

Topic: Low level Vision-1: Image Deblur/Super-resolution

Acknowledgement: The slides below is based on AZ's lecture note at Oxford VGG group. They are used here for ENGN8501 classroom teaching only. Please do not re-distribute to elsewhere.

Outline

- **Image degradations**
 - motion blur, focus blur, resolution
- The inverse filter
- The Wiener filter
- MAP formulation

Image Degradation



- original



- optical blur



- motion blur



- spatial quantization (discrete pixels)



- additive intensity noise

Overview – Deconvolution

The objective is to restore a degraded image to its original form.

An observed image can often be modelled as:

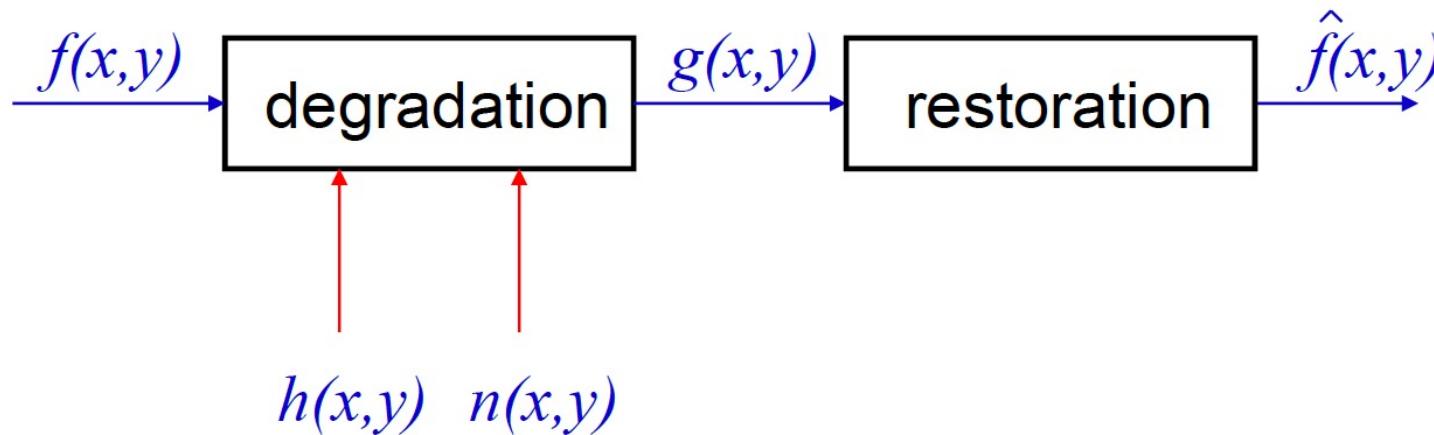
$$g(x, y) = \int \int h(x - x', y - y') f(x', y') dx' dy' + n(x, y)$$

where the integral is a convolution, h is the point spread function of the imaging system, and n is additive noise.

The objective of image restoration in this case is to estimate the original image f from the observed degraded image g .

Definitions

- $f(x,y)$ – image before degradation, ‘true image’
- $g(x,y)$ – image after degradation, ‘observed image’
- $h(x,y)$ – degradation filter
- $\hat{f}(x,y)$ – estimate of $f(x,y)$ computed from $g(x,y)$
- $n(x,y)$ – additive noise

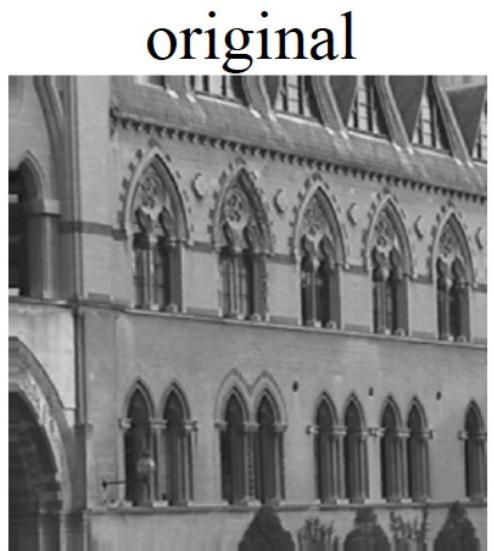


$$g(x,y) = h(x,y) * f(x,y) + n(x,y) \Leftrightarrow G(u,v) = H(u,v) F(u,v) + N(u,v)$$

Degradation model

Model degradation as a convolution with a linear, shift invariant, filter $h(x,y)$

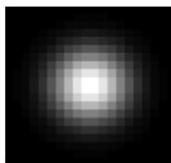
- Example: for out of focus blurring, model $h(x,y)$ as a Gaussian



