MATH1005/MATH6005 Semester 1 2021

Assignment 2

Workshop Details:

Number	Day	Time	Demonstrator name
16B	Friday	~~12~~ 19/03 2pm	

Student Details:

ID	Surname	Given name	Preferred name
u7235649	Zhang	Han	Han

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working**.

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature: Han Zhang .... Date: 21/03/2021

This document must be submitted by 11pm on the THURSDAY following your workshop.

Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question 1$^+$ For universal set $U = \{n \in \mathbb{N} : n \leq 15\}$ define sets S and T as follows:

$$\begin{aligned} S &= \{n \in U : n \text{ is divisible by 2 or 7 (or both)}\}, \\ T &= \{n \in U : n \text{ is divisible by 3 or 5 (or both)}\}. \end{aligned}$$

How many members have each of the following sets? Show your enumeration/calculation.

(a) S^c (b) $S \cap T$ (c) $S \cup T$

(d) $S \backslash T$ (e) $S \triangle T$ (f) $\mathcal{P}(S)$ [the power set of S]

$S = \{0, 2, 4, 6, 7, 8, 10, 12, 14\}$

$T = \{0, 3, 5, 6, 9, 10, 12, 15\}$

(a) $S^c = \{1, 3, 5, 9, 11, 13, 15\}$. 7 members.

(b) $\cancel{S \cap T = \{3, 5, 9, 15\}}$

$S \cap T = \{0, 6, 10, 12\}$ 4 members.

(c) $S \cup T = \{0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15\}$. 13 members.

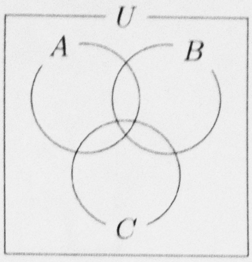
(d) $S \backslash T = \{2, 4, 7, 8, 14\}$ 5 members.

(e) $S \triangle T = (S \cup T) \backslash (S \cap T) = \{2, 3, 4, 5, 7, 8, 9, 14, 15\}$

9 members.

(f) $\cancel{\mathcal{P}(S) = \{\emptyset}$. $2^9 = 512$. 512 members.

Question 3#



(a) Using copies of the skeleton Venn diagram at right, draw four diagrams, one for each of the following:

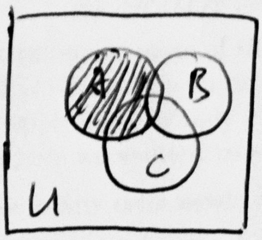
i) $A\setminus B$ ii) $(A\setminus B)\setminus C$

iii) $B\setminus C$ iv) $A\setminus(B\setminus C)$

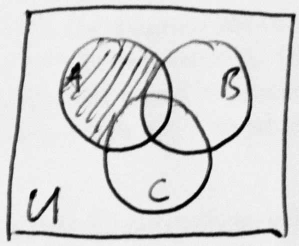
(b) Based on your answers to (a) decide whether $(A\setminus B)\setminus C = A\setminus(B\setminus C)$.

(c) Use logical equivalence or counterexample to prove your answer to (b) is correct.

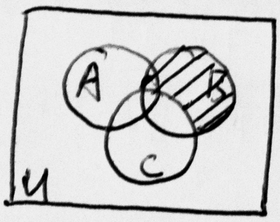
(a)



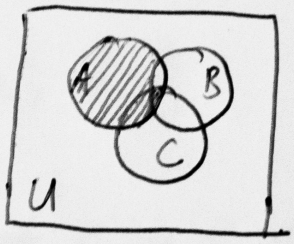
i)



ii)



iii)



iv)

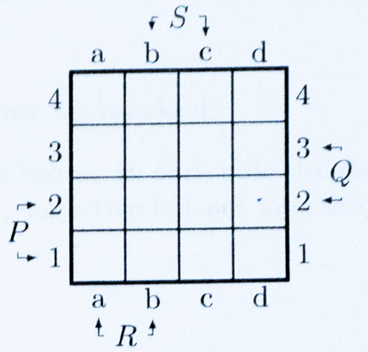
(b) No.

(c) Let $x \in A \cap C$.

~~$(A\setminus B)\setminus C$~~

$x \notin (A\setminus B)\setminus C$. ~~$x \notin A\setminus(B\setminus C)$~~. $x \in A\setminus(B\setminus C)$

$\therefore (A\setminus B)\setminus C \neq A\setminus(B\setminus C)$.



Question 4* Drawing a Venn diagram for four sets is not quite as straightforward as you might think. At right is one method. The complete square is the universal set U. Set P occupies rows 1 and 2, set Q rows 2 and 3, set R columns a and b, and set S columns b and c.

Then, for example, $P \cup Q \cup R \cup S$ occupies all the small squares except for square d4 [so square d4 represents $(P \cup Q \cup R \cup S)^c = P^c \cap Q^c \cap R^c \cap S^c$].

(a) Name or list the square(s) representing each of the follow subsets of U:

(i) $(P \cup Q)^c \cap R \cap S$ (ii) $(P \triangle R) \cap (Q \backslash S)$

(b) By marking one or more of the 16 'cells' (the little squares) in the diagram above, any subset of U defined by an expression like those in part (a) can be represented. (Such an expression involves only P, Q, R, S, set operation symbols and possibly brackets.) Now suppose we wanted to achieve a similar outcome when a fifth set T is added.

(i) How many cells would be needed?

(ii) [Challenge] Describe, or preferably draw, a diagram that displays these cells in a convenient way. (Think beyond the square!)

(a) (i) $(P \cup Q)^c \cap R \cap S = \{b4\}$

(ii) $(P \triangle R) \cap (Q \backslash S) = \{a3, d2\}$

(b) (i) 32 cells.

(ii) Assume there are n sets in total, arrange all the sets in order. For each set, use 1 and 0 to represent whether a square is occupied by the sets or not. All the situations can be represented by a n-bit binary integer.

For example, if there are 3 sets A, B and C, the situations will be:

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

If a cell is occupied by all the sets, it should be 111.

If a cell is occupied by sets A and C, it should be 101.

Question 6[†] Circle the correct answers.

For this question, working is not required and will not be marked.

For $U = \mathcal{P}(\mathbb{N})$, functions $a, b, c, d, e, f : U \to U$ are defined below. In each case decide whether the function is bijective, injective but not surjective, surjective but not injective, or neither injective nor surjective.

(a) $a : U \to U$; $a(X) = X^c$.

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER

(b) $b : U \to U$; $b(X) = 2X = \{2x : x \in X\}$.

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER

(c) $c : U \to U$; $c(X) = \frac{1}{2}X = \{\lceil \frac{x}{2} \rceil : x \in X\}$.

[Notation: $\lceil \frac{x}{2} \rceil$ denotes the "ceiling" of $\frac{x}{2}$, *i.e.* the least integer not les than $\frac{x}{2}$. *E.g* $\lceil \frac{7}{2} \rceil = 4$.]

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER

(d) $d : U \to U$; $d(X) = X \backslash \{1\}$.

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER

(e) $e : U \to U$; $e(X) = X_{+1} = \{x + 1 : x \in X\}$.

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER

(f) $f : U \to U$; $f(X) = X \triangle \{1\}$.

BIJECTIVE / INJECTIVE ONLY / SURJECTIVE ONLY / NEITHER