Lecture 9: Hypothesis testing part I

Statistical Methods for Data Science

Yinan Yu

Department of Computer Science and Engineering

December 01 and 05, 2022

Today

1. Terminology
    - Experiment and parameter of interest
    - Null hypothesis and alternative hypothesis
    - Test statistic
    - Null distribution $f(s \mid H_0)$
    - Significance level α, power and p-value

Learning outcome

- Be able to explain the following terminology
  - Null hypothesis H_0 and alternative hypothesis H_A
  - Test statistic s
  - Null distribution $f(s \mid H_0)$
  - Significance level α and power
  - p-value
- Be able to design and interpret the one-sample z-test
- Be able to explain the concept of p-hacking

Today

1. Terminology
   - Experiment and parameter of interest
   - Null hypothesis and alternative hypothesis
   - Test statistic
   - Null distribution $f(s \mid H_0)$
   - Significance level α, power and p-value

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?
- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?
- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?
- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?
- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?
- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?
- Now company B tested drug E on 30 ducks and the average weight loss after one month is 2.5 kg, while drug D results in 2.3 kg weight loss with the same setup, would you buy drug E instead of drug D?

Important example

If you control the diet of your ducks, they lose 2.1 kg after one month on average

- Company A has developed a drug D (aka. Duckyphanomin) to help duckies lose weight. They claim that **on average** the drug works better than diet control
- Company B has developed a drug E (aka. Everyduckyslim) and they claim that drug E is more effective than drug D **on average**

You NEED to help your chonker ducks lose weight. Which drug should you buy? Or should you just control their diet without drugs?

- If company A tested drug D on 30 ducks and the average weight loss after one month is 2.2 kg, would you buy drug D instead of regular diet control?
- What if company A tested drug D on 30 ducks and the average weight loss after one month is 2.3 kg? Would you buy drug D instead of regular diet control in this case?
- What if company A tested drug D on 100 ducks and the average weight loss after one month is 2.3 kg?
- Now company B tested drug E on 30 ducks and the average weight loss after one month is 2.5 kg, while drug D results in 2.3 kg weight loss with the same setup, would you buy drug E instead of drug D?

What would you do?

Hypothesis

- **Hypothesis:**
  - A proposed explanation for a phenomenon (Wikipedia)
  - An idea or explanation of something that is based on a few known facts but that has not yet been proved to be true or correct (Oxford dictionary)

Hypothesis

- **Hypothesis:**
  - A proposed explanation for a phenomenon (Wikipedia)
  - An idea or explanation of something that is based on a few known facts but that has not yet been proved to be true or correct (Oxford dictionary)
- **Statistical hypothesis:** a proposed distribution that explains a set of random variables

Hypothesis

- **Hypothesis:**
  - A proposed explanation for a phenomenon (Wikipedia)
  - An idea or explanation of something that is based on a few known facts but that has not yet been proved to be true or correct (Oxford dictionary)
- **Statistical hypothesis:** a proposed distribution that explains a set of random variables
- **Hypothesis testing in statistics:** we want to decide if it is likely that a random variable follows the proposed distribution

Hypothesis

- **Hypothesis:**
  - A proposed explanation for a phenomenon (Wikipedia)
  - An idea or explanation of something that is based on a few known facts but that has not yet been proved to be true or correct (Oxford dictionary)
- **Statistical hypothesis:** a proposed distribution that explains a set of random variables
- **Hypothesis testing in statistics:** we want to decide if it is likely that a random variable follows the proposed distribution
  - The test is based on sample statistics, which are computed from data

Hypothesis

- **Hypothesis:**
  - A proposed explanation for a phenomenon (Wikipedia)
  - An idea or explanation of something that is based on a few known facts but that has not yet been proved to be true or correct (Oxford dictionary)
- **Statistical hypothesis:** a proposed distribution that explains a set of random variables
- **Hypothesis testing in statistics:** we want to decide if it is likely that a random variable follows the proposed distribution
  - The test is based on sample statistics, which are computed from data
  - Hypothesis + data $\rightarrow$ decision on rejecting or not rejecting the hypothesis

Hypothesis testing: a list to go through

- **A default statement**
- **Experiment**
- **Data x, random variable X**
- **Parameter of interest θ**
- **Parameter estimate $\hat{\theta}$**
- **Null hypothesis H_0**
- **Alternative hypothesis H_A**
- **Test statistic s**
- **Null distribution $f(s \mid H_0)$**
- **Significance level α**
- **p-value**

Experiment and parameter of interest

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement?