$$f(x,y)$$

*

$$\frac{1}{2\pi\sigma^2}e^{-r^2/2\sigma^2}$$



$$h(x,y)$$

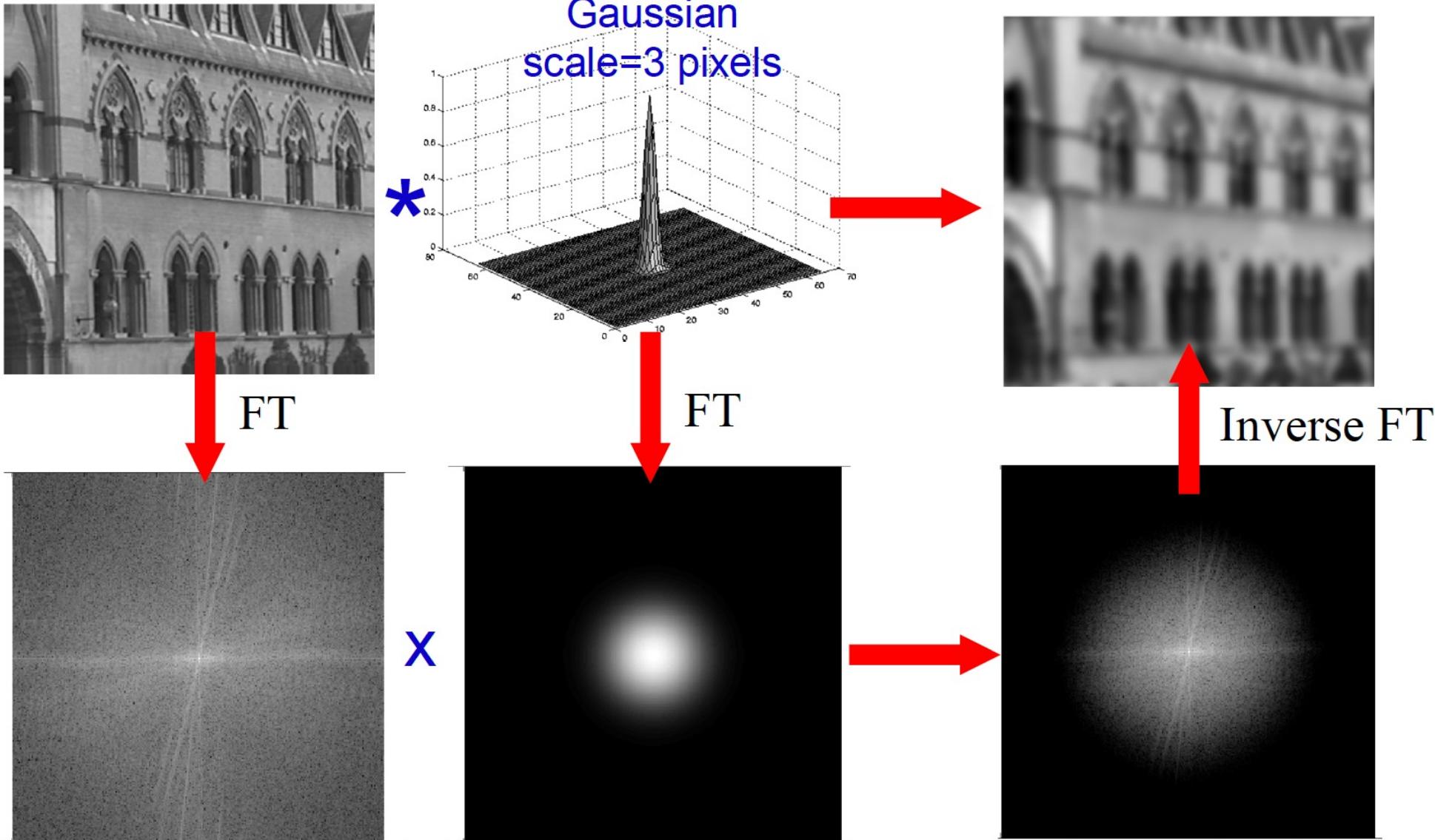


$$g(x,y)$$

i.e. : $g(x,y) = h(x,y) * f(x,y)$

$h(x,y)$ is the impulse response or point spread function of the imaging system

The challenge: loss of information and noise



Blurring acts as a low pass filter and attenuates higher spatial frequencies

The inverse filter

Start from the generative model

$$g(x,y) = h(x,y)*f(x,y) + n(x,y) \Leftrightarrow G(u,v) = H(u,v) F(u,v) + N(u,v)$$

and for the moment ignore $n(x,y)$, then an estimate of $f(x,y)$ is obtained from

$$\hat{F}(u,v) = G(u,v) / H(u,v)$$

Restoration with an inverse filter



Example : Deblurring (deconvolution)

Image blurred with Gaussian point spread function

$$h(x,y) = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

$n(x,y)$ = Normal distribution, mean zero



$f(x,y)$



blur $\sigma = 1.0$ pixels

noise $\sigma = 0.3$ grey levels

Restoration with an inverse filter

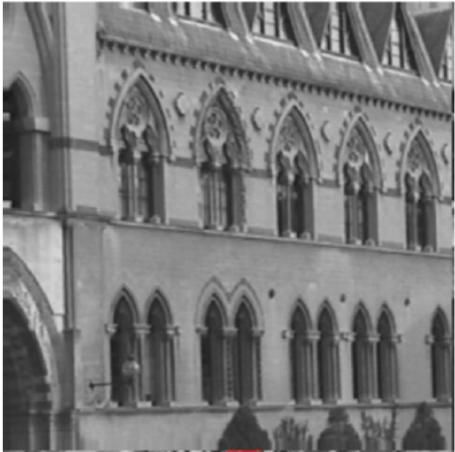
$\hat{F}(u,v) = G(u,v) / H(u,v)$ where $H(u,v)$, is the FT of the Gaussian

Deblurring with an inverse filter

noise $\sigma = 0.3$ grey levels

$$\hat{F}(u,v) = G(u,v) / H(u,v)$$

blur $\sigma = 0.5$ pixels



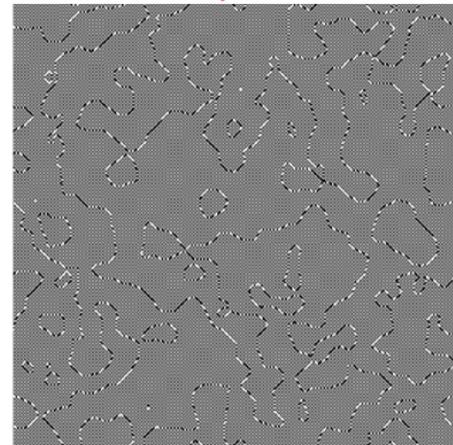
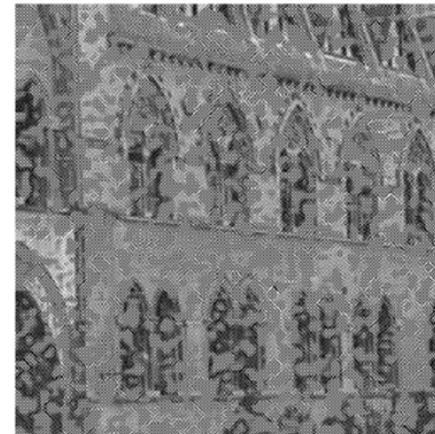
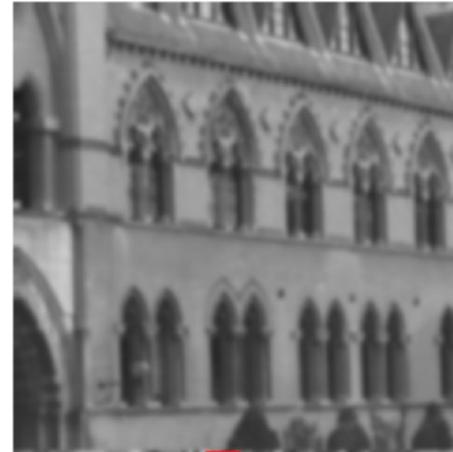
$$g(x,y)$$

