Announcements

Contents

- H1 posted. Due Next Tuesday

Nondeterministic Finite Automata

CS 536

Previous Lecture

Scanner: converts a sequence of characters to a sequence of tokens

Scanner and parser: master-slave relationship

Scanner implemented using FSMs

FSM: DFA or NFA

This Lecture

NFAs from a formal perspective

Theorem: NFAs and DFAs are equivalent

Regular languages and Regular expressions

NFAs, formally

For DFA, we have a string x1x2..xn in L(M) if:
δ(...δ(δ(q, x1), x2)..., xn) ∈F
For NFA, we just extend the single state to a set of states:
| δ(...δ(δ(q', x1), x2)..., xn) ∩ F | > 0
q' and the output of each δ is a set of states.

$$M \equiv (Q, \Sigma, \delta, q, F)$$

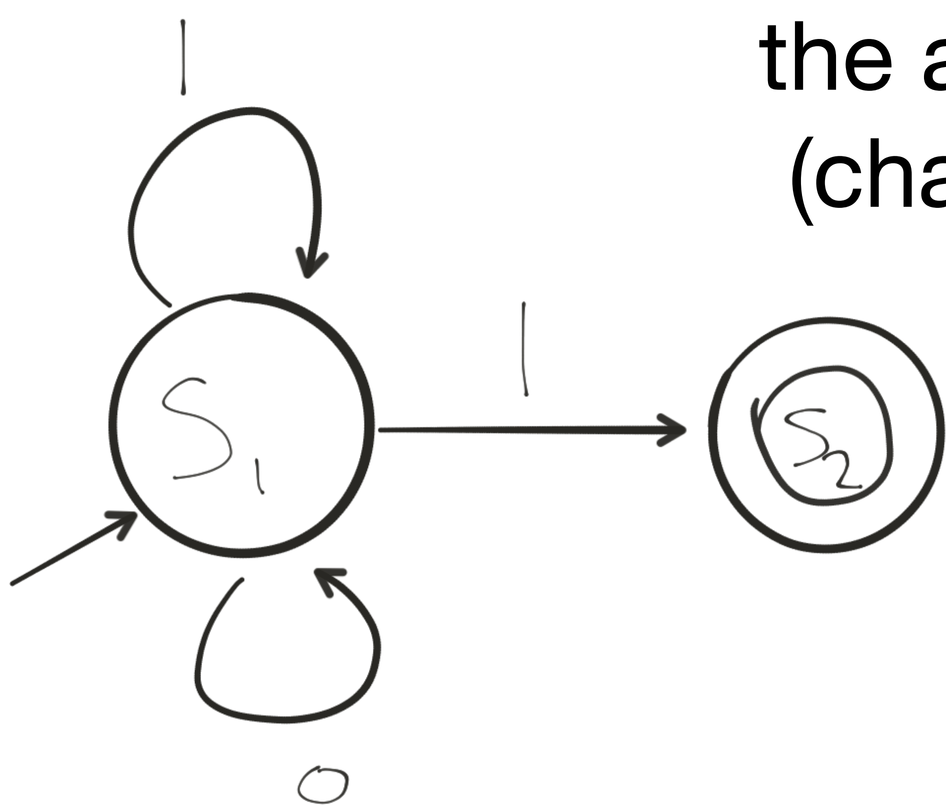
finite set of states

the alphabet (characters)

transition function
$$\delta : Q \times \Sigma \to \boxed{2^Q}$$

start state
$$q \in Q$$

final states
$$F \subseteq Q$$



δ is a relationship where:
(**An element of** Q) x (**An element of** Σ) -> (**An element of** the power set of Q)

