

INTRODUCTION AND RATIONALE

The applications of mathematics in real-life have always fascinated me. As an aspiring urban economist, I tend to be responsive to the happenings next to us that impact the lives of ordinary people. In this unusual year, my awareness of our circumstances has developed to the next stage where I would like to practically apply the knowledge of mathematics along with my analyzing skills as a future economist. My analysis would concentrate on this heatedly debated and impactful topic: COVID-19.

The economic and social disruption caused by COVID-19, viruses that trigger severe lung and heart diseases, were expanding globally. People's precious lives were threatened and their life qualities significantly dropped due to the pandemic. However, this difficult period inspired people to treasure natural resources and people around us, in addition to providing statistics to further enhance the understanding of the external world.

Although we cannot deny that our lifestyles were transformed since the pandemic had been outbreaken in March, 2020, it is essential to investigate each changing variable with respect to the fluctuation of COVID-19 cases to speculate a correlation in between, which leads us to a conclusion.

Having lived in a few major cities, I understood how transportations is important as the public depends on them. My interest in urban planning prompted me to pay extra attention on transporting passengers in airports of the world's two leading cities, New York City, US, and London, UK. The question arose to me: how much were they impacted; were the two biggest cities inevitably beaten as the rest of the world, or were they able to facilitate with incredible management and advanced technology.

To explore the transportation rates over pandemics, I chose to obtain the variables as numbers of passengers traveling directly between NYC & London and daily cases of COVID-19 in solely NYC or London and both. For the precision, I eliminated the possibilities of economic factors by utilizing the number of passengers instead of numbers of flights, which will more likely be influenced by airline companies and political limitations. Besides, I decided to analyze the yearly data, starting from March, 2020 and ending at February, 2021, to present a thorough analysis. In such cases, I think the comprehensive nature of an economist has been met. I am excited that I can finally apply multiple mathematical approaches that I learned in class into practical applications to conjecture the uncertainties in the world, especially under this time period.

In concrete terms, reflecting the urban economist as I am ambitious to become, I would employ the theoretical models of cases and passenger numbers to dissect with real-life considerations and inquire into their correlations to speculate the future.

AIM AND APPROACH

As established in the introduction, the aim of this exploration will be to determine whether the virus outbreak can speculate the traveling flows with uncertainties by rationalizing the mathematical models. The model will relate daily changing rate of COVID-19 cases within certain range of cities, corresponding to that of passenger numbers (which will both be interpreted in bi-monthly) in the recent year where the first three seasons will be analyzed for conclusive correlation and the last season will be speculated based on the previous.

Secondary data were collected on governmentally authorized reliable sources; however, I planned to heavily reinterpret those to effectively conduct my investigation.

I decided to pragmatically approach the problem of people traveling for vacation, especially during the summer months, by applying my knowledge on calculus and statistics. I realized that

