Data modeling: CSCI E-106

Applied Linear Statistical Models

Chapter 10 – Building the Regression Model II: Diagnostic

Overview

- a number of refined diagnostics for checking the adequacy of a regression model
  - detecting improper functional form for a predictor variable
  - outliers
  - influential observations
  - multicollinearity

Added Variable Plots

Previous

- Chap. 3,6: check whether a curvature effect for that variable is required in the model
- the residual plots vs. the predictor variables: determine whether it would be helpful to add one or more of these variables to the model

Limitation:

- not properly show the nature of the marginal effect of a predictor variable, given the other predictor variables in the model

Added Variable Plots, cont'd

- *partial regression plots* or *adjusted variable plots*;
  provide graphic information about the marginal importance of a predictor variable X_k , given the other predictor variables already in the model
- In an added-variable plot, both Y and X_k under consideration are regressed against the other predictor variables and residuals are obtained for each.
- the plot of these residuals:
  - the marginal importance of this variable in reducing the residual variability
  - provide information about the nature of the marginal regression relation for X_k under consideration for possible inclusion in the regression model

Added Variable Plots, cont'd

- Illustration: the regression effect for X_1, given that X_2 is already in the model

regress Y on X_2

$$\hat{Y}_i(X_2) = b_0 + b_2 X_{i2}$$
$$e_i(Y|X_2) = Y_i - \hat{Y}_i(X_2)$$

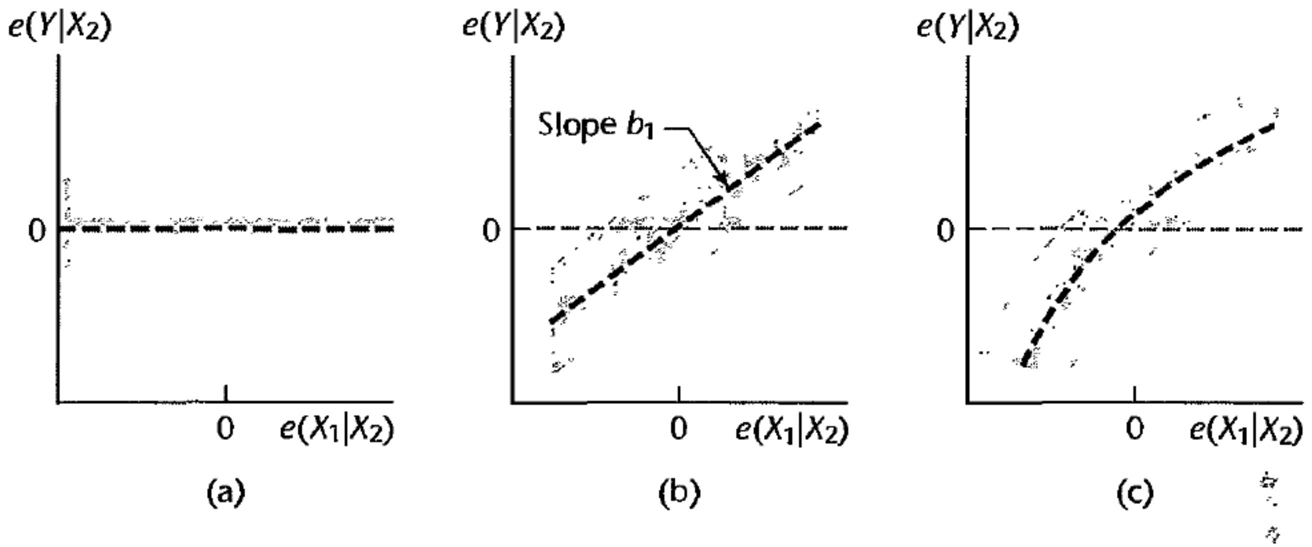
regress X_1 on X_2

$$\hat{X}_{i1}(X_2) = b_0^* + b_2^* X_{i2}$$
$$e_i(X_1|X_2) = X_{i1} - \hat{X}_{i1}(X_2)$$

$\Rightarrow$added variable for predictor variable X_1:

Plot $e(Y|X_2)$ vs $e(X_1|X_2)$

Added Variable Plots, cont'd



- Figure (a) shows a horizontal band, indicating that X_1 contains no additional information useful for predicting Y beyond that contained in X_2, so that it is not helpful to add X_1 to the regression model.
- Figure (b) shows a linear band with a nonzero slope. This plot indicates that a linear term in X_1 may be a helpful addition to the regression model already containing X_2.
- Figure (c) shows a curvilinear band, indicating that the addition of X_1 to the regression model may be helpful and suggesting the possible nature of the curvature effect by the pattern shown.

