins at zero speed and constantly accelerates westward toward Tristville. If they both reach Tristville in exactly one-half hour, what was Kevin's acceleration?

Results and Discussion

There are three important results from Experiment 3. First, unlike Experiment 1, students were equally successful in obtaining a correct solution in the concrete condition (47%) than the abstract condition (53%), $\chi^2(1) = .2$, $p = .6$. Second, Figure 3 shows the proportion of students obtaining the correct solution, separated out by High and Low Grade students. Inspection of Figure 3 reveals a pattern similar to the results in Experiment 2: High and Low grade students perform similarly on the Concrete question and the High Grade students perform significantly better on the abstract question than the Low Grade students. A two-way Generalized Linear Model (question type × grade rank) with a binary response analysis reveals a significant main effects of rank (Wald $\chi^2(1) = 9.9$, $p = .002$), and there is a reliable interaction between question type and grade rank, Wald $\chi^2(1) = 4.1$, $p = .04$. This interaction is mainly due to the High Grade students choosing the correct answer more often of the abstract question (87%) than for the concrete question (57%). While there may be a trend for students choosing the correct response in the concrete condition (41%) compared to the abstract condition (24%) for Low Grade students, the difference is not statistically reliable.

The third result comes from a closer examination of the solution methods which reveals an unanticipated and fairly frequent misunderstanding of the problem. In particular, 28% (10/36) of the students in the concrete condition made the same mistake of explicitly assuming that the two friends in the problem met *halfway* when in fact it is only stated in the problem that they met "in between" the starting points. None (0/32) of the students in the abstract condition made this erroneous assumption. Students' remarks in debriefing revealed that the most plausible explanation for this assumption is that the students in the concrete condition assumed that "friends meet halfway". In the abstract condition, the "friends" are replaced with "objects", and none of the students appeared to be cued to assume that the objects met halfway. Therefore, this is a case in which the concrete context appears to be interfering with the correct solution. It is important to note however, that the fraction of students in the concrete condition making this assumption did not depend on grade rank $\chi^2(1) = .8$, $p = .4$. Therefore, if this "misconception" (which could also be viewed as simply an incorrect assumption) is responsible for the difference in score between the abstract and concrete conditions for the High Grade students, it remains to be explained why the Low Grade students did not have a similar gain in score. It may be that the low grade students are such poor performers that switching from concrete to abstract simply involves switching from one incorrect method to another incorrect method.

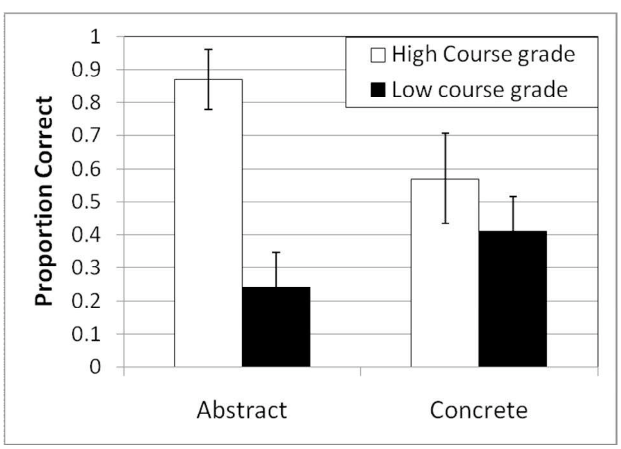

Figure 3: Student performance on Question Set 3. Error bars are 1 SEM.

General Discussion

There are four main results from this study. First, the advantage of concrete problem representation found in Koedinger et al. (2004, 2008) was replicated for a relatively simple physics problem. This experiment was different from the previous studies in that both of the representations were in "story" format. Nonetheless, the nature of the advantage in the concrete representation, namely better comprehension of the problem appears to be consistent with these previous studies.

The second finding was a significant interaction between abstract vs. concrete problem representation and student course grade for a simple conceptual question that is known to strongly elicit a well-known scientific misconception. In particular, the concrete, familiar representation was likely to elicit the misconception at a similarly high level for both High and Low Grade students; however, in the abstract representation, the High Grade students performed significantly better than the Low Grade students. This result could be interpreted as support for the idea that incorrect answering in the concrete representation is due to cueing of everyday, familiar knowledge common to all students while the abstract representation is less likely to cue such knowledge. The High Grade students may answer correctly more often in the abstract context because they may tend to use explicit reasoning skills rather than on relying on automatic, prior knowledge. Another possible explanation is that both High and Low Grade students began the course at floor on these questions, and the High Grade students have learned the abstract representation simply because the course may have presented more abstract than concrete examples, thus they would perform better on the abstract questions. In contrast, the Low Grade rank students may have simply not learned the material yet, thus they score low on both abstract and concrete questions. Understanding the distinction between these two possibilities is worth further investigation, and it is possible that both may be at work.

The third finding was a lack of difference in class averaged performance of the concrete and abstract representations for a problem that was slightly more complex (as measured by number of equations and unknowns to solve) than the problem in Experiment 1. This is somewhat consistent with the findings of Koedinger et al. (2008) in that they found an increase in advantage of the abstract representation when the problem difficulty was increased.

The fourth finding is similar to the second, but for a quantitative problem rather than a conceptual question. In particular, High and Low Grade students perform similarly in the concrete representation, but High Grade students perform significantly better in the abstract representation. A similar proportion of High and Low Grade students displayed an incorrect assumption in the concrete representation that was apparently caused by the familiar nature of the representation, and no students displayed this assumption in the abstract condition. Once again it is not clear what causes the difference in performance between the High and Low Grade students. The two explanations given above for the second finding remain viable. The concrete representation may be cueing familiar automatic problem solving processes and the abstract may be cuing (in High Grade students) more explicit, deliberative processes. On the other hand, it may be the case that (effectively) only relatively abstract examples are practiced in the course and only the High Grade students have learned them. In any case, it is clear that the concrete representation is cuing knowledge that inhibits the solution process, at least in High Grade students.

In sum, while some advantages of concrete representation have been replicated, this study identifies an important disadvantage for concrete representations: they can cue familiar knowledge that is contrary to scientific knowledge and this in turn can lead solvers to incorrect solution paths. This raises some practical issues about the relative usefulness of concrete and abstract questions as a diagnostic instrument. From a psychometric perspective, the abstract representation would be preferable over the concrete representation because the former is more successful at discriminating between students, at least as measured by final course grade. However, from a pedagogical perspective, the results from this study suggest that both high and low level students are susceptible to misconceptions cued by problems posed in the concrete representation format. Therefore, concrete problem representations are important for determining whether both High and Low Grade students have overcome the relevant scientific misconceptions. Once again, these results highlight the important tradeoffs between problems with concrete and abstract representations.

Endnotes

(1) Note that the two questions are not exactly isomorphic: the abstract question mentions forces in two directions and the concrete question mentions only one force in one direction. However, in our previous work in constructing items of this type we found that this difference does not reliably or consistently change student answering patterns.

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