blur $\sigma = 1.0$ pixels



$$\hat{f}(x,y)$$

blur $\sigma = 1.5$ pixels

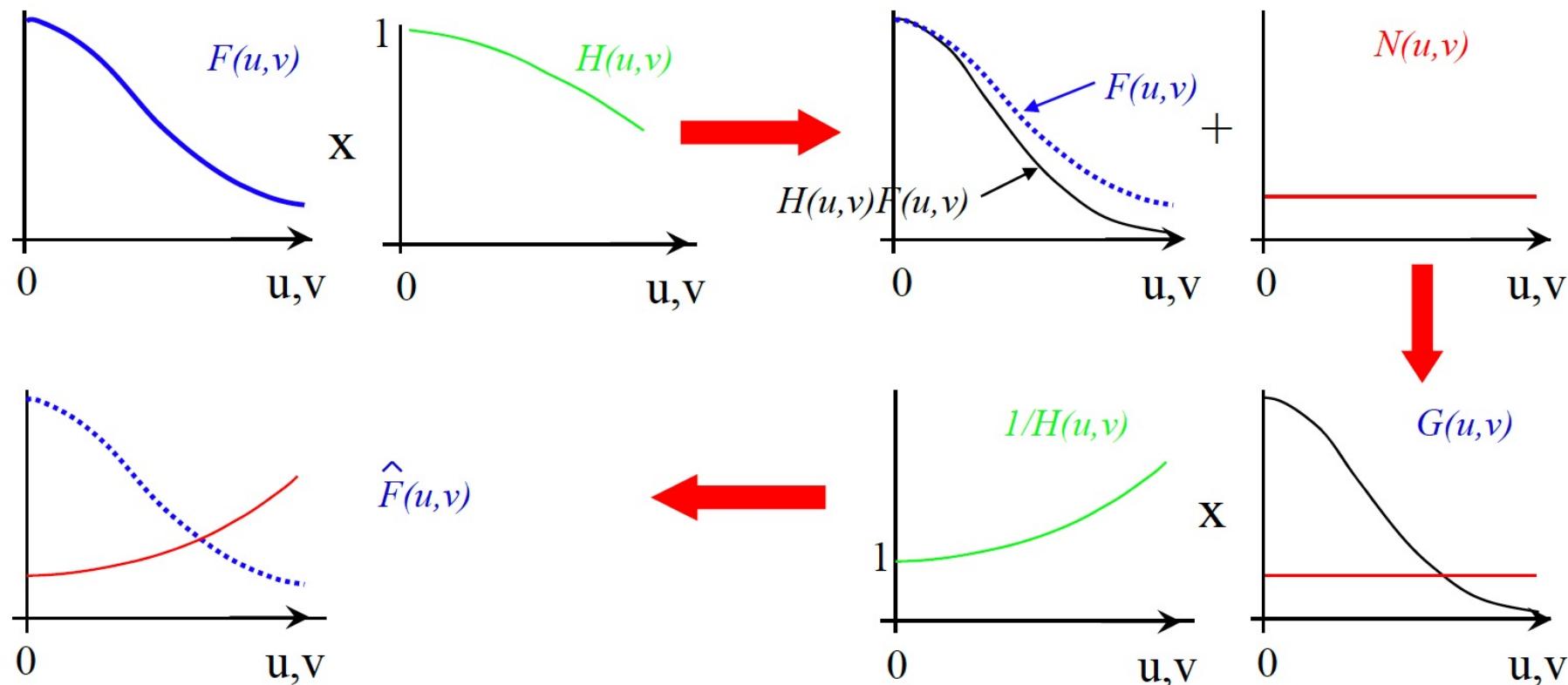


The problem of noise amplification

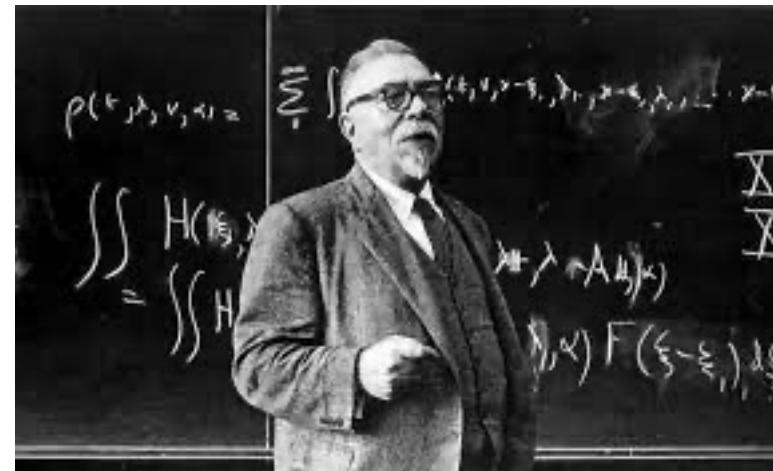
$$G(u,v) = H(u,v) F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = G(u,v) / H(u,v) = F(u,v) + N(u,v) / H(u,v)$$

Schematically ...



The Wiener filter



The Wiener filter

$$\hat{F}(u, v) = W(u, v) G(u, v)$$

$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K(u, v)}$$

where

$$K(u, v) = S_\eta(u, v)/S_f(u, v)$$

$S_f(u, v) = |F(u, v)|^2$ power spectral density of $f(x, y)$

$S_\eta(u, v) = |N(u, v)|^2$ power spectral density of $\eta(x, y)$

Frequency behaviour

$$\hat{F}(u, v) = W(u, v) G(u, v)$$

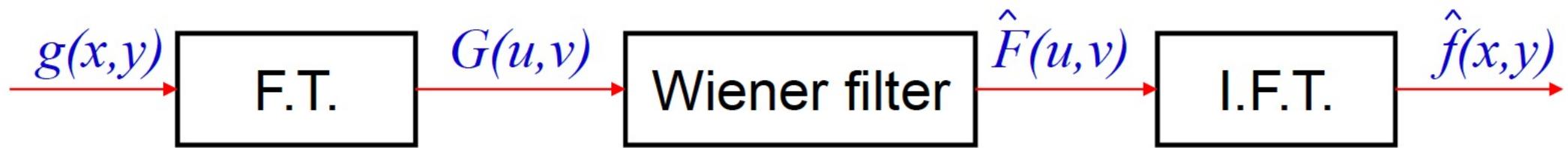
$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K(u, v)}$$

