COMP 2610/ COMP 6261

Tutorial 2

Theory

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{— (1)} \quad \Big\} \; P(A \cap B) = P(A,B).$$

Conditional Probability

$$P(A/B) = \frac{P(A,B)}{P(B)} \quad \text{— (2).}$$

$$P(A,B) = P(A/B).P(B) = P(B/A).P(A) \quad \text{— (2.1)}$$

Sum Rule

$$P(B) = \sum_{\forall x_i} P(B/x_i).P(x_i) \quad \text{— (3).}$$

Q_1

	R (0.2)	B (0.2)	G (0.6)	T
Apple	3	1	3	
Orange	4	1	3	
Limes	3.	0	4.	
	$P(F/r)$	$P(F/b)$	$P(F/g)$.	
T	10	2	10	

a. $P(A) = \sum_{\psi_i \in \{r,g,b\}} P(\psi_i) \cdot P(A/\psi_i)$.

$P(r) \times P(A/r) + P(b) \times P(A/b) + P(g) \times P(A/g)$

$= 0.2 \times 3/10 + 0.2 \times 5/10 + 0.6 \times 3/10$.

$= \frac{0.6}{10} + \frac{1}{10} + \frac{1.8}{10}$

$= \frac{3.4}{10} = 0.34$ //.

b. $P(g/o) = \frac{P(g \cap o)}{P(o)} = \frac{P(o/g) \times P(g)}{\sum P(\psi_i) P(o/\psi_i)}$

$P(o/g) = \frac{P(o \cap g)}{P(g)}$

$P(g \cap o) = 0.6 \times 3/10 = 0.18$

$P(o) = 0.2 \times 0.4 + 0.2 \times 0.5 + 0.6 \times 0.3 = 0.36$

$\therefore P(g/o) = 0.18/0.36 = 0.5$ //

Q3.

Table represent Number of occurance in 1000 trial.

		X = 0	X = 1	Tot
Y	0	100	250	350
	1	150	500	650
Tot		250	750	

a. $P(X=1, Y=1) = \frac{\text{Number of } (X=1, Y=1)}{\text{Total Occurance}} = \frac{500}{1000} = 0.5$

b. $P(X=1) = \frac{\text{Occurance of } X=1}{\text{Total Occurance}}$

$= \frac{750}{1000} = 0.75$

$$\left\{ \begin{aligned} P(X=1) &= P(X=1, Y=0) + P(X=1, Y=1) \\ &= \frac{250}{1000} + \frac{500}{1000} = 0.75 \end{aligned} \right\}$$

c. $E[X] = X \cdot p(X)$

$= 1 \times 0.75 + 0 \times 0.25$

$= 0.75$

d. $P(Y=1 / X=1) = \frac{P(Y=1 \cap X=1)}{P(X=1)}$

$= \frac{500/1000}{0.75} = \frac{0.5}{0.75} = \frac{2}{3}$

e. $P(Y=1/X=0) = \dfrac{P(Y=1 \cap X=0)}{P(X=0)}$

$= \dfrac{150/1000}{250/1000} = \dfrac{3}{5}$

f. $P(Z=1/[X=x, Y=y]) = \begin{cases} 0.9 & \text{if } (x,y)=(0,1) \text{ or } (1,0) \\ 0.1 & \text{if } (x,y)=(0,0) \text{ or } (1,1) \end{cases}$

A typical XOR

X	Y	Z	Perfect XOR Probability	Noisy XOR Probability
0	0	0	0	0.1
0	1	1	1	0.9
1	0	1	1	0.9
1	1	0	0	0.1

$P(X=1, Y=1/Z=1) = \dfrac{P(X=1, Y=1, Z=1)}{P(Z=1)}$

$P(X=1, Y=1, Z=1) = P(Z=1/X=1, Y=1) \times P(X=1, Y=1)$

$= 0.1 \times 0.5$

$= 0.05$

$P(Z=1) = \sum_{\substack{x=0,1 \\ y=0,1}} P(X=x, Y=y) \times P(Z=1/X=x, Y=y)$

$=$

$$P(Z=1) = P(X=0, Y=0) \times P(Z=1 / X=0, Y=0)$$

$$+ \quad \_ \; \_ \; \_ \; \_ \; .$$

$$= \frac{100}{1000} \times 0.1 + \frac{250}{1000} \times 0.9 + \frac{150}{1000} \times 0.9$$

$$+ \frac{500}{1000} \times 0.1$$

$$= 0.42$$

$$\therefore P(X=1, Y=1 / Z=1) = \frac{0.05}{0.42} = 0.1190$$

Q3. Placement

H → W.   T → R.

H → W.   T → R

3 drawn

All are red.

$$P(P2R/D3R) = \frac{P(P2R \cap D3R)}{P(D3R)}.$$

In the box probability of 1^{st} placement is red = 1/2

Similarly " " 2^{nd} place " = 1/2.

∴ In the box, probability of

$2R = \cancel{1}/2 \times 1/2 = 1/4$

$1R = 2[1/2 \times 1/2] = 1/2$

$0R = 1/2 \times 1/2 = 1/4.$

In Box	Probability of Red Drawing
2R	1/1
1R	1/2
0R	0.

Drawing Probabilities are same for all 3 drawings. ~~d~~ due to replacements.

$$P(P2R \cap D3R) = P\left(D3R \,/\, P2R\right) \times P(P2R)$$

$$= 1^3 \times \frac{1}{4}$$

$$= 1/4.$$

$$P(D3R) = \sum_{\psi_2} P\left(D3R/\psi_2\right) \times P(\psi_2)$$

$$= 1^3 \times 1/4 + (1/2)^3 \times 1/2 + 0 \times 1/4$$

$$= 1/4 + 1/16$$

$$= 5/16$$

$$\therefore\ P(P2R/D3R) = \frac{1/4}{5/16}$$

$$= 0.8$$

Q4.

a. $$P(X/Y) = \frac{P(X,Y)}{P(Y)} \quad -①$$

$$P(X/Y,Z) \overset{?}{=} \frac{P(Y/X,Z).P(X/Z)}{P(Y/Z)}$$

$$P(X/Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)} \overset{①\Rightarrow}{=} \frac{P(Y/X,Z)\,P(X,Z)}{P(Y,Z)}$$

$$\overset{①\Rightarrow}{=} \frac{P(Y/X,Y) \times P(X/Z).\cancel{P(Z)}}{P(Y/Z).\cancel{P(Z)}} = \frac{P(Y/X,Z).P(X/Z)}{P(Y/Z)}$$

b. $$P(X_1, \ldots X_n) = \sum_{x} P(X_1, \ldots X_i = x, \ldots X_n) \quad -②$$

$$P(X_1, \ldots, X_n/Y) \overset{?}{=} \sum_{x} P(X_1, \ldots X_i = x, \ldots X_n/Y)$$

$$P(X_1, \ldots X_n/Y) = \frac{P(X_1, \ldots X_n, Y)}{P(Y)}$$

$$③\Rightarrow \quad = \frac{\sum_{x} P(X_1, \ldots X_i = x, \ldots X_n, Y)}{P(Y)}$$

$$= \sum_{x} \left[\frac{P(X_1, \ldots X_i = x, \ldots X_n, Y)}{P(Y)} \right]$$

$$= \sum_{x} P(X_1, \ldots X_i = x, \ldots X_n/Y)$$

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