MATH1005/MATH6005 Semester 1 2021

Assignment 10

Workshop Details:

Number	Day	Time	Demonstrator name
16B	**Friday**	2pm	Cai Yang

Student Details:

ID	Surname	Given name	Preferred name
u7235649	Zhang	Han	

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working.**

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature: Han Zhang Date: 29/05/2021

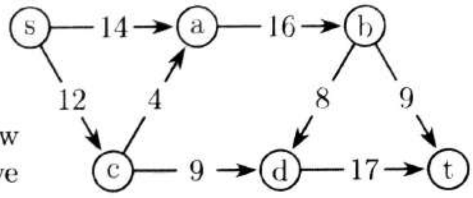
This document must be submitted by 11pm on the THURSDAY following your workshop.

Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

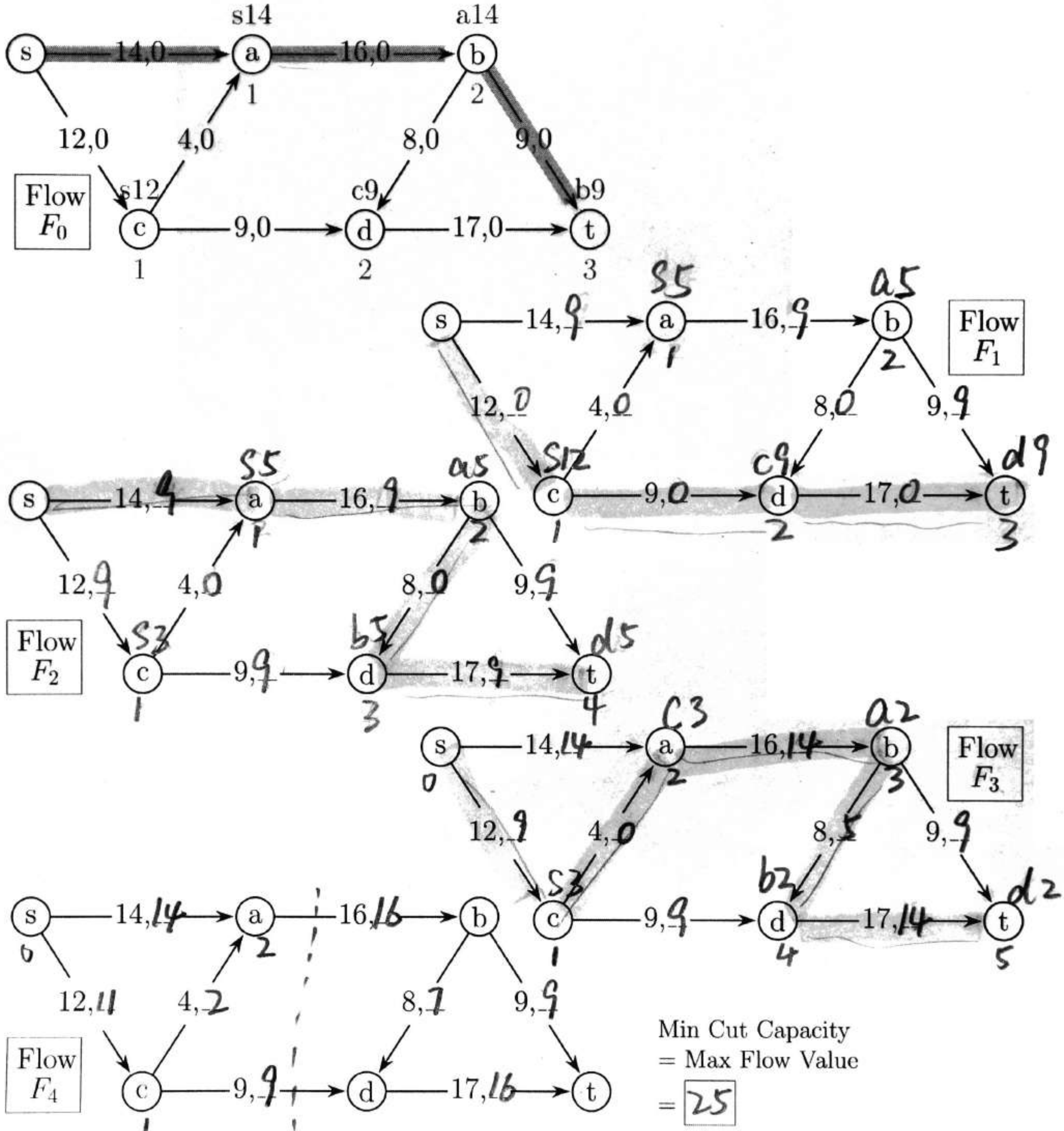
If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question 1# The diagram at right shows the capacities and directions of all links in a network with source s, target t and intermediate nodes a, b, c, d. Use the labelling algorithm to find the maximum flow through the network and how it can be achieved. Prove that your flow is maximum by finding a cut of equal value.



