Super Tangrams: A Child-Centered Approach to Designing a Computer Supported Mathematics Learning Environment

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In this paper we describe our experience in designing and evaluating a computer supported mathematics learning environment. This experience is part of an ongoing long-term study aimed at investigating design features of mathematical computer activities which promote reflective cognition and thinking in children and improve their conceptual understanding of and attitude towards formal school subjects. This experience covers a period of over two years of close collaboration between a university researcher, almost 50 students in grades 6 and 7 (10 to 12 year old children), and their teacher, and, we believe, can provide useful insights on issues of importance in the design of electronic learning environments for children.

One of the difficult tasks teachers face is to engage children in the learning of abstract mathematical concepts. Often many children find such subjects difficult to understand and boring. Two primary reasons for this are the cognitive challenge that many mathematical subjects present and the lack of motivation on the part of children. In recent years knowledge acquisition and motivation have been studied in the context of coherent, unified, and integrated learning activities. Researchers have found that motivational processes play a central role in any learning activity, and that motivation and learning interact very closely [Dweck, 1986; Snow, & Farr, 1987]. A number of researchers have suggested the use of computer games as a possible means to enhance children's motivation to learn school subjects such as mathematics [Lepper, & Malone, 1987; Klawe, 1992]. We decided to investigate this possibility in the context of a grade 6-7 French immersion class, many of whose students were not particularly interested in mathematics and found it boring. Our decision to use computer games as a means of motivating students to learn school subjects has since been supported by the input we have received from the students, as well as by our preliminary findings of a change in their attitude and an increase in their learning of quite sophisticated mathematics concepts. Our research forms one part of E-GEMS (Electronic Games for Education in Math and Science), a large university-industry project between the University of British Columbia, Queen's University, Electronic Arts, and several elementary schools, involving the design of both prototype and commercial educational computer games and evaluation of their effectiveness in the classroom and at home [Klawe, & Phillips, 1995].

From the outset of our research our intent was to create a game that would be a meeting place between the goals and objectives of educators and the needs of children. We wanted our design choices to be informed by children's expectations of a fun and engaging learning environment, but also wanted to make sure that we did not compromise the educational quality of the learning environment through making it all play and entertainment. Specifically, we wanted the game to be compatible with textbook materials in terms of mathematical substance, to be challenging, to be fun to play with, to promote reflective thinking and cognition, to help children construct deep conceptual understanding of the mathematical domain, and to provide a cognitive tool or a reference point by which classroom discussion could be generated. To meet these goals we implemented Super Tangrams, a computer game aimed at assisting children in learning 2-dimensional transformation geometry. This paper presents an account of the development of Super Tangrams, and the positive response that this game has invoked in the students with respect to both attitude and learning.

The Initial Investigations

Our investigation of electronic games began in Summer of 1993 when we observed thousands of children playing electronic games at an especially constructed exhibition in a science museum [Inkpen, et al., 1994; Lawry, et al., 1994]. In March of 1994 we placed 4 LC III Macintosh computers in our grade 6-7 class. We initially installed a few prototype games on negative numbers and coordinate systems that had been developed in Hypercard by the E-

GEMS team and a few commercial games. One of the authors (Sedighian) visited the class weekly for 1 to 2 hours. During these visits, we observed the students playing the games and conducted class discussions about the educational contents of the games, children's likes and dislikes, and what they thought they were learning. In addition, we asked the students to keep a journal of their computer activities. We found that the students were often very difficult to satisfy; rarely did the majority of them consider a game 'cool'. Furthermore, virtually all the games polarized the students along gender lines. In February and March of 1995 Sedighian interviewed all the students individually to find out what they expected from games, as well as what they considered to be attractive and engaging ways to learn mathematics. Points that were brought up by the majority of the students included: 1) mathematics is both difficult and boring when studied from a book (i.e., it is neither interactive nor fun); 2) unlike books, computer games provide a worthwhile and challenging goal which motivates students to do mathematics; 3) music in games is not only fun but also can act as a memory cue for remembering the mathematics content of the game at a later time; and 4) attractive colors and graphics are important motivating factors. The information gathered in the initial phase of our investigation provided the basis for the design of Super Tangrams.

