Group Meaning in Mathematical Discourse: A Multimodal Analysis of PreK Students Using Multi-Touch Virtual Manipulatives

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Abstract: For the current study, we applied a multimodal technique to examine discourse among PreK students (ages 4-5) as they used virtual manipulatives (tangrams) on a multi-touch surface under varying conditions of interdependence. Selecting heightened episodes of talk, gesture, and gaze in videotaped sessions, we identified coreferential chains that demonstrate how geometric puzzle solving can serve as examples of group cognition. Initial findings indicate distinct points of cohesion around topics and the possibility of differentiating mathematical talk (denoted by domain-specific concepts, relationships, and transformations) from project talk (denoted by problem solving and socio-cognitive aspects of discourse).

Overview

Our research is motivated by the construct of group cognition as a metaphor to investigate computer supported collaborative learning (CSCL), advances in early childhood mathematics education, and the design, use, and evaluation of virtual manipulatives for multi-touch/-user surface computing technologies. For the current paper, we focus on the development and use of virtual manipulatives for a multi-touch/-user system, the SmartTech SMART Table™. The table is specifically designed for PreK-5 learners and provides an attractive platform for our research and development activities. Consequently, in the following sections we discuss how priorities on group cognition have guided the design and development of our applications. We position our work in terms of recent calls for research and development in direct manipulation of digital interfaces for teaching and learning.

Virtual Manipulatives on Multi-Touch Tabletops: Advanced CSCL Technologies

As pointed out by Tapper (2007), "manipulatives, like tangrams, help students build on prior knowledge and expand both their math content knowledge and their problem solving skills" (p. 11). Combined with multi-touch, multi-user interactive tabletops such as the SmartTech SMART Table™, the design, implementation, and use of virual manipulatives opens the platform to possibilities worthy of research and development in CSCL.Multi-touch/-user tabletops and surfaces have become an area of interest for learning scientists, mathematics education researchers, and software developers. As research is beginning to reveal, students may focus more on the task at hand working in a more collaborative environment by freely using their hands and fingers to manipulate objects on interactive surfaces. Interactive whiteboards (IWB), a good case for multi-touch applications, have greatly improved presentation, motivated students, and enhanced learning with inbuilt programmed links and applications (Glover, Miller, Averis & Door, 2005; Romeo, Edwards, McNamara, Walker, & Ziguras, 2003). Thus, our work attempts to analyze and support group cognition in small-group settings where students informally explore ideas in mathematics. The combination of a theoretical ground (group cognition), pedagogical guide (informal geometry), and advanced technologies (multi-touch/-user surfaces) allows for a rich area of research and development.

Methods

Our methods aim to identify the communicative strategies of Pre-K children when faced with the task of solving geometric puzzles in small-group settings and the potential for exhibiting group cognition in mathematics learning contexts. By examining children's speech, gesture, gaze and actions, we investigated the points of discursive cohesion that structure children's collaborative reasoning throughout the problem solving process. Cohesive points were identified via "coreferences" after McNeill (2009). In its most basic sense, a coreference can be understood as the repeated expression of a single referent delineated in our work as follows:

- **Object-level coreferences** are references to an object or place in the physical world (e.g., "This triangle," "Here," or "Look at this.").
- **Meta-level coreferences** are references to the discourse itself or to the problem solving process, including specific references to the computer program, and time limits (e.g., "That wouldn´t work," [where that

represents a previous utterance] or “It goes there,” or “We need to start over,” or “No this way,” or “It’s my turn.”).

- **Para-level coreferences** are references to the participants themselves, the group, or emphasize a speaker’s viewpoint (e.g., “Now you go,” or “I know,” or “I got you.”).

For the current study, we conducted a series of trials with 4- and 5-year old students at a university-based early childhood education center. In these trials, students were instructed to solve geometric puzzles using the SMART Table^{TM} in a non-classroom, controlled setting. For the reported analyses, a group of three boys and a group of two girls each completed a series of five tangram puzzles. To encourage communication and negotiation of ideas, three different constraints were used. For the first puzzle, *free ownership* was offered, allowing all students to touch any of the puzzle pieces. For the second puzzle, *divided ownership* was offered; each student was restricted to manipulating only the pieces that matched his or her assigned color. For the final three puzzles, *single ownership* was offered; one student had permission to touch the pieces, while the others could only offer suggestions.

