Teacher Monitoring Routines: Understanding Pedagogical Judgments During Students' Collaborative Learning

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Abstract: In this paper we conceptualize *teacher monitoring routines* – a consequential yet understudied phase of instruction during students' collaborative work. We examine 5 lessons of experienced and equity-oriented secondary math teachers using a comparative case study design and analysis of their interactional routines. Our goal is to understand how teachers monitor groupwork and how they decide (intentionally or not) when, where, and how to intervene in student talk. We identified various patterns that make clearer the micro-judgments monitoring routines entail for teachers' (a) ways of moving among students and conversational *initiation*, (b) conversational *entry*; (c) *focus* of the interaction; (d) when and how to *exit* the interaction; and (e) conversation participation pattern; with an eye toward teachers' goals and instructional contexts. We end by considering implications for research on teaching and professional education in support of collaborative learning.

Purpose

As mathematics teachers shift toward what has been called "ambitious instruction" (Lampert et al., 2011), they frequently build on collaborative groupwork to promote students' sensemaking of mathematical ideas through interaction (Cohen & Lotan 2014; Horn, 2012; Smith & Stein, 2011). While teachers' judgments during groupwork involve managing tensions between intervention and student autonomy, these micro-judgments remain understudied and ill defined (Stein et al., 2008; Webb et al., 2009). Different scholars who study this phase of lessons suggest different roles for teachers, with some advocating that teachers primarily listen to conversations (Smith & Stein, 2011) or intervene only when a group is "hopelessly off task" (Cohen & Lotan 2014), while others suggest it is more productive for the teacher to be more active and probe students' thinking (Webb et al., 2009).

Our own interest in studying teacher monitoring routines did not arise as a theoretical gap to fill, but rather as an in vivo problem of practice we encountered as teacher educators. For the past several years, we have partnered with a professional development organization (PDO) to bridge formal professional development with classroom teaching through video-based coaching. As we have spent time in the teachers' classrooms and discussed their teaching with them, we have noticed two things: (1) There is a surprising variability across teachers' groupwork monitoring practices; and (2) The "shakiest" moments of lessons tend to occur in the monitoring phase. While tasks, launches, and instructional routines can be easily shared among teachers, monitoring is inherently interpretive and responsive, requiring more of the teachers' sensemaking in-the-moment. The uncertainties involved in this phase of the lesson, along with the imperative to support cognitive demand and productive engagement for all students, provoked our interest in identifying the sequence of judgments teachers make during this phase of their lessons. To this end, we offer a framework for teachers' monitoring routines as a sequence of moves, which we name *Initiation-Entry-Focus-Exit*, to better understand the choices teachers make. Building on examples from 5 secondary math teachers, this paper aims to probe and conceptualize teacher monitoring routines, and to make clearer the myriad micro-judgments (intentional or not) they entail for teachers.

Theoretical perspectives

Interactional routines

In their basic form, interactional routines are recurrent *moves* constituting the patterned ways conversations unfold within a certain social group, as within a classroom. However, while routines might have some recognizable structure, interactional routines are also *emergent*, as their details are contingent on the situation. In this way, interactional routines are both stable and performative, reflecting structure in their consistent aspects as well as the agency of those who use them (Feldman & Pentland, 2003; Horn & Little, 2010; Lavie et al., 2018).

Attuning to the tensions of structure and agency, our conceptualization of monitoring routines builds on two definitions. The first comes from Cazden (2001), who describes interactional routines as holding both sequential and selectional dimensions. Cazden compares these two dimensions to a Western restaurant menu, likening the sequential dimension to the set of categories (e.g. appetizer, entrée, dessert) that is culturally set and rarely varies, while the selectional dimension is the specific appetizer, entrée or dessert that a person orders. For

monitoring routines, the structural, sequential dimension is captured in our generic framework of initiation, entry, focus, and exit, coupled with students access to participation in the interaction. The agentic, selectional dimensions are then the particular choices teachers make as they initiate, enter, focus, and exit interactions, based on their interpretations and responses to the situation.

