Image Filtering

Announcement: Student Rep. & Lab Assign.& Lab Session

- Student Representatives
  - Duan Dao
  - Maitreyi Singh
- Lab Assignment One Released
  - Due date: due 11:59pm, 28th March 2021 (end of week 5)
- Lab Sessions for Lab Assignment One starts this week. Please attend the lab session for getting support.
  - Lab session: week 03 & week 04, no lab session at week 05
- About closing the discussion forum on wattle and move all discussions to Piazza.

Announcement: Practice questions, tutorial

- Introduce practice questions, type of questions for exam
  - Our tutor, Huiyu Gao will explain answers to these questions during the lab session.

- Tutorial: Mar. 10^{th}, about Image processing by our tutor: Jiayu Yang

Announcement

- Mathematics Reference Book: Linear algebra, probability
  - https://mml-book.github.io/book/mml-book.pdf
  - eg. CDF (cumulative distribution function, see chapter 6.2 discrete and continuous distribution function)

- Lecture Series by Prof. Shree Nayar, Professor of Computer Science in the School of Engineering at Columbia University, Senior researcher in computer vision with high profile.
  - https://fpcv.cs.columbia.edu/

Outline

- Continue: Linear filter
  - Gaussian filter
  - Edge detection
- non-linear filter
  - median filter
  - bilateral filter

Images as functions

- We can think of an **image** as a function, f, from R^2 to R:
  - $f(x, y)$ gives the **intensity** at position (x, y)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
    - $f: [a, b]\mathrm{x}[c, d] \rightarrow [0,1]$

Image Noise

Common types of noise

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian normal distribution

Original

Salt and pepper noise

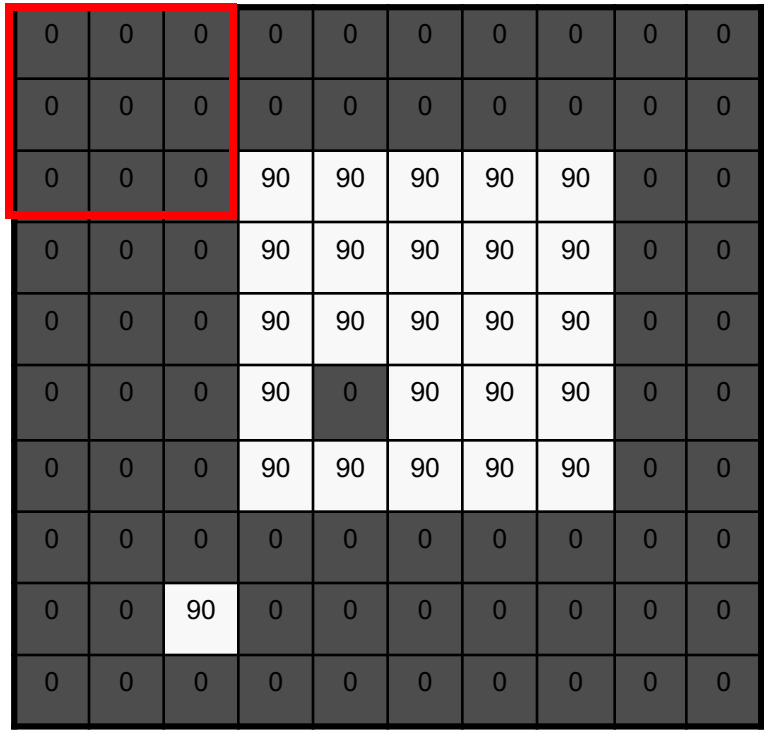
Impulse noise

Gaussian noise

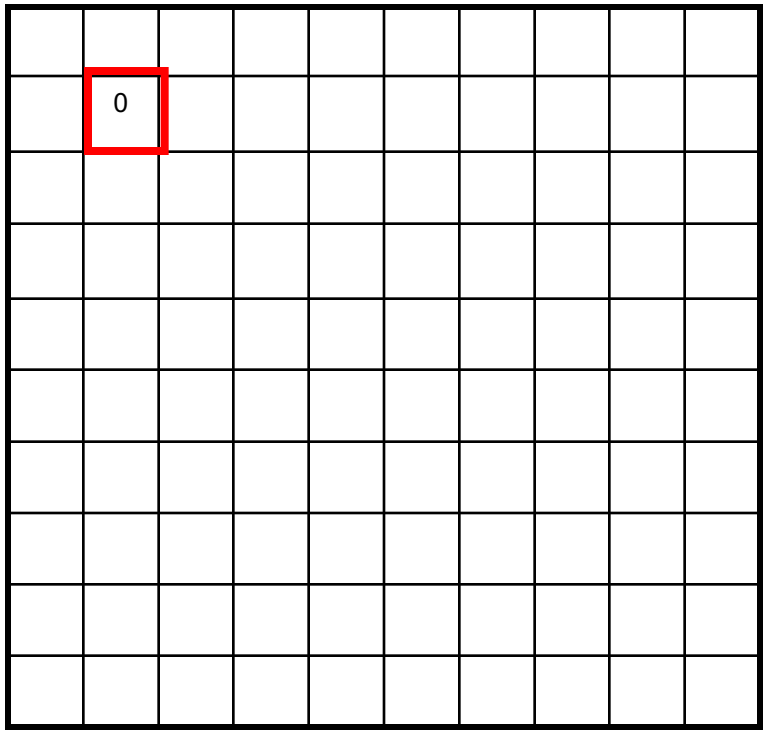
Source: S. Seitz

Moving Average In 2D

$F[x, y]$



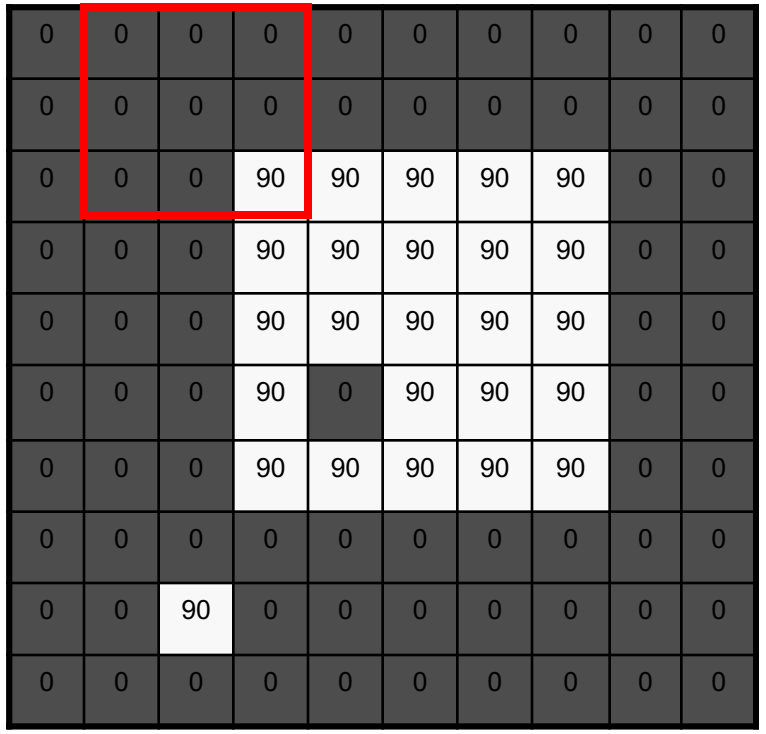
$G[x, y]$



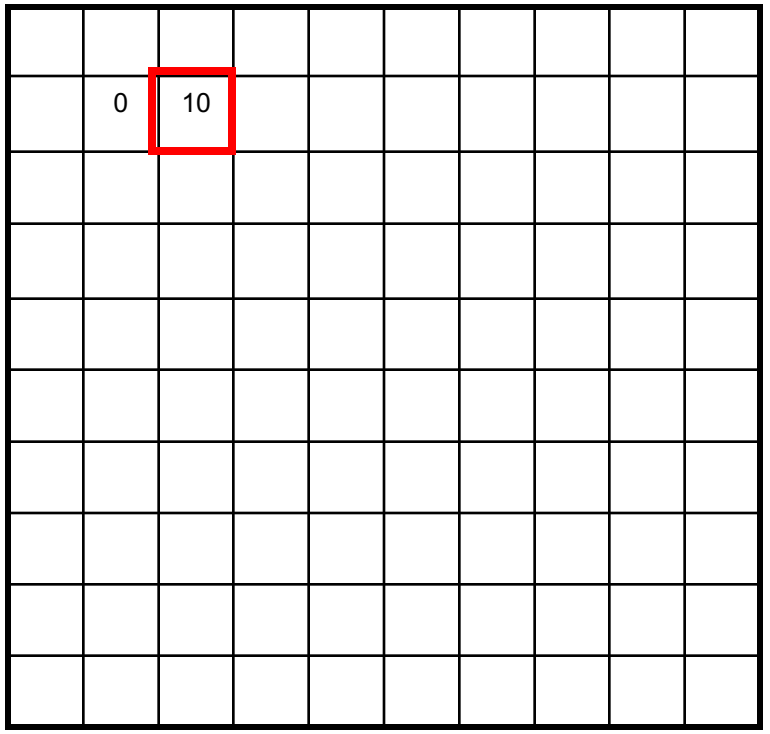
•Source: S. Seitz

Moving Average In 2D

$F[x, y]$



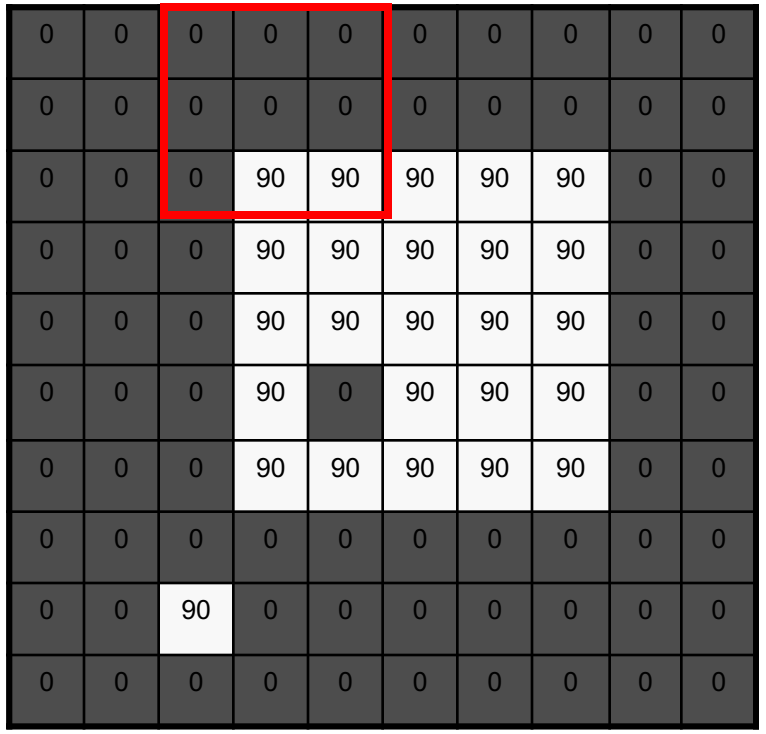
$G[x, y]$



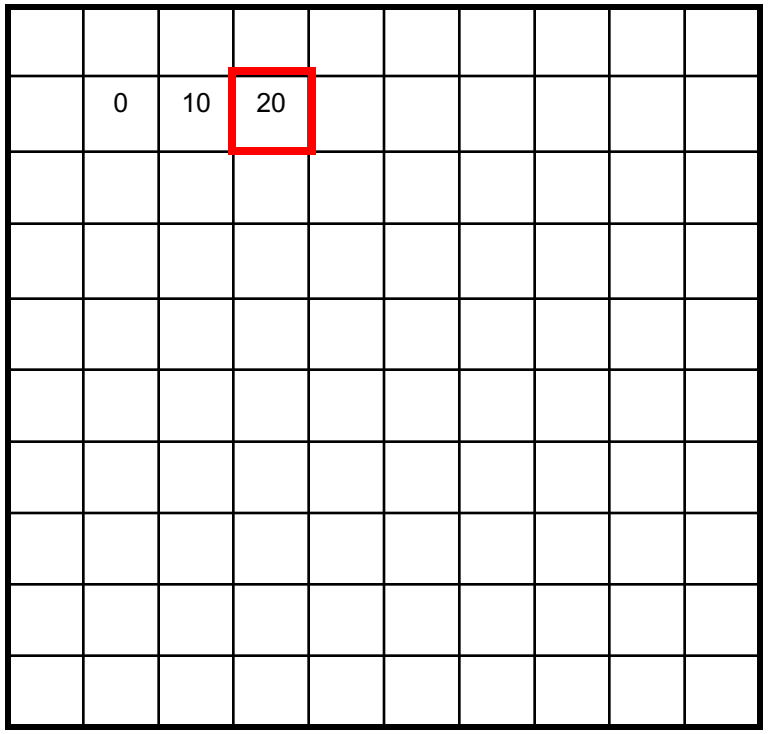
•Source: S. Seitz

Moving Average In 2D

$F[x, y]$



$G[x, y]$



•Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

•Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

•Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

•Source: S. Seitz

Correlation filtering

Say the averaging window size is (2k+1) x (2k+1):

$$G[x,y] = \underbrace{\frac{1}{(2k+1)^2}}_{\text{Attribute uniform weight to each pixel}} \underbrace{\sum_{u=-k}^{k}\sum_{v=-k}^{k} F[x+u, y+v]}_{\text{Loop over all pixels in neighborhood around image pixel F[x,y]}}$$

Now we can generalize to allow **different weights** depending on neighboring pixel's relative position:

$$G[x,y] = \sum_{u=-k}^{k}\sum_{v=-k}^{k} \underbrace{H[u,v]}_{\text{Non-uniform weights}} F[x+u, y+v]$$

Slide credit:
Kristen Grauman

Correlation filtering

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[x+u,y+v]$$

This is called cross-correlation, denoted $G = H \otimes F$

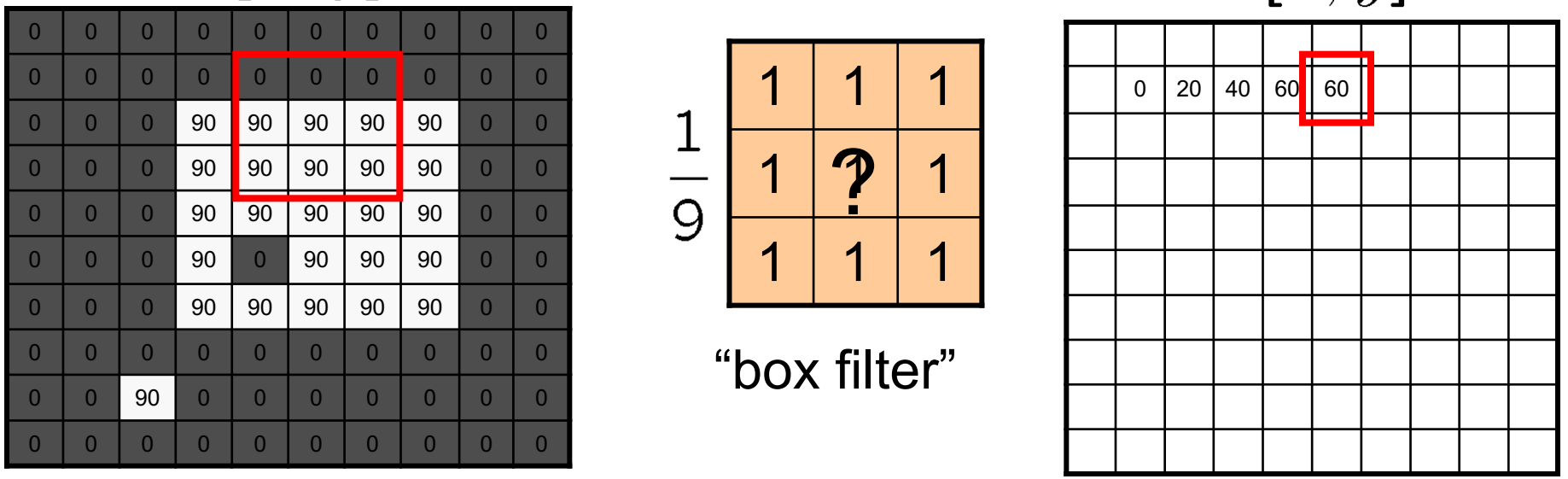
- Filtering an image: replace each pixel with a linear combination of its neighbors.
- The filter "kernel" or "mask" $H[u,v]$ is the prescription for the weights in the linear combination.

Slide credit:
Kristen Grauman

Averaging filter

- What values belong in the kernel *H* for the moving average example?

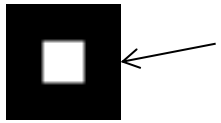
$F[x, y]$ $\otimes$ $H[u, v]$ $G[x, y]$



"box filter"

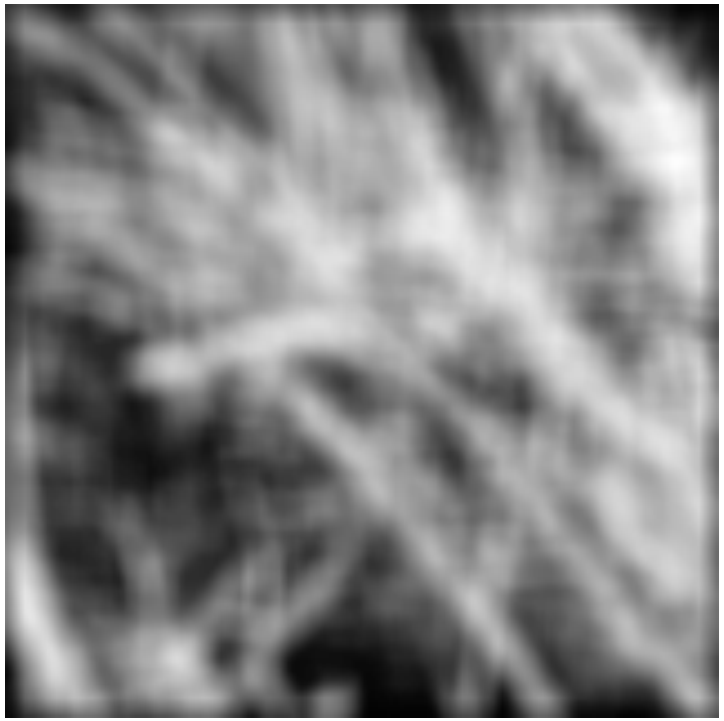
$$G = H \otimes F$$

Smoothing by averaging



depicts box filter:
white = high value, black = low value

original



filtered

We can see the ‘Block’ effect on the filtered image.

