Resources for Connecting Levels of Learning

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Abstract: CSCL research typically investigates processes at the individual, small-group and community units of analysis. However, CSCL analyses generally focus on only one of these units, even in multi-method approaches. Moreover, there is little data-based analysis of how the three levels are connected. This paper proposes that the levels of individual learning, group cognition and community knowledge building are connected by *interactional resources*, which can mediate between the levels. A theory of the connection of the levels is sketched. Then examples of such connections by interactional resources are presented from logs of several CSCL experiments. Finally, a curriculum for gradually providing math teachers and math students with a complex of resources relevant to dynamic geometry is described as an example of how to support the connection of small-group interaction with individual understanding and with cultural practices in a CSCL adaptation of geometry education.

The Problem of Connecting Levels

Learning, knowledge building and cognition can be analyzed at multiple units of analysis. For instance, analyses of CSCL are often conducted on one of three levels: individual learning, small-group cognition or community knowledge building. This tri-partite distinction is grounded in the nature of CSCL. With its focus on collaborative learning, CSCL naturally emphasizes providing support for dyads and small groups working together. In practice, CSCL small-group activities are often orchestrated within a classroom context by providing some initial time for individual activities (such as background reading or homework practice) followed by the small-group work and then culminating in whole-class sharing of group findings. Thus, the typical classroom practices tend to create three distinguishable levels of activity. Often, the teacher sees the group work as a warm-up or stimulation and preparation for the whole-class discussion, facilitated directly by the teacher. Conversely, the importance of testing individual performance and valuing individual learning posits the group work as a training ground for the individual participants, who are then assessed on their own, outside of the collaborative context. In both of these ways, group cognition is treated as secondary to either individual or community goals. By contrast, the role of intersubjective learning is foundational in Vygotsky (1930/1978), the seminal theoretical source for CSCL. Regardless of which is taken as primary, the three levels are actualized in CSCL practice, and the matter of their relative roles and connections becomes subsequently problematic (Dillenbourg et al., 1996; Rogoff, 1995; Stahl, 2006).

While these different units, levels, dimensions or planes are intimately intertwined, research efforts generally focus on only one of them, and current analytic methodologies are designed for only one (Stahl, 2013b; Suthers et al., 2013). Furthermore—and most importantly for this paper—there is little theoretical understanding of how the different levels are connected. To the extent that researchers discuss the connections among levels, they rely upon commonsensical notions of socialization and enculturation, popularizations of traditional social science. There are no explicit empirical analyses of the connections, and it is even hard to imagine where one would find data that would lend itself to conducting such analyses (Stahl et al., 2012).

The individual unit of analysis is the traditional default. It is supported by widespread training of researchers in the methods of psychology and education. In the era of cognitive science, analysis made heavy usage of mental models and representations (Gardner, 1985). With the "turn to practice" (Lave & Wenger, 1991; Schatzki, Knorr Cetina & Savigny, 2001), the focus shifted to communities-of-practice. Group cognition lies in the less-well-charted middle ground (Stahl, 2006). It involves the semantics, syntactics and pragmatics of natural language, gestures, inscriptions, etc. These meaning-making processes involve inputs from individuals, based on their interpretation of the on-going context (Stahl, 2006, esp. Ch. 16). They also take into account the larger social/historical/cultural/linguistic context, which they can reproduce and modify (Stahl, 2013b).

This paper will argue that the connections between the individual, group and community planes take place through the mediation of *interactional resources* (section on the theory)[1]. To provide specificity and to ground the presentation in empirical data, the paper then considers the resources that appear in recorded examples of mathematical work (section on the analysis). Applying this problematic to the learning of mathematics, the paper adopts a discourse-centered view of mathematical understanding as the ability to engage

[1] While the problem of connecting levels has recently been raised within the CSCL community—e.g., in the workshop at ICLS 2012 (Stahl et al., 2012) and in editorials in ijCSCL (Stahl, 2012b; 2013a)—this paper goes beyond those efforts to propose a central role for interactional resources and to review supporting analysis of empirical CSCL data. For further exploration since this paper was written, see (Stahl, 2013c).

in significant mathematical discussion (Sfard, 2008; Stahl, 2008). Here, "discourse" includes gesture, inscription, representation and symbol, as well as speech and text; these multiple modes are often closely interwoven in effective interaction (Çakir & Stahl, 2013; Çakir, Zemel & Stahl, 2009).

