Disciplinary Model-Based Reasoning and Metacognition Underlies Good Estimation Performance by Engineering Undergraduates

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Abstract: Engineers routinely make estimates of physical quantities, such as power, before they begin designing or making (Dym et al, 2005). However, there is very little clarity regarding the process of estimation problem solving. We need to understand the processes underlying good estimation performance before we design teaching-learning strategies for estimation. In this paper, we report on the second iteration of a design-based research project to understand and support estimation problem solving among engineering undergraduates. We designed a software environment with several resources and scaffolds for solving estimation problems. By analyzing student actions as they solve an estimation problem in this environment, we identified the problem-solving processes that lead to good estimation performance. We found that good estimators built models using a simulator, conceptual knowledge and disciplinary reasoning, and performed frequent evaluation and planning. These findings have implications for the teaching and learning of estimation.

Introduction

Consider this engineering problem: *"You are participating in an electric car race in which you are required to design an electric car of weight 5kg with wheel diameters of 4" that can traverse a track of 50m in less than 5 seconds. Estimate the electrical power needed to achieve this performance."* The engineer is required to estimate, to the correct order of magnitude, the power required by a car to attain certain performance specifications. This is an estimate because of the uncertainty about the operating conditions and the features of the car. However, this estimate allows the engineer to begin the car design process and move ahead in a situation of low information and resources (Linder, 1999). Such estimates require the disciplinary model-based reasoning (MBR) of engineering (Lehrer, 2009; Nersessian, 2012; Kothiyal et al, 2016). However, the exact process underlying good estimation is not understood, and research suggests that graduating engineering students cannot estimate simple quantities such as force and energy, while engineering practitioners can (Linder, 1999). This may be because engineering programs emphasize conceptual knowledge and well-structured analysis-based problems (Jonassen et al, 2006), while estimation requires an intuition about the complexity of engineering practice, physical quantities and their values (Mahajan, 2014). Therefore, the goal of this work is to understand the process of good estimation in order to explicitly support students in learning estimation problem solving.

Related work on engineering estimation

In order to estimate a quantity, a solver needs to create an equation of that quantity related to the parameters that dominate its value in the given problem context. This requires the solver to understand the problem system, how it behaves in the given operating conditions, which parameters will dominate the quantity in these conditions and which can be ignored without compromising on accuracy by making suitable assumptions and approximations, identify the inefficiencies in the system and incorporate all these to make the equation. Thus, the solver needs to make a simplified model of the problem context in the form of an equation (Mahajan, 2014; Linder, 1999; Kothiyal et al, 2016). Then they must substitute appropriate values for parameters in the equation, calculate and evaluate if the estimate is reasonable. This disciplinary MBR is challenging for students because in addition to conceptual knowledge, they require the disciplinary reasoning of reducing a complicated system to a small set of parameters which will dominate the estimate in the given context and making judgements regarding numerical values.

Linder (1999) proposed that most estimation activity can be explained using a set of effective actions, mediating characteristics like individual knowledge and compensation methods such as using external representations. Kothiyal et al (2016) identified that the expert estimation process is a three-phased MBR process, beginning with creating rudimentary mental models grounded in the context, which are then manipulated using mental simulation and coupling with external representations such as diagrams and equations, until the model is simplified for the estimation purposes. In a course titled "The Art of Approximation in Science and Engineering", Mahajan (2014) uses a five-step teaching strategy for teaching a set of nine estimation tools that can be applied on a variety of problems. Researchers (for instance, Linder, 1999, Shakerin, 2006) have offered recommendations and guidelines for teaching engineering estimation. The takeaway from these is to increase the number of estimation exercises with everyday objects where students have to select relevant information and balance different types of information, so that students practice estimation and become aware of its importance.

