Calculations for the real-world dataset ([Minnesota 2020 US house](#)):

Efficiency Gap

Map 1: Existing map

District	Votes		Wasted Votes	
	Democratic	Republican	Democratic	Republican
1	44.8	55.2	44.8	4.2
2	53.5	46.5	2.5	46.5
3	59.8	40.2	8.8	40.2
4	68.9	31.1	17.9	31.1
5	82.0	18.0	31	18
6	39.7	60.3	39.7	9.3
7	35.0	65	35	14
8	42.6	57.4	42.6	6.4
Total	426.3	373.7	222.3	169.7
EG	(222.3-169.7) / 800 = 6.6% - Pro-Republican			

Map 2: Murphy

District	Votes		Wasted Votes	
	Democratic	Republican	Democratic	Republican
1	46.4	53.6	46.4	2.6
2	56.1	43.9	5.1	43.9
3	60.4	39.6	9.4	39.6
4	68.4	31.6	17.4	31.6
5	82.5	17.5	31.5	17.5
6	39.4	60.6	39.4	9.6

District	32.7	67.3	32.7	16.3
7	32.7	67.3	32.7	16.3
8	43.7	56.3	43.7	5.3
Total	429.6	370.4	225.6	166.4
EG	(225.6-166.4) / 800 = 7.4% - Pro-Republican			

Map 3: Torkelson

	Votes		Wasted Votes	
District	Democratic	Republican	Democratic	Republican
1	43.9	56.1	43.9	5.1
2	47.4	52.6	47.4	1.6
3	64.7	35.3	13.7	35.3
4	70.0	30.0	19.0	30.0
5	82.1	17.9	31.1	17.9
6	43.0	57.0	43.0	6.0
7	36.1	63.9	36.1	12.9
8	42.8	57.3	42.8	6.3
Total	430	370	277	115
EG	(277.0-115.0) / 800 = 20.2% - Pro-Republican			

Map 4: Wattson

	Votes		Wasted Votes	
District	Democratic	Republican	Democratic	Republican
1	45.7	54.3	45.7	3.3
2	54.1	45.9	3.1	45.9
3	60.7	39.3	9.7	39.3
4	69.3	30.7	18.3	30.7

5	82.4	17.6	31.4	17.6
6	40.5	59.5	40.5	8.5
7	33.6	66.4	33.6	15.4
8	43.1	56.9	43.1	5.9
Total	429.4	370.6	225.4	166.6
EG	(255.4-166.6) / 800 = 7.4% - Pro-Republican			

Map 5: Sachs

	Votes		Wasted Votes	
District	Democratic	Republican	Democratic	Republican
1	45.3	54.7	45.3	3.7
2	59.6	40.4	8.6	40.4
3	62.8	37.2	11.8	37.2
4	67.5	32.5	16.5	32.5
5	77.5	22.5	26.5	22.5
6	38.4	61.6	38.4	10.6
7	34.3	65.7	34.3	14.7
8	43.4	56.6	43.4	5.6
Total	428.8	371.2	224.8	167.2
EG	(224.8-167.2) / 800 = 7.2% - Pro-Republican			

Map 6: Corrie

	Votes		Wasted Votes	
District	Democratic	Republican	Democratic	Republican
1	46.3	53.7	46.3	2.7
2	58.0	42.0	7.0	42

3	59.3	40.7	8.3	40.7
4	68.5	31.5	17.5	31.5
5	81.7	18.3	30.7	18.3
6	36.4	63.6	36.4	12.6
7	34.9	65.1	34.9	14.1
8	43.7	56.3	43.7	5.3
Total	428.8	371.2	224.8	167.2
EG	(224.8-167.2) / 800 = 7.2% - Pro-Republican			

Map 7: Anderson

	Votes		Wasted Votes	
District	Democratic	Republican	Democratic	Republican
1	44.6	55.4	44.6	4.4
2	54.1	45.9	3.1	45.9
3	60.9	39.1	9.9	39.1
4	69.0	31.0	18.0	31.0
5	82.2	17.8	31.2	17.8
6	41.3	58.7	41.3	7.7
7	35.0	65.0	35.0	14.0
8	42.1	57.9	42.1	6.9
Total	429.2	370.8	225.2	166.8
EG	(225.2-166.8) / 800 = 7.3% - Pro-Republican			