- If $K = 0$ then $W(u, v) = 1 / H(u, v)$, i.e. an inverse filter
- If $K \gg |H(u, v)|$ for large u, v , then high frequencies are attenuated
- $|F(u, v)|$ and $|N(u, v)|$ are often known approximately, or
- K is set to a constant scalar which is determined empirically
- A Wiener filter minimizes the least square error $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (f(x, y) - \hat{f}(x, y))^2 dx dy$

Restoration with a Wiener filter

$$G(u,v) = H(u,v) F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = W(u,v) G(u,v)$$



Example 1: Focus deblurring with a Wiener filter

blur $\sigma = 1.5$ pixels

noise $\sigma = 0.3$ grey levels

$$\hat{F}(u, v) = W(u, v) G(u, v) \quad W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K(u, v)}$$

$g(x, y)$



$\hat{f}(x, y)$



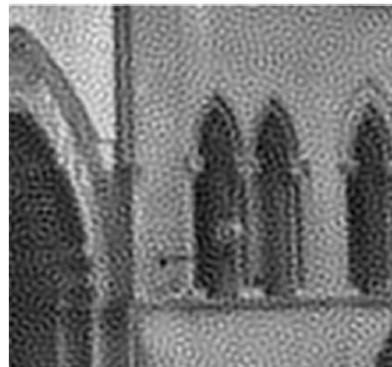
$K = 1.0 \text{ e } -5$



$K = 1.0 \text{ e } -3$



$K = 1.0 \text{ e } -1$



blur $\sigma = 3.0$ pixels

noise $\sigma = 0.3$ grey levels

$$f(x,y)$$



$$g(x,y)$$



$$\hat{f}(x,y)$$



$$K = 5.0 \text{ e -}4$$

Example 2: Motion deblurring

Suppose there is blur only in the horizontal direction
e.g. camera pans as image is acquired



Degradation model

$$g(x, y) = \frac{1}{T} \int_{-T/2}^{T/2} f(x - x_0(t), y) dt$$

Require $H(u, v)$ for Wiener filter

$$\begin{aligned}
G(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(ux+vy)} dx dy, \\
&= \frac{1}{T} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \int_{-T/2}^{T/2} f(x - x_0(t), y) dt \right\} e^{-j2\pi(ux+vy)} dx dy
\end{aligned}$$

interchange order of spatial and temporal integration

$$\begin{aligned}
G(u, v) &= \frac{1}{T} \int_{-T/2}^{T/2} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x - x_0(t), y) e^{-j2\pi(ux+vy)} dx dy \right\} dt \\
&\quad \text{Fourier transform of } f(x - x_0(t), y).
\end{aligned}$$

$$\begin{aligned}
G(u, v) &= \frac{1}{T} \int_{-T/2}^{T/2} F(u, v) e^{-j2\pi u x_0(t)} dt \\
&= F(u, v) \frac{1}{T} \int_{-T/2}^{T/2} e^{-j2\pi u x_0(t)} dt \\
&= F(u, v) H(u, v)
\end{aligned}$$

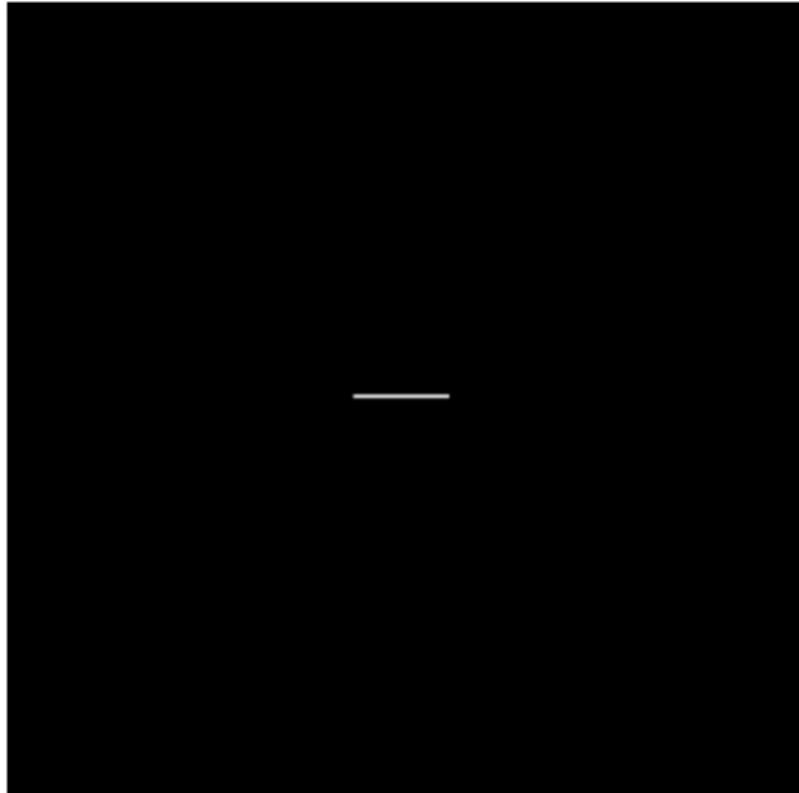
where

$$H(u, v) = \frac{1}{T} \int_{-T/2}^{T/2} e^{-j2\pi u x_0(t)} dt$$

suppose $x_0(t) = st$, and $sT = d$ pixels

FT of ... $h(x, y) = \text{hat}_d(x)\delta(y)$

$$\begin{aligned} H(u, v) &= \frac{1}{T} \int_{-T/2}^{T/2} e^{-j2\pi ust} dt \\ &= \frac{1}{T} \left[\frac{e^{-j2\pi ust}}{-j2\pi us} \right]_{-T/2}^{T/2} \\ &= \frac{1}{j2\pi ud} (e^{j\pi ud} - e^{-j\pi ud}) \\ &= \text{sinc}\pi ud \end{aligned}$$