The set of guidelines above, while valuable heuristics for instructors, have not been empirically validated for their effectiveness for learning estimation. Further, they do not include specific scaffolds to support student learning of disciplinary MBR for estimation. Research has shown that students face several difficulties while solving estimation problems (Kothiyal & Murthy, 2015). Thus, there is a need to design research-based learning environments and scaffolds for learning engineering estimation. Towards this goal, we designed a resource and scaffold-rich estimation problem-solving environment. We evaluated what problem-solving processes students employed to do good estimation. This is the first step in supporting student learning of estimation problem solving.

Design-based research

We adopted a design-based research (DBR) (McKenney & Reeves, 2014) in this work, and began with the goal of understanding how to support student solving of estimation problems, before moving on to student learning of estimation problem solving. In the first iteration of this DBR project reported elsewhere (Kothiyal & Murthy, 2018), we designed and evaluated a technology-enhanced learning environment (TELE) for estimation. We identified how students used the features of the TELE to solve the estimation problem, and how the learning design and features enabled students to adopt certain productive aspects of estimation problem solving, such as model-building by simulation. However, we also identified that students were unable to do certain important aspects of the disciplinary MBR such as identifying dominant parameters. In the second iteration, reported here, we refined the design of the TELE based on the findings from iteration 1. We incorporated additional features to support the aspects of disciplinary MBR that students had difficulty with and evaluated the design to identify the processes underlying good estimation performance.

Theoretical background

Supporting disciplinary model-based reasoning

Model-based reasoning or building models of a problem or concept or phenomenon, leads to “breakthroughs” in reasoning and problem solving (Nersessian, 2012; Lehrer, 2009; De Jong & van Joolingen, 2008). Estimation requires disciplinary MBR and students need a structure to do this disciplinary MBR and systematically reduce the complexities of a real-world problem context to a simplified model, namely an equation. MBR has been extensively and effectively adopted as a teaching-learning strategy in science, engineering and mathematics for learning goals such as improving conceptual understanding (Hamilton et al, 2008) and scientific inquiry (De Jong & van Joolingen, 2008). Specifically, causal model progressions (Sun, 2013) where students create progressively more sophisticated models, have been found to serve as a scaffold to students in creating the quantitative models (equations). This is also aligned with the observed three-phased expert MBR estimation process (Kothiyal et al, 2016). For these reasons, we adopted model progression as a way to structure (Quintana et al, 2004) estimation into a set of tasks. In model progression-based learning environments, students are provided with appropriate scaffolds for each stage of modelling (Sun, 2013), such as causal maps and simulations for qualitative modelling (Lindgren & Schwartz, 2009) and an equation builder for quantitative modelling. In addition, in scientific inquiry (Quintana et al, 2004), expert guidance and opportunities for epistemic reflection help students construct scientific explanations and learn about the epistemic aspects of inquiry. We similarly incorporated modeling scaffolds and guidance of disciplinary reasoning in our TELE.

Supporting metacognitive processes

Ill-structured problem-solving literature suggests that in order to obtain a good solution the solver must, periodically evaluate their solution and plan their solution process (Jonassen, 1997). In the case of estimation, Kothiyal et al (2016) observed that experts intermittently evaluate their models for their utility for the current estimation and plan their next steps. Research shows that students must be scaffolded in order to articulate and reflect on their inquiry (Quintana et al, 2004) and problem solving (Kim & Hannafin, 2011). Based on this literature, we conjecture that the problem-solving environment should have scaffolds to trigger students to evaluate their models and estimates, and plan/monitor their estimation process.

