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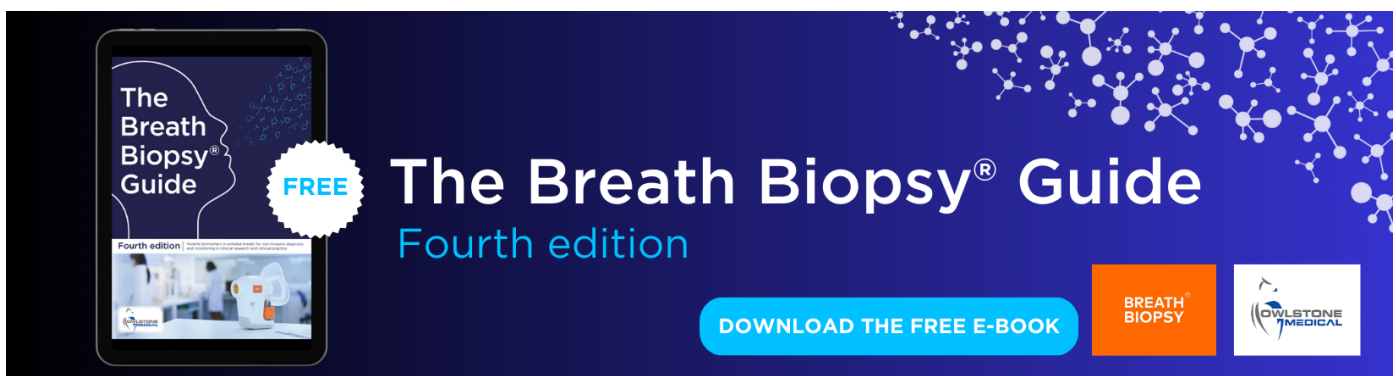
A novel training-free recognition method for SSVEP-based BCIs using dynamic window strategy

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A novel training-free recognition method for SSVEP-based BCIs using dynamic window strategy

Yonghao Chen¹ , Chen Yang^{1,2}, Xiaogang Chen³ , Yijun Wang⁴ and Xiaorong Gao¹¹ Department of Biomedical Engineering, School of Medicine, Tsinghua University, Beijing 100084, People's Republic of China² School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, People's Republic of China³ Institute of Biomedical Engineering, Chinese Academy of Medical Sciences and Peking Union Medical College, Tianjin 300192, People's Republic of China⁴ State Key Laboratory on Integrated Optoelectronics, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, People's Republic of ChinaE-mail: gxr-dea@mail.tsinghua.edu.cn**Keywords:** brain–computer interface (BCI), steady-state visual evoked potential (SSVEP), dynamic window, canonical correlation analysis (CCA), filter bank**Abstract**

Objective. Filter bank canonical correlation analysis (FBCCA) is a widely-used classification approach implemented in steady-state visual evoked potential (SSVEP)–based brain computer interfaces (BCIs). However, conventional detection algorithms for SSVEP recognition problems, including the FBCCA, were usually based on ‘fixed window’ strategy. That’s to say, these algorithms always analyze data with fixed length. This study devoted to enhance the performance of SSVEP-based BCIs by designing a new dynamic window strategy which automatically finds an optimal data length to achieve higher information transfer rate (ITR). **Approach.** The main purpose of ‘dynamic window’ is to minimize the required data length while maintaining high accuracy. This study projected the correlation coefficients of FBCCA into probability space by softmax function and built a hypothesis testing model, which took risk function as evaluation of classification result’s ‘credibility’. In order to evaluate the superiority of this approach, FBCCA with fixed data length (FBCCA-FW) and spatial temporal equalization dynamic window (STE-DW) were implemented for comparison. **Main results.** Fourteen healthy subjects’ results were concluded by a 40-target online SSVEP-based BCI speller system. The results suggest that this proposed approach significantly outperforms STE-DW and FBCCA-FW in terms of accuracy and ITR. **Significance.** By incorporating the fundamental ideas of FBCCA and dynamic window strategy, this study proposed a new training-free dynamical optimization algorithm, which significantly improved the performance of online SSVEP-based BCI systems.

1. Introduction

Brain computer interface (BCI) is a technique that provides a direct connection between human brain and external devices [1, 2]. BCI presents a promising approach that expresses user’s intention without any dependence on peripheral muscles and nerves, controls wearable devices without physical activities [3–5]. Electroencephalogram (EEG)-based BCI is a principal category of noninvasive BCIs. A variety of online EEG-based BCI systems were created for clinical rehabilitation systems, adaptive human-computer interfaces, communication apparatuses, etc [3, 6]. A series of experimental paradigms were

devised to build a practical and robust BCI system in real-world environments. Typically, steady state visual evoked potential (SSVEP)–based BCIs have elicited considerable attention because they are non-invasive, less dependent on training data and achieve extremely high information transfer rate (ITR) compared with other paradigms [6, 7]. SSVEP is a kind of evoked EEG signals, mostly exists in the occipital scalp region of human brain, induced by visual flickers flashing at a fixed frequency. Furthermore, SSVEP contains frequency components at the fundamental and harmonic frequencies of stimulus, which offers a convenient proposal to develop a visual encoding BCI by enciphering EEG spectrums. SSVEP can

easily be detected when participants fixated at stimulating interface, thus SSVEP-based BCIs are suitable and stable enough as communication systems. Many practical SSVEP-based BCIs have achieved significant performance and have been successfully applied on clinical environments with different subjects [8, 9].

One main research direction of BCI is to enhance its performance and applicability. To improve the performance of SSVEP-based BCIs, two main strategies were predominant: finding a more efficient paradigm or developing better target detection algorithms. Different stimulation methods tried different combinations of frequency and phase to arouse more strong response activities while maintaining the number of targets as large as possible. In other words, more targets offer more available commands but usually increase the difficulties of classification. Therefore, the stimulation system needs to compromise between classification accuracy and number of targets [10]. An efficient paradigm can adopt more targets and evoke strong response activities. A 40-targets hybrid frequency and phase coding paradigm, which was also adopted in this study, has been proposed as a capable and stable stimulation paradigm in SSVEP-based BCIs [11, 12].

Target detection methods also play an important role in SSVEP-based BCIs. Improvements toward recognition algorithms aim to boost classification accuracy and decrease average detection time. Those methods can be divided into training-free methods which do not require training data and training methods which definitely require extra training data [13, 14]. Training methods relatively achieve better performance with the assistance of training data, but training methods have to spend long time on obtaining training data of same experimental subjects before experiments. The process of acquiring training data is especially complex in multi-targets SSVEP-based BCIs because data for all targets have to be collected. In other sides, limited by indispensable requirement of training data, systems based on training methods are incapable to new subjects. While the training-free methods, which do not have any previous data requirements, are undeniably better choices than training methods in terms of practicality and convenience.

Early related algorithms detected targets by analyzing power spectral density of single channel EEG signal [15], but obviously, problems of low accuracy and inadequate utilization of multi-channel EEG signals appeared afterwards. Spatial filtering is the most widespread preprocessing method used in SSVEP-based BCI recognition problems. Canonical correlation analysis (CCA) is a well-known training-free classification approach based on spatial filtering. CCA finds two spatial filters corresponding to multi-channels signals and predefined sine-cosine templates, which maximize the correlation coefficient

between those two variables [16–18]. The spatial filtering process in CCA actually intensifies the task-related components. Most common spatial filtering methods such as Minimum energy combination (MEC) and Maximum contrast combination (MCC), are also based upon finding the optimal spatial filters to enhance the SNR of signals [19, 20]. Chen *et al* proposed filter bank canonical correlation analysis (FBCCA) which greatly improved the classification accuracy comparing with standard CCA [21]. FBCCA decomposes SSVEP into several sub-band components and calculates correlation coefficients on each of the sub-bands separately, then takes the weighted features for classification. What's more, filter bank technique has also been successfully operated in other spatial filtering-based SSVEP detection methods [22, 23].

