Representational Technology For Learning Mathematics: An Investigation of Teaching Practices in Latino/a Classrooms

Phil Vahey, Teresa Lara-Meloy, SRI International, 333 Ravenswood Dr. Menlo Park CA
philip.vahey@sri.com, teresa.lara-meloy@sri.com
Judit Moschkovich, Griselda Velazquez, University of California Santa Cruz, Education Department, 1156 High Street, Santa Cruz CA, jmoschko@ucsc.edu, gvelazqu@ucsc.edu

Abstract: This paper explores representation- and language-rich mathematics instruction in two classrooms with low-income Latino/a students. The two classroom teachers were part of a larger study investigating the use of SimCalc in middle school, and their classrooms had learning gains greater than the mean gains for the study overall. Prior analyses documented that these two teachers implemented exemplary classroom discourse practices: they engaged students in discussions with high intellectual work and showed high levels of responsiveness to student statements (Pierson, 2008). Such practices are consistent with both the research on representationally-rich mathematics environments, and instructional practices for students from non-dominant communities. We use transcript excerpts to illustrate these teachers' practices in detail. We found that these two teachers enacted some similar practices, but they also had different teaching styles, different approaches to connecting representations, and different ways of supporting mathematical discussions.

Introduction and Significance

As the population of Latino/a students increases in U.S. schools, so do concerns with their needs in mathematics classrooms. One concern is performance on assessments. Only 13% of Latino/a students scored at or above proficient on the 2005 8th grade NAEP exam, compared to 39% of white students and 47% of Asian students (Gándara & Contreras, 2009). Many schools attempt to address these concerns by placing large numbers of Latino/a students in remedial courses designed to address the needs of students who are "at risk," a decision which may exacerbate the situation. Research shows that the problem lies not in the students but in their access to quality teaching. In general, low-income Latino/a students attend racially segregated poor schools and have little access to the kind of teaching that supports the development of conceptual understanding or academic discourse (Gándara & Contreras, 2009). However, there are examples of classrooms where this is not the case. Quantitative data from the SimCalc study (Vahey, Lara-Meloy, Knudsen, 2009) and qualitative research with this student population has shown that Latino/a students can learn complex mathematics and participate in mathematical discussions (i.e. Moschkovich, 2002) when they have access to quality teaching.

In this paper we examine the teaching practices of two teachers, providing a sharp contrast to the perspective that Latino/a students need 'remedial" teaching. We analyze two instances of mathematics instruction that support conceptual understanding and mathematical discussion as students participate in representation-rich mathematics environments. There is reason to believe that there is consonance between the literature on improving instruction for low-income students from non-dominant linguistic backgrounds and the literature on the use of representationally rich technologies in mathematics (Vahey, Lara-Meloy, & Knudsen, 2009). Both highlight the use of multiple ways to represent ideas, point to supporting students as they make connections among these multiple representations, and point to the importance of language rich practices. However, there has not yet been a systematic attempt to integrate the representational affordances of technology with research on improving mathematics instruction for students from non-dominant linguistic backgrounds. This leaves educators with little guidance on how to use technology for this underserved student population.

This paper begins to address this gap by using two lenses to describe the instructional practices of two teachers who were successful in using technology to support students in their primarily high-poverty Latino/a classrooms in learning important conceptual mathematics. We explore the ways in which these successful teachers engaged in practices that were consistent with (or at odds with) research-based recommendations for mathematics instruction that is (a) representation-rich, and (b) supports mathematical discussions.