For that reason, we find it compatible to use Lavie and colleagues' (2018) definition of interactional routines as a *task-procedure pair*:

"a routine performed in a given task situation, by a given person, is the task as seen by the performer, together with the procedure she executed to perform the task." (Lavie, Steiner & Sfard, 2018, p. 9).

For example, in the case of groupwork monitoring routines, one teacher might see the *initiation move* as a task in which it is the teacher's responsibility to check-in with the groups. Another teacher might interpret the same task with different reasoning — for instance, as a situation where teacher-initiated conversation might interrupt students' thinking and thus would only approach a group if students had a question. Either way, some relatively stable structure of local classroom culture would be constructed, and students would know whether they need to raise their hands or simply wait for the teacher to come over. As we mentioned before, such teacher sensemaking is necessarily situated in the particularities of their teaching situations, including the teachers' instructional goals and their institutional constraints.

Situated view of teaching

Teaching is an irreducibly situated act, as teachers navigate complex relational and institutional terrain during instruction. By taking a situated view, we acknowledge teachers' instruction as co-constructed with the particularities of their teaching situations (Greeno, 1998; Horn & Kane, 2015; Lave & Wenger, 1991). As a consequence, teachers' monitoring practices are an interactional accomplishment between teachers and the particular students, content, and settings they are working with. Although routines may stay somewhat stable within individual teachers as they move between classrooms or groups, we also see teachers making microjudgments about whom they direct their attention to, in what ways, and for how long. For this reason, while abstract and idealized notions of monitoring practices may serve as a useful heuristic guide to practice, teachers necessarily exercise *pedagogical judgment* as they enact any instructional practice in their teaching situations (Horn, in press).

Applying this lens to the teachers in our study, we are especially interested in how their pedagogical judgment shapes the ways they monitor student worktime. By highlighting teachers' micro-judgments as they monitor students' work, we claim that teachers do more than simply implement known models of groupwork; teachers are constantly assessing, interpreting, and adjusting as groupwork unfolds in their particular classroom contexts.

Research question

Given our goal to probe and conceptualize teacher monitoring routines and considering our theoretical perspective, we ask: What interactional routines emerge during the focal teachers' conversations with student groups during the monitoring phase of the lessons? In the rest of the paper, we answer this question, with an eye towards the teacher micro-judgments that underlie these routines.

Data and methods

Research context and data

As we mentioned, this study comes from a research practice partnership between our university research team and a PDO. Our shared work centers on bridging formal professional development and classroom instruction, and we have worked to design a video-based formative feedback (VFF) coaching cycle to help elicit and engage secondary math teachers' pedagogical judgment. At the PDO, the secondary mathematics teachers have encountered different models of monitoring groupwork, including the aforementioned 5 *Practices* (Smith & Stein, 2011) as well as Complex Instruction (Cohen & Lotan 2014; Horn, 2012). In addition, they have attended workshops on the TRUmath Framework (Schoenfeld, 2014), which highlights issues of student agency and access to rich mathematics. In sum, our participants have had atypically rich opportunities to learn about ambitious instruction in formal professional development.

For this analysis, we looked at 5 lessons (see Table 1) from experienced secondary teachers in the PDO (5 to 18 years of teaching experience, mean = 8 years), in a large urban district in the western U.S. They were

selected because they included segments of 20 minutes or longer of teachers' monitoring students' mathematical groupwork (20 to 45 min segments, mean = 28.5 min). For all 5 lessons, we have two cameras recording the class session. Camera 1, a tablet camera on a robot tripod, captured the whole class, with a focus on the teachers' movements through the classroom. These video records also have student audio tracks captured through 4 separate microphones placed at students' desks. Camera 2, a point of view camera, was mounted on the teacher's head, shoulder, or chest to approximate what they were seeing as they moved through the classroom and interacted with students. In addition to these recordings, our data include lesson artifacts, conversations with the focal teachers before and after instruction, and email exchanges about the activities.