	0	1
s1	{s1}	{s1, s2}
s2		

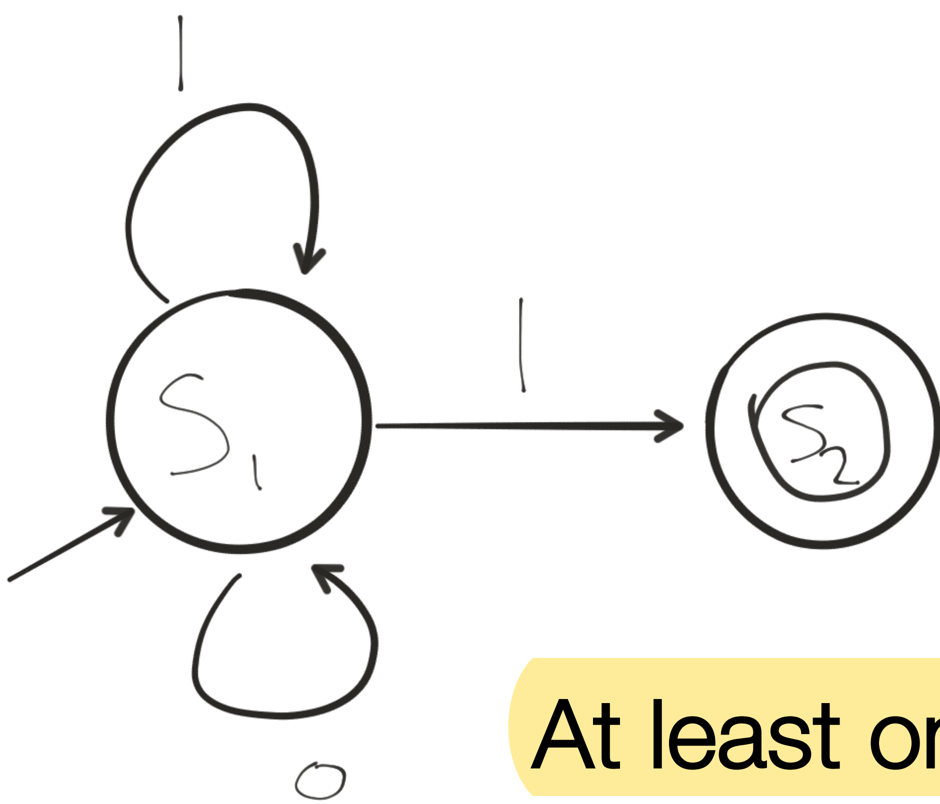
For DFA, δ: Q x Σ -> Σ. Each entry in the transition table is a single state (or stuck);

For NFA, since multiple edge with the same label is allowed, δ: Q x Σ -> {a set of states from Σ}. Each entry in the entry table is a set (or stuck).

NFA

Conceptually, you can think of the machine making different choices by choosing different paths in a **parallel** manner.

To check if string is in *L*(*M*) *of* NFA *M*, simulate **set** of **choices** it could make



1 1 1

s1	s2	st	st
s1	s1	s2	st
s1	s1	s1	**s2**
s1	s1	s1	s1

At least one sequence of transitions that:

Consumes all input (without getting stuck)

Ends in one of the final states

NFA and DFA are Equivalent

Two automata M and M’ are equivalent iff L(M) = L(M’)

Lemmas to be proven

Lemma 1: Given a DFA M, one can construct an NFA M’ that recognizes the same language as M, i.e., L(M’) = L(M)

Lemma 1 is trivial as each DFA can be considered as a NFA.

Lemma 2: Given an NFA M, one can construct a DFA M’ that recognizes the same language as M, i.e., L(M’) = L(M)

Proving lemma 2

Lemma 2: Given an NFA M, one can construct a DFA M' that recognizes the same language as M, i.e., L(M') = L(M)

Idea: we can only be in finitely many subsets of states at any one time

$2^{|Q|}$ possible combinations of states

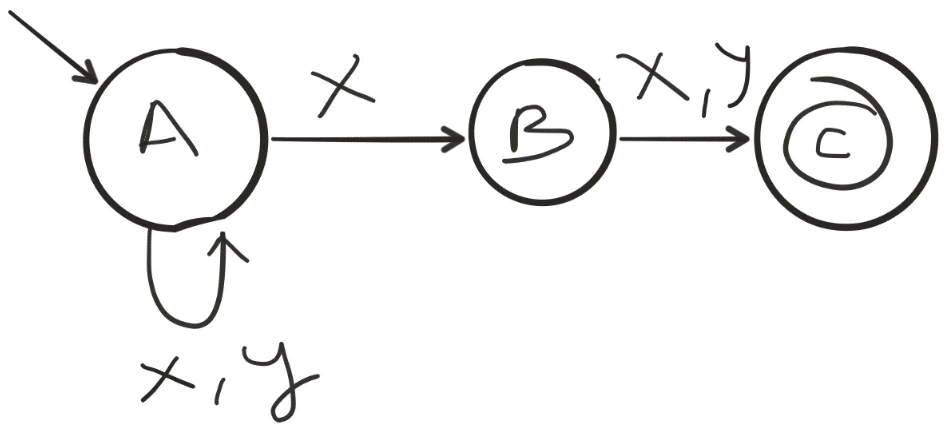
Why?

