title: “Exercises 2 Matrices” author: “” date: “January 25, 2018” output: pdf_document —

Matrix problems

1. Suppose

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $A^3 = \mathbf{0}$
(b) Replace the third column of A by the sum of the second and third columns

---

First, produce A

```
A1 <- matrix(c(1,1,3,5,2,6,-2,-1,-3), nrow = 3, byrow = TRUE)
A1
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    5    2    6
## [3,]   -2   -1   -3
```

```
A1%*%A1%*%A1
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    0    0    0
```

Then, add the columns 2 and 3 and assign the sum to the third column

```
A1[,3] <- A1[,2] + A1[,3]

A1
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    4
## [2,]    5    2    8
## [3,]   -2   -1   -4
```

---

2. Create the following matrix B with 15 rows

$$B = \begin{bmatrix} 10 & -10 & 10 \\ 10 & -10 & 10 \\ \dots & \dots & \dots \\ 10 & -10 & 10 \end{bmatrix}$$

```
B2 <- matrix(c(10,-10,10),nrow=15,ncol = 3, byrow = T)
B2
```

```
##       [,1] [,2] [,3]
##  [1,]   10  -10   10
##  [2,]   10  -10   10
```

```
##  [3,]   10  -10   10
##  [4,]   10  -10   10
##  [5,]   10  -10   10
##  [6,]   10  -10   10
##  [7,]   10  -10   10
##  [8,]   10  -10   10
##  [9,]   10  -10   10
## [10,]   10  -10   10
## [11,]   10  -10   10
## [12,]   10  -10   10
## [13,]   10  -10   10
## [14,]   10  -10   10
## [15,]   10  -10   10
```

Calculate the 3x3 matrix $B^T B$. You can make this calculation with the function crossprod(). See the documentaion.

```
t(B2)%*%B2
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

```
crossprod(B2)
```

```
##       [,1]  [,2]  [,3]
## [1,]  1500 -1500  1500
## [2,] -1500  1500 -1500
## [3,]  1500 -1500  1500
```

3. Create a 6 x 6 matrix matE with every element equal to 0. check what the functions row() and col() return when applied to matE.

Now, create the 6 x 6 matix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here is matE, a 6x6 matrix of 0's followed by row(matE) and col(matE)

```
matE <- matrix(rep(0,36), nrow = 6, byrow = TRUE)
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    0    0    0    0    0
## [2,]    0    0    0    0    0    0
## [3,]    0    0    0    0    0    0
## [4,]    0    0    0    0    0    0
## [5,]    0    0    0    0    0    0
## [6,]    0    0    0    0    0    0
```

```
# Note what the functions row() and col() do

row(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    1    1    1    1    1
## [2,]    2    2    2    2    2    2
## [3,]    3    3    3    3    3    3
## [4,]    4    4    4    4    4    4
## [5,]    5    5    5    5    5    5
## [6,]    6    6    6    6    6    6
```

```
col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    2    3    4    5    6
## [2,]    1    2    3    4    5    6
## [3,]    1    2    3    4    5    6
## [4,]    1    2    3    4    5    6
## [5,]    1    2    3    4    5    6
## [6,]    1    2    3    4    5    6
```

```
# With a little experimentation you would see
# that the specified pattern is in the |1|'s
row(matE)-col(matE)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0   -1   -2   -3   -4   -5
## [2,]    1    0   -1   -2   -3   -4
## [3,]    2    1    0   -1   -2   -3
## [4,]    3    2    1    0   -1   -2
## [5,]    4    3    2    1    0   -1
```

```
## [6,]    5    4    3    2    1    0
```

```
# so you use the locations of the 1's to modify matE
matE[abs(row(matE)-col(matE))==1] <- 1
matE
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    0    1    0    0    0    0
## [2,]    1    0    1    0    0    0
## [3,]    0    1    0    1    0    0
## [4,]    0    0    1    0    1    0
## [5,]    0    0    0    1    0    1
## [6,]    0    0    0    0    1    0
```

---

4. Look at the help for the function outer(). Now, create the following patterned matrix:

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
?outer()
a4 <- 0:4
a4
```

```
## [1] 0 1 2 3 4
```

```
A4 <- outer(a4,a4,"+")
A4
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    5
## [3,]    2    3    4    5    6
## [4,]    3    4    5    6    7
## [5,]    4    5    6    7    8
```

Use outer() a little more to make sure you get it.

```
B4 <- outer(a4,a4, "*")
B4
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    1    2    3    4
## [3,]    0    2    4    6    8
## [4,]    0    3    6    9   12
## [5,]    0    4    8   12   16
```

```
# and
b4 <- 5:10
C4 <- outer(a4,b4,"+")
C4
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    5    6    7    8    9   10
## [2,]    6    7    8    9   10   11
```

```
## [3,]    7    8    9   10   11   12
## [4,]    8    9   10   11   12   13
## [5,]    9   10   11   12   13   14
```

```
# and finally  --  make sure you check the values.
D4 <- outer(b4,a4, "%%")
D4
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   NA    0    1    2    1
## [2,]   NA    0    0    0    2
## [3,]   NA    0    1    1    3
## [4,]   NA    0    0    2    0
## [5,]   NA    0    1    0    1
## [6,]   NA    0    0    1    2
```

5. Create the following patterned matrices. Your solutions should be generalizable to enable creating larger matrices with the same structure.