Note, $H(u, v)$ has zeros – a problem for an inverse filter

Motion deblurring with a Wiener filter

blur = 20 pixels

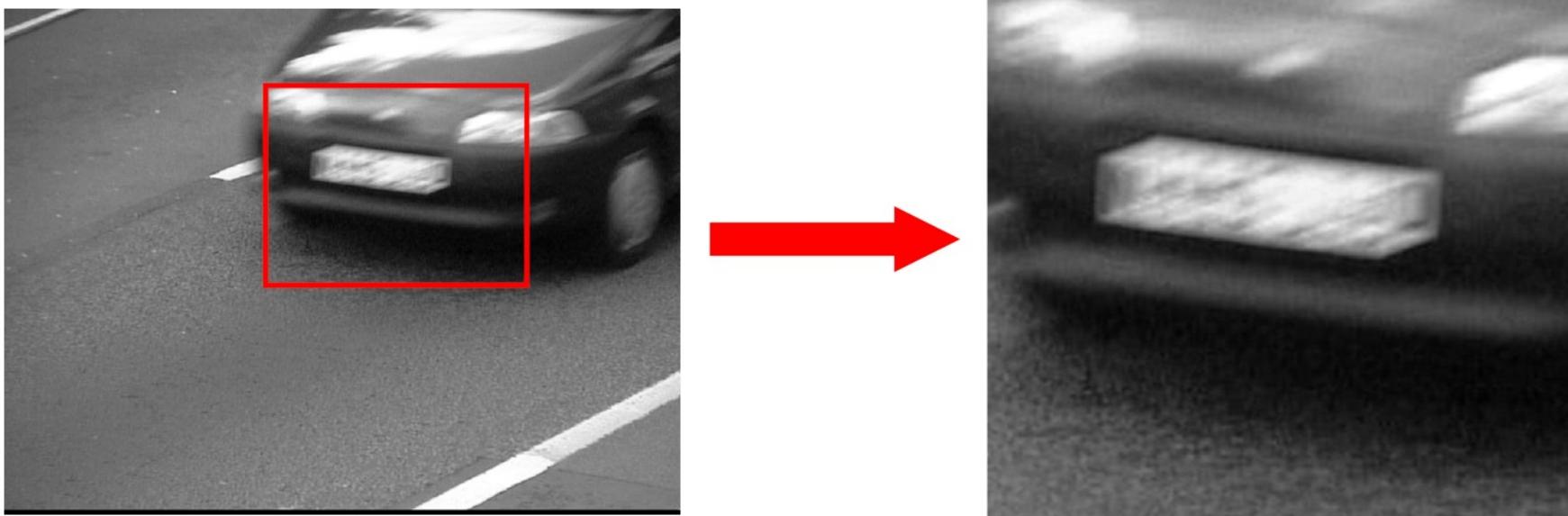
$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K(u, v)}$$



1. Compute the FT of the blurred image
2. Multiply the FT by the Wiener filter
3. Compute the inverse FT

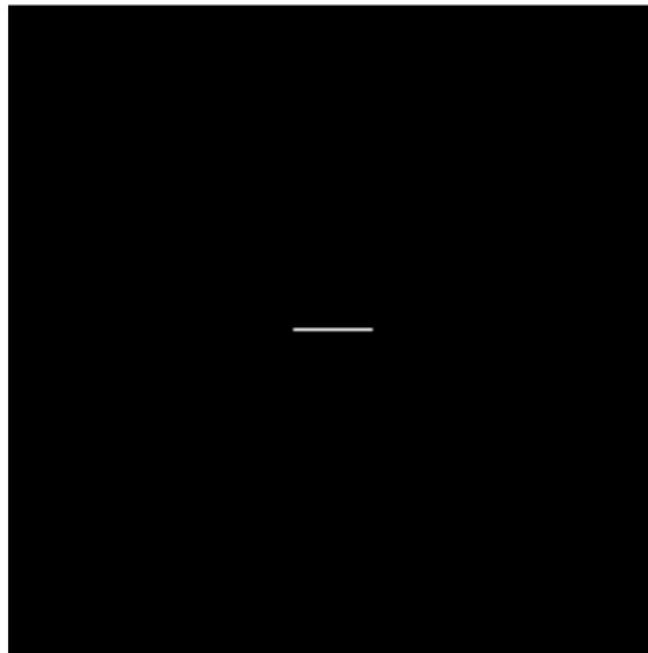
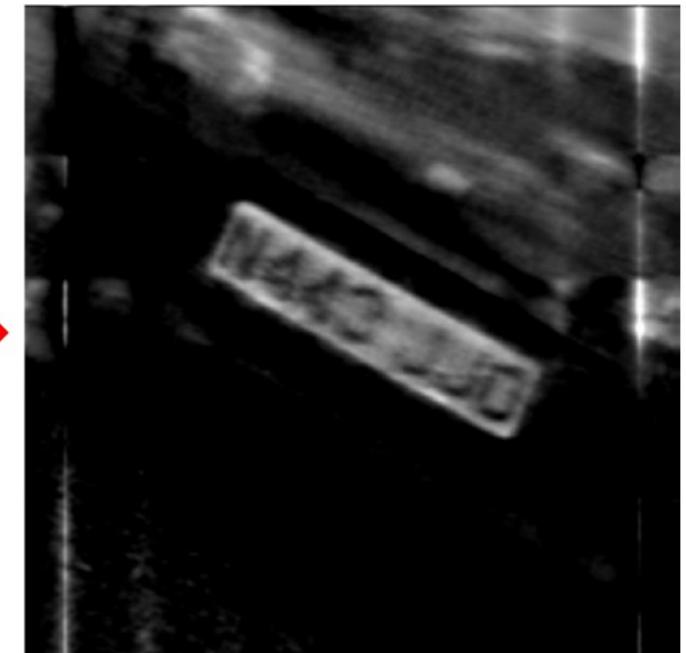
$$\hat{F}(u, v) = W(u, v) G(u, v)$$

Application: Reading number plates



Algorithm

1. Rotate image so that blur is horizontal
2. Estimate length of blur
3. Construct a bar modelling the convolution
4. Compute and apply a Wiener filter
5. Optimize over values of K

$f(x,y)$  $h(x,y)$  $\hat{f}(x,y)$ 

blur = 30 pixels

MAP image deblur

Maximum a posteriori (MAP)
Estimation

Generative model (forward process)



- original $f(x,y)$
- motion blur
- additive intensity noise

For an image with n pixels, write this process as

$$\hat{g} = Af + n$$

where \hat{g} and f are n -vectors, and A is an $n \times n$ matrix.

