Lecture 9: Hypothesis testing part I

Statistical Methods for Data Science

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Today

1. Terminology
   - Experiment and parameter of interest
   - Null hypothesis and alternative hypothesis
   - Test statistic
   - Null distribution $f(s \mid H_0)$
   - Significance level α, power and *p*-value
2. Example
3. *p*-hacking
4. Summary

Learning outcome

- Be able to explain the following terminology
  - Null hypothesis H_0 and alternative hypothesis H_A
  - Test statistic s
  - Null distribution $f(s \mid H_0)$
  - Significance level α and power
  - p-value
- Be able to design and interpret the one-sample z-test
- Be able to explain the concept of p-hacking

Today

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- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

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- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?

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- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?

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- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?

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- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?
- Now company B tested drug E on 30 ducks and the average weight loss after one month is 2.5 kg, while drug D results in 2.3 kg weight loss with the same setup, would you buy drug E instead of drug D?

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What would you do?

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  - Hypothesis + data $\rightarrow$ decision on rejecting or not rejecting the hypothesis

Hypothesis testing: a list to go through

- A default statement
- Experiment
- Data x, random variable X
- Parameter of interest θ
- Parameter estimate $\hat{\theta}$
- Null hypothesis H_0
- Alternative hypothesis H_A
- Test statistic s
- Null distribution $f(s \mid H_0)$
- Significance level α
- p-value

Experiment and parameter of interest

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
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  Then we test if $\bar{x}$ is greater than diet control (2.1 kg)

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average

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  Then we test if $\bar{x}$ and $\bar{y}$ are the same

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- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**

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- Intuition: "If the statement is true, then the evidence should support the statement", which is the same as ($\iff$) "if the evidence does not support the statement, the statement is considered false" , which is not the same as ($\not\Leftrightarrow$) "if the evidence supports the statement, the statement must be true"

Null hypothesis and alternative hypothesis

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution**

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  - Example 1: drug D is not more effective than regular diet on average

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  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis** H_A: an alternative hypothesis that is complementary (the opposite) to the null hypothesis

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  Answer: No
  Question 2.2: What are the choices for H_0 and H_A then?

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    For example, is classifier A better than classifier B? $H_0 : p_A = p_B$
- We have seen one-sample test and two-sample test in the Q-Q plot lecture
- In practice, you can narrow down your choice of hypotheses by looking at Q-Q plots

Choices for H_A

Given

$$H_0 : \theta = \beta$$

where β can be either a constant (one-sample test) or a parameter from another data distribution (two-sample test)

- **Alternative hypothesis H_A**: H_A can be **one-tailed** or **two-tailed**
  - **One-tailed**:

$$H_A : \quad \theta > \beta$$

or

$$H_A : \quad \theta < \beta$$

  - **Two-tailed**:

$$H_A : \quad \theta \neq \beta \iff \theta < \beta \text{ or } \theta > \beta$$

Summary: choices for H_0 and H_A

Putting everything together,

	One-sample test	Two-sample test
Two-tailed	$H_0 : \theta = c,\ H_A : \theta \neq c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 \neq \theta_2$
One-tailed	$H_0 : \theta = c,\ H_A : \theta > c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 > \theta_2$
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One-tailed	$H_0 : \theta = c,\ H_A : \theta > c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 > \theta_2$
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Note: this is the answer to question 1.1 (cf. page 14): if you choose the one-tailed test, then you are making the assumption $H_A : \mu_D > 2.1$; if you choose the two-tailed test, then you are not making this assumption

Test statistic

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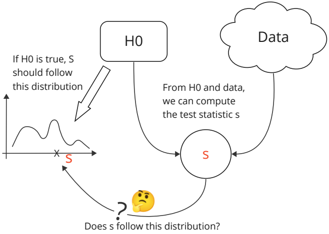
- **Test statistic** s (random variable S): a (typically standardized) statistic computed from data
- **Purpose:** to determine how plausible the null hypothesis H_0 is by observing s



