Total Marks: 25 Value: 5% of final grade

Due: 2 pm Friday 9 April 2021 (the Friday of the first week of teaching break)

This assignment is based on Part A (The language of mathematics and computer science).

Please upload your solutions in PDF format, using the link provided. If you write the solutions by hand, you will need to scan your work and save it as a pdf file.

Page 1 of your solutions document should be a 'cover page' containing **only**:

1. Title: "Graduate Assignment A"
2. Your full name, with surname in upper case.
3. Your ANU ID
4. The declaration: "I have read the ANU Academic Skills statement regarding collusion." (`https://www.anu.edu.au/students/academic-skills/academic-integrity/plagiarism/collusion`) "I have not engaged in collusion in relation to this assignment".
5. Your signature. (If you are typesetting rather than scanning a hand-written document, you can type your name and it will be deemed a signature.)
6. The date and approximate time of your submission.

Regarding item 4, I emphasise the last paragraph of the Academic Skills statement: *The best way people can help each other to understand the material is to discuss the ideas, questions, and potential solutions in general terms. However,* ***students should not draw up a detailed plan of their answers together. When it comes to writing up the assignment, it should be done separately. If collusion is detected, all students involved will receive no marks.***

There are five questions. You may find some questions more difficult/time-consuming than others, but nevertheless each question is worth the same (5 marks) and assessed against the same marking criteria. The marking criteria is detailed on the next page.

The following marking criteria will be applied to each question in this assignment.

Score	Description
5	Solutions are correct and complete; solutions are written in complete sentences; solutions are succinct and clearly communicated; notation is used accurately; statements to be justified are justified so well that the explanation or counterexample given constitutes a proof; any hypotheses/assumptions made are explicitly identified; any examples/counterexamples constructed are described effectively and how they serve the purpose at hand is made clear; any new variables used are introduced explicitly.
4	Solutions are correct and complete, except perhaps a minor error; solutions are written in complete sentences almost always; solutions are clearly communicated; notation is used accurately, except perhaps a minor misuse; statements to be justified are justified effectively; any hypotheses/assumptions made are explicitly identified; any examples/counterexamples constructed are described effectively and how they serve the purpose at hand is made clear; any new variables used are introduced explicitly.
3	Solutions are correct and complete, except for several minor errors or omissions; explanation is given for solutions; notation is used accurately most of time; statements to be justified are justified effectively; any hypotheses/assumptions made are identifiable; any examples/counterexamples constructed are described effectively; new variables may be used without introduction, but the role they play is discernible from the context.
2	Solutions do not meet the criteria for 3 points, but they provide evidence of partial understanding of the material and evidence of a substantial effort to answer the question.
1	Solutions do not meet the criteria for 2 points, but they provide evidence of a substantial effort to answer the question.
0	Solutions do not meet the criteria for 1 point.

Question 1

(A) Construct a circuit diagram corresponding to the input-output table below.

X	Y	Z	output
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

(B) Determine whether the following statement is true or false, and explain your reasoning:

> Every compound statement is logically equivalent to one in which the only symbols used are statement variables, '(', ')', '→' and '¬'.

(C) Determine whether the following statement is true or false, and explain your reasoning:

> Every compound statement is logically equivalent to one in which the only symbols used are statement variables, '(', ')', '∧' and '∨'.

Question 2

(A) Each of the variables in the following predicates is quantified over $\mathbb{Z}^+$:

$p(x)$: x is prime $\qquad$ $d(t,x)$: t divides x
$o(t)$: $t = 1$ $\qquad$ $q(t,x)$: $t = x$.

Using only quantifications, parentheses, logical connectives, variables and the predicates $d(t,x)$ and $o(t)$ and $q(t,x)$, write something in place of ... in the following to make a true statement.

$$\forall x \in \mathbb{Z}^+ \ \Big[p(x) \leftrightarrow \ldots \Big].$$

(B) Using only quantifications, parentheses, logical connectives, variables and the predicates $d(t,x)$ and $o(t)$ and $q(t,x)$ defined in part (A),write something in place of ... in the following to make a true statement.

$$\forall x \in \mathbb{Z}^+ \ \Big[\neg p(x) \leftrightarrow \ldots \Big].$$

(C) Let $g\colon \mathbb{Z} \to \mathbb{Z}$ be a function. Consider the following two statements, both assuming the universe of integers.

$$\text{Statement 1:} \quad \forall x \ \exists y \big([x \le y] \ \wedge \ [g(x) \ge g(y)] \big)$$
$$\text{Statement 2:} \quad \exists y \ \forall x \big([x \le y] \ \wedge \ [g(x) \ge g(y)] \big)$$

Without knowing any more about the function g, are you able to determine whether or not Statement 1 is true? How about Statement 2? Explain your answers.

Question 3

(A) Establish or refute the validity of the following argument:
If the Raiders are playing a home match, the traffic will be bad.
If the traffic is bad, we will be late to the Tina Arena concert.
$\therefore$ If we are late to the Tina Arena concert, it will be because the Raiders are playing a home match.

(B) Establish or refute the validity of the following argument:
Vika is a mathematics major or Vika is a computer science major.
If Vika is a computer science major, then Vika is required to take MATH1005.
$\therefore$ Vika is a mathematics major or Vika is required to take MATH1005.

(C) Sharky, a leader of the underworld, was killed by one of his own band of four minions. Detective Sharp interviewed the minions and determined that all were lying except for one. The detective's notes from the interviews included the following:

- Socko said "Lefty killed Sharky."
- Fats said"Muscles didn't kill Sharky."
- Lefty said "Muscles was shooting dice with Socko when Sharky was killed."
- Muscles said " Lefty didn't kill Sharky."

Who killed Sharky? Justify your answer.

Question 4

A relation $\mathcal{R}$ on a set S is said to be *transitive* if, and only if,

$$\forall x, y, z \in S \quad x \mathcal{R} y \ \wedge \ y \mathcal{R} z \implies x \mathcal{R} z.$$

Relations $\mathcal{R}_1$, $\mathcal{R}_2$, and $\mathcal{R}_3$ are defined on the power set $\mathcal{P}(\{a, b, c, d, e, f\})$ by the rules below. In each case, prove or disprove that the relation is transitive.

(A) $A \mathcal{R}_1 B \Leftrightarrow A \backslash B = \varnothing$.

(B) $A \mathcal{R}_2 B \Leftrightarrow$ there exists a bijective function $f : A \to B$.

(C) $A \mathcal{R}_3 B \Leftrightarrow$ there exists an injective function $f : A \backslash B \to B \backslash A$.

Question 5 Review the definition of countable and uncountable sets given in lectures. Prove or disprove each of the following statements:

(A) Every subset of a countable set is countable.

(B) If A and B are disjoint sets and both A and B are countable, then $A \cup B$ is countable.

(C) If $A \subseteq \mathbb{R}$ and

$$\forall a, b \in A \Big((a < b) \to (\exists c \in A \; a < c < b) \Big),$$

then A is uncountable.

End of Questions for Assignment A