Inverse problem

- Estimate $f(x,y)$ by optimizing a cost function:

$$\hat{f} = \arg \min_f (g - Af)^2 + \lambda p(f)$$

observed image generated image
 ↓
 { Likelihood/
 loss function } { prior/
 regularization }

Example

$$p(f) = (\nabla f)^2$$

to suppress high frequency noise

Example-3: multi-frame super-resolution

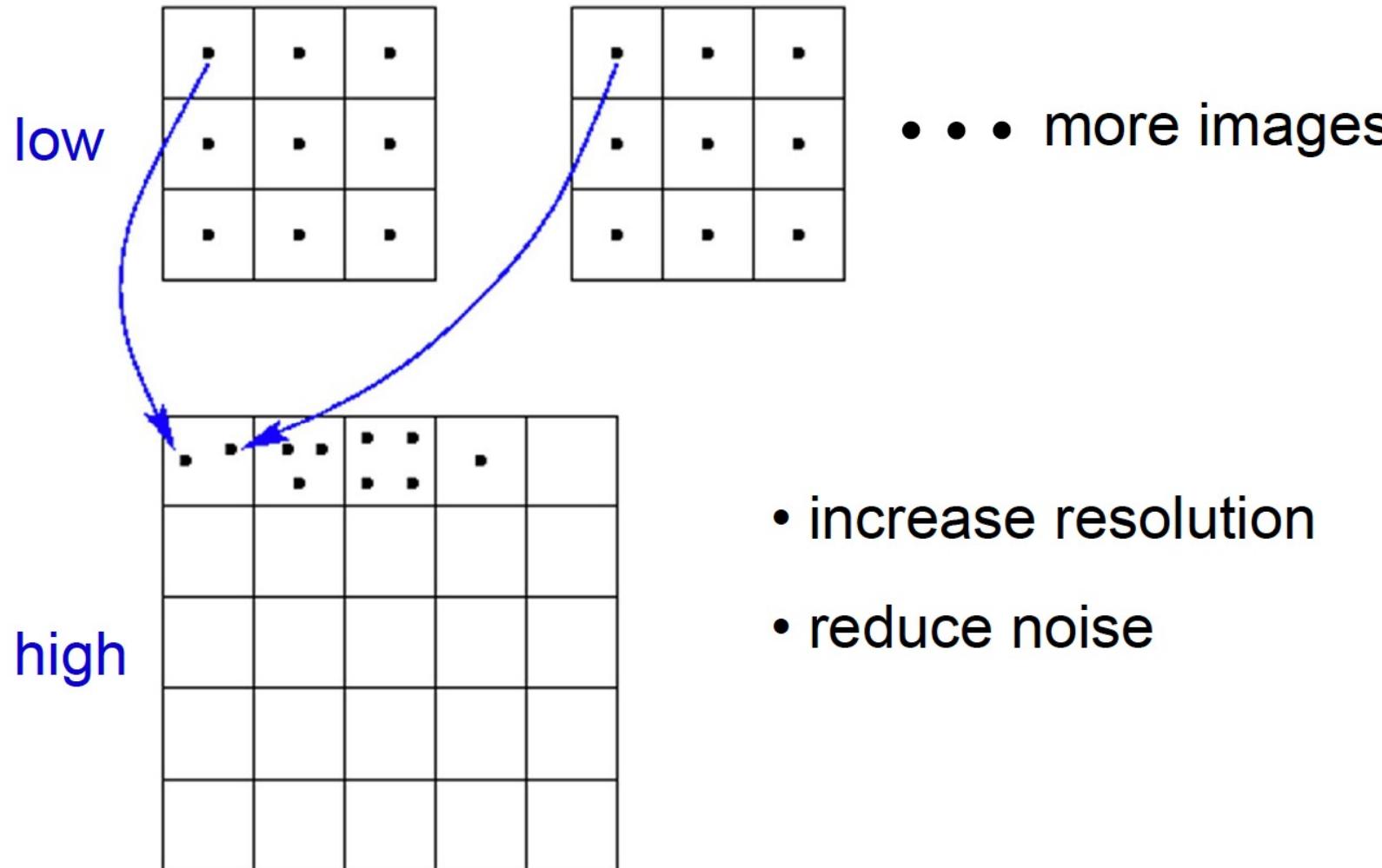
Suppose there are multiple images of the same scene
each displaced spatially ...



After registration the samples are not coincident
and this may be used to defeat the Nyquist limit.

Intuitive model

Treat images as point samples



Generative Model

High-resolution
image, f .



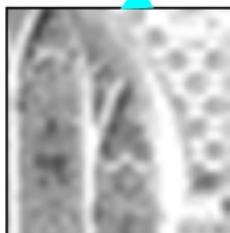
Registrations,
lighting and
blur.

M_1

M_2

M_3

M_4



g_1



g_2



g_3



g_4

Low-resolution images

- Estimate the super resolution image which minimizes the error between predicted and observed images.

Write the generative model for one image i as

$$\mathbf{g}_i = \mathbf{M}_i \mathbf{f} + \boldsymbol{\eta}_i$$

where \mathbf{M}_i combines registration, lighting and down-sampling.
Finally the generative models of all N images are stacked vertically to form an over-determined linear system

$$\begin{bmatrix} \mathbf{g}_0 \\ \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_{N-1} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_0 \\ \mathbf{M}_1 \\ \vdots \\ \mathbf{M}_{N-1} \end{bmatrix} \mathbf{f} + \begin{bmatrix} \boldsymbol{\eta}_0 \\ \boldsymbol{\eta}_1 \\ \vdots \\ \boldsymbol{\eta}_{N-1} \end{bmatrix}$$

$$\mathbf{g} = \mathbf{M}\mathbf{f} + \boldsymbol{\eta}$$

Maximum a posterior estimation

The MAP estimator has the form:

$$\mathbf{f}_{\text{MAP}} = \operatorname{argmin}_{\mathbf{f}} \underbrace{((\mathbf{g} - \mathbf{M}\mathbf{f})^2)}_{\text{likelihood}} + \underbrace{\lambda^2 p(\mathbf{f})}_{\text{prior}}$$