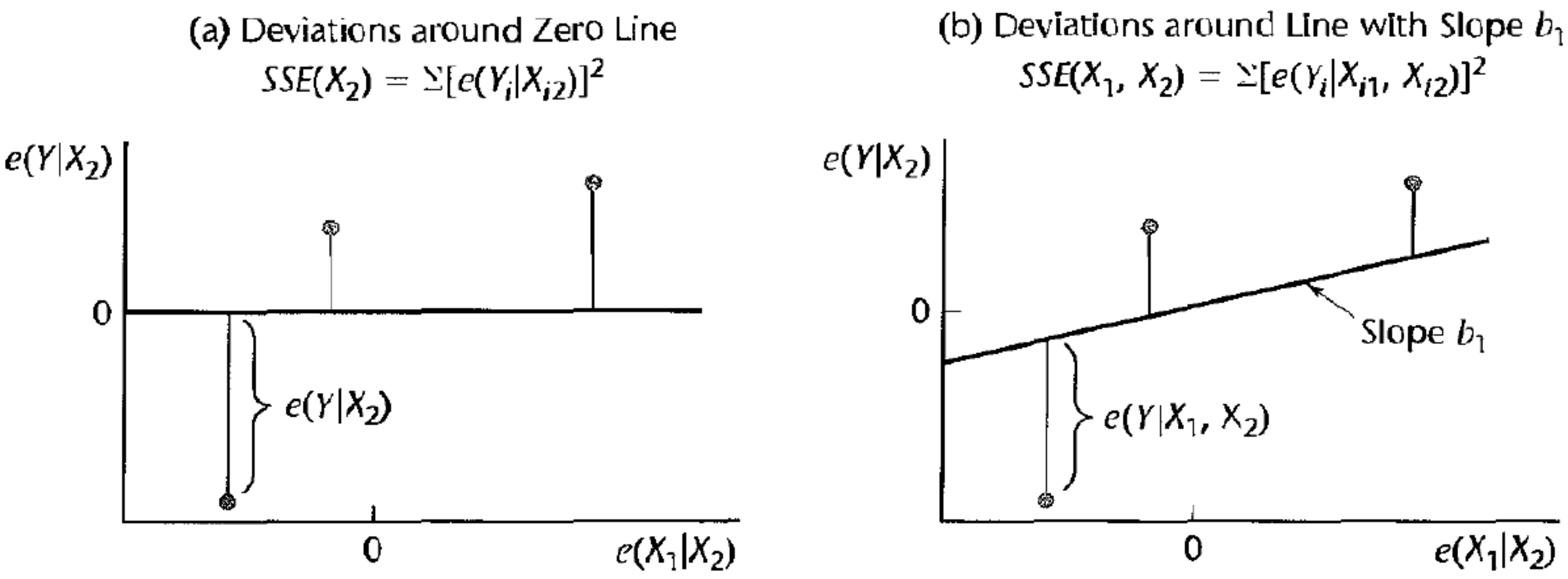
Added Variable Plots, cont'd

Added-variable plots:

- providing information about the possible nature of the marginal relationship for a predictor variable, given the other variables already in the regression model
- the strength of the relationship
- useful for uncovering outlying data points that may have a strong influence in estimating the relationship of the predictor variable X_k to the response variable

Added Variable Plots, cont'd

FIGURE 10.2 Illustration of Deviations in an Added-Variable Plot.



- $SSE(X_2)$
- $SSE(X_1, X_2)$
- Difference ($SSE(X_2) - SSE(X_1, X_2)$): $SSR(X_1|X_2)$; provides information about the marginal strength of the linear relation of X_1 to the response variable, given that X_2 is in the model

Added Variable Plots, cont'd

Example:

Manager i	Average Annual Income (thousand dollars) X_{i1}	Risk Aversion Score X_{i2}	Amount of Life Insurance Carried (thousand dollars) Y_i
1	45.01	6	91
2	57.20	4	162
3	26.85	5	11
...	...	...	...
16	46.13	4	91
17	30.37	3	14
18	39.06	5	63

- r_{12}=0.254
- $\hat{Y} = -205.72 + 6.288X_1 + 4.738X_2$
- Residual plot: a linear relation for X_1 is not appropriate in the model already containing X_2

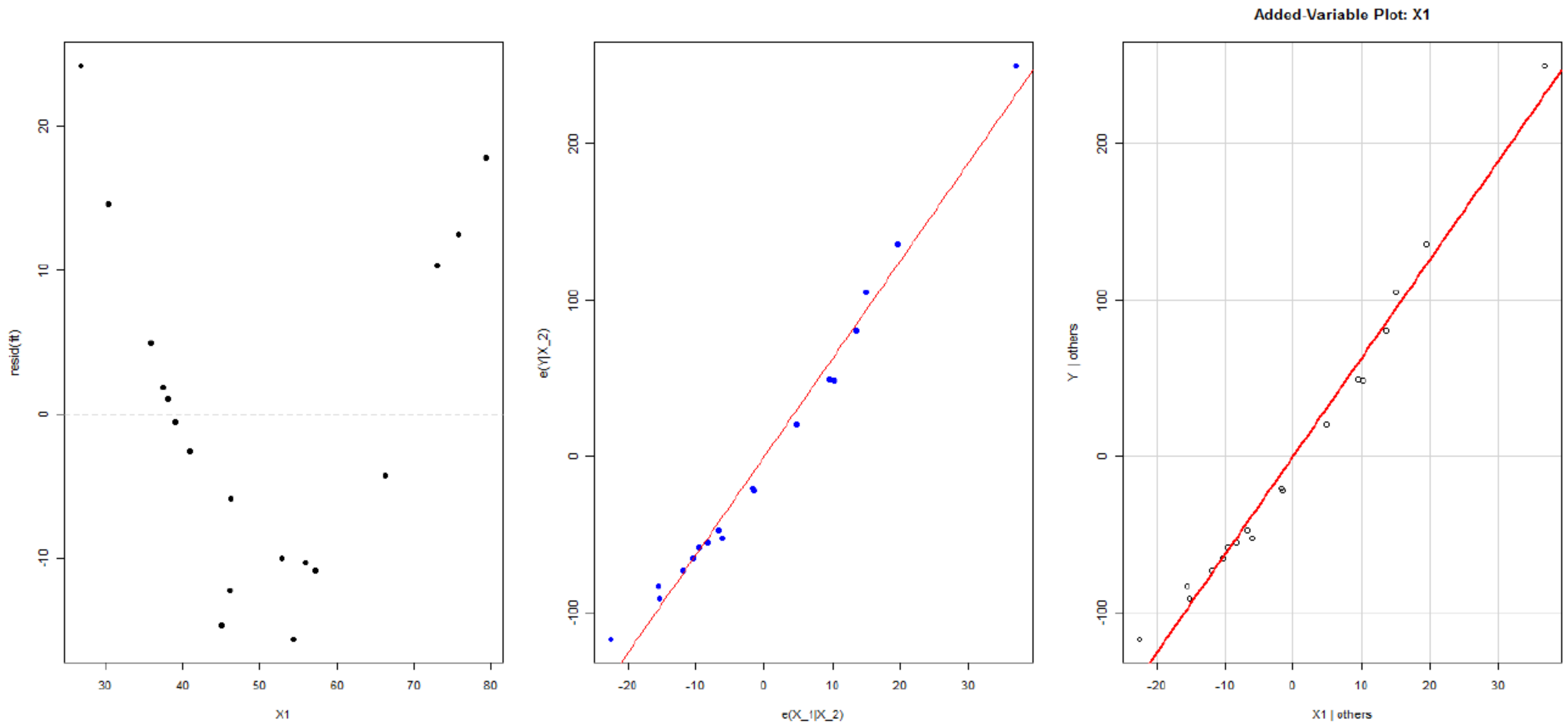
R Code

```
attach(Dataset_10TA01)
fit<-lm(Y~X1+X2)
par(mfrow=c(1,3))
plot(X1,resid(fit),pch=16)
abline(0,0,lty=2,col="gray")

## Method 1:
plot(resid(lm(Y~X2)) ~ resid(lm(X1~X2)),col="blue",pch=16,xlab="e(X_1|X_2)", ylab="e(Y|X_2)")
abline(lm(resid(lm(Y~X2))~resid(lm(X1~X2))),col="red")

## Method 2: using avPlot()
library(car)
avPlot( model=lm( Y~X1+X2 ), variable=X1 )
```

