Top-down parsing

Parsing: Review of the Big Picture (1)

- Context-free grammars (CFGs)
  - Generation: $G \rightarrow L(G)$
  - Recognition: Given w, is $w \in L(G)$?
- Translation
  - Given $w \in L(G)$, create a (G) parse tree for w
  - Given $w \in L(G)$, create an AST for w
    - The AST is passed to the next component of our compiler

Parsing: Review of the Big Picture (2)

- Algorithms
  - CYK
  - Top-down ("recursive-descent") for LL(1) grammars
    - How to parse, given the appropriate parse table for G
    - How to construct the parse table for G
  - Bottom-up for LALR(1) grammars
    - How to parse, given the appropriate parse table for G
    - How to construct the parse table for G

Last time

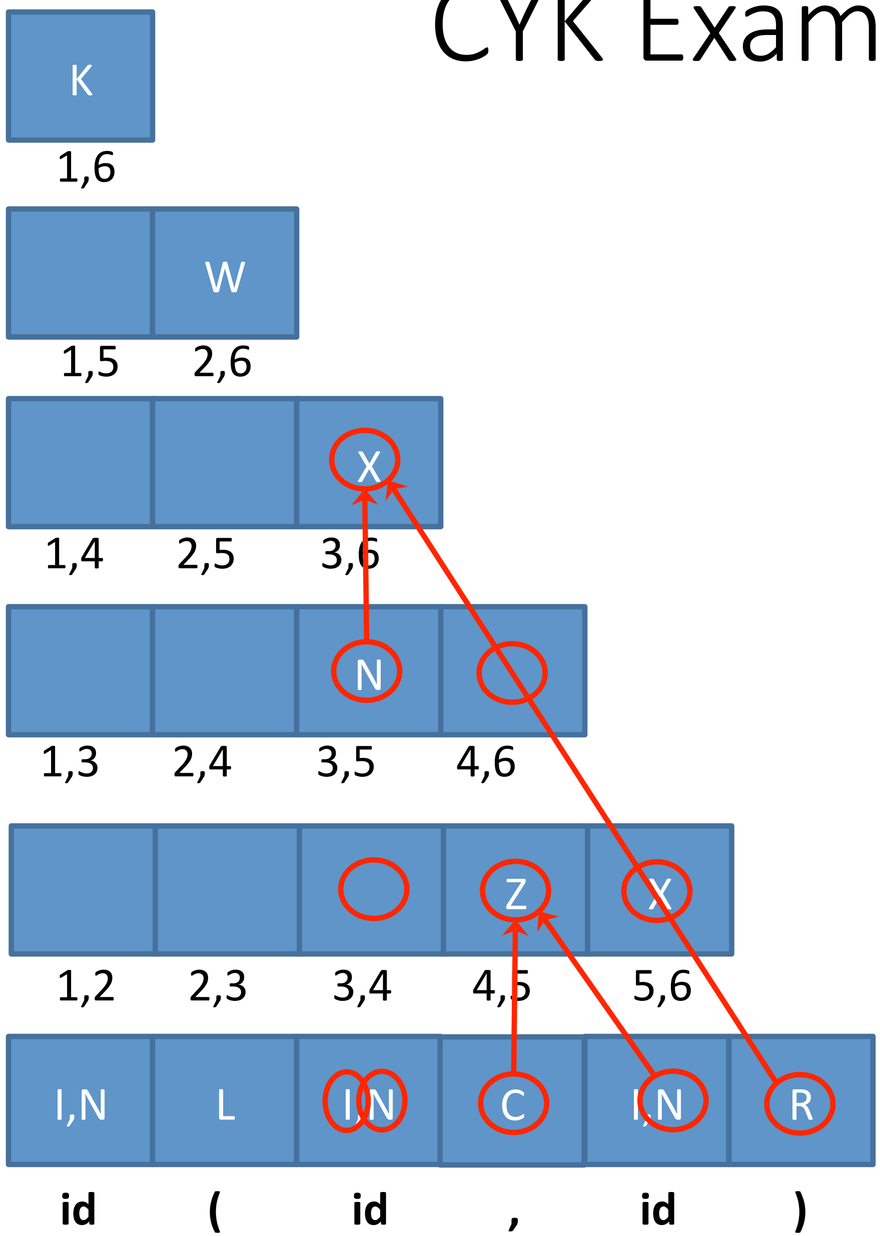
CYK

- Step 1: get a grammar in Chomsky Normal Form
- Step 2: Build all possible parse trees bottom-up
  - Start with runs of 1 terminal
  - Connect 1-terminal runs into 2-terminal runs
  - Connect 1- and 2- terminal runs into 3-terminal runs
  - Connect 1- and 3- or 2- and 2- terminal runs into 4 terminal runs
  - …
  - If we can connect the entire tree, rooted at the start symbol, we've found a valid parse

