Seeing Algebraic Thinking in the Classroom: A Study of Teachers' Conceptualizations of Algebra

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Abstract: Ideas about how students learn algebra have shifted over the years. Reform curricula have been developed to address these changes. Teachers' conceptualizations of algebra play a large role in how they view and implement reform curricula. This paper explores teachers' conceptualizations of algebra and investigates how they draw on their conceptualizations in the moments of instruction through the use of new methods that incorporate teacher-selected classroom video clips. Data revealed that teachers' verbal descriptions of student algebraic thinking did not always align with moments of instruction they selected as providing evidence of student algebraic thinking.

Introduction

Algebra is seen as a critical subject matter for students. In particular, algebra has been identified as the "gatekeeper" to advanced mathematics. Students need to have taken algebra in order to have access to upper-level mathematics courses. Such courses can then lead to further opportunities such as admissions to colleges and universities. Despite its importance, many students struggle in algebra classes (Kieran, 1992). This has prompted inquiry into the question of how to effectively support student learning of algebra.

Ideas about how students best learn algebra have recently shifted. Up until the last decade or so, algebra classes emphasized procedural activities such as symbol manipulation over more conceptual activities. In addition, procedural activities were generally separated from the conceptual sense-making activities (Kieran, 2004). This was in line with the "symbol precedence" view of algebra that most textbooks presented. According to Nathan and Koedinger (2000a) the belief underlying this view is that "arithmetic reasoning strictly precedes algebraic reasoning, and symbolic problem-solving develops prior to verbal reasoning" (Nathan & Koedinger, 2000a, p. 209). In contrast to the symbol precedence view, more popular today is an emphasis on the conceptual nature of algebra and of the importance of connecting symbolic and conceptual learning (Kaput, 2008). In response to this shift, reform curricula have been developed that are more in line with current views of algebra and algebraic thinking. However, research has shown that it can be difficult for teachers to teach reform curricula in the ways that they were intended (Cohen, 1990). One reason is that teachers' ideas about algebra may not align with the goals of reform. In this project, I investigate new methods for exploring teachers' conceptualizations of algebra and algebraic thinking and how these conceptualizations can inform practice.

Teachers' Conceptualizations of Algebra

There is much discussion in the literature about what constitutes algebra and algebraic activity (Battista and Borrow, 1998; Kaput, 1998; Kieran, 1996; NCTM, 1989; Usiskin, 1988). Kieran (1996) identified three forms of algebraic activity: generational, transformational, and global/meta-level. Generational activities include generating equations and expressions from data, situations, patterns of numbers, or number sequences. Transformational activity includes manipulating symbols following the rules of algebra. This includes typical classroom activities such as simplifying an expression, expanding a polynomial, and solving equations. Global/ meta-level activity includes using algebra to solve problems. This includes modeling, problem solving, providing evidence, and generalizing.

Traditionally, mathematics classes tend to give precedence to transformational activities. Such classrooms rely on highly procedural activities such as symbolic manipulation and focus little attention on more conceptual understanding that includes global/meta-level activities (Kieran, 2004). Recent shifts in the teaching of algebra recommend that transformational activities no longer be distinct from conceptual understandings. Therefore reform curricula have moved toward a balance among the forms of algebraic activity, emphasizing conceptual activities such as generalization, argumentation, and justification (Kaput, 1998; Carpenter, Franke & Levi, 2003; Kieran, 2004).

Teaching algebra from a more balanced perspective can be difficult for teachers for a number of reasons. First, while learning algebra themselves, most teachers experienced classes that emphasized procedures and symbol manipulation. As a result of this "dilemma of experience" (Doerr, 2004), teachers tend to hold on to this view and focus on procedural knowledge in their classrooms (Menzel & Clarke, 1998; Nathan & Koedinger, 2000b; Stephens, 2008). Second, because traditional methods are what we find in most algebra textbooks today, many teachers have not had the opportunity to explore a more conceptual approach to teaching algebra. Even when given reform materials, teachers are often asked to implement them without much guidance and they may end up interpreting the new materials in terms of their familiar approaches to the topic (Sherin, 2002). This is reminiscent of Cohen's (1990) description of Ms. Oublier, who interprets a reform curriculum in light of her traditional teaching methods.