Computer technologies play a central role in mediating the multi-level, intertwined problem-solving, learning and knowledge-building processes that take place in CSCL settings. From a CSCL perspective, emergent technologies should be designed to support this mediation. This involves considering within the design process of collaboration environments how to prepare groups, individuals and communities to take advantage of the designed functionality and to promote mathematical thinking at all levels. This paper reports on the design of a curriculum in dynamic geometry to support group cognition, individual learning and community practices in a coordinated way, based on how interactional resources are visibly used in analyzed excerpts of pilot case studies of the use of dynamic-geometry software (section on the pedagogy). The curriculum addresses both communication issues—such as effective collaboration practices—and mathematical issues—such as focusing on dependencies among math objects—as well as technological issues of software usage.

The Theory of Connecting Levels

The idea of viewing interactional resources as central to mathematical discourse around dynamic geometry was proposed by Öner (2013). This paper cited a number of distinctions drawn in the CSCL literature for contrasting social/collaborative/relational resources with content-related resources:

- Text chat versus shared-whiteboard graphics (Çakir, Zemel & Stahl, 2009);
- Building a joint problem space (JPS) versus solving a problem (Roschelle & Teasley, 1995);
- A relational space versus a content space (Barron, 2000);
- Diachronic content versus temporal dimensions of the JPS (Sarmiento & Stahl, 2008);
- Project discourse versus mathematical discourse (Evans et al., 2011);
- Spatio-graphical observation (SG) versus technical reflection (T) (Laborde, 2004).

Öner then generated some data to explore the interaction of the contrasting dimensions by having two people work together face-to-face in front of a shared computer on a particular dynamic-geometry problem whose solution required a mix of spatio-graphical observation and technical reflection involving mathematical theory—a mix of SG and T resources, to use the distinction she adopted from Laborde.

Inspired by Öner's experiment, Stahl (2013c) presented the same dynamic-geometry problem to two groups of people collaborating online in a CSCL system. We will review the sorts of resources that occur in the data generated in these two experiments after first considering the theoretical notion of resources as connections between levels.

Consider highway ramps or bridges used as resources for connecting road levels or landmasses. While we are more interested in linguistic interactional resources in this paper, it may be helpful to first consider the more intuitive physical case. A ramp or bridge often creates a possibility that did not otherwise exist for going from one level to another at a given point. To go from a local road to a limited-access superhighway, one must first find an available on-ramp. To cross a river from one side to the other, one may need a bridge. This is the individual driver's view. From a different vantage point—the perspective of the resource itself—the creation of a ramp or the building of a bridge "affords" connecting the levels (Dohn, 2009).

By "affords," we do not simply mean that the connecting is a happy characteristic or accidental attribute of the bridge, but that the bridge, by its very nature and design, "opens up" a connection, which connects the banks of the river it spans. In his early work, Heidegger (1927/1996) analyzed how the meaning of a tool was determined by the utility of the tool to the human user, within the network of meaning associated with that person's life and world. In his later writings, Heidegger (1935/2003) shifted perspective to focus on things like bridges, paintings, sculptures, pitchers and temples in terms of how they themselves opened up new worlds, in which people could then dwell—opening new opportunities or possibilities for living. In considering the intersubjective world in which collaboration takes place on multiple connected levels, we might say that the work of resources like bridges is to contribute the spanning of shores within the way that the world through which we travel together is opened up as a shared landscape of resources for discourse and action.

