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Self-assessment

COMP3670: Introduction to Machine Learning

Problem 1: Matrix addition and Multiplication
(1pt) We have three matrices: $\boldsymbol{A} \in R^{3\times 2}$, *i.e.*, real-valued 3 by 2 matrix; $\boldsymbol{B} \in R^{2\times 1}$; $\boldsymbol{C} \in R^{3\times 1}$.
$\boldsymbol{A} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & -1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} -3 \\ -1 \\ 0 \end{bmatrix}$. Calculate $\boldsymbol{AB}+\boldsymbol{C}$.

Solution.

$$\boldsymbol{AB}+\boldsymbol{C} = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}$$

Problem 2: Gaussian Elimination for System of Linear Equations
(2 pts) Solve the following system of linear equations. You can use any method you know of, such as intuitively solving it, or using the constructive Gaussian Elimination method.

$$\begin{cases} x_1 + x_2 + x_3 = 4 \\ 2x_2 + x_3 = 2 \end{cases}$$

Solution.
From the second equation

$$x_2 = \frac{2 - x_3}{2} = 1 - x_3/2$$

From the first equation, inserting the second:

$$x_1 + (1 - x_3/2) + x_3 = 4$$

$$\begin{aligned} x_1 &= 4 - (1 - x_3/2) - x_3 \\ &= 4 - 1 + x_3/2 - x_3) \\ &= 3 - x_3/2 \end{aligned}$$

Note that x_3 is a free variable. So the solution set is

$$\left\{ \begin{bmatrix} 3 - \lambda/2 \\ 1 - \lambda/2 \\ \lambda \end{bmatrix}, \lambda \in \mathbb{R} \right\}$$

Can also be solved via gaussian elimination.

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 1 & 2 \end{array}\right] \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 3 \\ 0 & 2 & 1 & 2 \end{array}\right] (R_1 := R_1 - 1/2R_2) \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & 1/2 & 3 \\ 0 & 1 & 1/2 & 1 \end{array}\right] (R_2 := 1/2R_2) \end{aligned}$$

At which point you can read off the solutions

$$\begin{aligned} x_1 + 1/2x_3 = 3 \\ x_2 + 1/2x_3 = 1 \end{aligned}$$

and hence

$$\begin{aligned} x_1 = 3 - 1/2x_3 \\ x_2 = 1 - 1/2x_3 \end{aligned}$$

Note x_3 is free, and we get the same solution obtained above.

Problem 3: Group
(1pt) Consider the set $\{1, -1\}$ together with the operation multiplication (*i.e.*, $\times$). Is this set a Group? Please explain.

Solution. Yes, it's a group.

Closure: Multiplying any of $\{-1, 1\}$ results in either -1 or 1.

Associativity: Trivial, as multiplication is an associative operator.

Identity: 1 is the identity, as anything multiplied by 1 is 1.

Inverse: Every element is it's own inverse.

Problem 4: properties of matrix transpose

(1pt) For $\boldsymbol{A} \in R^{m\times n}$, $\boldsymbol{B} \in R^{m\times n}$, prove that $(\boldsymbol{A}+\boldsymbol{B})^T = \boldsymbol{A}^T + \boldsymbol{B}^T$

Solution. We check the $i-j$th element, and verify they both match.

$$
\begin{aligned}
&(\mathbf{A}+\mathbf{B})^T_{i,j} \\
&= (\mathbf{A}+\mathbf{B})_{j,i} \\
&= \mathbf{A}_{j,i} + \mathbf{B}_{j,i} \\
&= \mathbf{A}^T_{i,j} + \mathbf{B}^T_{i,j} \\
&= (\mathbf{A}^T + \mathbf{B}^T)_{i,j}
\end{aligned}
$$

The above proof works as addition is performed elementwise.