These questions are for practice, in preparation for Workshop 1.

1. Which of the following sentences are statements?

(a) Australia is a country. **Yes – true.**

(b) Australia is the greatest country. **No – "greatest" is ambiguous.**
(With a broader definition of "statement" you could call this "an ambiguous statement")

(c) Is Australia a country? **No – question.**

(d) Tell me if Australia is a country. **No – directive.**

(e) Australia is in the Northern hemisphere. **Yes – false.**

(f) I always lie. **Yes – false. (If true, it would be contradictory.)**

If (f) is a statement, is it true or false?

2. Find the negation of the following statements.

(a) I am young. **I am not young. (NB: "not young" ≠ "old" !)**

(b) You are rich. **You are not rich. (NB "not rich" ≠ "poor".)**

(c) New Zealand always wins. **New Zealand doesn't always win.**
(NB: "doesn't always" ≠ "never".)

(d) This is not fair. **This is fair.**

(e) This is not what you think it is not. **This is what you think it is not.**

3. Rewrite the following statements symbolically as in the example.
Example: I am a mathematician, but I am not crazy.
p: I am a mathematician. q: I am crazy. r: I am a mathematician, but I am not crazy.
$r \equiv p \wedge \neg q$.

(a) Either I get it, or I don't. $\boldsymbol{p \vee \neg p}$; $\boldsymbol{p}$ **= "I get it."**

(b) I live in Australia or in France. $\boldsymbol{p \vee q}$; $\boldsymbol{p}$ **= "I live in Australia",**
$\boldsymbol{q}$ **= "I live in France"**

(c) There will be an election soon, and we will win. $\boldsymbol{p \wedge q}$;
$\boldsymbol{p}$ **= "There will be an election soon",** $\boldsymbol{q}$ **= "We will win (the election)"**

(d) Either you have earned enough points and have been a member since 2002, or you have earned enough points and paid \$100. $\boldsymbol{(p \wedge q) \vee (p \wedge r)}$; $\boldsymbol{r}$ **= "You paid \$100"**
$\boldsymbol{p}$ **= "You've earned enough pts",** $\boldsymbol{q}$ **= "You've been a mbr since 2002"**

4. Construct truth tables for the following statements.

(a) $(p \vee q) \wedge \neg r$.

(b) $(p \oplus q) \vee r$.

p	q	r	$(p \vee q) \wedge \neg r$	$(p \oplus q) \vee r$
T	T	T	$\cancel{T}$ $\boldsymbol{F}$ $\cancel{F}$	$\cancel{F}$ $\boldsymbol{T}$ $\cancel{T}$
T	T	F	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{T}$	$\cancel{F}$ $\boldsymbol{F}$ $\cancel{F}$
T	F	T	$\cancel{T}$ $\boldsymbol{F}$ $\cancel{F}$	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{T}$
T	F	F	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{T}$	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{F}$
F	T	T	$\cancel{T}$ $\boldsymbol{F}$ $\cancel{F}$	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{T}$
F	T	F	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{T}$	$\cancel{T}$ $\boldsymbol{T}$ $\cancel{F}$
F	F	T	$\cancel{F}$ $\boldsymbol{F}$ $\cancel{F}$	$\cancel{F}$ $\boldsymbol{T}$ $\cancel{T}$
F	F	F	$\cancel{F}$ $\boldsymbol{F}$ $\cancel{T}$	$\cancel{F}$ $\boldsymbol{F}$ $\cancel{F}$

5. Give an example of a tautology and an example of a contradiction.

Tautology: Either I succeed, or I don't. $(p \vee \neg p)$

Contradiction: $\sqrt{2}$ is both negative and non-negative. $(p \wedge \neg p)$

6. Are these two statements logically equivalent?

(a) $p \vee (q \wedge r)$.

(b) $(p \vee q) \wedge (p \vee r)$.

Yes. This is one of the distributive laws. It can be verified by truth table.

p	q	r	$p \vee (q \wedge r)$			$(p \vee q) \wedge (p \vee r)$	
T	T	T	$\boldsymbol{T}$	$\cancel{T}$	$\cancel{T}$	$\boldsymbol{T}$	$\cancel{T}$
T	T	F	$\boldsymbol{T}$	$\cancel{F}$	$\cancel{T}$	$\boldsymbol{T}$	$\cancel{T}$
T	F	T	$\boldsymbol{T}$	$\cancel{F}$	$\cancel{T}$	$\boldsymbol{T}$	$\cancel{T}$
T	F	F	$\boldsymbol{T}$	$\cancel{F}$	$\cancel{T}$	$\boldsymbol{T}$	$\cancel{T}$
F	T	T	$\boldsymbol{T}$	$\cancel{T}$	$\cancel{T}$	$\boldsymbol{T}$	$\cancel{T}$
F	T	F	$\boldsymbol{F}$	$\cancel{F}$	$\cancel{T}$	$\boldsymbol{F}$	$\cancel{F}$
F	F	T	$\boldsymbol{F}$	$\cancel{F}$	$\cancel{F}$	$\boldsymbol{F}$	$\cancel{T}$
F	F	F	$\boldsymbol{F}$	$\cancel{F}$	$\cancel{F}$	$\boldsymbol{F}$	$\cancel{F}$

▲——same——▲

7. Are these two statements logically equivalent?

(a) $\neg(p \vee q \vee r)$.