This transformation of perspective away from a human-centered or individual-mind-centered approach became characteristic for innovative theories in the second half of the 20th Century. It is a shift away from the individualistic, psychological view to a concern with how language, tools and other resources of our social life work. It is a post-cognitive move since it rejects the central role of mental models, representations and computations. The things themselves have effective affordances; it is not just a matter of how humans manipulate models in which the things are re-presented to the mind. In phenomenology, Husserl (1929/1960) called for a return to "the things themselves" (*die Sache selbst*) and Heidegger (1950/1967) analyzed "the thing" (*das Ding*) separate from our representation of it. In ethnomethodology, Garfinkel and Sacks (1970) followed Wittgenstein's (1953) linguistic turn to focus on the language games of words and the use of conversational resources (Koschmann, Stahl & Zemel, 2004). In distributed cognition, Hutchins (1996) analyzed the encapsulation of historical cognition in cultural artifacts. In actor-network theory, Latour (2007) uncovered the agency of various kinds of objects in how they move across levels in enacting social transformations. Our use of the term "resources" in the 21st Century is intended to carry forward these groundbreaking approaches into the study of how the various planes of human interaction are connected. Vygotsky (1930/1978) used the term "artifact" to refer to both tools and language as mediators of human cognition; we prefer to use the broader term "resource" as it is frequently used in sociocultural analysis (Furberg, Kluge & Ludvigsen, 2013; Linell, 2001) for references brought into discourse. Like artifacts, resources are identifiable units of the physical or linguistic world that are involved in meaning-making practices—spanning the classical mind/body divide.

A central research issue for CSCL is how collaborative knowledge building takes place. The main problem seems to be to understand the role of individual cognition and of societal institutions in the small-group meaning-making processes. At ICLS 2000, Stahl (2000) presented a diagram that was intended more to raise this question than to answer it. In this diagram, "cultural artifact" served to connect the three planes of meaning-making processes. The diagram was based on an eclectic combination of major theories influential in CSCL. It is now time to conduct empirical investigations of the connections suggested by these theories.

In recent years, the Virtual Math Teams (VMT) Project (Stahl, 2009; Stahl, Mantoan & Weimar, 2013) has conducted case studies of small-group interaction. In doing so, it has tried to focus exclusively on the small-group unit of analysis. It has done so based upon three observations:

1. That most CSCL studies have focused either on the individual (cognitive) plane or on the community (practices) plane. For instance, they code utterances of individuals (Strijbos & Stahl, 2007) and reduce interaction to contributions of individuals or else they view interaction as participation in community processes and institutions.
2. That the small-group unit is fundamental to learning; as Vygotsky (1930/1978) said, one learns most human skills in social interaction first, only then being able to do so individually.
3. That the multiple levels are so complexly intertwined that it is hard to imagine studying them all together without first understanding much of what takes place at each level, temporarily taken on its own.

A number of studies have recently analyzed the problem-solving activities of virtual math teams (Stahl, 2009). In these studies, the interaction of students is analyzed at the small-group unit of analysis as a sequential progression. The collaborative knowledge-building activity that takes place there is mediated by a variety of interactional resources.

The theory sketched in this paper is not meant to reify different levels or processes, but to suggest some of the constraints between different phenomena and possible flows of influence. The distinctions between levels and the identification of typical processes at each level are intended to operationalize an infinitely complex and subtle matter for purposes of concrete analytic work by CSCL researchers. We propose the term "resource" to name the entities that are involved in mediating these connections.

In the work of small groups typical in CSCL, the sequential interaction brings in resources from the individual, small-group and community planes and involves them in procedures of shared meaning making. This interaction requires co-attention to the resources and thereby shares them among the participants. The process results in generating new or modified resources, which are then retained at the various planes. The resources that are brought in and those that are modified or generated often take the form of designed physical artifacts and sedimented elements of language. In other words, "small groups are the engines of knowledge building. The knowing that groups build up in manifold forms is what becomes internalized by their members as individual learning and externalized in their communities as certifiable knowledge" (Stahl, 2006, p. 16).

Each of the sentences of the preceding paragraph could be taken as a research question: a hypothesis about how levels are connected and an agenda for exploration. The following sections begin that undertaking. They present examples of interactional resources in small-group discussions of dynamic-mathematics problem solving and then describe some illustrative resources that are being prepared to help students engage in collaborative dynamic-mathematics problem solving.