Here are some blank diagrams to fill in with **levels, labels** and **flows**. Use an outliner pen to mark the **incremental flow** dictated by the target's label. The first diagram is filled in as an example. You will need all diagrams. Draw a min cut on the last diagram.

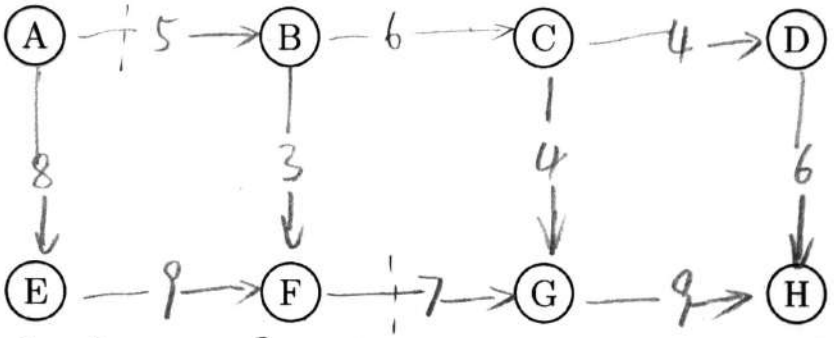


Draw a minimum cut on this diagram.

Question 2$^+$ The matrix at right shows the capacities for a network with source at vertex A and target at vertex H.

$$\begin{bmatrix} 0 & 5 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a minimum cut. Specify the partition of the vertices, the edges making up the cut, and the value of the cut.

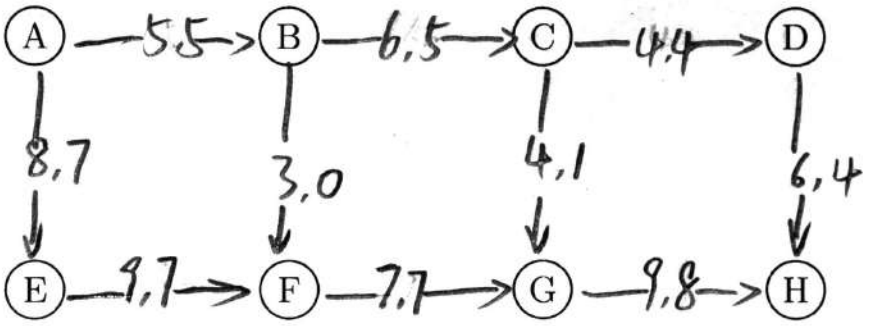


$S = \{A, E, F\}$

$T = \{B, C, D, G, H\}$

Cut edges: $\{(A,B), (F,G)\}$ value $= 5+7=12$

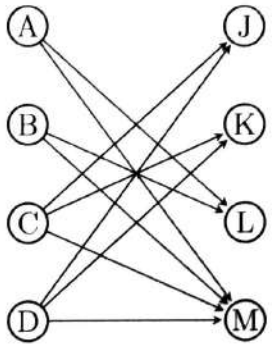
(b) Use the minimum cut to find a maximum flow, using only integer flows. There is no need to use the labelling algorithm.
Draw a diagram showing capacity and flow value for every directed edge.