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

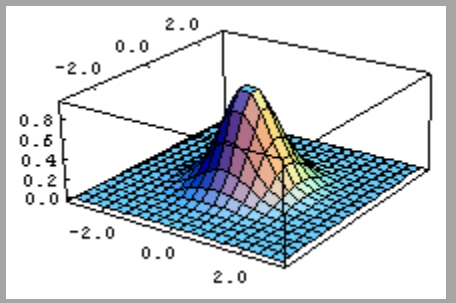
$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$H[u, v]$

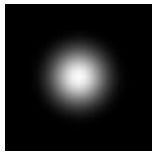
- This kernel is an approximation of a 2d Gaussian function:

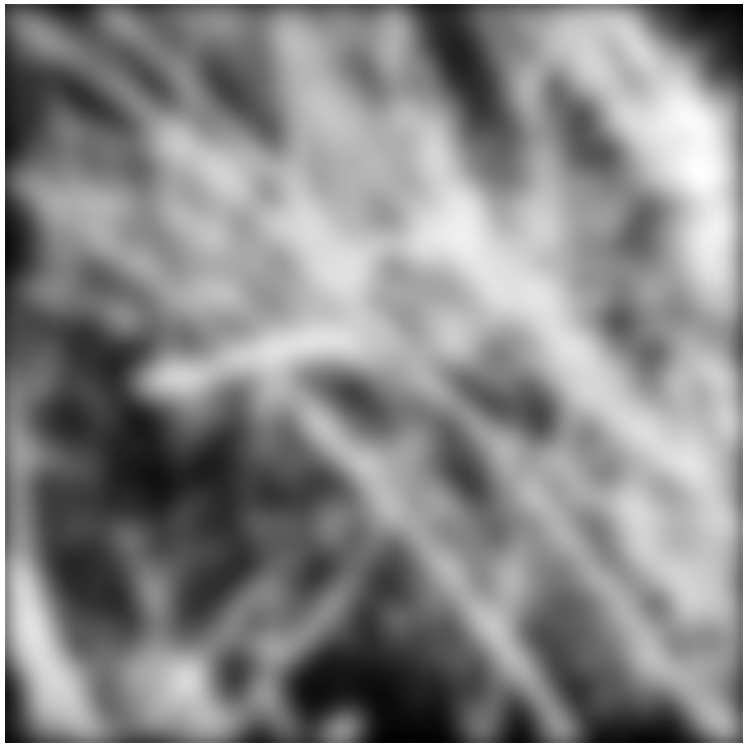
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



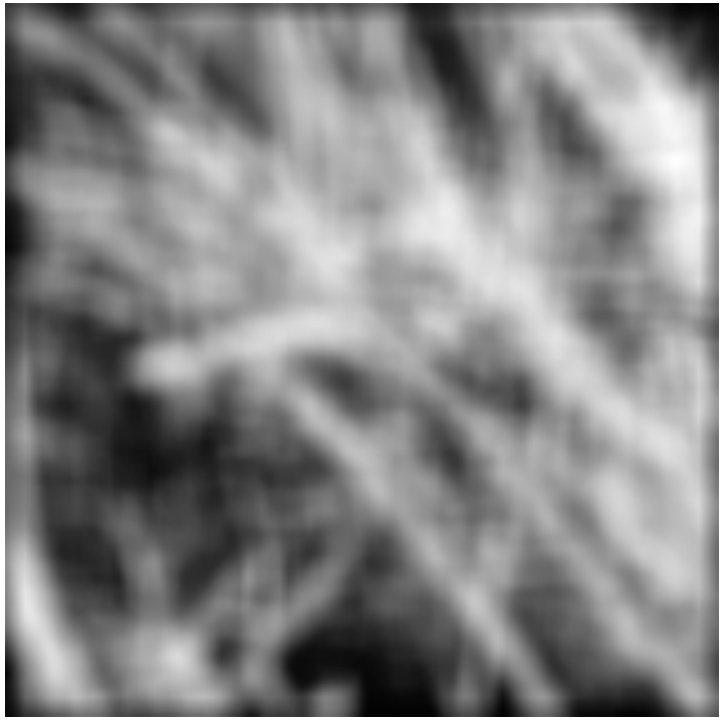
Source: S. Seitz

Smoothing with a Gaussian

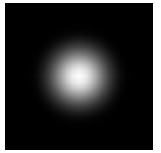


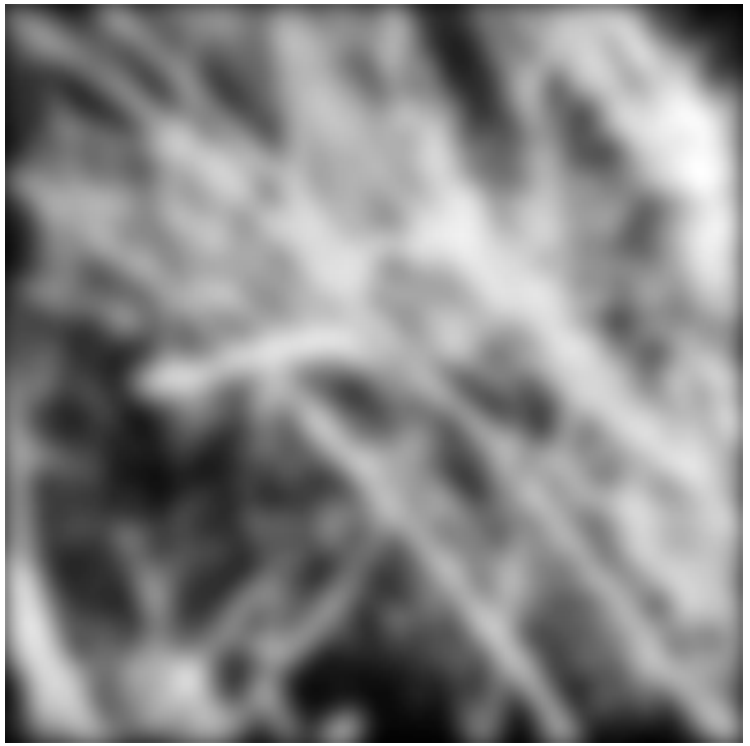


Compare box filter with Gaussian filter effects



Smoothed image by box filter



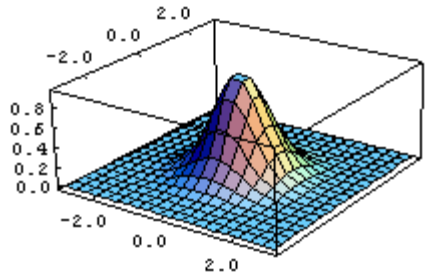


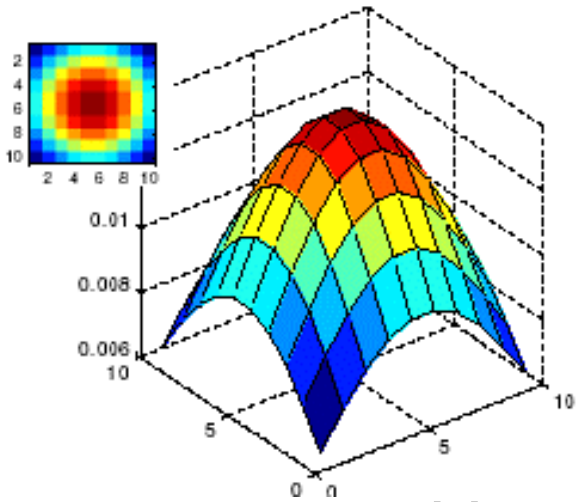
Smoothed image by Gaussian filter

Gaussian filters

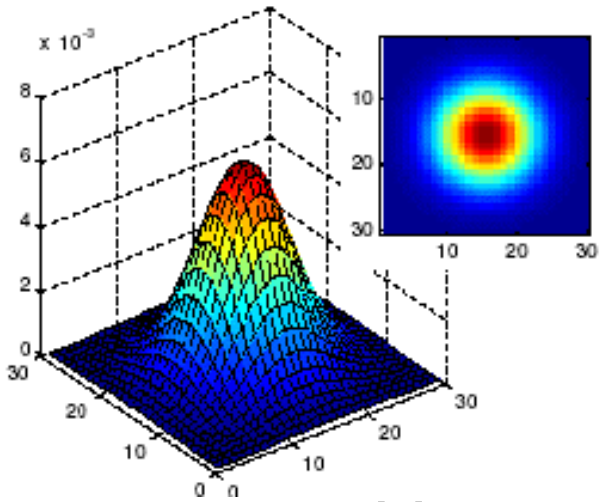
- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$





σ = 5 with 10 x 10 kernel

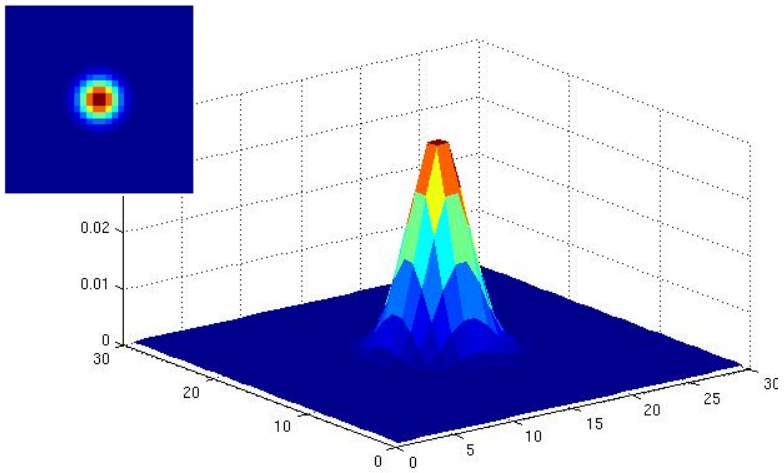


σ = 5 with 30 x 30 kernel

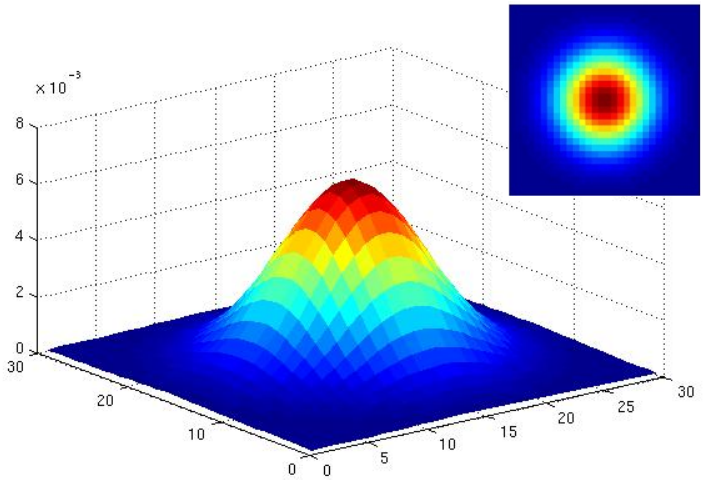
•Slide credit: Kristen Grauman

Gaussian filters

- What parameters matter here?
- **Variance** of the Gaussian function: determines extent of smoothing



σ = 2 with 30 x 30 kernel



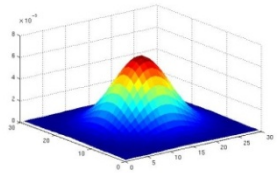
σ = 5 with 30 x 30 kernel

•Slide credit: Kristen Grauman

Matlab

```
>> hsize = 10;
>> sigma = 5;
>> h = fspecial('gaussian' hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```

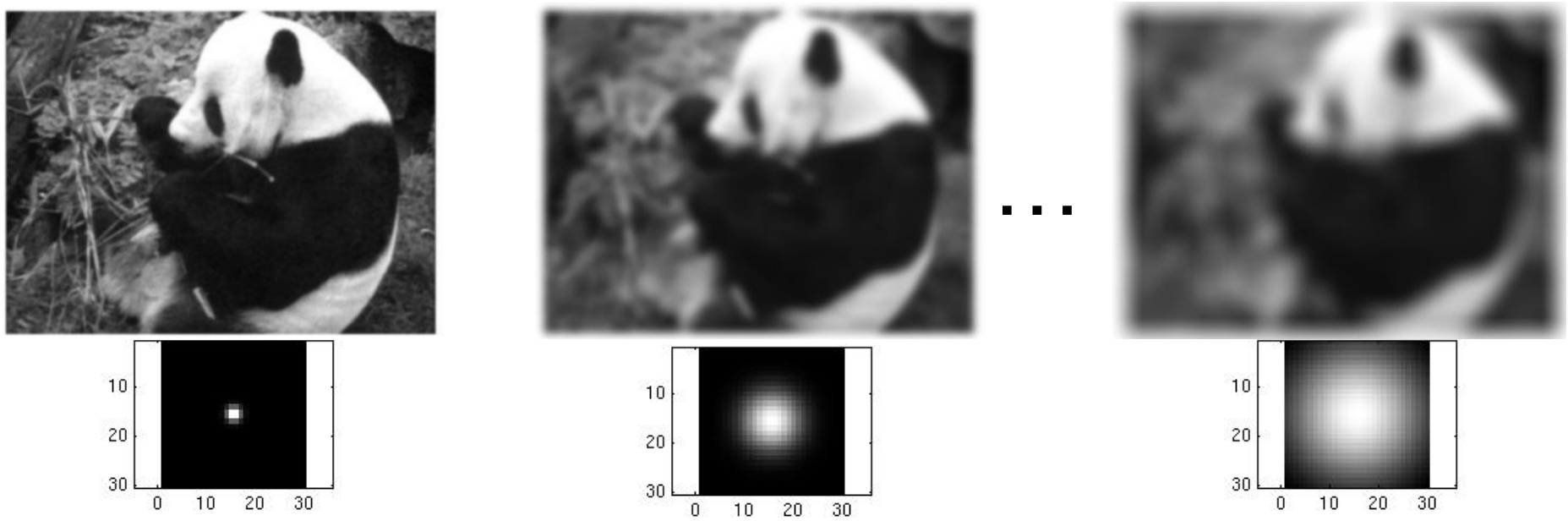
```
>> outim = imfilter(im, h); % correlation
>> imshow(outim);
```

outim

•Slide credit:
Kristen Grauman

Smoothing with a Gaussian

Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.