(a)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 2 & 3 & 4 & 0 & 1 \\ 3 & 4 & 0 & 1 & 2 \\ 4 & 0 & 1 & 2 & 3 \end{bmatrix}$$

```
a5 <- 0:4
a5
```

```
## [1] 0 1 2 3 4
```

```
A5 <- outer(a5, a5,"+") %%5
A5
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    1    2    3    4
## [2,]    1    2    3    4    0
## [3,]    2    3    4    0    1
## [4,]    3    4    0    1    2
## [5,]    4    0    1    2    3
```

(b)

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 8 & 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

```
b5 <- 0:9
b5
```

```
##  [1] 0 1 2 3 4 5 6 7 8 9
```

```
B5 <- outer(b5, b5,"+") %%10
B5
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]    0    1    2    3    4    5    6    7    8     9
##  [2,]    1    2    3    4    5    6    7    8    9     0
##  [3,]    2    3    4    5    6    7    8    9    0     1
##  [4,]    3    4    5    6    7    8    9    0    1     2
##  [5,]    4    5    6    7    8    9    0    1    2     3
##  [6,]    5    6    7    8    9    0    1    2    3     4
##  [7,]    6    7    8    9    0    1    2    3    4     5
##  [8,]    7    8    9    0    1    2    3    4    5     6
##  [9,]    8    9    0    1    2    3    4    5    6     7
## [10,]    9    0    1    2    3    4    5    6    7     8
```

(c)

$$\begin{bmatrix} 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 & 7 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 8 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

```
c5 <- 0:8
c5
```

```
## [1] 0 1 2 3 4 5 6 7 8
```

```
C5 <- (outer(c5,c5,"-")+9)%%9
C5
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##  [1,]    0    8    7    6    5    4    3    2    1
##  [2,]    1    0    8    7    6    5    4    3    2
##  [3,]    2    1    0    8    7    6    5    4    3
##  [4,]    3    2    1    0    8    7    6    5    4
##  [5,]    4    3    2    1    0    8    7    6    5
##  [6,]    5    4    3    2    1    0    8    7    6
##  [7,]    6    5    4    3    2    1    0    8    7
##  [8,]    7    6    5    4    3    2    1    0    8
##  [9,]    8    7    6    5    4    3    2    1    0
```

6. Solve the following system of linear equations by setting up and solving the matrix equation $\mathbf{Ax} = \mathbf{y}$.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 7 \\ 2x_1 + x_2 + 2x_3 + 3x_4 + 4x_5 &= -1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 + 3x_5 &= -3 \\ 4x_1 + 3x_2 + 2x_3 + x_4 + 2x_5 &= 5 \\ 5x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 &= 17 \end{aligned}$$

```
y6Vec <- c(7,-1,-3,5,17)
y6Vec
```

```
## [1]  7 -1 -3  5 17
```

```
A6Mat <- matrix(0,nr=5, nc=5)
A6Mat
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    0    0    0    0    0
## [2,]    0    0    0    0    0
## [3,]    0    0    0    0    0
## [4,]    0    0    0    0    0
## [5,]    0    0    0    0    0
```

```
A6Mat <- abs(col(A6Mat)-row(A6Mat))+1
A6Mat
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    3    4    5
## [2,]    2    1    2    3    4
## [3,]    3    2    1    2    3
## [4,]    4    3    2    1    2
## [5,]    5    4    3    2    1
```

```
?solve()
solve(A6Mat,y6Vec)
```

```
## [1] -2  3  5  2 -4
```

```
solve(A6Mat,matrix(y6Vec,nc=1) )
```

```
##      [,1]
## [1,]   -2
## [2,]    3
## [3,]    5
## [4,]    2
## [5,]   -4
```

```
solve(A6Mat)%*%y6Vec
```

```
##      [,1]
## [1,]   -2
## [2,]    3
## [3,]    5
## [4,]    2
## [5,]   -4
```

