Using Linear Programming Models to Maximize Profits

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Introduction

In this project, I conducted a monthly analysis using a linear programming model to maximize the net profit of a northern hardware company. Through this comprehensive analysis, the company aims to make informed decisions to ensure the success of its new distribution center, optimizing both inventory management and financial performance.

Project:

A northern hardware company is studying a plan to open a new distribution center in southeast. The company plans to rent a warehouse and an adjacent office and distribute its main products to the local dealers. The company has decided to initially start with four of its main products: Pressure washers, Go karts, Generators, and Water pumps. The table below describes how much each of the products will cost the company (including transportation costs):

Item	Cost (in Dollars)
Pressure washer	330
Go-kart	370
Generator	410
(Case of 5 Water Pumps)	635

Table 1: Costs of products in dollars

The company has set aside a purchasing monthly budget of $170,000 for the new location. The selling prices (per unit) for each item are given in the table below:

Item	Selling Price (in Dollars)
Pressure washer	499.99
Go-kart	729.99
Generator	700.99
Water pump	269.99

Table 2: Revenues of products in dollars

Other than the budget, another of the company's concern is the available space in the warehouse. The warehouse has 82 shelves, and each shelf is 30 ft long and 5 ft wide. Pressure washers and generators each are stored on 5 ft by 5 ft pallets whereas each Go Kart is stored on an 8 ft by 5 ft pallet. Furthermore, a 5 ft by 5 ft pallet is used to store four cases of water pumps

For promoting its brand products, the company's marketing department has decided to allocate at least 30% of its inventory to pressure washers and Go Karts and sell at least twice as many generators as water pumps.

Perform a monthly analysis using a linear programming model to maximize the company's net profit.

Analysis

1. In a Word document, write the mathematical formulation of the problem.

Mathematical Formulation

Decision Variables:

- x_1: Number of Pressure Washers to purchase
- x_2: Number of Go Karts to purchase
- x_3: Number of Generators to purchase
- x_4: Number of Water Pumps to purchase

Objective Function: Maximize Z (Net Profit)

$Z = (499.99-330)x_1 + (729.99-370)x_2 + (700.99-410)x_3 + (269.99-635/5)x_4$

The objective function can be simplified as:

$Z = 169.99x_1 + 359.99x_2 + 290.99x_3 + 142.99x_4$

Constraints:

1. **Budget Constraint:**

   $330x_1 + 370x_2 + 410x_3 + (635/5)x_4 \leq 170{,}000 \rightarrow 330x_1 + 370x_2 + 410x_3 + 127x_4 \leq 170{,}000$

2. **Space Constraints:**

   $(5x_1 + 8x_2 + 5x_3 + 0.25x_4) / 30 \leq 82 \rightarrow 0.1667x_1 + 0.2667x_2 + 0.1667x_3 + 0.0083x_4 \leq 82$

3. **Marketing Constraints:**

   $x_1 + x_2 \geq 0.3\ (x_1 + x_2 + x_3 + x_4) \rightarrow 0.7x_1 + 0.7x_2 - 0.3x_3 - 0.3x_4 \geq 0$

   $x_3 \geq 2x_4 \rightarrow x_3 - 2x_4 \geq 0$

4. **Non-negativity Constraints:**

$x_1, x_2, x_3, x_4 \geq 0$

2. Set up the linear programming formulation in an Excel workbook or R.

In Excel, I set up my data as follows:

Item	Cost per Unit($)	Selling Price per Unit($)	Profit per Unit($)	Decision Variable	**Purchased Units**
Pressure Water	330	499.99	169.99	X_1	**0.00**
Go-Kart	370	729.99	359.99	X_2	**155.18**
Generator	410	700.99	290.99	X_3	**237.77**
Water Pump	127	269.99	142.99	X_4	**118.88**

Constraints:

Constraints					Constraint LHS	Inequality	Constraint RHS
Budget	330.0000	370.0000	410.0000	127.0000	170000	≤	170000.0000
Space	0.1667	0.2667	0.1667	0.0083	82	≤	82.0000
Marketing	0.7000	0.7000	-0.3000	-0.3000	1.629179331	≥	0.0000
Marketing	0.0000	0.0000	1.0000	-2.0000	0	≥	0.0000

A numerical seed for decision variables was established to facilitate the solution process. This seed acted as an initial set of values for the decision variables, providing a starting point for the optimization algorithm. Setting up these numerical seeds is crucial as it can influence the efficiency and outcome of the solver's search for an optimal solution.

Additionally, constraint parameters were defined to streamline the solution process. They clearly ensured that all the restrictions and requirements of the problem are adequately addressed, guiding the solver to operate within the feasible region of the solution space.

