An Alternative Perspective for Developing a Mathematical Microworld

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Abstract

This article describes one attempt to develop an alternative perspective for designing a computer microworld by using a bottom-up approach to instructional design. This approach differs from traditional instructional design in two ways. First, the program was intended to augment an existing activity sequence rather than serve as an independent environment separated from the students' regular classroom work. Second, it is considered to be bottom-up in the sense that its aim was to engage students in activities that were consistent with their provisional ways of knowing. The first section of this paper describes how the constructs of reification and distributed intelligences were combined with Freudenthal's three design heuristics to guide the development of the microworld. The second section discusses some data from a project classroom in which the program was used.

Keywords — mathematical microworlds, collaborative learning, interface design.

1. An Alternative Approach to Designing a Microworld

The constructs of reification and distributed intelligences offer complementary ways of explaining how learning occurs as students use computers. On the one hand, psychological constructivists such as Kaput [5] suggest that when students are acting on dynamic symbol systems, they are able to reflect on and reify their actions into mathematical objects. On the other hand, Pea [7] assumes a socioculturalist perspective to view learning as a process in which students off-load their lower-level cognitive tasks to the surrounding environment. This off-loading frees the mind to engage in higher-level thinking and planning tasks. From an emergent point of view [1], a coordination of these two constructs suggests that there is a reflexivity between Kaput's view that learning originates in reflection on real-world actions and Pea's view of the computer as a cognitive reorganizer of mental activities.

The purpose of this article is to describe how the constructs of reification [8] and distributed intelligences [7] informed the development and research of a computer microworld. The program, which was developed as one part of a three year teaching experiment, served as one vehicle for exploring the reflexive relationship between the social and psychological components of learning. The guiding theoretical framework suggests that neither individual nor social components can adequately account for learning without considering the other [1]. Therefore, the goal for designing this microworld was to encourage whole-class and group discussions by building an environment in which students could distribute their intelligences to support their individual processes of reification. To this end, the microworld described in this paper offers an alternative approach to design for two reasons. First, it was designed to be used as part of a larger instructional sequence and therefore it was intended to support whole-class discussions rather than being used as a tutor helping individuals who are working alone. Second, it attempts to engage students in activities that are consistent with their current ways of knowing rather than having them interpret expert models. In this way, meanings are seen as emerging through engagement in the social interactions and mathematical practices of the classroom.

1.1. The Candy Factory and the MacCandy Factory

The Candy Factory instructional sequence was first used in a second-grade classroom in Indiana. Reflections on that project were used to inform the development of the present sequence and accompanying microworld which were used in a third-grade classroom in Tennessee. The intent of the sequence was to support students' construction of increasingly sophisticated conceptions of place value numeration and increasingly efficient algorithms for adding and subtracting three-digit numbers. The basic scenario involved students imagining that they were working in a candy factory in which candies were packed in rolls of ten, and rolls were packed in boxes of ten. During the teaching experiment described in this paper, the instructional activity was implemented in three phases. First, students were asked to "pack" and "unpack" Multi-Link cubes to build imagery for estimating quantities. After the students developed a sense of this activity, the first computer microworld was introduced to facilitate the process of packing and unpacking larger quantities. The two goals of this phase of the sequence were to engage students in activities that involved the composition and decomposition of arithmetical units, and to help them develop estimation skills for larger numbers.

The second phase of the sequence involved the introduction of an inventory form to keep track of transactions in the storeroom. The issue of how to record these transactions (and thus the composition and decomposition of arithmetical units) became an explicit focus of the children's activity. Students initially used drawings and other means of symbolizing to model their mathematical reasoning. After the class negotiated a common notation scheme, a second microworld that was consistent with the class's notation was introduced. This microworld contained a linked representation system which provided an opportunity for students to see how their packing and unpacking activities using the graphics were reflected on the inventory form, and, conversely, how changes on the digits of the inventory form were reflected in corresponding changes on the graphics. As a final phase of the sequence, the inventory form was modified and addition and subtraction tasks were presented in vertical column format. After several days of instruction using drawings to support their addition and subtraction models, a third microworld was introduced. This final program was designed to fit with the students' present ways of interpreting adding and subtracting as well as support the development of increasingly sophisticated conceptions.