Refining Categories of Meta-level Coreferences

In this study, we were able to identify an additional layer of analysis not used in previous work (Evans et al., 2011). In particular, for certain meta-coreferences, there may be a rule or principle that provides the context for the coreference instead of using a previous utterance within the current discourse episode as a meta-coreferent. For example, the rules included in the instructions given to the children before the start of the task. Indeed, so far we have found that many utterances and gestures contain implicit references to geometric principles (e.g., fitting larger pieces in first, staying within the lines, particular properties of the pieces) as well as implicit references to principles governing collaborative problem-solving (e.g., turn-taking or working together). Therefore, in coding, we distinguished between two types of meta-level coreferences: *mathematical* versus *project*. Differentiating between these two types of metacognition is useful in understanding the development of collaborative and problem-solving skills.

- **Mathematical coreferences** allude to geometric/mathematical principles and the properties of puzzle pieces. (e.g., “That fits.” or “It keeps leaving that white space.”);
- **Project coreferences** adhere to collaborative problem-solving strategies or cooperation. (e.g., “Let´s start over.” or “My turn goes next.”).

Our point is that mathematical and project coreferences are key to the organization of group cognition but function in different ways within the discourse. Mathematical-type meta-coreferences may be part of building skills in mathematical and geometric reasoning, as well as demonstrating an understanding of the geometric parameters of the task. Project-type meta-coreferences cohere to the group dynamics and the implicit social rules of cooperation, collaboration, and step-by-step group problem solving. Thus, pivotal moments of collaboration are identified as patterns in the structure of coreferences that drive the problem-solving forward.

Results

Video transcripts were independently coded by two research assistants for meta-, para-, and object-level coreferences. These coreferential chains are offered as incidents of group cognition. The following excerpt provides an example of a heightened episode in which two 5-year old girls, Emily and Gwen, work together on a puzzle constrained by a divided ownership puzzle condition. Recall that in divided ownership each student is assigned a color (red, green, or blue) and may only touch pieces of that color. They may, however, verbally negotiate placement of all of the pieces with the other member of the group. In the excerpts used below to illustrate our technique, as the children near the completion of the puzzle, they work together to negotiate the placement of the final piece. Both verbal and non-verbal coreferences were coded in accordance with the transcripts (see Table 1). Through multimodal analysis, we were able to identify the focused attention on the movement and position of the red triangle within the puzzle as a coalition between students. In Figure 1a, b, and c, we demonstrate three moments of cohesion within approximately 10 seconds of the puzzle in which the students exhibit features of group cognition.

Table 1: Transcription of girls’ puzzle with coreferences.

Time	Name	Talk	Gesture	Coreferences	Project/ Math Coreferences
0:00-0:01	Gwen	I have one more piece.	Gwen holds right forefinger on the blue square. Emily adjusts the red parallelogram.	Para (V) Obj (V/NV) Meta (V)	Project (V)

0:02-0:04	Ms. Lisa	Good job.	Gwen slides the blue square across the table and onto the puzzle	Obj (NV)	
0:04-0:05	Emily	Heh-heh.	Gwen rotates the blue square into place.	Obj (NV)	
0:06-0:07	Gwen	There. Now yours.	Gwen leans back, pointing briefly to the red triangle as she moves.	Obj (V/NV) Para (V) Meta(V)	Project (V)
0:08-0:10	Emily	Mine needs to be flipped.	Emily places both forefingers on the red triangle and rotates it 180 degrees.	Obj (V/NV) Para(V) Meta (V)	Math (V) Project (V)
0:11-0:13			Using 2 forefingers, Emily slides the red triangle to the white space at the top of the puzzle.	Obj (NV)	
0:14-0:16	Gwen	((giggles)) No not flip	Emily smiles; then rotates the red triangle 180 degrees.	Meta (V) Obj (NV)	Math (V) Project (V)
			Emily slides red triangle into place	Obj (NV)	

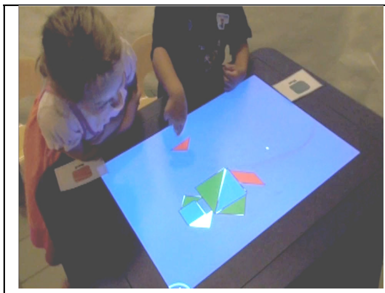

Figure 1a: Girls are focused on placement of small red triangle. Although Emily is responsible for the placement of the piece, Gwen is engaged verbally and physically. After inserting her final piece, Gwen prompts Emily by saying "Now yours." Then, points to the red triangle.