The Analysis of Connecting Levels

An early attempt within CSCL to present an extended argument for the centrality of the small-group unit of analysis appeared in (Stahl, 2006), with a preliminary draft in (Stahl, 2004). These lengthy discussions were grounded in a half-minute interaction among four students working with a computer simulation of model rockets. The excerpt involved the students coming to understand how to interpret a textual resource: a table of rocket components arranged to facilitate comparisons among differently configured rockets. At first, none of the students could see the designed affordance of the table, but after the half-minute, they could all see the shared artifact as a resource for their scientific discourse. The interaction analysis of this excerpt showed how aspects of the table artifact were brought in as resources for the group discourse; as were shared and repeated words like "*same*" and "*different*." The words of the dominant student, Chuck, were brought into the interaction by others in order to re-orient Chuck to a new, shared understanding of the co-attended-to table. The resource that emerged for the group's subsequent practice was a sophisticated understanding of the organization of the table (Stahl, 2006, Ch. 12 & 13). This locally achieved understanding was congruent to a standard scientific understanding, which the instructor had assumed in designing the table and offering it as a resource for the group task. Here we can see the use of interactional resources connecting ideas from novice *individual* and scientific *community* planes in the small-group discourse, which led to a significant advance in the *group's* meaning-making ability.

In the experiment reported by Öner (2013), two graduate students work on a dynamic-geometry task, using a shared computer running Geometer's Sketchpad software. The task was specifically selected because it tends to make visible a combination of exploring a figure to discover its dependencies and then duplicating the figure using those dependencies. Thus, it involved a combination of spatio-graphical (SG) observation and theoretical (T) mathematical construction. The task was to duplicate a given figure, consisting of an equilateral triangle inscribed within another equilateral triangle.

At the start of the group's work on this task, one of the students, Ayla, says, "*Are these equal, these distances?*" The group then points to and measures the short segments along the outer triangle up to the interior triangle, which look about the same length. They confirm that these line segments (EC, AD and FB) are always of equal length, even when the figure is dragged and the lengths change. The similar appearance of the three segments in the graphical view provides a perceptual resource, which Ayla brings into the discourse and points to both with her statement and with her finger on the computer screen, establishing co-attention to this resource.

Later, at the crucial point in the construction at which a second vertex of the inner triangle is to be specified, the earlier finding about the original figure is recalled as a resource for duplicating it. As Mete goes to position the second vertex on segment AB, Ayla points to segment EF and wonders quietly as if to herself, "*Hmm, the distance does not have to be always equal*." Then she says aloud, "*Does it? Look, EC and AD and FB are always equal in length,*" while pointing at the three segments on the screen. Mete immediately responds, "*Ha. Then we'll do the thing; we'll measure that gap*," and begins to do the corresponding construction. This is an instance of group memory (Sarmiento & Stahl, 2007), in which the group references a previous finding and re-situates it in the current interactional context, providing it as a shared resource for the current work. The subsequent 30 speech turns of the dyad are concerned with figuring out how to use the software tools to construct their equivalents of EC, AD and FB to be equal lengths. Geometer's Sketchpad provides a tool to do this simply in a couple of ways. However, the resourcefulness of the tool has to be reconstructed by the group interaction to be a usable and effective resource for the group effort. The reconstruction effort itself takes advantage of various interactional resources, such as the letters labeling the triangle vertices, which the group discusses in order to simplify the work of relating corresponding points between their duplicated figure and the original.

Examples of resources from the (Öner, 2013) analysis include those classified as theoretical (T)—such as the geometry problem, the software tools or the relevant concepts, definitions, axioms and theorems of geometry. There are also spatial-graphical (SG) resources—including various visual properties of the figure like segment lengths and point labels.

The experiment with reproducing the inscribed equilateral triangles was replicated within the VMT Project (Stahl, 2013c). Two teams (A and B) of three adults each spent about a half an hour in the online VMT collaboration environment including multi-user GeoGebra. The software supported text chat with graphical referencing and dynamic-geometry construction, providing a contrast to the face-to-face speech and finger pointing in the Öner scenario. The task was identical to Öner's, implying that many of the resources for group work were identical: concepts and theorems of geometry (to the extent that the participants had working knowledge of them) and the visual properties of the figure (as it was dragged in the dynamic-geometry software display).

Although Team A in the VMT experiment focused on observing the spatio-graphical behavior of the points under dynamic dragging, it took them a long time to make Ayla's key observation. Finally, Jan said, "*So I think F is CD units away from B on BC. Its not constructed as an equilateral triangle, it happens to be an equilaterl triangle because of the construction*." Here, the SG observation leads immediately to a T statement about the construction of the internal triangle, namely that it is not constructed by making its sides or angles equal, but rather their equality is a consequence of imposing a different dependency involving distances of the vertices of the interior triangle from those of the exterior triangle. Visual resources are turned into resources for construction and reflection.