2. Three Design Heuristics

In order to develop the Candy Factory instructional sequence and the *MacCandy Factory* microworlds, the investigators collaborated with designers at the Freudenthal Institute in the Netherlands. Their approach, entitled "Realistic Mathematics Education" (RME), has lead the mathematics reform movement in the Netherlands. The underlying assumption of this approach is that there is a reflexive relationship between design and research such that each phase informs the perspective of the other. The intent of this collaboration was to coordinate RME's three underlying design assumptions with the sociocultural framework of distributed intelligences [7] to guide the development of instructional sequences and computer learning environments that are consistent with reform-based instruction in mathematics in the United States [6]. The three basic tenets of RME form a strong core of beliefs about mathematics, pedagogy, and mathematics education itself [4]. These core assumptions constitute one example of a unified set of design heuristics that are consistent with the tenets of constructivism [1]. Each of these will be further described below in order to derive a framework for developing and researching other microworld designs.

2.1. Assumption #1: Mathematical learning occurs through mathematization and guided reinvention

This assumption is based on a view of mathematics as an activity rather than a static group of facts that must be assimilated. As students engage in and reflect on their activities, they reinvent mathematical concepts that are personally meaningful. In this way, their mathematical conceptions grow out of their own interpretations of their actions. Gravemeijer [4] uses the term "mathematization" to describe the process by which students begin to develop ways of organizing their situated activities into mathematical interpretations. Although each student interprets his or her activities individually, the teacher can guide this process by encouraging students to discuss their increasingly sophisticated mathematization strategies, with particular emphasis placed on those strategies that are efficient. In the case of the Candy Factory sequence, students' own situation-based activities of packing Multi-Link cubes arose out of their need to count large numbers of candies. Eventually, through whole-class discussions, the students negotiated meanings for the symbols they used to mathematize their activities using pencil and paper. Once these class conventions were established, the teacher introduced the *MacCandy Factory* microworld to support their mathematizing and symbolizing.

In order to facilitate these activities, the computer microworld was designed to include simplistic representations of boxes, rolls and pieces that resembled the students' previously documented notational schemes. The decision to use two-dimensional figures rather than more realistic drawings was deliberately made to support students' understandings of how their own imagery and symbols might be reflected in their microworld activities. Because the icons shown in the microworld were designed to be consistent with the taken-as-shared symbols used in the class discussions, the meanings of the symbols were not assumed to be built into the interface. Further, the interface was not viewed as "containing" meaning, or as a tool for conveying the concept of place value. Instead, it was designed to support students' mathematization activities from the bottom-up; that is, the program was designed to fit with their current ways of knowing and be consistent with the practices of the larger social environment of the classroom discussions.

2.2. Assumption #2: The process of development can be guided by didactic phenomenology

Didactic phenomenology refers to Freudenthal's notion that learning occurs as students create conceptions of mathematical objects by engaging in context problems. Contexts that are described as phenomenologically rich are those which contain situations that "beg to be organized." In the classroom, didactical phenomenology refers to the teacher's sense of how to recognize and capitalize on different students' mathematizations in order to lead to more formal mathematical conceptions. In the present teaching experiment, the program played a role in supporting this mathematization by providing an environment that enabled students to describe their informal strategies so that the teacher could guide discussions toward progressive mathematizations.

One design implication from this assumption is that the development of phenomenologically rich contexts can be informed by two sources, 1) a historical account of mathematics, and 2) observations of prior students' interpretations. These two sources can guide the design process by suggesting possible learning routes to be included in a learning sequence. Additionally, these sources help the designer to focus on the role of the student as an active interpreter. In the case of the Candy Factory sequence, both an historical account of the development of place value and previous research from other teaching experiments provided information regarding how students might develop, interpret, and act on the graphics included in the program. These insights were invaluable as the designers discussed various interface decisions such as the use of arrows and the configuration of the icons in groups of ten. By basing such design decisions on students' interpretations, the designer avoids the top-down assumption that mathematical concepts can be embodied in the software but does create the potential for increasingly sophisticated thinking to emerge.

A second implication from didactic phenomenology suggests that if developers create a virtual world that enables reflection on prior actions and makes alternative actions possible, then students might be able to progressively refine their strategies in sophisticated ways. However, it is also critical to note that many students are not necessarily inclined to develop more sophisticated approached on their own. Nevertheless, such thinking paths cannot be imposed by the program. In contrast, the construct of didactic phenomenology suggests that these processes are socially accomplished as the students and teacher engage in whole-class discussions. For example, as the students and teacher mutually negotiate the sociomathematical norms, efficiency might emerge as an implicit criteria for discussing solutions [2]. Thus, the intent of the *MacCandy Factory* program was to support whole-class discussions, but not to replace the teacher or serve as a tutor with a pre-given instructional route planned out.