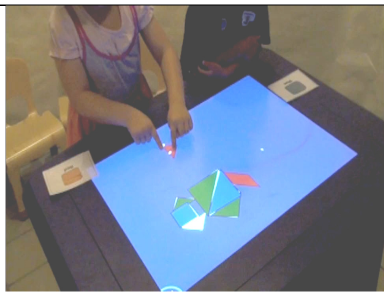

Figure 1b: Emily takes up Gwen's suggestion and moves the red triangle. She decides, however, that the piece needs to be rotated. Though she (incorrectly) uses the term "flip," her gestures indicates a rotation. She rotates the piece 180 degrees and slides it up to the top of the puzzle.

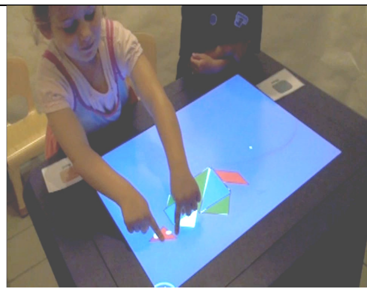

Figure 1c: When Emily brings the red triangle to the top of the puzzle, she and Gwen both realize that the rotation was not needed. Gwen giggles and shouts "No, not flipped!" while Emily rotates the piece to its original orientation before sliding it into place, completing the puzzle.

Figure 1. Screenshots and Descriptions of Girls' Puzzle.

The next excerpt demonstrates a heightened episode in which three 5-year old boys, Charlie, Jason, and Rico work together on a puzzle also constrained by divided ownership. The boys' attention is focused on the placement of a large blue triangle into the white space in the puzzle. Charlie is the only child permitted to touch the piece. Both verbal and non-verbal coreferences were coded in accordance with the transcripts (see Table 2). In the figure below (Figure 2), the three boys, Rico, Charlie and Jason have focused their attention on the movement and placement of the large blue triangle. As we can see from the screenshots, Jason's attention is drawn toward Charlie, who is rotating the triangle near the edge of the screen. After Charlie comments on the rotation of the piece, Jason suggests a location using both physical and verbal forms of communication.

Table 2: Transcription of boys' puzzle with coreferences.

Time	Name	Talk	Gestures	Coreferences	Math/ Project Coreferences
0:21 - 0.22	Charlie	I know we turn it this way.	Charlie uses his middle 3 fingers to adjust the large triangle.	Obj (V/NV) Para (V) Meta (V)	Math (V)
			Charlie slides triangle toward puzzle using index finger and thumb.		
0:23 - 0:24	Jason	Have to go. . . have to go here.	Jason points at the empty white section of the puzzle.	Meta (V) Obj (V/NV)	Project (V)

	Charlie	[Hey!]	Charlie stops moving the triangle but keeps his index finger touching		
0:24 - 0:26	Charlie	Oh, it goes there, Jason.	Jason taps the spot for the triangle twice with index finger then moves hand away slowly.	Obj (V/NV) Meta (NV)	Project (V)
			Charlie moves the triangle toward where Jason is pointing.		
0:27 - 0:30	Ms. Lisa	Yeah, oh good job, Charlie. Fantastic	Rico stops adjusting top triangle, pulls back from the table.	Obj (NV)	
			Charlie finishes placing triangle.		
0:31 - 0:31	Rico	How ‘bout mines.		Para (V) Obj (V)	

Figure 2a: Charlie rotates the blue triangle repeatedly, waiting for his turn. He stops spinning the piece to say, "I know we turn it this way," drawing attention of the other boys.

Figure 2b: Jason points to an area of the puzzle where he would like Charlie to place the triangle. He taps the area twice and says, "Have to go, have to go here."

Figure 2c: Prompted by Jason's gesture and comment, Charlie slides the large blue triangle toward the puzzle. Jason and Rico watch as the piece moves into place.

Figure 2. Screenshots and Descriptions of Boys’ Puzzle.