Team B took even longer to arrive at the key observation for constructing the inscribed triangles. They pursued multiple strategies, such as using geometric theorems about centers of triangles and correspondences of similar triangles. Finally Lauren said, "*I abandoned the center, and worked with the lengths of the sides*." Then she "*used the compass tool to measure the distance from D to C*" and constructed the circles around the two other vertices of the exterior triangle, each with radius equal to CD to locate the vertices for constructing the interior triangle.

The use of social conventions and other relationship-building resources in addition to the content-oriented phases of chats seem to play an important role in problem-solving interactions. As Mercer and Sams (2006, p. 517) put it, “while working in classroom groups, children use talk to do much more than engage in curriculum tasks: they form relationships, develop social identities, and pursue ‘off-task’ activities which may be more important to them than the tasks in which they officially engaged—and as Wegerif (2005) has argued, may be essential to the process of establishing good relationships so that effective ‘on-task’ activities result.” The use of social-discourse resources to build group cohesion may be even more pronounced, salient and varied in online interactions, which lack some of the social resources provided by physical presence.

Groups in CSCL contexts can be seen to be making considerable use of resources to accomplish their interactional work. Often, they bring in resources from their individual backgrounds or from a community plane (the classroom, the history of mathematics, the subculture of social texting, the practices common in society, the conventions of ordinary language). Frequently, they build local resources within the group, available for repeated use and for “internalization” into resources for the individuals or for “externalization” into disseminated resources for the larger community.

The resources must be shared—attended to by the group and similarly understood—for them to be effectively used. This may be achieved through pointing, questioning, explaining, drawing and illustrating (Stahl et al., 2011). In a problem-solving session, one of the first resources co-constructed by the group might be a formulation of the question that they will pursue, based perhaps on an assigned task, which they must understand and articulate collaboratively (Zemel & Koschmann, 2013). The use of resources can be accumulated in the sequentiality of interaction to produce larger group-cognitive accomplishments such as mathematical problem solving (Stahl, 2011). Across a somewhat larger time scale, resources can build on one another, much as Euclid’s proofs built upon previous proofs. Groups can use their earlier formulations of interactional resources to construct higher-level resources and to refine previous understandings, just as scientific knowledge advances by accumulation and revision (Kuhn, 1972). In each case, the group must enact the resource, coming to a shared understanding of it and situating it in the group-discourse context for it to function as a resource for them. In this sense, resources are emergent from the group interaction.

CSCL research can connect the levels in its research data by identifying the resources that are being enacted in collaborative interactions and by tracking how they are constituted, understood and applied in the meaning-making process. CSCL studies can contribute to our understanding of collaborative meaning-making processes by providing detailed analyses of the ways in which group discourses involve resources interactionally and how the resources are shared, interpreted, refined and preserved.

The Pedagogy of Connecting Levels

If resources play such an important role in collaborative learning, then how can CSCL designers support the use of resources? Clearly, it would be useful to make sure that students have access to relevant resources and that they understand how to use them. In situations where teachers play a central role in guiding the collaborative learning, it would similarly be important to ensure that the teachers have access to relevant resources and that they understand how to facilitate student use of them. Early attempts to support CSCL resources for teachers and students were proposed in (Stahl, Sumner & Owen, 1995) and (Stahl, Sumner & Repenning, 1995).

In the Virtual Math Teams (VMT) Project, we have learned through pilot trials of the VMT-with-GeoGebra environment that this relatively complex system requires careful preparation and training for teachers, students, online groups and classes to use effectively without encountering frustration. In response to this, we have drafted a set of dynamic-geometry curricular activities, interspersed with tutorial tours of the technology features (Stahl, 2012a). These materials are designed for use both by teachers in professional-development contexts and by student teams in online-classroom or after-school settings.