The Design

Super Tangrams involves the traditional Chinese tangram puzzles in which the player is challenged to put together 7 geometric pieces to fit an outline. Once a piece is selected, the player must choose one of three transformations to move it, namely, translation, rotation, or reflection. Figure 1 depicts a puzzle in which the player has chosen to use rotation to move the square piece. As indicated, the key elements in Figure 1 include: 1) the selected piece, 2) a formal representation of an arc of rotation with two handles (one for changing the center of rotation and the other for changing the angle of rotation), 3) a ghost image indicating the final position of the selected piece given the current settings for the angle and center of rotation. Once the player is satisfied with the current transformation settings, (s)he must click on the 'GO' button which results in an animation of the transformation of the selected piece to the location of the ghost image. Each transformation has a unique sound effect associated with it -- a whooshing sound for the animation of translation, a ticking sound for changing the angle of rotation and its animation, and a projectile plus thump sound for reflection.

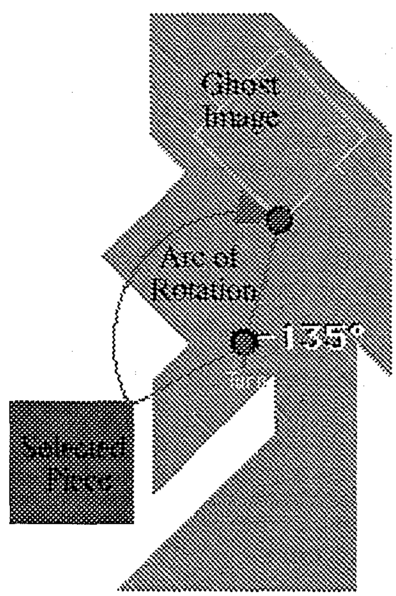

Figure 1

Our general objective in designing Super Tangrams is to assist students in moving beyond their informal and intuitive understanding of 2D transformations, and to stimulate them to think about the formal mathematics involved [Sedighian, & Klawe, 1995]. A few of the more specific learning goals include understanding that:

1) a rotation involves setting an angle as well as a center of rotation,
2) rotating an object both turns it and translates it,
3) a translation arrow indicates the distance and direction in which an object will move, and

4) composite reflections are sufficient for performing all transformations, i.e., that the effect of any transformation can be achieved by an appropriate sequence of reflections.

To make the puzzles progressively challenging, create cognitive dissonance as needed, and take the player through progressive zones of learning comfort, we have built the following features into the game:

1) some puzzles only allow the player the use of a subset of the transformations;
2) each puzzle's pieces have a unique initial configuration;
3) each puzzle allows the player to make a maximum number of moves;
4) each puzzle has a score which is inversely proportional to the number of moves made; and,
5) the game has 3 levels, where higher levels provide less visual feedback thus giving more cognitive responsibility to the player (i.e., level 1 displays the ghost image, level 2 hides the ghost image, and level 3 hides other elements of the mathematical representation).

As educators, we had initial reservations about the effect of giving scores and limiting the number of moves, but our investigations revealed very positive support from students for these features.

Next, we discuss the reasons for choosing transformation geometry and tangrams as the subject matter of our game. Russell and Bologna (1982) noted that the most neglected area of the elementary school mathematics curriculum is geometry. This is especially true of transformation geometry. Many teachers either leave this subject out of their curriculum or provide a very cursory treatment of it. Nevertheless, the National Council of Teachers of Mathematics [NCTM, 1989] emphasizes the importance of the study of this subject. Furthermore, according to Crowley, a mathematics educator:

> [transformations] serve as powerful problem-solving tools. They link traditionally compartmentalized areas of mathematics. And they have applications in areas outside of mathematics. All these characteristics promote the learning of mathematics! [Crowley, 1995].

A number of educators have suggested that tangrams are particularly useful for learning transformation geometry [Russell, & Bologna, 1982; Rahim, & Swada, 1989]. While solving tangram puzzles, the player is constantly moving the pieces about by sliding, turning, and flipping them. In other words, transformations are embedded in the puzzle-solving task in a very natural way. This is considered to be very important for the effectiveness of a learning environment by researchers in situated cognition [Brown, et al., 1989].

Among the games we used in our observations prior to designing Super Tangrams were a tangrams game and Geometric Golfer. Many children found the tangrams game boring, and almost all kids had problems with understanding the structural and operational representations of rotation and reflection in Geometric Golfer. Observing the students interact with these games provided us with valuable insights into the design of transformation geometry games, especially the need for appropriate user interfaces that assist children in constructing desirable mental models of the subject. Furthermore, the games provided a point of reference for the children to inform us of their motivational needs.

In creating Super Tangrams, we paid special attention to the following elements: interface representations of the transformations, color, graphics, sound effects, music, and the par and scoring system. We first developed numerous representations for each transformation and tested them for their educational effectiveness. We then created and evaluated dozens of sound effects for each transformation animation to find ones that provided the most appropriate auditory feedback. Moreover, we selected and tested numerous popular songs to match the mood of the puzzles. Similarly, in our search for interfaces that would retain children's interest without overly distracting their attention from the mathematical content, we experimented with a large variety of popular cartoon characters, color and texture combinations, exotic patterns, and interesting images. Finally, we developed a score and par system that would encourage thoughtful play (i.e., to get a high score, players have to think about which transformations to select and how to use them to solve puzzles in the fewest number of moves).

