# Problem set 1 Preferences, utility, and demand*

October 7, 2022

*Last updated: October 20, 2022*†

**Directions**: PS1 covers Lecture 0, 1 and 2. Please try your best to answer each question as completely as possible. Part of the solutions will be presented the on the seminar of Week 6 (time: October 21, 2022; venue: PB 205). Solutions that are not presented on the seminar will be posted on Moodle.

I. **(JR 1.3 & 1.4)**[1] Let $\succsim$ be a complete and transitive preference relation on set $X$, and define binary relations $\sim$ and $\succ$ as follows:

$$x \sim y \Leftrightarrow x \succsim y \text{ and } y \succsim x,$$
$$x \succ y \Leftrightarrow x \succsim y \text{ and } y \not\succsim x.$$

First, please show conditions (1) to (3) on $\sim$ hold:

(1) $x \sim x$ (for all $x$)

(2) $x \sim y \rightarrow y \sim x$ (for all $x, y$)

(3) $x \sim y$ and $y \sim z \rightarrow x \sim z$ (for all $x, y, z$)

Next, please show the conditions (4) to (6) on $\succ$ hold:

(4) $x \succ y$ and $y \succ z \rightarrow x \succ z$ (for all $x, y, z$)[2]

(5) $x \succ y \rightarrow y \not\succ x$ (for all $x, y$)[3]

(6) not $x \succ x$ (for all $x$)

*Instructor: Ying Chen. Email: *ying.chen2@nottingham.edu.cn* (or ask Ying anything anonymously). Office hour: Fridays 4.30-5.30 p.m. (w6-12); Trent 133 the staff lounge (next to Arabica).

†Do not hesitate to email me when you find errors like typos, *etc*. Any updates of this problem set will be notified via Moodle announcements.

[1]This mark refers to exercise 1.3 and 1.4 of G. Jehle and P. Reny, *Advanced Microeconomic Theory*, Prentice Hall, 2011.

[2]Hint: You might need to prove by contradiction.

[3]Hint: You might need to prove by contradiction as well.

Finally, please show condition (7) holds:
(7) If $x \succ y \succsim z$, then $x \succ z$ (for all $x, y, z$)[4]

II. **(JR 1.24)** Let $u(x)$ represent some consumers monotonic preferences over $x \in \mathbb{R}^n_+$. For each of the functions $f(x)$ that follow, state whether or not $f$ also represents the preferences of this consumer. In each case, be sure to justify your answer with either an argument or a counterexample.
(1) $f(x) = u(x) + [u(x)]^3$
(2) $f(x) = u(x) - [u(x)]^2$
(3) $f(x) = u(x) + \sum_{i=1}^n x_i$

III. Daniel's consumption bundle consists of two kinds of fruits, apples and bananas. His bundle $x$ contains $a$ apples and $b$ bananas, while his another bundle $x'$ contains $a'$ apples and $b'$ bananas. Consider he has the following lexicographic preferences:

$$x \succ x' \text{ if EITHER } a > a'$$
$$\text{OR } a = a' \text{ and } b > b'.$$

(1) Suppose that the set $a$ and $a'$ contains only two elements, 1 and 2. Suppose that the set $b$ and $b'$ also contains only 1 and 2. Cite an example of the utility function for Daniel that represents his lexicographic preferences.
(2) Now suppose that the set $a$, $a'$, $b$, $b'$ may be drawn contains all positive natural numbers: $1, 2, 3, \ldots$. Show that the utility function $u(a, b) = a + 1 - \frac{1}{b}$ can represent Daniel's lexicographic preferences.
(3) Now suppose that set $a$, $a'$, $b$, $b'$ may be drawn contains all positive real numbers. Can the utility function $u(a, b) = a + 1 - \frac{1}{b}$ still represents lexicographic preferences? Why?

IV. Emma consumes two goods, good 1 and good 2. The quantities of the two goods are denoted as $x_1$ and $x_2$, respectively. The prices of the two goods are denoted as $p_1$ and $p_2$, respectively. Emma has a budget $w$ at her disposal. Emma's utility is captured by a Cobb-Douglas function:

$$u(x_1, x_2) = x_1^a x_2^{1-a},$$

where $x_1$, $x_2 > 0$ and $0 < a < 1$.

Find each of the following for Emma:

[4] Hint: This proof has two parts. First, prove that $x \succsim z$; second, prove that $z \not\succsim x$.

(1) Assuming an interior solution, find her Marshallian demand function**s**.

(2) Make use of the solution to (1), find her indirect utility function.[5]

(3) Show that Roy's identity holds in her case.

(4) Set up the dual consumer problem for her utility function.

(5) Assuming an interior solution, derive her Hicksian demand functions from (4).

(6) Make use of the solution to (5), derive her expenditure function.[6]

(7) Make use of the solution to (6), derive her indirect utility function $v(p, w)$ from the expenditure function.

(8) Recall that the price elasticity of demand of good $i$ with respect to the price of good $j$ is given by

$$\epsilon_{ij} = \frac{\partial x_i(p, w)}{\partial p_j} \frac{p_j}{x_i(p, w)}.$$

Determine Emma's price elasticity of good 1 with respect to its own price and the price of good 2.

[5]Hint: the solution to (1) is $x_1(p, w) = \frac{aw}{p_1}$ and $x_2 = \frac{(1-a)w}{p_2}$. Now you can solve (2).

[6]Hint: the solution to (5) is $h_1 = \bar{u}\left(\frac{ap_2}{(1-a)p_1}\right)^{1-a}$ and $h_2 = \bar{u}\left(\frac{(1-a)p_1}{ap_2}\right)^{a}$. Now you can solve (5).