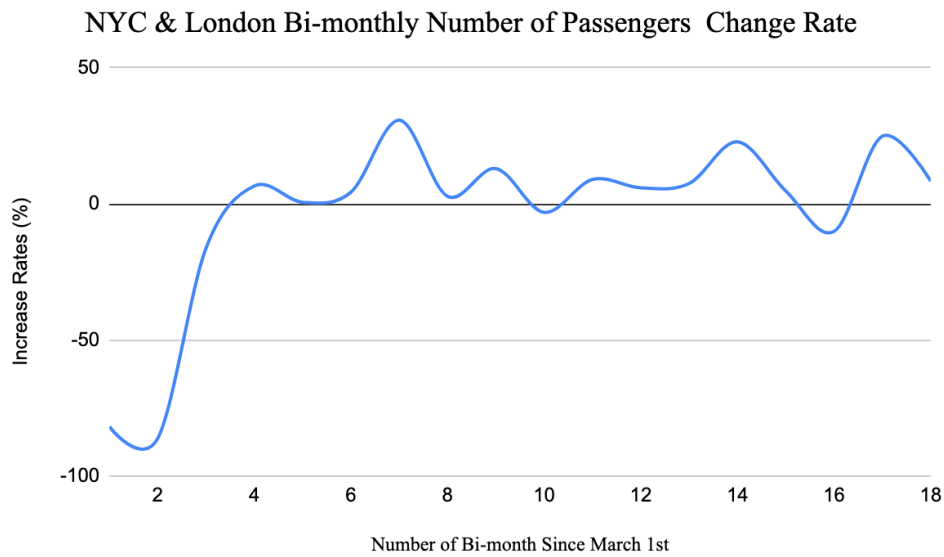
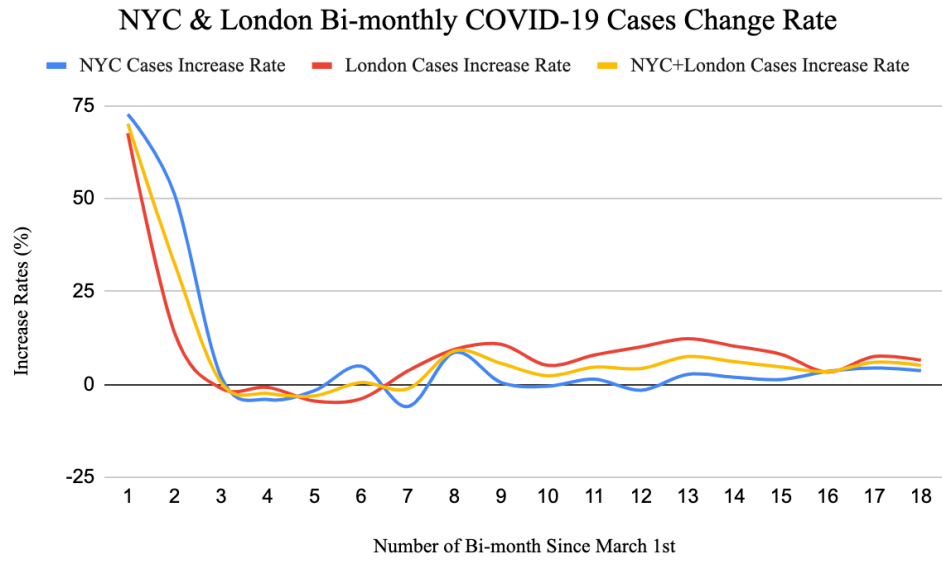
first I need to plot in the original secondary data to observe a trend of independent variable vs. time.

DATA COLLECTION AND RESULTS

Exactly one year's data of COVID-19 cases in NYC and London were collected and interpreted manually as cities' bi-monthly cases changing rate to concentrate on speculating correlation. Likewise, exactly one year's data of numbers of passengers and flights between NYC and London were collected and interpreted manually as bi-monthly cases changing rate to correspond to COVID-19 cases. To universalize the time zone difference, coordinated universal time (UTC) is regulated to avoid possible confusions. Although only the recent year data were used, the access to the last half of February, 2020, is necessary according to its involvement in the change rate over time formula.

In order to precisely speculate the trend of future traveling passengers, COVID-19 cases of NYC and London were separately interpreted before they combined. Data's change rates were interpreted in percentage to simplify the calculations. Due to the bi-month interval, the first three seasons were equally divided into 18 groups over time since March, 2020.

The data obtained summarised on the following shows daily COVID-19 cases change rates of NYC and London and the number of passengers traveling between NYC and London over the same period.



MODELING AND MATHEMATICAL APPLICATION OF RESULTS

I. Interpretation of original data (COVID-19 cases)

Formula of increase percentage over time:

$$A(x) = 100\% \times \frac{f(b) - f(a)}{|f(a)|}$$

COVID – 19 cases increase percentage present day

$$=100\% \times \frac{(\text{daily increased cases present day}) - (\text{daily increased cases last day})}{\text{daily increased cases last day}}$$

i.e. daily new cases in New York City, NY, US, on March

month/date/year (UTC)	Daily New Cases
3/11/2020	7
3/12/2020	13

$$\text{Cases increase percentage in New York City on March 12th} = 100\% \times \frac{13 - 7}{7} \approx 85.7\%$$

i.e. daily new cases in London, UK, on March

month/date/year (UTC)	Daily New Cases
3/11/2020	183
3/12/2020	164

$$\text{Cases increase percentage in London on March 12th} = 100\% \times \frac{164 - 183}{183} \approx -10.4\%$$

