Instructions: *See Worksheets 1 and 2*

For this workshop you are encouraged to make use of computers.

Question 1 In rôle-playing games, such as the original, *Dungeons and Dragons*, gamers use a variety of dice shapes, not just the standard cube. For example a 'd12' has the shape of a regular dodecadedron. A (fair) d12 has twelve faces numbered 1,2,. . . ,12, and each face is equally likely to become the top face (the face that is read) when the die is 'thrown'.

Suppose gamer Alice does not have a d12 but does have a d6 (*i.e.* a regular cubic die with six faces labelled 1,2,. . . ,6.) To simulate throwing a d12, Alice throws her d6 twice, resulting in a pair of values $(a, b) \in \{1, \ldots, 6\}^2$. She then combines the two values in some way to come up with a value in $\{1, \ldots, 12\}$.

(a) Write out the sample space S for this 'experiment'.

(b) One way to combine a and b would be to add them. But this would not accurately simulate a d12 throw because it would be impossible to score 1.
To avoid this problem Alice decides to use the formula: $v = \left\lceil \frac{ab}{3} \right\rceil.$
Write out the event $E \subseteq S$ coresponding to $v = 1$.

(c) What is the probability of event E above and in what way does it demonstrate that Alice's method also does not accurately simulate a d12 throw?

(d) How *could* Alice combine a and b to accurately simulate a d12 throw?
(Gamers actually do this, so you can Google an answer if you're desperate!)

Question 2 We have seen in lectures that if 50 people are chosen at random then there is a 97% chance that at least two of them share the same birthday. Use similar calculations to answer the following:

(a) What is the percentage chance that from 5 randomly chosen people at least two have their birthday in the same month? (Treat all months as if they had equal length.)

(b) What is the percentage chance that from 10 randomly chosen people at least two have their birthday in the same week? (Treat a year as exactly 52 whole weeks.)
NB: If your calculator cannot handle the large numbers involved, you could use WolframAlpha (`www.wolframalpha.com`) or some other on-line tool.

(c) By experimenting using WolframAlpha, or otherwise, find the minimum number N for which there is a better than even chance that from N randomly chosen people at least two have the same birthday. As a start, try $N = 25$.

Question 3 To evaluate the quality of a micro-finance program, some participants are selected, and their economic situation measured. In a given evaluation a sample of 100 participants, out of a total 10 000, is randomly selected. Every participant has the same probability of being selected.

Assume that 40 out of the 10 000 participants have actually seen their economic situation deteriorate. We seek the probability that at least one of these is in the sample.

(a) Describe a sample space S for this problem, and give a formula for $|S|$.

(b) Let E be the event "at least one of the participants in the random selection has seen her/his economic situation deteriorate". Give a formula for $|E^c|$.

(c) With the help of WolframAlpha, or some other electronic aid, calculate $\mathbb{P}(E)$.

Question 4 A *Binomial experiment* comprises a fixed number n of 'trials' where each trial has the same probability p of 'success'. The probability that a binomial experiment results in k successes is given by

$$\mathbb{P}(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}.$$

For example, what is the probability of scoring two 6s from twelve throws of a standard die? Here $n = 12$, $p = \dfrac{1}{6}$ and $k = 2$, so the the probability is $\dbinom{12}{2} \left(\dfrac{1}{6}\right)^2 \left(\dfrac{5}{6}\right)^{10}$.

(a) Use your calculator to find the probabilities of

  (i) two 6s from twelve throws,

  (ii) one 6 from twelve throws and

  (iii) no sixes from twelve throws

(b) Calculate the probability of scoring *at least two 6s* from twelve throws.

(c) In practice Binomial probabilities are usually found from a book of tables or from on-line tables or calculators. Both density and (cumulative) distribution values are available. With the help of an on-line statistical calculator such as *Stat Trek* (`http://stattrek.com/online-calculator/binomial.aspx`) find the probability of scoring more than two but less than eight 6s from 30 throws of a standard die. Give all the values you obtained, what they represented, and how you used them to obtain your answer.

Question 5 The income and education level of each person on the electoral roll for Queanberra is recorded as a pair $(x, y) \in \{1, 2, 3\}^2$, where 1 stands for low, 2 for average, and 3 for high, *e.g.* $(2, 3)$ represents a highly educated person with average income.

Let S denote the set of all people on the Queanberra electoral roll, and define random variables $X, Y : S \to \{1, 2, 3\}$ by $X(s), Y(s)$ are the income and educational levels of person s. Let $p_{i,j} = \mathbb{P}(\{(X(s) = i) \wedge (Y(s) = j)\})$ for $1 \leq i, j \leq 3$. Assume that

$$(p_{i,j})_{1\leq i,j\leq 3} = \begin{pmatrix} 0.05 & 0.10 & 0.05 \\ 0.10 & 0.20 & 0.10 \\ 0.15 & 0.20 & 0.05 \end{pmatrix}.$$

(a) Explain what $p_{1,2}$ represents.

(b) Find the probability of the event AI: "the person has an average income"?

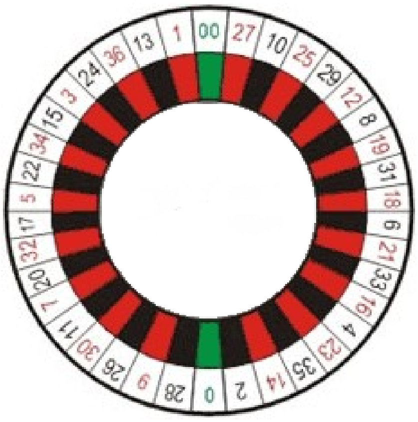
(c) Find the probability of the event AE: "the person has an average education level"?

(d) Describe the event $AI \cup AE$ and find its probability.

(e) Are the events AI and AE independent?

(f) Are the random variables X and Y independent?

Question 6 In the casino gambling game of American Roulette the wheel has 38 slots numbered $00, 0, 1, \ldots, 36$. Half of the slots numbered 1 to 36 are painted black, while the others are painted red. The slots numbered 00 and 0 are painted green. A ball is equally likely to land in any slot. Listed below are several of the many possible bets on where the ball lands, together with their winning payouts based on a \$1 stake. In each case calculate the expected return on a \$1 stake. (If you win, your return is your payout plus your stake; if you don't win, you lose your stake and so have no return.) The expected returns are of course all less than the the \$1 stake — the difference is the 'house edge'.



Bet	Specify	Win when ball lands in	Payout
'straight'	number in $\{00,\ldots,36\}$	pocket with that number	\$35
'first five'	—	pocket number 00, 0, 1, 2 or 3	\$6
'colour'	red or black	pocket of the specified colour	\$1