Added Variable Plots, cont'd



- $\hat{Y}(X_2) = 50.70 + 15.54X_2$; $\hat{X}_1(X_2) = 40.779 + 1.718X_2$
- Plots: through (0,0); $b_1 = 6.2880$;
- suggest the curvilinear relation between Y and $X_1|X_2$ is strongly positive; a slight concave upward shape
- $R^2_{Y1|2} = 0.984$

Added Variable Plots, cont’d

Comments:

- An added-variable plot only suggests the nature of the functional relation in which a predictor variable should be added to the regression model but does not provide an analytic expression of the relation.
- Added-variable plots need to be used with caution for identifying the nature of the marginal effect of a predictor variable.
  - may not show the proper form of the marginal effect of a predictor variable if the functional relations for some or all of the predictor variables already in the regression model are misspecified
  - the relations of the predictor variable to the response variable are complex
  - high multicollinearity among the predictor variables

Added Variable Plots, cont'd

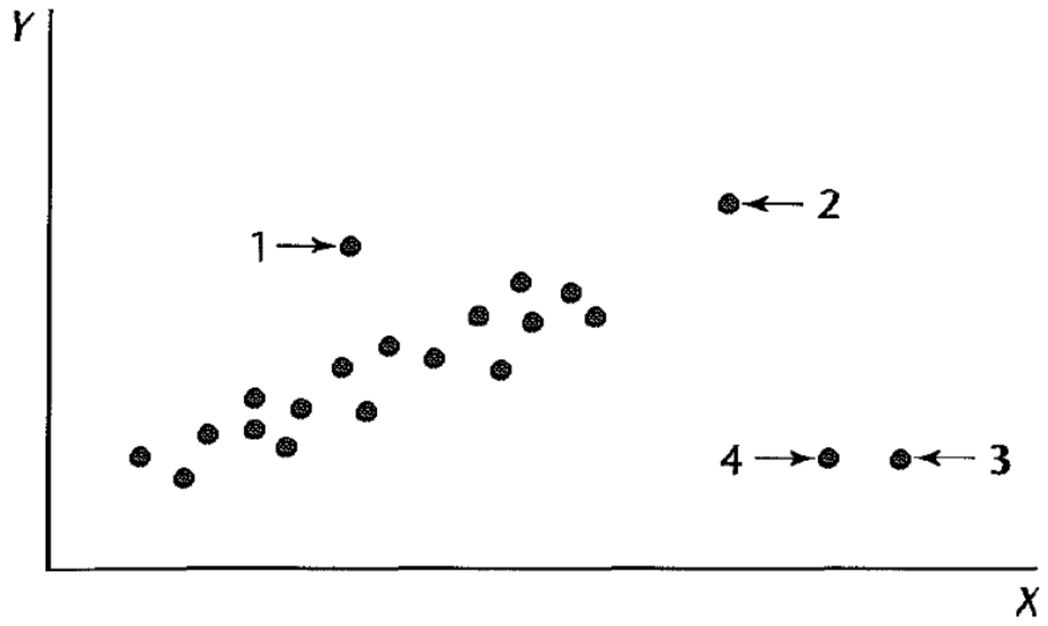
- Any fitted multiple regression function can be obtained from a sequence of fitted partial regressions. Ex: having $e(Y|X_2)$, $e(X_1|X_2)$

$\Rightarrow e(Y|X_2) = 6.2880[e(X_1|X_2)]$
$\Rightarrow [\, \hat{Y} - \hat{Y}(X_2) \,] = 6.2880[X_1 - \hat{X}_1(X_2)]$
$\Rightarrow \hat{Y} = -205.72 + 6.2880X_1 + 4.737X_2$

Identifying Outlying *Y* Observations

Outlying or Extreme:

- the observations for these cases are well separated from the remainder of the data
- large residuals; have dramatic effects



Identifying Outlying *Y* Observations, cont'd

- Outlying: *Y* value, *X* values or both
- Not all outlying cases have a strong influence on the fitted regression function.
- A basic step: determine if the regression model under consideration is heavily influenced by one or a few cases in the data set

Identifying Outlying Y Observations, cont'd

Two refined measures for identifying cases with outlying Y observations:

- Residuals, Semistudentized Residuals:

$$e_i = Y_i - \hat{Y}_i; \quad e_i^* = \frac{e_i}{\sqrt{MSE}}$$

- Hat matrix: $\boldsymbol{H} = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}' \Rightarrow \hat{\boldsymbol{Y}} = \boldsymbol{H}\boldsymbol{Y}$

$$\begin{aligned}
&\boldsymbol{e} = (\boldsymbol{I} - \boldsymbol{H})\boldsymbol{Y} \\
\Rightarrow &\sigma^2\{\boldsymbol{e}\} = \sigma^2(\boldsymbol{I} - \boldsymbol{H}) \\
&\sigma^2\{e_i\} = \sigma^2(1 - h_{ii}), \quad h_{ii}\text{: the } i\text{th elements on the diagonal of } \boldsymbol{H} \\
&\sigma\{e_i, e_j\} = -h_{ij}\sigma^2, \quad i \neq j \\
\Rightarrow &s^2\{e_i\} = MSE(1 - h_{ii}), \quad s\{e_i, e_j\} = -h_{ij}MSE
\end{aligned}$$

Identifying Outlying *Y* Observations, cont'd

- $h_{ii} = \boldsymbol{X}_i'(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}_i, \quad \boldsymbol{X}_i = [1\, X_{i,1} \cdots X_{i,p-1}]'$

Small data set: $n = 4$

(a) Data and Basic Results

i	(1) X_{i1}	(2) X_{i2}	(3) Y_i	(4) $\hat{Y}_i$	(5) e_i	(6) h_{ii}	(7) $s^2\{e_i\}$
1	14	25	301	282.2	18.8	.3877	352.0
2	19	32	327	332.3	−5.3	.9513	28.0
3	12	22	246	260.0	−14.0	.6614	194.6
4	11	15	187	186.5	.5	.9996	.2

(b) H

$$\begin{bmatrix} .3877 & .1727 & .4553 & -.0157 \\ .1727 & .9513 & -.1284 & .0044 \\ .4553 & -.1284 & .6614 & .0117 \\ -.0157 & .0044 & .0117 & .9996 \end{bmatrix}$$

(c) $s^2\{e\}$

$$\begin{bmatrix} 352.0 & -99.3 & -261.8 & 9.0 \\ -99.3 & 28.0 & 73.8 & -2.5 \\ -261.8 & 73.8 & 194.6 & -6.7 \\ 9.0 & -2.5 & -6.7 & .2 \end{bmatrix}$$

- $\hat{Y} = 80.93 - 5.84X_1 + 11.32X_2$
- $MSE = 574.9$
- $s^2\{e_1\} = 574.9(1 - 0.3877) = 352.0$

Identifying Outlying *Y* Observations, cont'd

- Deleted Residuals: The difference between Y_i and $\hat{Y}_{i(i)}$: (*PRESS prediction error*)

$$\text{delted residual: } d_i = Y_i - \hat{Y}_{i(i)} = \frac{e_i}{1 - h_{ii}}$$

- h_{ii}: the larger will be the deleted residuals as compared to the ordinary residual
- the estimated variance of d_i:

$$s^2\{d_i\} = MSE_{(i)}(1 + \boldsymbol{X}_i'(\boldsymbol{X}_{(i)}'\boldsymbol{X}_{(i)})^{-1}\boldsymbol{X}_i) = \frac{MSE_{(i)}}{1 - h_{ii}}$$

$$\Rightarrow \frac{d_i}{s\{d_i\}} \sim t((n-1) - p)$$

Identifying Outlying *Y* Observations, cont'd

- Studentized Deleted Residuals:

$$t_i = \frac{d_i}{s\{d_i\}} = \frac{e_i}{\sqrt{MSE_{(i)}(1 - h_{ii})}}$$

$$\left((n-p)MSE = (n-p-1)MSE_{(i)} + \frac{e_i^2}{1-h_{ii}}\right)$$

$$\Rightarrow t_i = e_i\left[\frac{n-p-1}{SSE(1-h_{ii}) - e_i^2}\right]^{1/2}$$

- h_{ii}: the larger will be the deleted residuals as compared to the ordinary residual

Identifying Outlying *Y* Observations, cont'd

- the estimated variance of d_i:

$$s^2\{d_i\} = MSE_{(i)}(1 + \boldsymbol{X}_i'(\boldsymbol{X}_{(i)}'\boldsymbol{X}_{(i)})^{-1}\boldsymbol{X}_i) = \frac{MSE_{(i)}}{1 - h_{ii}}$$

$$\Rightarrow \frac{d_i}{s\{d_i\}} \sim t((n-1)-p)$$

- Test for Outliers: whose studentized deleted residuals are large in absolute value
  - If the regression model is appropriate, so that no case is outlying. Each $t_i \sim t(n-p-1)$.
  - $|t_i|$: the appropriate Bonferroni critical value: $t(1-\alpha/2n; n-p-1)$