Modelling-based estimation learning environment (MEttLE)

We designed a TELE called **M**odelling-based **E**s**t**ima**t**ion **L**earning **E**nvironment (MEttLE) where students solve real world estimation problems. The estimation problem is structured into five tasks (three modelling tasks, calculation and evaluation) and each task is further structured into two or three sub-tasks. The three modelling tasks are, a) *functional modelling* - students describe how the system works, i.e., what are the various actions of the system that affect the quantity to be estimated, b) *qualitative modelling* - students identify the parameters which affect the quantity to be estimated and the dominant parameters (the parameters which have a large effect on the estimate), and the qualitative relations between the quantity to be estimated and the dominant parameters c) *quantitative modelling* – students use conceptual knowledge to create an equation connecting the quantity to be estimated and the dominant parameters. In each modelling task, students are triggered to evaluate their models and plan the rest of the estimation process. The calculation task consists of choosing realistic values for the equation parameters and calculating an estimate. The evaluation task requires evaluating the estimate on the basis of two independent criteria, namely, correctness to order-of-magnitude and on comparison with known values. Given the ill-structured nature of the problem and goal of this work to understand the processes of good estimation, we opted for a completely open-ended intervention in which students are free to use any resources and do any task/sub-task at any time, and can do the (sub-)tasks as many times as they wish, but the problem is considered complete only when their estimate passes the evaluation criteria, after which students reflect on the entire estimation process. A plausible workflow of a student working in MEttLE is shown in Figure 1.

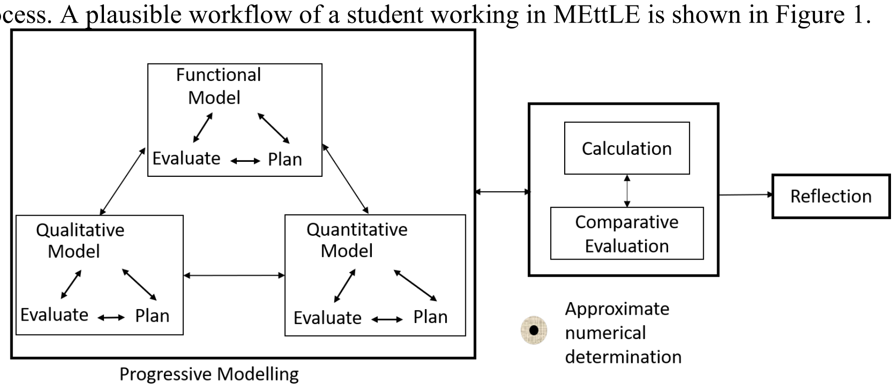

Figure 1. Workflow of MEttLE.

The central process management feature of MEttLE is the *Estimap* (Figure 2) which presents the five tasks of the estimation problem from where students can choose a task/sub-task and navigate through the problem. Each sub-task has a question prompt which introduces the goal of the task, such as *"What actions does the car need to do that requires power?"* for functional modelling. The scaffolds in MEttLE include a *Simulator* with animations, graphs and variable manipulation simulations which is known to provide implicit guidance, in addition to explicit guidance/feedback for disciplinary reasoning during modelling (Figure 3). Also, there are model-building affordances such as a causal mapping tool, question prompts for triggering model evaluation and planning/monitoring of the estimation process, and additional guidance/feedback for disciplinary reasoning during evaluation and planning/monitoring.

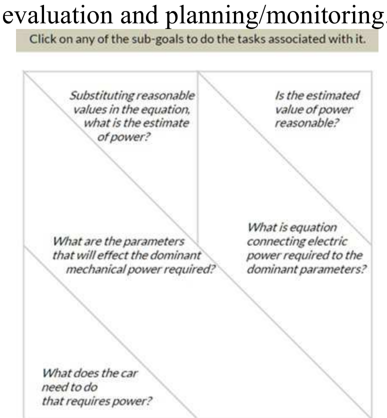

Figure 2. Estimap.