Background

We ground our exploration in the Scaling Up SimCalc study (Roschelle et al., in press) for three key reasons. First, SimCalc is based on a rich theoretical and empirical base, and so provides an excellent proxy for representationally rich technology use more generally. Second, the Scaling Up SimCalc study was a large scale randomized controlled experiment that not only found significant results across a wide range of student demographics, but also collected a rich array of data including many videotaped lessons. Finally, the study included large numbers of Latino/a(1) students (about half of the 1621 total students) from poor schools, many in the Rio Grande Valley, which borders Mexico and is one of the poorest regions in the United States (the

study denotes this area as Region 1, as it is served by the Region One Education Service Center). The SimCalc study is an ideal place to begin our exploration into the connections between research on representationally rich technologies and instructional recommendations for teaching students from non-dominant language groups.

The Scaling Up SimCalc Study

For over fifteen years the SimCalc project has had the goal of ensuring that all learners have access to complex and important mathematics (Kaput, 1994). To achieve this goal SimCalc places motion phenomena at the center of learning (see Figure 1), enabling students to build on existing cognitive and social competencies. Students in our studies, including traditionally low-achieving students, construct rich stories about motion over time, and use narratives as a resource for interpreting graphical and tabular representations of motion. SimCalc also allows students to play and replay a motion simulation as many times as they wish, providing more students access to these fundamental resources than is possible using traditional static media.
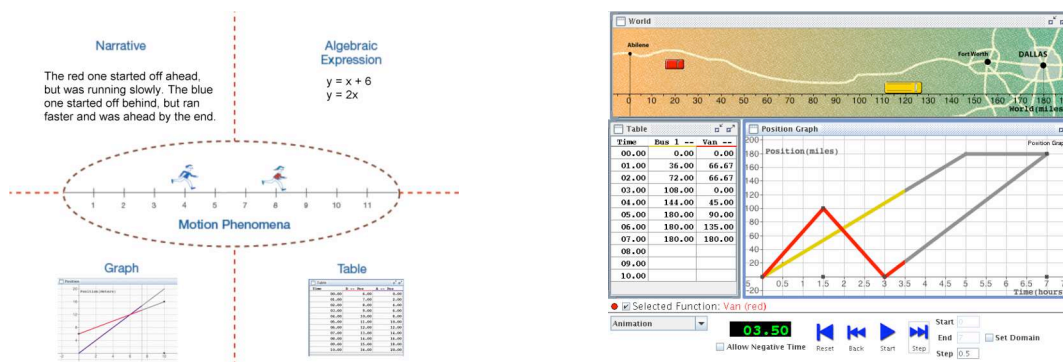
Figure 1. SimCalc linked representations (left); SimCalc MathWorlds® activity screenshot (right)

This engagement with motion leads to the study of functions through linked simulations, graphs, tables, and symbols. Students engaged with SimCalc can directly manipulate a mathematical representation such as a graph, and immediately see the effects on other linked representations (Roschelle et al., 2000). Formal mathematical vocabulary and symbolic forms can then be introduced *after* students have experiences with motion, narratives, tables, and graphs. In this way the vocabulary and symbols are *about* something, and can be understood as a compact and precise way of describing phenomena. The features of direct manipulation, multiple representations, and experience-before-formality may be particularly beneficial to students from non-dominant linguistic backgrounds, as these features alleviate some of the language demands found in mathematics classes, allowing students to work directly with mathematical objects (Vahey, et al., 2009).

These same features can allow students to more fully participate in classroom discussions. The linked representations provide a shared set of referents for students and teachers to explore: they can replay a motion or make changes in one representation to see the changes in the others. Students have opportunities to use a wider range of verbal and nonverbal communication acts, such as pointing: "See, right *here* the boy starts running faster." Students also have opportunities to use academic mathematical language for a communicative goal (e.g., Does *going longer* refer to *time* or *distance*?). This multi-faceted approach is consistent with recommendations for supporting mathematical discourse and developing vocabulary (Moschkovich, 2007a, 2007b) and it contrasts with traditional approaches to teaching academic language that rely on drill and practice or memorization.
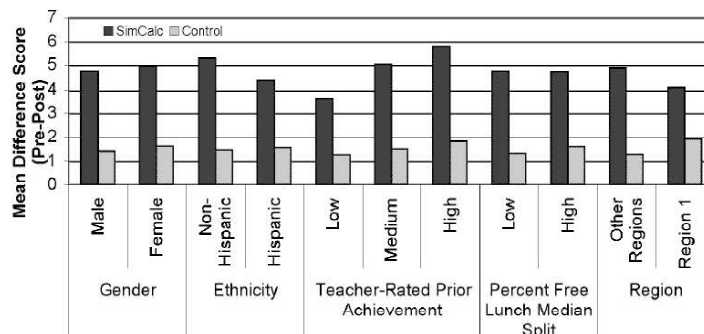
Figure 2. Mean student-learning gains by subpopulation group.

