Units of length: A notational system for conceptual understanding of size and scale

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Abstract: A cross-sectional study (101 grade 6-14 students) and teaching experiment (24 grade 6-8 students) suggest measurement units are important in conceptual understanding of size and scale. Students were significantly better at absolute size than relative size estimation after the teaching experiment. Students appear to off-load the requirement for fractions or decimals onto the notational system of measurement units. A planned cross-cultural study will indicate likely improvements if US students had greater schooling in metric units.

Introduction

The concepts of size and scale are important both in science and for science learning. Size is a characteristic of every object, and is the magnitude or extent of the object. The size of an object is established by comparing it to a standard, which functions as a scale. Scales are “the spatial, temporal, quantitative, or analytical dimensions used by scientists to measure and study objects and processes” (Gibson et al. 2000, p. 219). Tools that allow the exploration of new size regimes have opened entire scientific fields, e.g., the invention of the optical microscope led to microbiology and the development of atomic force microscopes to the nanotechnology revolution. Scale is a “common theme” that pervades science, and can be used to unify student learning (AAAS, 1993).

Theoretical Background

Vygotsky (1978, 1985) pointed out the importance of cultural tools to thinking and learning. More recently, Pea (1988) noted that “intelligence is often distributed by off-loading what could be elaborate and error-prone mental reasoning processes as action constraints of either the physical or symbolic environments.” (p. 48), including “notational systems such as algebra equations” (p. 54). Research revealed that experts and gifted seniors in high school use units as a notational system on which to distribute intelligence, by visualizing small units (e.g., the nm) “as a new unit (rather than as a fraction of a meter)”, with this unitizing possibly underlying conceptual understanding of spatial scales (Tretter et al., 2006, p. 1063). The research reported in this poster studies whether and how typical middle school students use units to construct understanding of size and scale.

Methods

The data used in this poster were obtained from a cross-sectional study with 101 students in grades 6-14 (Delgado, 2009), and a teaching experiment involving 24 students who had finished 6th through 8th grades at a diverse, low SES public school district in a small Midwestern city, during a summer camp (Delgado, Short, & Krajcik, 2009). The curriculum for the teaching experiment was built following a construct-centered design (CCD) approach (Pellegrino et al., 2008) and incorporating findings from the cross-sectional study. The instructional activities (Delgado et al., 2009) were designed to build conceptual understanding of size and scale by having students explore four aspects of size (ordering by size, grouping by size, relative scale or how many times bigger one object is than another, and absolute size) and the connections among the aspects. An engaging driving question – *How can nanotechnology keep me from getting sick?* – provided the context for the 12-hour curriculum. Participants were audiotaped while being individually interviewed; in the case of the teaching experiment, one week before camp, and after completion of the curriculum on size and scale. The interview includes questions about the smallest known unit of length, estimating the relative scale of atom, red blood cell, human, and Earth (relative to a 1-mm pinhead), and estimating the absolute size of the same four objects. The author coded the interviews according to a rubric; a second rater coded 10% of the interviews with inter-rater reliability above 90%. Paired-sample t-tests or McNemar tests (depending on the type of variable) were used to test for changes in the means of student performance pre- and post- instruction, as well as independent-sample t-tests or McNemar tests to compare groups or variables at a given point in time.

Findings

In the absence of the teaching experiment curriculum, next to no pre-university students knew of units of length smaller than the millimeter, yet students often felt a need for these. While any length can be expressed using any unit, many students asked what unit was smaller than a millimeter when prompted to estimate the absolute size of a red blood cell or atom. In the cross-sectional study, around 41% of students mentioned units larger than a millimeter (e.g., centimeter or inch) for the smallest unit they knew. Half of these students did not use the millimeter even after this unit had been provided to them in subsequent interview tasks. Students who continued to use the centimeter or inch scored significantly lower than those who did not initially respond with the millimeter but later used it ($p < 0.01$ for t-test, and effect sizes above 1 [calculated using pooled standard deviation]). Additionally, students became significantly more accurate at absolute size estimation compared to relative scale estimation at the end of the teaching experiment ($p < 0.01$), while there was no statistically significant difference at the beginning of the teaching experiment. This is noteworthy because previous research focusing on perception has shown that relative size is more accessible than absolute size (e.g., Vasilyeva et al., 2004), and research focusing on size estimation has found no consistent difference (Tretter et al., 2006).

Discussion and Implications

The power of units may reside in their allowing students to work with more familiar numbers. Many students struggled to find fractions, decimals, or powers of 10 to express the size of the cell or atom in millimeters, centimeters, or inches. Units may be allowing students to off-load the requirement for fractions or decimals onto the notational system (Pea, 1988) of measurement units. Despite having introduced students to micrometers and nanometers as fractions of a millimeter, most students resorted to micrometers and nanometers when expressing the size of a cell or an atom in the post-teaching experiment assessment, rather than using fractions of a millimeter. The few students who *did* use fractions of a millimeter mainly estimated grossly inaccurate sizes.

While scientists around the world near-universally employ the SI system, the United States still employs English units like the foot and mile for everyday use. Most other countries have long employed the metric system or are further along than the US in their transition from English to metric. Non-US students may be enjoying an advantage in their learning about size and scale by virtue of being more familiar with the metric system of units compared to US students. Finding better ways to help students construct their understanding of "common themes" (AAAS, 1993), including scale, through better curriculum and instruction may be a powerful way of improving US science education. A planned cross-cultural study will shed light on what kinds of improvements, if any, we might expect if US science and mathematics education included earlier and more extensive instructional experiences with the metric system, with the intention of supporting greater student learning and understanding of the common theme of scale. The data collection phase of this new study is scheduled to be completed shortly before ICLS 2010; preliminary findings will be included in the poster.

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