Mathematical Thinking and Representational Engagement of In-Service Secondary Mathematics Teachers in Pattern-Based Algebraic Growth Tasks

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Abstract: Mathematical practice standards ask teachers to support students in constructing and using mathematical models, but little is known about the ways teachers themselves engage in such practices. This paper examines the kinds of mathematical thinking and representational engagement exhibited by secondary mathematics teachers as they completed pattern-based algebra tasks. Think-aloud data was coded for mathematical thinking and representational engagement. Results suggest that teachers' thinking and representational approaches vary relative to their pattern-based task experience.

Introduction

Within the last twenty years the field of mathematics education has increasingly emphasized the role of students as users *and* creators of mathematical models. The *Common Core Standards for Mathematical Practice* require that a "mathematically proficient student" flexibly and strategically create and employ diagrams, tables, graphs, and other representations, as well as use appropriate concrete or digital modeling tools, in order to identify and make sense of mathematical relationships. Mathematical representations are complex tools to both use and study (Kaput, 1998), and much work has been done to help the field understand the ways in which teachers can use modeling practices and student-created representations to support learners (e.g. Dreher & Kuntze, 2015). However, limited work exists that considers the ways in which teachers themselves construct and make use of mathematical representations in the context of their own problem-solving practices.

In this study, I consider patterns in teachers' mathematical thinking and their engagement with external representations across a series of mathematical modeling tasks. I adopt Goldin and Kaput's (1996) definition of external representations as those that are "physically embodied, observable configurations such as words, graphs, pictures, equations or computer microworlds" (p. 400). External representations are cognitive tools made socially available to learners within the classroom community (White & Pea, 2011). Thus, the ways in which teachers model mathematical ideas and make these modeling practices available to students is an area rich for inquiry, as teacher-created external representations function as socio-cognitive tools for both teachers and their students.

I extend Bruner's (1966) framework for types of external representations – enactive, iconic, and symbolic – to study the ways in which teachers engage with and make use of representational types in mathematical modeling tasks. I also draw upon the mathematical thinking literature – specifically the work on arithmetic thinking (Carpenter, Franke & Levi, 2003), functional thinking (Blanton, Brizuela, Gardiner, Sawrey & Newman-Owens, 2015), relational thinking (Jacobs, Franke, Carpenter, Levi & Battey, 2007), and spatial reasoning (NCTM, 2000) – to consider how teachers make sense of the tasks. Together, these frameworks help examine the ways in which teachers-as-learners conceive of and enact mathematical modeling practices.

Methods

Ten adults who worked in the field of mathematics education were asked to think aloud while completing a series of five pattern growth algebraic tasks. All participants had formerly or were currently serving as secondary mathematics teachers, and all still worked in the field of mathematics education. Some had little experience with pattern-based tasks, and others had used pattern-based work as a cornerstone of their classroom curriculum. Each of the tasks was purposively structured differently so as to elicit varying approaches for entry into the tasks. All tasks ultimately either provided an equation that generalized the algebraic growth or asked for one, as part of the task solution. Similarly, all tasks either provided a pictorial representation of the growth or asked that a pictorial or manipulative-based representation be constructed

Participants were videoed and all paper artifacts were collected and cataloged. Think aloud videos were loaded into Angles software (fulcrumtec.com) for analysis. Video was then segmented in two phases. In the first phase, I selected and bounded meaningful units by shifts in participant strategy. In the second phase, I segmented units by shifts in representation. Beginning with the units bounded by shifts in strategy, the author and an additional coder individually worked through a segmented think-aloud video, utilizing *a priori* codes concerning types of mathematical thinking. Codes were developed from the literature, using definitions for arithmetic thinking, functional thinking, relational thinking, and spatial reasoning. Segmented units identified in the second

phase were also coded using *a priori* codes drawn from Bruner's (1966) framework, relating the type of external representation – enactive, iconic, or symbolic – to the ways in which participants engaged with them – creating, modifying, or referencing. Once twenty percent of the data had been coded individually by both coders, an inter-rater reliability test was conducted, achieving a pooled kappa of $\kappa = .91$. The author then coded the balance of the segmented data, identifying themes and patterns both within and between mathematical thinking and representational engagement codes. Additionally, patterns within and across participant task-experience subgroups were analyzed.

Findings and discussion

Analysis of both mathematical thinking and representational engagement codes surfaced differences between teachers' thinking and modeling practices relative to their level of experience with pattern-based algebraic growth tasks. Mathematical thinking codes revealed that the thinking of teachers who self-identified as having less experience with pattern-based tasks was often characterized by high concentrations of functional thinking. In contrast, teachers who self-designated as having much experience with pattern-based tasks often used more relational thinking in their work. Some segments were double coded, as teachers utilized multiple types of thinking. For example, one teacher isolated the n^2 term from a task's given expression to inform only the quadratic component of the visual pattern she was constructing for the cases N=1, N=2, and N=3. That is, she used the term's output as a tool to build the quadratic portion of the representation relationally across cases. Only after she completed constructing the quadratic portion of the representations for N=1, N=2, and N=3 did she consider the representation of the expression's linear term.

Codes for representational engagement also revealed noteworthy patterns. Teachers who self-designated as having much experience with pattern-based tasks utilized small proportions of symbolic written representations. Conversely, teachers who had little pattern-based task experience often utilized high proportions of written representations. Teachers with little to moderate experience with pattern tasks sometimes created a tertiary representation, outside of those given and requested by the task, in order to make sense of the given representation and work toward the one requested as a product.

These findings suggest that teachers' differential experiences with particular representations may inform the way in which they make sense of and construct mathematical models. Further, this paper reveals possible design implications for professional learning focused around mathematical modeling — first to consider teachers' prior modeling experiences and second to design for rich learning opportunities that make use of varied forms of representations. The different natures of the five think-aloud tasks may also contribute to the ways in which teachers modeled the mathematics. Therefore, questions about pattern-based task design in relation to teachers' mathematical thinking and representational engagement remains an area for future inquiry.

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