Identifying Outlying *Y* Observations, cont'd

Body fat example

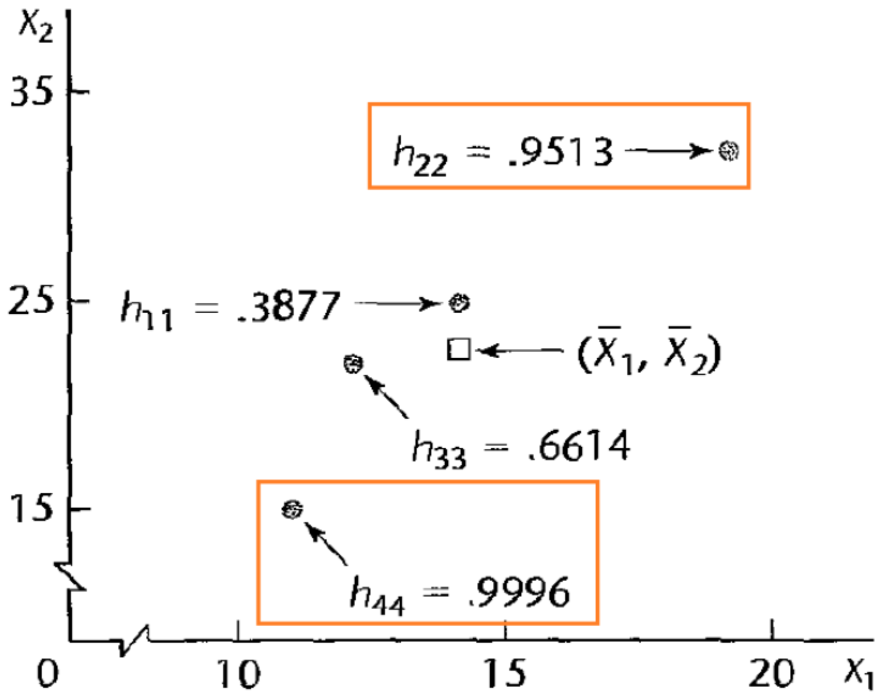
i	(1) e_i	(2) h_{ii}	(3) t_i
1	−1.683	.201	−.730
2	3.643	.059	1.534
3	−3.176	.372	−1.656
4	−3.158	.111	−1.348
5	.000	.248	.000
6	−.361	.129	−.148
7	.716	.156	.298
8	4.015	.096	1.760
9	2.655	.115	1.117
10	−2.475	.110	−1.034
11	.336	.120	.137
12	2.226	.109	.923
13	−3.947	.178	$\boxed{-1.825}$
14	3.447	.148	1.524
15	.571	.333	.267
16	.642	.095	.258
17	−.851	.106	.344
18	−.783	.197	.335
19	−2.857	.067	−1.176
20	1.040	.050	.409

- $|t| < 3.252 = t(1 - \alpha/2n; n - p - 1)$
- The Bonferroni procedure provides a Conservative test for the presence of an outlier.

Identifying Outlying X Observations

- Using $\boldsymbol{H}$ for identifying outlying X
- $0 \leq h_{ii} \leq 1 \quad \sum_{i=1}^{n} h_{ii} = p$
- h_{ii}: called *leverage*; measure the distance between X_i and $\bar{X}$
  - large $h_{ii} \Rightarrow X_i$ distant from the center of all Xs

FIGURE 10.6 **Illustration of Leverage Values as Distance Measures—Table 10.2 Example.**



Identifying Outlying X Observations, cont'd

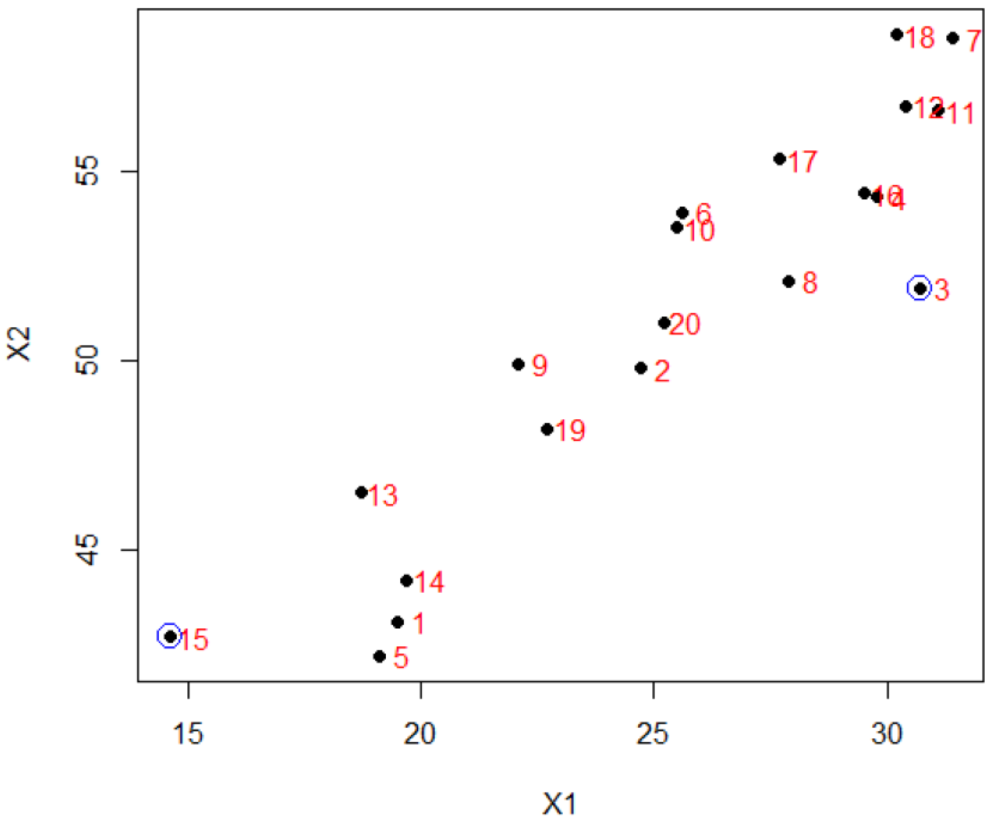
- If X_i is outlying$\Rightarrow$ has a large leverage h_{ii}
- $\hat{Y}_i$: a linear combination of Y ($\hat{\mathbf{Y}} = \boldsymbol{HY}$)
- h_{ii}: the weight of $Y_i \Rightarrow$ the larger is h_{ii}, the more important is Y_i in determining $\hat{Y}_i$
- The larger is h_{ii}, the smaller is $\sigma^2\{e_i\}$.
- $h_{ii} = 1 \Rightarrow \sigma^2\{e_i\} = 0$
- Rule:

$$h_{ii} > 2\bar{h} = 2\frac{\sum h_{ii}}{n} = 2\frac{p}{n} \quad (\frac{2p}{n} \leq 1)$$

$$\begin{cases} \text{very high leverage:} h_{ii} > 0.5 \\ \text{moderate leverage:} h_{ii} : 0.2 \sim 0.5 \end{cases}$$

Identifying Outlying X Observations, cont'd

Body fat example



- $2p/n = 0.30$
- $h_{33} = 0.372$; $h_{15,15} = 0.333$

Identifying Outlying *X* Observations, cont'd

```
attach(Dataset_07TA01)
fit<-lm(Y~X1+X2+X3)
n<-length(Y); p = 3
plot(X2~X1, pch=16 )
text(X1+0.5, X2,
labels=as.character(1:length(X1)),col="red")
hii<-hatvalues(fit)
index<-hii>2*p/n
points( X1[index], X2[index], cex=2.0, col="blue")
```

DFFITS measure

- Influence on single fitted value: *DFFITS*-measure the influence that case i has on $\hat{Y}_i$

$$(DFFITS)_i = \frac{\hat{Y}_i - \hat{Y}_{i(i)}}{\sqrt{MSE_{(i)} h_{ii}}} = t_i \left(\frac{h_{ii}}{1 - h_{ii}} \right)^{1/2}$$

- Rule:

$$\begin{cases} |DFFITS| > 1 & \text{for small to medium data sets} \\ |DFFITS| > 2\sqrt{p/n} & \text{for large data sets} \end{cases}$$

- If X_i is an outlier and has a high h_{ii}, $(DFFITS)_i$ will tend to be large absolutely.

Cook's Distance

- Influence on all fitted value: *Cook's distance*-consider the influence of the ith case on all n fitted values

$$D_i = \frac{\sum_{j=1}^{n}(\hat{Y}_i - \hat{Y}_{j(i)})^2}{pMSE} = \frac{(\hat{\mathbf{Y}} - \hat{\mathbf{Y}}_{(i)})'(\hat{\mathbf{Y}} - \hat{\mathbf{Y}}_{(i)})}{pMSE}$$

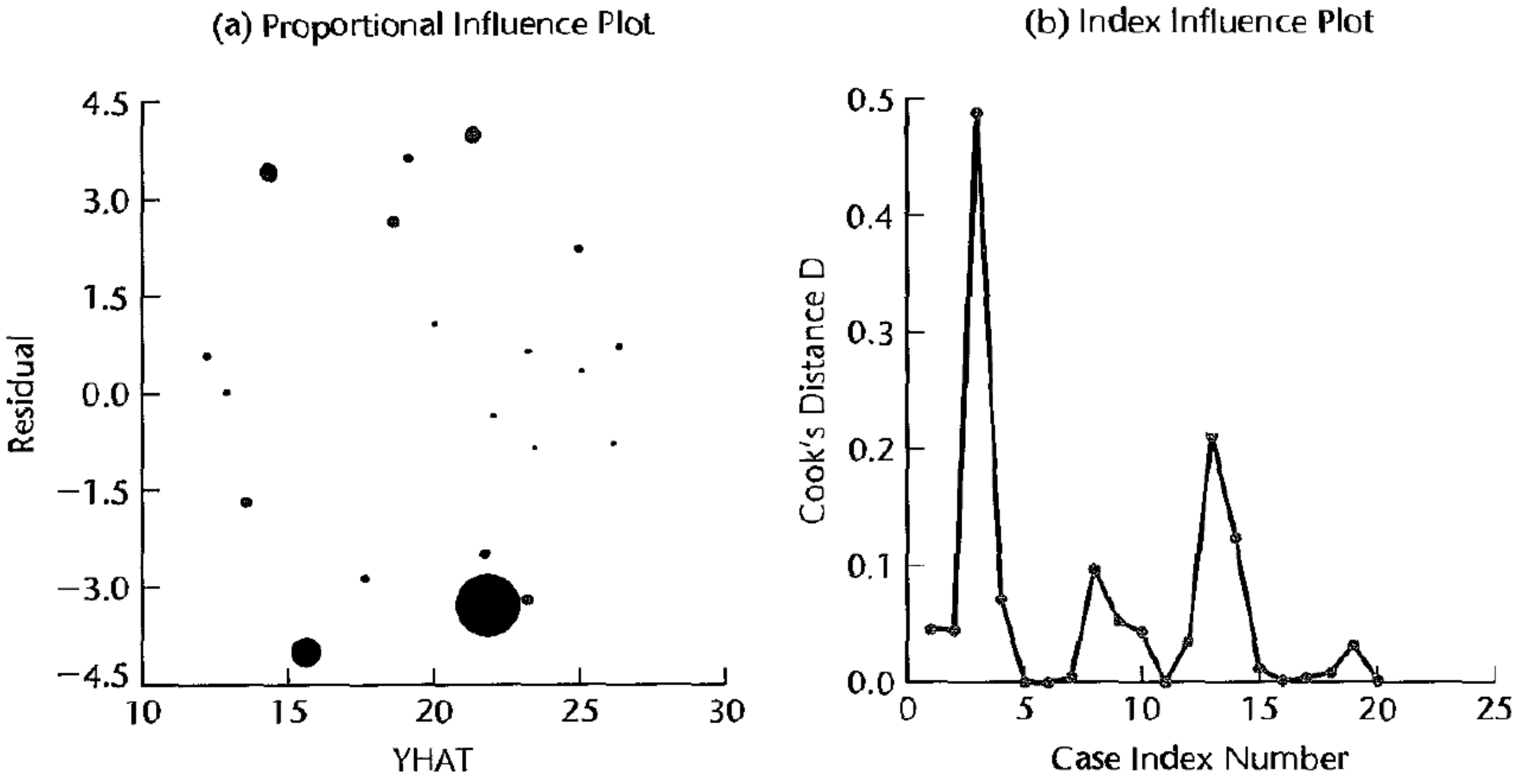
$$= \frac{e_i^2}{pMSE}\left[\frac{h_{ii}}{(1-h_{ii})^2}\right]$$

