INSTRUCTIONS

Before your workshop Revise the material from Week 2 and 3 (Set, Relations and Functions). Make a summary and learn the vocabulary. Then make your best attempt at Problems 1, 3 and 5 on the worksheet. Write your attempts neatly, on the worksheet or a separate piece of paper, and bring them to the workshop. At this stage, the correctness of your solution is not important. What is important is that you try your best, and write what you do know so that it is evident that you tried your best. You must take your attempts at Problems 1, 3 and 5 to the workshop. You may take along any summaries or other notes you have made about the material too. You should expect to spend about two hours revising material and making your attempts at Problems 1, 3 and 5.

During your workshop At the start of the workshop, your demonstrator will ask to see your attempts at Problems 1, 3 and 5. While your demonstrator looks these over, you will work collaboratively with your classmates on problems 2, 4 and 6. Your demonstrator will guide you through the workshop, and help you if you are stuck. The demonstrator and class members may comment and correct your work during the workshop. Your goal is to leave the workshop feeling like you understand how to solve as much of the worksheet as possible. The aim is for a collaborative mathematical experience.

You make take notes during a workshop for later reference. Some of you will prefer to take photos, rather than write notes. We need some rules for this. You may take photos of work written on the board, but only after the author of the work has given their explicit permission. You may not take photos of people during the workshops.

For MATH1005 students only, workshop participation is worth 10% of your overall grade. For a maximum participation mark you will need to bring satisfactory attempts at problems 1, 3 and 5 to class (where satisfactory does not have to mean correct) and participate productivity throughout the workshop. If illness or other circumstances prevent you attending one or two workshops, don't worry because we will only use the best eight out of ten workshop participation scores.

After your worshop At 8pm after your workshop your assignment document will become available on Wattle. This will contain several questions closely related to the questions on this worksheet. When you have worked out answers write up the solutions neatly and carefully in the spaces provided in the assignment document, using good English spelling and grammar and, most importantly, showing all necessary steps of calculations and reasoning (except on multiple choice questions). You have 6 days to submit your assignment (online). Your demonstrator will then grade it and you will be able to see marks and possible comments online. These assignments will contribute 10% to your overall grade.

Assignments will vary from one day to another, so you must download your own assignment, not rely on someone else who may have a workshop on a different day. Also note that late submissions will not be accepted, and the assignment document itself will disappear from Wattle after the submission deadline.

MY NAME IS: MY u NUMBER IS:

This week's questions mostly just check your understanding of notation and terminology.

Question 1 Recall that a natural number n is *prime* provided that $n > 1$ and n has no positive divisors other than 1 and itself (e.g. 17 is prime but 18 is not). For universal set $U = \{n \in \mathbb{N} : n \leq 15\}$ define sets P and H as follows:

$$P = \{n \in U : n \text{ is prime}\} \qquad H = \{n \in U : n > 10\}.$$

How many members have each of the following sets? Show your enumeration/calculation.

(a) P^c (b) $P \cap H$ (c) $P \cup H$

(d) $P \backslash H$ (e) $P \triangle H$ (f) $\mathcal{P}(P)$ [the power set of P].

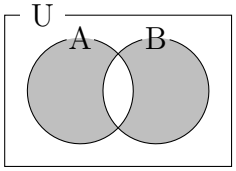
Question 2 Recall that a natural number n is *composite* if it is product of two smaller natural numbers (e.g. 18 is composite because $18 = 6 \times 3$). With U and P as in Question 1 and with $C = \{n \in U : n \text{ is composite}\}$, decide the truth or falsity of each of the following. Beware of deliberate traps! Briefly justify your answers.

(a) $\{1, 2, 3\} \subseteq P$ (b) $\{8, 9, 10\} \in C$ (c) $P \cap C = \emptyset$

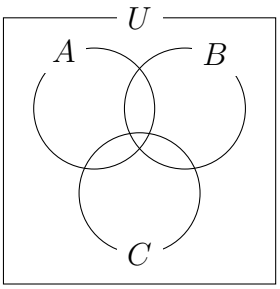
(d) $\{11, 12\} \in P \times C$ (e) $P \subset P \times C$ (f) $\{P, C\}$ is a partition of U

A **Venn diagram** is a graphical reprsentation of sets, subsets and elements. Inside a rectangle for the universal set are one or more circles (or shapes) representing sets. Dots inside the circles (or shapes) represent elements. A subset of the universe, and its relationship to the sets represented by circles, can be indicated by shading various regions on the Venn diagram.

For example, this Venn diagram represents the symmetric difference of A and B.



Question 3



(a) Using copies of the skeleton Venn diagram at right, draw five diagrams, one for each of the following:

i) $A \cup B$ ii) $(A \cup B) \backslash C$

iii) $A \backslash C$ iv) $B \backslash C$ v) $(A \backslash C) \cup (B \backslash C)$

(b) Based on your answers to (a) decide whether $(A \cup B) \backslash C = (A \backslash C) \cup (B \backslash C)$.

(c) Use a logical equivalence to prove your answer to (b).

Question 4

(a) Draw a Venn diagram showing four sets A, B, C, D in the most general configuration for which $A \subseteq C$ and $B \subseteq D$.

(b) By referring to your answer to (a), decide on the truth or falsity of the claim that, for all sets A, B, C, D,

$$[(A \subseteq C) \wedge (B \subseteq D)] \Rightarrow [(A \triangle B) \subseteq (C \triangle D)].$$

(c) [Challenge] Prove your answer to (b).

Question 5

	Location		Approximate air distance (000km) to:			
City	degrees North of Equator	degrees East of Gr'nwich	Auckland	Chennai	Marrakesh	Winnipeg
Auckland	-39	175	0	11	19	13
Chennai	13	80	11	0	9	13
Marrakesh	32	-8	19	9	0	7
Winnipeg	50	-97 ($\equiv 263$)	13	13	7	0

Relations R_1, R_2 and R_3 on the set of four cities tabulated above are defined below. Draw a directed graph for each relation (use arrows between 'A', 'C', 'M' and 'W') and say whether or not the relation is a function. If not a function, say why not.

(a) $xR_1y \Leftrightarrow x$ is more than 10 000km from y.

(b) $xR_2y \Leftrightarrow x$ is South of y.

(c) $xR_3y \Leftrightarrow x$ is between 80° and 100° East of y.

Question 6 Functions $a, b, c, d : \mathbb{N} \to \mathbb{N}$ are defined by the rules below. In each case decide whether the function is injective (one-to-one), surjective (onto), neither or both (bijective). Justify your answers.

$$a(n) = \begin{cases} 2n & \text{if } n \text{ is odd} \\ 3n & \text{if } n \text{ is even} \end{cases} \qquad b(n) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases} \qquad c(n) = n^2 \qquad d(n) = \lfloor \sqrt{n} \rfloor$$

For d, $\lfloor x \rfloor$ denotes the 'floor' of x, the greatest integer not greater than x. E.g. $\lfloor \sqrt{7} \rfloor = 2$.