The Scaling Up SimCalc study found the SimCalc approach to be successful in meeting the needs of a variety of teachers and their students. Ninety-five seventh grade teachers across varying regions in Texas participated in a randomized controlled experiment in which they implemented a SimCalc-based three-week replacement unit. An analysis of the results showed a large and significant main effect, with an effect size of 0.8

(Roschelle et al., in press). This effect was robust across a diverse set of student demographics including gender, ethnicity, teacher-rated prior achievement, and poverty level (measured by the percentage of the campus eligible for free or reduced price lunch) (Figure 2). Consistent with our other data, we see that the students in Region 1 (the Rio Grande Valley) who used SimCalc had greater learning gains than students in the control condition, and these learning gains included improvement on items that target conceptual understanding.

Theoretical Framing

We take a socio-constructivist theoretical approach. We assume that students learn through interacting with materials and technological artifacts while participating in discourse, that mathematical discourse is multi-modal and multi-semiotic (O'Hallaran, 2000), and that it includes not only talk, but also other tools such as inscriptions or animations and several modes such as oral, written, and gestural.

Recommended instructional practices for all students

Research in mathematics education describes teaching that promotes conceptual development as having two central features: one is that teachers and students attend explicitly to concepts, and the other is that students wrestle with important mathematics (Hiebert & Grouws, 2007). Teachers face a considerable challenge in balancing both of these features in their teaching. In particular, teachers working with Latinos/as often focus on procedural rather than conceptual aspects of mathematics (Gándara & Contreras, 2009). In contrast, effective environments for students from non-dominant linguistic backgrounds should provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge" (Garcia and Gonzalez, 1995, p. 424). These practices require that teachers view language as a resource, not a deficiency (Gándara & Contreras, 2009), and reject models of their students as intellectually disadvantaged (Garcia and Gonzalez, 1995).

Framing research on students from non-dominant linguistic backgrounds is complex. This student population is heterogeneous (e.g. in some geographical areas this population includes students from many countries who speak many languages). Within our relatively constrained population (students from two classes in the Rio Grand Valley) almost all students are labeled as Hispanic. Although we do not have an exact account of how many students in these classrooms were learning English, it is likely that students in these two SimCalc classrooms, like students in this geographic area in general, vary along a spectrum of monolingual to bilingual. We assume that some students in these classrooms may be learning English; some students may be first generation immigrants, and some families may have lived in Texas for several generations. With that complexity in mind, we examine teaching practices for these two teachers because we believe they are illustrative of teaching strategies that support gains in conceptual mathematics for Latino/a learners, some of whom are likely to be learning English.

Use of representations

A key feature of mathematics is the use of representations, not only to overcome the limitations of human memory (as in the use of simple lists), but also to embed computational rules into symbol systems (e.g. algebraic notation), and to re-represent complex relationships in ways that can be more easily perceived by those who have facility with the representational system (e.g. the graph of a function) (Ainsworth, 2006). The use of representations is both a requirement for full participation in everyday quantitative reasoning (e.g. it is necessary to interpret graphs commonly shown in daily newspapers), and an important tool that can allow students to engage with mathematical objects as they also grow in their mathematical skills and understanding (Kaput, 1994). Representations, and dynamic representations in particular, can be particularly powerful learning tools when they are part of mathematical discourse: they can support shared focus of attention, allow gestural and physical communication to supplement verbal communication, and provide meaningful feedback that is consistent with the mathematical phenomena under investigation (Moschkovich, 2008; Roschelle et al., 2000).