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec):

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.
  - **Parameter of interest** (5 sec):

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.
  - **Parameter of interest** (5 sec): the mean of the weight loss μ_D

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.
  - **Parameter of interest** (5 sec): the mean of the weight loss μ_D
  - **Parameter estimate** (5 sec):

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.
  - **Parameter of interest** (5 sec): the mean of the weight loss μ_D
  - **Parameter estimate** (5 sec): the sample mean $\hat{\mu_D} = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

Experiment design

- Before formulating the statistical hypothesis, we need to propose a **default statement**: a "boring" and unsurprising claim that we would like to **test**, e.g.,
  - Drug D is **not more effective** than regular diet on average
  - Drug E works the same as drug D on average

  In science, we are hoping for new discoveries and excitement, but we need to earn it by showing that the trivial explanation does not hold
- How do we test the default statement? We need to design and run **experiments** to collect evidence (**data**)
- Example 1: recall if you control the diet of your ducks, they lose 2.1 kg after one month on average
  - **A default statement**: drug D is not more effective than regular diet on average What experiments can we run to test if this statement is true?
  - **Experiment** (5 sec): give drug D to N chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):
    - Data: x_i weight loss after one month for $i = 1, \cdots, N$
    - Random variable: X_i i.i.d.
  - **Parameter of interest** (5 sec): the mean of the weight loss μ_D
  - **Parameter estimate** (5 sec): the sample mean $\hat{\mu_D} = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$

  Then we test if $\bar{x}$ is greater than diet control (2.1 kg)

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec):

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec):

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E
  - **Parameter of interest** (5 secs):

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E
  - **Parameter of interest** (5 secs): the mean μ_D and μ_E for drug D and E, respectively

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E
  - **Parameter of interest** (5 secs): the mean μ_D and μ_E for drug D and E, respectively
  - **Parameter estimate** (5 secs):

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E
  - **Parameter of interest** (5 secs): the mean μ_D and μ_E for drug D and E, respectively
  - **Parameter estimate** (5 secs): the sample mean $\hat{\mu}_D = \bar{x} = \frac{1}{N_D} \sum_{i=1}^{N_D} x_i$ and $\hat{\mu_E} = \bar{y} = \frac{1}{N_E} \sum_{j=1}^{N_E} y_j$

Experiment design (cont.)

- Example 2:
  - **A default statement**: drug E and drug D work the same on average
  - **Experiment** (5 sec): give drug D to N_D chonker ducks and record the average weight loss after one month; test drug E on another N_E chonker ducks and record the average weight loss after one month
  - **Data** and **random variable** (5 sec): data - x_i weight loss using drug D after one month for $i = 1, \cdots, N_D$; random variable - X_i i.i.d.; likewise, we have data y_j and random variable Y_j for drug E
  - **Parameter of interest** (5 secs): the mean μ_D and μ_E for drug D and E, respectively
  - **Parameter estimate** (5 secs): the sample mean $\hat{\mu}_D = \bar{x} = \frac{1}{N_D}\sum_{i=1}^{N_D} x_i$ and $\hat{\mu_E} = \bar{y} = \frac{1}{N_E}\sum_{j=1}^{N_E} y_j$

  Then we test if $\bar{x}$ and $\bar{y}$ are the same

Experiment design (cont.)

- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**

Experiment design (cont.)

- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**
- But we can never prove or accept the statement - we can only **reject** a statement by showing counterexamples

Experiment design (cont.)

- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**
- But we can never prove or accept the statement - we can only **reject** a statement by showing counterexamples
- Intuition: "If the statement is true, then the evidence should support the statement",

Experiment design (cont.)

- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**
- But we can never prove or accept the statement - we can only **reject** a statement by showing counterexamples
- Intuition: "If the statement is true, then the evidence should support the statement", which is the same as ($\iff$) "if the evidence does not support the statement, the statement is considered false"

Experiment design (cont.)