```
for sigma=1:3:10
  h = fspecial('gaussian', fsize, sigma);
  out = imfilter(im, h);
  imshow(out);
  pause;
end
```

•Slide credit: Kristen Grauman

Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[x-u,y-v]$$

$$G = H \star F$$

Notation for convolution operator

Convolution

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left)
  - Then apply cross-correlation

$$G(i,j) = \sum_{u=0}^{M} \sum_{v=0}^{N} F(u,v) H(i-u, j-v)$$

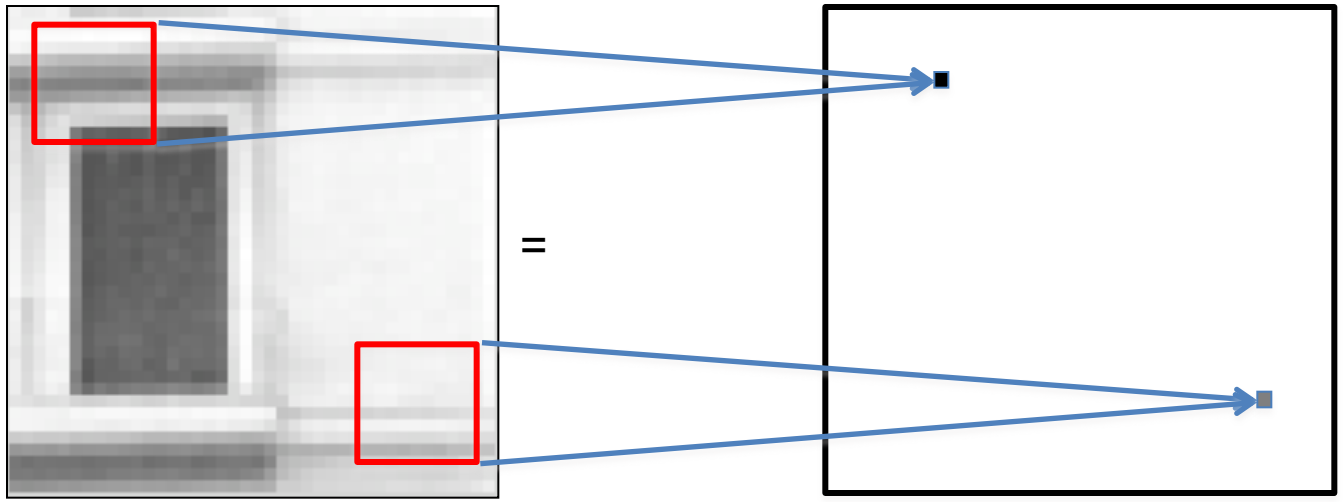
$$G = F \star H$$

Notation for convolution operator

Discrete linear system

In vision, many times, we are interested in operations that are spatially invariant.
For a linear spatially invariant system:

$$g[m,n] = h \star f = \sum_{k,l} h[m-k, n-l] f[k,l]$$



Properties of convolution

- **Shift invariant:**
  - Operator behaves the same everywhere, i.e. the value of the output depends on the pattern in the image neighborhood, not the position of the neighborhood.
- **Superposition:**
  - h * (f1 + f2) = (h * f1) + (h * f2)

Properties of convolution

- Commutative:

  $f * g = g * f$

- Associative

  $(f * g) * h = f * (g * h)$

- Distributes over addition

  $f * (g + h) = (f * g) + (f * h)$

- Scalars factor out

  $kf * g = f * kg = k(f * g)$

- Identity:

  unit impulse e = […, 0, 0, 1, 0, 0, …]. $f * e = f$

Matlab's Filter vs. Convolution

- 2D correlation filtering

h=filter g=image

  - `f=filter2(h,g);` or `f=imfilter(h,g);`

$$f[m,n] = \sum_{k,l} h[k,l]\, g[m+k, n+l]$$

- 2D convolution
  - `f=conv2(h,g);`

$$f[m,n] = \sum_{k,l} h[k,l]\, g[m-k, n-l]$$

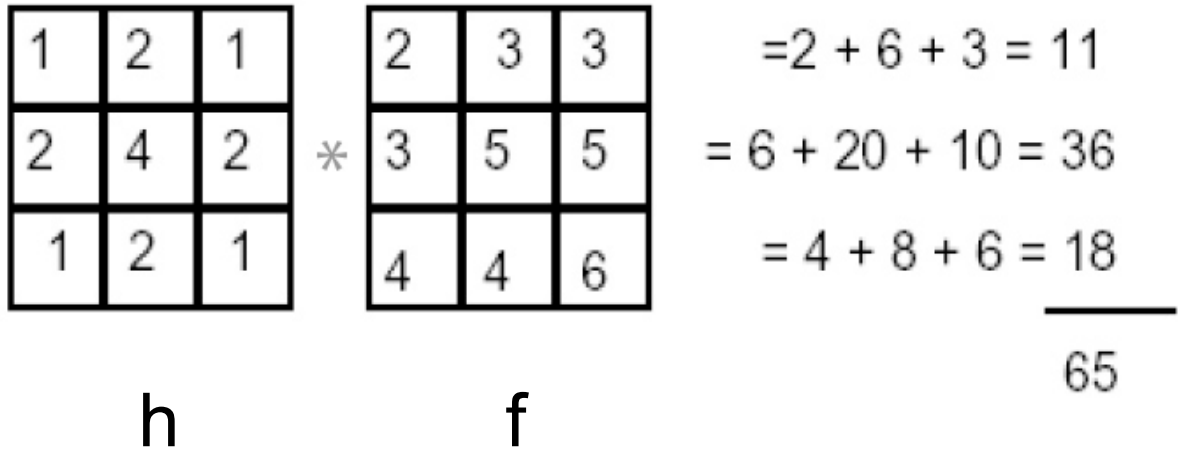
Separability

- In some cases, the filter is separable, and we can factor into two steps:
  - Convolve all rows with a 1D filter
  - Convolve all columns with a 1D filter

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

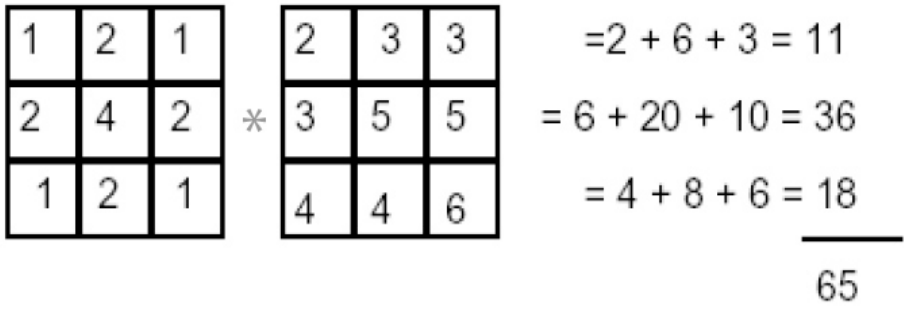
Separability example

- 2D convolution (center location only)(65)

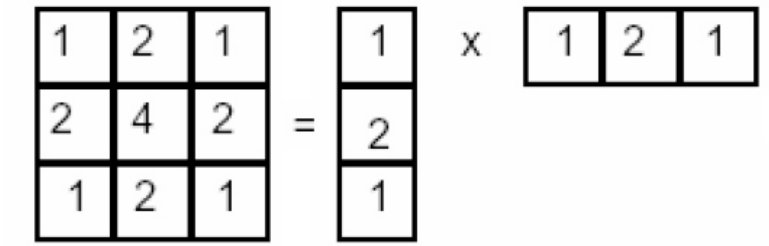


Source: K. Grauman

Separability example

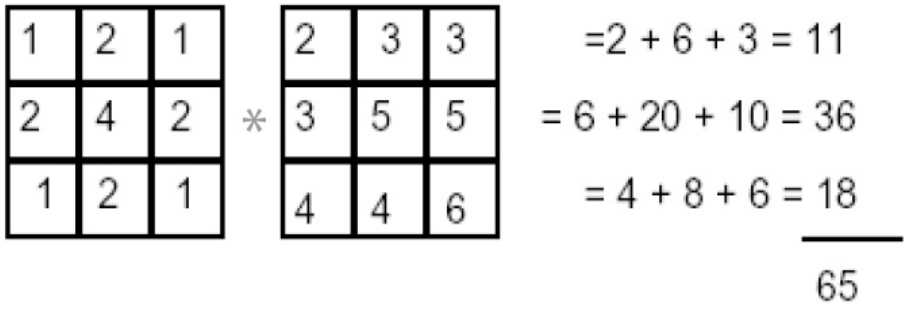


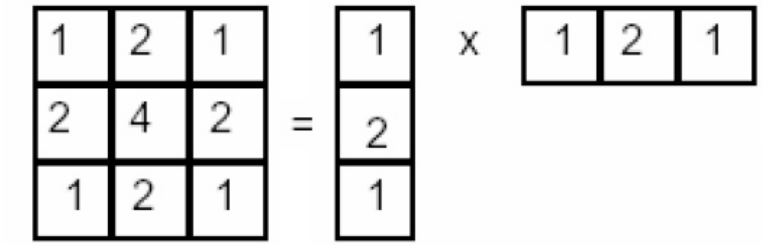
- The filter is factored into a product of 1D filters.



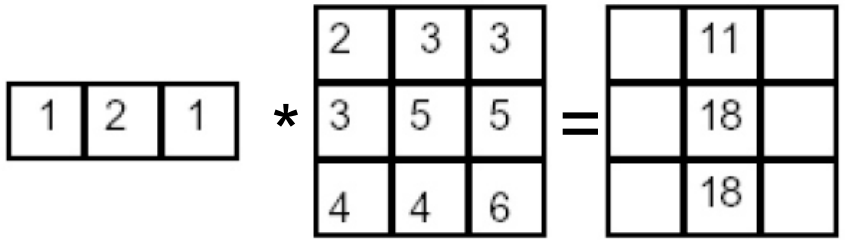
Source: K. Grauman

Separability example



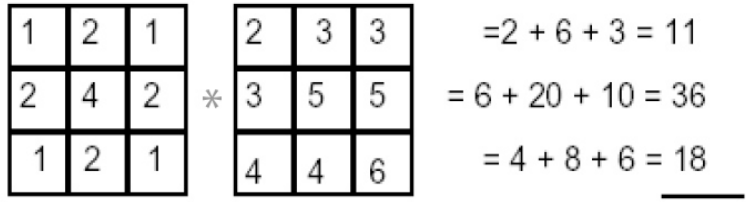


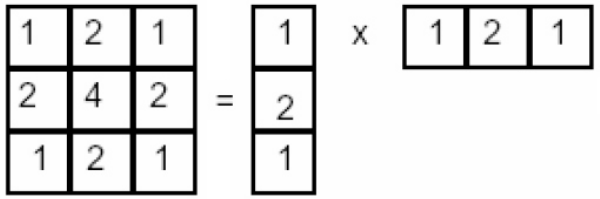
- Perform convolutions along rows.

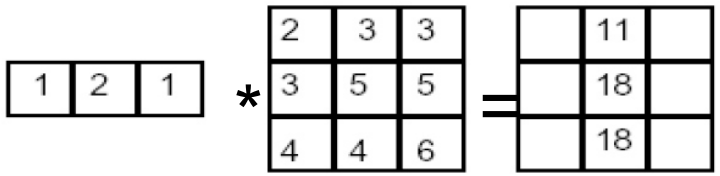


Source: K. Grauman

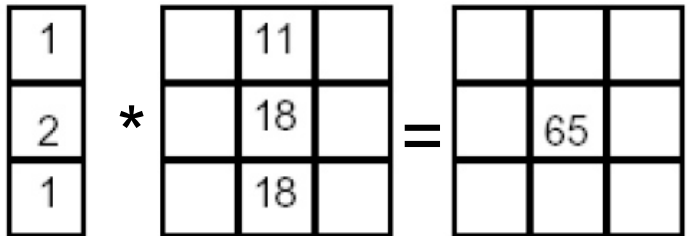
Separability example







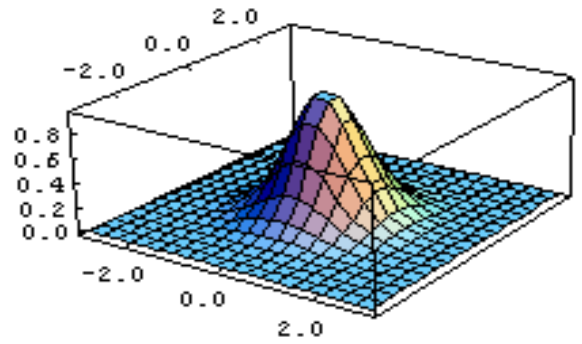
- Perform convolutions along columns.



Source: K. Grauman

This kernel is an approximation of a 2d Gaussian function:

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



Properties of Gaussian filter

- Rotational symmetry treats features of all orientations equally (isotropy).