The Response

The game was first tested at a UBC Open House attended by hundreds of people, both children and adults, in mid-October of 1995. As a result of our observations at the Open House, the interface was slightly modified to promote better understanding of its mathematical content.

In late October, we introduced Super Tangrams to our class and observed the students interact with it over a trial period of three weeks. The immediate reaction of all the students to the game was very enthusiastic. This was the first game for which we observed a unanimously positive response with no difference along gender lines. Despite the obvious mathematical content of Super Tangrams, every child wrote strongly positive comments about it in their journals, and almost all indicated that they appreciated learning mathematics in a fun way. Furthermore, from our observations of and interactions with the students, we found that they recognized that Super Tangrams was not simply a game for having fun, but that to advance through the game and get high scores they had to think hard and learn difficult transformation geometry concepts. Here are a number of typical journal entries from the first trial period:

> I think this game is superb. It is sooooooooo much fun. I love the new music and how they use cartoons for pieces of the puzzle. . . . I'm learning a lot about geometry but this game makes it much more fun and intersting. (grade 7 girl)

> When I was in grade 2 math was fun then in grade 3 math was not fun until grade six with Super Tangrams. Its a fun way of learning math, and a fun way to do math. (grade 6 boy)

> I think that super tangrams is one of the best games that I ever played. . . . The music gives it a nice rythem to work on. I think that this game is a good way to learn motion giomatry instead of reading it from a text book for a lot of ways. One you can not move thing in a text book. 2 a text book does not talk to you. 3 [ends here] (grade 7 boy)

> I think this game is neat. I liked the graphic and sound effects. The point of the game is simple but as the game progresses it gets harder. This game makes math (geometry) seem more fun and less boring. I liked this game because you're playing and learning at the same time. (grade 6 girl)

> It was really fun. I learned the real way to do a flip. But I thought it was a bit hard. But it was fun. . . . I really like the game. (grade 7 girl)

> I like this game. . . . It's little difficult for me. So I have to practice more. (grade 7 girl)

> It was really cool. . . . The music, graphics were really cool. . . . I learned that even though geometry is boring if you add something to it people will really like it alot. (grade 6 boy)

> Super Tangrams was very educational. You have to use your brains alot. (grade 6 boy)

After making a few minor modifications, we reinstalled Super Tangrams in our classroom in mid-November. All students regularly played the game until the Christmas holidays. During this time, we selected 4 students, considered by their teacher as weak achievers in mathematics, to study the effects of using Super Tangrams with adult mediation. In addition, three weeks before the holidays, we started a Super Tangrams tournament in the class as a means to encourage the students to tackle the more mathematically-challenging puzzles and to generate classroom discussion of transformation geometry. As expected, the tournament created a strong sense of challenge and a lot of excitement and social interaction in the class.

In addition to the fun and excitement, students have started learning some mathematical concepts that are rather sophisticated for their age group. For example, the first puzzle is very simple and only involves sliding the pieces into place. All students can easily solve it and are thus encouraged to try the next puzzle. By the time they get to the 4th puzzle, they are only allowed to use rotation to solve the puzzle, and many of them become very excited when they find out that they can use rotation to both turn and slide an object. By the time they get to puzzle 18, they can only use composite reflections to solve the puzzle. At this point, many students begin to articulate that any piece can be moved to a desired location using two or three flips, except for the 'funny shape' (i.e., the parallelogram which is asymmetric). In level two, since there is no ghost image, they have to know the exact angle of rotation as well as have a formal understanding of the translation arrow to solve the puzzles. By mid-December the 4 mediated students had reached level 2, and the rest of the class was close to the end of level 1. The following are some excerpts from the students' journals written during the second trial period:

> I've learned a lot such as when you use rotation you turn the object and slide it. You can use flip to get lots of things in if you use it more than once. (grade 7 girl, mediated)

> In my experience of playing ST I learned that you can use turn once instead of using turn then slide. You can also use flip twice to get a shape anywear. (grade 6 girl, mediated)

> In Super Tangrams I learned more about rotating shapes. I learned that when you rotate something it goes along the curved arch. I also learned that the + sign beside the angle number means counter-clockwise

and the - sign is clockwise. I learned more about the center of rotation. . . . I learned that the circle is called the arc of rotation, and when you rotate somthing it goes along that line. (grade 7 girl -- not mediated)