In most current studies, traditional spatial filtering-based recognition approaches only analyzed data of fixed length, in other words, most methods recognized targets by data of predetermined and unchanged length. In practical BCI applications, researchers often choose to search an optimal window length to achieve the highest ITR. However, the optimal window length is distinctive and susceptible from subject to subject and time to time because of spontaneous EEG's complexity and nonstationarity [24]. Apart from this, subjects' mental condition and environment variation are also undoubted factors of those distinctions [25]. To address these problems, 'dynamic window' methods were proposed to weaken the negative influence of those factors [26–29]. Yang *et al* proposed a dynamic window recognition algorithm using spatial-temporal equalizer (STE-DW) and demonstrated an online SSVEP-based BCI system adopting the strategy of dynamic window [26]. Jiang *et al* proposed a dynamic stopping method based on training data to enhance the performance of high-speed SSVEP-based BCI system [27]. Canonical correlation analysis reducing variation (CCA-RV) is another algorithm which finds an optimal window length, while it has to utilize additional training data and spend long time on training process [28]. An asynchronous SSVEP detection system with few numbers of targets was proposed, based on CCA [30], while the feasibility and robustness of the method remain uncertain when applied in multi-targets systems.

Motivated by the basic strategies of dynamic window detection methods and statistical knowledge. A novel training-free dynamic window recognition method based on FBCCA was proposed in this study, which can dynamically determine appropriate data length during recognition process. By considering that the classification problem in SSVEP-based BCI is similar with a hypothesis testing problem, the proposed method projects different correlation coefficients into a statistical probability model, and then

utilizes the normalized maximum and second maximum features of FBCCA to create a new strategy of threshold selection. This method was successfully applied to an online realistic system and significantly improved the system performance. The rest of this paper is organized as follows. Section 2 illustrates methods and mathematical models. Section 3 mainly describes the experimental environment and settings and two algorithms for comparison. Section 4 discusses the result of online experiments and offline experiments. Finally, section 5 contains further exploring about the result of test and discusses some details. At last, a brief conclusion was concluded in section 6.

2. Methods

2.1. Standard canonical correlation analysis

CCA is an effective spatial filter method used in SSVEP detection, which measures underlying correlation of two sets of multi-dimensional variables through statistical analysis. When implemented in actual multi-targets SSVEP-based BCI systems, standard CCA provides an approach of feature detection by calculating the correlation coefficients between EEG signals and generalized sinusoidal templates of different frequency [16].

Let N_c , N_s , N_h , N_b , f_s denote the number of EEG channels, the number of sampling points, the number of considered maximum harmonics, the number of stimulation targets, and the sampling frequency. Most current researches used 5 as the optimal number of N_h towards the highest classification accuracy [16]. One reason why CCA is so effective and robust is the clever design of unsupervised sinusoidal templates [31], which makes full use of the prior information in SSVEP stimulation sequence. Many CCA-related algorithms attempted to find better templates as substitutions of standard sinusoidal templates [31–33]. Considering multi-channels EEG signals $X \in R^{N_c \times N_s}$ and sine-cosine templates $Y \in R^{2N_h \times N_s}$, Y matching with stimulation frequency f is written as:

$$Y_f = \begin{pmatrix} \sin(2\pi ft) \\ \cos(2\pi ft) \\ \vdots \\ \sin(2\pi f N_h t) \\ \cos(2\pi f N_h t) \end{pmatrix} \quad t = \frac{1}{f_s}, \frac{2}{f_s}, \dots, \frac{N_s}{f_s} \quad (1)$$

CCA finds two spatial filters $W_x \in R^{N_c \times 1}$ and $W_y \in R^{2N_h \times 1}$ which maximize the correlation coefficient between filtered EEG signals and templates. It can be solved by addressing following problem:

$$CCA(X, Y_f) = \underset{W_x, W_y}{\operatorname{argmax}} \frac{E(W_x^T X Y_f^T W_y)}{E(W_x^T X X^T W_x) E(W_y^T Y_f Y_f^T W_y)} \quad (2)$$

Consequently, the coefficients between X and different templates Y are calculated as features for classification. Virtually, only the frequency of sinusoidal template, which corresponds the largest correlation coefficients, is considered to be the predicted frequency of stimulation. It can be described as:

$$f_{\text{test}} = \underset{f}{\operatorname{argmax}} CCA(X, Y_f) \quad (3)$$

2.2. Filter bank technique

Filter bank technique was proposed to enhance the classification accuracy of BCI systems [21, 34], and it was successfully promoted into the area of SSVEP-based BCIs. Filter bank analysis decomposes SSVEP signals into several sub-band components. For the i_{fb} th sub-band, the original signals are filtered by a band pass filter which has same upper cut-off frequency at 88 Hz and a different lower cut-off frequency at $i_{fb} \times 8$ Hz. Let N_{fb} denotes the number of filter banks. $X_{i_{fb}}$ indicates the filtered data of i_{fb} th sub-bands. Relatively, FBCCA calculates CCA between filtered data and different templates in different sub-bands.

$$\rho(i_{fb}, i_f) = CCA(X_{i_{fb}}, Y_{i_f}) \quad i_{fb} = 1, 2, \dots, N_{fb} \quad i_f = 1, 2, \dots, N_t \quad (4)$$

A weighted summation of squares of those correlation coefficients is calculated as the feature for target classification. The weighted matrix has been optimized in [21]. The weight corresponding with the m th sub-band was defined as:

$$w(i_{fb}) = (i_{fb})^{-1.25} + 0.25 \quad i_{fb} = 1, 2, \dots, N_{fb} \quad (5)$$

With the same idea inspired by the standard CCA, target class can be identified by the largest coefficient:

$$\begin{aligned} f_{\text{test}} &= \underset{f}{\operatorname{argmax}} \sum_{i_{fb}=1}^{N_{fb}} w(i_{fb}) * \rho(i_{fb}, i_f)^2 \\ &= \underset{f}{\operatorname{argmax}} \sum_{i_{fb}=1}^{N_{fb}} w(i_{fb}) * CCA(X_{i_{fb}}, Y_{i_f})^2 \end{aligned} \quad (6)$$

2.3. Dynamic window model and strategy

At first, we assumed that the problem of multi-targets SSVEP recognition task could similarly be abstracted as a hypothesis testing question. For each target, the relevant hypothesis can be rewritten as follows:

$$H_{i_f} : X = A_{i_f} Y_{i_f} + W_{\text{noise}} \quad i_f = 1, 2, \dots, N_t \quad (7)$$

where X denotes multi-channels EEG data, A_{i_f} denotes the aliasing matrix of sine template Y_{i_f} , and W_{noise} denotes background noises and spontaneous EEG signals. The aliasing matrix exists high variability between trials according to [26]. Obviously, those assumptions are mutually exclusive and every

assumption has identical prior probability. Inspired by main ideas about dynamic window in STE-DW [26], ‘reject hypothesis’ H_0 was taken into considered. This reject hypothesis reveals that current data are not convincible to determine a target and more data should be collected.

In deep learning area, multi-targets classification problems often normalize feature into probability by softmax function which has been applied successfully in different areas. Some researches about SSVEP-based BCIs also accepted softmax function as normalized function [35–37]. In this study, softmax function was used to build a project relation between indices calculated by FBCCA and probability distribution. Softmax function is monotonous and generalized into 1, that matches the background of SSVEP classification problems. What’s more important is that it strongly penalizes the most active incorrect prediction [38]. In a word, softmax is a suitable generalization function in this situation.