- Convolution with self gives another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into the product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

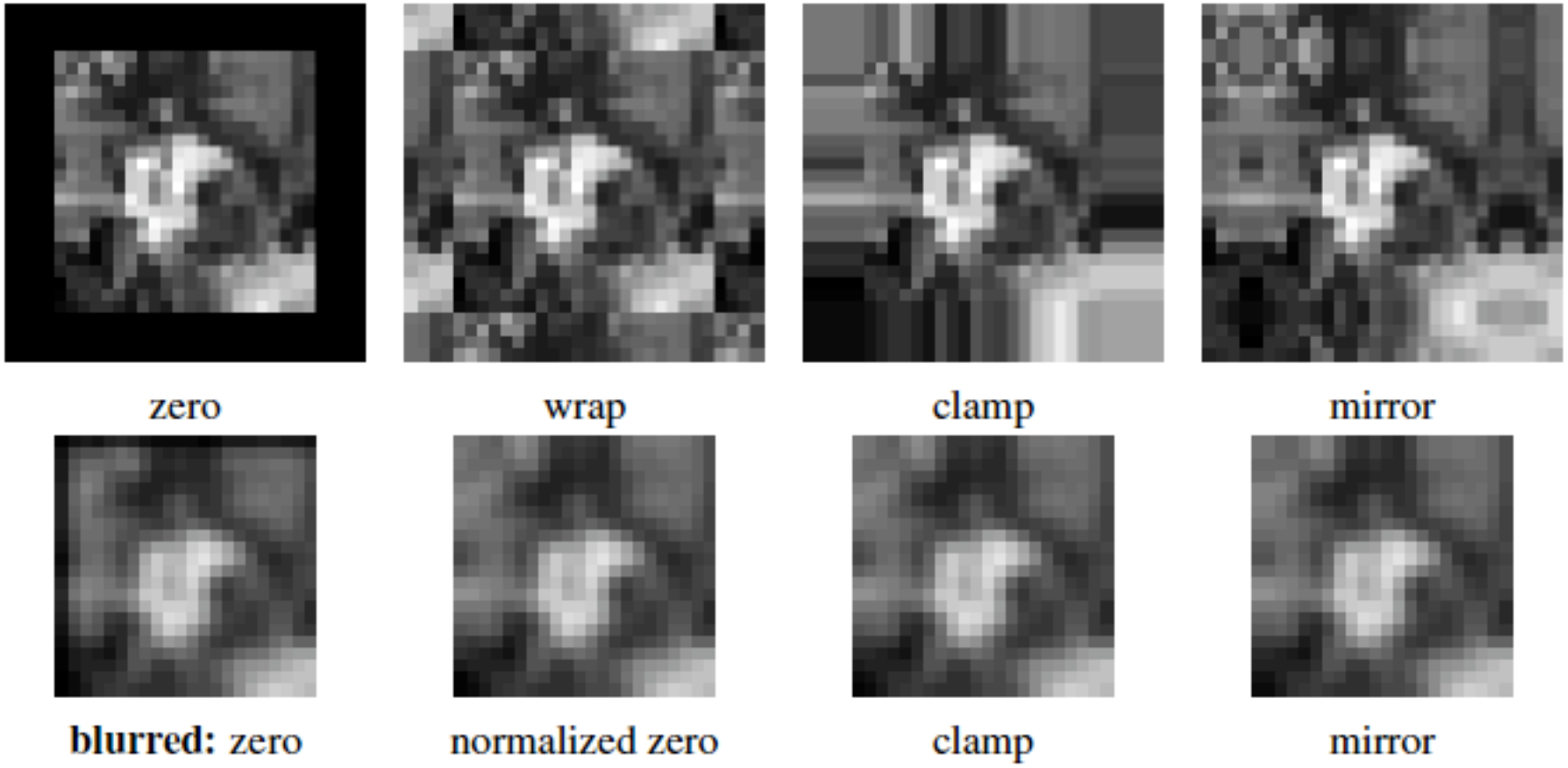
The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

- What is the complexity of filtering an n×n image with an m×m kernel?
    - $O(n^2 m^2)$
- What if the kernel is separable?
    - $O(n^2 m)$

How to handle borders



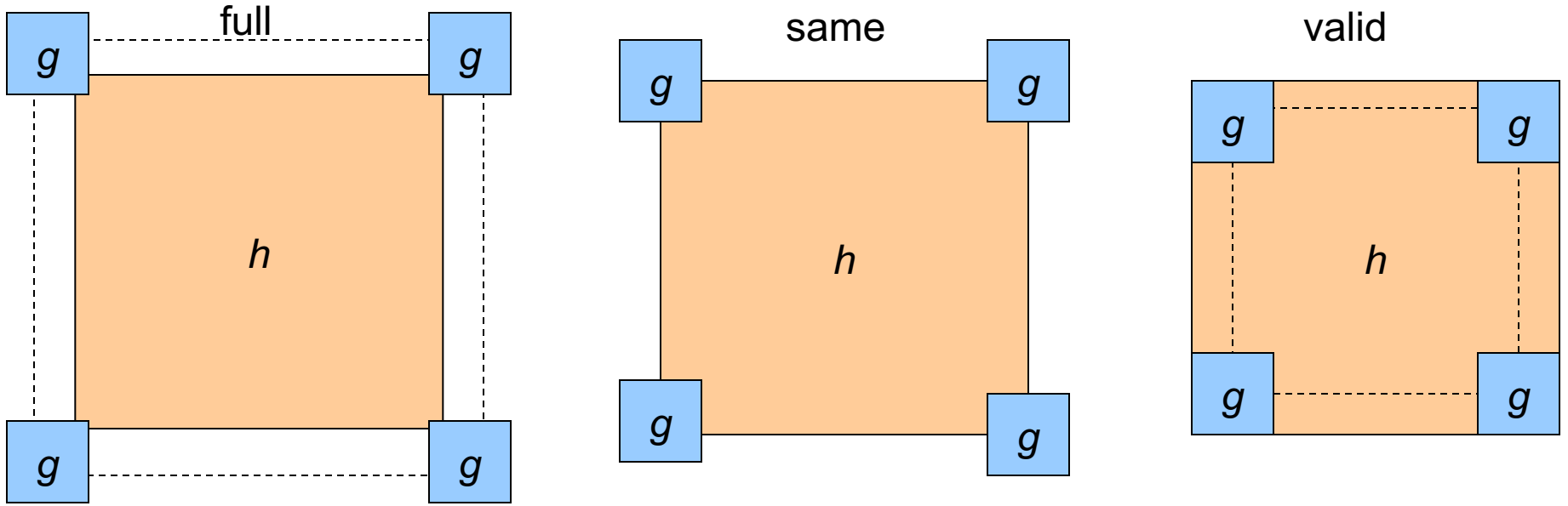
From Szeliski, Computer Vision, 2010

Example: Border handling

Source: S. Marschner

Matlab's border handling

- What is the size of the output?
- MATLAB: filter2(g, h, *shape*)
  - *shape* = 'full': output size is sum of sizes of h and g
  - *shape* = 'same': output size is same as h
  - *shape* = 'valid': output size is difference of sizes of h and g



Source: S. Lazebnik

Discrete Filtering

- Linear filter: Weighted sum of pixels over rectangular neighborhood—*kernel* defines weights

- Think of kernel as template being matched by correlation (Matlab: `imfilter, filter2`)

- Convolution: Correlation with kernel rotated 180° (Matlab: `conv2`)

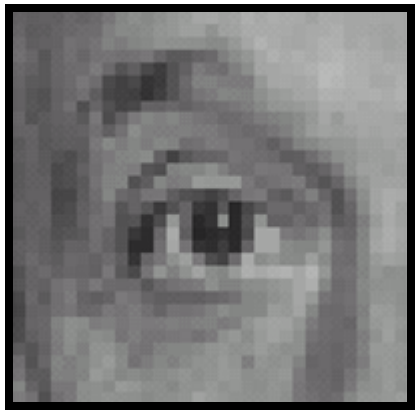
1	-1	-1
1	2	-1
1	1	1

Practice with linear filters

A taxonomy of useful filters

- Impulse, Shifts,
- Blur
  - Box filter
  - Gaussian filter
  - Bilateral filter
  - Asymmetrical filter: motion blur
- Edges
  - [-1 1]
  - Derivative filter
  - Derivative of a gaussian
  - Oriented filters
  - Gabor filter
  - Quadrature filters: phase and magnitude.
  - Elongated edges: filling gaps…

Practice with linear filters



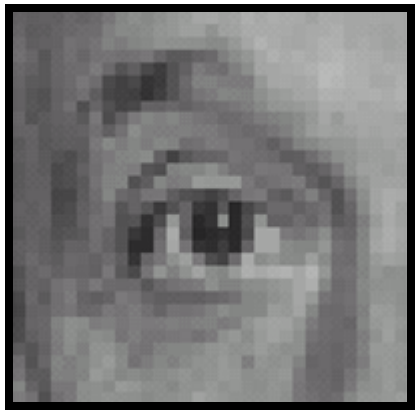
Original

0	0	0
0	1	0
0	0	0

?

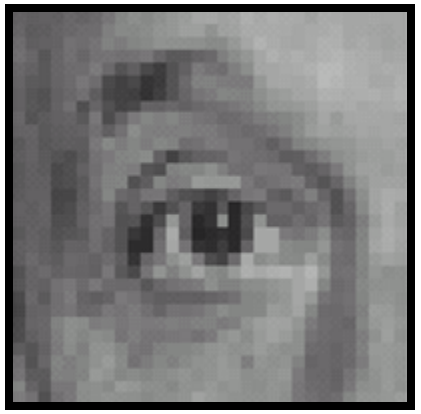
Source: D. Lowe

Practice with linear filters



Original

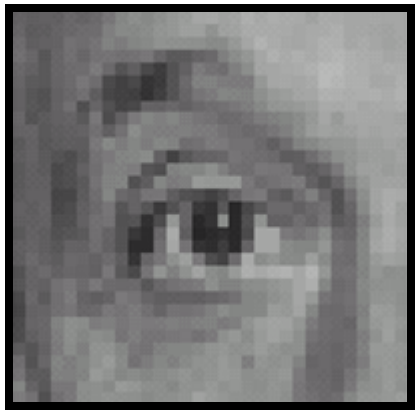
0	0	0
0	1	0
0	0	0



Filtered
(no change)

Source: D. Lowe

Practice with linear filters



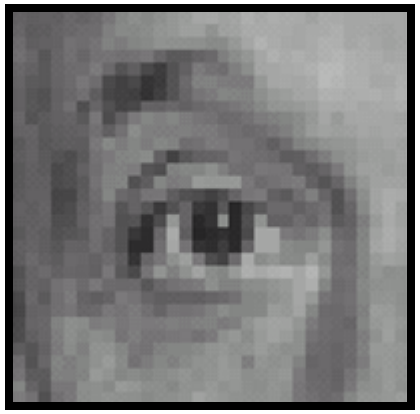
Original

0	0	0
0	0	1
0	0	0

?

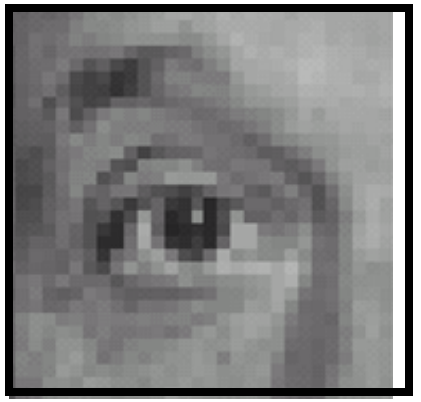
Source: D. Lowe

Practice with linear filters



Original

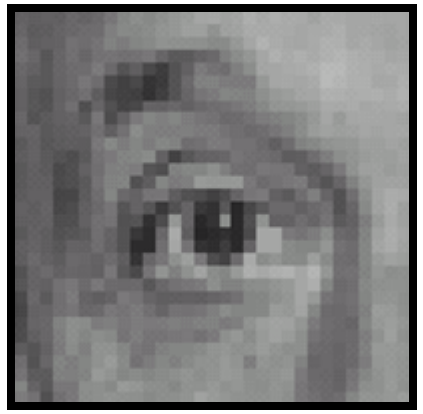
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Source: D. Lowe

Practice with linear filters



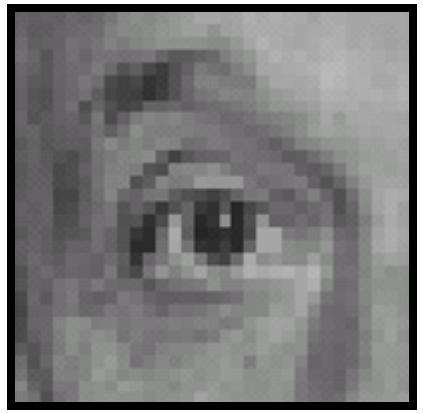
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters



Original

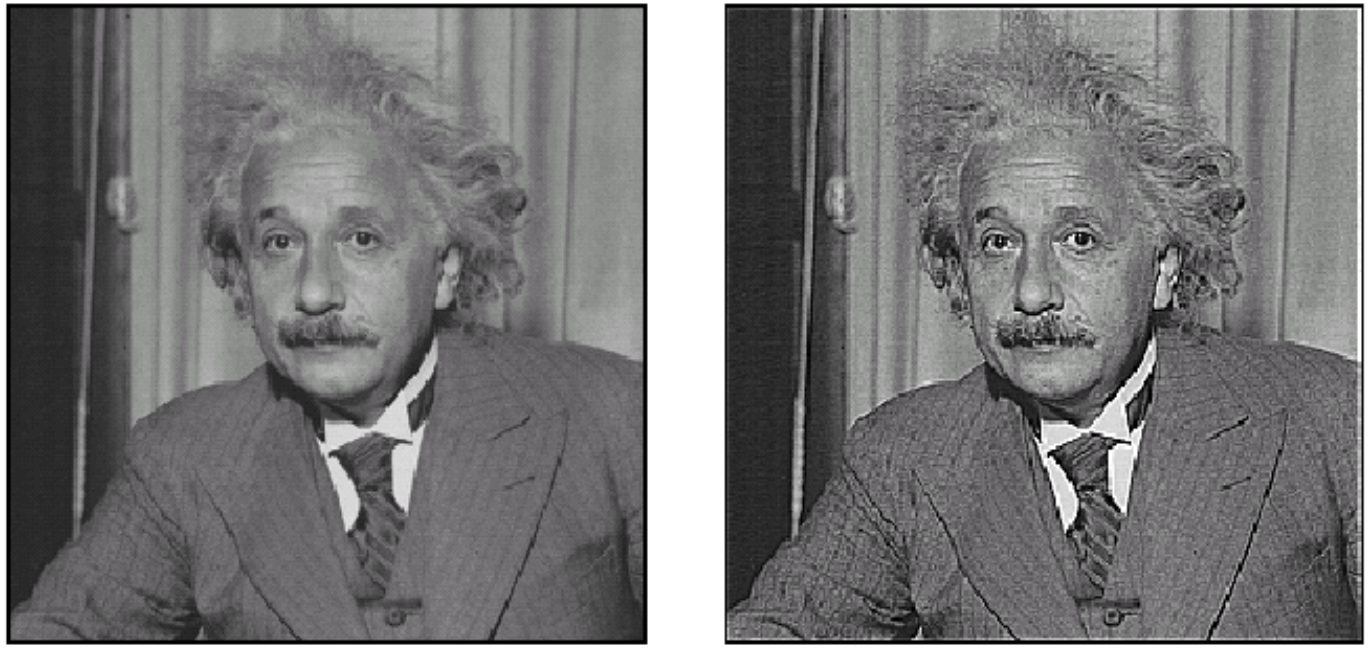
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

- Accentuates differences with local average

Source: D. Lowe

Sharpening

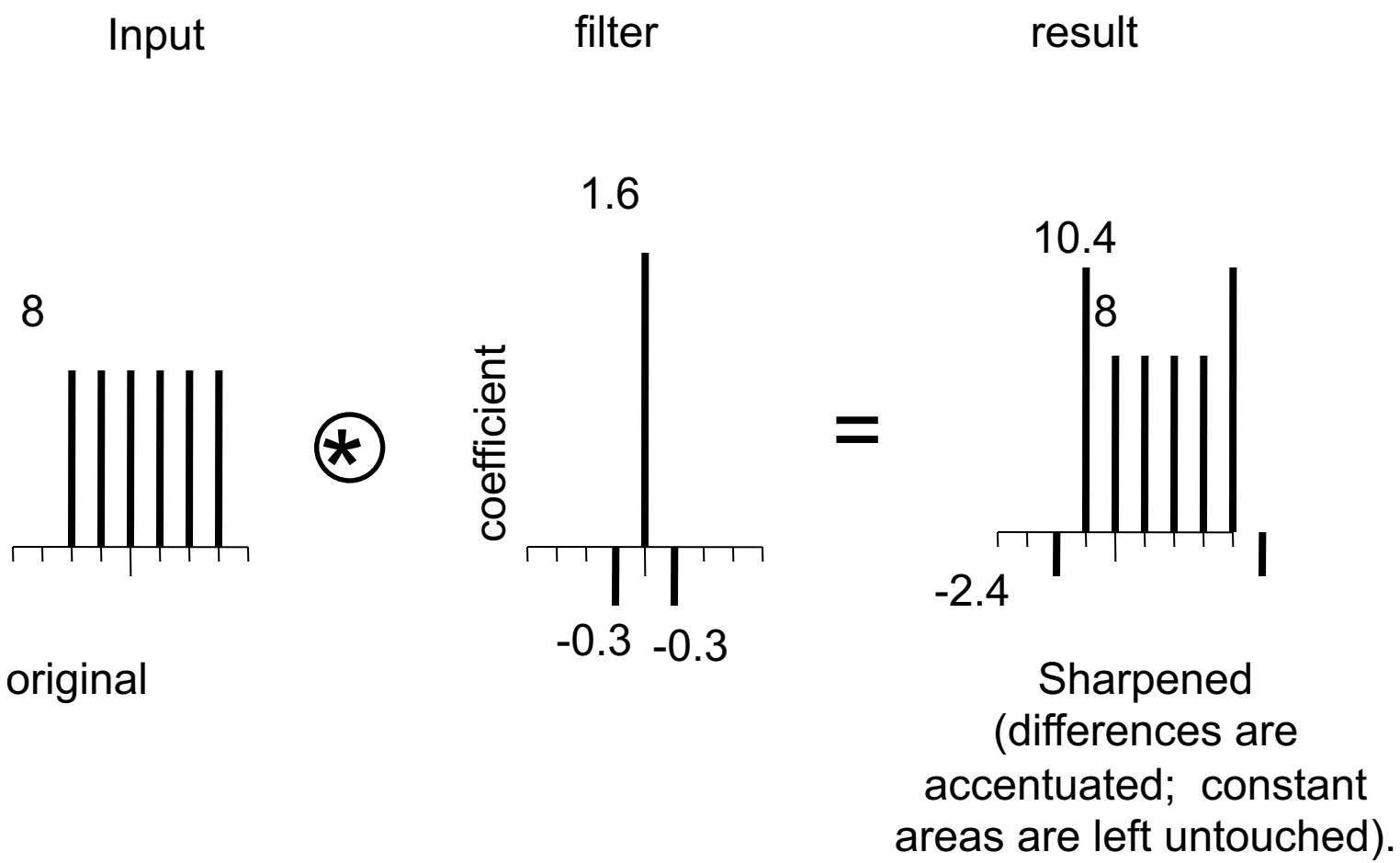


before

after

Source: D. Lowe

1-D sharpening example



[-1 1]

$$\frac{\partial \mathbf{I}}{\partial x} \simeq \mathbf{I}(x,y) - \mathbf{I}(x-1,y)$$

g[m,n]

$\otimes$

[-1, 1]

h[m,n]

=

f[m,n]

$[-1\ 1]^T$

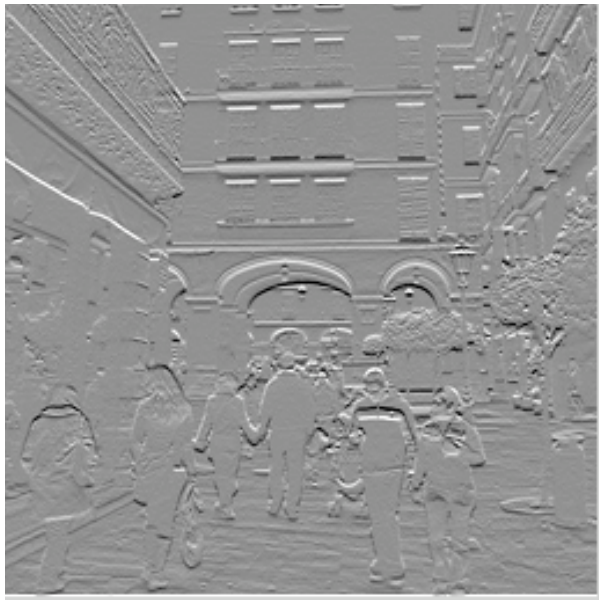
g[m,n]

$\otimes$

$[-1, 1]^T$

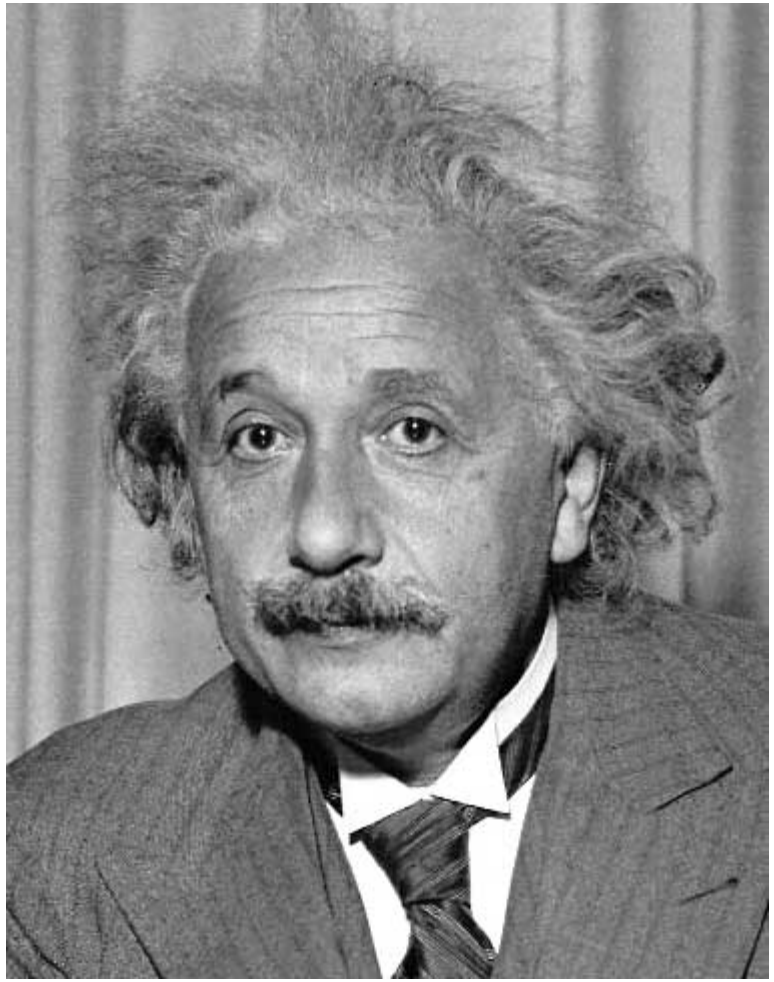
h[m,n]

=



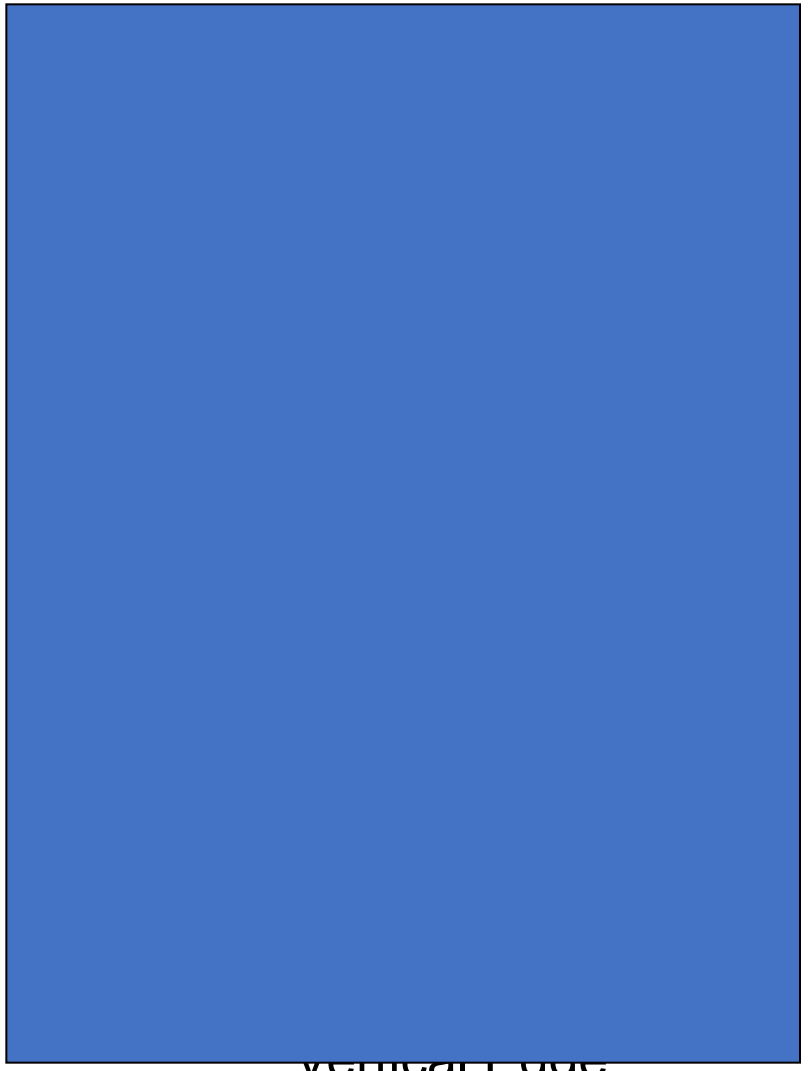
f[m,n]

Derivative filters