In our previous representation of DFAs, each circle in the FSM represents a single state. However, for NFA, if we think of the FSM as making different choices **in parallel**, the NFA can be in different states at the same time!
For example, in the following NFA, the machine would be in {S1, S2} after reading the first 'a'.

```
S -- a --> S1
 |
+- a --> S2
```

The path in a NFA can be infinitely long, thus it is infeasible to build a DFA that completely follows all path that the NFA takes.
However, the total number of **set of states** the NFA can be in is finite!

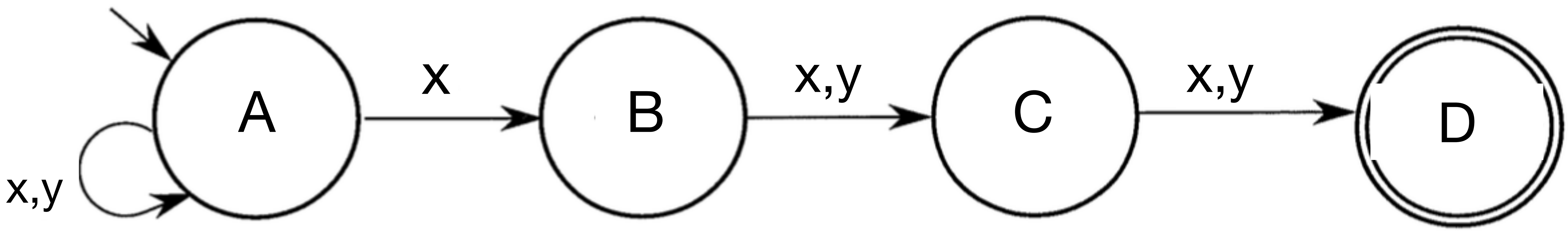
Why 2^|Q| states?



A	B	C		
0	0	0	=	{}
0	0	1	=	{C}
0	1	0	=	{B}
0	1	1	=	{B,C}
1	0	0	=	{A}
1	0	1	=	{A,C}
1	1	0	=	{A,B}
1	1	1	=	{A,B,C}

Build DFA that tracks set of states the NFA is in!

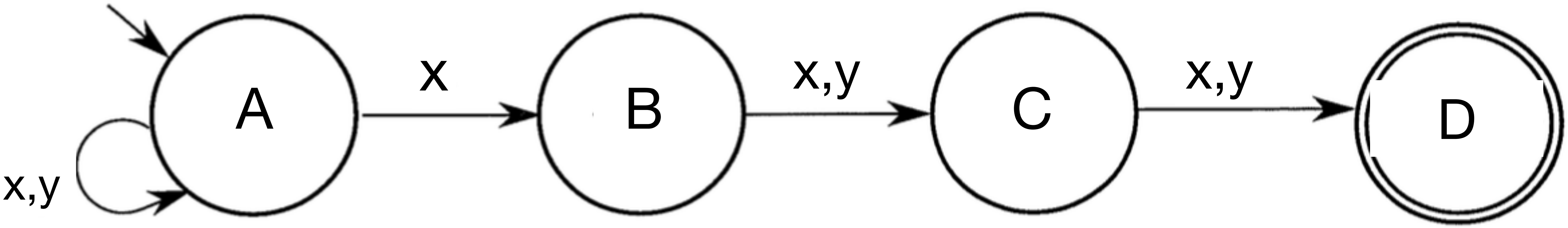
So we are guaranteed that the DFA is of finite size.



You can consider succ as δ.