Conceptualizations of Algebra in Practice

This paper seeks to understand teachers' conceptualizations of algebra and the relationship between these conceptualizations and teachers' instructional practices. In particular, the goal is to investigate the conceptualizations that teachers draw on while teaching. This is somewhat of a challenge, however. One way to explore a teacher's conceptualizations is through interviews where teachers are asked to explain their ideas. We know, however, that a teacher's espoused beliefs do not always align with their teaching practice (Cooney, 1985; Cohen, 1990). For example, Cooney (1985) describes an example of a teacher who, when interviewed, said he believed problem solving was essential to mathematics learning yet, due to a number of factors, did not incorporate much problem solving in his instruction. In order to explore the relationship between teachers' conceptualizations of algebra and their practice, we need to look at both espoused views and views as enacted.

One way to gain information about a teacher's enacted views is to observe her class instruction. However, one problem with observing a class and making inferences is that we do not know if our interpretation is the same as the teacher's. We might interview a teacher after class to try to access her thinking during instruction, but she may not remember what she was thinking in the moment of instruction. Many times teachers recall general things about a class based simply on overall feelings about how the instruction took place (Rosean, Lundeberg, Cooper, Fritzen, Terpstra, 2008). A related approach to exploring a teacher's practice is through the use of video.

Video to Explore Teacher Thinking in the Moments of Instruction

Video has been a valuable tool used to examine a teacher's practice (Lampert & Ball, 1998; Sherin & van Es, 2005; Sherin, 2007). Video offers the advantage of allowing observers to watch events as they took place instead of relying on memory or observation notes. This allows teachers the ability to "get back into" the moment of instruction. However, while the use of classroom video gives us valuable insights, it raises questions about whether a teacher's attention to student thinking after the fact is different from her attention in the moment of instruction. Relying on classroom video may draw a teacher's attention to things that they did not notice in the moments of instruction. Ainley and Luntley (2007) used video as a way to access teachers' in-the-moment reasoning. As they observed, however, "in some interviews, the effect of watching the video was to draw attention to aspects of the classroom that the teacher had previously been unaware of, rather than to help recall their perceptions of the lesson" (p. 9). So we cannot always be sure an interview based on classroom video taps into what a teacher was thinking while she was teaching.

To address this issue, new methods have been explored that make use of a camera that allows a teacher to record moments that catch her attention while she is teaching (Sherin, Russ, Sherin, & Colestock, 2008). This allows access to in-the-moment noticing that was not available before. Sherin, Russ, Sherin, & Colestock (2008) piloted the Camwear 100 (Reich, Goldberg, Hudek, 2004) that allows teachers to capture short video clips while they are teaching. It involves a small camera a teacher can wear on her person while she is teaching. When the teacher presses the record button, the camera captures the previous 30 seconds and saves it to a small digital video file to be viewed later. Using this technology allows me to study teachers' in-the-moment thinking in a way that has only recently become possible. By asking a teacher to press the button while teaching, I can identify moments the teacher was attending to in the midst of instruction and can save these moments to discuss later.

One question that might arise with this method is whether using this technology actually allows access to a teacher's in-the-moment noticing or whether it instead prompts an ad-hoc development of the vision of what is going on at the time. Sherin, Russ, and Colestock (in press) provide evidence that

interviewing a teacher about the clips they captured using this technology can in fact bring teachers back to the moment and does help them talk about their thinking during instruction.