Bi monthly cases increase percentage

$$= \frac{\Sigma (\text{daily increase percentage in the first/second half month})}{\text{numbers of dates in the first/second half month}}$$

i.e. daily increase percentage in New York City, US, and London, UK, on April

month/date/year (UTC)	NYC Daily Increase Percentage	London Daily Increase Percentage
4/1/2020	0.087	0.046
4/2/2020	-0.065	0.091
4/3/2020	0.113	-0.081
4/4/2020	0.017	-0.22
4/5/2020	0.046	-0.174
4/6/2020	-0.025	0.361
4/7/2020	0.144	0.107
4/8/2020	-0.05	-0.081
4/9/2020	0.033	-0.085
4/10/2020	0.058	-0.227
4/11/2020	0.027	-0.158
4/12/2020	0.021	-0.038
4/13/2020	-0.006	0.194
4/14/2020	-0.127	-0.002

4/15/2020	0.032	0.106
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The 3rd bi – monthly cases increase percentage in New York City since March 1st

$$= \frac{\text{increase (\%)}}{\text{increase (\%)}} = 100\% \times \frac{\Sigma (\text{daily increase percentage from 4/1–4/15})}{15 \text{ days}} = 100\% \times \frac{0.305}{15}$$

$$\approx 2.0\%$$

The 3rd bi – monthly cases increase percentage in London since March 1st

$$= \frac{\text{increase (\%)}}{\text{increase (\%)}} = 100\% \times \frac{\Sigma (\text{daily increase percentage from 4/1–4/15})}{15 \text{ days}} = 100\% \times \frac{-0.161}{15}$$

$$\approx -1.1\%$$

Merge increase percentage:

$$3\text{rd bi – monthly cases increase percentage in total} = 2.0\% + (-1.1\%) \approx 0.5\%$$

Thus:

Number of Bi-month	NYC Increase Percentage	London Increase Percentage	NYC+London Increase Percentage
3	2.0%	-1.1%	0.5%

II. Interpretation of original data (numbers of passengers cases)

Bi-monthly numbers of passengers traveling between NYC and London

$x =$

$$\frac{\Sigma (\text{weekly numbers of flights from NYC to London} + \text{weekly numbers of flights from London to NYC})}{\Sigma (\text{weekly numbers of flights from US to UK} + \text{weekly numbers of flights from UK to US})}$$

\times

$$\Sigma (\text{weekly numbers of passengers travel from NYC to London} + \text{weekly numbers of passengers travel from London to NYC})$$

i.e. bi-monthly numbers of passengers traveling between NYC and London

dates (weekly)	US flights to UK	UK flights to US	Flights between cities	US passengers to UK	UK passengers to US
2/15-2/21/2020	1245	1268	680	399065	403428
2/22-2/29/2020	1274	1176	680	396785	398015
3/1-3/7/2020	638	687	397	113280	100767
3/8-3/15/2020	227	236	136	50370	403765

February second bi-month numbers of passengers traveling between NYC and London

$$= \frac{(680+680)}{(1245+1268+1274+1176)} \times (399065 + 403428 + 396785 + 398015) = 437702$$

Likewise, March first bi-month numbers of passengers traveling between NYC and London

$$= \frac{(397+136)}{(638+687+227+236)} \times (113280 + 100767 + 50370 + 40765) = 90974$$

Formula of increase percentage over time (numbers of passengers)

$$A(x) = 100\% \times \frac{f(b)-f(a)}{|f(a)|}$$

$$\text{Change rate of number of passengers in the first bi-month} = 100\% \times \frac{90974-437702}{437702} \approx -79.2\%$$

Thus:

bi-monthly (UTC)	change rate of numbers of passengers traveling between NYC and London
1	-0.792

III. Calculation of the trendlines of both variables

The trendlines of bi-monthly change rate of COVID-19 cases, quartic regressions, are presented by using Google Spreadsheet due to the calculation complexity. (will talk about R Square and why chose quartic regression)

$$\text{NYC Cases Increase: } y_1 = 0.735 - 0.414x + 0.0749x^2 - 5.37E-3x^3 + 1.33E-4x^4$$

$$\text{London Cases Increase: } y_2 = 0.583 - 0.406x + 0.0832x^2 - 6.3E-3x^3 + 1.6E-4x^4$$

$$\text{Average of NYC \& London Cases Increase: } y_3 = 0.659 - 0.41x + 0.079x^2 - 5.84E-3x^3 + 1.46E-4x^4$$