Example

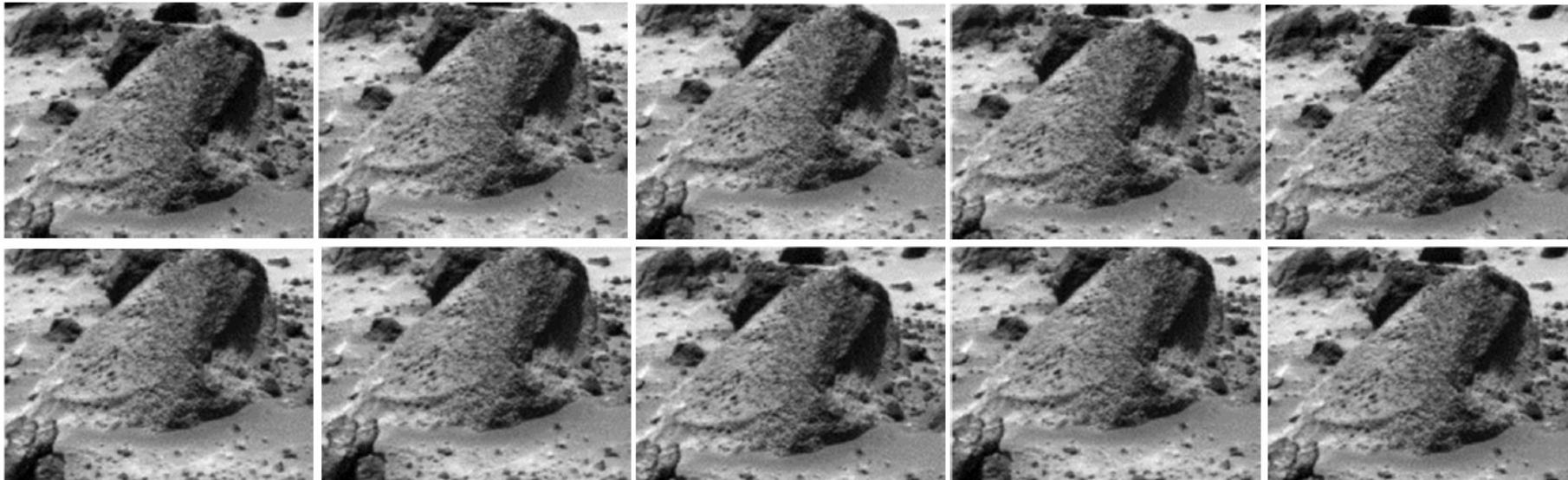
$$\mathbf{f}_{\text{MAP}} = \operatorname{argmin}_{\mathbf{f}} \left((\mathbf{g} - \mathbf{M}\mathbf{f})^2 + \lambda^2 \sum_{\forall x,y} p(\nabla f(x, y)) \right)$$

where the prior $p(x)$ is defined by the **Huber function**,

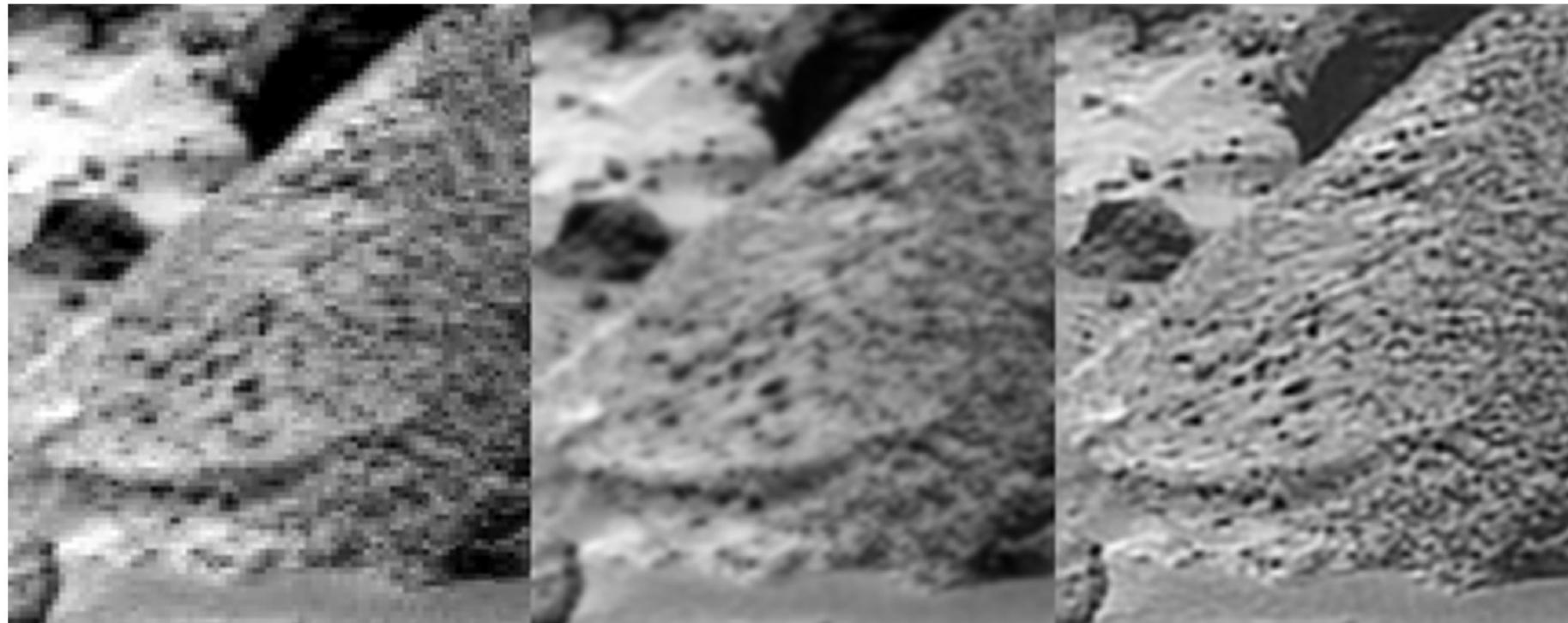
$$\begin{aligned} p(x) &= x^2, \text{ if } x \leq \alpha \\ &= 2\alpha |x| - \alpha^2, \text{ otherwise} \end{aligned}$$

Super resolution example I: Mars

25 JPEG images courtesy of the Mars lander
images are from different sweeps of a rotating camera



Super resolution result



Original frame

Average image

Super-resolution

2x zoom from 25 JPEG images.