(b) $p \wedge q \wedge \neg r$.

No. E.g. taking p and q true and r false provides a counterexample, because then statement form (a) is false but (b) is true. The only other counterexample comes from taking p, q, r all false.

8. Formalise the following statements as in the example.
Example: People who do not give up always succeed.
p: you do not give up. q: you succeed. $p \implies q$.

(a) You will get a discount if you apply early. $\boldsymbol{e \implies d}$
$\boldsymbol{d}$ = "You will get a discount" and $\boldsymbol{e}$ = "you apply early".

(b) Musicians are cool. $\boldsymbol{m \implies c}$
$\boldsymbol{m}$ = "You are a musician" and $\boldsymbol{c}$ = "you are cool".
[Alternatively, if M denotes the set of all musicians, and $\boldsymbol{c(m)}$ denotes the predicate "$\boldsymbol{m}$ is cool" then the sentence can be rendered $\forall \boldsymbol{x} \in \boldsymbol{M}\ \boldsymbol{c(x)}$.]

(c) No machine running Microsoft's Windows runs well. $\boldsymbol{m \implies \neg w}$
$\boldsymbol{m}$ = "The machine runs Microsoft Windows" and $\boldsymbol{w}$ = "the machine runs well".
[Alternatively, if $\boldsymbol{C}$ denotes the set of all computers and $\boldsymbol{m(c)}$ and $\boldsymbol{w(c)}$ denote the predicates "$\boldsymbol{c}$ runs MS Windows" and "$\boldsymbol{c}$ runs well", then the sentence can be rendered $\forall \boldsymbol{c} \in \boldsymbol{C}\ \boldsymbol{m(c) \implies \neg w(c)}$.]

9. Negate the following expressions.

(a) If GDP grows, people are happier. **GDP grows but people are no(t) happier.**

(b) If I have a coffee, I feel energetic. **I have a coffee but don't feel energetic.**
NB: Any answer involving "if" is wrong!

(c) You can get it if you really want. **You really want it but you can't get it.**
Again, any answer involving "if" is wrong.

10. Reasoning by contrapositive, what can be concluded from the following statements?
Guns don't kill people. I kill people.
Let $\boldsymbol{g}$ = "I am a gun" and $\boldsymbol{k}$ = "I kill people".
Then the statements become $\boldsymbol{g \implies \neg k}$ and $\boldsymbol{k}$.
Replacing the first statement by its (equivalent) contrapositive and applying the (valid) modus ponens argument we get the argument shown at right.

$$\begin{array}{r} \boldsymbol{k \implies \neg g} \\ \boldsymbol{k} \\ \hline \boldsymbol{\neg g} \end{array}$$

Thus we conclude "I am not a gun".

11. Find the condition in the following statements, and determine if it is necessary, sufficient, or both.

(a) To get a table you need to $\boxed{\text{have a reservation.}}$ **necessary.**

(b) Only people who $\boxed{\text{arrive early}}$ might get a ticket. **necessary.**

(c) If $\boxed{\text{you were a member of our earlier program,}}$ you are automatically a member of the new program. **sufficient.**

12. Are the following arguments valid or not?

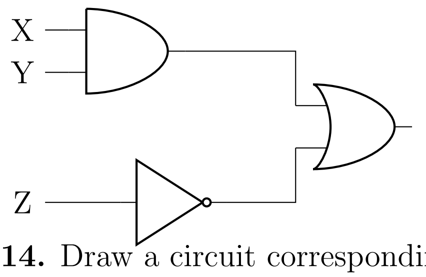
(a) If the user has been inactive for five minutes, then turn the display off. The display is on. Therefore the user has been active in the last five minutes. **Valid.**

With p = "user inactive for 5mins", q = "display off", this is modus tollens.

(b) If we are given more time to prepare a plan and have a representative on the committee, then we will reach a consensus. No consensus has been reached. This means that we were not given enough time to prepare our plan and did not have a representative on the committee. **Invalid. Using modus tollens and DeMorgan's law the correct conclusion is that we were not given enough time to prepare our plan $\boxed{\text{OR}}$ did not have a representative on the committee.**

(c) People who have done a lot of mathematics are logical. Therefore logical people have done a lot of mathematics. **Invalid. An implication does not imply its converse.**