In this study, I make use of this methodology to create a more complete picture of how teachers' conceptualizations of algebra inform a teacher's practice. By using methods that allow access to teacher-selected classroom video, I can identify the moments a teacher notices while teaching. I use information about what teachers notice and how they reflect on these moments, in addition to classroom observations, to help better explore their ideas about algebra and how they draw on these ideas in the moments of instruction.

Methods

This study explored high school teachers' ideas about student algebraic thinking. Four high school algebra teachers who teach in a local urban district volunteered to participate in this study. Two of the teachers, Dana and Maggie, taught from a traditional curriculum. Maggie taught ninth grade algebra and Dana taught eleventh grade algebra. Their teaching experience was four and nine years respectively. Nancy and Richard taught from a reform curriculum. Similar to the other pair, Nancy taught ninth grade algebra and Richard taught eleventh grade algebra. Their teaching experience was thirteen and seven years respectively. The traditional curriculum espoused a symbol precedence view whereas the reform curriculum placed a focus on student sense-making and emphasized the conceptual nature of algebra. The data consists of interviews, class observations, and selected excerpts of instruction chosen by the teachers.

The study protocol and instrumentation

Pre-observation interview

Each teacher participated in a pre-observation interview designed to investigate his or her ideas about algebra and algebraic thinking. The pre-interview had two sections, the *general question* section and the *concrete student response* section. Each section of the interview lasted around 20 minutes. The general question section consisted of questions about the teacher's ideas about algebra. I asked direct questions to try to get a sense of the teacher's ideas about algebraic thinking. An example of a question from this phase was, "What do you mean when you say that you want to get students to think algebraically?" In the student response section, I showed the teacher sample student responses to problems and asked the teacher whether he or she thought the student was thinking algebraically based on the solution presented.

Classroom observation

The Camwear 100 is a small, portable camera developed by Dejaview (Reich, Goldberg, & Hudek, 2004). When the camera is on it continually records and erases footage in a loop mode until the "record" button is pushed. When the user presses the record button, the camera saves the preceding 30 seconds. These clips can be downloaded, saved and viewed on a computer. I videotaped each classroom session from the back of the room. During the session, I asked teachers to wear the camera while teaching. I asked the teachers to press the record button whenever they saw evidence that their students were thinking algebraically.

Post-observation interview

After the classroom observation, I interviewed the teachers and asked them about each clip they captured and why they thought their students were thinking algebraically in each clip. I wanted to understand the nature of student thinking the teachers were paying attention to while they were teaching. Although I cannot claim that the camera is a direct window into the teachers' thinking, I can use the clips the teacher captured as a slice of evidence of what they were paying attention to while they were teaching. My intention for the post-interview was to help clarify or provide additional insight into the moments the teacher captured and why those moments stood out to the teacher. Furthermore, it would permit triangulation with the pre-interviews in order to gain a better understanding of the teachers' ideas about algebraic thinking.

Data analysis

In the first stage the type of activity the students were engaged in during the clips was coded. This was done by viewing the clip in the context of the whole class video that had been taken from the back of the room. This allowed me to see the context of each clip in the larger landscape of the class activities. There

were two broad codes, procedural manipulation and conceptual. The procedural manipulation code was applied when the student or students in the clip were in the midst of solving a symbolic algebra problem or discussing how they had solved the problem. An example is when Maggie asked her students to compute the side length of a triangle using the Law of Sines. The conceptual code was applied when the student or students in the clip were involved in a larger sense-making activity. An example is when the students in Richard's class extended a problem he asked them to do and discussed whether the procedures could be generalized from two dimensions to three or more dimensions and if so, what it might look like.

Then the post interview in which each teacher discussed the reasons for capturing each clip was reviewed. In this first pass, I was able to distinguish between the clips the teachers had a reason for capturing and those they did not. It turned out that in a few cases, upon further reflection of the clip, the teacher decided that he or she would not consider the thinking the students were engaged in during the clip to be algebraic so those clips were removed from the next stage.