Recommendations for instruction for students from non-dominant linguistic backgrounds (e.g. August and Shanahan, 2006; Echevarria et al., 2004) also include the use of representations. These recommendations focus on the use of visual artifacts to (a) offload demands on language, and (b) represent abstract ideas using illustrations or objects (e.g. holding up an illustration of a triangle when referring to a triangle in a geometry lesson), rather than as a way to provide insight into complex mathematical concepts.

Use of mathematical discussions

One way to engage students in both attending to and wrestling with important mathematics is through mathematical discussions. It is generally accepted that "mathematical discourse" and "academic language" are important for all students to develop, and are especially important for students from non-dominant communities. As students participate in these activities they are learning to communicate mathematically by making conjectures, presenting explanations, constructing arguments, and so on. When describing mathematical discourse we should not confuse "mathematical" with "formal" or "textbook." Textbook definitions and formal

ways of talking are only one aspect of school mathematical discourse. It is also important to avoid construing everyday and academic mathematical discourse as opposites (Moschkovich, 2007a), as some everyday experiences may provide resources for communicating mathematically.

Some recommendations for teaching reduce mathematical discourse to addressing vocabulary through direct instruction, drill and practice, or memorization. In contrast, other recommendations are based on research that shows that academic discourse is more than vocabulary, and that vocabulary is most successfully learned through instructional environments that are language rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000). Additionally, for mathematics instruction, learning to communicate mathematically is not primarily a matter of learning vocabulary, as students also need to develop conceptual understanding and learn to describe patterns, make generalizations, and use representations to support their claims (Moschkovich, 2002 and 2007b).

Analysis

In our exploration we examine and summarize the teaching strategies in one lesson selected from the classes of each of two teachers from Region 1, Teacher E and Teacher M. We chose these classes because these students are commonly labeled "at-risk" for poor mathematical performance: one class is 96% Latino/a (Teacher E) and the other is 100% Latino/a (Teacher M); and both classes are in schools where over 84% of the students are eligible for free or reduced price lunch. However, a previous quantitative study (Pierson, 2008) found that students in these two classrooms had gains greater than the mean gains for all classes using SimCalc. Pierson's analysis indicates that one possible reason for these gains may have been that these two teachers engaged their classes in discussions with high intellectual work, and showed high levels of responsiveness to student statements (Pierson, 2008). Intellectual work reflects the cognitive work requested from students with a given turn of talk. Responsiveness is an attempt to understand what another is thinking, displayed in how the teacher builds, questions, clarifies, takes up or probes that which another says.

Pierson (2008) shows that, for the 13 classes that had video data available, these two types of teacher moves were correlated with student gains. The two selected teachers stand in contrast with another teacher from a nearby school with a similar student population (95% Latino/a students and 93% students on campus on free and reduced lunch) who had lower than average intellectual work, responsiveness, and student gains. Our question moving forward is: in what ways did these two teachers, selected for exemplary mathematical discourse practices, engage in practices that were consistent (or at odds) with recommended instructional practices for the use of representationally rich mathematics environments and supporting mathematical discourse? Given the correlational findings in Pierson, our goal here is to provide a more detailed qualitative description of interactions in these two classrooms. These two descriptive cases ground both research and practice: while the correlational study showed *that* the quality of these two teachers' discourse was related to student gains, our goal here is to describe *how* these two teachers enacted these practices during a lesson.