13. Find the logic expression that corresponds to this circuit, and give its truth table.



$(X \wedge Y) \vee \neg Z$

14. Draw a circuit corresponding to the following truth table.

X	Y	Z	*output*
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

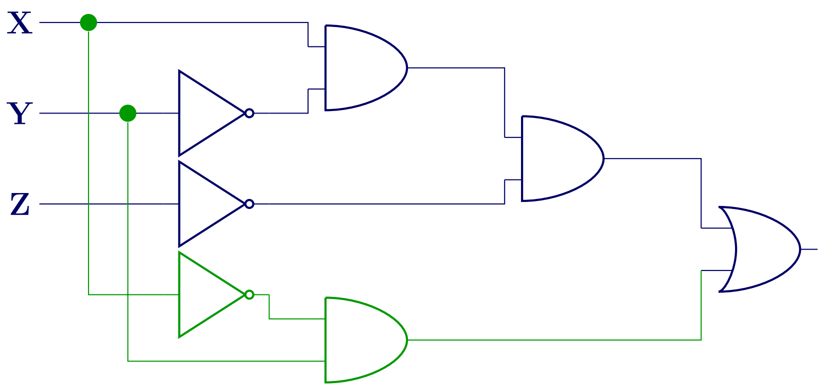
From lines 4,5,6 we read off that the table corresponds to the statement form

$(X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge Z) \vee (\neg X \wedge Y \wedge \neg Z)$

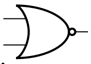
$\equiv (X \wedge \neg Y \wedge \neg Z) \vee [(\neg X \wedge Y) \wedge (Z \vee \neg Z)]$

$\equiv (X \wedge \neg Y \wedge \neg Z) \vee [(\neg X \wedge Y) \wedge T]$

$\equiv (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y)$

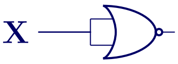


Several other circuits also work.

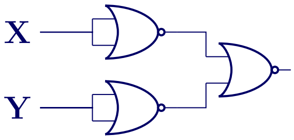
15. The NOR gate  has the truth table at right. Construct the following gates using only NOR gates.

X	Y	$NOR(X,Y) = X \downarrow Y$
1	1	0
1	0	0
0	1	0
0	0	1

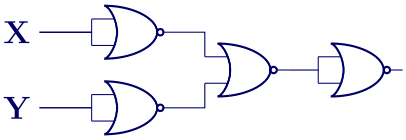
1. NOT **$\neg p \equiv p \downarrow p$**



2. AND **$p \wedge q$**
$\equiv \neg(\neg p \vee \neg q)$
$\neg p \downarrow \neg q$



3. NAND **$p \mid q$**
$\equiv \neg(p \wedge q)$



16. Which of these sentences could be predicates?

1. User x is not allowed to view this page. **Yes. Variable x is free.**
2. For every user x, the cache has been cleared. **No. The only variable, x, is bound by the universal quantifier "For every".**
3. This lecture is boring. **No. There is no free variable.**
4. For every user x, page y has been deleted. **Yes. Variable y is free.**

17. Negate the following statements.

(a) Snakes can't swim. ***Does "snakes" mean (i) "all snakes" or (ii) "some snakes" ?***
(i) Some snakes can swim. **(ii) All snakes can swim.**

(b) Fast growing countries are all in Asia. **Not all fast growing countries are in Asia. — or, mor directly — Some fast growing countries are outside Asia**.

(c) All sheep are black. **Not all sheep ar black. —or— Some sheep aren't black.**

18. Negate the following statements.

(a) $\exists x \quad p(x) \implies q(x)$. $\forall x \quad \neg(p(x) \implies q(x)) \quad \equiv \quad \boldsymbol{\forall x \quad p(x) \wedge \neg q(x)}$

(b) $\exists x \quad \forall y \quad p(y)$. $\forall x \quad \neg(\forall y \quad p(y)) \quad \equiv \quad \boldsymbol{\forall x \quad \exists y \quad \neg p(y)}$

(c) $\forall x \quad \exists z \quad \exists w \quad \forall t \quad p(x,z) \vee q(w,t) \implies \neg r(x,z,w,t)$

$$\exists x \quad \neg[\exists z \quad \exists w \quad \forall t \quad p(x,z) \vee q(w,t) \implies \neg r(x,z,w,t)]$$
$$\equiv \exists x \quad \forall z \quad \neg[\exists w \quad \forall t \quad p(x,z) \vee q(w,t) \implies \neg r(x,z,w,t)]$$
$$\equiv \exists x \quad \forall z \quad \forall w \quad \neg[\forall t \quad p(x,z) \vee q(w,t) \implies \neg r(x,z,w,t)]$$
$$\equiv \exists x \quad \forall z \quad \forall w \quad \exists t \quad \neg[p(x,z) \vee q(w,t) \implies \neg r(x,z,w,t)]$$
$$\equiv \boldsymbol{\exists x \quad \forall z \quad \forall w \quad \exists t \quad (p(x,z) \vee q(w,t)) \wedge r(x,z,w,t)}$$

19. Is the following argument valid?
It is false that snakes can't swim. Indeed, sea snakes can. So snakes can swim.
Let S be the set of all snakes and let $c(s)$ be the predicate "s can swim". Then the first premise of the argument is $\neg[\forall s \in S \ \neg c(s)] \equiv \exists s \in S \ c(s)$. Thus the second premise, c(sea snakes), adds nothing to the first premise. So the conclusion, $\forall s \in S \ c(s)$, does not follow from the premises, and consequently the argument is invalid. ***(This analysis makes assumption (i) in the answer to Q17. Other interpretations are possible.)***