Some Interesting properties of CYK

- Very old algorithm
  - Already well known in early 70s
- No problems with ambiguous grammars:
  - Gives a solution for *all* possible parse tree simultaneously

CYK Example



$K \longrightarrow I\ W$

$K \longrightarrow I\ Y$

$W \longrightarrow L\ X$

$\boxed{X \longrightarrow N\ R}$

$Y \longrightarrow L\ R$

$N \longrightarrow$ **id**

$N \longrightarrow I\ Z$

$Z \longrightarrow C\ N$

$I \longrightarrow$ **id**

$L \longrightarrow$ **(**

$R \longrightarrow$ **)**

$C \longrightarrow$ **,**

Thinking about Language Design

Balanced considerations

- Powerful enough to be useful
- Simple enough to be parsable

Syntax need not be complex for complex behaviors

- Guy Steele's "Growing a Language"
  https://www.youtube.com/watch?v=_ahvzDzKdB0

Restricting the Grammar

By restricting our grammars we can

- Detect ambiguity
- Build linear-time, O(n) parsers

LL(1) languages

- Particularly amenable to parsing
- Parseable by Predictive (top-down) parsers
  - Sometimes called recursive descent

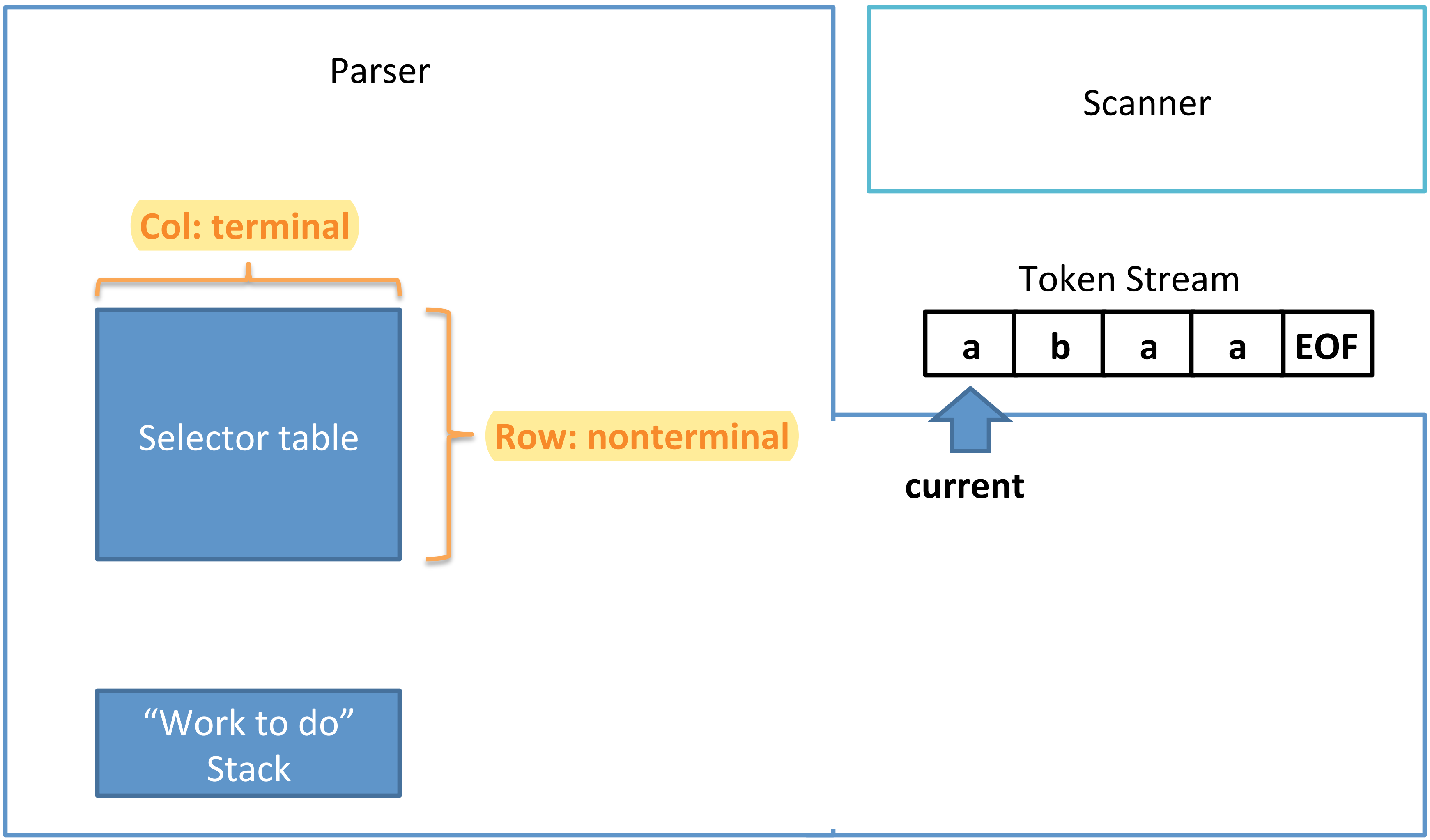
Top-Down Parsers

Start at the **Start** symbol

Repeatedly: “predict” what production to use

- Example: if the current token to be parsed is an **id**, no need to try productions that start with **intLiteral**
- This might seem simple, but keep in mind that a chain of productions may have to be used to get to the rule that handles, e.g., **id**