The trendline of number of passengers can be presented as a cubic regression, which can be solved by constructing the method of matrix: (will use DESMO to verify the regression equation)

$$A = \begin{bmatrix} x_1^3 & x_1^2 & x_1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ x_{18}^3 & x_{18}^2 & x_{18} & 1 \end{bmatrix} \quad \text{and let:} \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_{18} \end{bmatrix}$$

$$A^T = \begin{bmatrix} x_1^3 & x_2^3 & \dots & x_{17}^3 & x_{18}^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix}$$

$$\text{The formula } x = (A^*A)^{-1}A^*y = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \text{ can be used to determine the cubic regression } y = ax^3 + bx^2 + cx + d$$

Data substitution:

$$\begin{bmatrix} 1^3 & 1^2 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 18^3 & 18^2 & 18 & 1 \end{bmatrix} \quad \begin{bmatrix} -0.792 \\ -0.862 \\ \vdots \\ 0.249 \\ 0.084 \end{bmatrix} \quad \begin{bmatrix} 1^3 & 2^3 & \dots & 17^3 & 18^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix}$$

$$A = y = A^T =$$

$$x = \left\{ \begin{bmatrix} 1^3 & 2^3 & \dots & 17^3 & 18^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix} \begin{bmatrix} 1^3 & 1^2 & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 18^3 & 18^2 & 18 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1^3 & 2^3 & \dots & 17^3 & 18^3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \dots & 1 & 1 \end{bmatrix} \begin{bmatrix} -0.792 \\ -0.862 \\ \vdots \\ 0.249 \\ 0.084 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 8 & 27 & 64 \\ 1 & 4 & 9 & 16 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 15 & 40 & 85 \\ 15 & 85 & 259 & 585 \\ 40 & 259 & 820 & 1885 \\ 85 & 585 & 1885 & 4369 \end{pmatrix}$$

Details (Matrix multiplication)

Matrix multiplication: the rows of the first matrix are multiplied by the columns of the second one.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 8 & 27 & 64 \\ 1 & 4 & 9 & 16 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 & 1 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 1 \cdot 1 & 1 \cdot 27 + 1 \cdot 9 + 1 \cdot 3 + 1 \cdot 1 & 1 \cdot 64 + 1 \cdot 16 + 1 \cdot 4 + 1 \cdot 1 \\ 8 \cdot 1 + 4 \cdot 1 + 2 \cdot 1 + 1 \cdot 1 & 8 \cdot 8 + 4 \cdot 4 + 2 \cdot 2 + 1 \cdot 1 & 8 \cdot 27 + 4 \cdot 9 + 2 \cdot 3 + 1 \cdot 1 & 8 \cdot 64 + 4 \cdot 16 + 2 \cdot 4 + 1 \cdot 1 \\ 27 \cdot 1 + 9 \cdot 1 + 3 \cdot 1 + 1 \cdot 1 & 27 \cdot 8 + 9 \cdot 4 + 3 \cdot 2 + 1 \cdot 1 & 27 \cdot 27 + 9 \cdot 9 + 3 \cdot 3 + 1 \cdot 1 & 27 \cdot 64 + 9 \cdot 16 + 3 \cdot 4 + 1 \cdot 1 \\ 64 \cdot 1 + 16 \cdot 1 + 4 \cdot 1 + 1 \cdot 1 & 64 \cdot 8 + 16 \cdot 4 + 4 \cdot 2 + 1 \cdot 1 & 64 \cdot 27 + 16 \cdot 9 + 4 \cdot 3 + 1 \cdot 1 & 64 \cdot 64 + 16 \cdot 16 + 4 \cdot 4 + 1 \cdot 1 \end{pmatrix} = \begin{pmatrix} 4 & 15 & 40 & 85 \\ 15 & 85 & 259 & 585 \\ 40 & 259 & 820 & 1885 \\ 85 & 585 & 1885 & 4369 \end{pmatrix}$$

NOTE: needs more time to process since it is a long matrix and something not covered in the curriculum.