In the third stage the moments the teacher captured and the reasons they gave for capturing that moment were coded. The unit of analysis was the teacher's reason for capturing the clip. I was interested in why the teacher thought the student or students in the moment they captured were thinking algebraically. I engaged in the process of open coding where I looked at some of the teachers' responses and came up with some categories. I continued with these categories and the main reasons for identifying algebraic thinking that emerged were students (1) making connections, (2) generalizing, (3) justifying, articulating or testing, (4) understanding rules or properties of algebraic structures, (5) manipulating symbols or solving an equation, and (6) knowing or following rules of algebra. The first four were considered conceptual reasons and the last two were considered procedural reasons. Each teacher's response was assigned one code. After coding the teachers' reasons for capturing clips, I made a table containing a summary of the information. I looked for patterns in each teacher's responses. I also looked for patterns across teachers.

In the fourth stage I reviewed the pre-interviews. I was curious about whether or how ideas that emerged in the pre-interviews influenced the moments the teacher captured and how the teacher talked about those moments. I was interested in each teacher's ideas about algebra, goals for students, and feelings about their curriculum that emerged in the interviews. The analysis of the pre-interviews took place in three steps. In the first step, I segmented the transcripts and identified which portions dealt with ideas about algebra, goals for students, and thoughts on the curriculum. In the second phase, I looked at all the segments having to do with each of the three categories and I identified the central idea the teacher discussed in those segments. I did this iteratively and came up with a set of central themes the teacher discussed. In the last stage, I identified whether the themes corresponded with procedural or conceptual ideas about algebra.

Finally, I reviewed the class videos and created a summary. In the summary, I included three aspects of the class and the structure of the lesson. They were the way the class was organized, the mathematical topic that was covered, and the types of problems the students were asked to work on.

Results and Discussion

I found that two teachers' espoused ideas about algebra aligned with the clips they captured in the classroom, therefore adding evidence to their conceptualizations of algebra as described in their interviews. One of the teachers, Maggie, held a symbolic view of algebra and this was consistent in her pre-interview and in the clips she captured and the way she discussed them in the post-interview. For example, in one clip, while Maggie was presenting a problem on the overhead a student volunteered a solution to the problem. Maggie captured the clip and when asked why she captured the clip in the post-interview she responded, "(the student) was doing basic problem solving equations. They were solving for x. Solving for the variable."

Another teacher, Nancy held a conceptual view of algebra and this was consistent with the way she discussed algebra in the pre-interview and in the clips she captured and discussed in the post-interview. In one of Nancy's captured clips a student was explaining why he thought the quadratic function the class was graphing would not "bounce" at zero. When asked about the clip in the post-interview said she thought the student was thinking algebraically because he understood the properties of parabolic functions and understood how the graph should look. He understood how symmetry was connected to the shape of the graph and saw how the graph related to the equation.

While Maggie and Nancy captured clips that supported the ideas that emerge in their pre-interviews, the other two teachers were less consistent in the way they talked about algebra in the pre-interview and the clips they captured while teaching.

Dana

Dana taught eleventh grade algebra from a traditional curriculum. She talked about algebra from both a symbol manipulation and conceptual point of view in the pre- interview. When she discussed the clips she captured in class, however, she discussed 8 of the 11 clips with a focus on symbol manipulation.

The aspect of student thinking that stood out to Dana was the students' facility with the symbols and rules. Even though this would appear to be evidence that Dana's conceptualizations of algebra are dominated by algebraic procedures and symbol manipulation, her pre-interview told a different story.