7. Create a 6 x 10 matrix of random integers chosen from 1,2,. . . ,10 by executing the following two lines of code:

```
set.seed(75)
a7Mat <- matrix(sample(10, size=60, replace=TRUE), nr=6)
a7Mat
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,]    3    6    7    7    2    4    3    7    1     4
```

```
## [2,]    1    9    8    7    2    6   10    9    5     2
## [3,]    7   10    8    4   10    5    4    8    4     4
## [4,]    4    3    1    1    3    3    9    7    4     2
## [5,]    1    8    1    9    9    8    1    3    7     7
## [6,]    2    6    7    5    6   10    4    6   10     1
```

Use the matrix you have created to answer these questions:

(a) Find the number of entries in each row which are greater than 4.

```
?apply()
apply(a7Mat, 1, function(x){sum(x>4)})
```

```
## [1] 4 7 6 2 6 7
```

(b) Which rows contain exactly two occurrences of the number seven?

```
which( apply(a7Mat,1,function(x){sum(x==7)==2}) )
```

```
## [1] 5
```

(c) Find those pairs of columns whose total (over both columns) is greater tnan 75. The answer should be a matrix with two columns; so, for example, the row (1,2) in the output matrix means that the sum of columns 1 and 2 in the original matrix is greater than 75. Repeating a column is permitted; so, for exalmple, the final output matrix could contain the rows (1,2), (2,1), and (2,2).

```
a7MatColSums <- colSums(a7Mat)
a7MatColSums
```

```
##  [1] 18 42 32 33 32 36 31 40 31 20
```

```
outer(a7MatColSums,a7MatColSums,"+")
```

```
##       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##  [1,]   36   60   50   51   50   54   49   58   49    38
##  [2,]   60   84   74   75   74   78   73   82   73    62
##  [3,]   50   74   64   65   64   68   63   72   63    52
##  [4,]   51   75   65   66   65   69   64   73   64    53
##  [5,]   50   74   64   65   64   68   63   72   63    52
##  [6,]   54   78   68   69   68   72   67   76   67    56
##  [7,]   49   73   63   64   63   67   62   71   62    51
##  [8,]   58   82   72   73   72   76   71   80   71    60
##  [9,]   49   73   63   64   63   67   62   71   62    51
## [10,]   38   62   52   53   52   56   51   60   51    40
```

```
which( outer(a7MatColSums,a7MatColSums,"+")>75, arr.ind=T )
```

```
##      row col
## [1,]   2   2
## [2,]   6   2
## [3,]   8   2
## [4,]   2   6
## [5,]   8   6
## [6,]   2   8
## [7,]   6   8
## [8,]   8   8
```

```
?which()
```

What if repetitions are not permitted? Then only (1,2) from (1,2),(2,1) and (2,2) would be permitted.

```
?lower.tri()

a7MatColSums <- colSums(a7Mat)
logicalMat <- outer(a7MatColSums,a7MatColSums,"+")>75
logicalMat[lower.tri(logicalMat,diag=T)] <- F
which(logicalMat, arr.ind=T)
```

```
##      row col
## [1,]   2   6
## [2,]   2   8
## [3,]   6   8
```

8. Calculate

(a) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+j)}$

```
sum((1:20)^4) * sum(1/(3+(1:5)))
```

```
## [1] 639215.3
```

```
# or

sum(outer((1:20)^4, (3+(1:5)), "/"))
```

```
## [1] 639215.3
```

(b) $\sum_{i=1}^{20}\sum_{j=1}^{5}\frac{i^4}{(3+ij)}$

```
outer((1:20),(1:5), "*")
```

```
##       [,1] [,2] [,3] [,4] [,5]
##  [1,]    1    2    3    4    5
##  [2,]    2    4    6    8   10
##  [3,]    3    6    9   12   15
##  [4,]    4    8   12   16   20
##  [5,]    5   10   15   20   25
##  [6,]    6   12   18   24   30
##  [7,]    7   14   21   28   35
##  [8,]    8   16   24   32   40
##  [9,]    9   18   27   36   45
## [10,]   10   20   30   40   50
## [11,]   11   22   33   44   55
## [12,]   12   24   36   48   60
## [13,]   13   26   39   52   65
## [14,]   14   28   42   56   70
## [15,]   15   30   45   60   75
## [16,]   16   32   48   64   80
## [17,]   17   34   51   68   85
## [18,]   18   36   54   72   90
## [19,]   19   38   57   76   95
## [20,]   20   40   60   80  100
```

```r
sum( (1:20)^4 / (3 + outer(1:20,1:5,"*")))
```

```
## [1] 89912.02
```

(c) $\sum_{i=1}^{10} \sum_{j=1}^{i} \frac{i^4}{(3+ij)}$

```r
sum( outer(1:10,1:10,function(i,j){ (i>=j)*i^4/(3+i*j) }) )
```

```
## [1] 6944.743
```