Table 1: information about the focal teachers and the lessons

Focal teacher	Experience (years)	School	Class	Date	Groupwork (minutes) (%)
Veronica Kennedy	8	Rees	8 th grade math	Feb 1 2018	33 of 82 (40%)
Brad Miller	6	Noether	Algebra 1	May 7 2018	20 of 58 (34%)
Bridgette Campbell	7	Johnson	Algebra 1	Oct 05 2017	20 of 85 (23%)
Lee Bellver	14	Falconer	Algebra 1	Mar 1 2018	26 of 100 (26%)
Lizette McLoughlin	5	Fermat	AP Calculus	Feb 2 2018	45 of 55 (82%)

Data analysis

Phase 1: Data reduction and re-representation

We edited the video to isolate the monitoring sequences of each lesson, reducing 380 min of classroom footage to 144 min. Then, using Final Cut Pro X, we edited the two camera perspectives together as a picture-in-picture to support coordinated analysis of the teachers' actions and a view of where they are located in the classroom. (See Figure 1.)



Figure 1. Two camera perspectives on Lizette's classroom.

Phase 2: The sequential dimension of teachers' monitoring routines

Using the subset of video data from Phase 1, we used inductive coding (Strauss & Corbin, 1998) to identify the sequential dimension of the *monitoring routine*. We arrived at categories that explained all 5 cases, and they were 5 recurrent *moves* that described: (a) how teachers moved around the room and how conversations were initiated (*initiation*) (b) their opening moves of conversation (*entry*); (c) the main *focus* of the interaction; (d) when and how they *exit* the interaction; (e) whether the teacher interacts with students as a group or as individuals (see Table 2). This constituted the general task of the emergent monitoring routines.

Phase 3: The selectional dimension of teachers' monitoring routines

Once we arrived at the basic sequence of monitoring routines, we could delve into the interactional details — the particular judgments teachers made in their teaching situations. Using an inductive comparative case study design (Strauss and Corbin, 1998) and methods of interaction analysis (Jordan and Henderson, 1995), we looked closely at the videos to explore patterns of how the 5 teachers took up each of the 5 moves identified in Phase 3. The coding team consisted of two PhD students (including the first author) and a secondary math teacher. During the coding process, the team met regularly, maintained an online "coding dilemmas" document and a codebook with definitions and examples for each code. As coding dilemmas were discussed and resolved among the coding team

and the Principle Investigator (the second author), the definitions and examples were refined. Peer Debriefing was done with the larger research team (2 additional PhD students, a post-doctoral fellow, and an assistant professor) to ensure the ecological validity of the coding scheme (Lincoln & Guba, 1985).

Once the codes were more or less agreed upon, the team used Vosaic software to code the videos. Python code (in Pycharm framework) was used to process the data and to create representations that account for the teachers' time spent with each student group as well as the sequence of their visits (Figures 2,4,5).

Table 2: description of the 4 basic moves and participation pattern

Move	Description	Codes
Initiation	How the teacher approaches the group?	 Student-initiated: Student calls teacher over Teacher-initiated: The teacher approaches the group of their own accord
Entry	Teacher's first statement to the group	 Teacher asks for a summary or "what Ss are up to" in terms of math or participation norms Teacher answers students' question Teacher specifically asks about results or pacing Teacher redirects Ss interaction to a new topic
Focus	The substance of the interaction	 Teacher discusses mathematical ideas Probing (how? why?) Discussing results (what?) Hinting (pointing resources or evaluating students' ideas) Teacher discusses group dynamics Teacher discusses task directions Technical issues or brief comment Any combination of the above
Exit	The last directive the teacher gives before moving away	 Closed directive of "next steps" Open directive of "things to think about"
Participation pattern	Whom does the teacher interact with?	 Teacher discussing with whole group Teacher discussing with individuals or subgroup

Findings

As teachers monitor groupwork, we noted five distinct types of moves, generally following the sequence of *initiation, entry, focus,* and *exit,* as well as an overarching *participation pattern* summarized in Table 2. In the following section, we discuss each of these, using two contrasting cases from our data set to illuminate and span a space of possibilities by which these moves can be taken up.

