Learning inter-related concepts in mathematics from videogames

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Abstract: If students learn multiple inter-definable relational concepts, their resulting knowledge will be more stable, flexible, and likely to be applied on transfer tasks. A videogame has been developed to teach fractions. Based on analysis of Asian curricula in mathematics, the favored instructional sequence and relative emphasis on various topics have been incorporated into the game design. Through the game, students interact with different numbers and operations. Their performance on pre- and post-test are compared.

Objective

Learning mathematics involves recognizing relationships between different pieces of information including mathematical facts, ideas, and procedures, constructing new relationships between previously disconnected information or reorganizing already connected information, and finally building a network of representations (Brownell, 1935; Hiebert & Carpenter, 1992). To achieve this goal, most attempts to use games to teach mathematics have focused on practicing procedures. Some games have focused on teaching understanding of specific concepts (e.g., why the denominators cannot be added when adding fractions). Our approach is to use games to teach an inter-related set of concepts by helping students recognize relationships between different number systems and different operations. Our hypothesis is that if students learn a number of relational concepts, their resulting knowledge will be more stable, more flexible, and more likely to be applied on transfer tasks. Skemp (1976) distinguishes relational understanding from instrumental understanding in mathematics learning, and argues that relational understanding is more adaptable to new tasks and easier to remember because learners can remember inter-related concepts as parts of a connected whole, rather than as disconnected pieces of information.

Method

We have analyzed curricula of Asian countries that have shown high achievement levels in mathematics (Japan, Singapore, and South Korea), and defined what kinds of experiences enable students to acquire conceptual understanding of fractions. Based on our analysis, we provided game designers with the instructional sequence and relative emphasis on various topics related to operations with fractions.

Instructional game design and content goals

A videogame has been developed to teach fractions intending to be played by students who already have some prior experience with fractions (i.e., not a substitute for an introductory course). In this project, three major connections have been emphasized and incorporated into the game design process. First, students should be able to see relationships between concrete representations they manipulate and conceptual representation to be taught. Many teachers use concrete examples (e.g., visual diagrams, physical manipulatives) to help students understand abstract mathematical concepts and symbolic notations. Use of concrete materials, however, does not guarantee their effectiveness. Concrete materials seem to be useful in teaching only if learners recognize "the correspondence between the structure of the materials and the structure of the concept" (Boulton-Lewis, 1992, p. 10). Also, for an analogy between a concrete representation and a conceptual representation to be effective, the relations found in one representation should be clearly mapped to the relations present in the other representation (English & Halford, 1995). To achieve this goal, two distinct workspaces have been developed to teach fractions in the game: a realistic problem space which includes a concrete visual representation, and a symbolic number representation space. Players are allowed to manipulate simple visual forms of quantities using operator tools in the realistic problem space to test their hypotheses.

A potential issue with physical representations of quantities is that students may attend to superficial features of the representation rather than to the concept embodied by the representation. To address this issue, we have incorporated a design element into the game in which the intermediate visual representation fades out as players encounter "night-time" gameplay. Players learn early on in the game that night-time looms, and thus they must pay attention to the quantities being represented as well as to the visual representations of the quantities. Also, in a later phase of the game, the aid from the visual representation space fades out altogether so that students must work directly on the symbolic number representation. By having each manipulation tool in the visual representation space clearly correspond to one of the mathematical operations, students should be able to map unambiguously between different representations.

Second, students should be able to see relationships between different operations. Students should understand the way in which different operations are related, and how each operation works to change a quantity from one state to another. For example, students should understand that multiplication and division have an inverse relation to one another. Students will use the inverse relationship in two ways. One way is to reverse or undo an action and the other way is to use division to find an unknown scalar operator. By ensuring that students are exposed to problems that have the same structure but a different goal, inverse relations can be understood.

Third, students should be able to see relationships between different number systems. In most U.S. classrooms, operations with different types of numbers are taught separately. For example, after mastering operations with whole numbers, students move on to operations with decimal numbers or fractions. However, connections between different types of numbers, and subsequently operations with these different types, are not emphasized. When these number systems and operations with them are taught as distinct concepts, students may construct separate mental models for each of different number systems. This hypothesis is supported by the finding that students often treat the same word problems differently depending on the type of the numbers (Sowder, 1995). To prevent students from developing separate mental models, in this game, a common visual representation is used across different types of numbers. The main concepts common across different number systems include units, decomposing and recomposing numbers, and different operations and their inverse relations. For example, any number can be understood as a collection of units: whole numbers as a collection of 1's, fractions as a collection of 1/n's, and decimals as a collection of 0.1's. Besides emphasizing common mental models across different types of numbers, problem situations with different number systems were inter-mixed in the game sequence. Rather than mastering one number system with all kinds of operations and moving on to another number system, students have to operate with several different types of numbers together from a very early phase of the game.

In each level of the game, players are challenged to solve a problem involving lengths. This challenge makes students reflect on their own knowledge and strategies, and focuses their effort on construction of rich relationships among different representations and mathematical concepts. Students are not encouraged to simply memorize demonstrated rules or principles.

Results

To test the effectiveness of the video game for teaching fractions, the game is played by a group of sixth- through ninth-grade students. Students' performances on a pre-test and on a post-test of fraction operations are compared. Players are tested in terms of both conceptual understanding and procedural fluency with materials learned directly from the videogame, and with transfer tasks. Additional data are collected during game play, allowing evaluation of students' mistakes and their hypothesis testing throughout game play. The findings will be discussed in terms of advantages of teaching inter-related concepts, and the possibility of constructing a videogame-playing experience that results in increased relational understanding of mathematics.

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