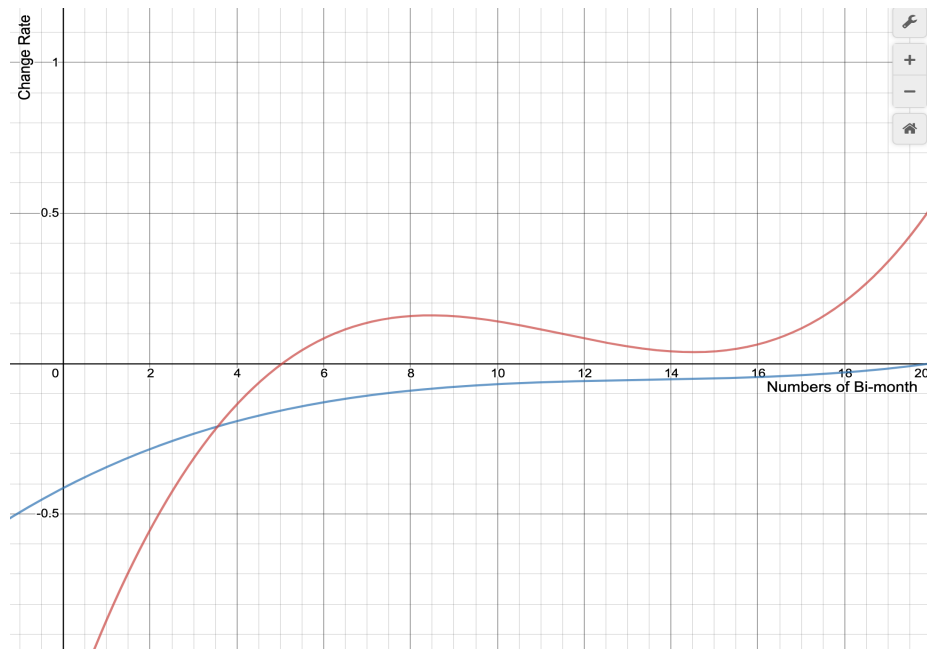
IV. Differentiation of quartic regression

Since the local maximum and local minimums of the calculated quartic regressions have a similar x coordinates as the roots of the calculated cubic regression, the differentiation of those will possibly be identical to the cubic regression. Therefore, a correlation can possibly be established based on the premises.

Change rate of NYC COVID-19 cases $= \frac{d}{dx}$

$$(0.735 - 0.414x^1 + 0.0749x^2 - 5.37E - 3x^3 + 1.33E - 4x^4)$$

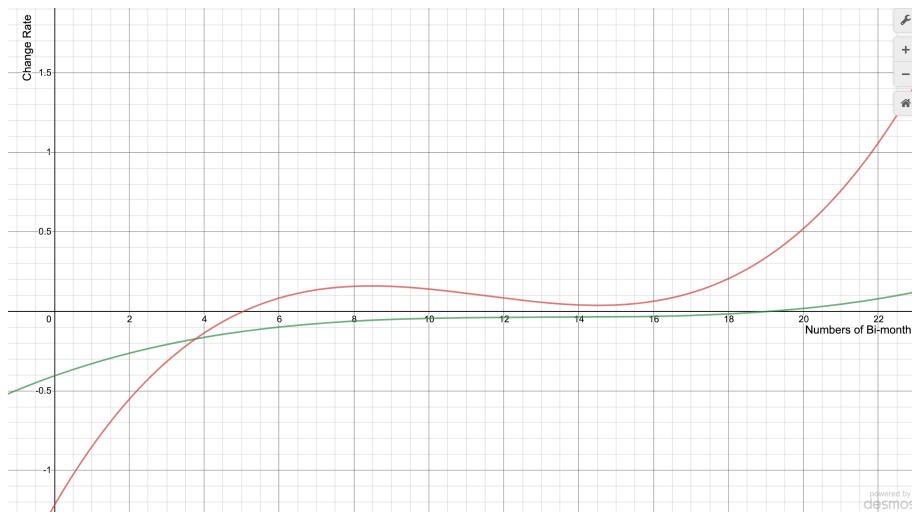
$$\text{Thus: } y_1' = 0.735 \times (0) - 0.414x^0 + 0.0749x^1 - 5.37E - 3x^2 + 1.33E - 4x^3$$