Suppose the correlation coefficients determined by FBCCA can be presented as a vector $\rho = [\rho_1, \rho_2, \rho_3, \dots, \rho_{N_t}] \in R^{N_t \times 1}$, the posterior probability of each hypothesis is:

$$P(H_i|\rho) = \frac{e^{\rho_i}}{\sum_{j=1}^{N_t} e^{\rho_j}} \quad i = 1, 2, 3, \dots, N_t \quad (8)$$

It can be obviously found that the maximum coefficient corresponds to the largest posterior probability, so the classification judgment is still the largest ρ . Cross-entropy is the most common cost function when softmax function is utilized to represent the probability function [38]. When the result hypothesis is decided as H_i , the cost can be expressed as:

$$Cost(H_i) = - \sum_{j=1}^{N_t} C(H_j, H_i) \log P(H_j|\rho) \quad (9)$$

Since only the largest probability hypothesis H_{result} and reject hypothesis H_0 are taken into considered, this algorithm just needs to make decisions by comparing the cost value of those two hypotheses. The most widely used function C was defined as 0 when $i \neq j$ and 1 when $i = j$. But it apparently neglected the occasion of H_0 , so it can be assumed that when $i = 0$, the function C is an unknown parameter ε :

$$C(H_j, H_0) = \varepsilon \quad \forall j = 1, 2, 3, \dots, N_t \quad (10)$$

Additionally, the main idea of penalization was introduced to enhance the robustness of this system. Some research papers showed that the difference between the first and the second largest feature value is also an available classification basis [31, 39]. The probability of correct classification is higher when this difference is more apparent. In most situations, the frequency detected by second largest feature is the ‘neighbor’ of the real stimulus frequency, whatever they are close in frequency domain or close

in visual domain. Those ‘neighbor’ frequencies often have the strongly negative impacts on detection, especially when the data length is specifically short or the participants did not concentrate enough. Actually, much more reasons of wrong identification results could be discussed, but we mainly focused on most an actively negative factor: the second largest feature.

Motivated by this consideration, we assume the largest probability hypothesis is H_q and the second largest probability hypothesis is $H_{q'}$. The function C was defined as:

$$C(H_j, H_q) = \begin{cases} 1 & j = q \\ 0 & j \neq q \neq q' \\ -1 & j = q' \end{cases} \quad (11)$$

The cost of hypothesis H_0 and H_q is:

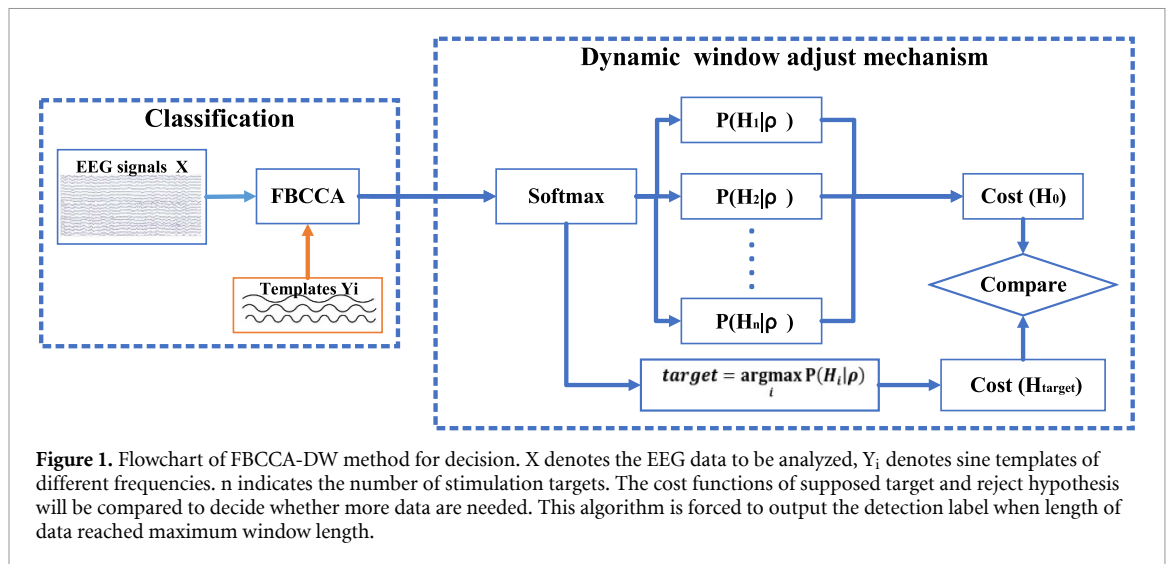
$$\begin{aligned} Cost(H_0) &= - \sum_{j=1}^{N_t} C(H_j, H_0) \log P(H_j|\rho) \\ &= -\varepsilon \sum_{j=1}^{N_t} \log \left(\frac{e^{\rho_i}}{\sum_{k=1}^{N_t} e^{\rho_k}} \right) \\ &= -\varepsilon \left(\sum_{k=1}^{N_t} \rho_k - N_t \log \left(\sum_{k=1}^{N_t} e^{\rho_k} \right) \right) \end{aligned} \quad (12)$$

$$\begin{aligned} Cost(H_q) &= - \sum_{j=1}^{N_t} C(H_j, H_q) \log P(H_j|\rho) \\ &= -\log \left(\frac{e^{\rho_{max}}}{\sum_{k=1}^{N_t} e^{\rho_k}} \right) + \log \left(\frac{e^{\rho_{2ndmax}}}{\sum_{k=1}^{N_t} e^{\rho_k}} \right) \\ &= -(\rho_{max} - \rho_{2ndmax}) \end{aligned} \quad (13)$$

When cost value of reject hypothesis is lower than cost value of recognition result, this algorithm believes that the current data are not enough to make a decision and the applied system needs to collect more data. It can be simplified to formula (14), and on the contrast, when this inequality is not true, the result is considered credible and the result should be outputted.

$$\varepsilon < \frac{-(\rho_{max} - \rho_{2ndmax})}{-(\sum_{k=1}^{N_t} \rho_k - N_t \log(\sum_{k=1}^{N_t} e^{\rho_k}))} \quad (14)$$

It can be noticed that the cost of H_0 and H_q are both negative numbers. Usually, as the increasing proportion of SSVEP signal components, the cost value of hypothesis H_q will decrease as increasing value of maximum coefficient. Therefore, the right side of formula (14) will decrease until this inequality is not satisfied. Whole detection process can be explained by figure 1, it contained two parts: classification and dynamic window model, classification part aims to find the most likely solution, whereas the dynamic window model justifies this solution feasibility. If the reject hypothesis was adopted, this process would repeat when next data package came. In this study, threshold refers to the parameter ε .



3. Experiments

3.1. Extensive algorithms for comparison

In order to validate the ascendancy of FBCCA-DW, other recognition methods were picked out for comparison. In this study, STE-DW and FBCCA-FW algorithms were chosen. STE-DW was designed as the reference method of dynamic window strategy. It is a training-free and dynamical window detection method, which achieved the highest performance recorded in current training-free dynamic window identification approaches in SSVEP-based BCI [26]. FBCCA with fixed window was introduced to evaluate the function of this hypothesis model. Those two compared algorithms maintain the same optimized parameters with previous studies [21, 26], so we neglected detailed parameter optimization processes.

3.1.1. Spatial-temporal equalization dynamic window (STE-DW).

STE-DW is a dynamic window recognition algorithm based on hypothesis testing and adaptive equalization, which could find an adaptive spatial-temporal equalizer to equalize signal from spatial and temporal domain to reduce the adverse effects of colored noise. Disregarding of classification approach itself, STE-DW presented a plausible and robust generative method to design dynamic window model and an extremely innovative idea of STE-DW was the introduction of reject hypothesis, which was inspired by sequential analysis [40]. This strategy is accessible to be transferred into other recognition methods and contributed a lot to the initiation of this research. Parameter selections and more details could be found in [26].

3.1.2. FBCCA with fixed window length (FBCCA-FW).

The main idea of FBCCA has been illustrated in section 2.2. To remove the influence from external

factors, the common basic parameters of fixed window methods maintained the same with dynamic window methods. The upper cutoff frequency and number of filter banks, the choosing of filter type, followed the optimized results in [21].