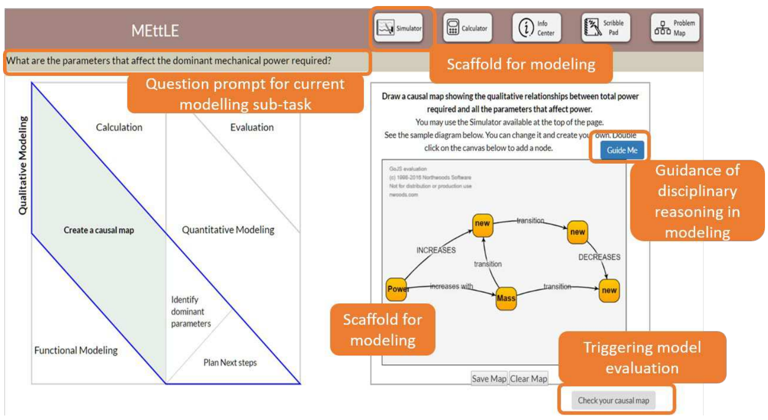

Figure 3. Modeling sub-task layout.

Methods

Our goal for this evaluation was to understand how students solve estimation problems in MEttLE. Our research question for this study was, "What is the student process of solving an estimation problem in MEttLE?"

Participants and data collection

We performed a field study in an engineering college with 72 second year students of mechanical (50) and electronics (22) engineering who solved a problem in MEttLE as part of a workshop. The average age of learners was 20 years and they were familiar with the use of computers through other courses and labs in their curriculum. After the problem solving, we recruited volunteers for a retrospective think aloud interview and had ten participants (four female, five mechanical engineering, convenience sampling) who constituted our sample for this study. The steps followed in the study were: 1) Participants were briefed about the study and its objectives and their consent for recording their audio and computer screen was obtained. 2) Participants solved one problem in MEttLE namely, *"You are participating in an electric car race in which you are required to design an electric car of weight 5kg with wheel diameters of 4" that can traverse a track of 50m in less than 5 seconds. Estimate the electrical power needed to achieve this performance."* While solving, they were not allowed to use the Internet. However, they were free to use all the resources in MEttLE all the time and ask the researcher any questions regarding how to use the resources in MEttLE, but not how to solve the problem. The context for the estimation problem was chosen based on our previous studies as it was relatable, motivating and engaging for engineering students. 3) Participants individually solved, on paper, an estimation problem similar to the one in MEttLE without any additional resources or help from any other person. Further, they were not allowed to consult MEttLE for any purpose while solving this problem. 4) We interviewed students using a semi-structured stimulated recall protocol wherein their screen capture was played back to them and we asked them to describe their thinking and actions at several points and reasons for their actions. The data collected included screen captures of their interactions in MEttLE captured using "CamStudio" (http://camstudio.org/), participant generated artefacts (including rough work on paper), audio recording of the interviews and solutions to the estimation problem on paper.

Data analysis

In order to assess participants' estimation performance, we used the criteria for good estimation defined in our earlier work (Kothiyal & Murthy, 2018) which include a) Estimate is of the right order of magnitude, b) The dominant parameters which affect the quantity to be estimated in the system are identified and c) The appropriate equation for the quantity to be estimated is used. We began by assessing students' estimation performance in MEttLE. We graded student solutions on the basis of whether their solution satisfied the three criteria of estimation with the following coarse rubric: Students who satisfied all three criteria got a high grade, those who satisfied two out of three criteria got a medium grade and those who satisfied one or none of the criteria got a low grade. This coarse grading of their solutions, provided a way to categorize students as we performed interaction analysis (Jordan & Henderson, 1995) to identify their estimation process in MEttLE.

We used the participants' screen captures and their interviews together to perform the analysis with the following steps: 1) We annotated the screen captures using "Elan" (https://tla.mpi.nl/tools/tla-tools/elan) in terms of the actions done in each page of MEttLE. The actions include reading, typing, clicking, changing values, drag and drop, drawing, adding, deleting and editing nodes and links in the causal map. We transcribed the participant interviews verbatim. 2) We interleaved the on-screen actions and their explanations of the actions to create each participants' workflow. This was the flow of actions as they were done and there was no inferencing at this point. 3) In the created workflows, we focussed on the interaction between the participant and the features of MEttLE. Using the actions and reported explanations for the actions we were able to abstract their process during each sub-task and thus their overall process. When they returned to a sub-task after the first pass through it, it was considered a separate event. 4) The data was viewed multiple times collaboratively by two researchers, comparing inferences against each other and refining them during each pass, in order to ensure their validity.

