1. Let $B = \{0, 1\}$ and $n \in \mathbb{N}$.

(a) Prove that $|B| = 2$. Of course it is completely obvious that $|B| = 2$, *i.e.* that B has cardinality (size) 2. But cardinality is defined in terms of bijections (one-to-one correspondences); in particular B has cardinality 2 if and only if there is a bijection between B and $\{1, 2\}$. Specify such a bijection, giving its signature and rule.

(b) Compute $|B^n|$.

(c) Compute the number of n digit binary numbers that start with a 1.

2. You have to deliver flyers to one side of part of a long street. Your houses have consecutive odd numbers starting at 37 and finishing at 251. How many flyers do you need? Prove your answer is correct by specifying a suitable function as in Q1(a) above, and justify that this function is a bijection.

3. Let S be any subset of $\{1, 2, \ldots, 12\}$ with $|S| = 7$. Prove that $\exists a, b \in S : \; a - b = 3$.

4. The digits 1, 2, . . . , 9 are divided into three groups. Prove that the product of the numbers in one of the groups must exceed 71.

5. Trying to break an 8-character password by intercepting internet packets, you have found all eight characters used in the password, but not their order. Your program can try one possible password every 2 seconds. How long do you need (at most) to break the password?

6. Jane has to choose 2 out of 6 maths subjects and 3 out of 10 computer science subjects. How many different combinations of maths and computer science subjects are there for Jane to choose from?

7. Using only the formula $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ prove

(a) $\forall n \in \mathbb{N} \quad \forall r \in \{0, ..., n\} \quad \binom{n}{r} = \binom{n}{n-r}$.

(b) $\forall n \in \mathbb{N} \quad \forall r \in \{1, ..., n\} \quad \binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$.

8. A PIN is a number with 4 decimal digits (*e.g.* 2357, 0922 etc).

(a) How many (different) PINs are there?

(b) How many (different) PINs contain the digit 0 at least once?

(c) How many (different) PINs contain at least one even digit?

(d) How many (different) PINs contain at least one even digit and at least one odd digit?

(e) How many (different) PINs start or end with 0 (or both)?

(f) How many (different) PINs contain no digit more than once?

(g) How many (different) PINs have their digits in increasing order (*e.g.* 0458)?

(h) How many (different) PINs have theirs digit in nowhere-decreasing order (*e.g.* 0448)?

(i) How many (different) PINs have digit sum 9?

(j) How many (different) PINs have digit sum 10?

(k) How many (different) PINs involve only two different digits?

(l) How many (different) PINs involve only two different digits, each used twice.