COMP2610/COMP6261 – Information Theory

Tutorial 1: Elementary Probability (Soln.)

Week 1, Semester 2, 2021

1. A spinner is divided into 5 equal sections, with sections labelled 1, 2, 3, 4 and 5. Compute the probability of:

Probability of event A [P(A)] = $\frac{Number\ of\ favourable\ outcomes\ (n(A))}{Total\ number\ of\ possible\ outcomes\ (n(S))}$

Sample space (S) = {1,2,3,4,5}

So, P(1) = P(2) = P(3) = P(4) = P(5) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes}$ = 1/5

a) spinning a 4 on the spinner.

P (4) = **1/5**

b) spinning an even number on the spinner.

Event of spinning an even number on the spinner (E1) = {2,4}

P (even number) = P(2) + P(4) = 1/5 + 1/5 = **2/5**

c) Spinning a prime number on the spinner.

Event of spinning a prime number on the spinner (E2) = {2,3,5}

P (prime number) = P(2) + P(3) + P(5) = 1/5 + 1/5 + 1/5 = **3/5**

2. Let us assume that ACT number plates have three letters followed by three numbers (e.g., YOA077). What will be the probability that a randomly chosen number plate will have an ACT with the number ending in a 7 (ACT##7)?

Probability of event A [P(A)] = $\frac{Number\ of\ favourable\ outcomes\ (n(A))}{Total\ number\ of\ possible\ outcomes\ (n(S))}$

Sample space for integers (E1) = {0,1,2,3,4,5,6,7,8,9}

⇨ P(0) = P(1) = P(2)……….. = P(9) = 1 / 10

And Sample space for alphabets (E2) = {A,B,C,D,E,F,G,H…………………………..,X,Y,Z}

⇨ P(A) = P(B) = P(C) = …………………………….. = P(Z) = 1/26

So basically, we have 6 positions (L_5 , L_4 , L_3, L_2, L_1, L_0) out of which four is fixed as ACT_ _7.

=> L_2 and L_1 can take any integer between 0 to 9.

Hence probability that a randomly chosen number plate will have an ACT with the number ending in a 7 will be P (E) = $(\frac{1}{26}) * (\frac{1}{26}) * (\frac{1}{26}) * (\frac{10}{10}) * \frac{10}{10}) * (\frac{1}{10}) = \frac{100}{(26)^3 * (10)^3} = \mathbf{\frac{1}{175760}}$

3. ACT Govt. plan to enforce speed limits during the morning rush hour on four different routes into the city. The traps on routes A, B, C, and D are operated 40% , 30%, 20%, and 30% of the time, respectively. Arya always speeds to work, and she has probability 0.2, 0.1, 0.5, and 0.2 of using those routes. Compute the probability of:

According to question,

P(A) = 0.2	P(B) = 0.1	P(C) = 0.5	P(D) = 0.2
P(traps_A) = 0.4	P(traps_B) = 0.3	P(traps_C) = 0.2	P(traps_D) = 0.3

a) Arya getting a ticket on any one morning.

To calculate Arya getting tickets on any one morning we need to sum the probabilities of getting a ticket by the frequencies with which she travels each route (P(event));

⇨ P(A) * P(traps_A) + P(B) * P(traps_B) + P(C) * P(traps_C) + P(D) * P(traps_D)

⇨ 0.2 * 0.4 + 0.1 * 0.3 + 0.5 * 0.2 + 0.2 * 0.3

⇨ 0.08 + 0.03 + 0.10 + 0.06

⇨ **0.27**

b) Arya will go five mornings without the tickets.

Since sum of probabilities of an event and its complementary event is 1 i.e., P(event) + $P(event)^C$ = 1

⇨ Probability that Arya will go without the tickets in any morning ($P(event)^C$) = 1 - P(event)

⇨ 1 - 0.27

⇨ **0.73**

⇨ **Probability that Arya will go Five mornings without the ticket will be = $(0.73)^5$ = 0.2073**

4. In an urn there are 5 blue, 3 red, and 2 yellow marbles. If you draw 3 marbles, what is the probability that less than 2 will be red if:

Blue	Red	Yellow	Total
5	3	2	10

Probability of blue marble (P (B)) = 5/10 = 0.5

Probability of red marble (P (R)) = 3/10 = 0.3

Probability of yellow marble (P (Y)) = 2/10 = 0.2

If three marbles are drawn simultaneously , then probability of drawing less than 2 red marbles when

a) the marbles are drawn with replacement.