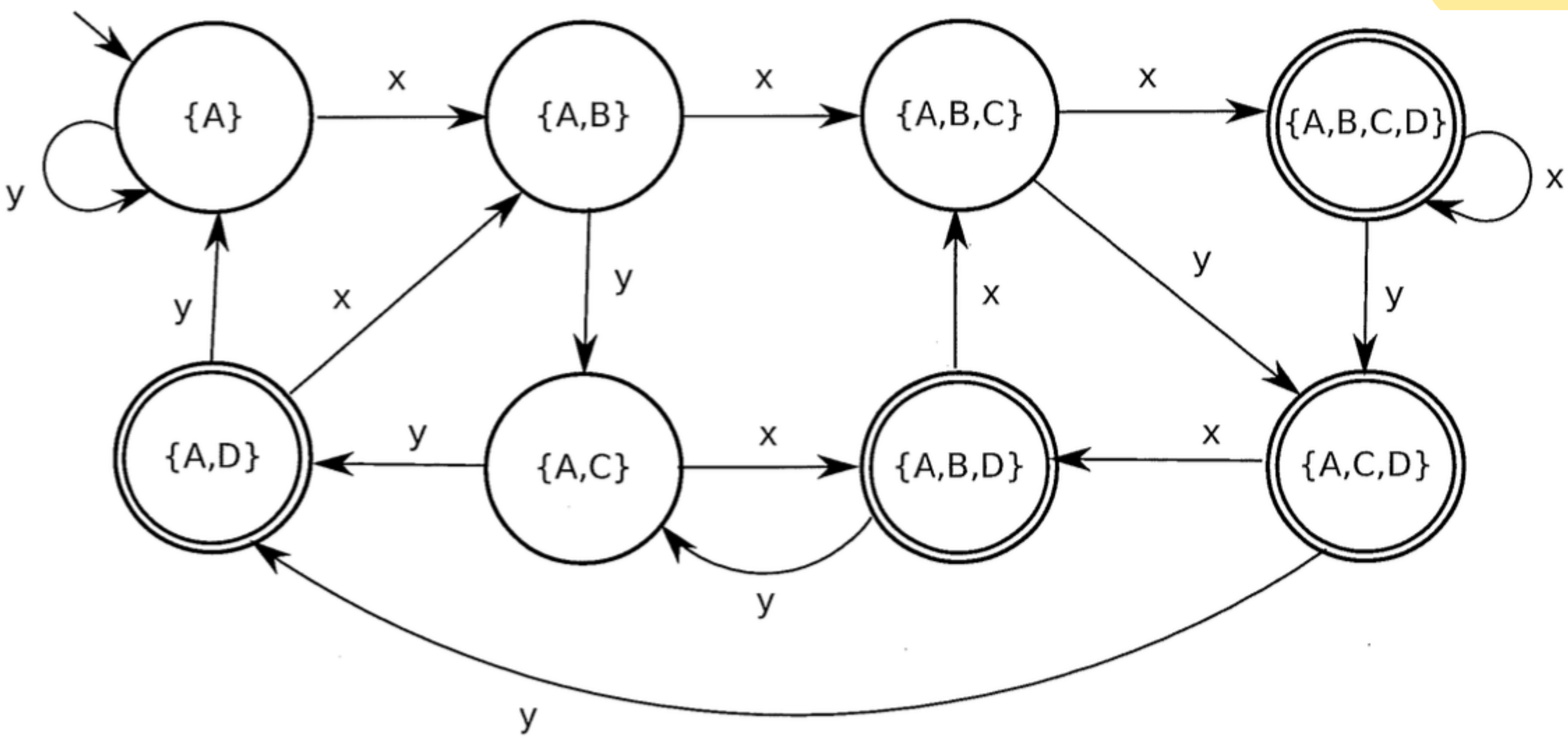
Defn: let succ(s,c) be the set of choices the NFA could make in state s with character c

succ(A,x) = {A,B}
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}



Build new DFA M' where Q' = 2^Q

Some of the elements in the powerset might not be present in M'.