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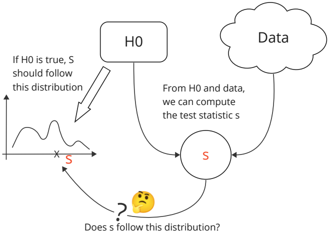

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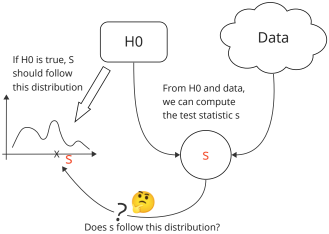

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Disclaimer: in this course, we only deal with null hypothesis where we are able to express the PDF/PMF $f(s \mid H_0)$, i.e. H_0 with an equal sign in them

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- Data: $x_1, \cdots, x_N$
- Random variable: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
- Parameter of interest: μ_D
- Parameter estimate: $\bar{x}$ ($\sim \mathcal{N}(\mu_D, \frac{\sigma^2}{N})$ - CLT)
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  - Why are we standardizing the statistic $\bar{x}$?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$ ($\sim \mathcal{N}(\mu_D, \frac{\sigma^2}{N})$ - CLT)
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: s = **standardized** $\bar{x}$ **assuming the null hypothesis**
  - What are we trying to do here? - To decide if we can reject the null hypothesis - if the null hypothesis is true, we should be able to see evidence that supports it - if we do not see evidence, we reject the null hypothesis
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Standardize $\bar{x}$ (15 sec)

Test statistic (cont.)

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Standardize $\bar{x}$ (15 sec)

$$s = z = \frac{\bar{x} - 2.1}{\sigma/\sqrt{N}}$$

Test statistic (cont.)

Example 2. two-sample test

- **Data**: $x_1, \cdots, x_{N_D}$ and $y_1, \cdots, y_{N_E}$
- **Random variable**: $X_1, \cdots, X_{N_D}$ i.i.d. **Gaussian with known σ_D**; $Y_1, \cdots, Y_{N_E}$ i.i.d. **Gaussian with known σ_E**; X_i and Y_j independent
- **Parameter of interest**: μ_D, μ_E
- **Parameter estimate**: $\bar{x}$, $\bar{y}$
- **Null hypothesis**: $H_0 : \mu_D = \mu_E \iff H_0 : \mu_D - \mu_E = 0$
- **Test statistic**: standardized $\bar{x} - \bar{y}$ assuming the null hypothesis

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\sigma_D^2/N_D + \sigma_E^2/N_E}}$$

Note: $\bar{x}$, $\bar{y}$ Gaussian (CLT); awesome properties of Gaussian from Lecture 6

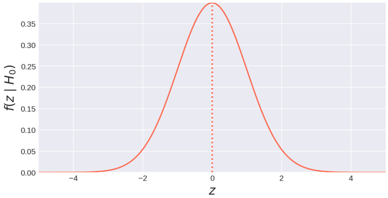
Null distribution $f(s \mid H_0)$

Null distribution

- **Null distribution** $f(s \mid H_0)$: the distribution of the **test statistic** S given the null hypothesis H_0
- Example:
  - **Data**: $x_1, \cdots, x_N$
  - **Random variable**: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
  - **Parameter of interest**: μ
  - **Parameter estimate**: $\bar{x}$
  - **Null hypothesis**: $H_0 : \mu = \mu_0$
  - **Test statistic**:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{N}}$$

  - **Null distribution**: standard Gaussian distribution



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Do I need to come up with the test statistic and NULL DISTRIBUTION MYSELF!!!??

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No, but you need to be able to choose and apply existing tests to given scenarios

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Do I need to come up with the test statistic and NULL DISTRIBUTION MYSELF!!!??

No, but you need to be able to choose and apply existing tests to given scenarios

Okay, noice! Sounds easier!