- We make our decision by observing data; if the evidence does not support the default statement, we **reject the statement**; otherwise, we **do not reject the statement**
- But we can never prove or accept the statement - we can only **reject** a statement by showing counterexamples
- Intuition: "If the statement is true, then the evidence should support the statement", which is the same as ($\iff$) "if the evidence does not support the statement, the statement is considered false" , which is not the same as ($\not\Leftrightarrow$) "if the evidence supports the statement, the statement must be true"

Null hypothesis and alternative hypothesis

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution**

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis** H_0: the default statement translated into a mathematical expression

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis** H_0: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis** H_0: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis H_0**: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis** H_0: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis** H_A: an alternative hypothesis that is complementary (the opposite) to the null hypothesis

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis H_0**: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis H_A**: an alternative hypothesis that is complementary (the opposite) to the null hypothesis
  - Example 2 (5 sec):

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis** H_0: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis** H_A: an alternative hypothesis that is complementary (the opposite) to the null hypothesis
  - Example 2 (5 sec): drug E and drug D do not work the same on average (5 sec)

$$H_A : \mu_D \neq \mu_E$$

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis H_0**: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis H_A**: an alternative hypothesis that is **complementary** (the opposite) to the null hypothesis
  - Example 2 (5 sec): drug E and drug D do not work the same on average (5 sec)

$$H_A : \mu_D \neq \mu_E$$

  - Example 1 (5 sec):

Hypotheses H_0 and H_A

- **Statistical hypothesis**: a proposed **distribution** - a statement about the **parameter of interest**
- **Null hypothesis H_0**: the default statement translated into a mathematical expression
  - Example 1: drug D is not more effective than regular diet on average

$$H_0 : \mu_D = 2.1$$

  - Example 2: drug E and drug D work the same on average (5 sec)

$$H_0 : \mu_D = \mu_E$$

- **Alternative hypothesis H_A**: an alternative hypothesis that is **complementary** (the opposite) to the null hypothesis
  - Example 2 (5 sec): drug E and drug D do not work the same on average (5 sec)

$$H_A : \mu_D \neq \mu_E$$

  - Example 1 (5 sec): drug D is more effective than regular diet on average (5 sec)

$$H_A : \mu_D > 2.1$$

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?
  Answer: In a few slides

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?
  Answer: In a few slides
  Okay

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?
  Answer: In a few slides
  Okay
- Question 2: Can H_0 and H_A be ANYTHING I want? Like a magic mirror!?

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?
  Answer: In a few slides
  Okay
- Question 2: Can H_0 and H_A be ANYTHING I want? Like a magic mirror!?
  Answer: No

Hypotheses H_0 and H_A (cont.)

Questions:

- Question 1: Why are $H_A : \mu_D > 2.1$ and $H_0 : \mu_D = 2.1$ complementary to each other? What about $H_A : \mu_D < 2.1$?
  Answer: One implicit assumption here is that μ_D will not be smaller than 2.1
  Question 1.1: Do I need to make this assumption?
  Answer: No
  Question 1.2: Could you elaborate on that?
  Answer: Yes
  Question 1.3: When?
  Answer: In a few slides
  Okay
- Question 2: Can H_0 and H_A be ANYTHING I want? Like a magic mirror!?
  Answer: No
  Question 2.2: What are the choices for H_0 and H_A then?

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis H_0**: two cases

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis** H_0: two cases
  - **One-sample test**: to test a data distribution against a theoretical probability distribution, i.e. for a given constant c

$$H_0 : \theta = c$$

For example, is this (binary) classifier more accurate than random?

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis** H_0: two cases
  - **One-sample test**: to test a data distribution against a theoretical probability distribution, i.e. for a given constant c

    $$H_0 : \theta = c$$

    For example, is this (binary) classifier more accurate than random? $H_0 : p = 50\%$

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis H_0**: two cases
  - **One-sample test**: to test a data distribution against a theoretical probability distribution, i.e. for a given constant c

    $$H_0 : \theta = c$$

    For example, is this (binary) classifier more accurate than random? $H_0 : p = 50\%$
  - **Two-sample test**: to test a data distribution against another data distribution, i.e.

    $$H_0 : \theta_1 = \theta_2$$

    For example, is classifier A better than classifier B?