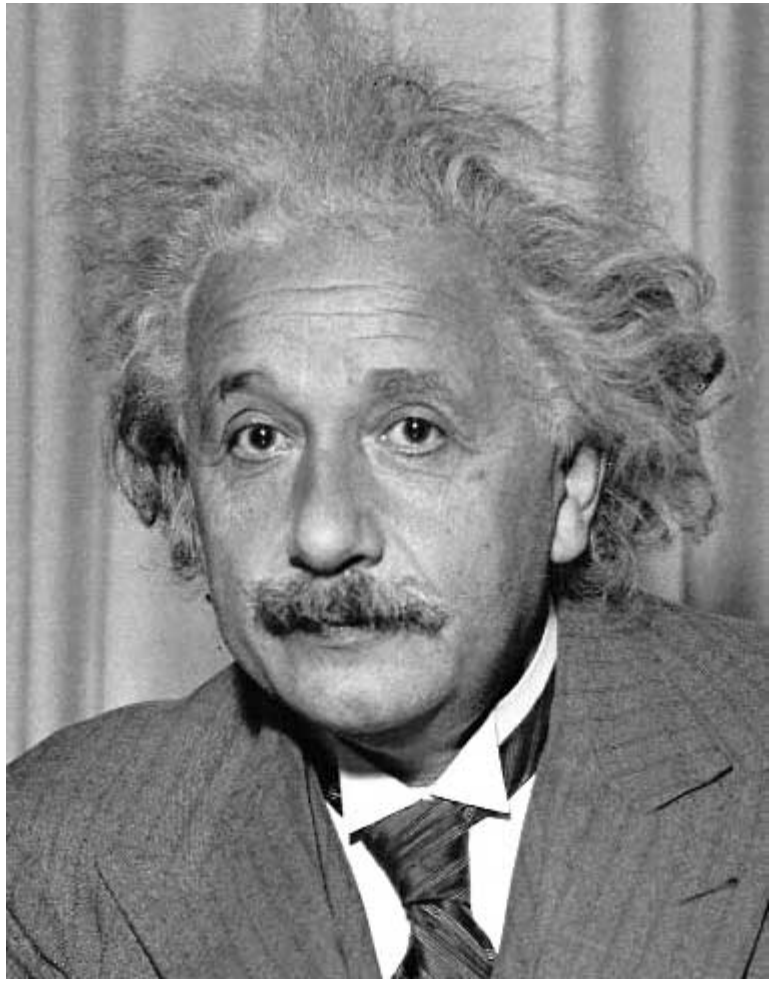
1	0	-1
2	0	-2
1	0	-1

Sobel



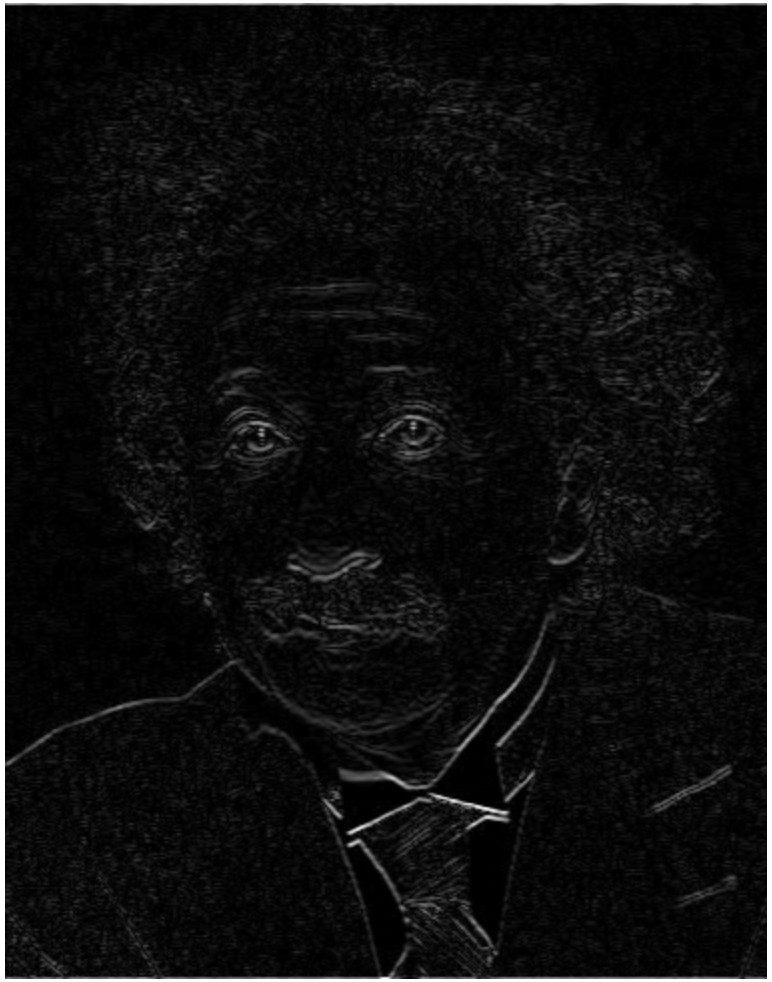
Vertical Edge
(absolute value)

Derivative filters



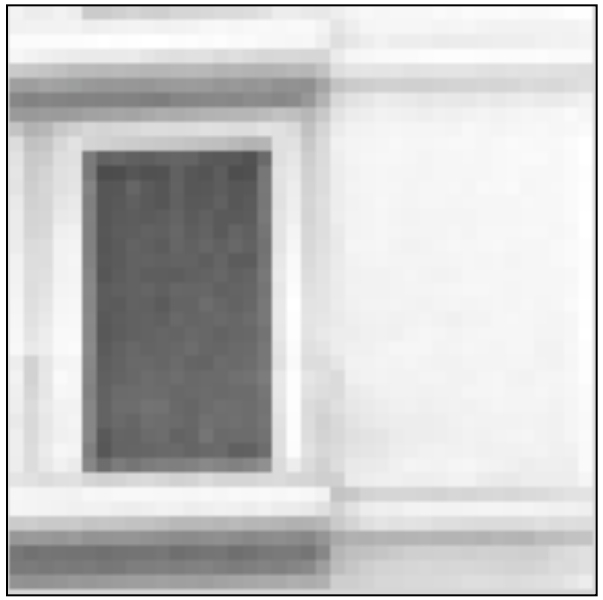
1	2	1
0	0	0
-1	-2	-1

Sobel



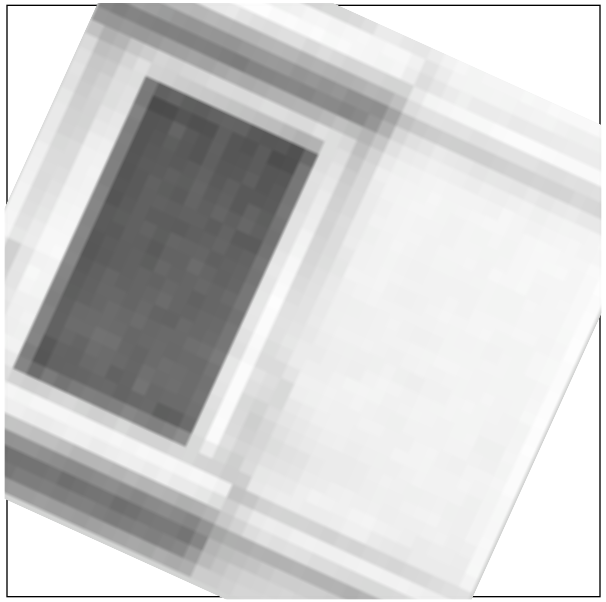
Horizontal Edge
(absolute value)

Image rotation



g[m,n]

$\otimes$? = h[m,n]

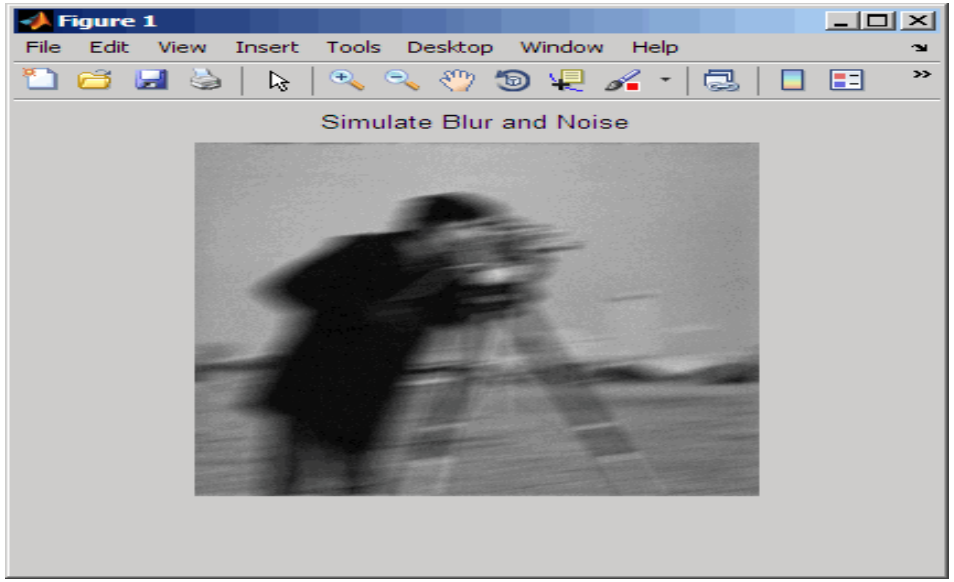


f[m,n]

It is linear, but not a spatially invariant operation. There is not convolution.

How could we synthesize motion blur?

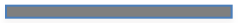
```
theta = 30; len = 20;
fil = imrotate(ones(1, len), theta, 'bilinear');
fil = fil / sum(fil(:));
figure(2), imshow(imfilter(im, fil));
```



Motion blur filter

g[m,n]

$\otimes$



h[m,n]

=

f[m,n]

Motion blur filter

g[m,n]

$\otimes$

h[m,n]

=

f[m,n]

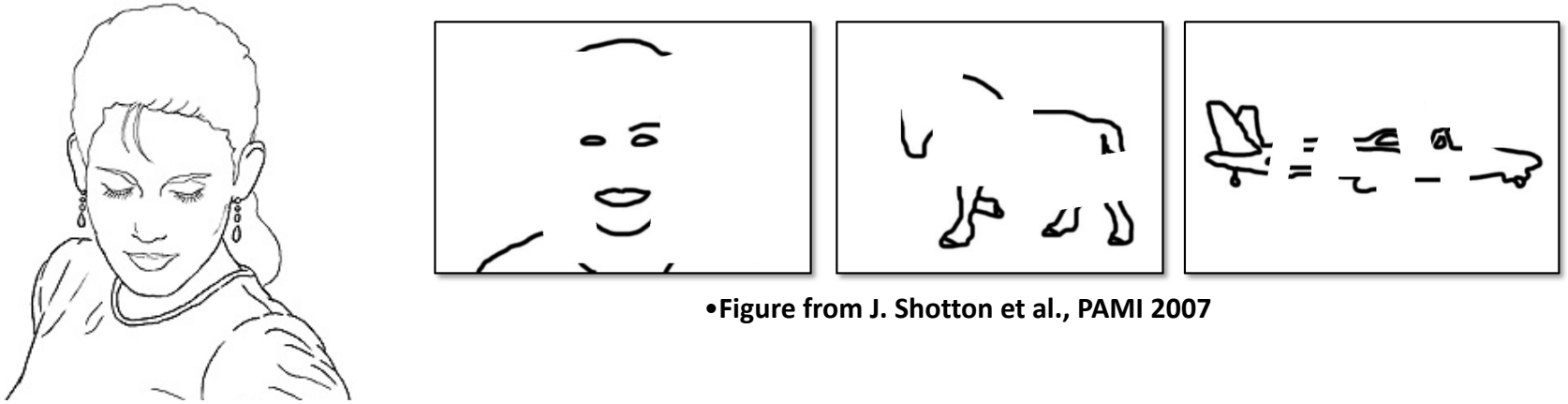
Summary

- Linear filters and convolution useful for
  - Enhancing images (smoothing, removing noise)
    - Box filter
    - Gaussian filter
- Separable filters more efficient

Edge Detection

Edge detection

- **Goal**: map image from 2d array of pixels to a set of curves or line segments or contours.
- **Why?**



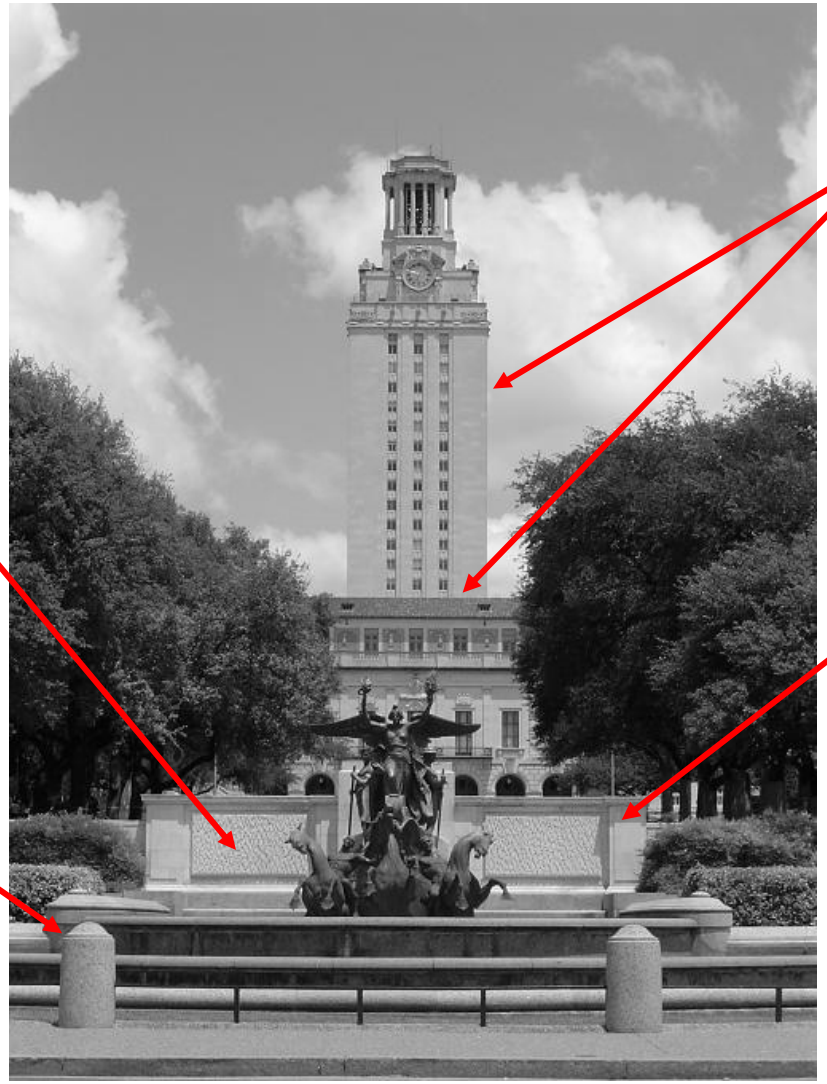
•Figure from J. Shotton et al., PAMI 2007

- **Main idea**: look for strong gradients, post-process

What causes an edge?

•Reflectance change: appearance information, texture

•Change in surface orientation: shape

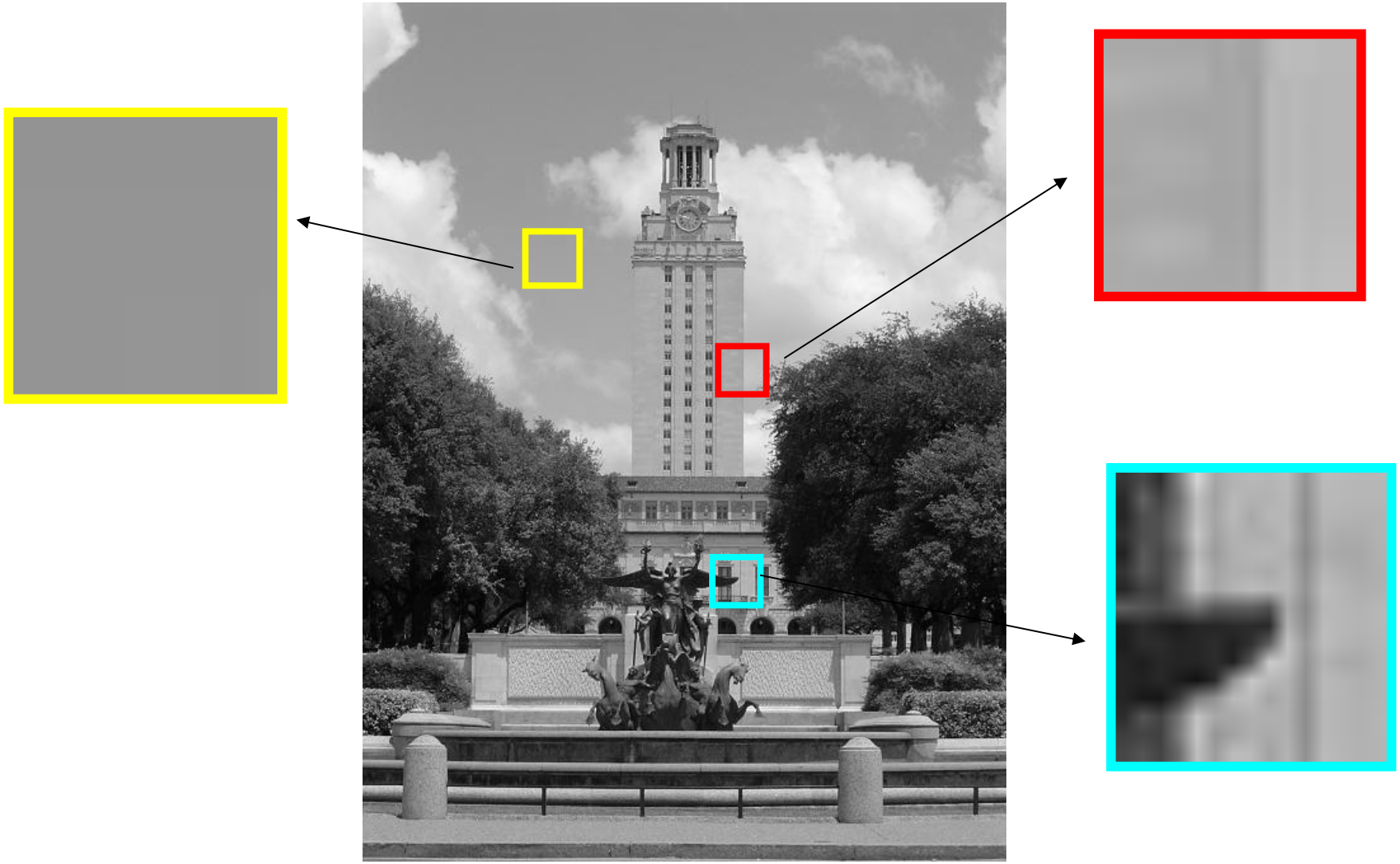


•Depth discontinuity: object boundary

•Cast shadows

•Slide credit: Kristen Grauman

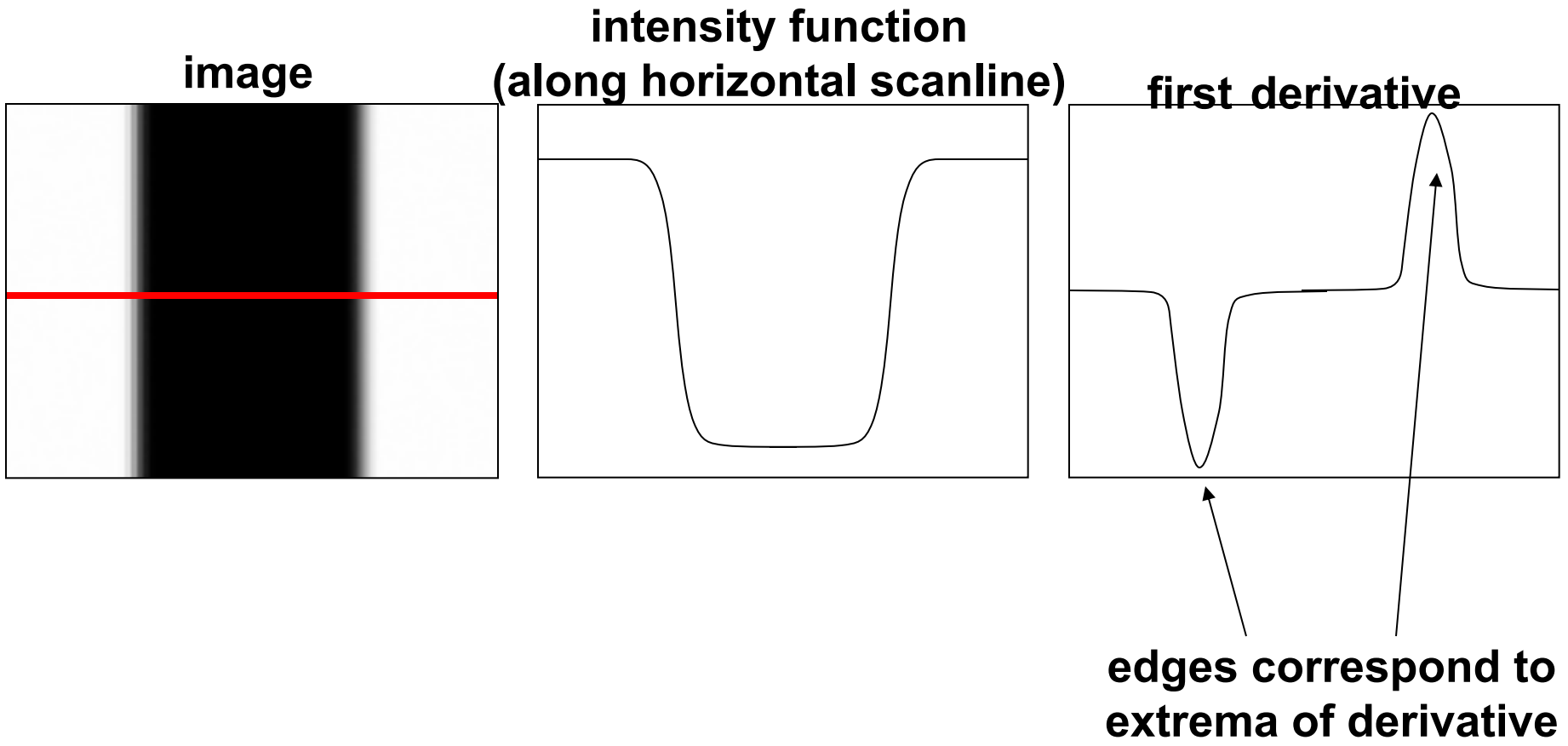
Edges/gradients and invariance



•Slide credit:
Kristen Grauman

Derivatives and edges

•An edge is a place of rapid change in the image intensity function.



•Source: L. Lazebnik

Methods

- Using first order derivative: *Look for extrema*
  - Sobel operator : `edge(I, 'sobel');`
  - Prewitt, Roberts,…
  - Derivative of Gaussian
- Using second order derivative: *Look for zero-crossings*
  - Laplacian : isotropic
  - Second directional derivative
  - LOG/DOG (Laplace of Gaussian/Difference of Gaussians)

$$\nabla^2 \mathbf{I}$$

Common edge detectors

- Basic gradient edge detectors
  - **Sobel**, Prewitt, Gaussian derivative …
- **LoG/Marr** edge detector
- **Canny** edge detector

Gradient Edge Detectors

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

; $M_y =$

1	2	1
0	0	0
-1	-2	-1

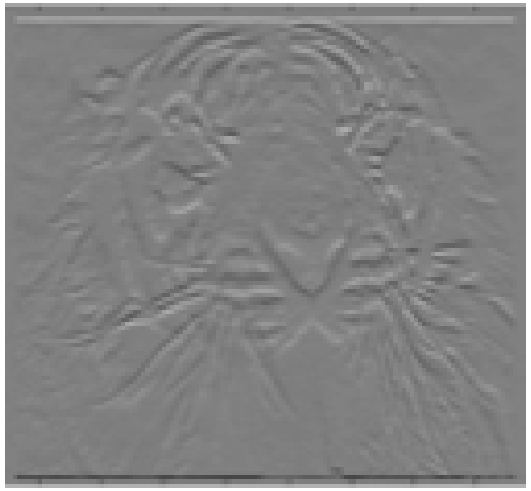
Roberts: $M_x =$

0	1
-1	0

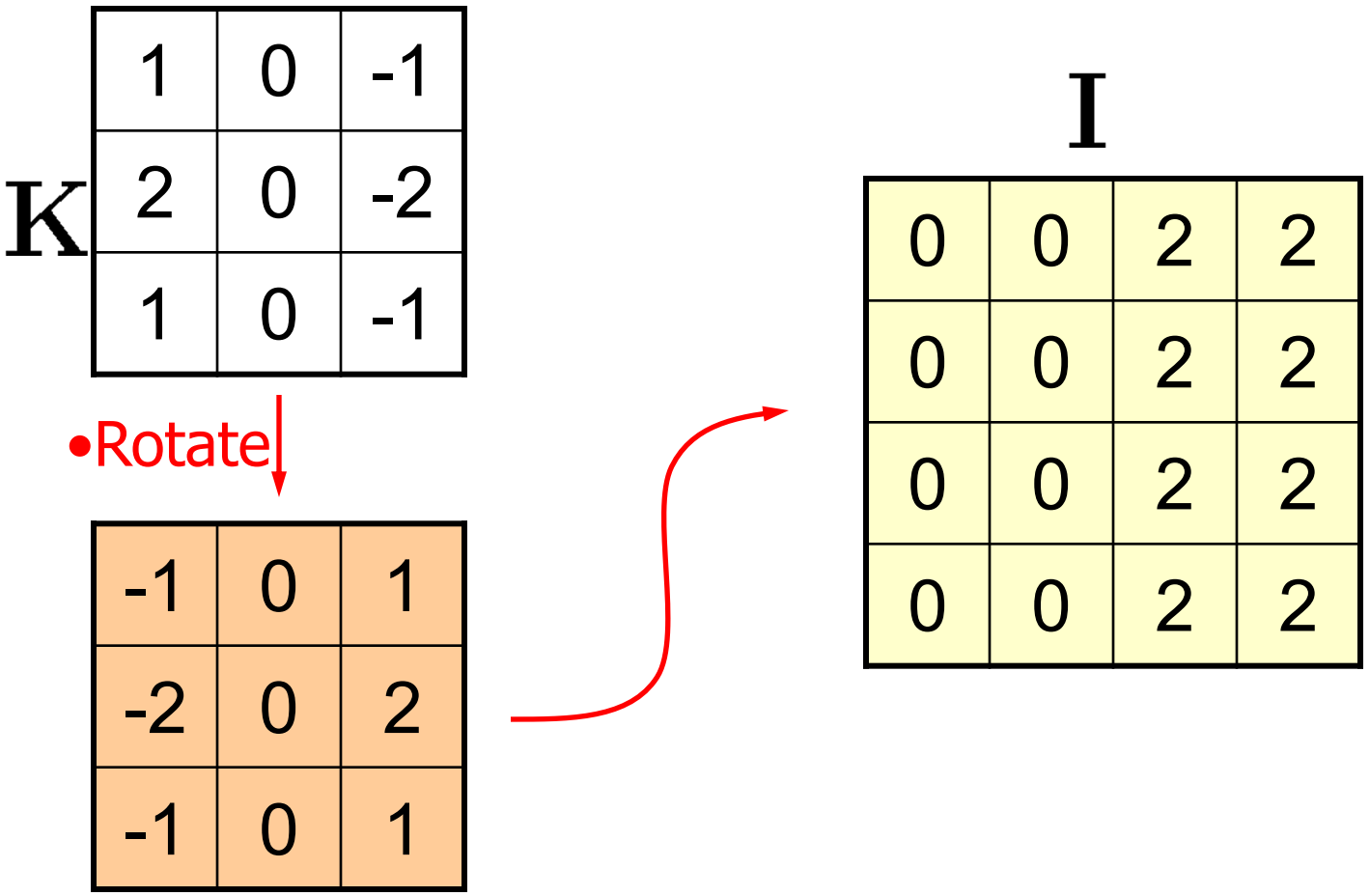
; $M_y =$

1	0
0	-1

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```



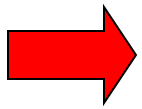
Eg: Convolute an image with a Sobel kernel



Step

-1	0	1
-2	0	2
-1	0	1

-1	0	1		
-2	0	0	2	2
-1	0	0	2	2
	0	0	1	2
	0	0	2	2



0			

$\mathbf{I}$

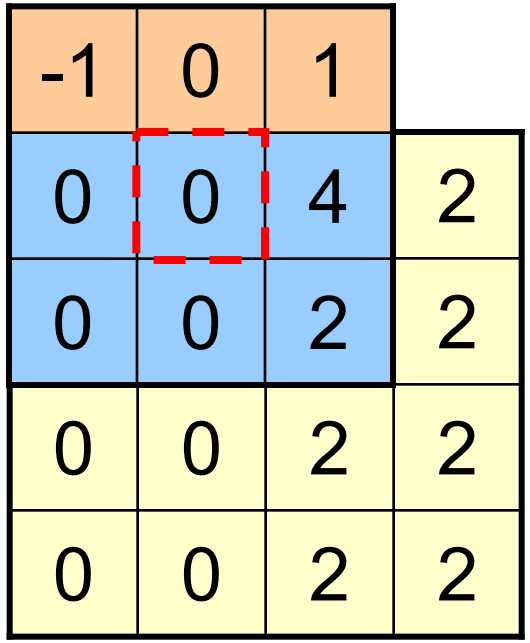
$\mathbf{I}'$

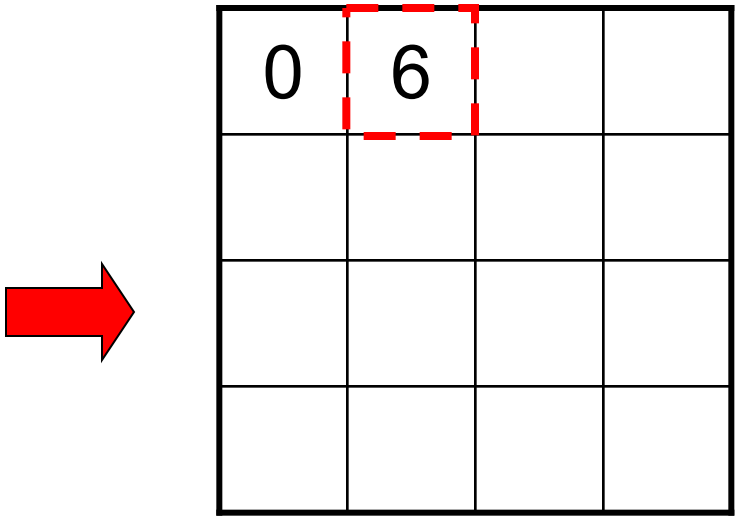
0	0	2	2
0	0	2	2