Super resolution example II: car sequence

rotating DV camera

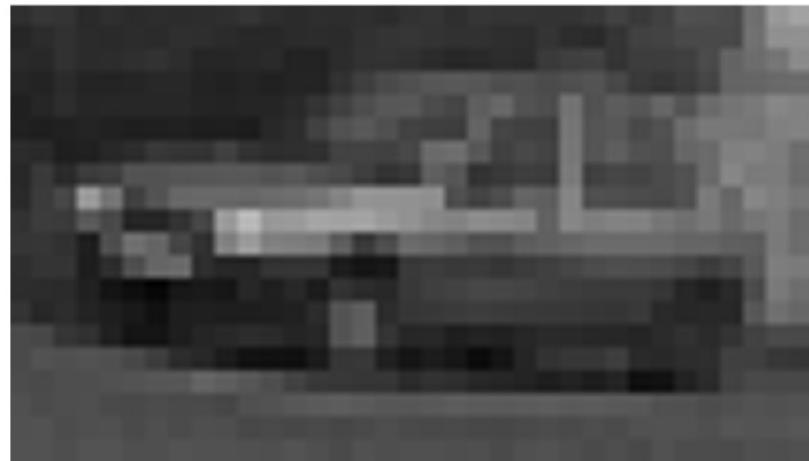


Mosaic



Super-resolution result for ROI

85 JPEG images



original ROI

35 x 20 pixels

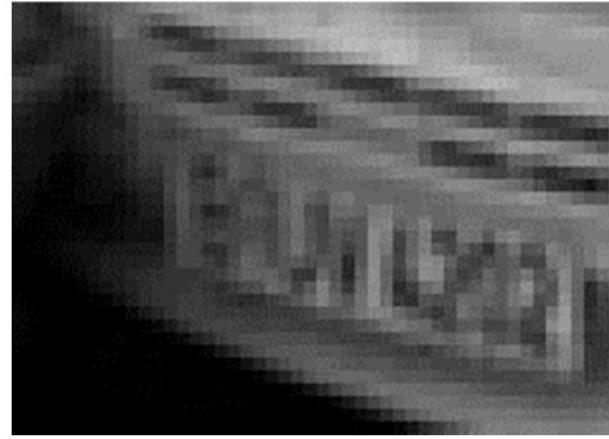
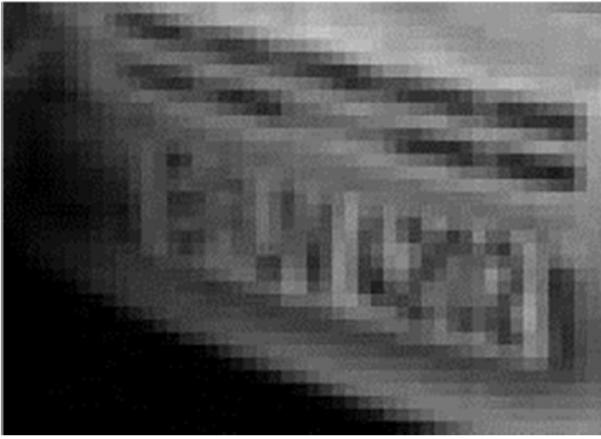
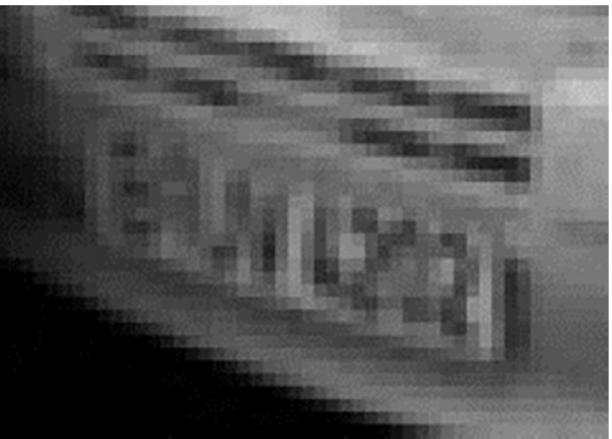


four times resolution

Super resolution example III: Run Lola Run



Input – low resolution



Super-resolution output



Blind Deblur/blind deconvolution

Blind deblurring

Non-examinable

So far we have assumed that we know the generative model, e.g.

$$g = A(h) f$$

$$G = H F$$



i.e. that $h(x,y)$ is known, so that given the observed image $g(x,y)$, then the original image $f(x,y)$ can be estimated (restored)

Consider if only the observed image $g(x,y)$ is known.
This is the problem of **blind estimation**.

Blind deblurring continued

- Estimate $f(x,y)$ and $h(x,y)$ by optimizing a cost function:

$$\min_{f,h} \underbrace{(g - A(h)f)^2}_{\text{Likelihood/loss function}} + \underbrace{\lambda p_f(f)}_{\text{image prior}} + \underbrace{\mu p_h(h)}_{\text{blur prior}}$$

observed image generated image

↓

blue bracket under $(g - A(h)f)^2$

blue bracket under $\lambda p_f(f)$

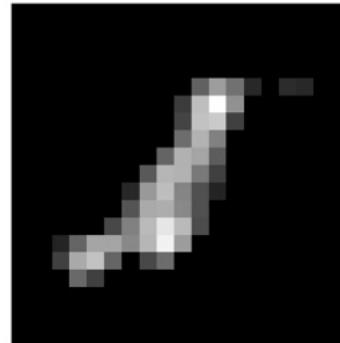
blue bracket under $\mu p_h(h)$

Example I: Blind deblurring

blurred image



estimated
blur filter



restored image



More examples of blind deblurring



Paper reading exercise

- Pre-lecture reading.
- Paper Summary
- Critical Analysis
- Breakout room Group discussion
- Report back via zoom.