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis H_0**: two cases
  - **One-sample test**: to test a data distribution against a theoretical probability distribution, i.e. for a given constant c

$$H_0 : \theta = c$$

    For example, is this (binary) classifier more accurate than random? $H_0 : p = 50\%$

  - **Two-sample test**: to test a data distribution against another data distribution, i.e.

$$H_0 : \theta_1 = \theta_2$$

    For example, is classifier A better than classifier B? $H_0 : p_A = p_B$

Choices for H_0

- In this course, we only deal with null hypotheses **with an equal sign** in them - only one fixed choice for the distribution proposed by H_0
- **Null hypothesis H_0**: two cases
  - **One-sample test**: to test a data distribution against a theoretical probability distribution, i.e. for a given constant c

    $$H_0 : \theta = c$$

    For example, is this (binary) classifier more accurate than random? $H_0 : p = 50\%$
  - **Two-sample test**: to test a data distribution against another data distribution, i.e.

    $$H_0 : \theta_1 = \theta_2$$

    For example, is classifier A better than classifier B? $H_0 : p_A = p_B$
- We have seen one-sample test and two-sample test in the Q-Q plot lecture
- In practice, you can narrow down your choice of hypotheses by looking at Q-Q plots

Choices for H_A

Given

$$H_0 : \theta = \beta$$

where β can be either a constant (one-sample test) or a parameter from another data distribution (two-sample test)

- **Alternative hypothesis H_A**: H_A can be **one-tailed** or **two-tailed**
  - **One-tailed**:

$$H_A : \quad \theta > \beta$$

or

$$H_A : \quad \theta < \beta$$

  - **Two-tailed**:

$$H_A : \quad \theta \neq \beta \iff \theta < \beta \text{ or } \theta > \beta$$

Summary: choices for H_0 and H_A

Putting everything together,

	One-sample test	Two-sample test
Two-tailed	$H_0 : \theta = c,\ H_A : \theta \neq c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 \neq \theta_2$
One-tailed	$H_0 : \theta = c,\ H_A : \theta > c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 > \theta_2$
	$H_0 : \theta = c,\ H_A : \theta < c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 < \theta_2$

where θ, θ_1, θ_2 are the parameters of interest and c is a constant

Summary: choices for H_0 and H_A

Putting everything together,

	One-sample test	Two-sample test
Two-tailed	$H_0 : \theta = c,\ H_A : \theta \neq c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 \neq \theta_2$
One-tailed	$H_0 : \theta = c,\ H_A : \theta > c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 > \theta_2$
	$H_0 : \theta = c,\ H_A : \theta < c$	$H_0 : \theta_1 = \theta_2,\ H_A : \theta_1 < \theta_2$

where θ, θ_1, θ_2 are the parameters of interest and c is a constant
Note: this is the answer to question 1.1 (cf. page 14): if you choose the one-tailed test, then you are making the assumption $H_A : \mu_D > 2.1$; if you choose the two-tailed test, then you are not making this assumption

Test statistic

Test statistic

- **Test statistic** s (random variable S): a (typically standardized) statistic computed from data
- **Purpose:**
  - Assume the null hypothesis is true, we can calculate the sampling distribution of S
  - Then we observe s; s will indicate how plausible this sampling distribution

Test statistic

- **Test statistic** s (random variable S): a (typically standardized) statistic computed from data
- **Purpose:**
  - Assume the null hypothesis is true, we can calculate the sampling distribution of S
  - Then we observe s; s will indicate how plausible this sampling distribution
- What is needed for computing the test statistic?
  - Assumptions on random variables X_i
  - We only need the null hypothesis H_0 (not H_A) to choose the test statistic

Test statistic

- **Test statistic** s (random variable S): a (typically standardized) statistic computed from data
- **Purpose:**
  - Assume the null hypothesis is true, we can calculate the sampling distribution of S
  - Then we observe s; s will indicate how plausible this sampling distribution
- What is needed for computing the test statistic?
  - Assumptions on random variables X_i
  - We only need the null hypothesis H_0 (not H_A) to choose the test statistic

Disclaimer: in this course, we only deal with null hypothesis where we are able to express the PDF/PMF $f(s \mid H_0)$, i.e. H_0 with an equal sign in them

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: standardized $\bar{x}$ assuming the null hypothesis

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
    - Recall: what is **standardization**?
        - Random variable X: $Y = \frac{X-\mu_X}{\sigma_X}$
        - Data x: $y = \frac{x-\mu_X}{\sigma_X}$
    - What are we trying to do here?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X-\mu_X}{\sigma_X}$
    - Data x: $y = \frac{x-\mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?
    - Gaussian distribution with standard deviation σ and mean $\mu_D = 2.1$

