Two Optimization Problems

Yinan Zhou

29th June 2024

Table of Contents

Part 1: Rockhill Shipping & Transport Company Analysis

Introduction

The goal of this part is to minimize the transportation costs for Rockhill Shipping & Transport Company when transporting hazardous waste from six plants to three disposal sites. The analysis involves two scenarios: direct shipping from plants to disposal sites and shipping with intermediate drop-offs at other plants and disposal sites. The Excel data input and Solver setup have been used to derive the optimal routes and respective costs.

Analysis

(i) Direct Shipping Model

In the direct shipping model, the costs are calculated for shipping directly from each plant to each disposal site. The following steps were followed:

1. **Data Input**:

- **Shipping Costs**: The cost per barrel from each plant to each disposal site was entered into the Excel table.

Cost Matrix from Plants to Waste Disposal Sites:

	Waste Proposal Site		
Plant:	*Orangeburg*	*Florence*	*Macon*
Denver	$12	$15	$17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Table 1: Shipping costs, per barrel of waste from six plants to three waste disposal sites

- **Waste Generated**: The weekly waste production for each plant was listed.

Supply (Waste Generated):

Plant:	*Waste per Week (bbl)*
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38

- **Disposal Site Capacity**: The maximum capacity of each disposal site was specified.

Capacity of Waste Disposal Sites:

Waste Disposal Site:	*Waste per Week (bbl)*
Orangeburg	65
Florence	80
Macon	105

2. **Solver Setup**:

- **Objective Function**: The total transportation cost was set as the objective to minimize.

  Function in Excel: =SUMPRODUCT(CostMatrix, DecisionVariables)

- **Decision Variables**: The number of barrels shipped from each plant to each disposal site.

  A numerical seed for decision variables was established to facilitate the solution process. This seed acted as an initial set of values for the decision variables, providing a starting point for the optimization algorithm. Setting up these numerical seeds is crucial as it can influence the efficiency and outcome of the solver's search for an optimal solution.

- **Constraints**:

- The total barrels shipped from each plant must equal the waste generated by that plant.
- The total barrels received at each disposal site must not exceed its capacity.

	Transportation						
Solution:	**Decision Variables:**						
		Orangeburg	Florence	Macon	**Ship From**	**Constraints:**	**Supply**
	Denver	42	1	2	45	=	45
	Morganton	0	0	26	26	=	26
	Morrisville	0	0	42	42	=	42
	Pineville	0	53	0	53	=	53
	Rockhill	23	0	6	29	=	29
	Statesville	0	26	12	38	=	38
	Ship To	65	80	88			
	Constraints:	<=	<=	<=			
		65	80	105			
	Objective Function:						
	Minimize the total transportation cost	**$2,988.00**					

3. **Results**:

The Solver determined the optimal number of barrels to be shipped from each plant to each disposal site, minimizing the total cost to **$2,988.00**.

Optimal Shipping Routes:

- From Denver:
- To Orangeburg: 42 barrels
- To Florence: 1 barrel
- To Macon: 2 barrels
- From Morganton:
- To Macon: 26 barrels
- From Morrisville:
- To Macon: 42 barrels
- From Pineville:
- To Florence: 53 barrels
- From Rockhill:
- To Orangeburg: 23 barrels
- To Macon: 6 barrels
- From Statesville:
- To Florence: 26 barrels
- To Macon: 12 barrels

(ii) Intermediate Shipping Model

In the intermediate shipping model, additional routes were considered, where waste could be dropped off at other plants or disposal sites before reaching the final destination. The following steps were followed:

1. **Data Input**:

- **Shipping Costs**: The cost per barrel for shipping between plants and between disposal sites was added to the Excel table. Notably, since there are no routes between the same points, an arbitrary large number (in this case, $1000) is entered for the cost. This ensures that Solver will avoid selecting those routes.

