MATH1005/MATH6005 Semester 1 2021

Assignment 5

Workshop Details:

Number	Day	Time	Demonstrator name
	Friday		

Student Details:

ID	Surname	Given name	Preferred name

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working**.

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature:.. Date:................

This document must be submitted by 11pm on the THURSDAY following your workshop.

Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question $1^{\#}$ (*Matrix algebra*) Let A, B, C be the matrices shown:

$$A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 1 \\ 6 & 7 \end{bmatrix}$$

Compute those of the following that are defined:

(a) $A + B$

(b) $C + D$

(c) AB

(d) BA

(e) CD

(f) $\det(A)$

(g) $\det(C)$

(h) $\det(D)$

(i) C^{-1}

(j) D^{-1}

Question 2* (*Systems of Linear equations*)

(a) Express the linear system

$$\begin{array}{rcrcrcr} x & - & 3y & + & 4z & = & 11 \\ x & + & 3y & - & z & = & 1 \\ x & - & 2y & + & 3z & = & 9 \end{array}$$

in matrix form $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is a 3×3 matrix and $\mathbf{x}$, $\mathbf{b}$ are 3×1 matrices.

(b) Verify that $\mathbf{A}^{-1} = \begin{bmatrix} -7 & -1 & 9 \\ 4 & 1 & -5 \\ 5 & 1 & -6 \end{bmatrix}$.

(c) Use $\mathbf{A}^{-1}$ to solve the linear system.

Question 4[†]

For this question, working is not required and will not be marked.

For parts (a)-(c) circle the correct answers:

(a) For arbitrary finite sets A, B, C a counterexample to the claim that

$$|A \cup B \cup C| = |A + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A|$$

is $A = \{a, b, c\},\ B = \{b, c\},\ C = \{c\}$ TRUE / FALSE

(b) For cetain sets A, B, C it is known that

$|A| = |B| = |C| = 8$, $|A \cap B| = 6$, $|B \cap C| = 5$, $|C \cap A| = 4$, $|A \cap B \cap C| = 3$.

The value of $|A \cup B \cup C|$ is

10 / 11 / 12 / 13 / 14 / 15 / 16 / 17 / 18 / 19 / 20 / 21

(c) For cetain sets A, B, C it is known that

$|A| = |B| = |C| = 38$, $|A \cap B| = 12$, $|B \cap C| = 14$, $|C \cap A| = 16$, $|A \cup B \cup C| = 77$.

The value of $|A \cap B \cap C|$ is

0 / 1 / 2 / 3 / 4 / 5 / 6 / 7 / 8 / 9 / 10 / 11

In parts (d)-(f) a PIN is a string of four decimal digits, *e.g.* 2357, 0944 etc.

Write the correct numbers in the boxes:

(d) (*Complementary counting*) The number of PINs that contain at least one digit more than once (*e.g.* 2327, 0330 etc) is $\boxed{}$.

(e) (*Inclusion-exclusion*) The number of PINs that contain at least one sequence of three consecutive digits $n, n+1, n+2$ (*e.g.* 2340, 5678 etc) is $\boxed{}$.

(f) (*Inclusion-exclusion*) The number of PINs that contain at least one sequence of two consecutive digits $n, n+1$ (*e.g.* 7340, 5671 etc) is $\boxed{}$.

Question 6^+ (*The pigeon hole principle*)

(a) A total of 260 students are spread across 15 tutorials. (Each student belongs to exactly one tutorial.) Explain why each of the two statements below must be true.

(i) At least one tutorial has 18 or more students.

(ii) Five of the tutorials have 90 or more students between them.

(b) Twenty numbers, each greater than 1, are picked from the set $\{1, 2, 3, \ldots, 70\}$ of the first seventy natural numbers. Prove that amongst the twenty numbers picked it is guaranteed that two of them, say a and b have a common factor greater than 1.

Hint: If a and b have a common factor $d > 1$ then they also have a common factor p for each prime divisor of d. How many relevant primes are there?