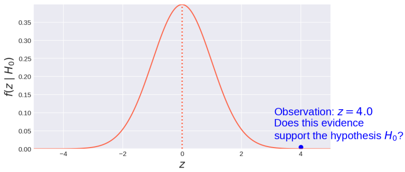
Significance level α, power and p-value

Significance level

Given a null hypothesis $H_0 : \mu = 2.1$ and the null distribution $f(s \mid H_0)$, we decide if we reject the hypothesis or not by observing data

- Run some experiments and collect data $x_1, \cdots, x_N$
- Compute the test statistic from data, e.g.

$$z = \frac{\bar{x} - 2.1}{\sigma/\sqrt{N}} = 4.0$$



- Does this evidence support the hypothesis H_0?

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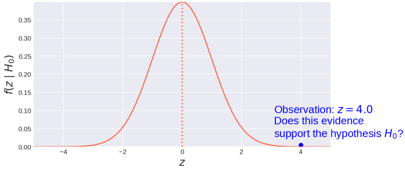
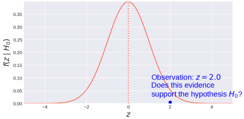


- Does this evidence support the hypothesis H_0? Probably not since it's so far away from the center?

Significance level (cont.)

- What about this observation?



Significance level (cont.)

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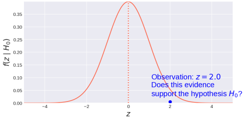

- To be able to answer the question, you need to decide where you draw the line (quite literally)

Significance level (cont.)

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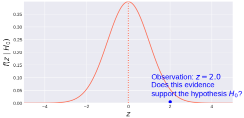

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Significance level (cont.)

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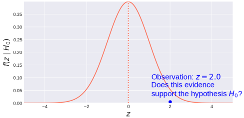

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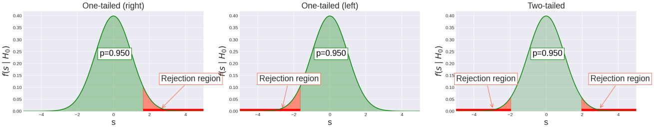
- **Significance level** α: red area under the curve



In these three images, $\alpha = 0.05$

Significance level (cont.)

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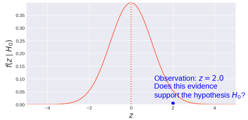

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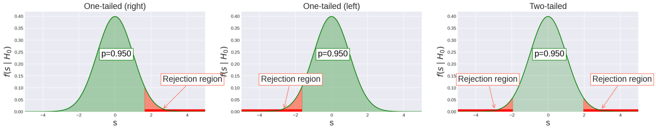

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More conservative $\Rightarrow$ less probable to reject H_0, which indicates a smaller rejection region

Significance level (cont.)

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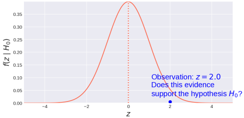

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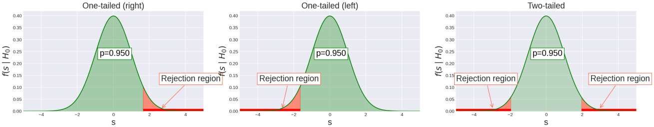

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More conservative $\Rightarrow$ less probable to reject H_0, which indicates a smaller rejection region
Two-tailed H_A is more conservative

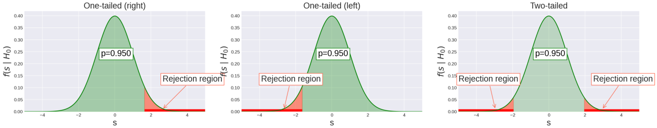
Significance level (cont.)

What is needed for choosing a meaningful α?