The data for this study are records of teacher activity at the front of the classroom over a single lesson. Thus, our analyses will focus on teacher's uses of talk and inscriptions. Our data will not let us investigate non-observable features such as teacher beliefs, or those features that take an extended time to develop, such as the initial setting of classroom discourse norms. The classroom videos and transcripts we analyzed for teachers E and M show the same lesson from the SimCalc 7th grade curriculum, entitled "On the Road." During this lesson, students are expected to write stories explaining the motion of a bus and a van over several trips from Abilene to Dallas. Students must make sense of piecewise linear functions (see Figures 3 and 4) of increasing complexity. Students are asked to interpret functions that consist of multiple slope lines, including some with zero and negative slopes. In the first situation, students observe a position graph and corresponding computerized simulation of the motion of a bus and a van, and then write an explanatory story. In the second situation, students are asked to predict the motion of the bus and the van based on the graph before viewing the simulation to verify their predictions.

In our analysis we first partitioned the classroom videos into a set of curricular segments. These segments are based on curricular topics, as dictated by the teacher (and, of course, heavily influenced by the SimCalc materials). Within each segment we investigated episodes in which the participants focused on a particular mathematical topic. While the materials influence these topics, episodes were typically driven by the particular classroom interactions, and we expect episodes to vary significantly by classroom. In this paper we focus on representative classroom episodes that illustrate how these teachers (a) made significant use of representations or (b) engaged students in significant mathematical classroom discourse.

Our analysis shows that Ms. M and Ms. E used various strategies during these lessons. Since some of the teaching strategies were similar across the teachers and others differed, we emphasize that no single aspect of the teaching accounts for student gains, that there is no single way to enact best practices, and that there is no simple formula for teaching this student population. In fact, a strength of this analysis is to show that teachers with different teaching styles can instantiate best practices in such a way that students commonly labeled "at

risk" can and do learn complex and conceptual mathematics. Additionally, although we present examples of how the teacher used representations and supported discussion separately, these two are dialectically related, and this separation is only for the purpose of our illustration.

Teaching Practices that Connect Representations

As the teachers led class discussions based on position graphs, we note that both teachers consistently pushed the students to make connections between the two functions represented on each graph, and across multiple representations (simulation, narrative, and graph). In addition, after several attempts to have the students describe these connections, the teacher would then explicitly state the connection she considered most salient to the activity. Both teachers did this on occasions where the students had already described the desired connection, as well as cases in which the students had not, presumably so that all students had some exposure to the desired connection.
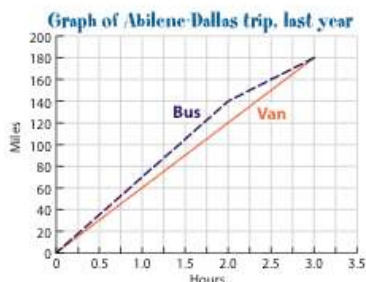

Figure 3. Last year's graph of the Abilene-Dallas trip.

Figure 4. The trip from three years ago.

Example 1: Ms. E (see Figure 3)

Ms. E conducted this class is in a computer lab, with a 1:1 student to computer ratio. Students were seated facing the computers, lined up along the wall. The teacher had a document camera projecting her workbook in the center of the room. During whole group discussions, she often pointed or traced her pencil over the graphs or text in her workbook projected via the document camera. This type of gesturing is a common strategy for helping students connect what is being said with the activity tasks. The following episode took place early in the lesson. The students worked in pairs at computers as they viewed the simulation that is represented by the graph in Figure 3. As this episode begins, the teacher was orienting the students to a whole-class discussion.

T:	Ok. Did everybody get a chance to go through the simulation?
Ss:	Yes.
T:	It says: (Reading from the workbook) "What information can you get about last year's trip from looking at and analyzing the graph? Write down everything you can think of and be sure to include the speeds of the vehicles." Ok. So the first thing that we noticed is that we have two graphs. One represents who and who?
Ss:	The bus and the van.
T:	Ok. Very good. The bus and the van. Ok. What can you tell me by looking at the graph?
Yuri:	The bus was going faster --
Amy:	-- The bus was going faster but then it slowed down and the van caught up.
T:	Ok. Yuri and Amy said that the bus was going faster but then what happened Amy?
Amy:	The bus slowed down and the van caught up.
T:	Ok. [Writing in her workbook] "The bus was going faster." So let's write that down.

