Wiki

We calculated the following measures for the gerrymandering example on the Wikipedia page.

A. Perimeter to area ratio [10]
B. Ranked-marginal deviation (similar to standard deviation) [11]
C. Distance to mean [12]
D. Ratio of the number of seats won to the percent of votes a party got [13,14]
E. Area Ratio Convexity Measure: Ratio of the area of partition (P) to the area of the convex hull of P [14]
F. Efficiency gap [15-17]

Perimeter to area ratio (P/A):
area = 10

perimeter: (14, 14, 14, 14, 14) -> P/A = 14/10 * 5 = 7
perimeter: (16, 22, 20, 22, 16) -> P/A = 16/10 * 2 + 22/10 * 3 = 9.8
perimeter: (22, 22, 22, 22, 22) -> P/A = 22/10 * 5 = 11
perimeter: (16, 16, 14, 16, 16) -> P/A = 16/10 * 4 + 14/10 = 7.8

Ranked-marginal deviation (deviation):
Mean percentages for the collection of redistricting plans:
(1.0, 1.0, 1.0, 1.0, 1.0)

Sorted winning percentages (top left):
(0.6, 0.6, 0.6, 0.6, 0.6) ->

$$(0.6 - 1.0)^2 + (0.6 - 1.0)^2 + (0.6 - 1.0)^2 + (0.6 - 1.0)^2 + (0.6 - 1.0)^2 = 0.8$$

Gerrymandering Index = $\sqrt{0.8} = 0.89$

Sorted winning percentages (top right):
(0.9, 0.9, 0.4, 0.4, 0.4) ->

$$(0.9 - 1.0)^2 + (0.9 - 1.0)^2 + (0.4 - 1.0)^2 + (0.4 - 1.0)^2 + (0.4 - 1.0)^2 = 1.1$$

Gerrymandering Index = $\sqrt{1.1} = 1.05$

Sorted winning percentages (bottom left):
(1.0, 1.0, 1.0, 0, 0) ->

$$(1.0 - 1.0)^2 + (1.0 - 1.0)^2 + (1.0 - 1.0)^2 + (0 - 1.0)^2 + (0 - 1.0)^2 = 2.0$$

Gerrymandering Index = $\sqrt{2.0} = 1.41$

Sorted winning percentages (bottom right):
(0.8, 0.8, 0.6, 0.4, 0.4) ->

$$(0.8 - 1.0)^2 + (0.8 - 1.0)^2 + (0.6 - 1.0)^2 + (0.4 - 1.0)^2 + (0.4 - 1.0)^2 = 0.92$$

Gerrymandering Index = $\sqrt{0.92} = 0.96$

[**Ranked-marginal deviation (deviation) (Alternative method, based on method proposed by Herschlag et al [5]):**
Mean percentages for the collection of redistricting plans (unsorted, averaged district-wise):
(0.73, 0.7, 0.65, 0.58, 0.35)

Sorted winning percentages (top left):
(0.6, 0.6, 0.6, 0.6, 0.6) ->
Gerrymandering Index = $\sqrt{0.23} = 0.48$

Sorted winning percentages (top right):
(0.9, 0.9, 0.4, 0.4, 0.4) ->
Gerrymandering Index = $\sqrt{0.08} = 0.28$

Sorted winning percentages (bottom left):
(1.0, 1.0, 1.0, 0, 0) ->
Gerrymandering Index = $\sqrt{0.43} = 0.65$

Sorted winning percentages (bottom right):
(0.8, 0.8, 0.6, 0.4, 0.4) ->
Gerrymandering Index = $\sqrt{0.01} = 0.10$]

Distance to mean:
Blue wins: 5, 2, 3, 3
mean_blue = (5+2+3+3)/4 = 3.25

Distance to mean (top left) = |5 - 3.25| = 1.75
Distance to mean (top right) = |2 - 3.25| = 1.25
Distance to mean (bottom left) = |3 - 3.25| = 0.25
Distance to mean (bottom right) = |3 - 3.25| = 0.25

Seats to votes ratio (S/V):
Seats per partition = 1 if won, 0 o.w.

