MATH1005/MATH6005 Semester 1 2021

Assignment 6

Workshop Details:

Number	Day	Time	Demonstrator name
16B	Friday	2pm 30/04	Cai Yang

Student Details:

ID	Surname	Given name	Preferred name
u7235649	Zhang	Han	

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working.**

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature: Han Zhang .... Date: 02/05/2021

This document must be submitted by 11pm on the THURSDAY following your workshop.

Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question 2# We have seen in lectures that if 50 people are chosen at random then there is a 97% chance that at least two of them share the same birthday. Use similar calculations to answer the questions below. Assume that a random number generator generates all integers within a specified range with equal likelihood.

(a) A random number generator generates six numbers in the range 0 – 19 inclusive. Calculate the probability that at least two of them are equal.

$$M = \{0, \cdots, 19\}$$

$$|S| = |M^6| = |M|^6 = 20^6$$

$$P(E^c) = \frac{|E^c|}{|S|} = \frac{P(20,6)}{20^6} = \frac{20!}{20^6 \times 14!} = 0.43605$$

$$P(E) = 1 - P(E^c) = 0.56395$$

(b) Use WolframAlpha (`www.wolframalpha.com`) to test your answer to (a). Do this by making the request "six random numbers less than 20" ten times. Write out the ten lists and mark those that contain two or more equal values. Do you get the proportion you expect?

{16, 3, 15, 13, 2, 1} {15, 9, 19, 5, 8, 18}
{14, 10, 7, 5, 8, 0} {19, 12, 6, 0, 6, 18} ✓
{4, 17, 16, 6, 7, 6} ✓ {10, 2, 16, 16, 14, 1} ✓
{6, 8, 4, 4, 9, 16} ✓ {5, 19, 3, 3, 19, 4} ✓
{2, 17, 2, 14, 5, 11}
{18, 0, 1, 16, 1, 4} ✓ $P = \frac{6}{10} = 0.6$

The proportion is similar with P(E).

(c) By trial calculations using WolframAlpha, or otherwise, find the minimum number N for which there is a 95% chance that from N randomly chosen numbers in the range 0 – 19 inclusive at least two are equal. As a start, try $N = 15$.

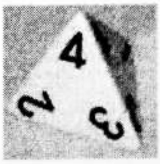
$N = 11$:

$$P(E^c) = \frac{P(20,11)}{20^{11}} = \frac{20!}{20^{11} \times 9!} \sim 0.033$$

$$P(E) = 1 - P(E^c) \sim 0.967$$

Question 4⁺ A *Binomial experiment* comprises a fixed number n of 'trials' where each trial has the same probability p of 'success'. The probability that a binomial experiment results in k successes is given by

$$\mathbb{P}(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}.$$



The probability of scoring a 4 on one throw of a d4 die is 0.25.

(a) Use your calculator to find the probabilities of scoring, with a d4 die,

(i) four 4s from six throws,

$$P = \binom{6}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^2 = \frac{6!}{4!2!} \cdot \frac{1}{4^4} \cdot \frac{3^2}{4^2} = \frac{6480}{196608} \sim 0.033$$

(ii) five 4s from six throws and

$$P = \binom{6}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^1 = \frac{6!}{5!1!} \cdot \frac{1}{4^5} \cdot \frac{3}{4} = \frac{3\times 6}{4^6} = \frac{18}{4096} \sim 0.004$$

(iii) six 4s from six throws

$$P = \binom{6}{6}\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^0 = \frac{1}{4^6} = \frac{1}{4096} \sim 0.00024$$

(b) Calculate the probability of scoring *at most three 4s* from six throws of a d4.

$$P = 1 - \frac{6480}{196608} - \frac{18}{4096} - \frac{1}{4096} \sim 0.921$$

(c) In practice Binomial probabilities are usually found from a book of tables or from on-line tables or calculators. Both density and (cumulative) distribution values are available. With the help of an on-line statistical calculator such as *Stat Trek* (`http://stattrek.com/online-calculator/binomial.aspx`) find the probability of scoring more than four but no more than ten 4s from 25 throws of a d4. Give all the values you obtained, what they represented, and how you used them to obtain your answer.

$P(x=4) \sim 0.1175$ $P(x<4) \sim 0.0962$ $P(x \le 4) \sim 0.2137$ $P(x>4) \sim 0.7863$

$P(x \ge 4) \sim 0.9038$. $P(x=10) \sim 0.0417$ $P(x<10) \sim 0.9287$ $P(x \le 10) \sim 0.9703$

$P(x>10) \sim 0.0297$ $P(x \ge 10) \sim 0.0713$

$$P(4<x<10) = 1 - P(x \le 4) - P(x \ge 10) \sim 1 - 0.2137 - 0.0713 = 0.7150$$

Question 5† Write the correct values in the boxes.

For this question, working is not required and will not be marked.

According to ABS census data from 2011, the number of children ever born to women (females older than 14) living in NSW, Victoria or SA (our sample space S) was distributed as shown in the table at right.

A woman from one of these states is chosen at random. Let V be the event "the woman is from Victoria" and for $n \in \{0, 1, 2\}$ let C_n denote the event "the woman has had exactly n children.

	number of children 0	1	2	> 2	all
NSW:	16%	6%	14%	14%	49%
Vic:	13%	5%	11%	10%	39%
SA:	4%	1%	3%	3%	12%
All:	33%	12%	28%	27%	100%

(Percentages rounded to integer.)

For (a) - (d) below round all answers to a whole number percentage.
For example: $\mathbb{P}(V)\mathbb{P}(C_0) = 13\%$

(a) $\mathbb{P}(C_2^c)$: $\boxed{72\%}$

(b) $\mathbb{P}(C_2 \cap V)$: $\boxed{11\%}$

(c) $\mathbb{P}(C_2 \cup V)$: $\boxed{56\%}$

(d) $\mathbb{P}(C_2)\mathbb{P}(V)$: $\boxed{11\%}$

(e) To the level of accuracy used above, are the events C_2 and V independent?

YES / NO.

(f) For the women of NSW, Victoria and SA in 2011, and to the same level of accuracy as above, are home state and number of children independent random variables?

YES / NO .

Question 6* In the casino gambling game of European Roulette the wheel has 37 slots numbered $0, 1, \ldots, 36$. Half of the slots numbered 1 to 36 are painted black, while the others are painted red. The slot numbered 0 is painted green. A ball is equally likely to land in any slot. Listed below are several of the many possible bets on where the ball lands, together with their winning payouts based on a \$1 stake. In each case calculate the expected return on a \$1 stake. (If you win, your return is your payout plus your stake; if don't win, you lose your stake and so have no return.)

The expected returns are of course all less than the the \$1 stake — the difference is the 'house edge'.

Bet	Specify	Win when ball lands in	Payout
'split'	two numbers on the wheel	pocket with either specified number	\$17
'dozen'	$n = \underset{1}{34}, \underset{2}{35}$ or $\underset{0}{36}$	pocket with non-zero number $\equiv n \pmod 3$	\$2
'odd'	—	an odd-numbered pocket	\$1

split: $\frac{2}{37} \times (17+1) = \frac{36}{37}$

dozen: $\frac{12}{37} \times (2+1) = \frac{36}{37}$

odd: $\frac{18}{37} \times (1+1) = \frac{36}{37}$