In this excerpt we see the teacher explicitly asking the students to connect the graph to the simulation they had just viewed ("One represents who and who?", "What can you tell me by looking at the graph?"). However, it is not obvious (to the teacher or to the observer) whether the students were relying on their recollection of the simulation, the markings on the graph, or both. For example, the teacher asked for information that could be derived from the graph, but the source of Yuri and Amy's response is unclear, as they could have been referring to the simulation. Although the teacher accepted these responses in this interaction, she followed this conversation with a detailed analysis of the speeds of the bus and the van, using statements that connected the information they had gathered from the simulation with the details in the graph. This is shown in Example 3, in which we analyze how the teacher leveraged student language as a resource for not only connecting representations, but also for supporting conceptual understanding and language development.

Example 2: Ms. M (See Figure 3)

Ms. M's class was also held in a computer lab, with one student at each computer facing the front of the class. The relevant SimCalc software file was displayed for the class with a projector. Ms. M seemed to have a good rapport with her class, often teasing them and being willing to entertain their outlandish stories. In this example we see Ms. M asking students to read their stories to the class (as in Example 1, the class had already viewed the corresponding motion simulation). Ms. M challenged Dan's story not by questioning the (possibly inappropriate) content, but by questioning the relationship between the story and the graph.

> T: Everybody listen to Dan's story. Go ahead Dan.
> Dan: Alright. The bus driver is going 70 miles per hour. But then a monster got in the way and the bus driver had to slow down to 40 miles per hour. And, then he shot him with an emergency shotgun, (student laughter) but not before the monster ate him. But at the end they played in memory of him. But I had to take over and drive the bus--
> T: -- But the bus -- was already there, if I'm interpreting correctly, you already said that they got there and that's when the monster attacked him?
> Dan: No! [unintelligible]
> T: Read it again. Go ahead.
> Dan: "The bus was going 70 miles per hour. But then all of a sudden, a monster gets in the way (he makes a monster growl)."
> T: And it slowed down?
> Dan: Yeah. And the bus, it slowed down, he got scared, and took the emergency shotgun and shot him but not- not-
> T: While he was driving?
> Dan: Yeah, while he was- like this (pretends to shoot a gun, students laugh).
> T: Okay. That's kind of scary.
> Dan: But not before the monster ate him. So I had to take over and drive the bus. And the game is played in memory of him (reads imagined name and date of birth and death).
> T: But okay, in the real world, okay, if they eat something, how is it staying constant? (gestures to the graph projected on the board) How does it stay constant (gestures to the graph of the bus)? 40 miles per hour. Unless you were there while he was being eaten, and you put your foot, and you stayed steady. Understand?
> Dan: Yeah
> T: So maybe if you would have arrived in Dallas (points to Dan) and then the monster could have eaten the bus driver.

In this excerpt we see the teacher challenging Dan to describe exactly how his story related to the graph. She points out that in Dan's story the driver is eaten en route, without the bus stopping She pushes Dan to make the connection between his story and the fact that the graph shows the bus slowing from a constant speed of 70 mph to another constant speed of 40 mph. While she was apparently willing to accept that the monster scaring the driver would result in the bus slowing down in a manner consistent with the graph ("And it slowed down?...Okay"), she was not willing to accept that the monster could then eat the driver with the bus staying at the constant speed of 40 mph. Instead she provided an alternative narrative ("So maybe if you would have arrived in Dallas and then the monster could have eaten the bus driver").