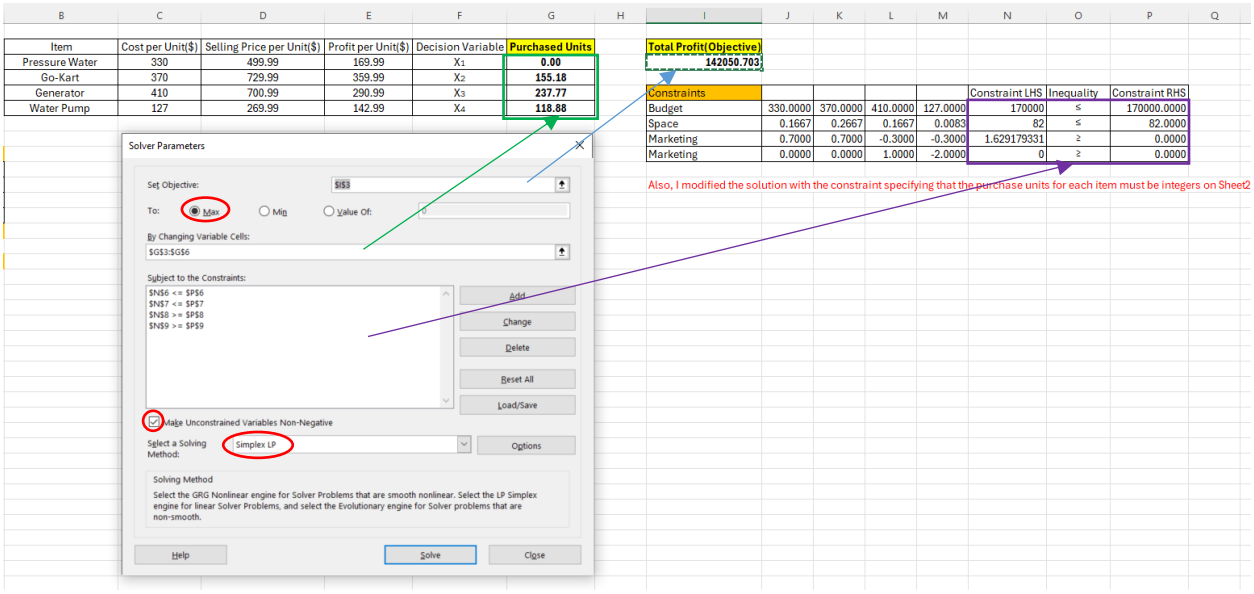
By initializing the decision variables and constraint parameters, I enhanced the solver's ability to navigate the problem landscape effectively, leading to a more efficient and accurate optimization process.

3. Use the Excel Solver or R to solve the problem, and generate a sensitivity report.

The objective function for total profit was calculated using the “SUMPRODUCT” function.

To use Excel Solver for solving the optimization problem, I followed these steps:

- Indicate the cell containing the objective function.
- Specify the problem type (maximization).
- Identify the changing variables (decision variables).
- Add constraints one at a time.
- Ensure that non-negativity constraints are included by selecting the appropriate option.
- Use the Simplex method for linear programming problems and solve the problem.



Also, I generated a sensitivity report as follows:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1]Sheet1
Report Created: 6/23/2024 3:24:38 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
G3	X1 Purchased Units	0	-110.0715237	169.99	110.0715237	1E+30
G4	X2 Purchased Units	155.179067	0	359.99	205.8402439	76.73878564
G5	X3 Purchased Units	237.7692613	0	290.99	98.20490541	131.8664063
G6	X4 Purchased Units	118.8846306	0	142.99	196.4098108	89.11965734

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
N6	Budget Constraint LHS	170000	0.557648341	170000	428.8	56225
N7	Space Constraint LHS	82	576.2254262	82	40.52252252	0.206312548
N8	Marketing Constraint LHS	1.629179331	0	0	1.629179331	1E+30
N9	Marketing Constraint LHS	0	-33.68339104	0	27.91666667	974.1201949

4. Describe the optimal solutions obtained in the Word document. These will consist of the inventory level for all four products and the optimal monthly profit.

Based on the optimization analysis performed using Excel Solver, the following inventory levels and monthly profit have been determined:

Inventory Levels:

- Pressure Washers: 0 units
- Go-Karts: 155.18 units
- Generators: 237.77 units
- Water Pumps: 118.88 units

Optimal Monthly Profit:

- Total Profit**:** $142,050.70

The optimal solution indicates that the company should not purchase any pressure washers. Instead, the focus should be on acquiring approximately 155 Go-Karts, 238 generators, and 119 water pumps. This allocation maximizes the company's profit while adhering to the constraints set for the purchasing budget and warehouse space. By following this optimal purchasing strategy, the company is projected to achieve a monthly profit of $142,050.70.

5. One of the decision variables has an optimal value of zero. Use the Solver sensitivity report to determine the smallest selling price for that item so that this optimal zero solution value changes to a non-zero value.

To determine the smallest selling price for Pressure Washers (x_1) so that the optimal zero solution value changes to a non-zero value, we need to consider the reduced cost from the sensitivity report.

From the sensitivity report:

- The reduced cost for x_1 (Pressure Washers) is -110.0715237.
- The current selling price for x_1 is 169.99.