- Null distribution
- H_A one-tailed or two-tailed

Interpretation of α

- $\alpha = P(\textit{reject } H_0 \mid H_0 \textit{ is true})$ - the probability of making such a mistake



  - The rejection region indicates that H_0 is **unlikely**, but the probability is not zero
  - It is possible that H_0 is true, but our observation happens to fall in the rejection region
  - If H_0 is true and our observation falls in the rejection region, we will **mistakenly** reject H_0
  - The probability of making this type of mistakes is α
- Similar to the confidence interval, $1 - \alpha$ is called the **confidence level**

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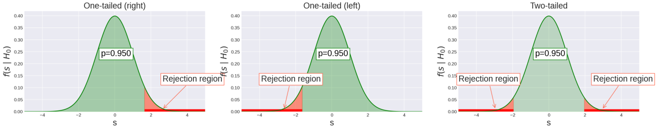

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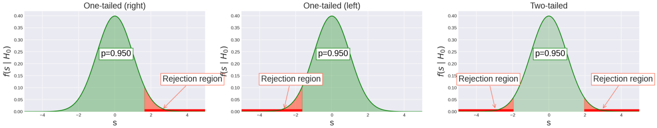

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  - The probability of making this type of mistakes is α
- Similar to the confidence interval, $1 - \alpha$ is called the **confidence level** - "with 95% confidence, rejecting H_0 is the right thing to do"
- Define the significance level **before you run the experiments** so that you can't cheat!

Significance level and power

- Contingency table:

	$y = H_A$	$y = H_0$
$\hat{y} =$ reject H_0	**TP**	**FP** (Type I error)
$\hat{y} =$ do not reject H_0	**FN** (Type II error)	**TN**

Significance level and power

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$$\alpha = P(\text{type I error})$$

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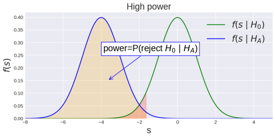

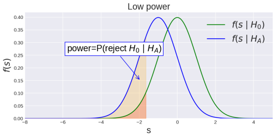

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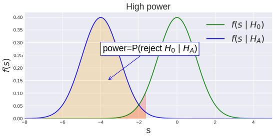

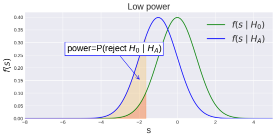

- What is needed for computing power (20 sec)?

Significance level and power

- Contingency table:

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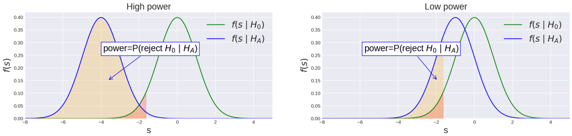


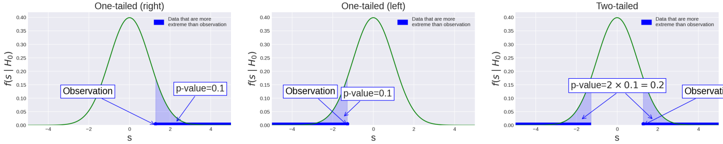
- What is needed for computing power (20 sec)? $f(s \mid H_0)$, $f(s \mid H_A)$, α

p-value

- p-value:
  - One-tailed:
    - Right tail: $p = P(S \geq s \mid H_0)$, e.g. 1-stats.norm.cdf(s, 0, 1)
    - Left tail: $p = P(S \leq s \mid H_0)$, e.g. stats.norm.cdf(s, 0, 1)
  - Two-tailed:
    - $p = 2\min\left(P(S \leq s \mid H_0), P(S \geq s \mid H_0)\right)$, e.g. 2*min(stats.norm.cdf(s, 0, 1), 1-stats.norm.cdf(s, 0, 1))

    Note: for example, if $f(s \mid H_0)$ is symmetric around zero and $s < 0$,

$$p = 2P(S \leq s \mid H_0)$$



- What is needed for computing the p-value? (10 sec)

p-value

- ***p*-value**:
  - One-tailed:
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    - Left tail: $p = P(S \leq s \mid H_0)$, e.g. stats.norm.cdf(s, 0, 1)
  - Two-tailed:
    - $p = 2\min\left(P(S \leq s \mid H_0), P(S \geq s \mid H_0)\right)$, e.g. 2*min(stats.norm.cdf(s, 0, 1), 1-stats.norm.cdf(s, 0, 1))