Seats to votes ratio

Map 1: Existing map

District	D Win	R Win

	Percentage %	Percentage %
1	0.0%	55.2%
2	53.5%	0.0%
3	59.8%	0.0%
4	68.9%	0.0%
5	82.0%	0.0%
6	0.0%	60.3%
7	0.0%	65.0%
8	0.0%	57.4%

Sum of seat to vote for Democrat party: $1 / 0.535 + 1 / 0.598 + 1/ 0.689 + 1 / 0.820 = 6.21$

Map 2: Murphy

District	D Win Percentage %	R Win Percentage %
1	0.0%	53.6%
2	56.1%	0.0%
3	60.4%	0.0%
4	68.4%	0.0%
5	82.5%	0.0%
6	0.0%	60.6%
7	0.0%	67.3%
8	0.0%	56.3%

Sum of seat to vote for Democrat party: $1 / 0.561 + 1 / 0.604 + 1/ 0.684 + 1 / 0.825 = 6.11$

Map 3: Torkelson

District	D Win Percentage %	R Win Percentage %
1	0.0%	56.1%

2	0.0%	52.6%
3	64.7%	0.0%
4	70.0%	0.0%
5	82.1%	0.0%
6	0.0%	57.0%
7	0.0%	63.9%
8	0.0%	57.3%

Sum of seat to vote for Democrat party: 1 / 0.647 + 1 / 0.700 + 1/ 0.821 = 4.19

Map 4: Wattson

District	D Win Percentage %	R Win Percentage %
1	0.0%	54.3%
2	54.1%	0.0%
3	60.7%	0.0%
4	69.3%	0.0%
5	82.4%	0.0%
6	0.0%	59.5%
7	0.0%	66.4%
8	0.0%	56.9%

Sum of seat to vote for Democrat party: 1 / 0.541 + 1 / 0.607 + 1/ 0.693 + 1 / 0.824 = 6.15

Map 5: Sachs

District	D Win Percentage %	R Win Percentage %
1	0.0%	54.7%
2	59.6%	0.0%
3	62.8%	0.0%

4	67.5%	0.0%
5	77.5%	0.0%
6	0.0%	61.6%
7	0.0%	65.7%
8	0.0%	56.6%

Sum of seat to vote for Democrat party: 1 / 0.596 + 1 / 0.628 + 1/ 0.675 + 1 / 0.775 = 6.04

Map 6: Corrie

District	D Win Percentage %	R Win Percentage %
1	0.0%	53.7%
2	58.0%	0.0%
3	59.3%	0.0%
4	68.5%	0.0%
5	81.7%	0.0%
6	0.0%	63.6%
7	0.0%	65.1%
8	0.0%	56.3%

Sum of seat to vote for Democrat party: 1 / 0.580 + 1 / 0.593 + 1/ 0.685 + 1 / 0.817 = 6.09

Map 7: Anderson

District	D Win Percentage %	R Win Percentage %
1	0.0%	55.4%
2	54.1%	0.0%
3	60.9%	0.0%
4	69.0%	0.0%
5	82.2%	0.0%

6	0.0%	58.7%
7	0.0%	65.0%
8	0.0%	57.9%

Sum of seat to vote for Democrat party: 1 / 0.541 + 1 / 0.609 + 1/ 0.690 + 1 / 0.822 = 6.16

Ranked-marginal deviation

Existing map = Map1; Murphy = Map2; Torkelson = Map3; Wattson = Map4; Sachs = Map5; Corrie = Map6; Anderson = Map7. (Source)

Mean percentages for the collection of redistricting plans:
(45.17, 54.69, 61.23, 68.8, 81.49, 39.81, 34.51, 43.06)

Sorted mean percentages = (81.49, 68.8, 61.23, 54.69, 45.17, 43.06, 39.81, 34.51)

Sorted winning percentages (Map1):
(82.0, 68.9, 59.8, 53.5, 44.8, 42.6, 39.7, 35.0)
Gerrymandering Index = $\sqrt{4.33}$ = 2.08

Sorted winning percentages (Map2):
(82.5, 68.4, 60.4, 56.1, 45.6, 43.7, 39.4, 32.7)
Gerrymandering Index = $\sqrt{7.9}$ = 2.81

Sorted winning percentages (Map3):
(82.1, 70.0, 64.7, 47.4, 43.9, 43.0, 42.8, 36.1)
Gerrymandering Index = $\sqrt{80.08}$ = 8.95