The reduced cost tells us how much the objective coefficient (selling price) of a variable would need to improve before that variable's value in the optimal solution would become positive.

Calculation:

The reduced cost is the amount by which the selling price of x_1 needs to increase for it to become part of the optimal solution. Therefore, the new selling price (SP) must be:

New Selling Price ≥ Current Selling Price + Reduced Cost

Substituting the values:

New Selling Price ≥ 169.99 + 110.0715237

New Selling Price ≥ 280.0615237

In conclusion, the smallest selling price for Pressure Washers that would result in a non-zero optimal value is approximately $280.06.

6. In the word document explain whether, in addition to the $170,000 allocated to the purchasing budget during the first month, the company should allocate additional money. If yes, how much additional investment do you recommend, and how much should the company expect its net monthly profit to increase as a consequence of this increase?

To determine whether the company should allocate additional money beyond the $170,000 purchasing budget, we need to consider the shadow price associated with the budget constraint. The shadow price indicates how much the objective function (profit) will increase with an additional unit of resource (in this case, dollars).

From the sensitivity report, we have the following information:

- Shadow Price for Budget Constraint: $0.557648341
- Current Budget: $170,000
- Allowable Increase for Budget Constraint: $428.8

Analysis:

The shadow price tells us that for every additional dollar added to the budget, the company's net monthly profit is expected to increase by approximately $0.557648341.

Maximum Additional Investment:

The maximum allowable increase for the budget constraint is $428.8. This means that the budget can be increased by up to $428.8 without changing the optimal basis of the solution.

Calculation:

To calculate the potential increase in net monthly profit with this additional investment:

Additional Investment = $428.8

Increase in Profit = Additional Investment × Shadow Price

Increase in Profit = 428.8×0.557648341

Increase in Profit = $239.12

Recommendation:

Yes, the company should consider allocating additional money beyond the initial $170,000 budget. The recommended additional investment is $428.8. With this increase, the company can expect its net monthly profit to increase by approximately $239.12.

7. In the word document, explain whether you recommend that the company should rent a smaller or a larger warehouse. In any case, indicate the ideal size of your recommended warehouse in square feet, and indicate how much this change in the size of the warehouse will contribute to the monthly profit.

To determine whether the company should rent a smaller or larger warehouse, we need to examine the shadow price associated with the space constraint. The shadow price tells us how much the objective function (profit) will increase with an additional unit of the resource, in this case, the shelf space.

From the sensitivity report, we have the following information:

- Shadow Price for Space Constraint: $576.2254262
- Current Space (Number of Shelves): 82
- Shelf Dimensions: 30 ft long and 5 ft wide
- Allowable Increase for Space Constraint: 40.52252252 shelves

Analysis:

The shadow price indicates that for each additional shelf, the company's net monthly profit will increase by approximately $576.23. Each shelf is 30 ft by 5 ft, so we can calculate the increase in square footage and the corresponding increase in profit.

Calculation:

- **Increase in Shelves:**

  Maximum allowable increase: 40.52252252 shelves

- **Increase in Square Footage:**

  Each shelf's area: 30 ft * 5 ft = 150 sq ft

  Total increase in square footage: 40.52252252 shelves * 150 sq ft/shelf = 6,078.378378 sq ft

- **Increase in Profit:**

  Increase in Profit: 40.52252252 shelves * $576.2254262 = $23,350.10

Recommendation:

Based on the shadow price, the company should rent a larger warehouse to maximize its monthly profit. The recommended increase in warehouse size is 40.52252252 shelves or approximately 6,078.38 square feet.

Contribution to Monthly Profit:

By increasing the warehouse size by 6,078.38 square feet, the company's net monthly profit is expected to increase by approximately $23,350.10.

Ideal Warehouse Size:

- **Current Warehouse Size:** 82 shelves * 150 sq ft/shelf = 12,300 sq ft
- **Recommended Increase:** 6,078.38 sq ft
- **Ideal Warehouse Size:** 12,300 sq ft + 6,078.38 sq ft = 18,378.38 sq ft

In conclusion, the company should rent a larger warehouse with an ideal size of 18,378.38 square feet. This expansion will contribute significantly to the monthly profit, making it a valuable investment.

Conclusion

In this analysis, I examined the optimal purchasing strategy and warehouse size for a northern hardware company planning to open a new distribution center in the southeast. Using a linear programming model and the Excel Solver, I aimed to maximize the company's net monthly profit while considering various constraints, such as budget, warehouse space, and marketing requirements. The analysis of the linear programming model indicates that the optimal purchasing strategy for the company entails acquiring 155 units of go-karts, 238 units of generators, 119 units of water pumps, and no units of pressure washers. Implementing this strategy is projected to generate a net monthly profit of around $142,050.70. The sensitivity report has been instrumental in providing actionable recommendations. By leveraging these insights, the company can enhance its operational efficiency, maximize profits, and effectively manage its new distribution center in the southeast.