	Transshipment													
Data Input:			Intermediates									Destinations		
Shipping costs			Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Orangeburg	Florence	Macon
	Origins	Denver	1000	3	4	9	5	4	12	15	17	12	15	17
		Morganton	6	1000	7	6	9	4	14	9	10	14	9	10
		Morrisville	5	7	1000	3	4	9	13	20	11	13	20	11
		Pineville	5	4	3	1000	3	11	17	16	19	17	16	19
		Rockhill	5	9	5	3	1000	14	7	14	12	7	14	12
		Statesville	4	7	11	12	8	1000	22	16	18	22	16	18
	Intermediates	Denver	1000	3	4	9	5	4	12	15	17	12	15	17
		Morganton	6	1000	7	6	9	4	14	9	10	14	9	10
		Morrisville	5	7	1000	3	4	9	13	20	11	13	20	11
		Pineville	5	4	3	1000	3	11	17	16	19	17	16	19
		Rockhill	5	9	5	3	1000	14	7	14	12	7	14	12
		Statesville	4	7	11	12	8	1000	22	16	18	22	16	18
		Orangeburg	12	14	13	17	7	22	1000	12	10	1000	12	10
		Florence	15	9	20	16	14	16	12	1000	15	12	1000	15
		Macon	17	10	11	19	12	18	10	15	1000	10	15	1000

2. **Solver Setup**:

- **Objective Function**: Similar to the direct shipping model, the total transportation cost was minimized.

  Function in Excel: =SUMPRODUCT(IntermediateCostMatrix, IntermediateDecisionVariables)

- **Decision Variables**: The number of barrels shipped between all possible routes, including intermediate points.

- **Constraints**:

- The total barrels shipped from each plant and through intermediate points must equal the waste generated.

- The total barrels received at each disposal site must not exceed its capacity.

- For each intermediate point, the total barrels shipped from origins must equal the total barrels shipped to destinations.

Solution: **Decision Variables:**

		Intermediates									Destinations					
		Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Orangeburg	Florence	Macon	**Ship From**	**Constraints:**	**Supply**
Origins	Denver	0	45	0	0	0	0	0	0	0	0	0	0	45	=	45
	Morganton	0	0	0	0	0	0	0	0	0	0	0	26	26	=	26
	Morrisville	0	0	0	0	0	0	0	0	0	0	0	42	42	=	42
	Pineville	0	0	17	0	36	0	0	0	0	0	0	0	53	=	53
	Rockhill	0	0	0	0	0	0	0	0	0	29	0	0	29	=	29
	Statesville	0	0	0	0	0	0	0	0	0	0	38	0	38	=	38
Intermediates	Denver	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Morganton	0	0	0	0	0	0	0	0	0	0	42	3	45		
	Morrisville	0	0	0	0	0	0	0	0	0	0	0	17	17		
	Pineville	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Rockhill	0	0	0	0	0	0	0	0	0	36	0	0	36		
	Statesville	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Orangeburg	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Florence	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Macon	0	0	0	0	0	0	0	0	0	0	0	0	0		
	Ship To	0	45	17	0	36	0	0	0	0	65	80	88			
										Constraints:	<=	<=	<=			
											65	80	105			

Objective Function:

Minimize the total transportation cost	**$2,674.00**

3. **Results**:

The Solver determined the optimal number of barrels to be shipped between each possible route, including intermediate drop-offs, reducing the total cost to **$2,674.00**.

Optimal Shipping Routes:

Origins to Destinations	**Origins to Intermediates**	**Intermediates to Destinations**
From Morganton: • To Macon: 26 barrels From Morrisville: • To Macon: 42 barrels From Rockhill: • To Orangeburg: 29 barrels From Statesville: • To Florence: 38 barrels	From Denver: • To Morganton: 45 barrels From Pineville: • To Morrisville: 17 barrels • To Rockhill: 36 barrels	From Morganton: • To Florence: 42 barrels • To Macon: 3 barrels From Morrisville: • To Florence: 17 barrels From Rockhill: • To Orangeburg: 36 barrels

In the intermediate shipping solution, the flexibility to drop and pick up loads at various plants and disposal sites allowed for a more cost-effective transportation strategy.

