Environments for Collaborating Mathematically: The Middle-School Mathematics through Applications Project

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Abstract

Math reforms call for deeper understandings of concepts, more collaboration among students, and use of technologies. The Middle School Mathematics through Applications Projects (MMAP) is developing computer technologies to support mathematics learning by middle schoolers. We outline the premises underlying our approach to software design and describe three software environments and the mathematical activities they support. Two features stand out: (1) the computer environments are resources in a larger activity structure—the group-based project; and (2), the work students do inside the software environments helps generate students discourse and mathematical activities.

Keywords — Computer-based learning, collaborative group work, mathematics education.

1. Introduction

Computer technologies can be powerful tools for doing mathematical work; they can also be powerful tools for learning mathematics. They can perform complex and tedious computations, quickly graph and transform curves (Confrey and Smith, 1992; Schoenfeld, 1990), display multiple representations of a problem situation (Kaput, 1989; Pea, 1987), display empirical data (Rubin, 1994), and support dynamic visualization of geometric objects (Lampert, 1988; Olive, 1994). Mathematics educators and researchers are aware of the potential that computers can bring to the math learning enterprise and have developed many different kinds of software environments.

Curricular guidelines and research in mathematics education have recommended the development of classroom activities that focus on the conceptual aspects of mathematics and that support conversations as a context for improving conceptual learning in mathematics classrooms (NCTM, 1989). The charge is for deeper understandings of mathematical ideas and more collaborative processes in classrooms. The expectation is that freeing students from the tedium and procedural burden of excessive calculating or graphing will enable them to turn their attention to mathematical objects and concepts.

Student collaboration has also been proposed as useful for improving conceptual learning in mathematics classrooms. Group work is expected to encourage an environment in which students can "explore, formulate and test conjectures, prove generalizations, and discuss and apply the results of their investigations" (NCTM, 1989). Researchers have explored peer collaboration and mathematical discussions, as well as discussions between students while using software (Hooper and Smith, 1994; Moschkovich, 1993; Goldman, 1995).

We are committed to the development of computer technologies to support mathematics learning by middle- school students. We have been actively engaged in a research and development initiative for four years, the Middle-School Mathematics through Applications Project (MMAP).[1] While MMAP is not solely a technology development project, the development of computer-based environments is a key component of our work.

When we began our work, we considered many ideas about how computer technologies might help

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children be more successful at learning math. First and foremost was that any computer technologies we developed would have to be useful tools for meeting the learning needs of students who have been underserved by more traditional approaches to school math. Our theories and ideas about how children learn mathematics would inform design decisions and eventually become embodied in our software (Bowers, 1988). We wanted to create modeling and simulation environments that would encourage a collaborative kind of mathematics in the classroom. We wanted to test the possible roles that technologies might play in creating a more applications-based and collaborative version of school mathematics. Our belief that learning is a social activity meant that we wanted to create classroom environments where students would become equally engaged with each other and mathematics.

In this paper we tell a short story of how we conceive and design math projects and computer environments. The story highlights two important points about the reciprocity of the MMAP software environments: (1) the computer environments are interactional and mathematical resources in a larger activity structure—the group-based project; and (2) design work students do inside the simulation and modeling environments help to generate opportunities for mathematical activities and student discourse. We outline the premises embedded in our approach to design and describe three software environments and the kinds of mathematical engagement they support.

2. Environments for Collaborative Math Learning

Our view is that mathematics is crucial to everyday living and to current scientific practice. Hence, our materials are designed to reflect that view through a group-based applications approach to learning mathematics. Central to our applications project focus is the placing of students in design problems that require the use of mathematics concepts as tools for their solutions. By experiencing the utility of applying these concepts as tools, students have a chance to appreciate the power of the mathematics. The students can compare and deepen their evolving understanding and mastery of the tools-in-use.

Our charge is to design mathematics learning environments that integrate real-world problem solving, collaborative group work, and material resources that include computer-based tools. MMAP software must help us meet several goals, and our design and development work has evolved continually. From the start the process has been cyclical and iterative. The process relies on cycles consisting of four main activity components: design, prototyping, field testing, and analysis and redesign. The most characteristic feature of the design process is that it is heavily school-based. All development stages are informed by the real everyday conditions of middle school teachers, students and classrooms. MMAP team members observe and videotape in classrooms and interview teachers and students at each stage of the development process, first to validate application ideas, and then to test prototype designs against desired interaction with mathematics.

When we began development of our first software prototype, ArchiTech©, our criterion was that it had to pass a student engagement test and a math "yield" test. After one full year of prototyping and field testing the design environment for creating floor plans, we were able to settle on a design premise that met the engagement and math yield criterion. We settled on the idea that the computer environment and the activity environment needed to become symbiotic; one was only useful and engaging if it was totally interlocked with the other. This was a major departure from the ways we had seen other software for mathematics learning come into development. We do not develop "mathematics" software with some suggested activities for their use. We first decide on the kinds of interactions and collaborations we want students to have with mathematics, design activities to realize them, and finally, decide whether supports for the interactions are best left in the software or in the classroom activity structures.

These design decisions mean that our software environments are quite consciously not stand-alone in any sense of the term. To complete unit activities a group of students have to use some component of the software, and while using some part of the software, emergent classroom math activities are generated. Using the software is only satisfying in yielding the range of middle school mathematics we expect if the students are engaged in solving particular kinds of problems with corresponding constraints. The computer applications support designing solutions to these problems by providing multiple representations, levels of operation, and analytic opportunities.