3.2. Offline experiments

Before the beginning of online experiments, uncertain parameters need to be predetermined. The publicly available benchmark SSVEP dataset [41] was taken as the data source of offline experiments. Its participants contain 35 subjects (17 females and 18 males), and were demanded to take part in a standard 40-targets SSVEP-based BCI speller experiment. The stimulation interface consisted of 5×8 English speller keyboard which was coded by a joint frequency-phase (JFPM) method [6]. The experiments' stimulus frequencies differed from 8 Hz to 15.8 Hz with an interval of 0.2 Hz. Every experiment contained six blocks and each block contained 40 trials corresponding to 40 targets with a random sequence. The sampling rate was 250 Hz and each trial lasted 6 s (0.5 s for cue appeared, 5 s stimulation, 0.5 s after stimulation offset). Considering of 140 ms visual latency, all algorithms just analyzed data that were extracted in $[0.64, 0.64 + d]$, where d was the length of time window. EEG data were acquired by a Synamps2 system, from 64 channels that were placed on standard position of international 10–20 system and event triggers were recorded simultaneously. All the electrode impedances were kept below 10 K Ω . In addition to that, all recorded data passed a 50 Hz notch filter and an infinite impulse response (IIR) band pass filter. More details about this dataset could be found in [41].

The offline procedure simulated the real-time data sending process of the online system, which will be further explained in section 3.4. The data segments were sent as a 65×10 points (40 ms) package each turn. 65 channels contain 64 channels of EEG and

>>										>>									
8Hz 0	8.8Hz 0.5 π	9.6Hz π	10.4Hz 1.5 π	11.2Hz 0	12Hz 0.5 π	12.8Hz π	13.6Hz 1.5 π	14.4Hz 0	15.2Hz 0.5 π	1	2	3	4	5	6	7	8	9	0
8.2Hz π	9Hz 1.5 π	9.8Hz 0	10.6Hz 0.5 π	11.4Hz π	12.2Hz 1.5 π	13.0Hz 0	13.8Hz 0.5 π	14.6Hz π	15.4Hz 1.5 π	Q	W	E	R	T	Y	U	I	O	P
8.4Hz 0	9.2Hz 0.5 π	10.0Hz π	10.8Hz 1.5 π	11.6Hz 0	12.4Hz 0.5 π	13.2Hz π	14.0Hz 1.5 π	14.8Hz 0	15.6Hz 0.5 π	A	S	D	F	G	H	J	K	L	
8.6Hz π	9.4Hz 1.5 π	10.2Hz 0	11.0Hz 0.5 π	11.8Hz π	12.6Hz 1.5 π	13.4Hz 0	14.2Hz 0.5 π	15.0Hz π	15.8Hz 1.5 π	<	Z	X	C	V	B	N	M	,	.

Figure 2. Online stimulation interface. (a) Frequency and phase of 40 targets. (b) The keyboard of user interface.

an additional channel for synchronous event trigger. The detection algorithms chose to return classification label or wait for another data package. The maximal calculation time was set as 4.6 s because of the limited stimulation time and restriction of calculation complexity.

3.3. Online data acquisition

3.3.1. Participants.

Fourteen healthy subjects (6 females and 8 males, aged 18–30 years) with normal or corrected to normal vision participated in this test section. All those subjects were demanded to participate in real-time online experiment. Six of fourteen subjects' data were acquired during the BCI competition of 2019 world robot conference (S1–S6), which can be found in [42], and others were acquired in laboratory extra after the conference (The data can be downloaded from this website: <http://bci.med.tsinghua.edu.cn/download.html>). To reduce the negative impact of visual fatigue, participants had enough time to relax after every block and everyone successfully completed the tasks. Besides, participants were asked to avoid blink and focus attention during stimulation process as much as they can. They were required to read and sign an informed consent form approved by the Research Ethics Committee of Tsinghua University before the experiment and received economical compensation after the experiment.

3.3.2. EEG recording.

During online experiments, EEG data were acquired using a 64-channel Neuracle EEG system at a sampling rate of 1000 Hz. All channel impedances were reduced less than 20 K Ω . Event triggers were synchronously recorded to identify the stimulus onsets. During the online experiments, EEG data and event trigger were both recorded and transported to online analysis programs simultaneously.

In all experiments, original data were down sampled into 250 Hz and all the EEG signals passed a 50 Hz notch filter for eliminating power-line noise. Since the latency delay of visual system, only data

after 140 ms visual delay were analyzed. Filter pre-processing was completed by the filter() function in MATLAB. All three algorithms only considered nine electrodes over parietal and occipital areas (Pz, PO5, PO3, POz, PO4, PO6, O1, Oz, O2), which have been reported to be the most ideal locations.

3.4. Online experimental design

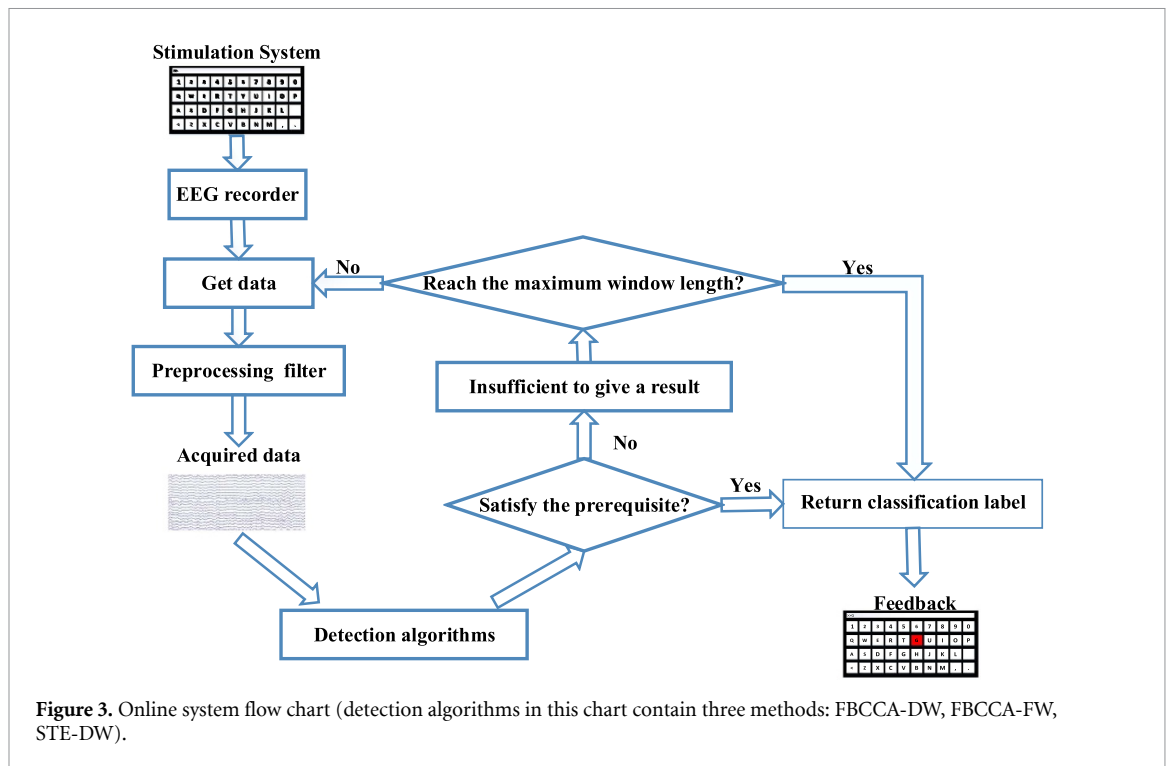
3.4.1. Stimulus system design.

This stimulus system utilized the sampling sinusoidal stimulation method to provide visual flickers coded by the JFPM method on a liquid-crystal display (LCD) monitor [12]. This experiment chose a 27-inch ASUS MG279Q monitor as stimulator. Its refresh rate was set as 60 Hz and the resolution was 1920 \times 1080 pixels. The stimulation interface contained 40 targets flashing with different frequencies and phases, constituting a 4*10 keyboard (10 digits, and 26 English alphabet letters, 4 punctuations). An extra part on the top of monitor was added to display the contents of inputted characters. Specific frequency and phase values of each keyboard are shown in figure 2. All the stimulation programs were developed with psychophysics toolbox under MATLAB2018b [43].