Conclusion

The analysis indicates that considering intermediate shipping routes significantly reduces the total transportation costs for Rockhill Shipping & Transport Company. While the direct shipping model resulted in a total cost of $2,988.00, the intermediate shipping model brought the total cost down to $2,674.00. The optimal routes involve strategic use of intermediate points, ensuring that waste is transported efficiently and cost-effectively from the plants to the disposal sites.

Part 2: Investment Allocations

Introduction

In this part of the project, I focus on optimizing the investment allocation for an investor who aims to achieve a minimum baseline expected return with the least risk. The investment portfolio consists of six asset types: bonds, high-tech stocks, foreign stocks, call options, put options, and gold. We utilize historical data to estimate the expected returns and covariance matrix of these assets' returns. The goal is to find the optimal investment fractions that minimize the portfolio's risk for various baseline returns and to analyze the relationship between risk and expected return.

Analysis

(i) Investment Allocation for Minimum Baseline Return of 11%

1. **Data Input**

- **Expected Returns**

The expected returns for each asset type are as follows:

	Expected Returns
Bonds	7%
High tech stocks	12%
Foreign stocks	11%
Call options	14%
Put options	14%
Gold	9%

- **Covariance Matrix**

The covariance matrix indicates the variance of each asset (diagonal entries) and the covariances between pairs of assets (non-diagonal entries):

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	-0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

2. Solver Setup:

- **Objective Function**: Minimize the total risk.

  The risk (variance) is calculated using the covariance matrix of the assets' returns and the investment fractions: Risk = $x^T \Sigma x$

  where x is the vector of investment fractions and Σ is the covariance matrix.

  Function in Excel: =MMULT(TRANSPOSE(Fractions), MMULT(CovarianceMatrix, Fractions))

- **Decision Variables**: Investment fractions for each asset type.

  A numerical seed for decision variables was established to facilitate the solution process. This seed acted as an initial set of values for the decision variables, providing a starting point for the optimization algorithm. Setting up these numerical seeds is crucial as it can influence the efficiency and outcome of the solver's search for an optimal solution.

- **Constraints**:

- The sum of the investment fractions must equal 100%.
- The total expected return must be at least 11%.

Solution:	**(i) Investment Allocation:**			
	Decision Variables:			
		Investment Fraction		
	Bonds	19%		
	High tech stocks	11%		
	Foreign stocks	27%		
	Call options	5%		
	Put options	25%		
	Gold	13%		
	Sum	100%		
	Constraint:	=		
		100%		
			Constraint:	
	Total Expected Return	0.11	>=	11%
	Objective Function:			
	Minimize the total risk	0.00074		

The Solver in Excel was used to find the optimal investment fractions by setting the objective function to minimize the total risk and adding constraints for the expected return and the total investment fraction.

3. **Results**:

- The Solver determined the optimal investment fractions for achieving a minimum baseline return of 11% with minimized risk.

Optimal Investment Fractions:

- Bonds: 19%
- High tech stocks: 11%
- Foreign stocks: 27%
- Call options: 5%
- Put options: 25%
- Gold: 13%

Total Expected Return: 11%

Minimized Risk: 0.00074

(ii) Successive Baseline Returns and Risk

In this section, I examined how different baseline returns affect the investment allocation and risk. By using baseline returns of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13%, and 13.5%, I determine the optimal investment fractions that minimize risk while meeting these returns. This analysis helps understand the risk-return trade-off more comprehensively.

1. **Data Input:**

- **Expected Returns**

The expected returns for each asset type are as follows:

	Expected Returns
Bonds	7%
High tech stocks	12%
Foreign stocks	11%
Call options	14%
Put options	14%
Gold	9%

- **Covariance Matrix**

The covariance matrix indicates the variance of each asset (diagonal entries) and the covariances between pairs of assets (non-diagonal entries):

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	-0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

2. **Solver Setup:**

- **Objective Function**: Minimize the total risk.