While Rico does not suggest a placement for the piece, his engagement is confirmed by his concern for the placement of his own piece (“How ‘bout mines.”). We found that talk alone was not a comprehensive indicator of group cognition. For example, in the two excerpts above, we found 17 coreferences in the video clip of the girls puzzle and 11 coreferences in the boys’ clip. However, 8 of the girls’ coreferences and 5 of the boys’ coreferences were non-verbal. Looking at gestures in addition to talk nearly doubled the number of coreferences found in both girls’ and boys' video clips. In expanding our data set, multimodal analysis grants us the types and amounts of data required to discuss group cognition with greater confidence and detail (Strijbos & Stahl, 2007).

Discussion

Multi-modal Analytical Techniques and Group Cognition

Multi-modal techniques that examine talk, gesture, gaze, and activity are not unknown to the CSCL research literature. Works by Cakir, Zemel, and Stahl (2010) and Strijbos and Stahl (2007) have demonstrated the benefits of using multimodal techniques to examine collaborative learning. Though Cakir et al. (2010) were investigating the use of a digital whiteboard in a virtual mathematics chat room setting, results from this work corroborate our emphasis on focusing on coreferential or joint problem solving moments in the discourse to find traces or evidence of group cognition. Moreover, the techniques adopted by Cakir et al. (2010) justify our adoption of microgenetic ethnographic methods to identify discrete moments of group cognition. Where our work is distinguished from theirs is the emphasis of co-located interaction and collaboration. The virtual chat room space decreases, or entirely removes, indications rendered by gesture. Though our work aligns in emphasizing gaze and how it might establish a dual space, we extend these efforts by including the gestural component that has been found critical in conveying mathematical ideas (McNeill, 2009).

Mathematical Talk versus Project Talk: Group Cognition and Discourse

As indicated, our analyses of the preschool children working with virtual manipulatives on the multi-touch/multi-user tabletop has allowed us to further refine the category of meta-level coreferences. We are now able to distinguish between what we have categorized as *mathematical* talk and *project* talk, providing further insights into how children deal with multiple layers of multimodal discourse as they engage in problem solving activities in mathematics. Sfard (2008) has proposed the concept of *commognition* (merging the ideas of communication and cognition) to describe the multi-levelness of mathematical thinking and reasoning. Her point is that to understand the development of higher order thinking processes in a developmental fashion, particularly when investigating the emergent levels of thinking in young children, it is important that mathematical thinking be conceptualized as multi-level and multi-referential. In our data sets from girls and boys, we are able to detect the levels of communication (or discourse) taking place among the children as they refer, and co-refer, to persons, objects, and activities. Our analyses point to evidence of commognition among the students as indicated in mathematical and project talk. Where differences lie, as we have detailed in our excerpts, is how levels are differently emphasized among peers and how boys and girls differently appropriate these levels to achieve or exhibit a distributed cognition of problem identification and, eventually, solution. Whereas group cognition serves well as a general framework for our analytical techniques, commognotion provides additional reference as we explore the distinctions between mathematical and project talk.

Conclusions and Implications

A multimodal approach to analyzing mathematical discourse allows us to "see" potentially what is conceptualized as group cognition. By combining gestural analysis with conventional discourse analysis, we are able to recognize an elaborate system of verbal and non-verbal communication used by Pre-K students to engage in collaborative problem-solving in a virtual environment. In the two examples detailed in this paper, much of the talk and many of the gestures are directed toward another child who is also engaged in completing the puzzle, leading us to the conclusion that mathematical learning is not merely an individual pursuit, but also social endeavor. The results from the currently reported work include the following: 1) earlier work on the relative affordances of physical and virtual manipulatives has the *virtualization* of social cues and artifacts to promote collaborative, co-constructive reasoning; 2) out-of-the box applications and the teacher toolkit included with the SMART Table™ insufficiently support collaboration and co-construction; and 3) the ecology of design of virtual manipulatives demands the meditational role of the teacher for more productive efforts that better serve the classroom. Furthermore, the ideas gleaned from this method of analysis are being embedded into the design of further SMART Table applications for the classroom. These data, along with feedback from facilitators and teachers, provided essential information regarding usability of the application, which were then instrumental in the redesign of the tangram puzzles. We feel the method described is a promising one to detect group cognition. Our goal is to contribute empirical evidence to this mission at the micro level.

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