MATH1005/MATH6005 Semester 1 2021

Assignment 8

Workshop Details:

Number	Day	Time	Demonstrator name
16B	**Friday**	2pm 14/05	Cai Yang

Student Details:

ID	Surname	Given name	Preferred name
47235649	Zhang	Han	

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working.**

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature: Han Zhang ..... Date: 14/05/2021

This document must be submitted by 11pm on the THURSDAY following your workshop.

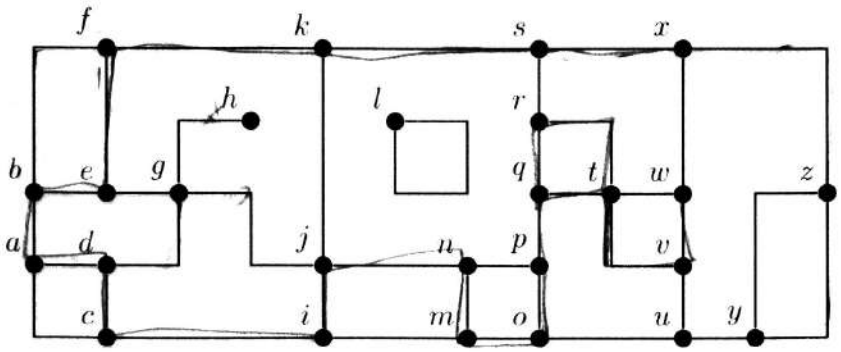
Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question 1† Circle the correct answers.

For this question, working is not required and will not be marked.



Here are twelve statements about the graph G depicted above.
By circling the appropriate word, mark each statement as either true or false.
Beware of deliberate traps!

(a) $|E(G)| = 26$. TRUE / (FALSE)

(b) Vertex h is isolated. TRUE / (FALSE)

(c) Vertex l is isolated. (TRUE) / FALSE

(d) G has vertices of degrees $1, 2, 3$ and 4. (TRUE) / FALSE

(e) G has no parallel edges. TRUE / (FALSE)

(f) G has no loops. TRUE / (FALSE)

(g) G is simple. TRUE / (FALSE)

(h) G is connected. TRUE / (FALSE)

(i) There is a path of length 4 from q to t. TRUE / (FALSE)

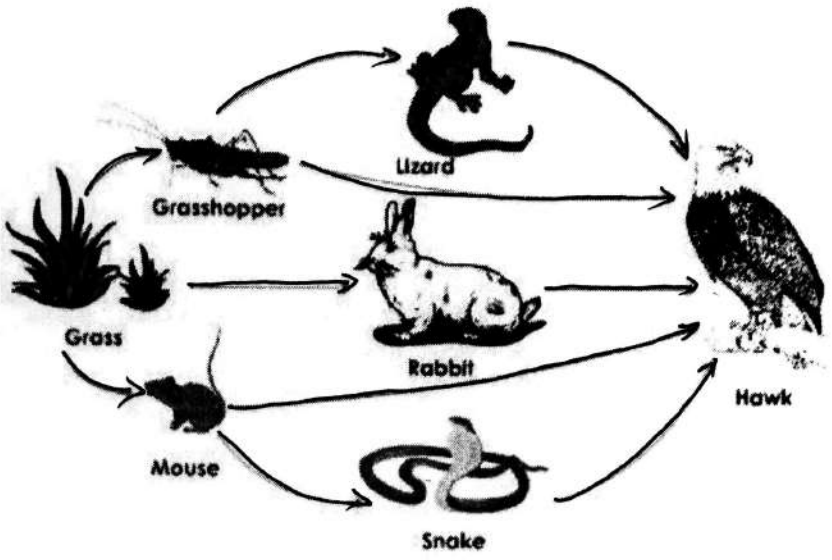
(j) G has a simple path of length 25. (TRUE) / FALSE

(k) G has a circuit of length 23. (TRUE) / FALSE

(l) G has a simple circuit of length 21. (TRUE) / FALSE

Question 3A#

In a *food web matrix* there is one column for each *consumer* under study and one row for each studied *resource* (food item) for these consumers. Some consumers are food for other consumers. A 1 in column c row r means c consumes r.
The web shown has six consumers L, GH, H, R, M and S, and six resources L, GH, G, R, M and S.



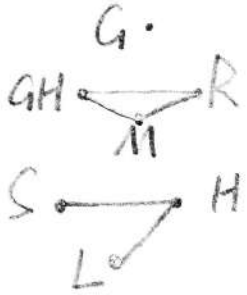
A Food Web in a Grassland Ecosystem With Five Possible Food Chains

An arrow from X to Y means X is consumed by Y.
(Arrows have opposite meanings compared to the lecture example).