Same with the benchmark dataset, the frequencies of this experiment ranged from 8 Hz to 15.8 Hz with an interval of 0.2 Hz. Each subject was requested to complete online experiments which contained 4 blocks. Each block contained 45 trials of random target stimulation and each trial lasted 3 s. Before a new trial started, the target block would become temporarily red to provide hints.

3.4.2. Online system processing.

The whole flowing process of online system is shown in figure 3. EEG recorder kept recording data and 10-points data package was sent to three computers, which implemented three algorithms, by TCP/IP in a wired network every 40 ms. For a fair comparison, all three algorithms came up with the same detection tasks, operating with three same type PC at the same time, in other words, different algorithms confronted exactly the same situation, so acquired data were filtered by the same preprocessing filters. For



the fixed window algorithm, the feedback result was given only when the received data length was larger than fixed window length. While for dynamic window length algorithms, it would decide whether the feedback was returned when each data package was sent, if prerequisites were satisfied, the stimulation system would receive the returned label of classification frequency. The results were shown only when all detected labels of three algorithms were outputted. Moreover, each separate trial lasted 3 s even if all detected labels were shown. Without a doubt, this system was forced to output when the received data length reached maximal window length.

To compare the performance of three algorithms more directly, another computer was added to display the detailed information. When a new trial started, this monitor would show the correct stimulus labels; detection results and cost time of three algorithms; the average ITR of three algorithms on the current block. All results of online methods would be recorded. Nevertheless, this stimulation interface only presented result on the top of interface by one specific method. From S1 to S6 during the world robot competition, they received feedbacks came from STE-DW, while S7 to S14 received feedbacks from FBCCA-DW. In fact, because the results were merely provided on stimulation interface after three algorithms completed current detection task, so distinctive algorithms for judgments would not affect the process of stimulation.

Similar to offline experiments, to guarantee the success calculation in the real-time online system, the minimal window length and maximum window length were set as 0.7 s and 2.6 s respectively

(the visual latency and gazing shifting time were not included). The minimal window length was designed for declining the instability of short time stimulation, while the maximum was for preventing the situation of overtime.

3.5. Performance evaluation

Enhancing performance is one of the most important motivations to propose new recognition methods. ITR is an authoritative and widely used evaluation criterion for BCI performance and it is estimated by accuracy of target classification P , number of targets N_t , average time for detection T (seconds) [44]:

$$ITR = \left(\log_2 N_t + P \log_2 P + (1 - P) \log_2 \left[\frac{1-P}{N_t-1} \right] \right) \times \frac{60}{T} \quad (15)$$

Accuracy is defined as number of correctly trials divided by total number of trials and it is the most intuitive measure of BCI performance. In order to simulate the real-world BCI system situations, T was obtained by considering average equivalent data length and gaze shifting time. The equivalent data length only took the data after 140 ms visual latency into consideration, then the data length was transformed into seconds with 250 Hz sampling frequency, which indicated that the detection time did not contain calculation delay and visual latency. During practical application processes, gaze shifting time is different from subject to subject. Online and offline experiments both considered 0.5 s gaze shifting time in this study. In fact, actual ITRs in BCI application always have some difference with estimated ITRs [45], which will be further discussed in section 5.1.

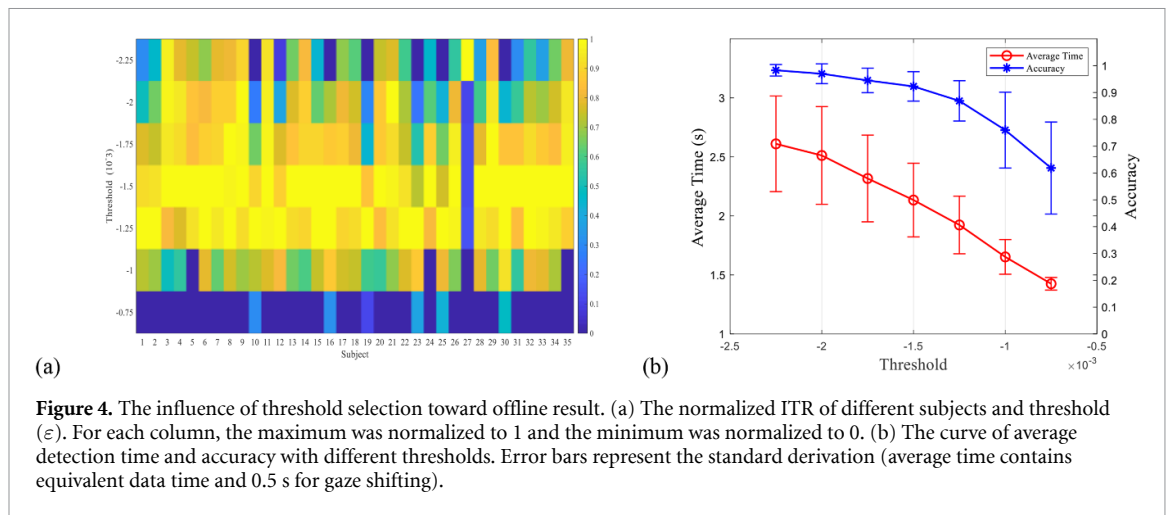


Figure 4. The influence of threshold selection toward offline result. (a) The normalized ITR of different subjects and threshold (ε). For each column, the maximum was normalized to 1 and the minimum was normalized to 0. (b) The curve of average detection time and accuracy with different thresholds. Error bars represent the standard derivation (average time contains equivalent data time and 0.5 s for gaze shifting).

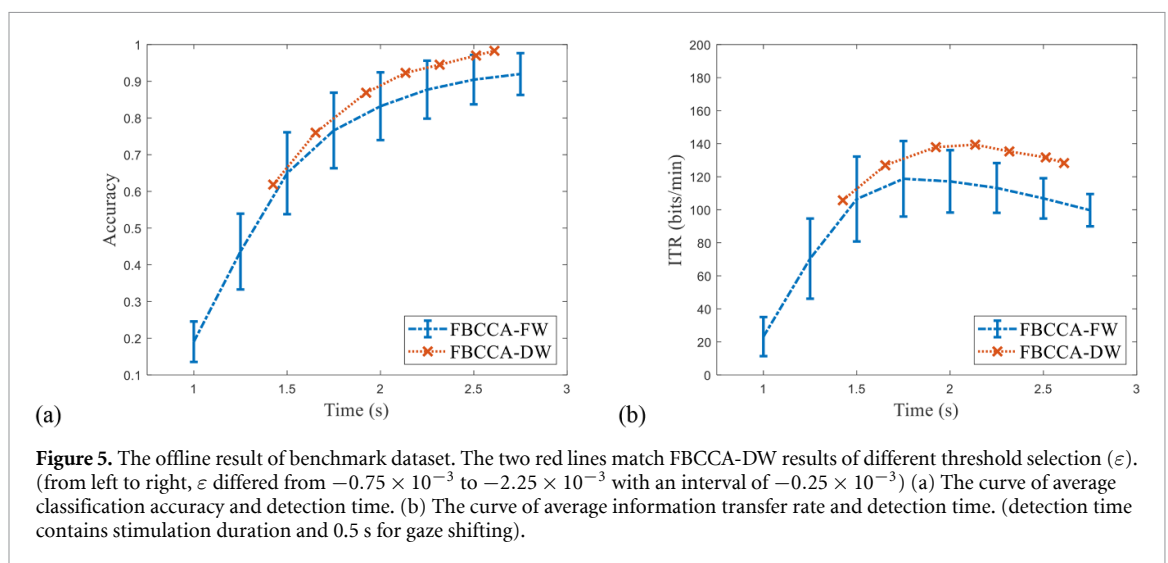


Figure 5. The offline result of benchmark dataset. The two red lines match FBCCA-DW results of different threshold selection (ε). (from left to right, ε differed from -0.75×10^{-3} to -2.25×10^{-3} with an interval of -0.25×10^{-3}) (a) The curve of average classification accuracy and detection time. (b) The curve of average information transfer rate and detection time. (detection time contains stimulation duration and 0.5 s for gaze shifting).