Results

We began by grouping the students based on their estimation performance. Of the ten students, two students (S1, S2) scored a high grade because they obtained got an order of magnitude estimate, identified the dominant parameters and wrote the correct equation, five students (S3-S7) scored a medium grade because they didn't obtain an order of magnitude estimate, but identified the dominant parameters and wrote the correct equation and three students (S8-S10) scored a low grade because they did not satisfy any of the estimation criteria. Next, we report the MEttLE estimation process of each category of students.

Estimation process of high performers

While solving the estimation problem, high performers did functional, qualitative and quantitative modeling followed by calculation and evaluation and satisfied all the criteria of good estimation (Figure 4). As they satisfied the evaluation criteria, they did not iterate after evaluation. However, we found that they concentrated most of their efforts in obtaining good functional and qualitative models by repeatedly applying disciplinary reasoning and evaluating their models, as seen from the green links in Figure 4. They created models, evaluated them and based on the feedback/guidance, revised their models. They reported that the questions in MEttLE *"forced them to think about why they had (for example) chosen a particular set of parameters."* These students used the simulator continuously at this stage as described by S1, *"So, that simulator helped us to know what is the exact data, so we can figure it out and build up the relation, so, without that, I don't think it was possible for us to make that, because we didn't have the proper data for it."* Specifically, they used different tabs of the simulator for different modeling tasks to extract the relevant information. For instance, they understood the actions of the car using animations and identified dominating parameters using graphs as elaborated by S1, *"I selected mass and I increased the parameters and velocity and then I compared where it is varying and then the maximum change that I got to know, I put that in one category and the smaller changes were in one category, from there I got to know that which will dominate the power requirements of the simulator."* When they were unsure of what to do, the high performers used the guidance/feedback on disciplinary reasoning.

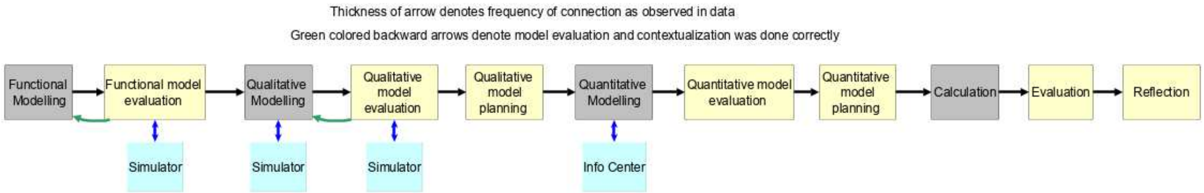

Figure 4. Estimation process of high performers.

Finally, the high performers applied their conceptual knowledge correctly to create an equation incorporating all the dominating parameters by starting from the basic equation of power and manipulating it. While their responses to the planning questions show that they were unable to abstract the purpose of the functional modelling, we found that after the qualitative modelling they examined all their activities and were able to understand the purpose of qualitative modelling. However, they were still unable to plan their next steps. By the time they had completed quantitative modelling, they were able to abstract out the purpose of the task and in addition, plan their next steps. In the calculation task, they substituted suitable values for the parameters and obtain an order of magnitude estimate of power because their models were simplified. Finally, they used the feedback/guidance in the evaluation task to assess if their estimate was reasonable by both criteria.