We offer three caveats about the scope of this analysis. First, our intention is not to be exhaustive: We do not claim that these contrasts capture the full breadth of how teachers might use the moves to monitor groupwork. Second, we do we intend to claim that these routines capture an essence of any individual teacher. In fact, we have examples in our data set of the same teachers leading different lessons where their monitoring routines look quite different. Finally, we do not seek global, normative statements about monitoring routines. Returning to our situative perspective, we see the monitoring routine's structure as capturing a wide range of instruction across a variety of teaching situations. Instead of highlighting "best practices," we identify these phases to help teachers reflect on and deliberate on the range of micro-judgments they can make during this crucial phase of the lesson.

Initiation

We used three codes for the initiation phase of each group interaction describing who initiated the teachers' involvement: student(s), teacher, or unclear. (This third code was not used much.) Within each monitoring case, we saw patterns of mostly student initiation, mostly teacher initiation, or mixed initiation. These top-level patterns capture the general tendency of different teachers to intervene in student groupwork as well as the degree of student autonomy in the class.

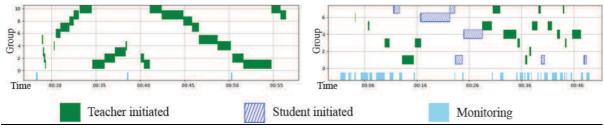
To illustrate how initiation patterns vary, we contrast Veronica and Lizette (Figures 2a and 2b). Because these are nonstandard representations, we offer this guide to understanding what they mean: the x-axis represents

time, the y-axis represents each unique group in the class, with -1 representing moments when the teacher did not directly interact with a group. Thus, the rectangles represent discrete interactions, and their lengths represent the amount of time the teacher spent with a particular group. Green rectangles signal teacher-initiated interactions, while blue rectangles signal student-initiated ones. Immediately we see that Veronica's top-level pattern was mostly teacher initiated, while Lizette's was mixed initiation.

Using these representations, we note a few differences between Veronica and Lizette's initiation routines. First, while Lizette spent 31% of her time *quietly monitoring* (-1 on the y-axis, light blue rectangles), Veronica only spent 2% of hers in the same way. Turning to the underlying pedagogical judgment, we surmise that this may be due to the age difference between the students and, relatedly, their degree of autonomy: Veronica taught 8th grade math, while Lizette taught older students AP Calculus. From our conversations with the teachers, we also see these differences as arising from their conceptions of their role as a teacher during groupwork. For example, when we discussed Veronica's monitoring pattern with her, she shared that she did not think about standing back and watching.

Second, Veronica walked around the room in a fixed pattern from group to group, represented by the sinusoidal curve, whereas Lizette constantly changed the order that she approached student groups. Turning to the underlying pedagogical judgments, we can interpret Veronica's fixed pattern as informed by her judgment that she needs to distribute her attention evenly and help keep the students on task, whereas Lizette's irregular pattern might signal her responsiveness to students' initiation, which may be less predictable.

To reiterate, we do not seek to evaluate these moves as good or bad in an absolute sense. However, we are curious about the consequence of teachers' moves for students' opportunities for mathematical engagement. As we listened to the student audio tracks during Veronica's lesson, for instance, we noted several times that, as a consequence of her fixed pattern of teacher initiation, students' vibrant mathematical talk got interrupted by Veronica's intervention. At the same time, the fixed pattern results in a relatively equal time allocation among the groups, which, as the diagrams show, was not true for Lizette's students. For example, Lizette conversed with Group 2 briefly exactly once and conversed with Group 6 only twice, with the second time lasting much longer. Of course, equal time allocation is not necessarily equitable or even productive for learning. A group that is working well might not need the teacher intervention. But all this points back to the centrality of pedagogical judgment during monitoring: What 'working well' means, for whom, and when, is a matter of interpretation. However, knowing that *not to intervene* is a sound possibility — and one that hadn't occurred to Veronica — might help support the complexity of this (intentional or not) micro-judgment.



Figures 2a & 2b. Veronica's (left) & Lizette's (right) initiation routines.

