1. 

A fair coin is tossed four times.

(a) Write out the sample space S.

(b) Write out the event E: 'exactly half the tosses show heads'.

(c) Evaluate $|E|$ and $|S|$ and hence find the probability $\mathbb{P}(E)$ of E.

(d) If the fair coin is tossed six times instead of four, would you expect $\mathbb{P}(E)$ to increase, decrease or stay the same?
To test you intuition, calculate the probability.

2. Two fair D4 dice are thrown, showing values $a, b \in \{1,2,3,4\}$.

(a) Write out the sample space S.

(b) Write out the event E: $\{|a-b| = 1\}$

(c) Evaluate $|E|$ and $|S|$ and hence find the probability $\mathbb{P}(E)$ of E.

(d) Draw a histogram for the probability density of $X = |a-b|$.
Is it symmetrical?
(Technically, X is a random variable $X : \{1,2,3,4\}^2 \to \mathbb{Q}_+$; $X(a,b) = |a-b|$.)

3. A pack of playing cards contains fours suits ($\clubsuit, \diamondsuit, \heartsuit, \spadesuit$) each of thirteen cards of which four are honours (A,K,Q,J). A card is drawn at random from the pack. What is the probability it is:

(a) a heart?;

(b) an honour?;

(c) a heart *and* an honour?;

(d) a heart *or* an honour?

4. [Challenge] At a Queanberra school two thirds of the girls, and seven eighths of the boys, are Australian-born.
A student is selected at random from the school.
The probability that this student is an overseas-born girl is 20%.

What is the probability that this student is an an overseas-born boy?

5. If you pick a random student at the Havana Acadamy there is a 32% chance that she/he is good at maths, a 27% chance that her/his favourite drink is a mojito, and a 6% chance that she/he is good at maths and favours mojitos.

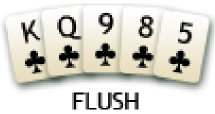
(a) Are the events "the student is good at maths", and "the student's favourite drink is a mojito" independent?

(b) Should you conclude that drinking mojitos makes you good at maths, or that maths makes you drink mojitos?

6. A 'poker hand' is a set of five cards drawn from a pack of playing cards. (See Q3.)

(a) What is the probability, to two significant figures, that a poker hand:

(i) is a 'flush'? *i.e* all five cards are in the same suit;



(ii) is a 'straight'? *i.e* five consecutive values from list A,K,Q,J,10,9,8,7,6,5,4,3,2,A (suits irrelevant);

(iii) is a straight flush?

(b) Are the events 'flush', and 'straight', independent?

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A fair D8 die is tossed eight times.
Use the binomial density function to compute the probability that:

(a) half the tosses give an even result (2, 4, 6 or 8);

(b) exactly one of the tosses give result 8;

(c) exactly three of the tosses give a result of 3 or less.

8. The owner of an ice cream shop was asked about the proportions of customers buying 1-, 2- and 3-scoop ice creams with wafer or waffle cones. The table records his estimates. Based on these, is the choice of cone type independent of the number of scoops?

Cone	scoops		
	1	2	3
wafer	7%	8%	20%
waffle	13%	15%	37%

9. Stephan has capital invested in a portfolio of highly volatile shares on the stock market. In his current monthly performance model there is a 10% chance that the capital will grow by 20%, a 20% chance that it will grow by 10%, a 20% chance that it will grow by 5%, a 30% chance that it will lose 5%, and a 20% chance that it will lose 20%.

(a) Let $S = \{1, 2, 3, 4, 5\}$. Define a probability density function $\mathbb{P} : S \to \mathbb{Q}_+$ and a random variable $X : S \to \mathbb{Q}$ modelling this situation, such that $\mathbb{E}(X)$ is the expected growth (or loss) after a month. Evaluate $\mathbb{E}(X)$.

(b) The random variable $Y = 1 + X$ represents the monthly 'multiplier' for Stephan's capital. (*e.g.* if he has \$10 000 invested at the beginning of the month and $Y = 1.05$ then he will have $1.05 \times \$10\,000 = \$10\,500$ at the begining of next month.)
Prove that $\mathbb{E}(Y) = 1 + \mathbb{E}(X)$. Hence evaluate $\mathbb{E}(Y)$.

(c) The random variable $Z : S^2 \to \mathbb{Q}$ defined by $Z(s, t) = Y(s)Y(t)$ represents the two-monthly multiplier.
Prove that $\mathbb{E}(Z) = (\mathbb{E}(Y))^2$, stating any required assumptions. Hence evaluate $\mathbb{E}(Z)$.

(d) Is it true that $(\mathbb{E}(Y))^2 = \mathbb{E}(Y^2)$? Prove or disprove.