The primary theme that emerged in Dana's pre-interview was that she believed there were two types of algebra, school algebra and more "sophisticated algebra" that is not taught in schools. When presented with a student's less traditional, more conceptual solution in the pre-interview, she commented, "(it's) logical reasoning. And it's algebra but it isn't the kind of thing that gets taught… explicitly taught." In response to another student's similar solution she said, "(the solution) is more logic than…official algebra. But I still think they're a thinking person's approach to the problem… yeah, I think that's algebra. It's just not the rote algebra we have taught." She continued, "We teach them algorithms so they can develop that. I think (the solution the student had is) really sophisticated. But this (pointing to a more symbolic answer) is what I teach." So Dana is capable of recognizing two types of algebraic thinking-procedural and conceptual algebraic thinking. In the classroom, however, she tends to focus on the procedural aspects of algebraic thinking. For example, she captured a moment in her class in which a couple of students were trying to explain why negative numbers are not in the domain of the log function. In the post-interview she said that she thought the students were thinking algebraically because they were "defining how you do algebra" and they were discussing the rules of the game. Instead of focusing on the conceptual aspects of the conversation, she focused on the students' knowledge of the rules. In another clip she captured a student answering a problem she posed on the overhead projector. The problem had more variables than the previous problems had. Dana said the student was thinking algebraically because she was able to manipulate the symbols to solve the more difficult type of problem. She commented:

> …(some students) may have said '… there are so many pieces here, how do I plug this into where it goes and this into where it goes? I've got four pieces and I've only got three slots, what am I going to do?' So I was relieved to see that most of (the students in this class) just looked at it and went, 'oh', and just did it. And saw that it was actually a simple problem. It's a little bit rote, I mean I've got- here's this equation and here's this equation and here's how you transform them. It may not be super meaningful, but… but it's algebra *(laughs)*.

Her focus was primarily on the students' realization that they could manipulate this problem as they had done with others in the past. She did not focus on the student's conceptual understanding of the problem. Overall, Dana tended to focus on students' knowledge and manipulation of the rules of algebra during instruction.

Dana exhibited dual views of algebraic thinking in the pre-interview. One is more conceptual and she believes is not the type they explicitly teach in schools. The other is more procedural and is emphasized in her curriculum. Even though she is capable of recognizing the more conceptual algebraic thinking she did not use that to inform the moments she captured in class or the way she discussed those moments. She could have discussed the same clips she captured in a more conceptual way. In the next case, we will see an example of a teacher who, similar to Dana, primarily captured clips where students were engaged in procedural activity but who discussed them in a more conceptual way.

Richard

Richard taught eleventh grade algebra from a reform curriculum. The students' desks were arranged in groups and they spent a large part of the class working in their groups. The class started with a set of five problems that involved factoring and expanding polynomials, a highly symbolic type of problem. For the first 30 minutes of class the students worked on these opening problems in groups and then five students presented their solution to one of the problems to the class on a whiteboard. For the remainder of the class period, the topic shifted to probability and students discussed more open-ended problems from the curriculum in groups. Occasionally groups would present their solutions to the problems.

Richard captured a total of 10 clips in which he saw his students thinking algebraically. All but three of the clips were captured during the first 30 minutes of class when the class was working on the symbolic algebra problems. Richard's ideas about algebra and the symbolic nature of algebra prompted him to capture most of the clips where students were working through problems with symbols and manipulating expressions or equations using algebraic structures. For example, he captured one clip while listening to a groups' presentation of their solution to one of the symbolic problems he posed at the beginning of class. However, when asked why he thought he saw his students thinking algebraically in the clips, he discussed nine of the ten clips in terms of students' sense-making and only one of them was discussed from a procedural standpoint. For example, Richard captured the clip of students symbolically solving a problem, yet when he discussed the clip further, he said that the student was thinking algebraically, not because of symbol use, but because of the students' "high-level understanding of exponents". Richard's focus went beyond mere knowledge of the rules to a concern with students' understanding and sense-making. When he discussed the clip, he did not mention the symbol manipulation or procedures but focused on the depth of the student's understanding.

I found it interesting that most of the clips were captured during the very symbolic algebraic part of the lesson yet he discussed most of them from a conceptual standpoint. I would have thought that if he limited his attention to the traditional algebra problems, he might focus on the symbolic or procedural nature of the students' thinking, as Dana and Maggie did. Why did Richard discuss procedural clips differently than the other two teachers? What lead to his focus on the conceptual nature of his students' thinking? I turn to Richard's pre-interview to answer some of these questions.