At the end, the high performers reflected on their entire estimation process, thinking about the purpose of each step and reported that identifying the parameters and their relationships, and then connecting the electrical and mechanical powers were the most important steps. These students reported that they began by being unsure of which path to take but by the end recognized a logic to the steps they took as elucidated by S2, *"I didn't find so much difficulty for solving it, first of all we have to examine what we need to find, then okay, now I'll have to find power, then what is the problem with power, we have to, then we came into two types of power, electrical and mechanical power, then we find the solution for how to manage the mechanical power, how to manage loss, so it was step by step. So, I felt that okay, now we are going the right way, so I continued in that way."*

Estimation process of medium performers

Overall, the medium performers (S3-S7) solved the estimation problem by identifying the parameters involved and then creating the equation, but did not obtain order of magnitude estimates (Figure 5). This was because their models were not grounded in the problem context and so they were unable to substitute the correct values and calculate the estimate. These students began by doing functional modelling and used the simulator to understand the system. However, they did not do the model evaluation well. This was confirmed from the quality of their answers to the questions in MEttLE and depicted using orange links in Figure 5. They did not determine the operating conditions, and did not use the feedback/guidance of disciplinary reasoning and the simulator appropriately to make decisions regarding which actions will dominate power. Some students reported that they had ignored some model evaluation questions because they could not see their purpose in solving the problem.

After functional modelling, there was variation in the process within the group. All students except S6 showed jumps in the process as seen in Figure 5, such as from functional modelling to calculation (because it is the most familiar task)/evaluation (because it is the last task), because they reported being confused and stuck, and so used exploration to come unstuck. When they were unable to do calculation or evaluation, they returned to qualitative or quantitative modelling and continued. S6 continued with qualitative and then quantitative modelling after functional modelling. These students used the simulator to bridge the conceptual knowledge gap

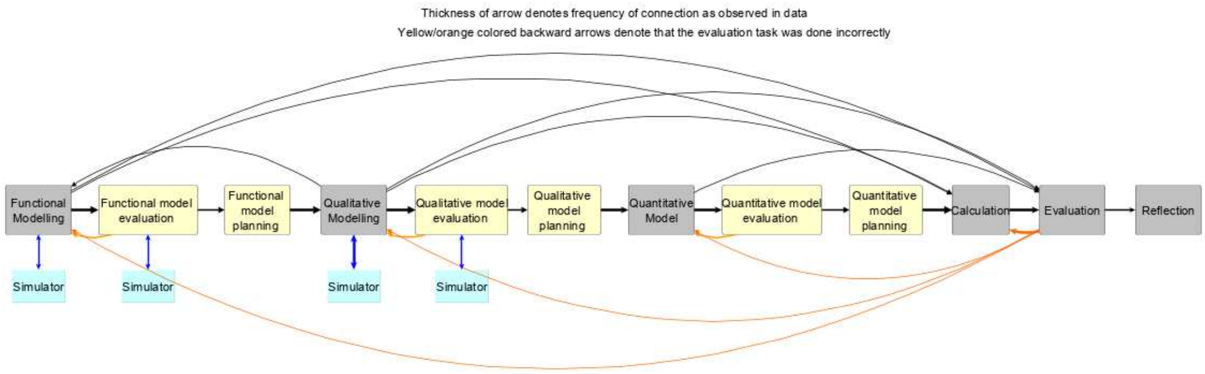

Figure 5. Estimation process of medium performers.

while building the qualitative model, but not to do disciplinary reasoning during model evaluation. As before, their responses to the questions show that they did not do model evaluation well, and this was for two reasons. Firstly, they were unable to do the disciplinary reasoning required, i.e., identifying which parameters dominate, even though the simulator was available. They used their intuitive reasoning rather than systematically exploring the graphs in the simulator to experiment, compare and decide. Secondly, they did not answer the model evaluation questions or act upon the feedback/guidance, because they did not perceive them useful to solving the problem. When they did not satisfy the evaluation criteria, they iterated back to calculation first and then to each modelling task, often multiple times. Most often though, they only read but made no changes in the model because they were unable to self-assess their models and make changes. However, they persisted in their attempts to refine their models and estimate, going back and forth between the sub-tasks.