0	0	2	2
0	0	2	2

Step

-1	0	1
-2	0	2
-1	0	1

0	0	2	2
0	0	2	2
0	0	2	2
0	0	2	2





$\mathbf{I}$

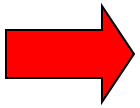
$\mathbf{I}'$

Step

-1	0	1
-2	0	2
-1	0	1

0	0	2	2
0	0	2	2
0	0	2	2
0	0	2	2

	-1	0	1
0	0	0	4
0	0	0	2
0	0	2	2
0	0	2	2



0	6	6	

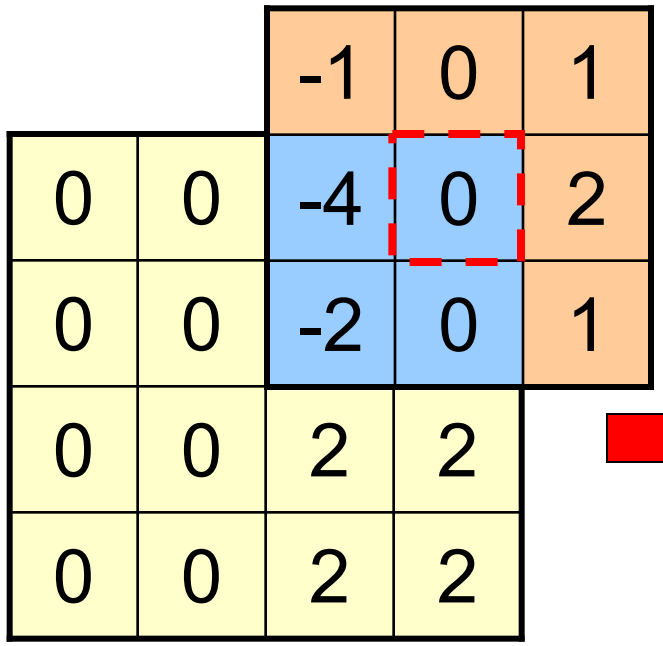
$\mathbf{I}$

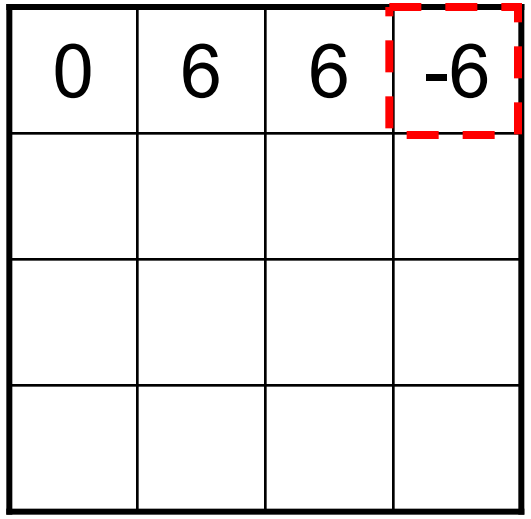
$\mathbf{I}'$

Step

-1	0	1
-2	0	2
-1	0	1

0	0	2	2
0	0	2	2
0	0	2	2
0	0	2	2

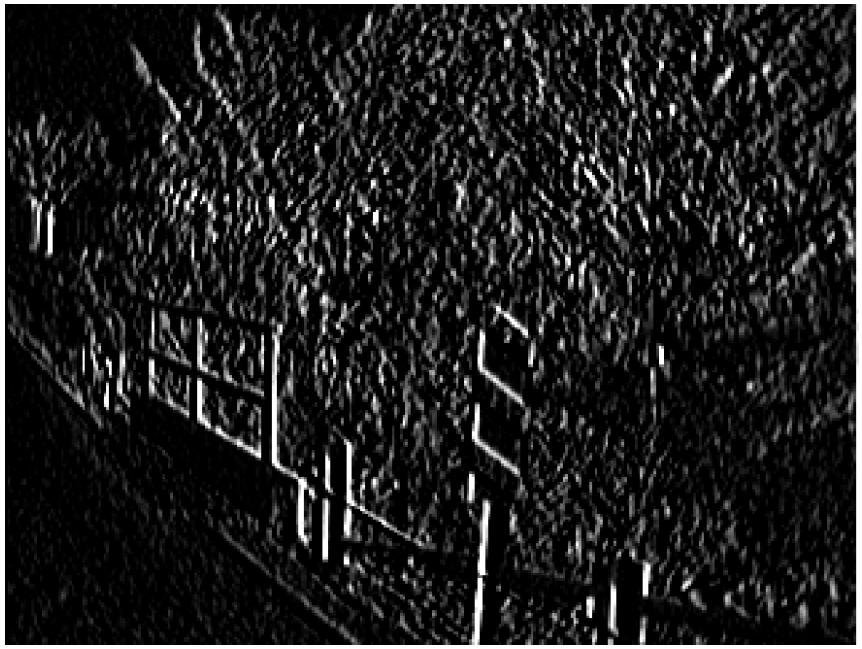


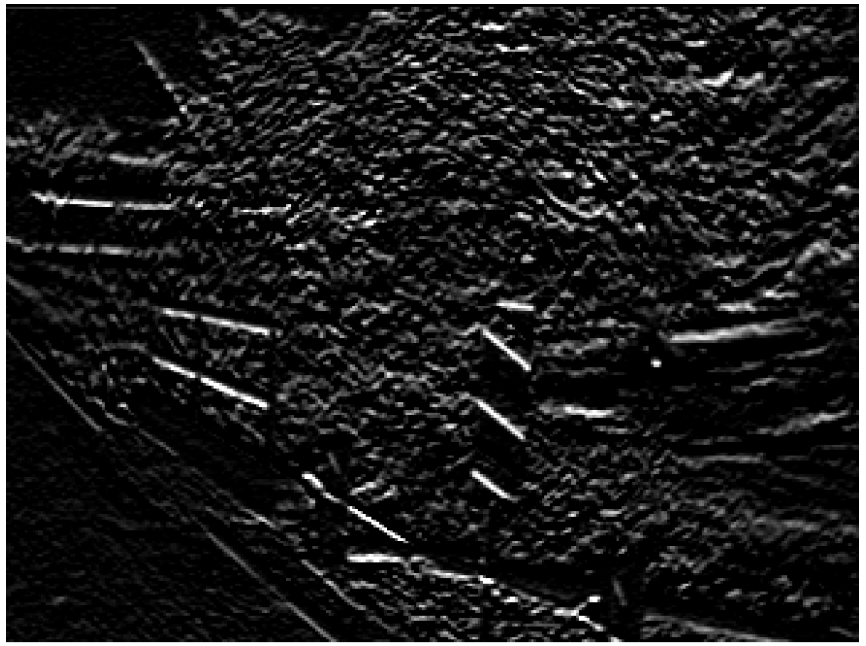


border artifect

$\mathbf{I}$ $\mathbf{I}'$

Sobel edge detection: example





- Vertical edges
- Horizontal edges

Derivatives with convolution

For 2D function, f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

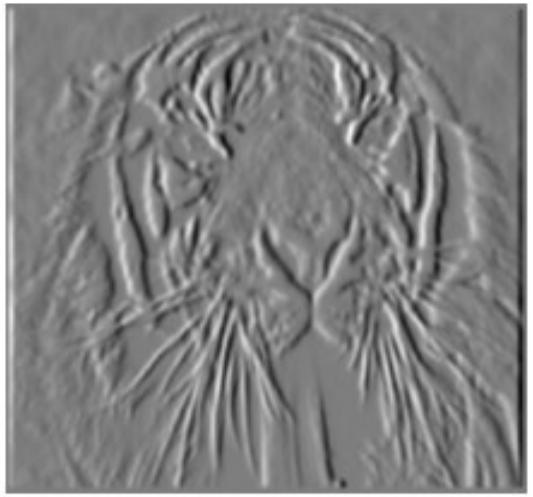
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement above as convolution, what would be the associated filter?

•Slide credit:
Kristen Grauman

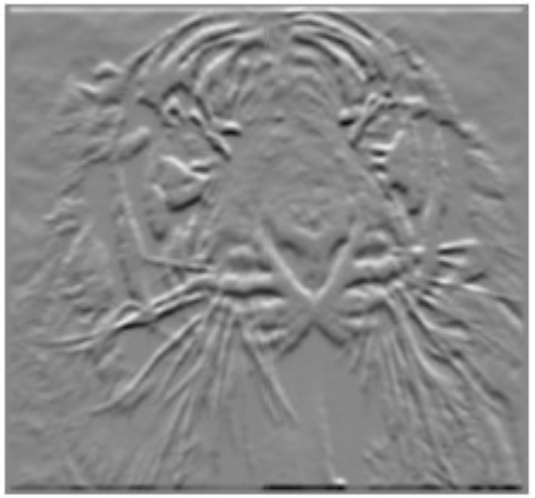
Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$



-1	1

$$\frac{\partial f(x,y)}{\partial y}$$



-1
1

or

1
-1

- Which shows changes with respect to x?

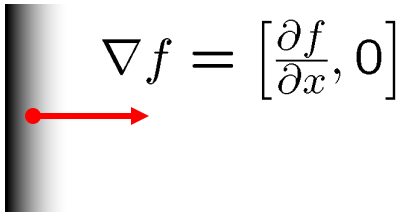
- (showing filters for correlation)

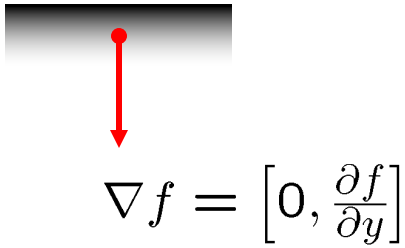
Image gradient

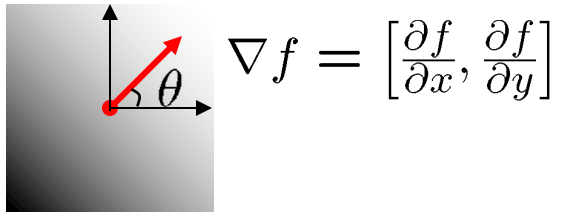
The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid change in intensity





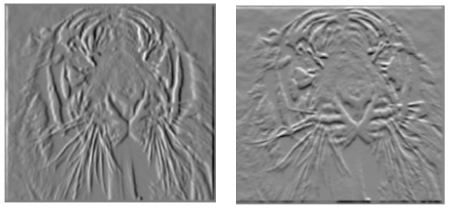


- The **gradient direction** (orientation of edge normal) is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- The **edge strength** is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

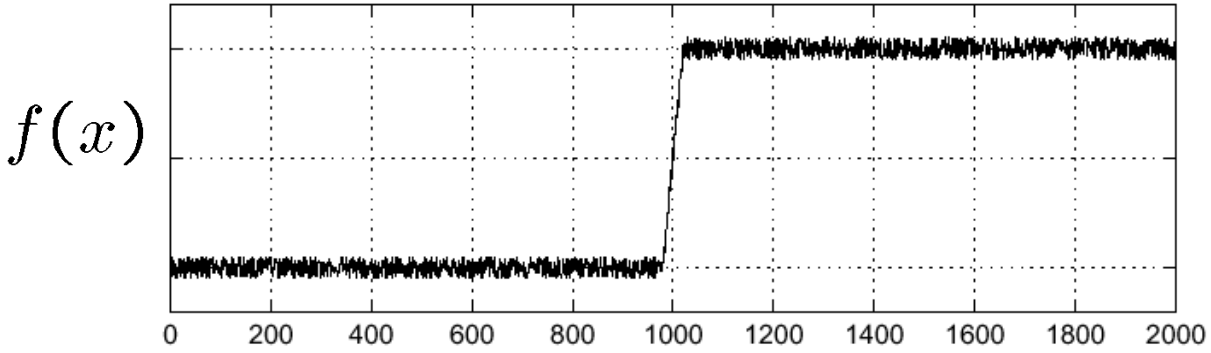


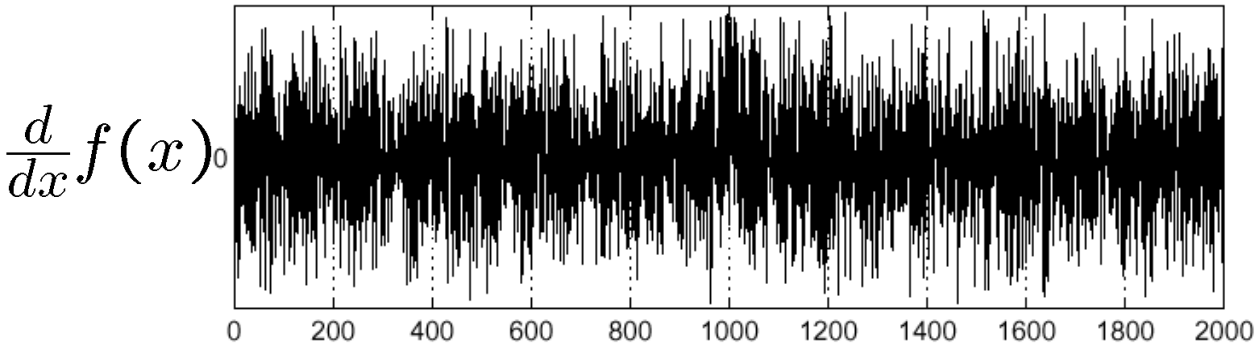
•Slide credit Steve Seitz

Effects of noise

Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal





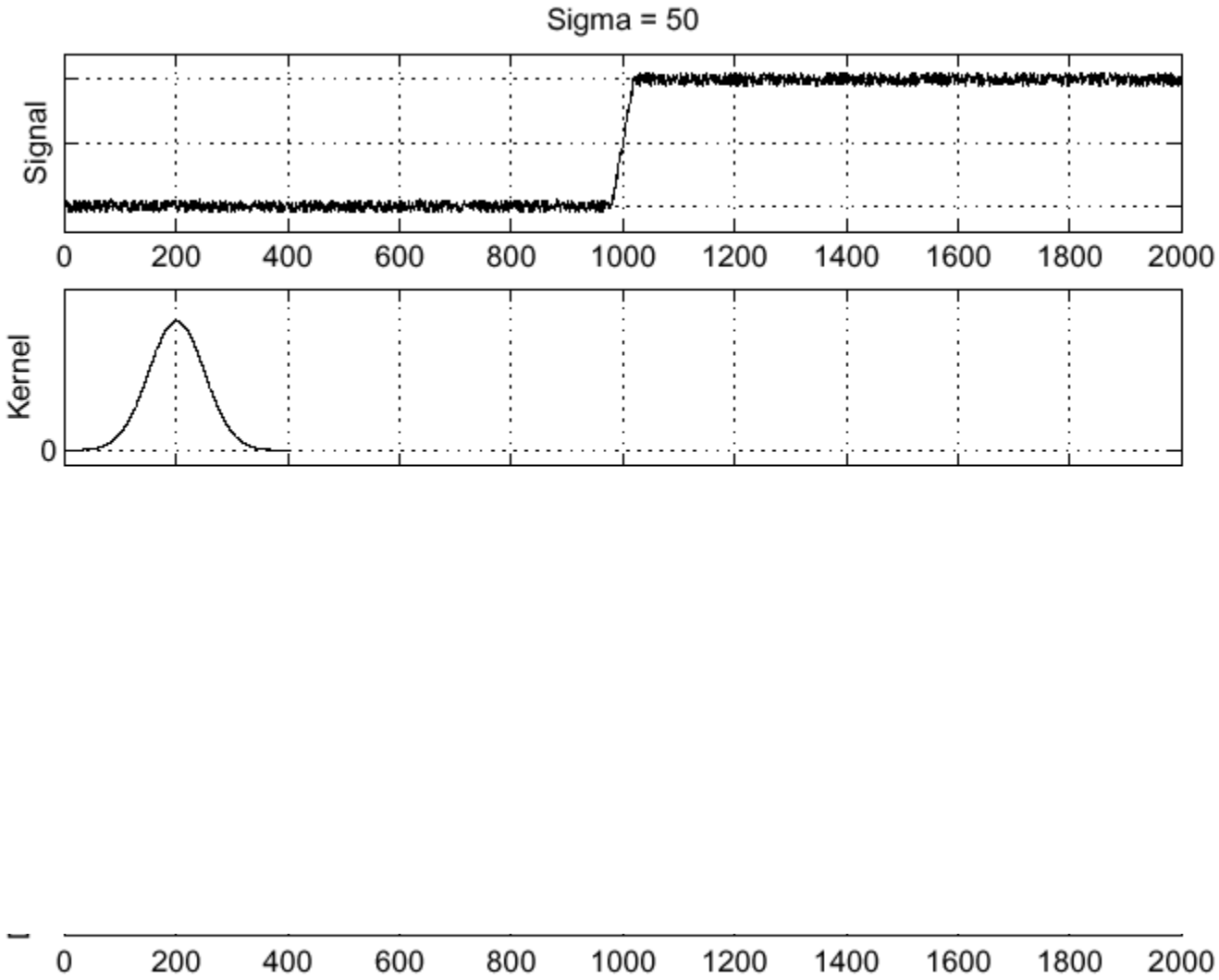
- Where is the edge?

•Slide credit Steve

Solution: smooth first

f

h



- Where is the edge?
- Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

Derivative theorem of convolution

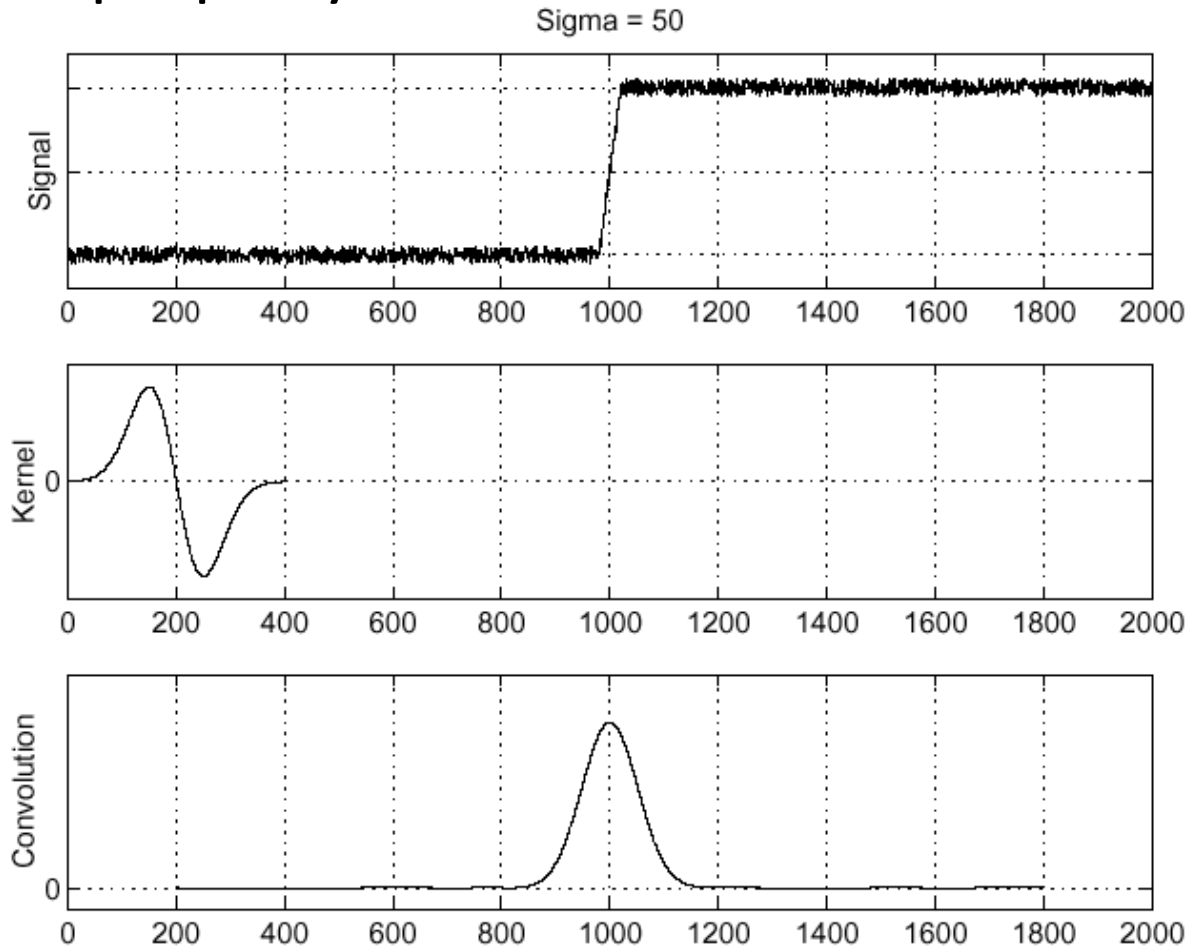
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

Differentiation property of convolution.

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$

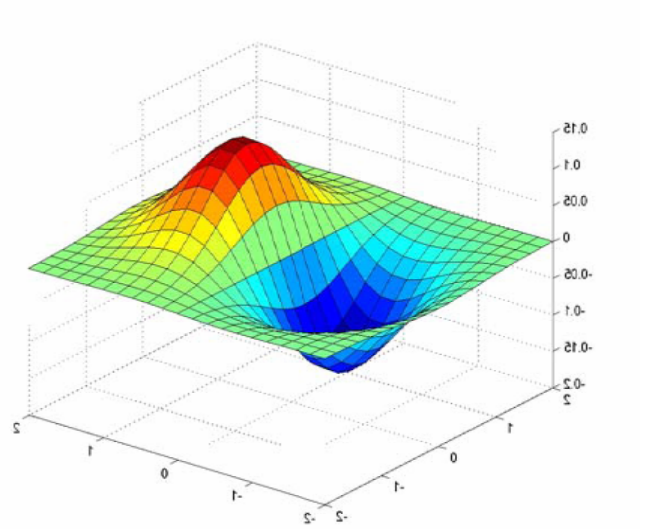


•Slide credit Steve

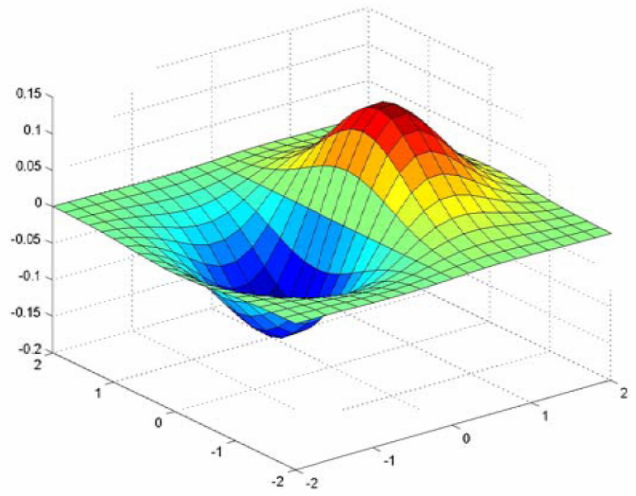
Derivative of Gaussian filters

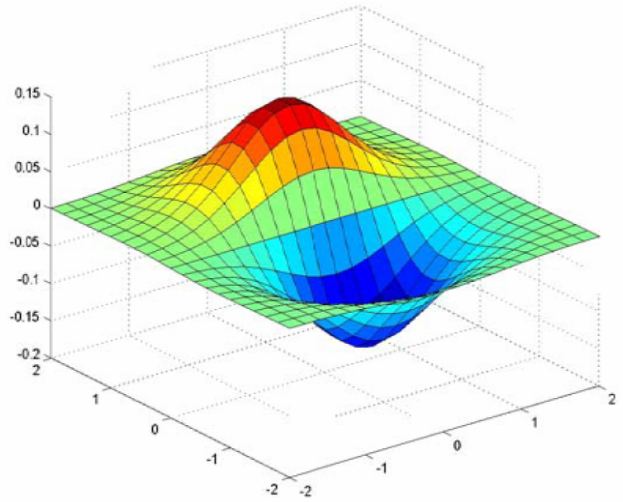
$$(I \otimes g) \otimes h = I \otimes (g \otimes h)$$

$$\begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix} \otimes \begin{bmatrix} 1 & -1 \end{bmatrix}$$

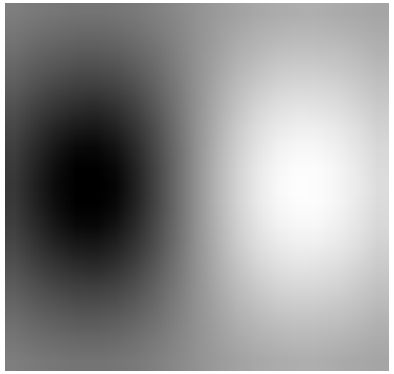


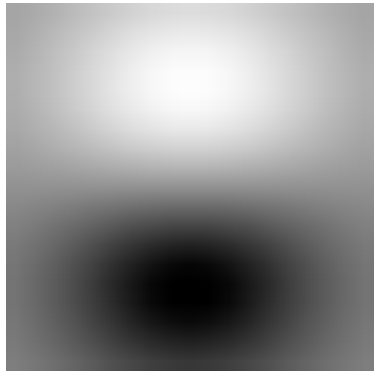
Derivative of Gaussian filters





- ***x*-direction**
- ***y*-direction**





- **Source: L. Lazebnik**

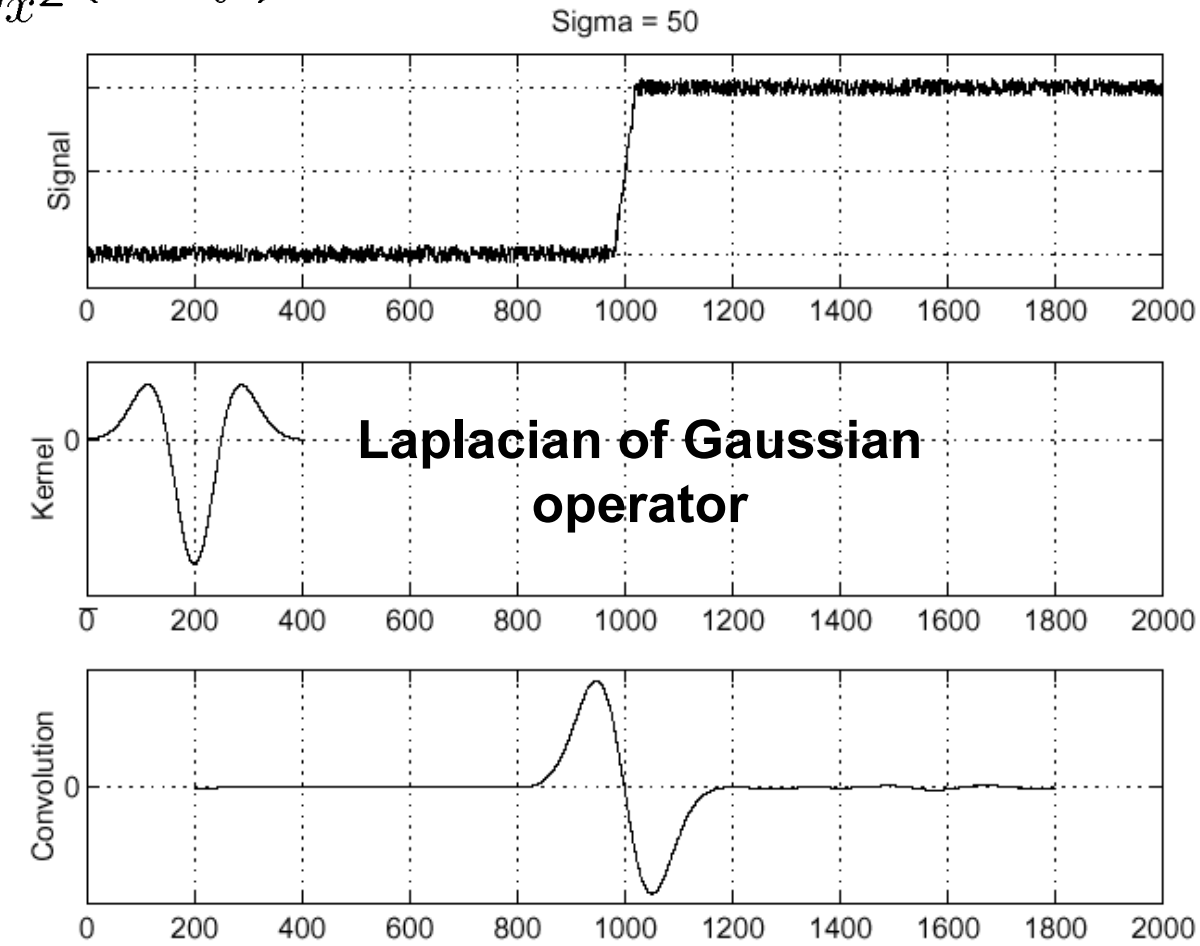
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

f