1. the size of e_i
2. the leverage h_{ii}
3. $e_i \uparrow$ or $h_{ii} \uparrow \Rightarrow D_i \uparrow$

- Rule: $D_i \sim F(p, n-p)$

$$\begin{cases} \text{little influence} : P(F(p, n-p) \leq D_i) > 0.1 \text{ or } 0.2 \\ \text{major influence} : P(F(p, n-p) \leq D_i) > 0.5 \end{cases}$$

Cook’s Distance, cont’d



DFBETAS

- Influence on regression coefficients: *DFBETAS*-the difference between b_k and $b_{k(i)}$

$$(DFBETAS)_{k(i)} = \frac{b_k - b_{k(i)}}{\sqrt{MSE_{(i)} c_{kk}}}, \quad k = 0, 1, \ldots, p-1$$

where c_{kk}: the kth diagonal element of $(\boldsymbol{X}'\boldsymbol{X})^{-1}$

- Rule:

$$\begin{cases} |DFBETAS| > 1 & \text{for small to medium data sets} \\ |DFBETAS| > 2\sqrt{n} & \text{for large data sets} \end{cases}$$

DFBETAS, cont'd

```
# Body fat example (Table 10.4)
> influence.measures(fit)
Influence measures of
	 lm(formula = Y ~ X1 + X2) :

      dfb.1_    dfb.X1    dfb.X2     dffit cov.r   cook.d    hat inf
1  -3.05e-01 -1.31e-01  2.32e-01 -3.66e-01 1.361 4.60e-02 0.2010
2   1.73e-01  1.15e-01 -1.43e-01  3.84e-01 0.844 4.55e-02 0.0589
3  -8.47e-01 -1.18e+00  1.07e+00 -1.27e+00 1.189 4.90e-01 0.3719   *
4  -1.02e-01 -2.94e-01  1.96e-01 -4.76e-01 0.977 7.22e-02 0.1109
5  -6.37e-05 -3.05e-05  5.02e-05 -7.29e-05 1.595 1.88e-09 0.2480   *
6   3.97e-02  4.01e-02 -4.43e-02 -5.67e-02 1.371 1.14e-03 0.1286
7  -7.75e-02 -1.56e-02  5.43e-02  1.28e-01 1.397 5.76e-03 0.1555
8   2.61e-01  3.91e-01 -3.32e-01  5.75e-01 0.780 9.79e-02 0.0963
9  -1.51e-01 -2.95e-01  2.47e-01  4.02e-01 1.081 5.31e-02 0.1146
10  2.38e-01  2.45e-01 -2.69e-01 -3.64e-01 1.110 4.40e-02 0.1102
11 -9.02e-03  1.71e-02 -2.48e-03  5.05e-02 1.359 9.04e-04 0.1203
12 -1.30e-01  2.25e-02  7.00e-02  3.23e-01 1.152 3.52e-02 0.1093
13  1.19e-01  5.92e-01 -3.89e-01 -8.51e-01 0.827 2.12e-01 0.1784
14  4.52e-01  1.13e-01 -2.98e-01  6.36e-01 0.937 1.25e-01 0.1480
15 -3.00e-03 -1.25e-01  6.88e-02  1.89e-01 1.775 1.26e-02 0.3332   *
16  9.31e-03  4.31e-02 -2.51e-02  8.38e-02 1.309 2.47e-03 0.0953
17  7.95e-02  5.50e-02 -7.61e-02 -1.18e-01 1.312 4.93e-03 0.1056
18  1.32e-01  7.53e-02 -1.16e-01 -1.66e-01 1.462 9.64e-03 0.1968
19 -1.30e-01 -4.07e-03  6.44e-02 -3.15e-01 1.002 3.24e-02 0.0670
20  1.02e-02  2.29e-03 -3.31e-03  9.40e-02 1.224 3.10e-03 0.0501
```

DFBETAS, cont'd

Some final comments:

- Analysis of outlying and influential cases: a necessary component of good regression analysis
  - neither automatic nor foolproof
  - require good judgment by the analyst
- Methods described: ineffective
- Extensions of the single-case diagnostic procedures: computational requirements

Variance Inflation Factor-VIF

Some problems: multicollinearity

- Adding or deleting X: change the regression coefficients
- Extra sum of squares: depending upon which other X_k variables are already included in the model
- X_k highly correlated with each other $\Rightarrow s\{b_k\}\uparrow$
- the estimated regression coefficients individually may not be statistically significant