The risk (variance) is calculated using the covariance matrix of the assets' returns and the investment fractions: Risk = $x^T \Sigma x$

where x is the vector of investment fractions and Σ is the covariance matrix.

Function in Excel: =MMULT(TRANSPOSE(Fractions), MMULT(CovarianceMatrix, Fractions))

- **Decision Variables**: Investment fractions for each asset type, based on a series of baseline returns starting from 10% and increasing in increments of 0.5% up to 13.5%.

  A numerical seed for decision variables was established to facilitate the solution process. This seed acted as an initial set of values for the decision variables, providing a starting point for the optimization algorithm. Setting up these numerical seeds is crucial as it can influence the efficiency and outcome of the solver's search for an optimal solution.

- **Constraints**:

- **Total Investment Constraint:** The sum of the investment fractions must equal 100%.
- **Expected Return Constraint:** The weighted sum of the expected returns of the assets must be greater than or equal to the baseline return.

(ii) Investment Allocation:

Baseline return = 10%

	Investment Fraction
Bonds	37%
High tech stocks	7%
Foreign stocks	23%
Call options	2%
Put options	20%
Gold	11%
Sum	100%
Constraint:	=
	100%

Baseline return = 10.5%

	Investment Fraction
Bonds	28%
High tech stocks	9%
Foreign stocks	25%
Call options	3%
Put options	23%
Gold	12%
Sum	100%
Constraint:	=
	100%

Baseline return = 11%

	Investment Fraction
Bonds	19%
High tech stocks	11%
Foreign stocks	27%
Call options	5%
Put options	25%
Gold	13%
Sum	100%
Constraint:	=
	100%

Baseline return = 11.5%

	Investment Fraction
Bonds	10%
High tech stocks	13%
Foreign stocks	29%
Call options	6%
Put options	28%
Gold	14%
Sum	100%
Constraint:	=
	100%

Baseline return = 12%

	Investment Fraction
Bonds	1%
High tech stocks	14%
Foreign stocks	32%
Call options	8%
Put options	31%
Gold	15%
Sum	100%
Constraint:	=
	100%

Baseline return = 12.5%

	Investment Fraction
Bonds	0%
High tech stocks	16%
Foreign stocks	31%
Call options	13%
Put options	35%
Gold	5%
Sum	100%
Constraint:	=
	100%

Baseline return = 13%

	Investment Fraction
Bonds	0%
High tech stocks	14%
Foreign stocks	24%
Call options	22%
Put options	39%
Gold	0%
Sum	100%
Constraint:	=
	100%

Baseline return = 13.5%

	Investment Fraction
Bonds	0%
High tech stocks	10%
Foreign stocks	10%
Call options	36%
Put options	44%
Gold	0%
Sum	100%
Constraint:	=
	100%

Objective Function:

Minimize the total risk

r (risk)	**e (expected return)**	**Constraints:**	**Baseline return**
0.00051	10.00%	>=	10.00%
0.00060	10.50%	>=	10.50%
0.00074	11.00%	>=	11.00%
0.00091	11.50%	>=	11.50%
0.00113	12.00%	>=	12.00%
0.00146	12.50%	>=	12.50%
0.00210	13.00%	>=	13.00%
0.00350	13.50%	>=	13.50%