$\frac{\partial^2}{\partial x^2}h$

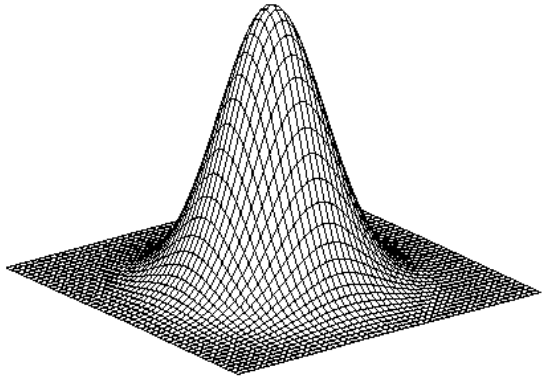
$(\frac{\partial^2}{\partial x^2}h) \star f$



- Where is the edge?
- Zero-crossings of bottom graph

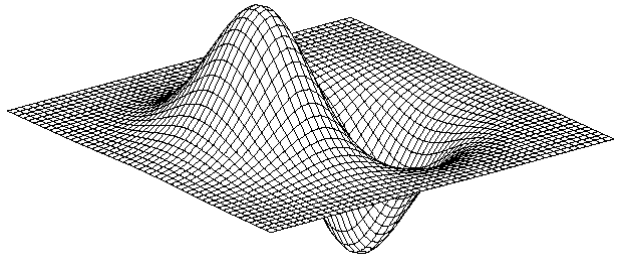
•Slide credit: Steve Seitz

2D edge detection filters



Gaussian

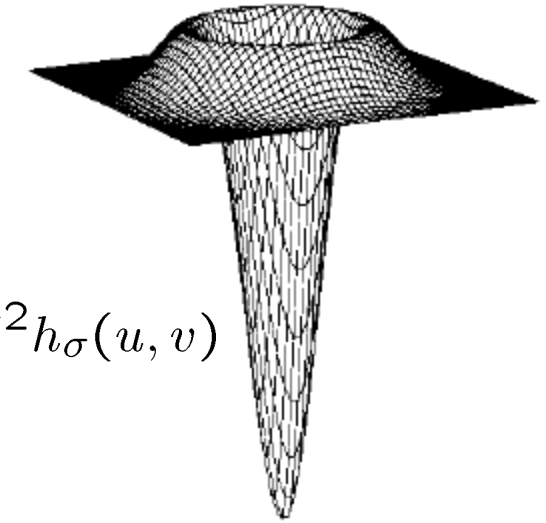
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

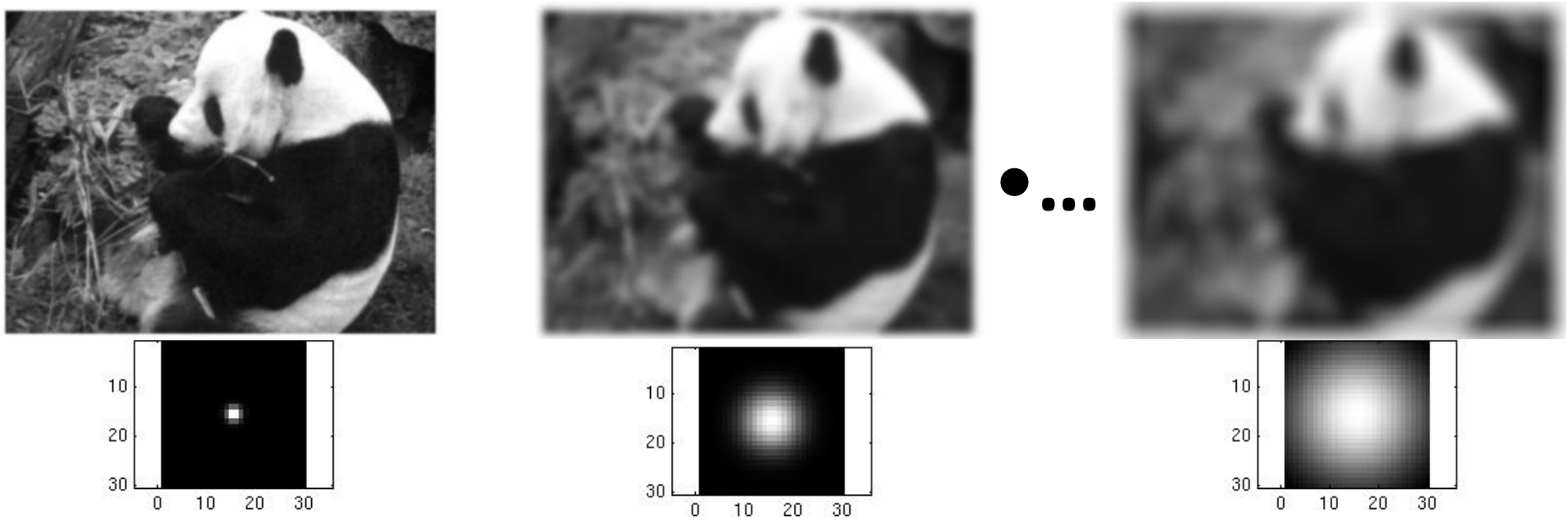
- $\nabla^2$is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

•Slide credit: Steve Seitz

Smoothing with a Gaussian

- Recall: parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel, and controls the amount of smoothing.



- Slide credit: Kristen Grauman

Effect of σ on derivatives

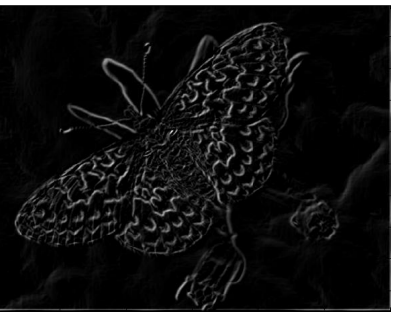
σ = 1 pixel

σ = 3 pixels

- The apparent structures differ depending on Gaussian's scale parameter.

- Larger values: larger scale edges detected
- Smaller values: finer features detected

- Slide credit: Kristen Grauman



Gradients -> edges

Primary edge detection steps:

1. Smoothing: suppress noise
2. Edge enhancement: filter for contrast
3. Edge localization

   Determine which local maxima from filter output are actually edges vs. noise

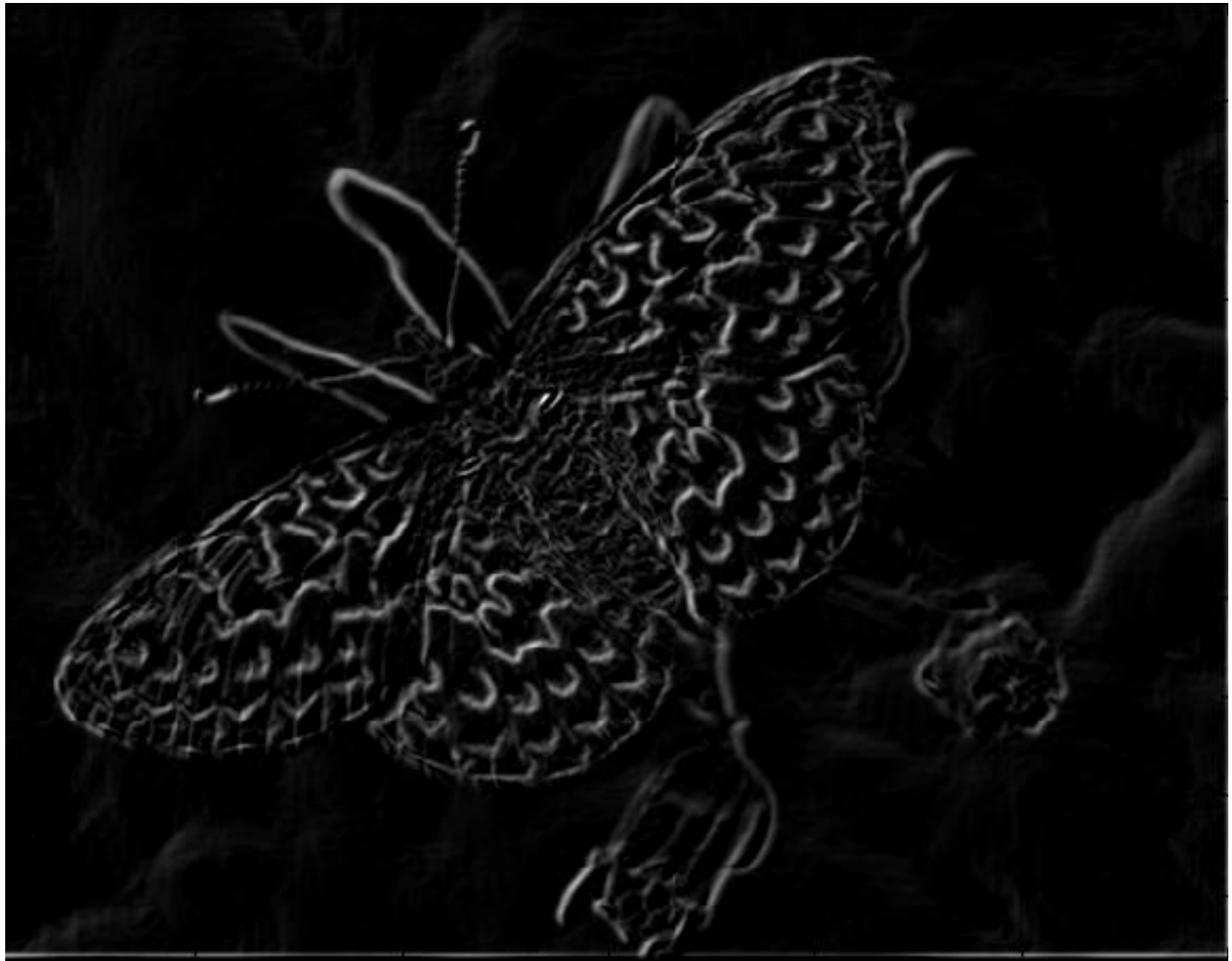
   - Threshold, Thin

Thresholding

- Choose a threshold value t
- Set any pixels less than t to zero (off)
- Set any pixels greater than or equal to t to one (on)

Original image

Gradient magnitude image



Thresholding gradient with a lower threshold

Thresholding gradient with a higher threshold

Canny edge detector

- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- **Non-maximum suppression**:
  - Thin wide “ridges” down to single pixel width
- **Linking and thresholding** (**hysteresis**):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: `edge(image, 'canny');`
- `>>help edge`

•Source: D. Lowe, L. Fei-Fei

The Canny edge detector

original image (Lena)

Slide credit: Steve Seitz

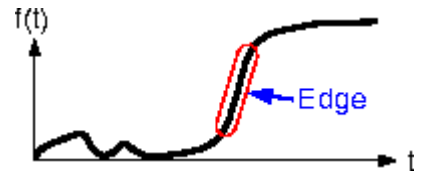
The Canny edge detector

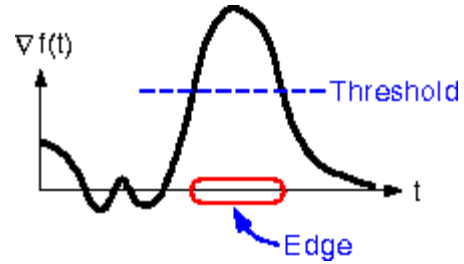
- norm of the gradient

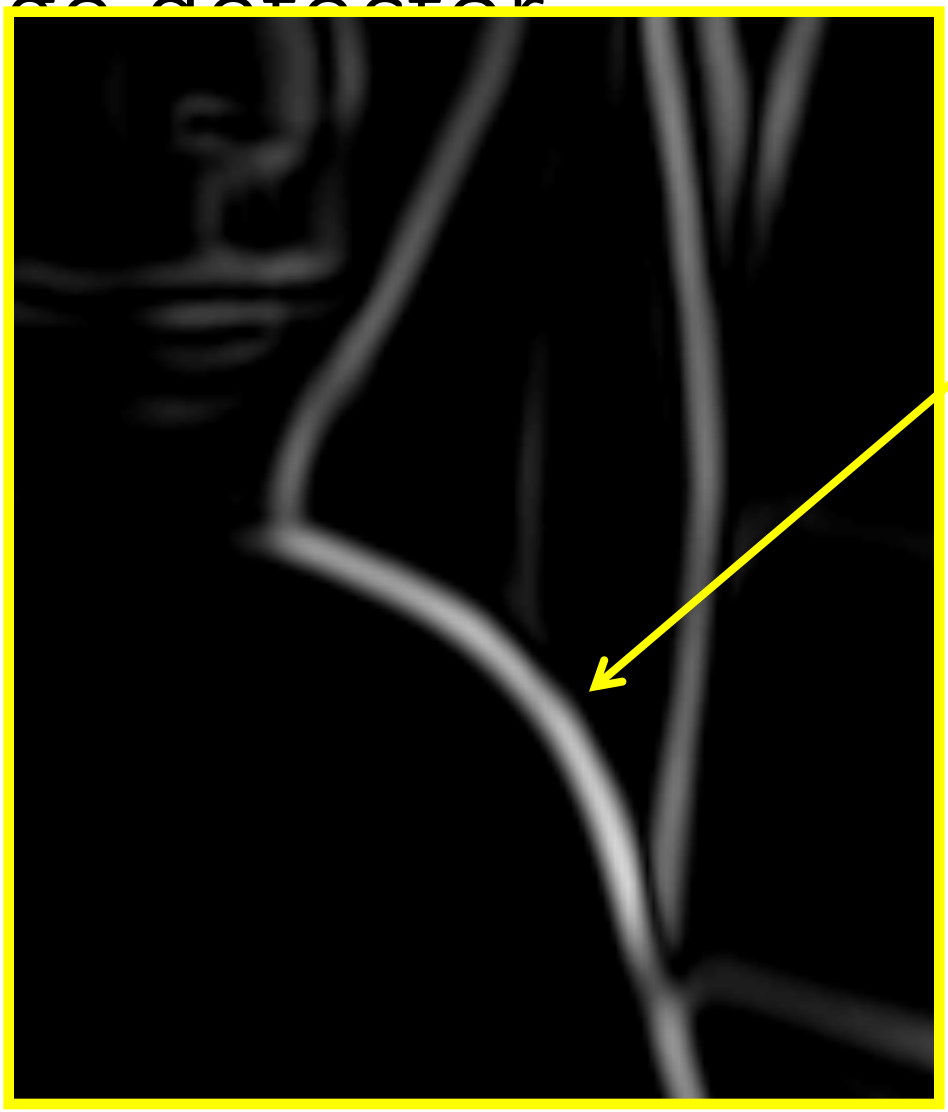
The Canny edge detector

- thresholding

The Canny edge detector

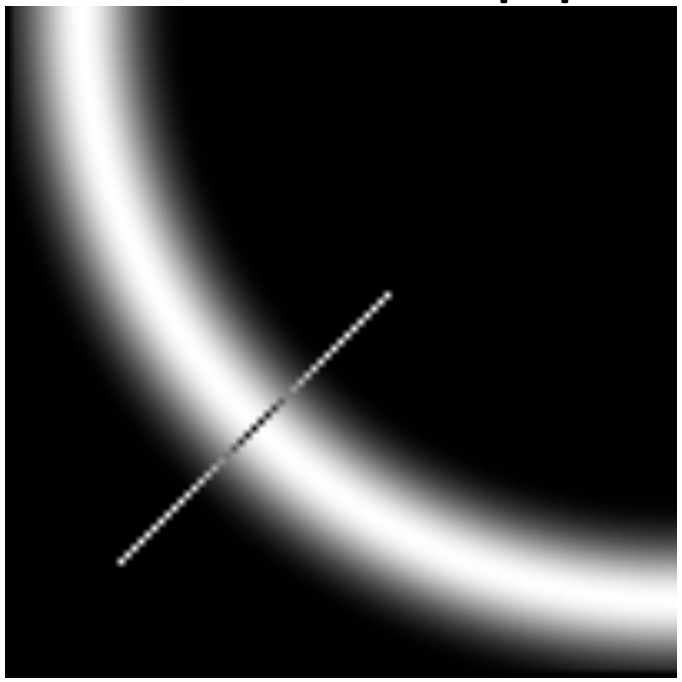


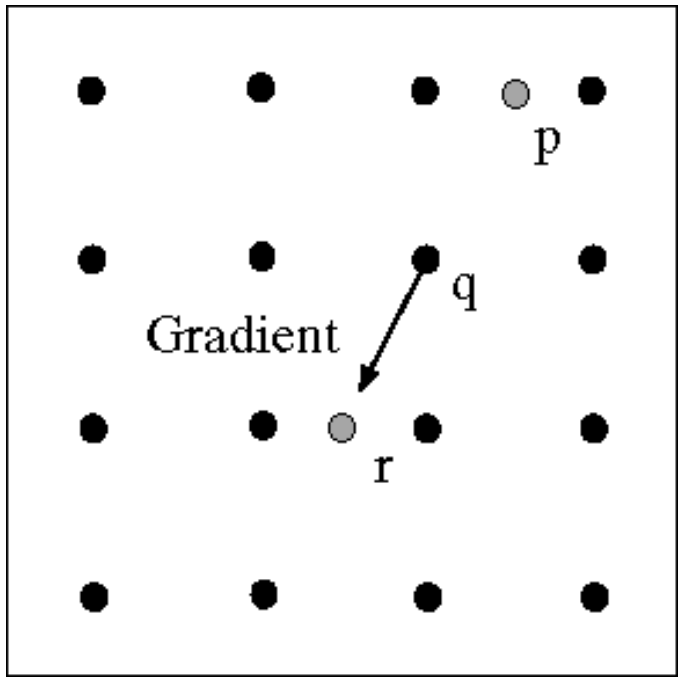




- How to turn these thick regions of the gradient into curves?

Non-maximum suppression





Check if pixel is local maximum along gradient direction, select single max across width of the edge

- requires checking interpolated pixels p and r

The Canny edge detector

- Problem: pixels along this edge didn’t survive the thresholding

- thinning
- (non-maximum suppression)

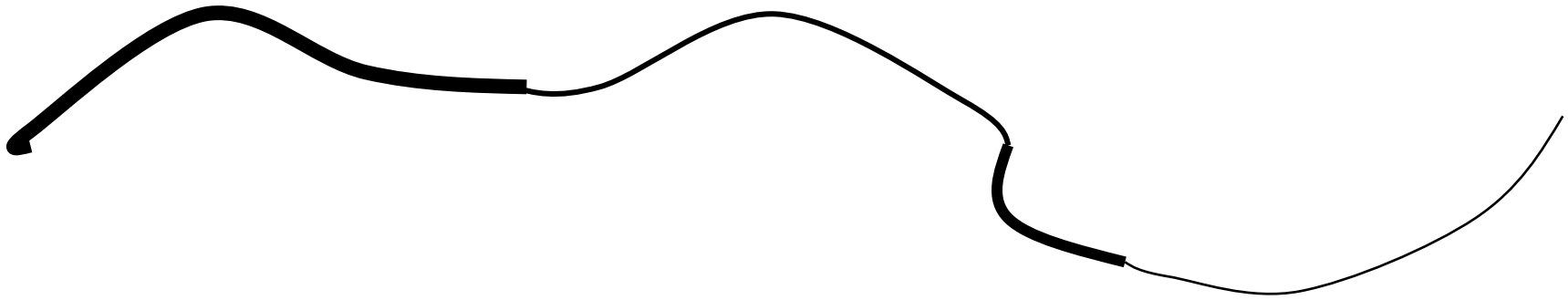
Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

•Credit: James Hays

Hysteresis thresholding

- Use a high threshold to start edge curves, and a low threshold to continue them.



•Source: Steve Seitz