In sum, we see that both teachers leverage the simulation capabilities of SimCalc to provide a shared context of motion that can be referred to by all students. They also used SimCalc's position graphs to provide a shared mathematical representation that grounded the discussion, and a set of stories that were used to explain the graphs and simulations. The teachers then spent considerable effort pushing students to make connections between these representations. In terms of their teaching practices for connecting representations, Ms E does this during the whole group discussion before writing stories, and Ms. M does this principally in response to students' stories. Noticeably absent is the use of representations as a purely visual aid designed to make abstract ideas concrete to language learners. In the next section we analyze how these teachers scaffolded the use of language in ways suitable for students from non-dominant linguistic backgrounds, and how this language use helped students interpret the representations as well as the target mathematical concepts.

Teaching Practices that Support Mathematical Discussion

In the lessons observed, neither teacher used explicit vocabulary instruction, vocabulary drill and practice, or memorization for vocabulary. Instead, both teachers addressed vocabulary in the context of language-rich instruction focused on making sense of the mathematics. Thus, they used many of the research based recommendations for vocabulary development: environments that are language rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000). We also found that both teachers

wove together everyday and academic meanings, vocabulary, and ways of talking. Both teachers accepted, built on, and used student contributions to introduce academic language. They often used and built on students' informal contributions and their own informal descriptions to include technical language in their discussions, and used colloquial language to further illustrate their own formal descriptions.

Despite these similarities, the two teachers addressed vocabulary using different approaches. Ms. M began her lesson by clarifying colloquial language used in the problem, for example "traffic tie-ups," and by asking students to describe what they saw on the map (roads, cities, towns, etc). In contrast, Ms. E asked her students to go straight into the simulation without previous discussion of the map or of the meaning of terms used in the problem. Ms. E also made heavy use of gesturing and pointing to the workbook (which was projected on a screen). She used her finger to point to the problem text as she read out loud, and frequently pointed to segments of the graph as the class discussed the problem. Below we provide two examples of the ways that each teacher's practices reflected recommendations for supporting mathematical discussions and the development of academic discourse.

Example 3: Ms. E (see Figure 3)

This excerpt follows immediately where Example 1 ends. After asking the students to determine "how fast" the bus was going, the teacher used student responses to introduce the word "speed," using the phrase "constant speed" as represented in the graph:

T:	How fast was the bus going? Ok we're going hours and miles. Miles and hours.
S:	60 miles per hour.
T:	Ok. How did you determine that?
S:	Divided the 180 miles by 3 hours.
T:	Ok. If I divide the final distance divided by the final time. Ok. That's gonna give me a speed. You're right. But you told me first he was going fast and then he slowed down. So was this a constant speed?
Ss:	No.
T:	No. Ok. So I need to determine the speed to see how fast he was going before he started slowing down. Ok?

In this example, the teacher moved from a general description of "the bus was going faster then slowed down" (shown in Excerpt 1) to asking students to find the specific speeds at which the bus was traveling. She used the students' responses to her questions to introduce into the discussion mathematical concepts and phrases such as speed, constant speed, and changing speed, all grounded in a discussion of how these three were represented in the graph.

Example 4: Ms. M (see Figure 4)

The following segment takes place towards the end of the lesson, after students have written stories about two motion situations. This example illustrates how Ms. M moved back and forth between formal and informal descriptions of a horizontal segment and clarified the mathematical significance of that segment on the graph:

T:	Now look at the bus. The bus looks pretty normal this time, right? Because we know that right here is a straight, horizontal line right. And what does the flat line tell us?
S:	It's going nowhere.
T:	It's going nowhere. And what kind of slope is it?
S:	Zero.

In this example, Ms. M combined formal academic language to describe the line as "horizontal" with a more informal description of the line as a "flat line." She repeated the student's contribution and followed it with a question about the slope, introducing the formal mathematical concept of a zero slope. Ms. M used colloquial language to lead the class to academic language, interweaving both types of language while grounding the discussion in the graph. In sum, neither teacher used explicit instruction of technical terms, but rather wove together both everyday and academic language to support a discussion about mathematical concepts. Our continuing analysis shows that both teachers' discourse was oriented to mathematical concepts rather than to vocabulary definitions.