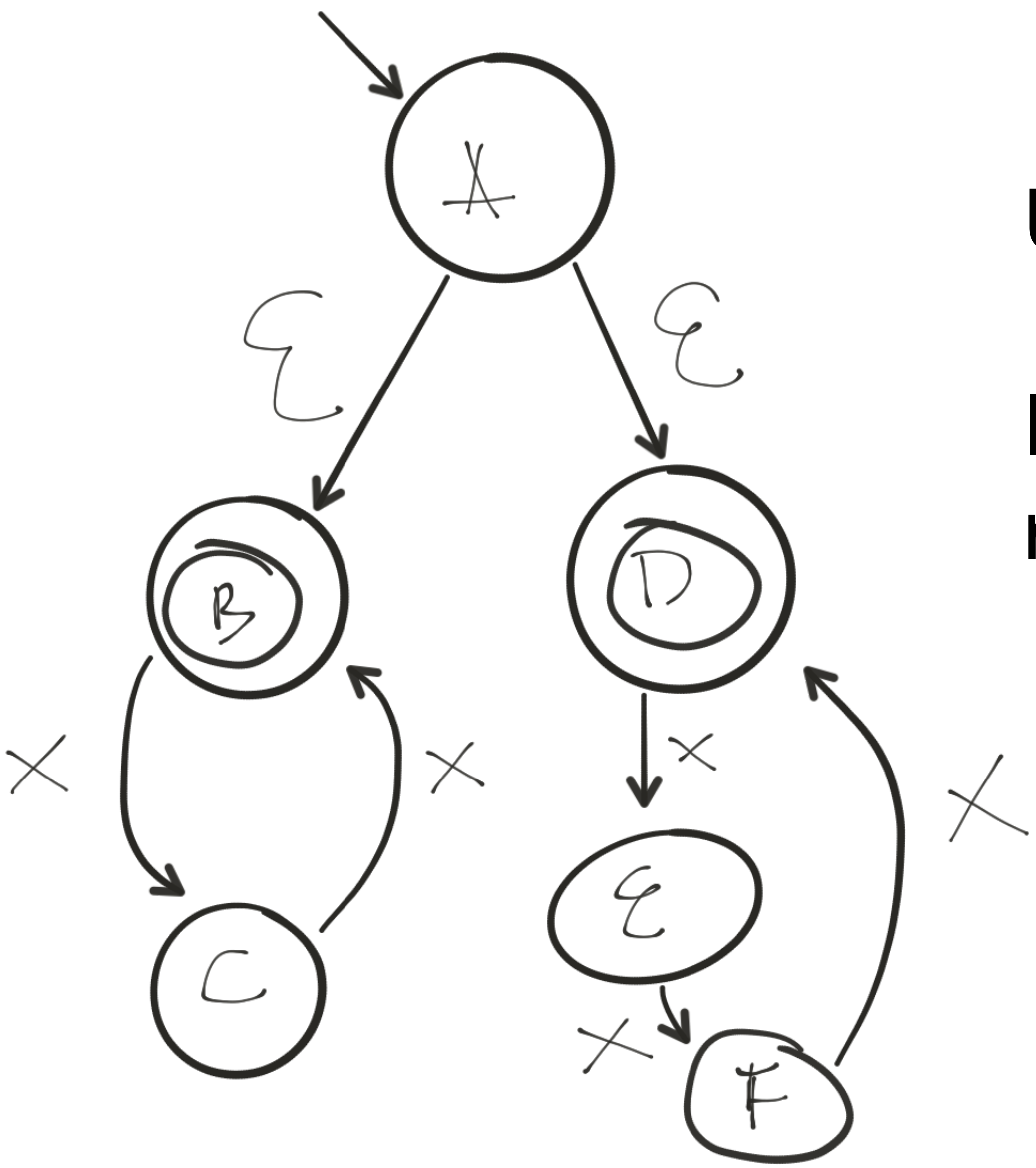


succ(A,x) = {A,B}
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}

To build DFA: Add an edge from state S on character c to state S' if S' represents the union of states that all states in S could possibly transition to on input c

ε-transitions

Eg: x^n, where n is even **or** divisible by 3



The "OR" operator.

Useful for taking union of two FSMs

In example, left side accepts even n; right side accepts n divisible by 3

x x
AB C B
AD E F
A

Eliminating ε-transitions

We want to construct ε-free FSM M' that is equivalent to M

Definition:

eclose(s) = set of all states reachable from s in zero or more epsilon transitions

including the state itself.

