The Function of Mathematical Terminology: The Case of 'Slope'

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Abstract: This poster presents a case study of one Grade 6 student to illustrate and consider the role of introducing key mathematical terminology as a semiotic means to objectify mathematical ideas at hand (Radford, 2003). The poster describes how in two classrooms mathematical terminology provided a context for discussing graphs of linear functions, whereas everyday language used for the same mathematics resulted in ambiguous or unproductive discussion. As a component of an eight-day unit, students (n=50) were introduced to the word form *slope* to describe and compare linear functions. The case reported here presents a student before and after the introduction of the word *slope*. Interviews provide reason to consider the productive role of mathematical terminology in the development of mathematical understandings.

In this poster, I provide an example of a mathematical term, *slope*, functioning as a semiotic means of objectifying (Radford, 2003) the mathematical properties of slope. The case described here gives examples of one student providing responses that are mathematically normative when using technical terms, yet ambiguous or incorrect when using everyday terms to describe the same mathematics. While to many having an appropriate word to describe an applicable phenomenon may seem obvious or straightforward, there is a complex phenomenon in mathematics education. Symbol systems and other semiotic forms, including terminology, may be difficult to teach to students who have not yet grasped the concepts that the symbols or terms represent; nevertheless, the concepts may be difficult to teach to students who have not yet mastered the symbols or terms (Uttal, Scudder & DeLoache, 1997). One task of mathematics educators is to address this tension and determine instructional techniques to overcome this paradox. The poster presents a case study in which a Grade 6 student, Carolyn, uses mathematical and everyday terminology to describe linear functions on a graph. In particular, the word form *slope* provided a way to talk about the rate of change of each function in a way that everyday terms—such as words like *faster* or *steeper* that proliferated throughout this instructional sequence—left unclear. The case illustrates the semiotic role of specific mathematical terminology in mathematics meaning making and conversely the ambiguity everyday language can bring to bear on the same mathematics.

For the purposes of this study, slope was defined as the ratio, of one variable to another. Instruction often fails to connect the value of slope to qualitative descriptions of linear functions. Slope is often taught in a strictly procedural way, the result of which is a reduction of the concept of *slope* to a formula without any foundation in patterns of change. Caddle and Earnest (2009) reported on Grade 7 and 8 students reciting various formula for slope who then failed to connect these ideas to a ratio of change in a particular story context, displaying some of the pitfalls of having learned slope only as a procedure without conceptual grounding. Likewise, Moschkovich (1996) described Algebra 1 students successfully refining everyday terminology involving the ideas of slope without the technical term; at the same time, she acknowledged that everyday terms, such as *steeper* or *less steep*, "that may be sufficiently precise for everyday purposes proved to be ambiguous for describing lines in the context of a mathematical discussion" (273). Yet unanswered is the question of *when* the introduction of such terminology may behoove students in their inquiry into graphing. The introduction of the word form *slope* to the classroom discourse served as a semiotic means of objectifying (Radford, 2003) the rate of change of a function, with student interviews serving as data points to determine how students might have appropriated such a term.

Since slope is a core idea to the study of linear functions, I focus on this concept and the possible semiotic means that can be used by students to objectify their emerging understandings of this concept. The research question guiding the design and subsequent analysis of this study included: How might the technical term *slope* serve as a semiotic means to objectify mathematical ideas of slope?

Theoretical Framing

Given the underlying focus on semiotics, the analysis demands a framework that allows for detailed exploration of meaning ascribed to particular representational features. For this reason, I use Saxe's form-function framework (Saxe, 2004; Saxe & Esmonde, 2005), a cultural development framework that provides an analytic foothold on various semiotic means of objectification as they emerge and shift in activity. The framework provides a way to describe how the same ostensible form may have varied meanings across individuals at a point in time, while at the same time, one particular function, such as comparing trends in two linear functions, may be satisfied by various forms across individuals.

Forms are the various artifacts that can be found and used in a particular setting; in a mathematics classroom, forms can be various symbols or artifacts. They may range from components of written representations, such as a tickmark on a number line or an axis on a graph, to a specific word form, for example, *even* or *odd*, or in this case study, *slope*, *faster*, or *steepness*. These forms have meanings that are negotiated in social activity. *Function* refers to the use of a form in activity. The functions of such forms may vary across individuals. At the same time, there may be a culturally normative function associated with a given form in the context of a particular activity. The analysis makes use of this framework by considering word forms and the various functions the forms may be serving to the focal student.

Methods and Materials

The case study was drawn from a larger data corpus, a study that involved a sample of 50 Grade 6 students in two classrooms in Northern California with a predominantly Latino and African American population. One teacher provided instruction in both classrooms. *Individual cognitive interviews* were given to 10 students, including the focal student presented here, to further understand student approaches to solving these particular tasks and reveal learning hurdles. These interviews drew upon ideas that came up during classroom observations, and materials included particular problems from written assessments about which the interviewer asked the student to reflect. These interviews sought to probe more deeply students' written and oral responses and test the stability or fragility of such responses. *Data Sources* included videotapes of 8 classroom lessons and cognitive interviews, as well as copies of written student artifacts. Videos were transcribed and written responses were coded for response frequencies and the functions of representational forms.

Preliminary Findings, Conclusions and Implications

I chose Carolyn as a case study because the pattern of her responses well-reflected overall trends in student responses over the unit, making her a typical case. She was also one of ten students that participated in a cognitive interview, providing further insight into her reasoning. This analysis treated technical terminology a semiotic means to both enable and constrain the focus of individuals' meaning making.

The poster will present Carolyn's interviews regarding the same problems given before and after the classroom intervention. Results show that Carolyn interpreted two interview questions designed to address the same content in different ways. For prompts using the word *slope*, Carolyn drew upon the graph as a resource to talk about the slopes of two different linear functions. However, for prompts that made use of everyday language such as *faster*, Carolyn provided different answers that communicated a less sophisticated understanding of the mathematics at play. The use of particular words in the given prompt resulted in Carolyn interpreting the graph in different ways.

The case study seeks to demonstrate the complexity of language, both everyday and mathematical, with which students grapple as they engage in mathematics. While the larger study was exploratory in nature, Carolyn's interview gives reason to consider technical terminology as a key mechanism by which instruction may focus on particular mathematical concepts. In this case, such concepts seemed to remain ambiguous absent of that terminology. Findings have potential implications for curriculum development and instruction, in addition to considering in future research what various semiotic means, such as technical vocabulary, bring to bear on student understanding.

References

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