Variance Inflation Factor-VIF, cont'd

informal diagnostics for multicollinearity:

1. Large changes in b_k when X_k is added or deleted, or when an observation is altered or deleted
2. Nonsignificant results in individual tests on the regression coefficients for important predictor variables.
3. Estimated regression coefficients with an algebraic sign that is the opposite of that expected from theoretical consideration or prior experience.
4. Large $\boldsymbol{r}_{XX}$
5. Wide confidence interval for β_k

Variance Inflation Factor-VIF, cont'd

Important limitations:

- do not provide quantitative measurements
- may not identify the nature of the multicollinearity
- sometimes the observed behavior may occur without multicollinearity being present

Variance Inflation Factor-VIF, cont'd

Variance Inflation Factor (VIF)

- a formal method: detecting multicollinearity; widely accepted
- measure how much the variances of b_ks are inflated as compared to when the predictor variables are not linearly related.
- Illustration:
- Variance-covariance matrix of ***b***: $\sigma^2\{b\} = \sigma^2(X'X)^{-1}$

Variance Inflation Factor-VIF, cont'd

Using the standardized regression model:

Variance-covariance matrix of $\boldsymbol{b}^*$: $\sigma^2\{\boldsymbol{b}^*\} = (\sigma^*)^2 r_{xx}^{-1}$

$(\sigma^*)^2$= the error term variance for the transformed model
$(\text{VIF})_k$= the kth diagonal element of r_{xx}^{-1}

$$\Rightarrow \sigma^2\{b_k^*\} = (\sigma^*)^2 (VIF)_k = \frac{(\sigma^*)^2}{1 - R_k^2}$$

VIF for b_k^*: $(VIF)_k = (1 - R_k^2)^{-1}, \quad k = 1, 2, \ldots, p-1$

R_k^2: X_k is regressed on the $p-2$ other $X_{k'}$s

- $R_k^2 = 0 \Rightarrow (VIF)_k = 1$: X_k is not linearly related to $X_{k'}$s
- $R_k^2 \neq 0 \Rightarrow (VIF)_k > 1$: indicate inflated variance for b_k^* as a result of the intercorrelations among the X variables

Variance Inflation Factor-VIF, cont'd

Perfect linear association with X_ks $\Rightarrow R_k^2 = 1 \Rightarrow VIF_k\ and\ \sigma^2\{b_k^*\}$ are unbounded

Rule: largest *VIF* value among all *X*s $\Rightarrow$ as an indicator of the severity of multicollinearity

$\max\{VIF_1, \ldots, VIF_{p-1}\} > 10$

Variance Inflation Factor-VIF, cont'd

- If $\overline{VIF} > 1 \Rightarrow$ serious multicollinearity problems

$$\because E\left\{\sum_{k=1}^{p-1}(b_k^* - \beta_k^*)^2\right\} = (\sigma^*)^2 \sum_{k=1}^{p}(VIF)_k$$

$\Rightarrow$large $\overline{VIF} \Rightarrow$ larger differences between b_k^* and β_k^*

- If no linearly $R_k^2 \equiv 0 \Rightarrow (VIF)_k = 1$

$$\Rightarrow E\left\{\sum_{k=1}^{p-1}(b_k^* - \beta_k^*)^2\right\} = (\sigma^*)^2(p-1)$$

$$\Rightarrow \overline{VIF} = \frac{(\sigma^*)^2 \sum_{k=1}^{p}(VIF)_k}{(\sigma^*)(p-1)} = \frac{\sum_{k=1}^{p}(VIF)_k}{(p-1)}$$

Variance Inflation Factor-VIF, cont'd

Variable	b_k^*	$(VIF)_k$
X_1	4.2637	708.84
X_2	−2.9287	564.34
X_3	−1.5614	104.61

Maximum $(VIF)_k = 708.84$ $\quad (\overline{VIF}) = 459.26$

Figure : VIF-Body Fat Example with three Xs

- $VIF_3 = 105$
- $r_{13}^2 = 0.458^2 = 0.209764$, $r_{23}^2 = 0.085^2$: not large
- X_3 : $R_3^2 = 0.990$; strong related to X_1, X_2

Variance Inflation Factor-VIF, cont'd

Body Fat Example:

```
> summary(fit)
Call:
lm(formula = Y ~ X1 + X2 + X3)

Residuals:
    Min      1Q  Median      3Q     Max
-3.7263 -1.6111  0.3923  1.4656  4.1277

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  117.085     99.782   1.173    0.258
X1             4.334      3.016   1.437    0.170
X2            -2.857      2.582  -1.106    0.285
X3            -2.186      1.595  -1.370    0.190

Residual standard error: 2.48 on 16 degrees of freedom
Multiple R-squared:  0.8014,	Adjusted R-squared:  0.7641
F-statistic: 21.52 on 3 and 16 DF,  p-value: 7.343e-06
```

```
> anova(fit)
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
X1         1 352.27  352.27 57.2768 1.131e-06 ***
X2         1  33.17   33.17  5.3931   0.03373 *
X3         1  11.55   11.55  1.8773   0.18956
Residuals 16  98.40    6.15
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> vif(fit)
      X1       X2       X3
708.8429 564.3434 104.6060
```

Variance Inflation Factor-VIF, cont'd

Comments

- Some program: using $1/VIF_k$ = $1 - R_k^2$ < 0.01 (0.001, 0.001)
- Limitation: cannot distinguish between several simultaneous multicollinearities
- Other methods: more complex than *VIF*