M' components

s is an accepting state of M' iff eclose(s) contains an accepting state

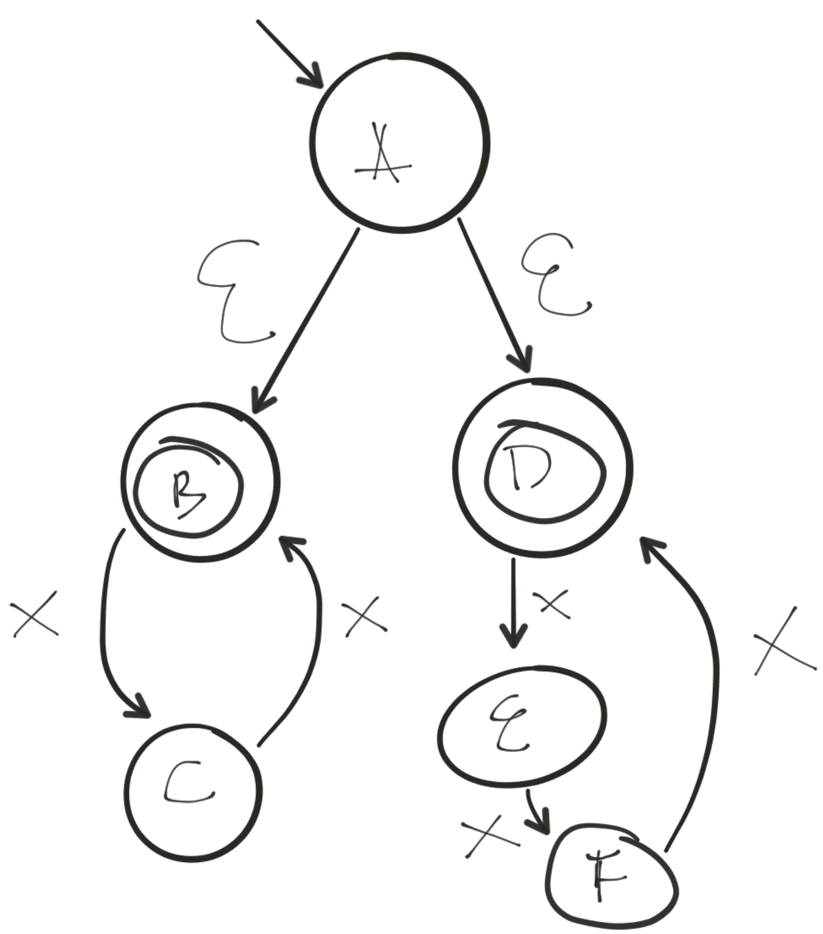
s —c—> t is a transition in M' iff
q —c—> t for some q in eclose(s)

Eliminating ε-transitions

We want to construct ε-free NFA M’ that is equivalent to M

Definition: Epsilon Closure

eclose(s) = set of all states reachable from s using zero or more epsilon transitions



	eclose
A	{A, B, D}
B	{B}
C	{C}
D	{D}
E	{E}
F	{F}

Def: eclose(s) = set of all states reachable from s in zero or more epsilon transitions