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?
    - Gaussian distribution with standard deviation σ and mean $\mu_D = 2.1$
  - **Assuming the null hypothesis**: data are assumed to be generated from the distribution described by the null hypothesis

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?
    - Gaussian distribution with standard deviation σ and mean $\mu_D = 2.1$
  - **Assuming the null hypothesis**: data are assumed to be generated from the distribution described by the null hypothesis - $X_i \sim \mathcal{N}(2.1, \sigma^2)$

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- Data: $x_1, \cdots, x_N$
- Random variable: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
- Parameter of interest: μ_D
- Parameter estimate: $\bar{x}$
- Null hypothesis: $H_0 : \mu_D = 2.1$
- Test statistic: standardized $\bar{x}$ assuming the null hypothesis
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?
    - Gaussian distribution with standard deviation σ and mean $\mu_D = 2.1$
  - **Assuming the null hypothesis**: data are assumed to be generated from the distribution described by the null hypothesis - $X_i \sim \mathcal{N}(2.1, \sigma^2)$

Standardize $\bar{x}$ (15 sec)

Test statistic (cont.)

Example 1. one-sample test (is drug D more effective than diet control)

- **Data**: $x_1, \cdots, x_N$
- **Random variable**: $X_1, \cdots, X_N$ i.i.d. **Gaussian with known σ**
- **Parameter of interest**: μ_D
- **Parameter estimate**: $\bar{x}$
- **Null hypothesis**: $H_0 : \mu_D = 2.1$
- **Test statistic**: **standardized** $\bar{x}$ **assuming the null hypothesis**
  - Recall: what is **standardization**?
    - Random variable X: $Y = \frac{X - \mu_X}{\sigma_X}$
    - Data x: $y = \frac{x - \mu_X}{\sigma_X}$
  - What are we trying to do here? - To test if we can reject the null hypothesis by asking does data follow the **distribution described by the null hypothesis**?
  - Why are we standardizing the statistic $\bar{x}$? We want to use standard tools for our analysis
  - What is the **distribution described by the null hypothesis**?
    - Gaussian distribution with standard deviation σ and mean $\mu_D = 2.1$
  - **Assuming the null hypothesis**: data are assumed to be generated from the distribution described by the null hypothesis - $X_i \sim \mathcal{N}(2.1, \sigma^2)$

Standardize $\bar{x}$ (15 sec)

$$z = \frac{\bar{x} - 2.1}{\sigma/\sqrt{N}}$$

Test statistic (cont.)

Example 2. two-sample test

- **Data**: $x_1, \cdots, x_{N_D}$ and $y_1, \cdots, y_{N_E}$
- **Random variable**: $X_1, \cdots, X_{N_D}$ i.i.d. **Gaussian with known σ_D**; $Y_1, \cdots, Y_{N_E}$ i.i.d. **Gaussian with known σ_E**; X_i and Y_j independent
- **Parameter of interest**: μ_D, μ_E
- **Parameter estimate**: $\bar{x}$, $\bar{y}$
- **Null hypothesis**: $H_0 : \mu_D = \mu_E \iff H_0 : \mu_D - \mu_E = 0$
- **Test statistic**: standardized $\bar{x} - \bar{y}$ assuming the null hypothesis

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\sigma_D^2/N_D + \sigma_E^2/N_E}}$$

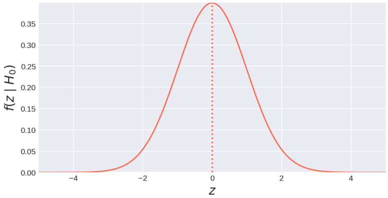
Null distribution $f(s \mid H_0)$

Null distribution

- **Null distribution** $f(s \mid H_0)$: the distribution of the test statistic given the null hypothesis
- Example:
  - **Data**: $x_1, \cdots, x_N$
  - **Random variable**: $X_1, \cdots, X_N$ i.i.d. Gaussian with known σ
  - **Parameter of interest**: μ
  - **Parameter estimate**: $\bar{x}$
  - **Null hypothesis**: $H_0 : \mu = \mu_0$
  - **Test statistic**:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}}$$

  - **Null distribution**: standard Gaussian distribution



Significance level α, power and p-value