One example of how the principle of didactic phenomenology informed the design of the *MacCandy Factory* microworld was the decision of how to handle incorrect answers. Previous experience using the sequence in another teaching experiment revealed that students working with Multi-Link cubes often miscounted as they were packing. As a consequence, the class discussions often consisted of having students recognize their procedural errors. Using the dynamic feedback capability of the microworld, the students' numeric errors were easily caught and rectified. Consequently, the students were able to distribute this low-level counting procedure to the computer so that the class discussions could focus on higher level conceptions such as how packing and unpacking did not change the total number of candies in the storeroom. In this way, the computer supported higher-level discussions in which all students could participate.

2.3. Assumption #3: Self-developed models of actions can be reified to models for mathematical reasoning

This third principle discusses how self-developed (or emergent) models serve to bridge the gap between informal and formal knowledge [4]. When students initially attempt to make sense of context problems, they begin to mathematize their actions by forming models of their informal activity. Through the process of guided reinvention, students' models *of* their informal activities become models *for* more formal mathematical strategies. This approach, which is consistent with Sfard's theory of reification [8], varies widely from a representational view which suggests that students use pre-existing models (such as pre-structured manipulatives) that contain mathematical concepts embodied in their structure [3]. Through a process of reification, meanings and models reflexively emerge as the students engage in the activities and (re)interpretations. Further, if the situations are experientially real for the students, then certain conventions of the models can be introduced because the students understand the need for them. This varies from some traditional instruction wherein certain notation conventions appear (to the students) to be imposed with little rationale.

It may appear incongruous to suggest that students will develop their own models if they are interacting with pre-given representations in a computer environment. Indeed, in many sequences (even those that are not computer-based), the models are not literally invented by the students, but great care is taken to ensure that the models fit with the students' informal activities. This distinction can be used to illustrate the difference between the top-down and bottom-up views. If the students' task becomes one of guessing how the model works, then it becomes a top-down activity, even if this was not the original design intent. In contrast, if the students use the representations to form their own models of their situated activities, then the activity becomes realized in a bottom-up way.

In the case of the *MacCandy Factory*, the intent was that the students' drawings and the icons on the screen could serve as models *of* their situated packing and unpacking activity. As the sequence progressed, these models *of* their actions could become models *for* the more formal mathematical strategies of transforming quantity representations within the base ten system. The working assumption guiding this design process was that the meanings and mathematical understandings could not be conveyed through the program itself. Instead, meanings were intended to emerge from the bottom-up; that is, rooted in the students' models of the context. In this way, their thinking was implicit in their strategies and guided by the activities in which they engaged.

3. Preliminary Findings

Preliminary results from this teaching experiment have revealed that students' models were supported by their use of the *MacCandy Factory* software within the classroom environment. The constructs of reification and distributed intelligences provide insight for guiding our observations of how the students interpreted the activities. For example, after the students became more familiar with the interface, they were able to manipulate and conceptualize increasingly larger quantities of candies. Thus, their efforts to pack the candies were distributed to the computer enabling them to engage in the higher-order thinking skills of estimation and planning. For example, during one whole class discussion, the class debated different strategies for showing the canonical form of 3 boxes, 36 rolls, and 25 pieces by anticipating what would happen if all boxes and rolls were unpacked.

When using the LCD panel to project the *MacCandy Factory,* the type of language used in whole-class discussions shifted from the abstract language of "carrying" to the more concrete language of "packing." Over time, this shift in language reflected the students' shift in viewing the representations on the screen as boxes and rolls to more sophisticated conceptual units. This indicates that students may have been able to reify their models of packing and unpacking to develop increasingly sophisticated models for place value numeration. For example, several students developed the ability to estimate large quantities by building on their imagery of the conceptual units in the candy factory.

One of the most frequent topics of conversation centered around negotiating various notational schemes. It became evident that there was a great debate over the actual *use* of notation: Some students wanted a record of their actions, while others only wanted a representation of the final amount of candies. It is significant to note that the rationales used in these discussions were often rooted in the students' imagery of the candy factory. Further, the quality of the students' reasonings illustrated both their growing understandings of the inherent conventions of standard notation, and their increasingly sophisticated conceptions of place value.

The present data analysis suggests that although there was a wide variety of different interpretations by the end of the teaching experiment, all students had successfully developed more flexible conceptions of place value. There is a strong likelihood that this occurred because of the teacher's attempts to fold back the discussions to the imagery of the Candy Factory and the students' use of the *MacCandy Factory* in order to situate their informal activities. It appears that this alternative approach to software design combined with the teacher's expertise provided a way of building on students' informal strategies from the bottom-up to support their more formal mathematical conceptions of place value numeration.

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