Change rate of London COVID-19 cases $= \frac{d}{dx}$

$$(0.583 - 0.406x^1 + 0.0832x^2 - 6.3E - 3x^3 + 1.6E - 4x^4)$$

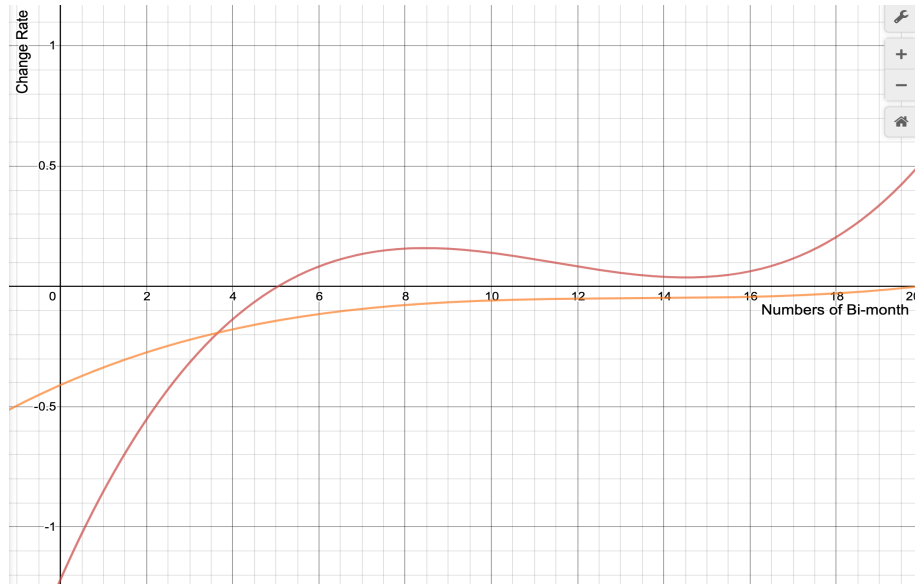
$$\text{Thus: } y_2' = 0.583 \times (0) - 0.406x^0 + 0.0832x^1 - 6.3E - 3x^2 + 1.6E - 4x^3$$



Change rate of total COVID-19 cases $= \frac{d}{dx}$

$$(0.659 - 0.41x^1 + 0.079x^2 - 5.84E - 3x^3 + 1.46E - 4x^4)$$

$$y_3' = 0.659 \times (0) - 0.41x^0 + 0.079x^1 - 5.84E - 3x^2 + 1.46E - 4x^3$$



V. Intersections of the cubic regression and the differentiations

In order to further investigate the cubic equations, calculations of intersecting points of those are necessary.

Let

$$-0.414 + 0.0749x - 0.00537x^2 + 0.000133x^3 = -1.22 + 0.405x - 0.0379x^2 + 0.0011x^3$$

$$\text{Relocate corresponding components, so } 0.000967x^3 - 0.03253x^2 + 0.3301x - 0.806 = 0$$

Since none of them are perfect cubic equation, Cardano Cubic Formula will be applied:

$$x = \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)} + \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3} + \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)} - \sqrt[3]{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3} - \frac{b}{3a}$$

$$\text{Where } \frac{-b^3}{27a^3} = \frac{-(-0.03253)^3}{27(0.000967)^3}, \frac{bc}{6a^2} = \frac{(-0.03253) \times 0.3301}{6(0.000967)^2}, \frac{d}{2a} = \frac{(-0.806)}{2(0.000967)}, \frac{c}{3a} = \frac{0.3301}{3(0.000967)},$$

$$\frac{b^2}{9a^2} = \frac{(-0.03253)^2}{9(0.000967)^2}, \frac{b}{3a} = \frac{(-0.03253)}{3(0.000967)}, \text{ followed:}$$

$$\begin{aligned}
x &= \sqrt[3]{1409.967 - 1913.927 + 416.753 + \sqrt{(1409.967 - 1913.927 + 416.753)^2 + (113.788 - 125.740)^3}} \\
&\quad \sqrt[3]{1409.967 - 1913.927 + 416.753 - \sqrt{(1409.967 - 1913.927 + 416.753)^2 + (113.788 - 125.740)^3}} + 11.213 \\
&= \sqrt[3]{-87.207 + 76.797} + \sqrt[3]{-87.207 - 76.797} + 11.213
\end{aligned}$$

Thus: $x_1 = 3.556$, $x_2 = 15.04196 + 2.84958i$, $x_3 = 15.04196 - 2.84958i$ (will explain the imagery roots later to involve more topic)

Similarly,

let

$$-0.406 + 0.0832x - 0.0063x^2 + 0.00016x^3 = -1.22 + 0.405x - 0.0379x^2 + 0.0011x^3$$