Final Canny Edges

•Credit: James

Recap: Canny edge detector

- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- **Non-maximum suppression**:
  - Thin wide “ridges” down to single pixel width
- **Linking and thresholding** (**hysteresis**):
  - Define two thresholds: low and high
  - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB: `edge(image, 'canny');`
- `>>help edge`

•Source: D. Lowe, L. Fei-Fei

Mask properties

- Smoothing
  - Values positive
  - Sum to 1 → constant regions same as input
  - Amount of smoothing proportional to mask size
  - Remove “high-frequency” components; “low-pass” filter

- Derivatives
  - Opposite signs used to get high response in regions of high contrast
  - Sum to 0 → no response in constant regions
  - High absolute value at points of high contrast

•Slide credit:
Kristen Grauman

Non-linear Filter

- Median Filter
- Bilinear Filter

Nonlinear Image Filtering

Median Filter (a nonlinear filter)

Median filter

- The median filter is a sliding-window based spatial filter.

- It replaces the value of the center pixel with the median of the intensity values in the neighborhood of that pixel.

- Median filtering is a nonlinear operation often used in image processing to reduce "salt and pepper" noise. A median filter is more effective than convolution when the goal is to simultaneously reduce noise and preserve edges.

- **Median filters** are particularly effective in the presence of *impulse noise*, also called '***salt – and – pepper***' noise because of its appearance as white and black dots superimposed on an image.

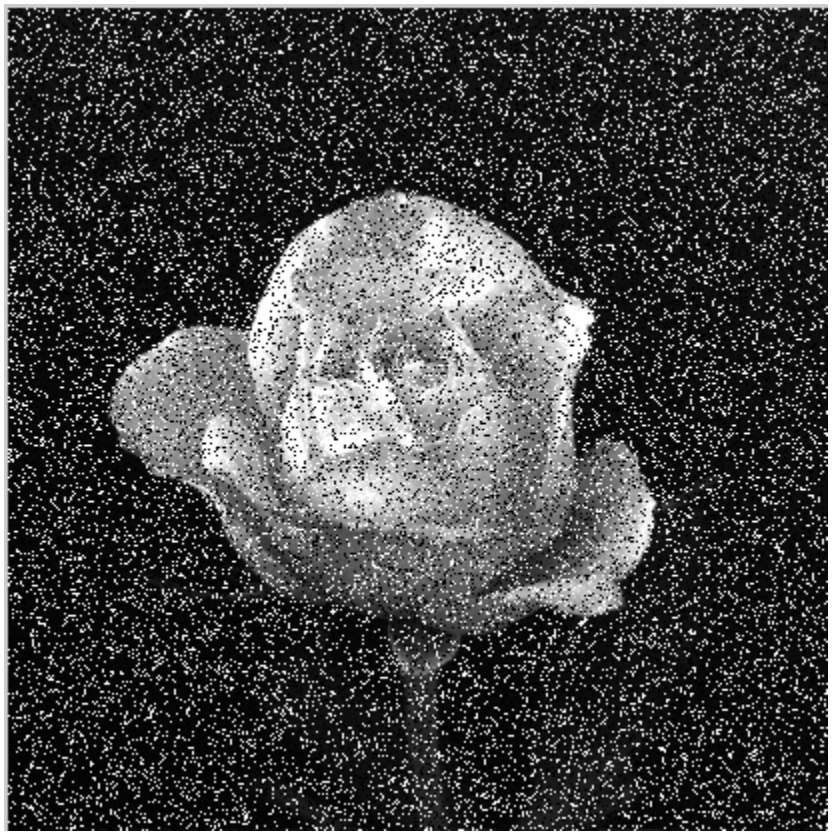
- For every pixel, a 3x3 neighborhood with the pixel as center is considered. In **median filtering**, the value of the pixel is replaced by the median of the pixel values in the 3x3 neighborhood.

Questions

What is the value of the yellow box after 3x3 median filtering?

1	2	2	2	3
2	5	2	4	3
2	2	2	2	3
4	3	7	5	3
3	3	3	1	1

Example: Median filter result



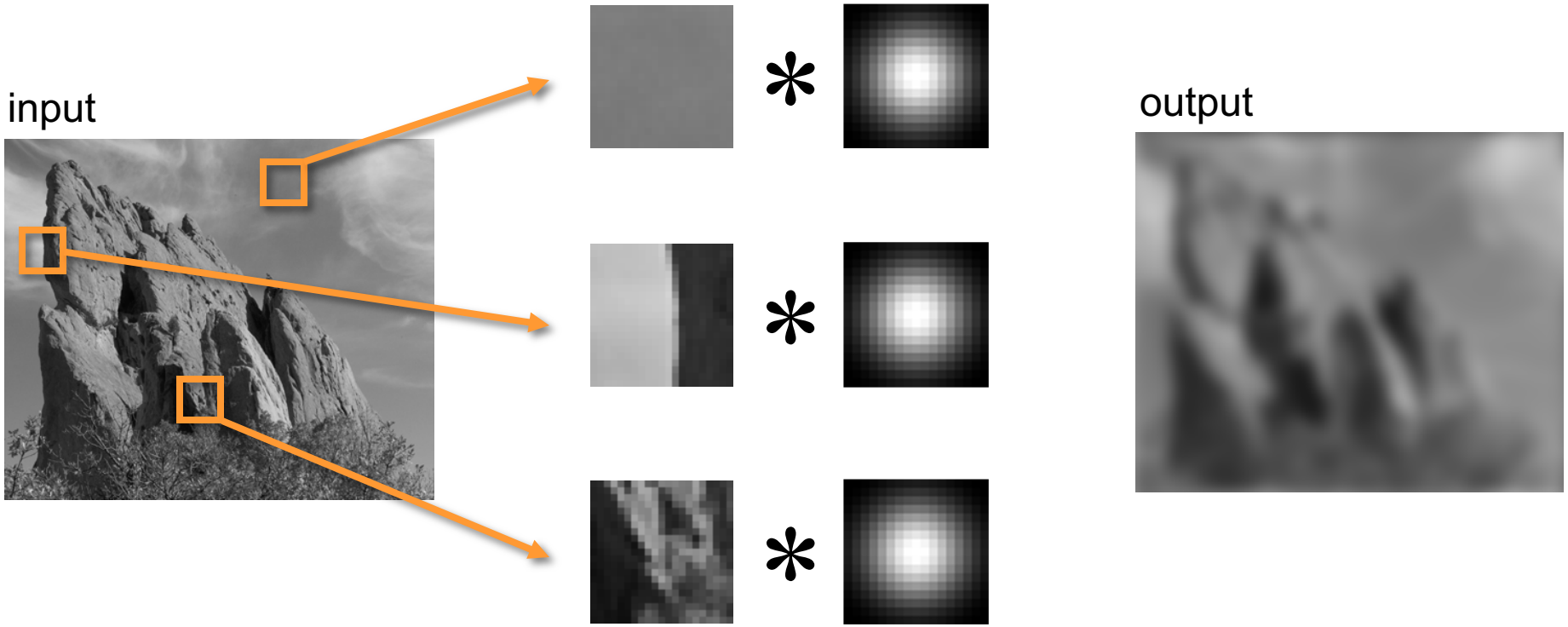
Bilateral filter (a nonlinear filter)

How to smooth an image without causing excessive edge-blur ?

Bilateral Filter (Tomasi et al.1998)

- It smooths regions while preserving edges

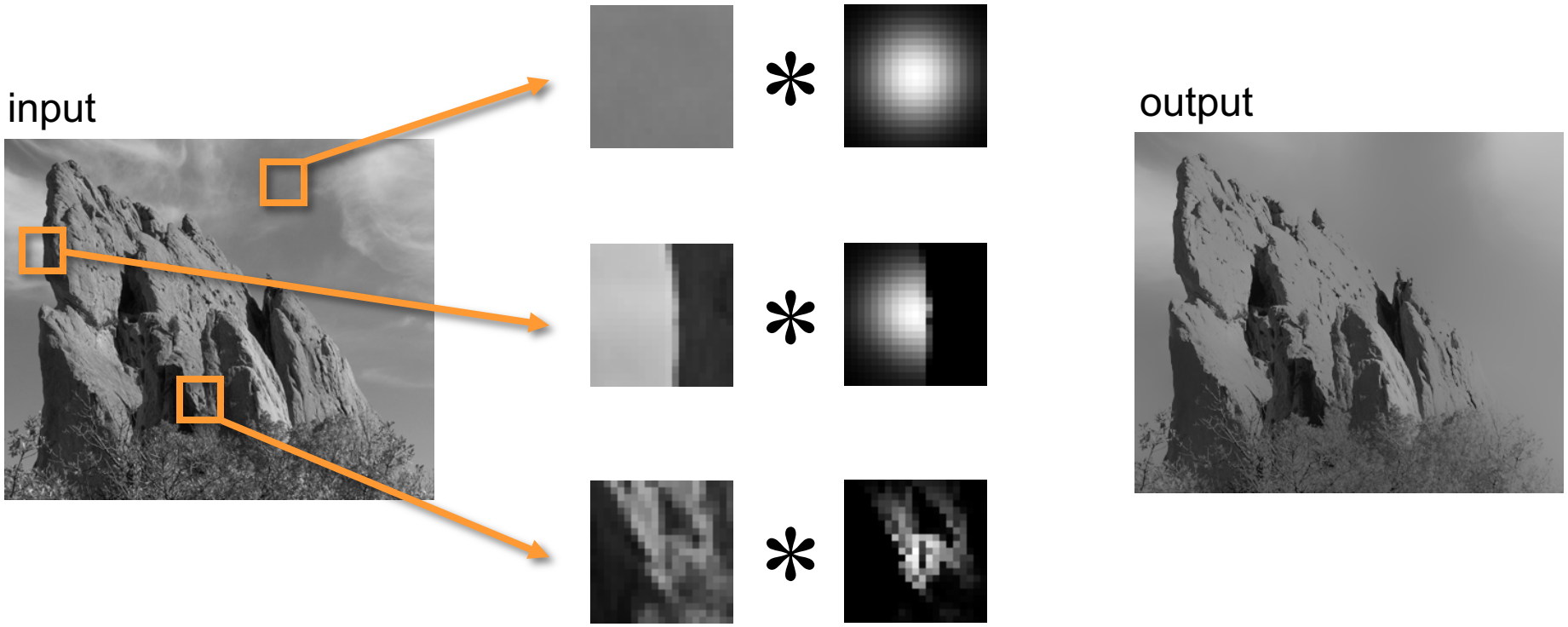
Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

Bilateral Filter
No Averaging across Edges

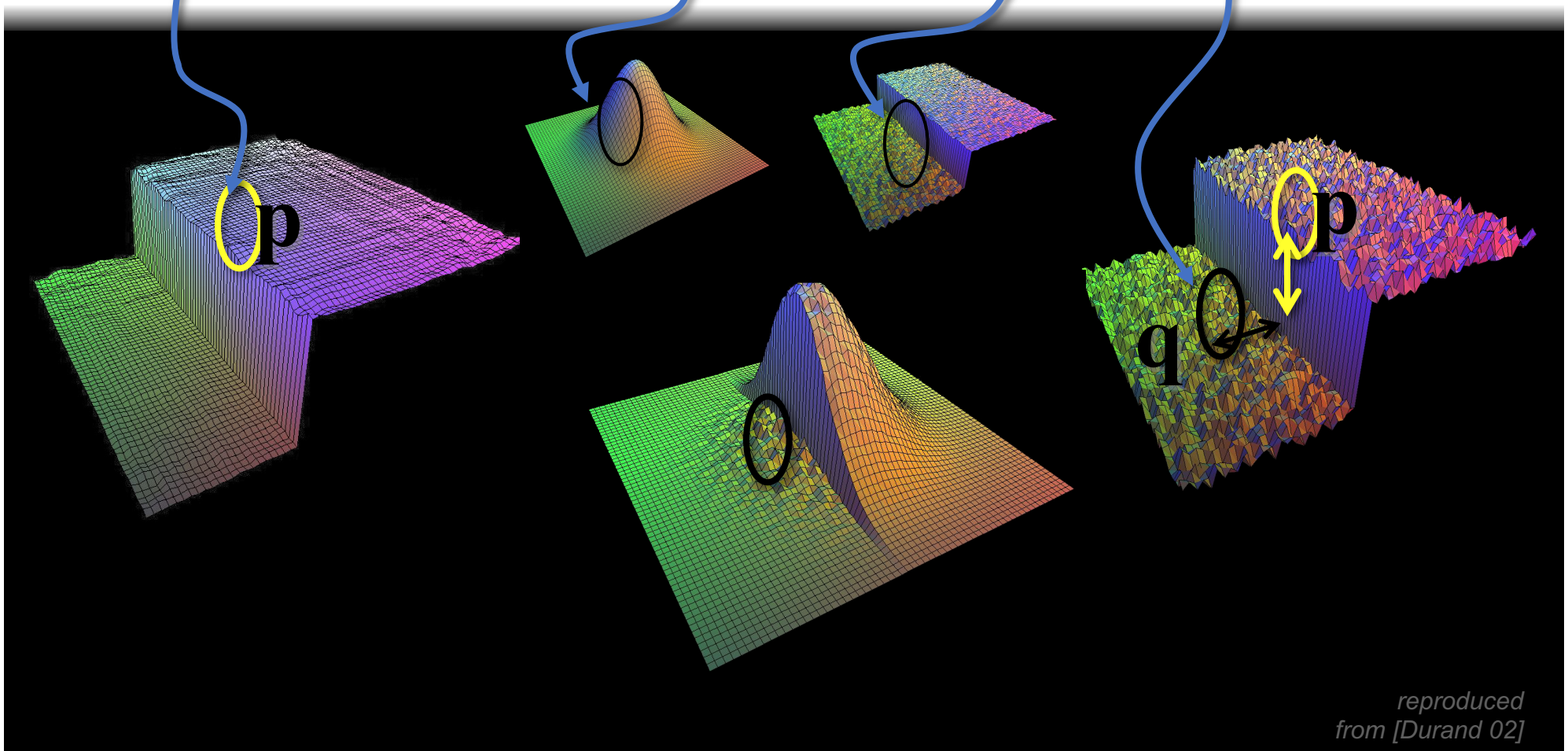
[Aurich 95, Smith 97, Tomasi 98]



The kernel shape depends on the image content.

Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$



reproduced from [Durand 02]

Bilateral Filter on a Height Field

- Domain filter: G_{σ_s}
- Range filter: G_{σ_r}

Varying the Space Parameter

input

$\sigma_r = 0.1$

$\sigma_r = 0.25$

$\sigma_r = \infty$ (Gaussian blur)

$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$

Readings

- Computer Vision: Algorithms and Applications, Chapter 3.1, 3.2 and 3.3