Currently, there are three MMAP software environments. In each technology supported project, groups of students role-play adult occupations: architectural designers (using ArchiTech©), cryptographers and cryptanalysts (using Coding Toolbox©), or biologists and policy designers (using HabiTech©). Each individual plays a role in a group, and each group in class–wide activities. The goal is to create situations where the math can be helpful in solving a common problem, and student conversations and discussions help to bring about new mathematical understandings. The computer simulation tools allow students to immediately design solutions to their problem without getting bogged down in unnecessary technical details, computation, or symbolic forms. The environments give students access to multiple representations of their work and information about the different designs they create. The technology also helps students organize data in certain ways, structure their reportability, and use the data in explanations.

3. Three Software Environments for Engaging Students in Math Learning

In *The Antarctica Project* unit students take on the role of architectural designers who are creating a research station for a scientific expedition to Antarctica. Students use ArchiTech© to create floor plans and retrieve information on their structures' heating and building costs. Students have opportunities to develop and apply measurement and scale concepts. They work with independent and dependent variables and controlling variables situations through math opportunities which utilize hands–on experiences with the software.

With the ArchiTech© environment the group's floor plan becomes a conversation piece in two ways. First, because the group shares a design, their design decisions need to be negotiated, made explicit and justified. These aspects of design are also supported by the curriculum and the classroom activities for this unit. Second, students can make conjectures about how changing a variable, such as insulation, might affect the cost to heat a structure. They can then test these conjectures by changing the value of insulation in their design and seeing how other quantities are affected by this change.

In the *LifeLines* unit, students work as population-biologist advisors to policy makers regarding wildlife issues. Students use HabiTech© to create and run dynamic mathematical models of wildlife interactions and graph or chart changes in those interactions over time. They translate from verbal descriptions of animal behavior to mathematical descriptions and create relationships among variables such as animal birth rates and death rates. Students explore how different kinds of functions produce different patterns of growth (linear, logistical, and sinusoidal).

In the HabiTech© environment, students experience the connection between two representations, natural language and algebra, as they make and test conjectures about different rates of birth and death. They also alternate between tabular, algebraic, and graphical representations of different relationships.

In the *Codes Inc.* unit, students take on the role of cryptographers and cryptanalysts, designing and analyzing codes for various clients. Using the Coding Toolbox©, with its three different tools, students work with patterns and algebraic functions in various representations, use tables, verbal rules and graphs to represent number patterns, and are able to see and use both linear and non–linear functions. The Matrix Tool lets students work with transposition ciphers, rearranging the letters in a message by using a matrix and using number theory (factors, primes and multiples) to construct and analyze these codes. The Matrix Tool requires students to use patterns based on factors and remainders. The Function Tool lets students use a subset of algebraic expressions with a single variable to create replacement ciphers, where each letter in the alphabet is assigned a symbol using functions. The Analysis Tool, useful for "breaking" codes, requires that students look for and use patterns both in the frequency of symbols and letters and in the code key table.

These environments provide some of the usual types of support that one considered useful for software environments in mathematics. For example, while using Coding Toolbox© students use the connections between the graphical and algebraic representations of functions to decide whether the equation for a code is a first or higher degree equation. Most unusually, these environments are designed to support early access to some mathematical concepts. For example, the Coding Toolbox© environment can be useful without students generating their own symbolic forms or graphs. It thus supports the development of qualitative understandings of functions and the connections between two representations. These serve as an entry point to a conceptual view of functions while being used in the service of solving a larger problem. Likewise, HabiTech© allows the creation of models based on either natural language or symbolic expressions, thus allowing earlier entry into the processes of mathematical modeling.

4. Summary

Our goal is to develop technologies that become seamless components of the classroom activities. We have strong ideas about what we want computer technologies to do for middle-school aged children and even stronger ideas about the kind of collaborative math learning we want to make possible. Our orienting criteria are that the computer-based environments need to:

- be easy to use and invite all students to engage in mathematical activities, regardless of past school math history or level of mathematical sophistication;
- reflect the mathematics standards and frameworks by supporting collaborative learning, complex problem solving, understanding and use of multiple representations, analysis, explanations, etc.; and,
- bridge the gap for students between school math and the real world where mathematics is useful.

Today, three software environments are up, running, and robust enough to be field tested in a variety of classrooms across the country. Our research team has observed and documented in field notes and videotape over 300 hours of collaborative work, software use and mathematics conversations by middle schoolers involved in MMAP. One focus of the project's research program is to examine if the environments support conceptual learning and collaboration among students.

To date, two findings seem promising. The environments generate and support conversations between students. These conversations can serve as the sites for negotiating and transforming students' descriptions of mathematical situations (Moschkovich, 1993). The students' computer- generated designs and supporting data serve as the referents for conversations, provide a medium in which students can make, record, and evaluate competing conjectures, and provide entry points into mathematical topics and teachable moments.

The environments also shift the locus of control in the design work from the teacher to the students. Each environment allows students to create their own design. Whether they are being questioned about their progress or about the features of the design, the students must explain their work to the teacher before next steps are negotiated. The responsibility for explanation lies with the students even when they are seeking the help of their teachers. They must "bring the teacher up to speed" in order to make it possible for the teacher to help them.

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