Rule 1

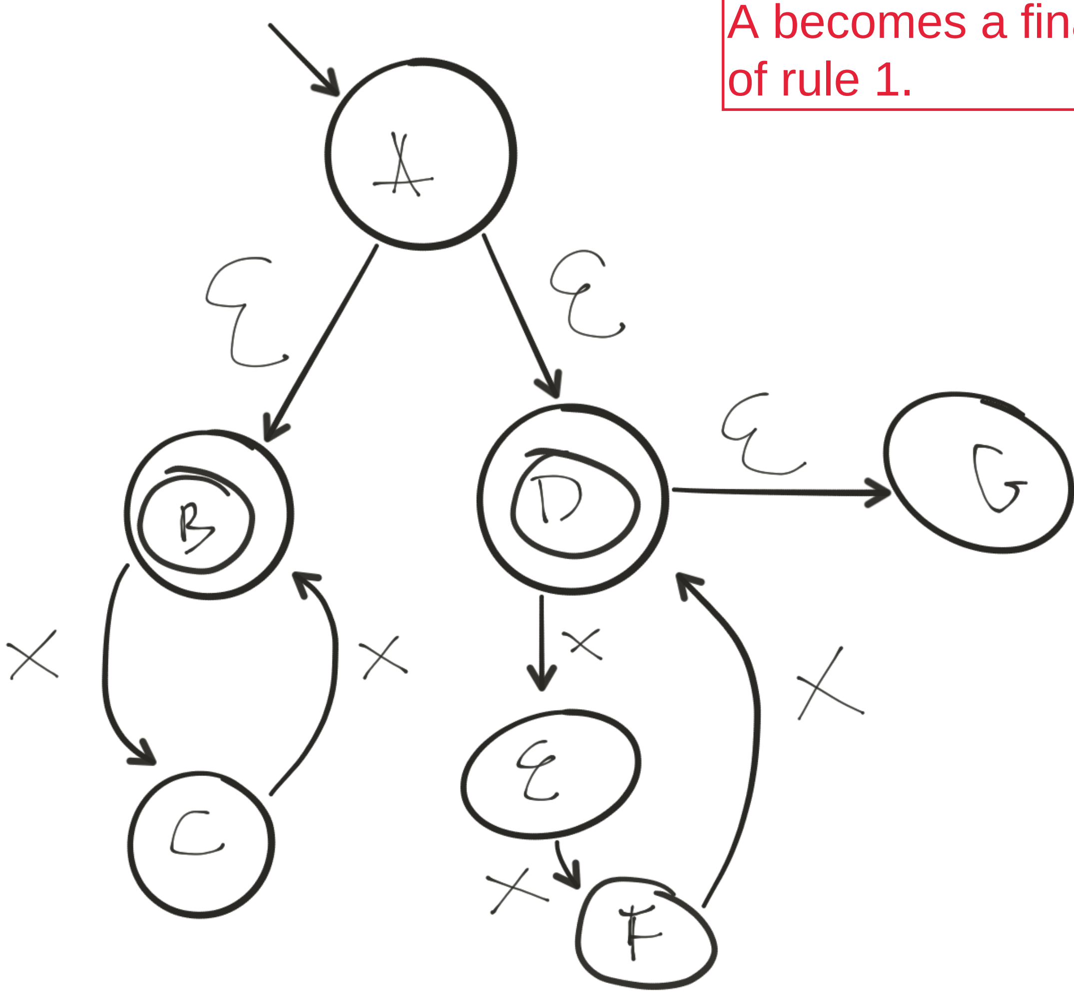
s is an accepting state of M’ iff eclose(s) contains an accepting state