Question 3† Circle the correct answers.

For this question, working is not required and will not be marked.

Brothers Alan, Brian, Carl and Dan agree that they each need a dog to encourage them to get outside more. They visit the Pound and as luck would have it, exactly four dogs are available, Joey, Katie, Lucky and Molly. The diagram at right indicates which of the dogs each of the boys said they would be happy to re-home.



(a) How many of the 4! = 24 possible allocations of dogs to brothers would satisfy all the boys preferences?

2 / 3 / (4) / more than 4

On a separate piece of paper, not to be submitted, use the matching algorithm (the one derived from the labelling algorithm for Max flow) to find a satisfactory matching. This question is mostly about the use of the algorithm, so be sure to stick strictly to the rules about levels and alphabetical order.

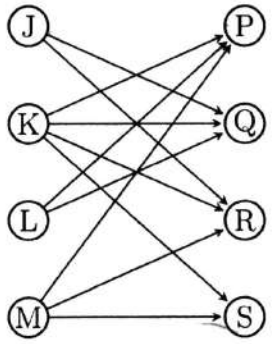
(b) The first brother to be (tentatively) allocated a dog by the algorithm is

(ALAN) / BRIAN / CARL / DAN

(c) At the completion of the allocation Molly would go home with

ALAN / (BRIAN) / CARL / DAN

Not to be out done, the brothers' partners, Jane, Kate, Lucy and Mary, also visit the Pound, but they are after a cat each. Available are Princess, Queenie, Rusty and Simba, and the girls preferences are indicated in the diagram at right.



(d) How many of the 4! = 24 possible allocations of cats to the girls would satisfy all their preferences?

2 / 3 / 4 / (more than 4)

On another piece of paper, again not to be submitted, use the same matching algorithm to find a satisfactory matching of girls to cats. Remember that this question is mostly about the use of the algorithm, so be sure to apply it strictly according to the rules.

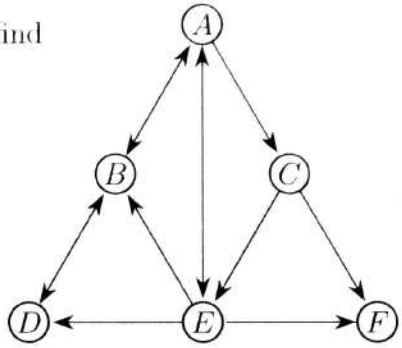
(e) The first partner to be allocated a cat by the algorithm is

(JANE) / KATE / LUCY / MARY

(f) At the completion of the allocation Kate would go home with

PRINCESS / QUEENIE / (RUSTY) / SIMBA

Question 5* For the webgraph shown at right, find the PageRanks of each page, assuming no damping. You will need to use the computer.



$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1/3 & 1/3 & 0 & 1/3 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 1/4 & 0 & 1/4 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 \end{bmatrix}$$

$$[T'-I \mid 0] = \begin{bmatrix} -1 & 1/2 & 0 & 0 & 1/4 & 1/5 & 0 \\ 1/3 & -1 & 0 & 1 & 1/4 & 1/5 & 0 \\ 1/3 & 0 & -1 & 0 & 0 & 1/5 & 0 \\ 0 & 1/2 & 0 & -1 & 1/4 & 1/5 & 0 \\ 1/3 & 0 & 1/2 & 0 & -1 & 1/5 & 0 \\ 0 & 0 & 1/2 & 0 & 1/4 & -1 & 0 \end{bmatrix}$$

$$S = \frac{35}{487} \times \begin{bmatrix} 99/35 \\ 22/5 \\ 8/7 \\ 99/35 \\ 12/7 \\ 1 \end{bmatrix} \approx \begin{bmatrix} 0.2033 \\ 0.3162 \\ 0.0821 \\ 0.2033 \\ 0.1232 \\ 0.0719 \end{bmatrix}$$

∴ Page Ranks:

A: 0.2033
B: 0.3162
C: 0.0821
D: 0.2033
E: 0.1232
F: 0.0719