MATH1005/MATH6005: Discrete Mathematical Models

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Section A: The language of mathematics and computer science

Bringing it all together

Let’s start with some practice

Proving set theoretic identities ... by unpacking and repacking the logic

Prove that, for all subsets A, B of a universal set U,

$$(A \cap B)^c = A^c \cup B^c.$$

Proving that a function is injective/surjective/bijective

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = 5x + 13.$$

Prove that f is bijective.

Proving/disproving

Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = 10 - 2x^2.$$

Determine whether or not f is bijective.

Capturing an idea of 'same size'

Let U be a universal set, and let A, B be subsets of U. We say that $A \sim B$ when there exists a bijection $f : A \to B$.

EXAMPLE: $\{\text{cat}, \text{dog}, \text{chicken}\} \sim \{50, 60, 25\}$.

EXAMPLE: $\{\text{cat}, \text{dog}, \text{chicken}\} \not\sim \{50, 60, 25, 110\}$.

Some awesome results (due to Cantor (1845-1918))

$$\mathbb{Z}^+ \sim \mathbb{N}$$

$$\mathbb{Z}^+ \sim \mathbb{Z}$$

$$\mathbb{Z}^+ \sim \mathbb{Q}$$

$$\mathbb{Z}^+ \not\sim \mathbb{R}$$

No Bijection:
Not surjective

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Some important definitions

Definition: A set A is **countable** when there exists a bijection from A to a subset of $\mathbb{Z}^+$.

Definition: A set A is **countably infinite** when there exists a bijection from A to $\mathbb{Z}^+$.

Definition: A set A is **uncountable** when it is not countable.

What is this course about again? (see lecture 1)

Discrete mathematical models are abstract representations of processes and objects, the steps or units of which can be indexed by the non-negative integers. In particular, we avoid continua (like the open interval $(0, 1)$).

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Can be restated

Discrete mathematical models are abstract representations of processes and objects, the steps or units of which form a countable set.