

Section 1.

1. In \mathbb{R}^2 , $\because V_3 \cdot V_1 = 0$ and $V_3 \cdot V_2 = 0$, $\therefore V_1$ and V_2 are orthogonal, also for V_3 and V_2 .

$\therefore V_1 = \lambda V_2$, $\lambda \in \mathbb{R}$, which is a contradiction with " $\{V_1, V_2\}$ is linearly independent."

$$\begin{aligned} 2. \quad P_A(\lambda) &= \det(A - \lambda I) = \det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) \\ &= \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} \end{aligned}$$

$$= (a - \lambda)(d - \lambda) - bc.$$

$$= \lambda^2 - (a + d)\lambda + (ad - bc).$$

For unique λ , $(a + d)^2 - 4(ad - bc) = 0$.

$$a^2 + 2ad + d^2 - 4ad + 4bc = 0.$$

$$(a - d)^2 = -4ac.$$

If a, b, c, d are all positive, then $-4ac < 0$.

However $(a - d)^2 \geq 0$, which is a contradiction.

\therefore Not exists such A .