Medium performers did not build on their models, effectively treating each task as an independent entity that needed to be completed without realizing its purpose. This was evident from their answers to the planning questions, which show that they were thinking in terms of the parameters associated with the system, rather than in terms of the purpose of each model in estimation. Only two out of these five students completed the reflection and their responses during the interview show that they did not see much value to planning and reflection. They abstracted out a two-phased MBR process for solving such power estimation problems, as described by S4, *"...actually there was a table, in that table there were different things differentiated such as the first part. That included the identification of those parameters which would affect the power actually such as velocity, mass, acceleration. So first I properly identified those parameters and then further I went to the next step. My next step was to draw a general diagram which would relate all these parameters together. Then my third step was to provide a formula which could properly implement this using the previous knowledge that I have studied. Such as the linear equations of motion and all. My fifth step was to implement that formula using numerical values, in order to get a proper output. But actually, I wasn't successful in obtaining the proper result..."*

Estimation process of low performers

The low performers (S8-S10) broadly followed the process of doing functional, qualitative and quantitative modelling, followed by calculation and evaluation (Figure 6). However, they did not obtain order of magnitude estimates or identify the correct parameters and equation. They reported that they did not see the value of doing functional modelling and so concentrated their efforts on creating qualitative and quantitative models. However similar to the medium performers they were unable to do model evaluation appropriately and did not use the feedback/guidance to revise their models, as shown using thick orange backward links in Figure 6. This was because, as their models at each stage show, these students had very weak conceptual knowledge, and so were unable to interpret and follow the feedback/guidance. Even though they used the simulator often, and read the guidance/feedback of disciplinary reasoning, their retrospective reports show that the inferences that they drew were incorrect or incomplete. Thus, weak conceptual knowledge was a limiting factor for their incorrect models.

Low performers went back to previously completed tasks when they were unable to satisfy the evaluation criteria. However, unlike the medium performers they did not persist on revising their models and iterating, choosing instead to end the problem solving. They reported that this was because they were unfamiliar with the required conceptual knowledge which, as they reported, constrained their reasoning and led to frustration. Similar to medium performers, low performers did not build on their models in the sense that the functional model did not lead to the qualitative model, and the latter did not lead to the quantitative model. Their answers to the planning and reflection questions show that they were able to identify "what" they needed to do, as described by S8, *"Later* *when I read carefully, I noticed what I needed to do. So, the hints that were there were useful. After that, I understood that I was going by a wrong sequence, and if I follow the right sequence I would be able to get an answer. So, I understood the right sequence and started proceeding like that."* However, the lack of conceptual knowledge made it difficult for them to understand and implement the components of the "right sequence" and was the reason for their poor estimation performance. This was also the reason for their being unable to understand the three-phased MBR process for estimation. While they recognized the sequence, they did not understand how to do its components and how to integrate them.

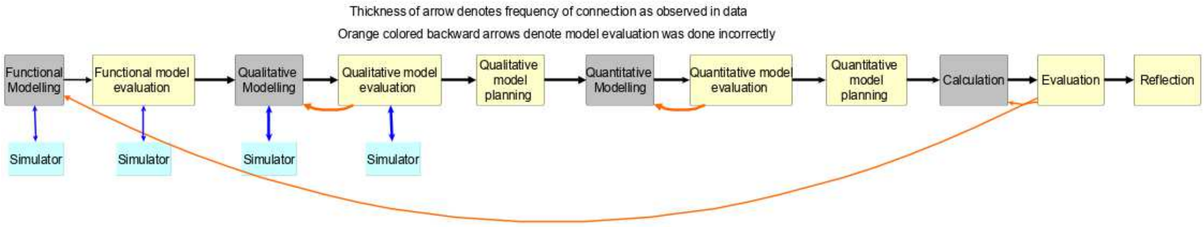

Figure 6. Estimation process of low performers.