4. Results

4.1. Offline result

At first, benchmark dataset was introduced to evaluate the performance of dynamic window algorithm and detect optimal thresholds. At first, the result of different parameter selection has been shown in figure 4, with threshold (ε) from -0.75×10^{-3} to -2.25×10^{-3} with an interval of -0.25×10^{-3} . Figure 4(a) demonstrates the relation of normalized ITR and threshold. Except for S27, optimal thresholds of other common subjects were usually -1.25×10^{-3} or -1.5×10^{-3} . While figure 4(b) suggests that smaller threshold increases average accuracy at the cost of increased judgment time.

According to the pre-optimized parameters on this dataset in previous researches, FBCCA-FW selected 1.75 s (contains gaze shifting time) as fixed window length [41]. The result of this offline test is shown in figure 5. It can be seen that FBCCA-DW had better performance in terms of accuracy and ITR. According to figure 5(b), generally optimized parameter for FBCCA-DW is $\varepsilon = 1.5 \times 10^{-3}$ (the fourth point on

the red line from left side to right side). Regarding ITR, the optimal window length of FBCCA-FW is 1.75 s, which was consistent with previous study about this dataset [41]. The highest average performance achieved by FBCCA-DW was $140.38 \text{ bits min}^{-1}$ while that by FBCCA-FW was $118.75 \text{ bits min}^{-1}$. Generally speaking, average ITR of FBCCA-DW was 20% better than FBCCA-FW (paired t-test, $p < 0.01$). Because of high adaptability existed in this algorithm, 33 of 35 subjects' accuracy were higher than 80%, while FBCCA with fixed window only contained 18 subjects with accuracy higher than 80%.

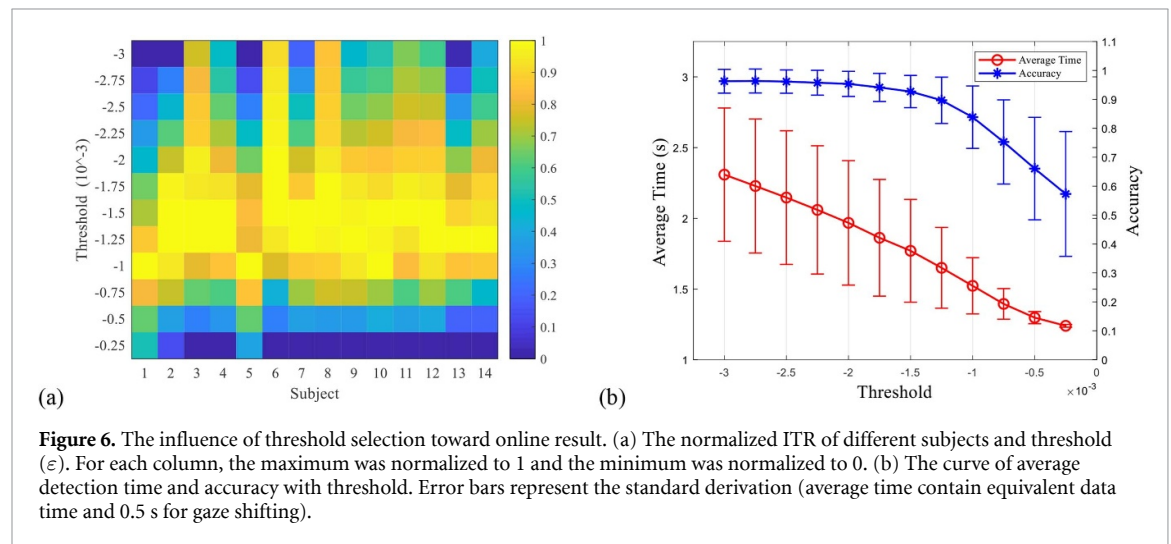
4.2. Online result

4.2.1. BCI performance.

Because stimulus frequency range in online tests were same with Benchmark dataset. So parameters applied on benchmark dataset were generic. During the online test, three compared algorithms of SSVEP-based BCIs detection were set as: FBCCA-DW ($\varepsilon = 1.5 \times 10^{-3}$), STE-DW ($\varepsilon = 10^{-6}$) [26], FBCCA with fixed window length (1.75 s). In the same way, the actual T was calculated by considering equivalent

Table 1. The online experiments result (ACC represents accuracy, T represents detection time: s, and ITR indicates information transfer rate: bits/min, Avg means average and Std means Standard deviation. Bold numbers indicate the highest ITR of each subject).

Subjects	FBCCA-DW			STE-DW			FBCCA-FW	
	ACC	T	ITR	ACC	T	ITR	ACC	ITR
S1	0.983	1.382	222.25	0.983	1.500	203.26	0.989	176.26
S2	0.983	1.321	233.09	0.983	1.568	195.94	0.983	174.27
S3	0.900	1.692	154.66	0.928	1.863	147.82	0.933	157.50
S4	0.950	1.845	157.03	0.850	2.044	116.42	0.833	129.51
S5	0.911	1.517	175.20	0.889	1.840	138.16	0.938	159.02
S6	0.806	2.419	90.10	0.778	2.485	83.57	0.528	63.54
S7	0.978	1.418	214.76	0.961	1.590	184.96	0.827	134.17
S8	0.828	2.491	90.91	0.683	2.457	67.22	0.5	58.15
S9	0.967	1.656	179.18	0.927	1.927	143.73	0.95	163.02
S10	0.894	1.938	133.16	0.794	2.220	95.38	0.811	123.24
S11	0.911	2.077	128.42	0.767	2.333	85.25	0.694	96.20
S12	0.950	1.818	159.08	0.850	2.234	105.38	0.867	138.27
S13	0.961	1.480	199.35	0.906	1.741	151.59	0.939	158.88
S14	0.956	1.727	168.90	0.806	2.114	102.70	0.9	147.48
Avg	0.927	1.770	164.72	0.864	1.994	130.10	0.835	134.25
Std	0.056	0.363	44.33	0.088	0.316	42.21	0.158	37.75



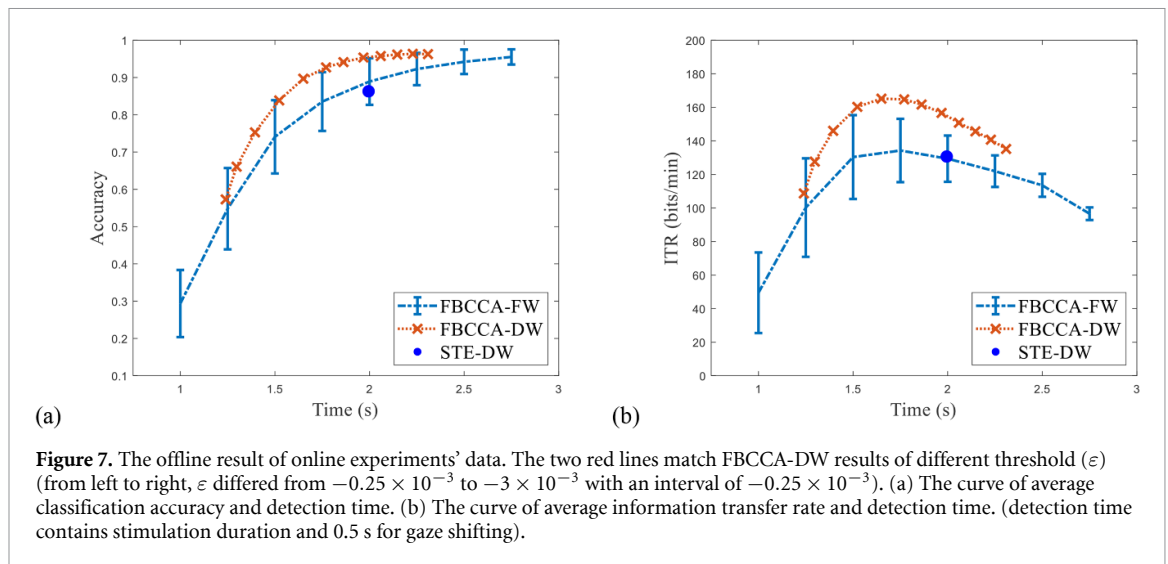
data length and 0.5 s for gaze shifting. Every subject needs to complete 4 blocks and ITR was calculated with each block separately. Table 1 shows the averaged 4 blocks online results of 14 subjects. There is no denying that FBCCA-DW notably outperformed STE-DW and FBCCA-FW both on accuracy and ITR. The average detection time of FBCCA-DW is 1.77 s, closely to fixed window length, but accuracy was significantly higher than fixed widow method (paired t-test, $p < 0.05$). In general, ITR acquired by FBCCA-DW is 25% better than FBCCA-FW and STE-DW (paired t-test, $p < 0.01$).

