MATH1005/MATH6005 Semester 1 2021

Assignment 9

Workshop Details:

Number	Day	Time	Demonstrator name
	Friday		

Student Details:

ID	Surname	Given name	Preferred name

Instructions:

This assignment has four questions. Write your solutions in the spaces provided. Hand-writing is preferable to typesetting unless you are fast and accurate with LaTeX, and even then typesetting will take you longer. Also, be aware that typesetting will not be allowed on exams.

Except for multiple-choice questions, or where answer boxes are provided (one question of this type each week), **show all working**.

Question numbers indicate the corresponding questions from the workshop. Since the workshop has six questions, questions on this assignment may not be sequentially numbered.

Declaration:

I declare that while I may have discussed some or all of the questions in this assignment with other people, the write-up of my answers herein is entirely my own work. I have not copied or modified the written-out answers of anyone else, nor allowed mine to be so used.

Signature:.. Date:................

This document must be submitted by 11pm on the THURSDAY following your workshop.

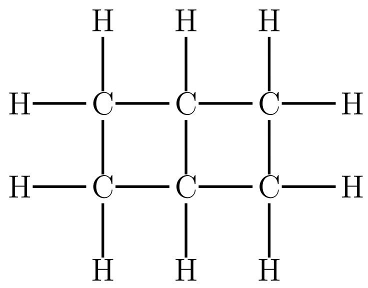
Sign, date, then scan this completed document (5 pages) and save as a pdf file with name format uXXXXXXXAssXX.pdf (*e.g.* u6543210Ass01.pdf).
Upload the file via the link from which you downloaded this document.

If copying is detected, and/or the document is not signed, no marks will be awarded.

This document has five pages in total.

Question 2# At right is a graph diagram showing the structure of a molecule of an isomer of an unsaturated hydrocarbon, C_6H_{10}, called bicyclo[2,2,0]hexane.

The molecule can be saturated to form various isomers of C_6H_{14} by breaking suitable C-C bonds and adjoining H atoms at the breaks.



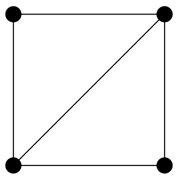
(a) How many different (non-isomorphic) isomers of C_6H_{14} can be made in this way? Draw a structure diagram for each and show which C-C bonds you broke to make it. Hint: One isomer is obtained by breaking the top two horizontal C-C bonds in the diagram. But note that breaking the bottom two horizontal C-C bonds does not create a different isomer because the relevant trees are isomorphic.

(b) Are there any isomers of C_6H_{14} that cannot be made in this way from bicyclo[2,2,0]hexane? Draw a structure diagram for any such isomer.

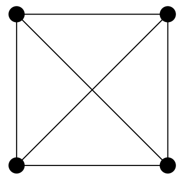
(c) With the help of a ChemSpider (`www.chemspider.com`) or other reference, give the full chemical name of each of the molecules whose tree you drew for (a). Write the name directly under the relevant diagram.

Question 4† Write the correct integer values in the boxes.

For this question, working is not required and will not be marked.

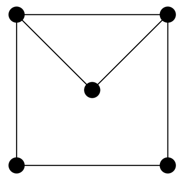


This question is about the number of spanning trees of a graph. In a lecture we used complementary counting to calculate that the graph depicted at left has exactly eight spanning trees. By adding just one more edge to this graph we arrive at the complete graph K_4 depicted at right.



A spanning tree has $n-1 = 3$ edges and so we first count the total number of sets of three edges; our extra edge takes this from $\binom{5}{3}$ to $\binom{6}{3}$. Now we count the number of sets to discard because they do not create a tree. The extra edge means there are a couple more of these. This leads to:

(a) The number of spanning trees of K_4 is $\boxed{}$.

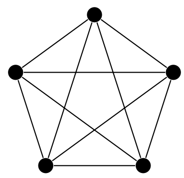


Moving up to $n = 5$ vertices, we next consider the graph G depicted at right, which has five vertices and six edges. Again using complementary counting we need to find two numbers:

(b) The total number of sets of edges we need to consider is $\boxed{}$ [number, not formula]

(c) The number of these sets that do not provide a tree is $\boxed{}$.

By subtracting answer (c) from answer (b) you will get the number of spanning trees for G. You can check your answers by drawing and counting all these trees.



Finally consider the case of the complete graph K_5 depicted at right. It again has five vertices, but now ten edges, so there will be a lot more spanning trees. Counting as before:

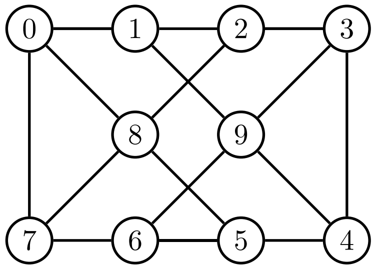
(d) The total number of sets of edges we need to consider is $\boxed{}$ [number, not formula]

(e) The number of these sets which need to be discarded because they form **disconnected** subgraphs (and hence cannot be trees) is $\boxed{}$.