Predictive Parser Sketch

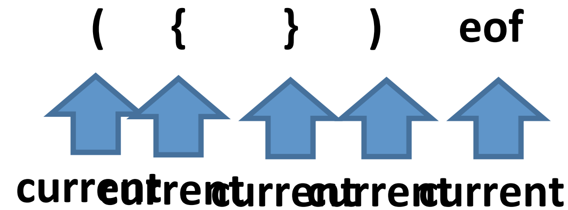


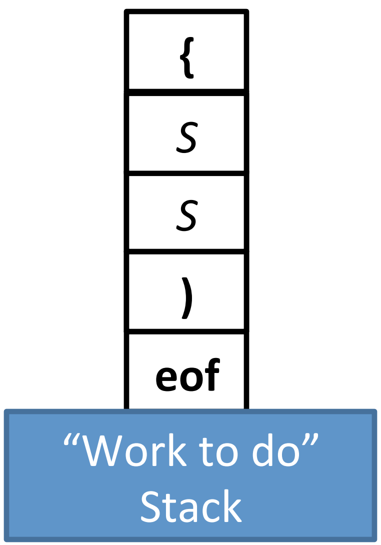
Example

$S \to (S) \mid \{ S \} \mid \varepsilon$

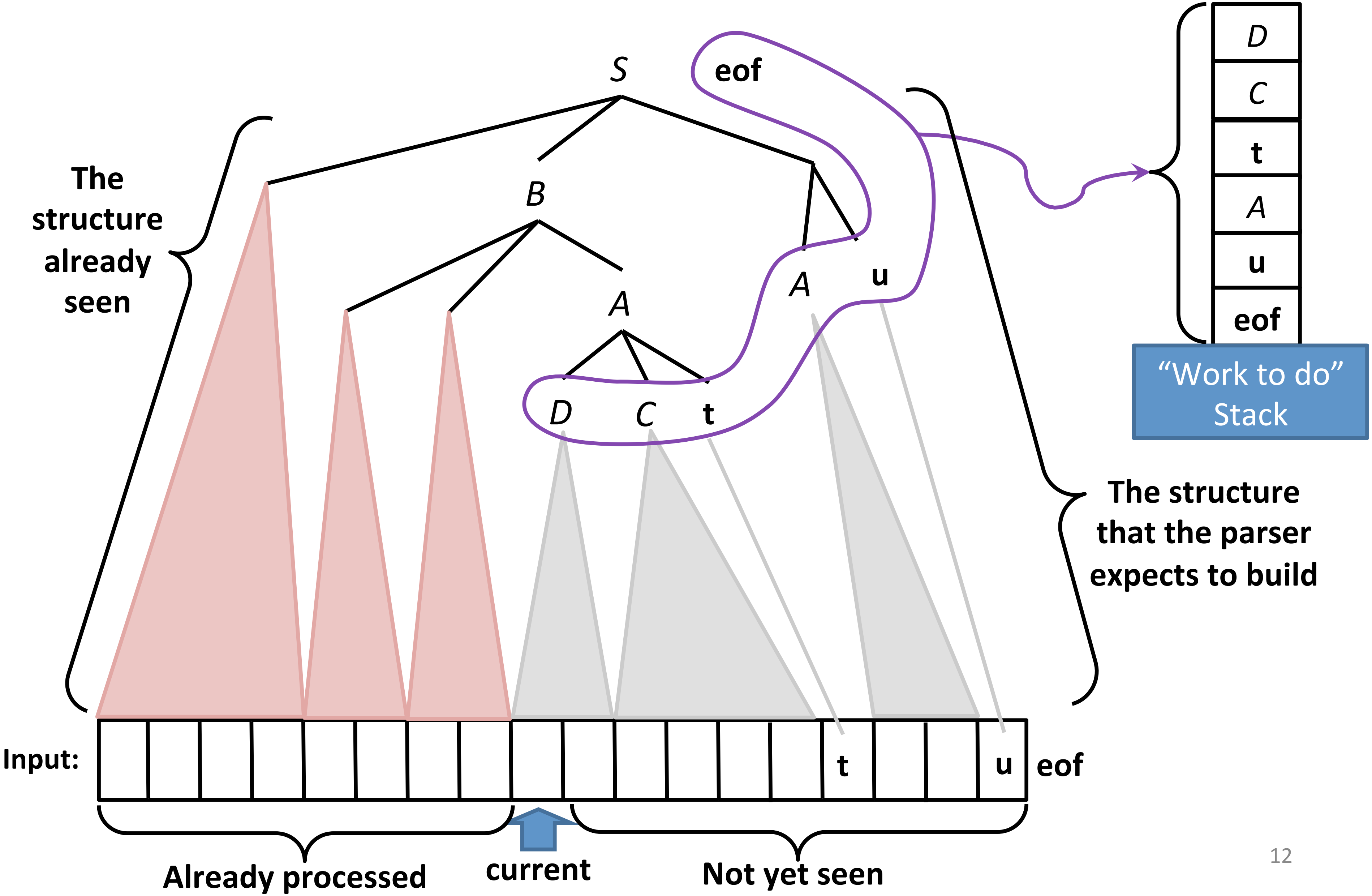
	(	)	{	}	eof
S	(S)	ε	{ S }	ε	ε

The selector table, with rows being non-terminal and cols being terminals.





A Snapshot of a Predictive Parser



Algorithm

```
stack.push(eof)
stack.push(Start non-term)
t = scanner.getToken()
Repeat
  if stack.top is a terminal y
    match y with t
    pop y from the stack
    t = scanner.next_token()
  if stack.top is a nonterminal X
    get table[X,t]
    pop X from the stack
    push production’s RHS (each symbol from Right to Left)
Until one of the following:
  stack is empty                                    accept
  stack.top is a terminal that doesn’t match t
  stack.top is a non-term and parse table entry is empty
reject
```

Example 2, bad input: You try

$S \rightarrow$ **(** S **)** | **{** S **}** | **ε**

	(	)	{	}	eof
S	(S)	ε	{ S }	ε	ε

INPUT

((} eof

This Parser works great!

Given a single token we always knew exactly what production it started

	(	)	{	}	eof
S	(S)	ε	{ S }	ε	ε

Two Outstanding Issues

1. How do we know if the language is LL(1)
   – Easy to imagine a Grammar where a single token is not enough to select a rule

Any Idea? $S \rightarrow \mathbf{(}\ S\ \mathbf{)} \mid \mathbf{\{}\ S\ \mathbf{\}} \mid \mathbf{(\ )}$

The rows of the table are indexed by non-terminals, while the columns of the table are terminals.
Let T be the table, X be the non-terminal and t be the terminal.
In rule "S = (S)", X = S, t = '(', T[X][t] = T[S]['('] = "(S)". If each cell of the table contains at most one production rule, we can do predictive parsing. And this grammar is LL(1) and can be parsed in O(n) time.