4.2.2. Parameter optimization.

To consolidate the effectiveness of predefined parameters, the data collected during the online experiment were used as new materials for offline experiment. The result of different threshold selection has been shown in figure 6. In figure 6(a), this chart shows the normalized ITR results for different subjects and

different threshold selections. For every column, the values of ITR were linearly normalized from 0 to 1. It is easy to discover that the optimal threshold is common across subjects. Thresholds from -1.75×10^{-3} to -1×10^{-3} achieved almost the same result. As shown in figure 6(b), identical with the offline experiment results, smaller threshold increases average accuracy at the cost of increased judgment time. An interesting phenomenon is that the average detection time almost has a linear relation with threshold. Because of constraints of upper and low limits of window length, higher or lower threshold are meaningless.

Similar with offline conclusion which has been mentioned in section 4.2, figure 7 shows the result that compared three algorithms in online experiments. The red lines match results of different threshold selections for FBCCA-DW. It can be inferred that curve of FBCCA-DW always exceeded FBCCA-FW, although they classified with the same method. In other words, with a common average

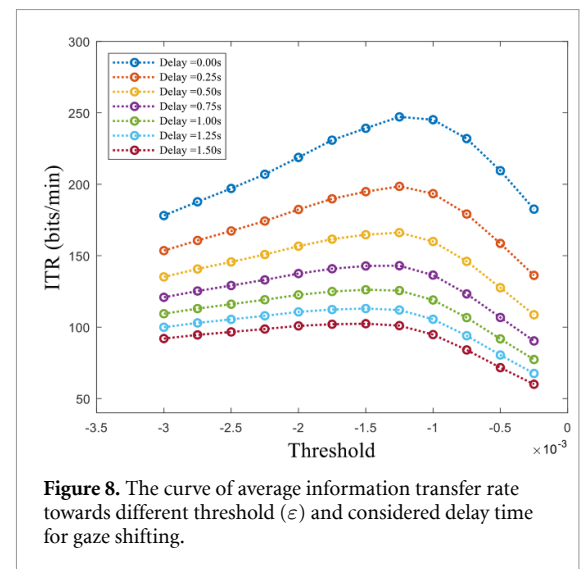


detection time, dynamic window algorithm is always better than the fixed window strategy. Of course, STE-DW also own this advantage, but in general, it is worse than FBCCA-DW. The optimized window length of FBCCA-FW is still 1.75 s, so FBCCA-FW preserved the generally best result during the online tests. However, for specific users or other databases, the most suitable window length might vary. As a result, when applied on a larger group of subjects, the gap between those two methods may be more noticeable if the optimal window length is inconsistent with predefined value. Except of that, STE-DW did not show enough difference when compared with FBCCA-FW and even was a little bit worse, which could probably be caused by subjects' variance, but STE-DW also achieved relatively high accuracy towards all subjects. Except of S8, all subjects in online tests got accuracy higher than 75% by STE-DW (see table 1).

5. Discussions

5.1. BCI performance

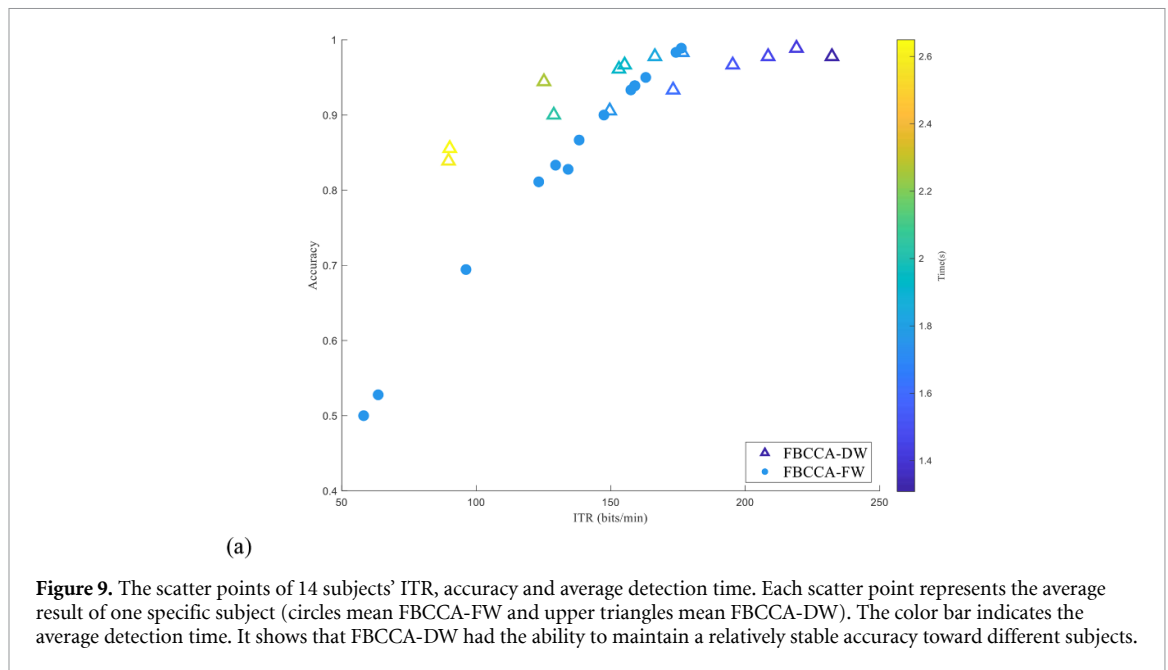
The main purpose of this algorithm was to enhance the performance of online SSVEP-based BCI systems, in which the average detection time was calculated by averaged data length and 0.5 s for gaze shifting. But real communication systems' performance often existed discrepancy with evaluated performance. Usually, users' eye can move faster when they were accustomed to this experiment interface. Shorter the delay time for gaze shifting, higher the calculated ITR. So it did make sense that offline experiment on same dataset acquired slightly different ITR with previous study. Figure 8 explains the relation between threshold and ITR under different delay time. If we assume the delay time is 0 s, the average ITR of FBCCA-DW is 247.2 bits min^{-1} , while FBCCA-FW is 188.14 bits min^{-1} . Besides that, higher the delay time, the divergence of distinctive threshold



appeared more ambiguous, because the influence of accuracy was more perceptible. Briefly, the optimized threshold could be slightly different when facing different applied situation and subjects. Last but not least, the online test calculated ITR of each block separately and then found the average value. In fact, ITR which was calculated for all four blocks together would not cause much obvious difference.

Although ITR is the most authoritative performance evaluation used in BCI system, in some specific applications, accuracy is a more valuable index. For example, BCI communication systems, which are specially devised for disabled patient, emphasize the preciseness more than speed. Some researches argued that the a BCI system is effective only when the mean accuracy is higher than 70% [46]. Needless to say, both FBCCA-DW and STE-DW were much more effective than FBCCA-FW on this dataset.

Analogous with offline experiments, compared with the fixed window strategy, the dynamic window strategy had the ability to maintain a relatively stable



accuracy toward different subjects. It can determine longer or shorter window length according to the quality of signal. FBCCA-DW chose a longer window length for subjects who showed relatively weak response (the upper left corner of figure 9). In figure 9, discrete points of dynamic approach are more concentrated. Taking the typical subject S6 as an example, FBCCA-FW approach only achieved 50% accuracy but FBCCA-DW approach achieved 82.8%, which indicates that 1.75 s for detection is insufficient for S6. From the opposite direction, S2 is an exceptional subject so 1.75 s for recognition is overlong, but average detection time for FBCCA-DW is 1.382 s while maintaining the same detection accuracy with FBCCA-FW.