Relocate corresponding components, so $0.00094x^3 - 0.0316x^2 + 0.3218x - 0.814 = 0$

$$\text{Where } \frac{-b^3}{27a^3} = \frac{-(-0.0316)^3}{27(0.00094)^3}, \frac{bc}{6a^2} = \frac{(-0.0316) \times 0.3218}{6(0.00094)^2}, \frac{d}{2a} = \frac{(-0.814)}{2(0.00094)}, \frac{c}{3a} = \frac{0.3218}{3(0.00094)},$$

$$\frac{b^2}{9a^2} = \frac{(-0.0316)^2}{9(0.00094)^2}, \frac{b}{3a} = \frac{(-0.0316)}{3(0.00094)}, \text{ followed:}$$

$$\begin{aligned}
z &= \sqrt[3]{1407.064 - 1918.078 + 432.979 + \sqrt{(1407.064 - 1918.078 + 432.979)^2 + (114.113 - 125.567)^3}} \\
&\quad \sqrt[3]{1407.064 - 1918.078 + 432.979 - \sqrt{(1407.064 - 1918.078 + 432.979)^2 + (114.113 - 125.567)^3}} + 11.206 \\
&= \sqrt[3]{-78.035 + 67.726} + \sqrt[3]{-78.035 - 67.726} + 11.206
\end{aligned}$$

Thus: $z_1 = 3.767$, $z_2 = 14.92523 + 2.6729i$, $z_3 = 14.92523 - 2.6729i$

Similarly,

let

$$-0.41 + 0.079x - 0.00584x^2 + 0.000146x^3 = -1.22 + 0.405x - 0.0379x^2 + 0.0011x^3$$

Relocate corresponding components, so $0.000954x^3 - 0.03206x^2 + 0.326x - 0.81 = 0$

$$\text{Where } \frac{-b^3}{27a^3} = \frac{-(-0.03206)^3}{27(0.000954)^3}, \frac{bc}{6a^2} = \frac{(-0.03206) \times 0.326}{6(0.000954)^2}, \frac{d}{2a} = \frac{(-0.81)}{2(0.000954)}, \frac{c}{3a} = \frac{0.326}{3(0.000954)},$$

$$\frac{b^2}{9a^2} = \frac{(-0.03206)^2}{9(0.000954)^2}, \frac{b}{3a} = \frac{(-0.03206)}{3(0.000954)}, \text{ followed:}$$

$$\begin{aligned}
p &= \sqrt[3]{1405.664 - 1913.961 + 424.528 + \sqrt{(1405.664 - 1913.961 + 424.528)^2 + (113.906 - 125.484)^3}} \\
&\quad \sqrt[3]{1405.664 - 1913.961 + 424.528 - \sqrt{(1405.664 - 1913.961 + 424.528)^2 + (113.906 - 125.484)^3}} + 11.206
\end{aligned}$$

$$= \sqrt[3]{-83.769 + 73.927} + \sqrt[3]{-83.769 - 73.927} + 11.202$$

$$\text{Thus: } p_1 = 3.656, p_2 = 14.97475 + 2.82301i, p_3 = 14.97475 - 2.82301i$$

Eliminating all imaginary roots since they are not real numbers of intersections.

VI. Integration for the area between curves

To investigate the similarities of the differentiations to the original cubic regression, the method of integration for area between curves can be calculated to compare their outcomes.