2. How do we build the selector table?

It turns out that there is one answer to both:

If selector table has <=1 production per cell, then grammar is LL(1)

LL(1) Grammar Transformations

Necessary (but not sufficient) conditions for LL(1) Parsing:

- Free of left recursion
  - No nonterminal loops for a production
  - Why? Need to look past list to know when to cap it
- Left factored
  - No rules with common prefix
  - Why? We'd need to look past the prefix to pick rule

Left-Recursion

Recall, a grammar such that is left recursive

A grammar is immediately left recursive if this can happen in one step:

$$A \rightarrow A\ \alpha \mid \beta$$

Fortunately, it's always possible to change the grammar to remove left-recursion without changing the language it recognizes

Why Left Recursion is a Problem (Blackbox View)

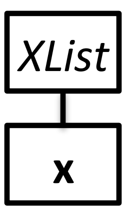
CFG snippet: $\mathit{XList} \longrightarrow \mathit{XList}\ \mathbf{x}\ \mid\ \mathbf{x}$

Current parse tree: $\boxed{\mathit{XList}}$

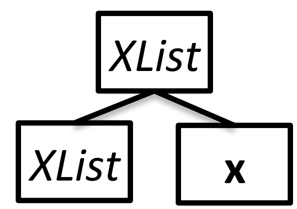
Current token: **x**

When seeing an "XList", two predictions available. However, if left recursion exists, either prediction would be wrong in some cases.
If it predicts "x", we would be wrong if there're more x's; if it predicts "XList x", we would be wrong if there's no more x's.
In other words, the prediction is not guaranteed to be correct.

How should we grow the tree top-down?



(OR)



Correct if there are no more **x**s

Correct if there <u>are</u> more **x**s

We don't know which without more lookahead

Why Left Recursion is a Problem (Whitebox View)

CFG snippet: $XList \longrightarrow XList\ \mathbf{x}\ \mid\ \mathbf{x}$

Current parse tree: $\boxed{XList}$

Current token: **x**

Parse table:

	x	**eof**
XList	*XList* **x**	ε

Stack
XList
x
x
x
eof

x

Current

(Stack overflow)

Removing Left-Recursion

(for a single immediately left-recursive rule)

$A \rightarrow A\,\alpha \mid \beta$

$$\begin{aligned} A &\rightarrow \beta\, A' \\ A' &\rightarrow \alpha\, A' \\ &\mid\ \varepsilon \end{aligned}$$

Where β does not begin with A

Example

The language is not changed: One beta followed by one or more alphas.

$$A \rightarrow A\ \alpha \mid \beta$$

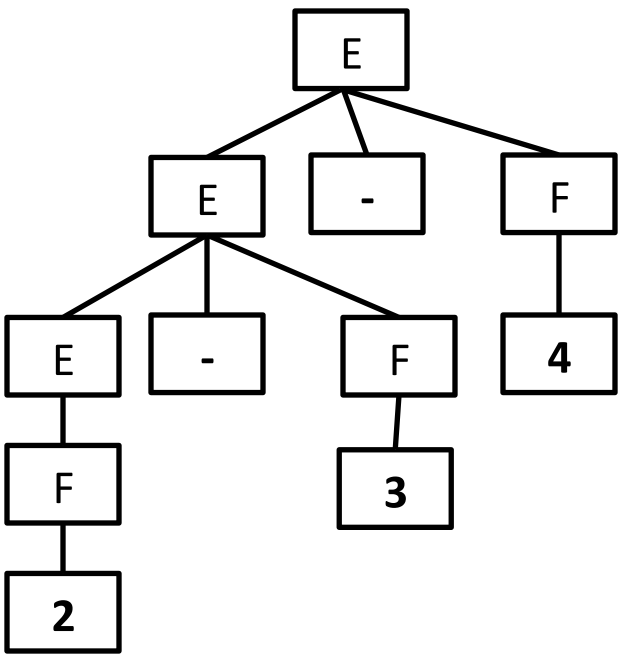
$$\begin{aligned} A &\rightarrow \beta\ A' \\ A' &\rightarrow \alpha\ A' \\ &\mid\ \varepsilon \end{aligned}$$

$$\begin{aligned} \mathit{Exp} &\rightarrow \mathit{Exp}\ -\ \mathit{Factor} \\ &\mid\ \mathit{Factor} \\ \text{Factor} &\rightarrow \textbf{intlit} \mid (\ \text{Exp}\ ) \end{aligned}$$