One particularly encouraging aspect of our study has been the impact of Super Tangrams on students with a history of low achievement in mathematics. In particular, it seems to have increased their confidence in their ability to do mathematics. The following excerpts are from the journal entries of one of the grade-6 mediated students recorded over a period of one month, playing one hour per week. This student is described by his teacher as particularly weak in all subject areas, and as someone who has difficulty understanding abstract concepts:

> On the first day of Super Tangrams I though[t] that the game was hard, but then I though[t] it was pretty cool. . . . The music, graphics were really cool. . . . I enjoyed how you can't use flip or turn and only use slide on some puzzles. . . . Level [2] was really good even though we didn't get past the first puzzle. . . . I learned how to find the right angle without the ghost image, which is all the lines have to be parrarel to where you're going to put the object(s). And each point has to go in the right place. . . . I found out that if you're on a negative angle and you go to 0° and you turn one more you'll get a positive angle. . . . I've learned that with turn you can rotate and translate in one move. . . . I've spoke to some of my friends and they love the game, alot of them have learned alot just like myself. Even if the puzzles are hard I still like it. Some other things I learned about were the arc of rotation, angle of rotation and center of rotation. The arc of rotation is that arrow shows how much you're going to rotate the object. The first time I saw an arc of rotation arrow was in this game. The angle of rotation shows how much you've turned or fliped your object. The center of rotation is the green dot on the radius, here's an example: [drawing of the rotation arc]. If the object can't go in you must use two moves[.] if the point of the object is in its place only turn to angle and leave it. Now I can visulize angles really good in my head. Every move someone makes I think in my head and tell the person if its wrong or right. . . . This game makes me always do more. . . . Last couple of times that I played this game I was pretty slow on my angles, but Monday I was so quick I even felt it inside of me. (grade 6 boy, mediated)

Written comments from two of the other mediated students also indicate the development of a sense of visualization of angles and a feeling of self-assurance.

> I'm learning a lot about geometry. Such as angles like 270° is the same as -90°. . . . I learned that the arc of rotation show's how far and how much to turn it. I can see the angles in my head how far it should go. I don't really know why it's just that I can see it's just something I can see now. (grade 7 girl)

> Today I learned how to rotait an image without a ghost image. I had not learned to do this till I played this game. Most if not all of the thing in this game I did not know ontill I played this game. I can now gess angles without a protracter. (grade 7 boy)

It is important to note that these students find it very difficult to record their understanding of their learning. For instance, the boy quoted above has very poor writing skills and shows little initiative towards general classroom activities. However, in our close interactions with him, we have been astounded by his ability to quickly grasp difficult mathematics concepts and by the degree to which he has been captivated and motivated by the game. Often the verbal comments made by the students reveal much more than their written comments, thus we have been videotaping all the mediated sessions.

Finally, we found that successfully solving a puzzle with the number of moves allowed, beating the par, and getting a high score were very important to the students. We were impressed by how much reflection and reasoning they would put into every move in order to solve a puzzle with the best combination of transformations.

Conclusions and Future Work

Our preliminary experience with Super Tangrams shows that computer games can provide motivating mathematics learning environments for children. Super Tangrams has also been tested in a grade 7-8 classroom in Ontario with similarly positive responses. Our findings reinforce the importance of child-centered design in which children's motivational needs are taken into account. This, however, does not imply a compromise in the educational quality of the game environment. From our experience it seems it is feasible to combine fun with the learning of formal mathematical subjects in a computer game. This can help change children's attitude towards the learning of mathematics and create a social environment in the classroom where the students are eager to discuss mathematics.

Some of our preliminary findings indicate that the inclusion of colorful textures and patterns, popular music, cartoon characters, and sound effects play an important role in motivating children to play educational games. Moreover, pars and scores are important in creating a sense of challenge and in promoting thoughtful action. Last but not least, user interfaces are critically important in assisting with the learning of the mathematics subject.

To examine our preliminary findings in greater depth, we will conduct a 6-week experimental study with approximately 150 grade 6 students who have not previously been involved in our research. The purpose of this study will be to investigate the effect of different interfaces on children's learning, the effect of sensory stimuli (such as color, graphics, sound effects and music) on children's attitude towards the game and the mathematics subject, and the effect of adult intervention on both children's learning and attitude. The children will be divided into a control group plus 5 other groups which will each use a different version of Super Tangrams. All children will be given pre- and post-tests before and after the required number of game treatments to measure their knowledge of transformation geometry, their attitude towards transformation geometry and geometry as a whole, and their attitude towards Super Tangrams itself.

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