(f) The number of these sets which need to be discarded because they form **connected but not circuit-free** subgraphs (and hence cannot be trees) is $\boxed{}$. Hint: Two types of circuits are involved. There are a lot.

By subtracting answers (e) and (f) from answer (d) you will get the number of spanning trees for K_5. You might spot a pattern by comparing the corresponding values for K_4 and K_3. The simple formula that applies, and especially the proof of that formula, is one of the jewels of graph theory. Try Googling it.

Question 5* This question introduces the idea of using a travelling salesman algorithm to search for a Hamilton circuit in any simple graph.



(a) Find a Hamilton circuit for the graph G indicated by the diagram at right. Do this 'by eye', without using any particular algorithm. Answer by drawing heavy lines over each edge on your circuit. There are many correct answers.

(b) TSP algorithms usually work on a complete weighted graph. One way to transform G into such a graph $G^\star$ is as shown.

$$\begin{aligned} V(G^\star) &= V(G) \\ E(G^\star) &= \{\ \{u,v\}\ :\ u,v \in V(G),\ u \neq v\ \} \\ \mathrm{wt}(\{u,v\}) &= \text{length of shortest path in } G \text{ from } u \text{ to } v \\ & \text{(length as defined for an un-weighted graph)} \end{aligned}$$

(i) How many weight-1 edges has $G^\star$?

(ii) How many weight-3 edges has $G^\star$?

(iii) How many edges has $G^\star$ in total?

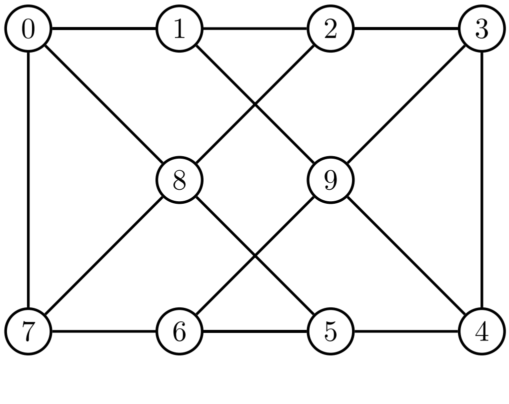
(c) Suppose the Nearest Neighbour algorithm were applied to $G^\star$. How could you tell from the reported tour weight whether a Hamilton circuit of G had been found?

(d) When applying Nearest Neighbour to $G^\star$, there will be situations where the current vertex has more than one nearest neighbour. We could then just pick one of them at random, but for grading purposes it is better to have a rule. Please use this:

> If the current vertex u has two or more equally 'close' available neighbouring vertices choose the one with the lowest compass bearing relative to u.

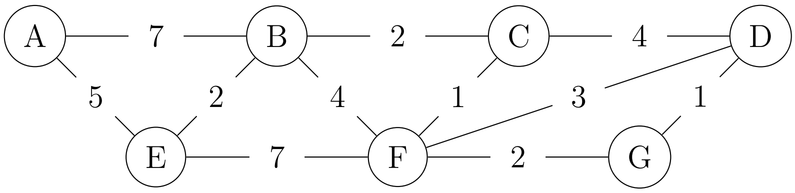
(The compass bearing of a vertex v relative to u is the angle made by the the line uv to line uy where y is an imaginary point directly above u, and angles are measured in a clockwise direction. For example vertex 0 of G has bearing 315° relative to vertex 8.)

Starting at the vertex determined by the last digit of your ANU ID, apply the Nearest Neighbour algorithm to $G^\star$. Draw your circuit on the enlarged diagram below. If necessary, used curved lines for 'virtual edges' (*i.e.* edges of $G^\star$ that are not edges of G) to avoid these edges passing through other vertices.



Tour weight =

Question 6* Let G be the weighted graph



(a) Use Dijkstra's algorithm to find the shortest path from A to G. You can do all the work on a single diagram, but, to show that you have used the algorithm correctly, if an annotation needs updating do not erase it — just put a line through it and write the new annotation above that.

(b) In what order are the vertices added to the tree?

(c) Notice that the algorithm does not, in this instance, generate a spanning tree. Which vertex or vertices are missing?

(d) Extend the use of the algorithm until the shortest distance from A to each other vertex is established, and a spanning tree is thereby generated. Draw this tree.

(e) Is the spanning tree you have generated using Dijkstra's algorithm a minimal spanning tree? Justify your answer.