Entry and focus

Entry and focus capture the heart of the teachers' interactions with student groups. First, the four codes that captured teachers' entry into the conversation answered the question: how does the teacher verbally begin the interaction with the students? Our codes were: (1) listening/asking for a summary; (2) asking about results; (3) answering student questions; and (4) redirecting interaction. The focal teachers typically started their entry by asking where the group stands (codes 1 and 2). Code 1, listening or asking for a summary, can center on either in terms of their mathematical thinking ("so tell me about what you're doing") or participation patterns ("are they helping you out?").

Next, to capture the focus of the conversations, we asked, what is the nature of the sensemaking throughout the conversation? We had six codes for the focus, 3 of which involved math talk and 3 of which involved not-math talk. The three math codes were: probing students' explanation, results-centered talk, and teacher hints/scaffolds. The three not-math codes were: norms of participation, assignment centered, and technical issues/brief asides. Of our four moves in the monitoring routine, focus was the only one that was often double coded, as teachers commonly focused on more than one thing over the course of an interaction.

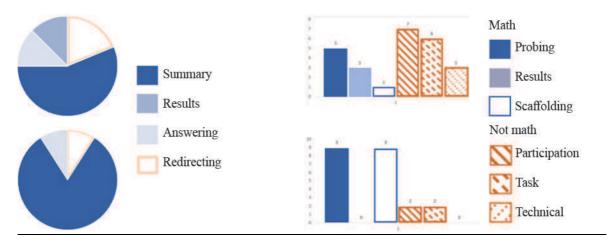
To illustrate how entry and focus can vary, we contrast Bridgette and Brad. Bridgette (Figure 3a) was the only teacher who spoke about participation more than she did about math. In comparison, Brad's entry and

focus (Figure 3b), was more typical, since most of his focus was mathematical, with a constant tension between probing students thinking and scaffolding it.

Like Lizette, Bridgette quietly monitored her classroom more than the other teachers in our sample. However, unlike Lizette (see Figure 2b), Bridgette seldom initiated interactions during the monitoring phase (only 3 out of 16 interactions were teacher-initiated). Bridgette also assigned group roles to students and was very consistent about redirecting student questions back to their peers. In sum, at a top-level description, Bridgette's entry and focus aimed for *equitable participation*.

Brad, on the other hand, like most of our focal teachers, primarily spoke about math with student groups. Looking into the types of math talk, we see a tension between *probing students' thinking* and *hints/scaffolds*. As we mentioned above, Brad's conversations generally started with an entry move (move 2) of asking for a summary of what they have done, followed by a focus (move 3) on *probing* students' questions and thinking, concluding with a strong *hint/scaffold* by either evaluating their thinking or directing students to a next step to continue with. In sum, at a top-level description, Brad's entry and focus aimed for *guided work completion*.

By contrasting Bridgette's equitable participation approach with Brad's guided work completion approach, we again abstain from evaluation of their actions. Instead, we emphasize the tensions they were navigating in making the micro-judgments that inform their practice. For instance, Bridgette's emphasis on equitable participation pressed on student involvement and centered their mathematical thinking. However, in the class session we observed, not all groups completed their task for the day, a valued outcome for many teachers, In contrast, Brad's students, guided by his responsive hints and scaffolds, mostly completed their work, although its production may not have been as equally distributed across students. How and how much to balance probing and scaffolding, and for whom and when, again is ultimately a matter of teachers' pedagogical judgment.



<u>Figures 3a & 3b.</u> Bridgette's (upper charts) & Brad's (lower charts) entry and focus routines.

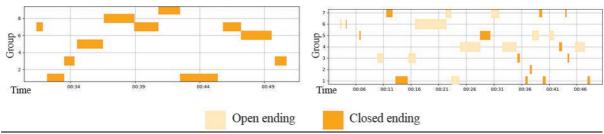
Exit

To capture the exit moves, we asked, how does the teacher end the interaction with students? with an eye toward how varying exits set students up for continued conversation. For this reason, we distinguished between open exits that point toward further exploration, and closed exits, which involved clearer directives. For example, an open ending might offer an issue to think about, while a closed exit would tell a next step.