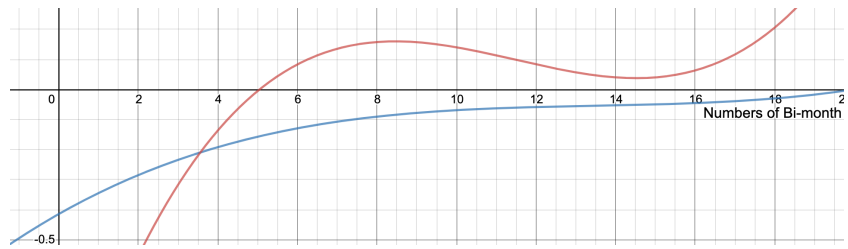
Within the domain of $0 \leq x \leq 18$

, the first interval is

$$0 \leq x \leq 3.556 (\text{first intersection}),$$

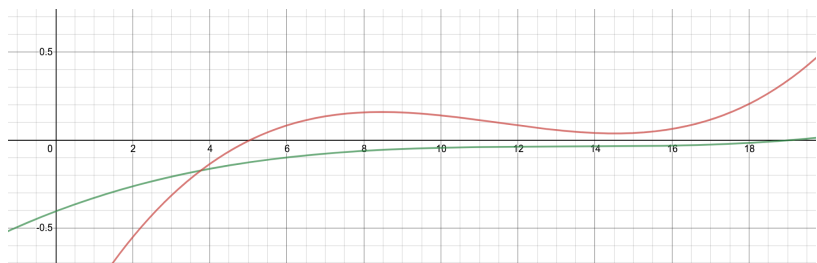
and the second interval is

$$3.556 \leq x \leq 18.$$



$$\begin{aligned} AREA_1 &= \left| \int_0^{3.556} [(-0.414 + 0.0749x - 0.00537x^2 + 0.000133x^3) - (-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3)] dx \right. \\ &\quad \left. + \int_{3.556}^{18} [(-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3) - (-0.414 + 0.0749x - 0.00537x^2 + 0.000133x^3)] dx \right| \\ &= \left| \int_0^{3.556} (0.000967x^3 - 0.03253x^2 + 0.3301x - 0.806) dx + \int_{3.556}^{18} (-0.000967x^3 + 0.03253x^2 - 0.3301x + 0.806) dx \right| \\ &= \left| \left[\frac{0.000967}{4}x^4 - \frac{0.03253}{3}x^3 + \frac{0.3301}{2}x^2 - 0.806x + C \right]_0^{3.556} + \left[-\frac{0.000967}{4}x^4 + \frac{0.03253}{3}x^3 - \frac{0.3301}{2}x^2 + 0.806x + D \right]_{3.556}^{18} \right| \\ &= |-1.267 - 2.336| \approx 3.602 \end{aligned}$$

Likewise, within the domain of $0 \leq x \leq 18$, the first interval is $0 \leq x \leq 3.767$ (first intersection), and the second interval is $3.767 \leq x \leq 18$.



$$AREA_2 = \left| \int_0^{3.767} [(-0.406 + 0.0832x - 0.0063x^2 + 0.00016x^3) - (-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3)] dx \right.$$

$$\begin{aligned}
& + \left| \int_{3.767}^{18} [(-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3) - (-0.406 + 0.0832x - 0.0063x^2 + 0.00016x^3)] dx \right| \\
& = \left| \int_0^{3.767} (0.00094x^3 - 0.0316x^2 + 0.3218x - 0.814) dx + \int_{3.767}^{18} (-0.00094x^3 + 0.0316x^2 - 0.3218x + 0.814) dx \right| \\
& = \left| \left[\frac{0.0094}{4}x^4 - \frac{0.0316}{3}x^3 + \frac{0.3218}{2}x^2 - 0.814x + C \right]_0^{3.767} + \left[-\frac{0.0094}{4}x^4 + \frac{0.0316}{3}x^3 - \frac{0.3218}{2}x^2 + 0.814x + D \right]_{3.767}^{18} \right| \\
& = |-1.299 - 2.017| \approx 3.316
\end{aligned}$$

Likewise, within the domain of $0 \leq x \leq 18$, the first interval is $0 \leq x \leq 3.767$ (first intersection), and the second interval is $3.767 \leq x \leq 18$.



$$\begin{aligned}
AREA_3 & = \left| \int_0^{3.656} [(-0.41 + 0.079x - 0.00584x^2 + 0.000146x^3) - (-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3)] dx \right| \\
& + \left| \int_{3.656}^{18} [(-1.22 + 0.405x - 0.0379x^2 + 0.0011x^3) - (-0.41 + 0.079x - 0.00584x^2 + 0.000146x^3)] dx \right| \\
& = \left| \int_0^{3.656} (0.000954x^3 - 0.03206x^2 + 0.326x - 0.81) dx + \int_{3.656}^{18} (-0.000954x^3 + 0.03206x^2 - 0.326x + 0.81) dx \right| \\
& = \left| \left[\frac{0.00954}{4}x^4 - \frac{0.03206}{3}x^3 + \frac{0.326}{2}x^2 - 0.81x + C \right]_0^{3.656} + \left[-\frac{0.00954}{4}x^4 + \frac{0.03206}{3}x^3 - \frac{0.326}{2}x^2 + 0.81x + D \right]_{3.656}^{18} \right| \\
& = |-1.262 - 2.206| \approx 3.469
\end{aligned}$$

The differentiation of the quartic regression of London COVID-19 cases bi-monthly change rate is chosen for further data analysis due to its calculation of the smallest area.

To BE CONTINUED...