The VMT curriculum activities have been designed to promote collaborative learning, particularly as it occurs in significant mathematical discourse about geometry. We do this by providing a carefully structured set of resources for use by teachers and students. These include the following:

1. Resources for engaging in significant mathematical *discourse*; to collaborate on and discuss mathematical activities in supportive small online groups. This includes suggested uses of linguistic and interactional resources for coordinating collaboration, as well as tutorials in using the communication tools of the VMT software.
2. Resources to collaboratively *explore* mathematical phenomena and dependencies; to make mathematical phenomena visual in multiple representations; and to vary their parameters. This includes scaffolded exercises in noticing visual characteristics of dynamic-geometric figures being dragged and in wondering in chat postings about their dependencies.
3. Resources for *constructing* mathematical diagrams—understanding and exploring their structural dependencies. This consists primarily of a semester-long sequence of construction activities, initially with step-by-step instructions and tutorials about GeoGebra tools.

4. Resources to notice, wonder about and form conjectures about mathematical relationships; to justify, explain and *prove* mathematical findings. This involves discussion prompts and situated examples of explanations or proofs.
5. Resources to understand core concepts, relationships, theorems and constructions of basic high-school *geometry*. The included materials and activities cover central conceptual and procedural resources from Euclid's first book of propositions and from the Common Core standards for beginning geometry.

The presentation of resources is organized developmentally, so that understanding of the resources presented first can be used to build understanding of resources presented subsequently. Concomitant with this is a progressive shift from scaffolded explanation of basic resources (like software tools) to open-ended inquiry of more complex resources (like mathematically interesting micro-worlds).

There is a theoretical basis for gradually increasing skill levels in terms of both geometric understanding and deductive proof. The van Hiele theory (see deVilliers, 2003, p. 11) specifies several levels in the development of students' understanding of geometry resources. The implication of van Hiele's theory is that students who are at a given level cannot properly grasp ideas presented at a higher level until they work up to that higher level. That means that unprepared groups will fail to enact available resources in a meaningful way. Thus, a developmental series of activities pegged to the increasing sequence of levels is necessary to effectively present the various resources of geometry, such as, eventually, the formal structure of deductive proof. Failure to lead students through this developmental process is likely to reinforce student feelings of inadequacy and consequent negative attitudes toward geometry.

A particularly important resource for understanding and working in dynamic geometry is the concept of *dependency*. GeoGebra allows one to construct systems of inter-dependent geometric objects. The dependencies built into dynamic-geometry constructions are intimately related to proofs illustrated by those constructions. Often, to understand a dependency and to be able to implement it in a construction is tantamount to being able to articulate a proof and to explore its validity dynamically (Stahl, 2013c). Students have to learn how to think in terms of these dependencies. They can learn through use of resources like visualizations, manipulations, constructions and verbal articulations. These can all be modeled by examples, and these resources can be provided gradually.

The VMT Project is now drafting and piloting versions of curricular activities designed to develop significant mathematical discourse focused on dependencies among geometric objects (Stahl, 2012a). Concomitantly, it is implementing software support for teachers and students to explore the dependencies and assembling materials for professional development to prepare teachers to enact this curriculum with their students (Stahl & Powell, 2012). The set of activities is designed to provide the most important basic geometry resources to math teachers and students, taking them from a possibly novice level to a more skilled level, at which they will have a sufficient portfolio of resources for engaging in significant mathematical discourse without continuing scaffolding. The resources of classical Euclidean geometry were decisive in the historical development of rational thinking by literate individuals and of scientific culture in the modern world (Netz, 1999; Stahl, 2013c). We hope to adapt these resources to the CSCL context, where they may enter into small-group collaborative online interactions and thereby influence both individual understanding and classroom practices.

In on-going experiments within the VMT Project and elsewhere, our colleagues and we will be logging the use of the resources by teachers and students in order to analyze how resources connect levels of learning in a CSCL setting. We will track individual and group performance in significant mathematical discourse as resources and practices from community levels are taken up in sequential small-group interaction. Perhaps we will witness the formation of local practices and group interactional resources, which can influence individual and community levels over time. In these ways, we will study resources for connecting levels of learning in CSCL. More generally, through analysis of the nature and work of resources in case studies of a broad variety of CSCL interactions, the CSCL research community can expect to reach a better understanding of the nature of different levels of analysis in CSCL research and how the levels may be connected in terms of their mediation by diverse resources. Gradually, we will discover how resources are enacted, understood, shared, designed, adapted and preserved—and how they mediate connections among levels of learning through social interaction.

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