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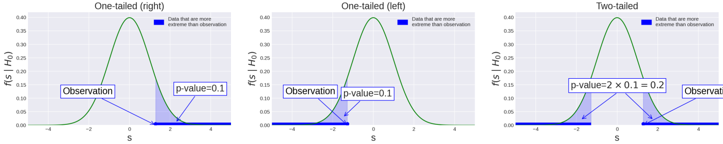

- What is needed for computing the p-value? (10 sec)
  - Null distribution
  - Alternative hypothesis H_A to know one-tailed or two-tailed
  - Observation - test statistic computed from data

Summary: steps for hypothesis testing

- Step 1 Make a default statement
- Step 2 Design an **experiment**
- Step 3 Describe the **data** generated from the experiment and the corresponding random variables
- Step 4 Describe the parameter of interest and their estimates
- Step 5 Translate the default statement into a statistical hypothesis and call it the **null hypothesis** H_0
- Step 6 Find the expression for the **test statistic** s
- Step 7 Find the expression for the **null distribution**
- Step 8 Define **an alternative hypothesis** H_A: one-tailed or two-tailed
- Step 9 Choose a **significance level** α (the tail), which defines the **rejection region**
- Step 10 Run experiments and collect **data**
- Step 11 Compute the test statistic from data
- Step 12 Compute the p-value
- Step 13 If p-value$< \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0; otherwise, we fail to reject H_0.

Today

Example

Recall example: if you control the diet of your ducks, they lose 2.1 kg after one month on average. Company A has developed a drug D (Duckyphanomin!) to help ducks lose weight. They claim that on average the drug works better than diet control.

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Step 1 Make a default statement (5 secs): **drug D works the same as diet control**

Step 2 Design an **experiment** (choose $N = 30$) (10 secs):

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Example (cont.)

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$$z_0 = \frac{2.2 - 2.1}{0.6/\sqrt{30}} = 0.91$$

Example (cont.)

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- **For** $H_A : \mu_D > 2.1$ **(one-tailed):** $p = P(Z \geq z_0 \mid H_0) = 0.1807 > \alpha$

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- **For** $H_A : \mu_D \neq 2.1$ **(two-tailed):** $p = 2P(Z \geq z_0 \mid H_0) = 0.3613 > \alpha$

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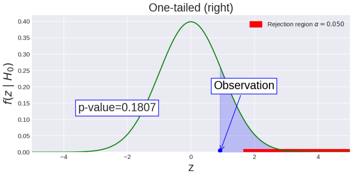
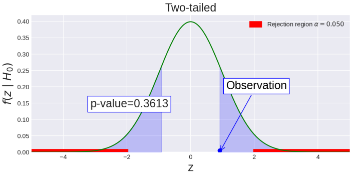
Step 13 If p-value$< \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0





Do not reject H_0 for both one-tailed and two-tailed H_A

Example (cont.)

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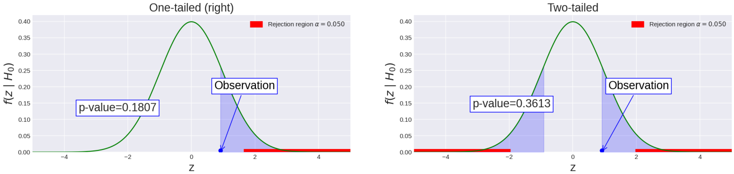

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What does it mean?

Example (cont.)

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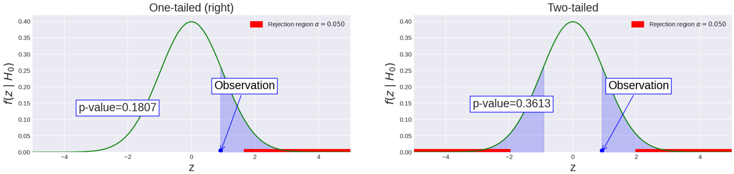

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What does it mean? - Based on this test, you will stick to diet control instead of buying Duckyphanomin

Example (cont.)

What if $\bar{x} = 2.3$?