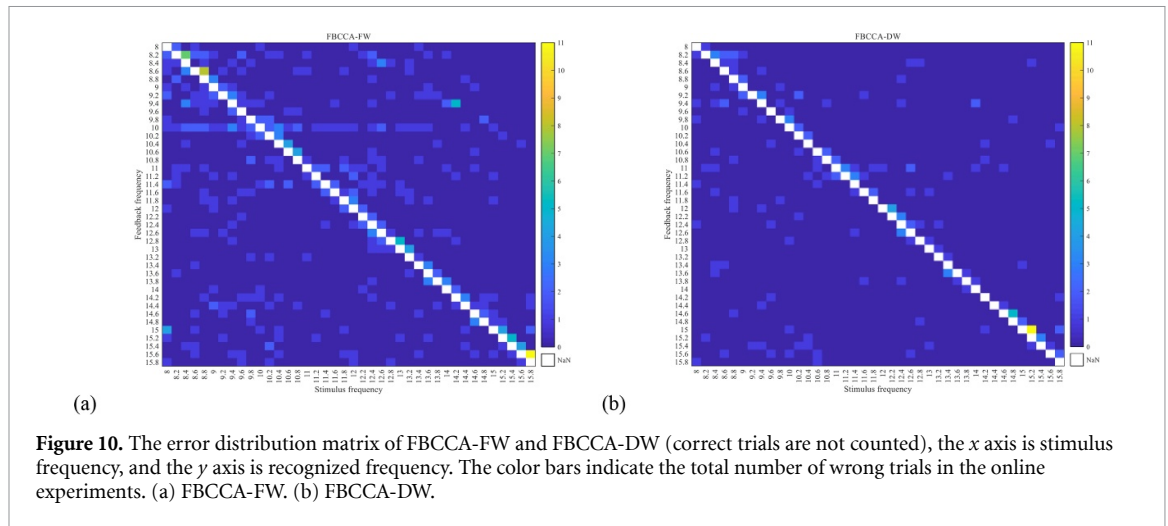
5.2. The exploration of cost function

The similar form of cross-entropy was placed as the cost function. As a matter of fact, regularization is a basic idea used in the machine learning area [47], and FBCCA-DW adopted the second largest classification feature as penalization of cost function. Multi-targets SSVEP recognition problems exactly own the issue that interferences from irrelevant stimulation are inevitable. From one aspect, the size of stimulation monitor is restricted, subjects can hardly fully gaze at target without any glimpse of other targets. To solve the problem, better stimulus paradigms were proposed and they certainly declined the impact from other stimulation. On the other hand, different recognition algorithms have different interference factors. As we have illustrated in section 2.3, one important strategy of CCA is the exploitation of sinusoidal templates, which definitely brings many conveniences in training-free SSVEP-based BCI systems and has acquired highly praised reflection. Essentially, sinusoidal templates are not completely orthogonal with each other when data length is short, so the nearby

frequency is an affected factor responsible for mistaken judgments. That's why regularization is essential and indispensable, yet the main reason choosing the penalization to be -1 is to simplify the expression and make the judgment criteria to be more intuitive. Figure 10 shows the error distribution matrix of the two algorithms, indicating that the error often appears at surrounding of matrix's diagonal line. It confirms our assumption that neighbor frequencies always have strongest negative influence on actual response of stimulation. If we only consider the error trials, which were detected as neighbor frequency (0.2 Hz interval), FBCCA-FW has 150 such trials and FBCCA-DW has 87 trials (total 2520 trials). Without a doubt, penalty factor cannot distinguish those error situations, but commonly, it reduced those occasions greatly.

Thus this penalty factor is essential and indispensable. Without it, the result could easily be decided wrongly when data length is not long enough and the optimal threshold of each subject will show a large difference. Needless to say, regularizations improve a lot towards system stability, nonetheless, the negative number in (10) linked with penalization merits more investigation. Obviously, smaller this value, the influence of penalty will be more influential, vice versa, the influence of penalty grows more unaffected when this factor is larger.

Previous studies such as [27, 48] have proposed reasonable and feasible dynamic stopping schemes, of which the whole frames share some similarity with our work. However, the methods in [27, 48] heavily relied on enough training data, which suggests that it cannot be applied to unfamiliar users. Meanwhile, additional time is required for acquiring training data before the formal experiment. Whereas this study proposed a uniform and training-free method



for all subjects without requiring additional information. What's more, the method in [48] was based on a 12-targets SSVEP-based BCI, whether it is suitable for 40-targets SSVEP-based BCIs remains unknown. Moreover, the generalization methods applied in this study are non-linear, which is distinctive with preceding studies [27, 48]. The introduction of softmax function as generalization formula and the penalization factors effectively decreased the probability of error judgment.

5.3. Computational complexity

The online experiments have shown that FBCCA-DW is an accessible and reasonable recognition approach which can be applied in realistic SSVEP-based BCI systems. Essentially, FBCCA itself is an algorithm with high computational complexity in SSVEP training free recognition methods, especially because several band pass filtering processes cost too much time. We must ensure that the calculation delay would not affect the practical performance of the online system. Therefore, FBCCA-DW has to adopt some low-cost strategy to guarantee the feasibility of online system. The filter() function in MATLAB can filter signal iteratively, i.e. every time a new data package received by calculation module, only the latest 10 points are calculated and then connected with pre-processing data segments. Except of that, the core step of canonical correlation analysis is to find SVD decomposition and it also can be simplified by iteration procedure. Those details guarantee the feasibility of online experiments.

5.4. Future work

Previous investigation has proved that spatial temporal equalization with fixed window strategy (STE-FW) was not better than FBCCA-FW, but jointing of dynamic window strategy made STE-DW superior than FBCCA-FW [26]. Naturally, we combined the main idea of dynamic window and FBCCA, and then proposed a new dynamic window recognition method based on FBCCA and hypothesis testing.

This method won the first place of training-free SSVEP algorithm in the BCI competition of 2019 world robot conference and significantly outperformed other training-free algorithms. While the practical applications of free-spelling SSVEP-based BCI systems are different with cue-guided competition platform, this algorithm need to be more robust and stable in free-spelling tasks. What's more, other parameter optimizations are also helpful for enhancing the system performance, such as the maximum and minimum window length; design of filter banks; optimization of penalization; selection of channel. This study chose a same threshold for different subjects, although this threshold was robust and relatively stable, a dynamic selected threshold is also a potential approach to enhance performance. Dynamic window strategy is based on the variance of experimental subjects and environmental factors, nevertheless, variances of different blocks also cannot be ignored. Transfer learning [31] or dynamically training methods are also efficient solutions.

Besides the algorithm itself, an asynchronous speller system based on this algorithm, which can selectively output only when individual data deserve outputting, seems feasible. Asynchronous systems meet the actual requirements and usage habit of users' communication [49]. On the other side, dry electrode-based BCI systems provide another accessible solution to improve the user experience [50].

6. Conclusions

A novel dynamic window recognition method based on FBCCA and hypothesis testing was proposed and evaluated in this research. The strategy in this algorithm can dynamically select proper data length to acquire higher performance without compulsive need of training data. Besides that, this algorithm retains advantages of low computational complexity and high adaptability towards different subjects,

which satisfies requirements of real world BCI systems. Offline results showed that FBCCA-DW significantly outperformed STE-DW and FBCCA-FW in ITR and recognition accuracy. The feasibility and adaptability were also validated by online experiments, the result showed that this algorithm achieved high robustness and high accuracy with subjects of different signal qualities. So this method is an appropriate and efficient candidate of detection algorithms for SSVEP-based BCI system.

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ORCID iDs

Yonghao Chen  <https://orcid.org/0000-0002-8788-830X>

Xiaogang Chen  <https://orcid.org/0000-0002-5334-1728>

Yijun Wang  <https://orcid.org/0000-0002-8161-2150>

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