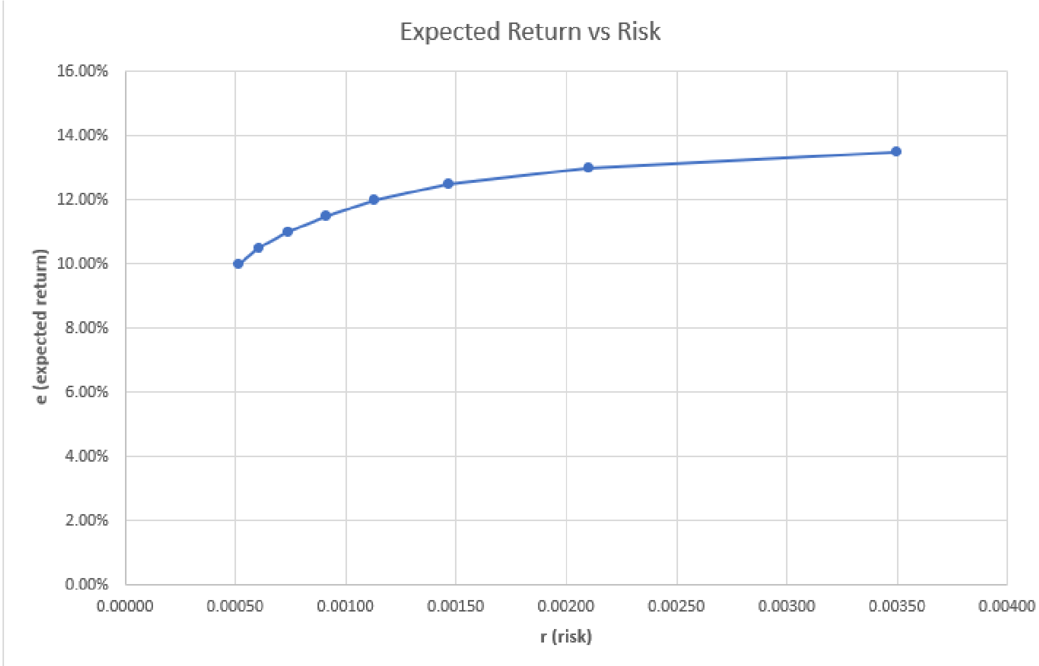
3. Results

I repeated the analysis for baseline returns of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13%, and 13.5%. The results are summarized as follows:

Baseline Return	Minimized Risk	Investment Fractions
10%	0.00051	Bonds: 37%, High tech stocks: 7%, Foreign stocks: 23%, Call options: 5%, Put options: 20%, Gold: 11%
10.5%	0.00060	Bonds: 28%, High tech stocks: 9%, Foreign stocks: 25%, Call options: 3%, Put options: 23%, Gold: 12%
11%	0.00074	Bonds: 19%, High tech stocks: 11%, Foreign stocks: 27%, Call options: 5%, Put options: 25%, Gold: 13%
11.5%	0.00091	Bonds: 10%, High tech stocks: 13%, Foreign stocks: 29%, Call options: 6%, Put options: 28%, Gold: 14%
12%	0.00113	Bonds: 1%, High tech stocks: 14%, Foreign stocks: 32%, Call options: 8%, Put options: 31%, Gold: 15%
12.5%	0.00146	Bonds: 0%, High tech stocks: 16%, Foreign stocks: 31%, Call options: 13%, Put options: 35%, Gold: 5%
13%	0.00210	Bonds: 0%, High tech stocks: 14%, Foreign stocks: 24%, Call options: 22%, Put options: 39%, Gold: 0%
13.5%	0.00350	Bonds: 0%, High tech stocks: 10%, Foreign stocks: 10%, Call options: 36%, Put options: 44%, Gold: 0%

4. Plot and Analysis

I plotted the expected return (e) against the minimized risk (r) for each baseline return value:



Observations:

- The plot demonstrates a concave relationship between expected return and risk.
- As the baseline return increases, the risk also increases, but at an increasing rate.
- This indicates that higher returns can only be achieved by accepting significantly higher risks, showcasing the trade-off between risk and return.

Conclusion

This analysis highlights the importance of balancing risk and return in investment portfolios. By using optimization techniques, investors can achieve desired returns while managing their risk exposure effectively. The concave relationship between risk and expected return underscores the need for careful consideration of the risk-return trade-off when making investment decisions.