Sorted winning percentages (Map4):
(82.4, 69.3, 60.7, 54.1, 45.7, 43.1, 40.5, 33.6)
Gerrymandering Index = $\sqrt{3.29}$ = 1.82

Sorted winning percentages (Map5):
(77.5, 67.5, 62.8, 59.6, 45.3, 43.4, 38.4, 34.3)
Gerrymandering Index = $\sqrt{46.35}$ = 6.81

Sorted winning percentages (Map6):
(81.7, 68.5, 59.3, 58.0, 46.3, 43.7, 36.4, 34.9)

Gerrymandering Index $= \sqrt{28.28} = 5.32$

Sorted winning percentages (Map7):
(82.2, 69.0, 60.9, 54.1, 44.6, 42.1, 41.3, 35.0)
Gerrymandering Index $= \sqrt{4.71} = 2.17$

Distance to mean:

Democrats win: 4, 4, 3, 4, 4, 4, 4
mean_blue = (4+4+3+4+4+4+4)/7 = 3.86

Distance to mean (Map1) = |4 - 3.86| = 0.14
Distance to mean (Map2) = |4 - 3.86| = 0.14
Distance to mean (Map3) = |3 - 3.86| = 0.86
Distance to mean (Map4) = |4 - 3.86| = 0.14
Distance to mean (Map5) = |4 - 3.86| = 0.14
Distance to mean (Map6) = |4 - 3.86| = 0.14
Distance to mean (Map7) = |4 - 3.86| = 0.14

Map1: Existing map

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	1108.8	31538.8	38027.4	0.04	0.83
2	505.2	6552.2	8811.9	0.08	0.74
3	262.5	1509.4	1982.1	0.17	0.76
4	144.7	944.5	1075.5	0.15	0.88
5	113.4	367.8	444.9	0.31	0.83
6	661.6	7864.8	12627.9	0.08	0.62
7	2288.4	91651.9	125821.2	0.02	0.73
8	1804.6	78075.5	103292.0	0.02	0.76
Total	6889.32	218504.87	292082.85	0.88	6.15

Perimeter to area ratio (P/A):

0.04+0.08+0.17+0.15+0.31+0.08+0.02+0.02=0.88

Area Ratio Convexity Measure (ARCM):

0.83+0.74+0.76+0.88+0.83+0.62+0.73+0.76=6.15

Map2: Murphy

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	867.4	25020.8	27364.1	0.03	0.91
2	276.2	1872.2	2549.1	0.15	0.73
3	258.9	1335.9	1897.6	0.19	0.70
4	174.1	824.2	1126.9	0.21	0.73
5	116.5	351.1	439.6	0.33	0.80
6	702.6	7512.3	12291.4	0.09	0.61
7	1733.4	67729.5	81325.0	0.03	0.83
8	2297.2	120488.7	155311.5	0.02	0.78
Total	6426.44	225134.64	282305.34	1.06	6.10

Perimeter to area ratio (P/A):

0.03+0.15+0.19+0.21+0.33 +0.09 +0.03+0.02 = 1.06

Area Ratio Convexity Measure (ARCM):

0.91+0.73+0.70+0.73+0.80+0.61+0.83+0.78 = 6.10

Map3: Torkelson

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	1297.1	42267.2	50479.1	0.03	0.84
2	646.5	10057.4	13382.5	0.06	0.75

3	212.5	837.4	1368.4	0.25	0.61
4	150.4	701.9	852.4	0.21	0.82
5	99.9	349.3	392.8	0.29	0.89
6	423.3	5186.4	6481.1	0.08	0.80
7	2138.3	69746.0	99736.5	0.03	0.70
8	2005.1	95989.2	131058.0	0.02	0.73
Total	6972.91	225134.64	303750.89	0.98	6.15

Perimeter to area ratio (P/A):
0.03+0.06+0.25+0.21+0.29+0.08+0.03+0.02 = 0.98

Area Ratio Convexity Measure (ARCM):
0.84+0.75+0.61+0.82+0.89+0.80+0.70+0.73 = 6.15