To illustrate exit moves, we contrast Brad and Lizette. Using diagrams constructed similar to those illustrating the initiation move, Figures 4a and 4b show the teachers' movement around their classroom, the time spent with each group, with yellow rectangles signaling open exits and orange rectangles signaling closed exits. Here, we see that Brad's exit routines (Figure 4a) provide us with important insights regarding his overall monitoring routine. On the one hand, Brad elicited students thinking and had rich mathematical conversations with the groups (which we omit here for lack of space). On the other hand, all of Brad's 11 group interactions had a closed ending (e.g., "so the vertex should come in the middle"). In contrast to Brad, Lizette (Figure 4b) had mostly open exit moves, which included less directive hinting, such as pointing to a resource (e.g. "anything we've talked about last week that might help here?"), generally leaving the group with something to think about rather than something to do.

Once again, the teachers' understanding of their role during monitoring shapes these moves. The distinction between open and closed endings are critical here, because it might have important implications for how students' conversation will unfold after the teacher walks away. In Lavie and colleagues' (2018) terms, if

teachers interpret their task as supporting students' collaboration in their absence, they should privilege open exits. If teachers interpret their task as wrapping up the conversation, ensuring work completion, or supporting student pacing, then they should privilege closed endings.



Figures 4a & 4b. Brad's (left) and Lizette's (right) exit routines.

Participation pattern

As we were coding the moves of the monitoring routine, we noticed differences in the overarching participation pattern between the teacher and student groups. Specifically, whom the teacher interacts with during intervention and in what ways. To illustrate participation patterns, we turn to the contrasting cases of Bridgette and Lee, two outliers in our sample. Most of the other cases show teachers moving between interacting with all students in a student group or only with some. In many cases, teachers start a conversation with certain students and slowly (and seemingly unintentionally) exclude other students from the discussion through their gaze or body positioning.

Bridgette, as we have already described, was very intentional about equitable participation. Of our 5 focal teachers, Bridgette was the only one who consistently addressed all students in the group (16 of 16 interactions). Other teachers were either less intentional about this participation pattern or clearly facilitated a different one. Lee, for example, walked between groups and made sure to talk to every student; however, he addressed them one at a time. We see this as evidence that Lee's interpretation of his own task was less about promoting collaboration and more about supporting individual students' understanding of content.

Discussion

Teachers' monitoring practices are consistent enough to be captured as emergent routines involving initiation, entry, focus, and exit, with general overarching participation patterns. We note two things about these routines. First, the different teacher monitoring routines described above have implications for sustaining students' mathematical engagement. Our claim is not that one mode of monitoring is always appropriate, but rather that monitoring groupwork entails complex interactions, some of which teachers may not be aware of. Second, the differences in monitoring routines seem to reflect the teachers' understandings of their role during groupwork. By examining and conceptualizing this understudied phase of instruction, we illuminate how different monitoring routines both shape the learning environment and are shaped by it.

Although we see monitoring practices as reflecting teachers' conceptions of their role, we also stress that they are not teachers' fixed "teaching style." For example, while Lizette was quietly monitoring 31% of the time in this class session and mainly used open-ended exits, she was teaching an AP Calculus class. If she, like Veronica, had been teaching a middle school mathematics class, she may not have had a chance to circulate quietly, and she may have exited conversations in a content oriented way to work toward task completion — or to quell the anxieties of the younger students. Other aspects of the teaching situation surely influence monitoring routines, such as the extent to which the tasks are groupworthy (Lotan, 2003) and thus invite open ended exploration, as well as the physical space of the classroom. For instance, some of our teachers used vertical whiteboards on the perimeter of the room for groupwork. In these classrooms, the sinusoidal pattern emerged fairly consistently as teachers circulated around the room's perimeter. (See Veronica [Figure 2a] and Lee [Figure 5b] for examples.) In contrast, teachers who are walking between desks could circulate quietly without officially being part of some group discussion.

While supports exist to aid teachers in building student-centered classrooms, this analysis highlights key points in this work, uncovering the range of possibilities for monitoring groupwork, and the complexity of the micro-judgments required in monitoring. As we better understand these routines, we can develop clearer ways to support productive equitable mathematical discourse during this crucial phase of lessons.

Endnotes

(1) The authors contributed equally to this paper.

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