**Instructions:** *See Worksheets 1 and 2*

These questions relate to the first 75% or so of the notes for D1: Graph Theory; all the material covered in lectures in Week 9.
The questions mostly just test your understanding of terminology and basic properties. More interesting and challenging questions relating to graphs will be on next week's worksheet.

Question 1   For the 'fish' graph G at right find:

(a) $V(G)$.

(b) $|E(G)|$.

(c) All edges incident on vertex 2.

(d) All vertices adjacent to vertex 2.

(e) All edges adjacent to edge c.

(f) A loop.

(g) An isolated vertex.

(h) A pair of parallel edges.

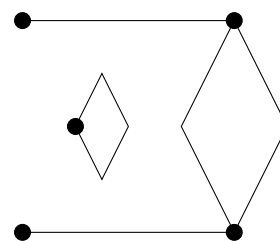(i) The adjacency matrix of G.
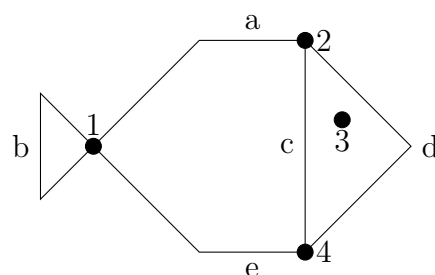
Question 2

(a) Draw diagrams for the graphs with adjacency matrices below:

(i) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
(ii) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

(b) Are the graphs of (a) isomorphic? Justify your answer.

(c) Is the graph (a)(i) isomorphic to the graph with diagram at right? Justify your answer.

Question 3A   The digraph W at right represents a foodweb.
An edge from a to b indicates that a eats b.
Let k, l, f, i denote kookaburras, lizards, frogs and insects.

(a) Which of these are edges of W: $(k, l), (l, f), (f, i), (i, k)$?

(b) Draw the niche overlap graph for W.
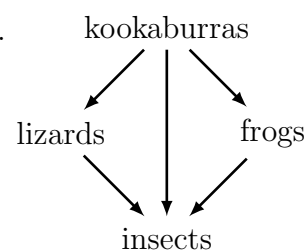
Question 3B   A relation $R \subseteq \{1, 2, 3, 4, 5\}^2$ is defined by

$xRy \iff 0 < x - y < 3$.

(a) Draw a digraph representing R.

(b) Write out the adjacency matrix for R.

Question 4A   For the graph G at right:

(a) State the degree of each vertex.

(b) Exhibit subgraphs of G isomorphic to K_{15}, K_{24}, K_{33} and K_5.

(c) Prove that G does not have a subgraph isomorphic to K_{34}.
Hint: Concentrate on vertices G and H.
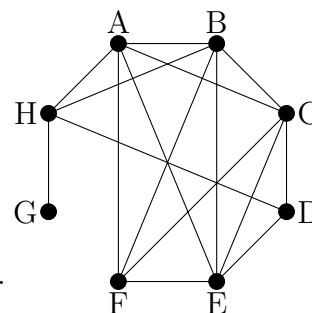
Question 4B   Draw a graph with five vertices, each of degree 3,

or say why you believe this to be impossible.

Question 5   For the graph H at right:

(a) Is the graph simple? Justify your answer.

(b) Is the walk abcdb

   (i) a path?

  (ii) closed?

 (iii) simple?

(c) Find simple circuits of lengths 3, 4 and 5.

(d) Find an Euler circuit.

(e) Find a Hamilton circuit.
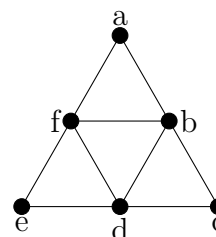
Question 6   For the graph J at right:

(a) Prove that J has no Euler circuit.

(b) Prove that J has no Hamilton circuit.

(c) Suppose that Fleury's algorithm is used to find an Euler path from p to r. What feature of the algorithm prevents the path starting ptvr?