Discussion

The goal of this study was to investigate the student process of solving estimation problems in a resource and scaffold rich problem-solving environment called MEttLE in order to identify the processes of good estimation. We found that high performers used the model-building affordances, simulator, guidance of disciplinary reasoning and evaluation questions to build, evaluate and revise functional, qualitative and quantitative models and solved the estimation problem. Thus, the resources and scaffolds in MEttLE supported them in building simplified models and then integrating the three models in order to obtain a good estimate. Further, we found that high performing students were able to report a broadly applicable estimation process and its components needed for solving problems of power estimation. This suggests that MEttLE supported high performers in learning a method that they could apply in the future for solving power estimation problems.

Medium performers built functional, qualitative and quantitative models, but did not systematically use the simulator, model evaluation questions or act on the guidance/feedback of disciplinary reasoning to evaluate and revise their models. As a result, they were unable to integrate all the three models and obtain a good estimate. Further, their reflection shows that they abstracted out a two-phased MBR process because they did not understand the role of functional modelling. Thus, they did not do or understand all the components of the estimation process because they were not systematic and reflective while solving. These students reported that they would need to practice a few times in MEttLE to learn all the components of estimation problem solving. Low performers attempted to build functional, qualitative and quantitative models, but they did not have the conceptual knowledge needed to use the simulator, model evaluation questions and the guidance/feedback of disciplinary reasoning to evaluate and revise their models. As a result, their models were incorrect and they did not obtain a good estimate. These students reported that they would need a better grasp over the conceptual knowledge and practice several times in MEttLE to learn all the components of estimation problem solving.

Together, the findings indicate that the interaction path of building, evaluating and revising functional, qualitative and quantitative models, followed by reflectively integrating all three models together leads to good estimation performance and understanding a three-phased MBR process for solving similar estimation problems. On the other hand, not evaluating and revising the models, and not reflectively integrating all three models leads to poor estimation performance, and not understanding the three-phased MBR process for solving similar estimation problems. The findings also suggest that the features of MEttLE including the model progression structure, the question prompts for model building, evaluation and planning, multiple visualizations of the problem system including variable manipulation simulations for implicit guidance of disciplinary reasoning, and explicit guidance of disciplinary reasoning scaffold the processes of good estimation.

This study reaffirmed a result intuitively known to every domain instructor - poor conceptual knowledge is a limiting condition for obtaining good estimates (Linder, 1999; Shakerin, 2006). Estimation requires disciplinary MBR which was supported by the features of MEttLE, but accessible only to students with good conceptual understanding. As the process of low performers showed, poor conceptual knowledge makes it difficult for students to understand and act upon the scaffolds (question prompts, simulator, guidance/feedback). Thus, the findings of this study elaborate on the findings with estimation experts (Kothiyal et al, 2016) and show that conceptual knowledge, disciplinary MBR, evaluation and reflection are all necessary components of estimation. Further, the process underlying good estimation performance is an intertwining of model-building, evaluation and planning, all of which use knowledge (informal and conceptual) and the problem context as inputs.

Based on our results we conjecture that when students solve problems in MEttLE by executing the model-building processes, take action based on the question prompts and guidance of disciplinary reasoning to evaluate and revise their models, reflect on the role of each model and integrate them into the MBR process, and deliberately practice (Litzinger et al., 2011) this process on multiple estimation problems, they will learn a method for solving similar estimation problems in the future. In the next iteration of this DBR project, we will introduce more problems in MEttLE for practice and evaluate this learning conjecture.

While our results are based on a small sample, we reiterate that the goal of this study is not generalizability, but identification of a process underlying good estimation which may be systematically supported among students so that they learn to do good estimation. The results of the in-depth interaction analysis highlight the importance of supporting disciplinary MBR, model evaluation and reflective integration of models among students so that they learn estimation. This work also highlights the need for learning analytics (for instance, Biswas et al, 2019) to identify student processes as they are solving problems and provide adaptive support to ensure that they learn the processes of good estimation. In future work, we plan to incorporate such learning analytics within MEttLE to better support students in learning estimation problem solving.

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