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Step 12 Compute the p-value (20 secs):

- **For** $H_A : \mu_D > 2.1$ **(one-tailed):** $p = P(Z \geq z_0 \mid H_0) = 0.0339 < \alpha$
- **For** $H_A : \mu_D \neq 2.1$ **(two-tailed):** $p = 2P(Z \geq z_0 \mid H_0) = 0.0679 > \alpha$

Example (cont.)

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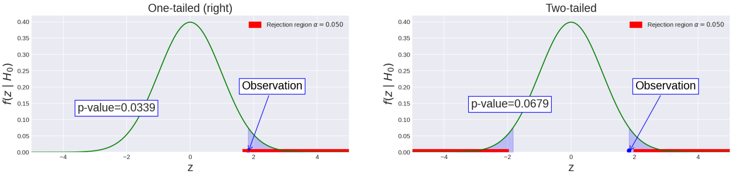
Example (cont.)

Step 13 If p-value$< \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0



Reject H_0 for one-tailed H_A; do not reject H_0 for two-tailed H_A for the same confidence level $1 - \alpha = 95\%$

Example (cont.)

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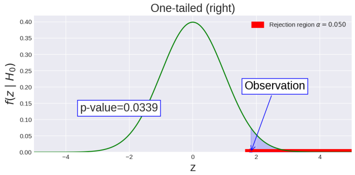

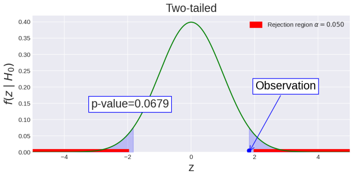

Reject H_0 for one-tailed H_A; do not reject H_0 for two-tailed H_A for the same confidence level $1 - \alpha = 95\%$

Note: the two-tailed test is more conservative - if the data passes a two-tailed test, it is more conclusive than one-tailed test for the same confidence level

Example (cont.)

What if $\bar{x} = 2.3$ with $N = 100$?

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Example (cont.)

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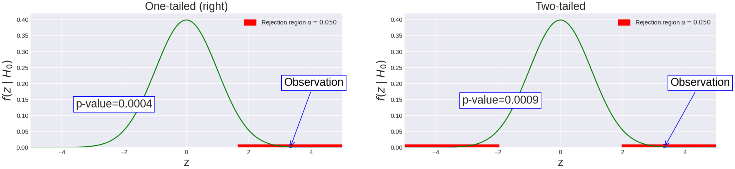
Step 12 Compute the p-value (20 secs):

- **For** $H_A : \mu_D > 2.1$ **(one-tailed):** $p = P(Z \geq z_0 \mid H_0) = 0.0004 < \alpha$
- **For** $H_A : \mu_D \neq 2.1$ **(two-tailed):** $p = 2P(Z \geq z_0 \mid H_0) = 0.0009 < \alpha$

Example (cont.)

Step 13 If p-value$< \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0



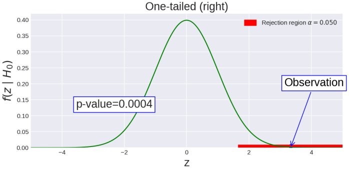
Reject H_0 for both one-tailed and two-tailed H_A

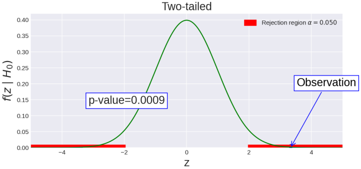
Note:

- With more data, it becomes more certain that we should reject H_0 in favor of H_A given the observation $\bar{x} = 2.3$

Example (cont.)