Conclusions

This exploration into the practices of two teachers shows some connections between instruction that is both representation and language rich, and thus supports both conceptual mathematics learning and discussions that are oriented to mathematical concepts. The analysis shows that these two teachers shared some practices but also had different styles, approaches to connecting representations, and ways of addressing vocabulary. We find

it promising that this data shows that teachers can have an impact on the mathematics learning of students mistakenly labeled "at-risk." This beginning analysis is also promising in that these teachers are enacting many of the research-based teaching practices for providing representation- and language-rich learning environments.

Endnotes

(1) We use the term "Latino/a" to be consistent with current usage, although "Hispanic" is the term commonly used in Texas to designate people of Latin American—specifically Mexican—descent.

References

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183-198,

August, D. and Shanahan, T., eds. 2006. *Developing Literacy in Second-Language Learners: Report of the National Literacy Panel on Language-Minority Children and Youth*. Mahwah, NJ: Lawrence Erlbaum.

Blachowicz, C., & Fisher, P. (2000). Vocabulary instruction. In M. Kamil, P. Mosenthal, P. D. Pearson, & R. Barr (Eds.), *Handbook of Reading Research* (vol. III, pp. 503-523). Mahwah, NJ: Lawrence Erlbaum Associates.

Echevarria, J. Vogt, M. and Short, D. 2004. *Making Content Comprehensible for English Learners: The SIOP Model*. Needham Heights, MA: Allyn & Bacon.

Education Trust. (2003a). *Latino achievement in America*. Retrieved July 14, 2004, from http://www2. edtrust.org/NR/rdonlyres/7DC36C7E-EBBE-43BB-8392-CDC618E1F762/0/LatAchievEnglish.pdf

Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge, MA: Harvard University Press.

Garcia, E., & Gonzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record, 96* (3), 418-431.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: NCTM.

Kaput, J. (1994). Democratizing access to calculus: New routes using old roots. In A. Schoenfeld, (Ed.), *Mathematical thinking and problem solving*. Hillsdale, NJ: Erlbaum.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2 & 3), 189-212.

Moschkovich, J. N. (2007a). Examining mathematical discourse practices. *For the Learning of Mathematics, 27*(1), 24–30

Moschkovich, J. (2007b). Using two languages while learning mathematics, *Educational Studies in Mathematics*, 64(2), 121-144.

Moschkovich, J. N. (2008). "I went by twos, he went by one:" Multiple interpretations of inscriptions as resources for mathematical discussions. *The Journal of the Learning Sciences*, 17(4), 551-587.

O'Halloran, K. L. (2000). Classroom discourse in mathematics: A multisemiotic analysis. *Linguistics and Education, 10*(3), 359-388.

Pierson, J. (2008). *The relationship between teacher follow-up moves and mathematics learning*. Austin: University of Texas. Unpublished doctoral dissertation.

Roschelle, J., Kaput, J., & Stroup, W. (2000). SimCalc: Accelerating students' engagement with the mathematics of change. In M. J. Jacobson & R. B. Kozma (Eds.), *Innovations in science and mathematics education: Advanced designs for technologies of learning* (pp. 47-75). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Roschelle, J., Shechtman, N., Tatar, D., Hegedus, S., Hopkins, B., Empson, S., Knudsen, J., & Gallagher, L. (in press). Scaling up SimCalc: Results from random-assignment field trials in seventh and eighth grade mathematics. *American Educational Research Journal*.

Vahey, P. Lara-Meloy, T., & Knudsen, J. (2009). Meeting the Needs of Diverse Student Populations: Findings from the Scaling Up SimCalc Project. In Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 416-424). Atlanta, GA: Georgia State University.

Acknowledgments