The theme that emerged from Richard's pre-interview was the dual nature of his goals for students. Because his goals were influenced by both his pre-teaching experiences and his curriculum, he had two distinct goals for his students. He described one of his goals for student learning as helping students master as much of the content as they could. This idea seemed to drive many of the clips he captured. This idea of mastery of the concepts seems to me to be in line with a more procedural idea of algebra and algebraic thinking.

Richard's learning goals for students seemed to be influenced equally by his ideas about algebra and the reform curriculum he uses. When I asked him about his goals for students, he answered, "I think that I'm probably a product of this curriculum. I'm not saying I know what they were thinking but I know my opinion is highly influenced by what I taught over the past five years." His curriculum emphasizes the conceptual nature of student thinking. The dual nature of Richard's goals for students becomes apparent when he talks about his experiences.

Richard's ideas about algebra and the curriculum influenced each other, however they were also in conflict with each other. There was kind of a trade-off, back-and-forth kind of interaction between the two and how they would affect his learning goals for students. His preconceived ideas about algebra affected the way he viewed the curriculum. He appreciated the conceptual nature of the curriculum but also believed it needed to be supplemented with more procedural material. On the other hand, the curriculum affected his ideas about algebra. The dual nature of his influences was apparent again when he said, "I also think that, however…that (the curriculum) is much more on the 'it's really important to conceptualize and understand end of the spectrum' than 'let's do these equations and understand'. You know? So I feel like I'm kind of trying to bridge the two." So his ideas about algebra before he started teaching influenced how he worked with the curriculum and the curriculum he uses influenced his preconceived ideas about algebra. Both factors were influential in his goals for students. This dichotomy might explain the fact that even though Richard tended to record clips in which students were involved in procedural activities, he focused on the deeper, conceptual understanding the students exhibited.

Richard's preconceived ideas about algebra were focused on procedures and skills and he believed that the probability lesson might not provide many instances of this type of thinking. The fact that all but one of the clips he captured was from the part of class where students were engaged in manipulating symbols might illustrate his need for symbols and equations as *signifiers* of algebraic thinking. On the other hand, influenced by curriculum he teaches, when he discussed the clips he captured, he did so in a conceptual way, meaning that he did not discuss the symbol manipulation. Instead he discussed the deeper, conceptual thinking his students were engaged in. Dana also captured clips where students were engaged in manipulating symbols, but when she talked about them, she discussed symbol manipulation and following procedures as the primary reason for identifying the students' thinking as algebraic.

Conclusion

Using a variety of methods allows us to draw a more complete picture of teachers' conceptualizations of algebra. In particular by using a video camera that allows teachers to capture footage in the moments of instruction, I was able to go beyond what is typically available through interviews and classroom video alone. Classroom activity is complex and watching video of an entire class does not necessarily give us insight into what a teacher was attending to while they were teaching. However, allowing a teacher the ability to capture and save video clips in the moments of instruction has the potential to give us this insight. Furthermore, interviewing the teacher about the clips they captured allows us access to the teacher's insight about the moment they captured and why that moment stood out to them.

I also extended prior research by exploring the relationship between teachers' professed and enacted views about algebra in particular. In line with previous research that has noted how difficult it can be to balance procedural and conceptual views I found that teachers continue to struggle with this dilemma.

In this study I showed that teachers can hold more than one idea about algebra and it is difficult to predict which aspects of their conceptualizations they will draw on from moment to moment while teaching. If we continue to explore teachers' ideas using methods described in this study we may be better able to understand their conceptualizations- where teachers' ideas come from and how they drive what a teacher notices in the classroom. This, in turn will help us facilitate teachers' transitions to a more balanced approach to algebra teaching that includes conceptual as well as procedural activity.

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