Step 13 If p-value$< \alpha$, i.e. the test statistic falls in the rejection region of the null distribution, then we reject the hypothesis H_0





Reject H_0 for both one-tailed and two-tailed H_A

Note:

- With more data, it becomes more certain that we should reject H_0 in favor of H_A given the observation $\bar{x} = 2.3$

This test is called **one-sample z-test** (one of the established tests you choose from)

Today

1. Terminology
2. Example
3. p-hacking
4. Summary

Recall: one-tailed vs two-tailed tests

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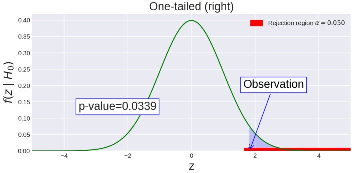

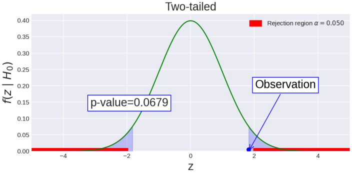

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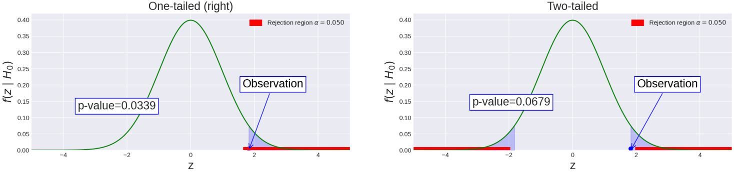

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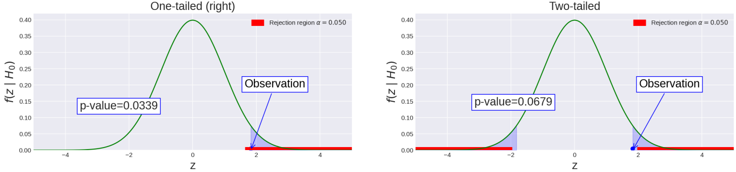

- In this example, if we use the two-tailed test, we will not claim that we have observed potential novelty with the experiment, whereas if we use the one-tailed test, we claim that we do observe potential novelty
- The conclusion we draw depends on which test we conduct

Variation of the p-value

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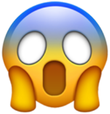
- p-value is computed from data
- Data is random - ***p*-value is random**
- With the same experiment set up, if we switch to a different sample, p-value will be different

p-hacking

- Many factors can result in a different *p*-value
- *p*-hacking refers to situations where researchers are **trying multiple things until they get the desired result**
- This action can be a conscious decision, a subconscious decision or even an accident
- *p*-hacking can be tricky to identify
- Suggestions to avoid *p*-hacking, e.g. one should always **report effect sizes and confidence intervals**
- Reference:
  - `https://www.nature.com/news/scientific-method-statistical-errors-1.14700`
  - Why Most Published Research Findings Are False?

p-hacking (cont.)

What should I do!? 

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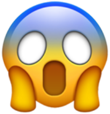
What should I do!? 

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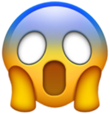
p-hacking (cont.)



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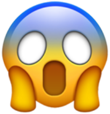
What should I do!? 

- Be honest and explicit about your assumptions
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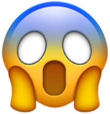
p-hacking (cont.)



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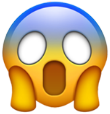
What should I do!? 

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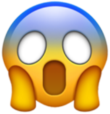
p-hacking (cont.)



What should I do!?

- Be honest and explicit about your assumptions
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- Assume the first success is always **too good to be true** - **try to prove yourself wrong** - be a proper scientist

Today

Summary

So far:

- Data types and data containers
- Descriptive data analysis: descriptive statistics, visualization
- Probability distributions, events, random variables, PMF, PDF, parameters
- CDF, Q-Q plot, how to compare two distributions (data vs theoretical, data vs data)
- Modeling
- Parameter estimation: maximum likelihood estimation (MLE) and maximum a posteriori estimation (MAP)
- Classification, multinomial naive Bayes classifier, Gaussian naive Bayes classifier
- Central limit theorem, interval estimation
- Clustering, cluster tendency
- Centroid clustering, k-means, parameter estimation, SSE, Silhouette score
- Gaussian Mixture Models, AIC/BIC
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- More examples and test statistics

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Before next lecture:

- Steps for hypothesis testing

Screw diet! I'm perfect $p = 100\%$!

That's not how p-value works...