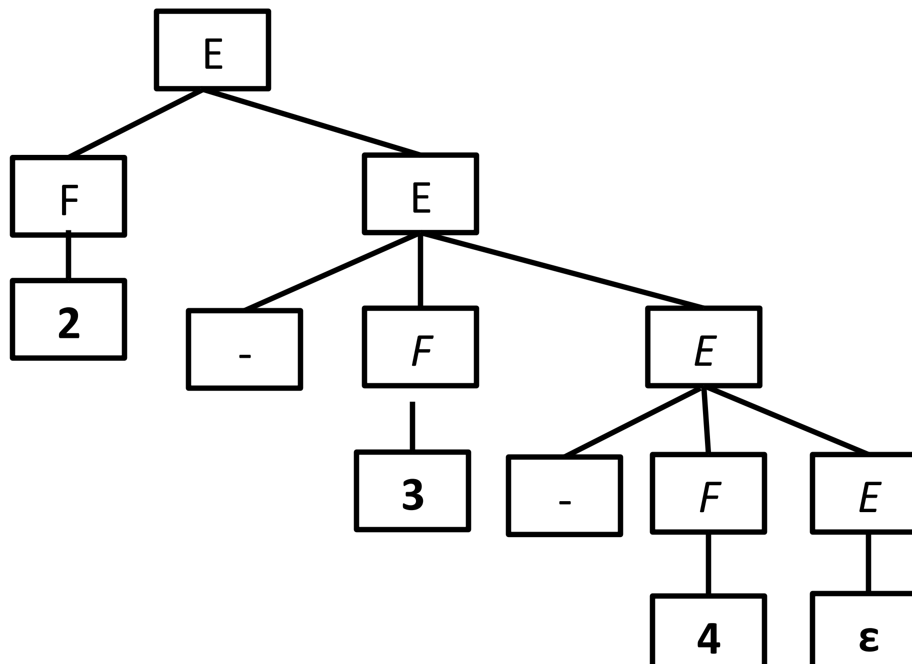
$$\begin{aligned} \mathit{Exp} &\rightarrow \mathit{Factor}\ \mathit{Exp}' \\ \mathit{Exp}' &\rightarrow -\ \mathit{Factor}\ \ \mathit{Exp}' \\ &\mid\ \varepsilon \\ \text{Factor} &\rightarrow \textbf{intlit} \mid (\ \text{Exp}\ ) \end{aligned}$$

Let’s check in on the Parse Tree…

Exp → *Exp* – *Factor*
| *Factor*
Factor → **intlit** | (Exp)



Exp → *Factor Exp’*
Exp’ → - *Factor Exp’*
| ε
Factor → **intlit** | (Exp)



… We'll fix that later

General Rule for Removing Immediate Left-Recursion

$$A \rightarrow \alpha_1 \mid \alpha_2 \mid \dots \mid \alpha_n \mid A\,\beta_1 \mid A\,\beta_2 \mid \dots A\,\beta_m$$

$$A \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_n A'$$
$$A' \rightarrow \beta_1 A' \mid \beta_2 A' \mid \dots \mid \beta_m A' \mid \varepsilon$$

Left Factored Grammars

If a nonterminal has two productions whose RHS have common prefix

→ Grammar it is not left factored and not LL(1)

$$Exp \rightarrow (\ Exp\ )\ |\ (\ )$$

Not left factored

Left Factoring

Given productions of the form

$$A \rightarrow \alpha\ \beta_1 \mid \alpha\ \beta_2$$

$$A \rightarrow \alpha\ A'$$
$$A' \rightarrow \beta_1 \mid \beta_2$$

Combined Example

$Exp \rightarrow \mathbf{(}\ Exp\ \mathbf{)} \mid Exp\ Exp \mid \mathbf{(\ )}$

Remove
Immediate left-recursion

$Exp \rightarrow \mathbf{(}\ Exp\ \mathbf{)}\ Exp' \mid \mathbf{(\ )}\ Exp'$

$Exp' \rightarrow Exp\ Exp' \mid \varepsilon$

Left-factoring

$Exp \text{ -> } \mathbf{(}\ Exp''$

$Exp'' \text{ -> } Exp\ \mathbf{)}\ Exp' \mid \mathbf{)}\ Exp'$

Exp' -> Exp Exp' | ε

Where are we at?

We’ve set ourselves up for success in building the selection table

- Two things that prevent a grammar from being LL(1) were identified and avoided
  - Not Left-Factored grammars
  - Left-recursive grammars
- Next time
  - Build two data structures that combine to yield a selector table:
    - FIRST set
    - FOLLOW set