(a) Write out the food web matrix for this web:[1]

$$\begin{array}{c} \\ L \\ GH \\ G \\ R \\ M \\ S \end{array} \begin{array}{c} \begin{array}{cccccc} L & GH & H & R & M & S \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

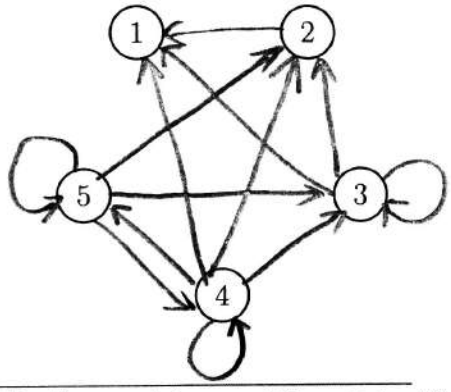
(b) Draw a niche overlap graph for this food web. [For this graph, two consumers are adjacent if and only if they share a resource.]



Question 3B#

A relation $R \subseteq \{1,2,3,4,5\}^2$ is defined by $xRy \Leftrightarrow x \leq y+3 \leq 2x$.

(a) Draw a digraph representing R.

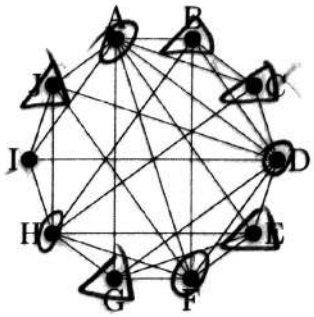


(b) Write out the adjacency matrix for R.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

[1]The diagram is borrowed from `http://biology.tutorvista.com/ecology/food-web.html`

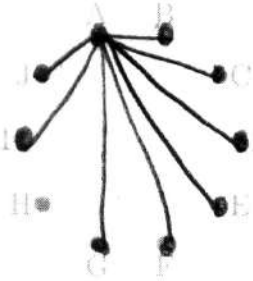
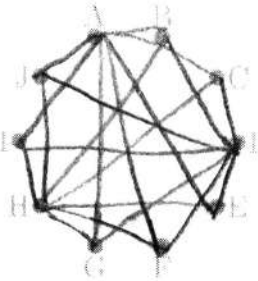
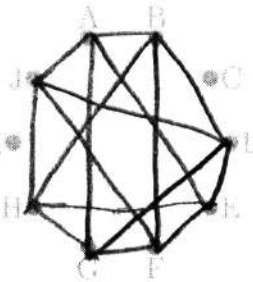
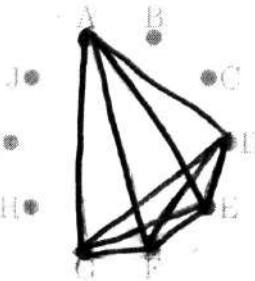
Question 4A# For the graph G at right:



(a) State the degree of each vertex.

Vertex:	A	B	C	D	E	F	G	H	I	J
Degree:	8	5	4	8	5	7	5	7	4	5

(b) Exhibit subgraphs of G isomorphic to K_{18}, K_{37}, K_{44} and K_5.

K_{18}	K_{37}	K_{44}	K_5
			

(c) Prove that G does not have a subgraph isomorphic to K_{45}.

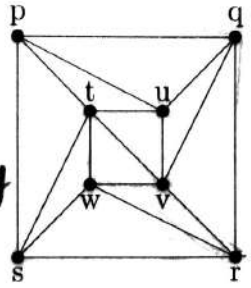
If I is in the subgraph, J.A.D.H should be included in the other set and there should be no edges between them, then J has only 2 edges left, which should be 5. So I should not in the subgraph. Then J has 4 degree, so H.F.D.A should be in one set and J.G.E.B.C in the other set. For C it should be 4 degree to the other set but it is only 2. so G does not have a subgraph isomorphic to K_{45}.

Question 4B# Draw a simple graph with six vertices of degrees 0, 1, 2, 3, 4 and 5, or say why you believe this to be impossible.

Impossible.

In any graph there should be an even number of vertices of odd degree, but in this graph there are 3.

Question 6⁺ For the graph J at right:



(a) Prove or disprove that J has an Euler circuit.

A connected graph has an Euler circuit only when each of its vertices has even degree, but v and t have 5 degree.

So J has no Euler circuit.

(b) Prove or disprove that J has an Euler path.
(By convention, Euler paths are non-closed.)

A connected graph has an Euler path only when it has 2 vertices of odd degree. For J, only v and t have odd degree, so J has an Euler path.

(c) Prove or disprove that J has a Hamilton circuit.

J has a Hamilton circuit:

P → S → r → q → u → v → w → t → P

(d) Prove or disprove that J has a Hamilton path.
(By convention, Hamilton paths are non-closed.)

J has a Hamilton path:

P → S → r → q → u → v → t → w