Method 1 : Conventional approach

The probabilities are fixed. Hence the probability of no red at all is P(not R) = 1 – P(R) = 1 – 0.3 = 0.7.

⇨ Probability that no red will be drawn in all three drawing of marble, P (event : R = 0) = P(not R)^3

⇨ **P (event : R = 0)** = $(0.7)^3$ = **0.343**

Since there are three ways to get one red - in the first draw, second draw or third draw.

In all three cases, the probability will be the same. Hence,

⇨ **P (event : R = 1)** = 3 * (P(R)*P(not R)* P(not R)) = 3 * (0.3 * 0.7 * 0.7) = 3 * (0.147) = **0.441**

⇨ **P (event : R < 2) =** P (event : R = 0) + P (event : R = 1) **=** 0.343 + 0.441 **= 0.784**

Method 2 : Binomial distribution

$$P_X = {}^nC_x \, p^x \, q^{n-x}$$

Where, **n** = number of trials
p = probability of success on a single trial
q = probability of failure on a single trial = 1 – p

P (event : R < 2) = P (event : R = 0) + P (event : R = 1) = ${}^3C_0\, (P(R))^0\, (1-P(R))^{3-0} + {}^3C_1\, (P(R))^1\, (1-P(R))^{3-1}$

⇨ **P (event : R < 2) =** ${}^3C_0\, (0.3)^0\, (0.7)^3 + {}^3C_1\, (0.3)^1\, (0.7)^2 = (0.7)^3$ + 3 * (0.3 * 0.7 * 0.7) = 0.343 + 0.441

⇨ **P (event : R < 2) = 0.784**

b) the marbles are drawn without replacement.

So, 3 of the 10 marbles are red. The probability of drawing less than two is the sum of the probabilities of drawing either 1 or none:

$$P\,(\text{Red} < 2) = \frac{(\text{No.of ways to select(0 red and 3 other marbles)} + \text{No.of ways to select(1 red and 2 other marbles)})}{\text{No.of ways to select 3 marbles out of 10}}$$

$$= ({}^3C_0)\,({}^7C_3) + ({}^3C_1)\,({}^7C_2)\,/\,({}^{10}C_3) = \frac{1*(35)+3*21}{120} = \frac{98}{120} = \mathbf{\frac{49}{60}}$$

5. Nick will miss an important Cricket match while taking his Information theory exam, so he sets both his VCRs to record it. The first VCR has 70% chances to successfully record the match and the second VCR has 60% chances to successfully record the match . What is the probability that he gets home after the exam and finds? (Note: Here we assume that events A and B are independent, so with P(A) = 0.7 and P(B) = 0.6 and their set complements A^c and B^c occurring with probabilities 0.3 and 0.4 respectively).

According to question,

Probability of VCR 1 recording successfully (P(A)) = 0.7

⇨ Probability of VCR 1 not recording successfully ($P(A^c)$) = 1 - P(A) = 1 - 0.7 = 0.3

Similarly, Probability of VCR 2 recording successfully (P(B)) = 0.6

⇨ Probability of VCR 2 not recording successfully ($P(B^c)$) = 1 - P(B) = 1 - 0.6 = 0.4

Let E1 be the event when no copies of the Cricket match will be available,

E2 be the event when 1 copy of the Cricket match will be available, and

E3 be the event when two copies of the Cricket match will be available.

a) No copies of the Cricket match?

P (E1) = $P(A^c$ and $B^c)$ = $P(A^c)P(B^c)$ = (0.3) * (0.4) = **0.12**

b) One copy of the Cricket match?

Here we need to account that any one VCR (out of the two) needs to record. So,

P(E2) = P(A and B^c) + P(A^c and B) = P(A) * P(B^c) + P(A^c) * P(B) = 0.7 * 0.4 + 0.3 * 0.6

⇨ P(E2) = 0.28 + 0.18

⇨ P(E2) = **0.46**

c) Two copies of the Cricket match?

Here we need to account that both VCRs are recording simultaneously. So,

P(E3) = P (A and B) = P (A) * P (B)

⇨ P(E3) = 0.7 * 0.6 = **0.42**