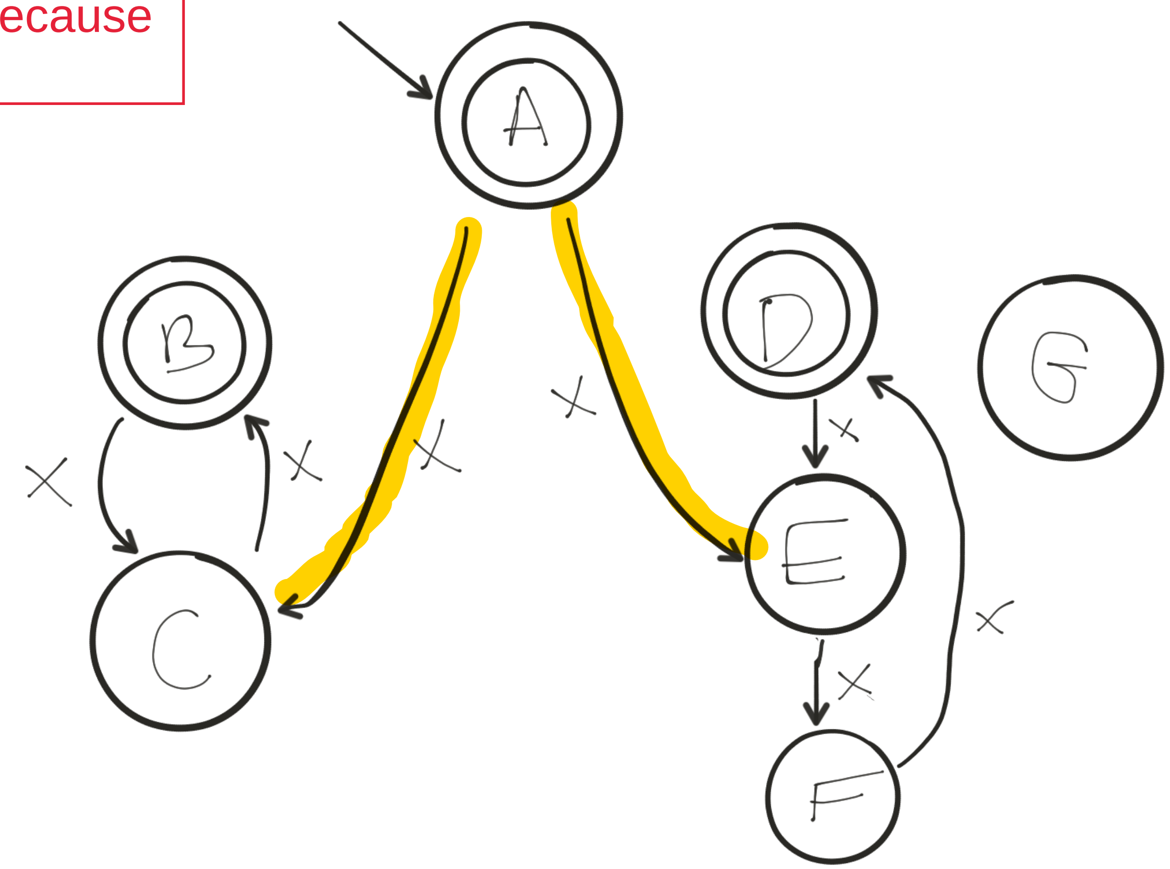
Rule 2

s —c—> t is a transition in M’ iff
q —c—> t for some q in eclose(s)

G is connected in the old machine but not in the new machine. The reason is that in the new epsilon-free machine, we only have transitions when reading **real** characters, not epsilons.

The two highlighted transitions exist due to rule 2;
A becomes a final-state because of rule 1.





Recap

NFAs and DFAs are equally powerful

any language definable as an NFA is definable as a DFA

ε-transitions do not add expressiveness to NFAs

we showed a simple algorithm to remove epsilons

Regular Languages and Regular Expressions

Regular Language

The language can be described by a DFA or NFA.

Any language recognized by an FSM is a regular language

Examples:

- Single-line comments beginning with //
- Integer literals
- {ε, ab, abab, ababab, abababab, …. }
- C/C++ identifiers

Regular expressions

Pattern describing a language

operands: single characters, epsilon

operators: from low to high precedence

alternation “or”: a | b

The dot is usually omitted by convention.

catenation: a.b, ab, a^3 (which is aaa)

iteration: a* (0 or more a’s) aka Kleene star

Why do we need them?

Each token in a programming language can be defined by a regular language

Scanner-generator input: one regular expression for each token to be recognized by scanner

Regular expressions are inputs to a scanner generator

Regexp, cont'd

Conventions:

a+ is aa*

letter is a|b|c|d|…|y|z|A|B|…|Z

digit is 0|1|2|…|9

not(x) all characters except x

. is any character

parentheses for grouping, e.g., (ab)*

ε, ab, abab, ababab

Regexp, example

Hex strings

- start with 0x or 0X
- followed by one or more hexadecimal digits
- optionally end with l or L

0(x|X)hexdigit+(L|l|ε)

- where hexdigit = digit|a|b|c|d|e|f|A|…|F

Regexp, example

Single-line comments in Java/C/C++

// this is a comment

//(not('\n'))*'\n'

not('\n') to ensure that we only match a **single-line.**

Regexp, example

C/C++ identifiers: sequence of letters/digits/underscores; cannot begin with a digit; cannot end with an underscore

Example: a, _bbb7, cs_536

Regular expression

letter | (letter|_)(letter|digit|_)*(letter|digit)

Edge case: single-char identifier.

Recap

Regular Languages

- Languages recognized/defined by FSMs

Regular Expressions

- Single-pattern representations of regular languages
- Used for defining tokens in a scanner generator

Creating a Scanner