Map4: Wattson

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	1130.2	31407.7	38146.4	0.04	0.82
2	485.9	5522.7	7526.1	0.09	0.73
3	273.2	1212.0	1764.6	0.23	0.69
4	163.9	911.7	1046.7	0.18	0.87
5	123.7	355.3	459.9	0.35	0.77
6	709.3	6286.5	10685.4	0.11	0.59
7	2421.2	80156.9	117206.3	0.03	0.68
8	2070.7	99281.7	139220.8	0.02	0.71
Total	7378.21	225134.64	316056.23	1.04	5.87

Perimeter to area ratio (P/A):
0.04+0.09+0.23+0.18+0.35+0.11+0.03+0.02 = 1.04

Area Ratio Convexity Measure (ARCM):
0.82+0.73+0.69+0.87+0.77+0.59+0.68+0.71 = 5.87

Map5: Sachs

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	989.8	28761.4	33542.1	0.03	0.86
2	255.7	2046.7	2421.9	0.12	0.85
3	201.9	1357.5	1518.1	0.15	0.89
4	176.1	1253.3	1465.4	0.14	0.86
5	130.2	401.4	564.1	0.32	0.71
6	737.1	12807.0	17131.3	0.06	0.75
7	2508.2	102830.6	137541.4	0.02	0.75
8	1688.3	75676.9	99087.5	0.02	0.76
Total	6687.25	225134.64	293271.81	0.88	6.42

Perimeter to area ratio (P/A):
0.03+0.12+0.15+0.14+0.32+0.06+0.02+0.02 = 0.88

Area Ratio Convexity Measure (ARCM):
0.86+0.85+0.89+0.86+0.71+0.75+0.75+0.76 = 6.42

Map6: Corrie

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	778.2	22849.9	25098.0	0.03	0.91
2	238.5	1228.3	1625.5	0.19	0.76
3	220.4	1295.3	1577.7	0.17	0.82

4	162.5	1040.6	1238.8	0.16	0.84
5	103.9	370.9	442.3	0.28	0.84
6	689.2	13194.8	16702.9	0.05	0.79
7	1552.3	61451.3	74704.2	0.03	0.82
8	2256.9	123703.6	154802.1	0.02	0.80
Total	6001.85	225134.64	276191.58	0.93	6.58

Perimeter to area ratio (P/A):
0.03+0.19+0.17+0.16+0.28+0.05+0.03+0.02 = 0.93

Area Ratio Convexity Measure (ARCM):
0.91+0.76+0.82+0.84+0.84+0.79+0.82+0.80 = 6.58

Map7: Anderson

District	Perimeter (P) (km)	Area (A) (km^2)	Convex polygon area (C) (km^2)	P/A	A/C (= ARCM)
1	1164.8	32684.8	39586.3	0.04	0.83
2	444.1	4834.6	6842.6	0.09	0.71
3	249.1	1328.0	1887.0	0.19	0.70
4	148.0	907.9	1027.7	0.16	0.88
5	103.6	354.4	424.6	0.29	0.83
6	605.5	6492.7	10199.2	0.09	0.64
7	2366.0	92904.5	126481.4	0.03	0.73
8	1737.3	85627.8	108612.0	0.02	0.79
Total	6818.32	225134.64	295060.74	0.91	6.11

Perimeter to area ratio (P/A):
0.04+0.09+0.19+0.16+0.29+0.09+0.03+0.02 = 0.91

Area Ratio Convexity Measure (ARCM):

0.83+0.71+0.70+0.88+0.83+0.64+0.73+0.79 = 6.11

Table 2. Quantifying gerrymandering for proposed Minnesota redistricting plans based on frequently used estimators

	Perimeter to area ratio (P/A)	Ranked-marginal deviation	Distance to mean	Seats to votes ratio (S/V)	Area Ratio Convexity Measure (ARCM)	Efficiency gap (EG)
Map1 Existing	0.88	2.08	0.14	6.21	6.15	6.6%
Map 2 Murphy	1.06	2.81	0.14	6.11	6.10	7.4%
Map 3 Torkelson	0.98	**8.95**	**0.86**	**4.19**	6.15	**20.2%**
Map 4 Wattson	1.04	1.82	0.14	6.15	5.87	7.4%
Map 5 Sachs	0.88	6.81	0.14	6.04	6.42	7.2%
Map 6 Corrie	0.93	5.32	0.14	6.09	6.58	7.2%
Map 7 Anderson	0.91	2.17	0.14	6.16	6.11	7.3%