Blue votes:
Votes: (0.6, 0.6, 0.6, 0.6, 0.6) -> S/V = 1/0.6 * 5 = 8.33
Votes: (0.9, 0.4, 0.4, 0.4, 0.9) -> S/V = 1/0.9 * 2 + 0/0.4 * 3 = 2.22
Votes: (0, 0, 1.0, 1.0, 1.0) -> S/V = 0/0 * 2 + 1/1 * 3 = 3.0
Votes: (0.8, 0.4, 0.6, 0.8, 0.4) -> S/V = 1/0.8 * 2 + 1/0.6 + 0/0.4 * 2 = 4.17

Area Ratio Convexity Measure (ARCM):
Area = 10

Convex polygon area (top left): (10, 10, 10, 10, 10) ->
ARCM = 10/10 * 5 = 5.0

Convex polygon area (top right): (13, 18, 15, 18, 13) ->
ARCM = 10/13 * 2 + 10/18 * 2 + 10/15 = 3.32

Convex polygon area (bottom left): (10, 10, 10, 10, 10) ->
ARCM = 10/10 * 5 = 5.0

Convex polygon area (bottom right): (12, 12, 10, 12, 12) ->
ARCM = 10/12 * 4 + 10/10 = 4.33

Efficiency gap (EG): The efficiency gap (EG) might also be a valid identifier:

Top left:

	Votes		Wasted Votes	
District	Blue	Yellow	Blue	Yellow
1	60	40	10	40
2	60	40	10	40
3	60	40	10	40
4	60	40	10	40
5	60	40	10	40
Totals	300	200	50	200
EG	(200-50)/500 = 30% - Pro-Blue			

Top right:

District	Votes		Wasted Votes	
	Blue	Yellow	Blue	Yellow
1	40	60	40	10
2	40	60	40	10
3	40	60	40	10
4	90	10	40	10
5	90	10	40	10
Totals	300	200	200	50
EG	(200-50)/500 = 30% - Pro-Yellow			

Bottom left:

District	Votes		Wasted Votes	
	Blue	Yellow	Blue	Yellow
1	0	100	0	50
2	0	100	0	50
3	100	0	50	0
4	100	0	50	0
5	100	0	50	0
Totals	300	200	150	100
EG	(150-100)/500 = 10% - Pro-Yellow			

Bottom right:

District	Votes		Wasted Votes	
	Blue	Yellow	Blue	Yellow
1	80	20	30	20
2	40	60	40	10
3	60	40	10	40

4	80	20	30	20
5	40	60	40	10
Totals	300	200	150	100
EG	(150-100)/500 = 10% - Pro-Yellow			

Table 1. Quantifying gerrymandering for different exemplary maps based on frequently used estimators

	Perimeter to area ratio (P/A)	Ranked-marginal deviation	Distance to mean	Seats to votes ratio (S/V)	Area Ratio Convexity Measure (ARCM)	Efficiency gap (EG)
Figure 1 (top left)	7.0	0.89	**1.75**	8.33	5.0	**30%**
Figure 2 (top right)	9.8	1.05	**1.25**	2.22	3.32	**30%**
Figure 3 (bottom left)	11.0	1.41	0.25	3.0	5.0	10%
Figure 4 (bottom right)	7.8	0.96	0.25	4.17	4.33	10%

Follow-up question:
Q. Which measure do you think can be used to identify disproportionate outcomes. If none of the measures works, can you propose or identify measures that can be used to identify disproportionate outcomes.

A. Among the first three measures, distance to mean differentiates disproportionate versus proportionate outcomes. Lower distance to mean measure seems to be descriptive of the proportional outcome, whereas large measures indicate disproportionate outcomes.