QI JR 4.8

1,2 $\boxed{p = a - b(q_1 + q_2)}$, $0 \leq c_1 < c_2$

$$\pi_1 = p \cdot q_1 - c_1 \cdot q_1 = \left(a - b(q_1 + q_2)\right) q_1 - c_1 q_1$$

$$= (a - bq_1 - bq_2) \cdot q_1 - c_1 q_1$$

$$= (a - bq_2 - c_1) \cdot q_1 - b q_1^2$$

$$\pi_2 = p \cdot q_2 - c_2 q_2$$

$$= (a - bq_1 - c_2) \cdot q_2 - b q_2^2$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \implies a - bq_2 - c_1 - 2b q_1 = 0 \implies q_1^* = \frac{a - bq_2^* - c_1}{2b}$$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \implies a - bq_1 - c_2 - 2bq_2 = 0 \implies q_2^* = \frac{a - bq_1^* - c_2}{2b}$$

$$q_1^* = \frac{a - \cancel{b}\,\dfrac{a - b q_1^* - c_2}{2\cancel{b}} - c_1}{2b}$$

$$2b \cdot q_1^* = a - \frac{a - b q_1^* - c_2}{2} - c_1$$

$$4b q_1^* = 2a - a + b q_1^* + c_2 - 2c_1$$

$$3b q_1^* = a + c_2 - 2c_1$$

$$q_1^* = \frac{a + c_2 - 2c_1}{3b}$$

$$\frac{(a - c_1) + (c_2 - c_1)}{3b}$$

$c^1 < c^2$

$$q_2^* = \frac{a + c_1 - 2c_2}{3b} = \frac{(a - c_2) + (c_1 - c_2)}{3b}$$

$q_1^* > q_2^*$

$$p^* = a - b(q_1^* + q_2^*)$$

$$= a - \cancel{b}\left(\frac{a+c_2-2c_1}{3\cancel{b}} + \frac{a+c_1-2c_2}{3\cancel{b}}\right)$$

$$= a - \frac{a+c_2-2c_1+a+c_1-2c_2}{3}$$

$$= \frac{3a}{3} - \frac{2a-c_1-c_2}{3}$$

$$= \frac{a+c_1+c_2}{3}$$

$$\pi_1 = pq_1 - c_1q_1 = (p-c_1)q_1 = \left(\frac{a+c_1+c_2}{3} - c_1\right)\cdot\frac{a+c_2-2c_1}{3b}$$

$$= \frac{a-2c_1+c_2}{3}\cdot\frac{a+c_2-2c_1}{3b}$$

$$= \frac{(a-2c_1+c_2)^2}{9b} = \frac{(a-c_1+c_2-c_1)^2}{9b}$$

$$\pi_2 = p \cdot q_2 - c_2 q_2 = \frac{(a-2c_2+c_1)^2}{9b} = \frac{(a - c_2 + c_1 - c_2)^2}{9b}$$

$c_1 < c_2, \quad \pi_1 > \pi_2$

$\underline{c_1 < c_2}$

$a - c_1 > a - c_2$

QII JR. 4.13

$$P_1 = 20 + \frac{1}{2} P_2 - q_1$$

$$P_2 = 20 + \frac{1}{2} P_1 - q_2$$

$$q_1 = 20 + \frac{1}{2} P_2 - P_1$$

$$\pi_1 = P_1 \cdot q_1 - 20 \cdot q_1 = (P_1 - 20) \cdot q_1 = (P_1 - 20) \cdot (20 + \frac{1}{2} P_2 - P_1)$$

$$= -P_1^2 + (40 + \frac{P_2}{2}) \cdot P_1 - (400 + 10 P_2)$$

$$\frac{\partial \pi_1}{\partial P_1} = 0 \implies -2P_1 + 40 + \frac{P_2}{2} = 0 \implies P_1^* = \frac{80 + P_2^*}{4} \quad ①$$

$$P_2^* = \frac{80 + P_1^*}{4} \quad ②$$

use ①, ②

$$P_1^* = \frac{80 + \frac{80 + P_1^*}{4}}{4}$$

$$4P_1^* = 80 + \frac{80 + P_1^*}{4}$$

$$16 \cdot P_1^* = 80 + 80 + P_1^*$$

$$15 \cdot P_1^* = 160 \implies P_1^* = \frac{160}{15} = \frac{80}{3} = P_2^*$$

$$q_1^* = 20 + \frac{1}{2} p_2^* - p_1^*$$

$$= 20 + \frac{1}{2} \cdot \frac{80}{3} - \frac{80}{3} = \frac{20}{3} = q_2^*$$

QIII. Auction

(1). 2nd price sealed bid auction

- two players, $i, j \in \{1, 2\}$
- Bidder i values the good as v_i
- v_i, v_j are iid $\sim U[0,1]$

As a Bayesian game of incomplete information,

① action space : bidder i needs to submit a bid, b_i $\rightarrow A_i = [0, \infty)$

② type space : her valuation, v_i ; $\rightarrow T_i = [0, 1]$

③ beliefs : bidder i believes that v_j is $\sim U[0,1]$, no matter what the realization of v_j. $\rightarrow$ Valuation v_i v_j are private info.

④ payoff function :

$$u_i(b_1, b_2, v_1, v_2) = \begin{cases} v_i - b_j & \text{if } i \text{ submits winning bid} \\ 0 & \text{if } i \text{ submits losing bid.} \end{cases}$$

QIV Adverse selection

(1)

gains from the trade

seller

buyer

$g_s = \underbrace{p - \text{exp. value of the car}}_{} \geq 0$

X L, M, H

$g_b = \underbrace{\text{exp. value of the car} - p}_{} \geq 0$

X L, M, H

$$g_s = p - \left(\frac{1}{3}\cdot s(L) + \frac{1}{3}\cdot s(M) + \frac{1}{3}\cdot s(H)\right)$$

$$= p - \left(\frac{1}{3}\cdot 0 + \frac{1}{3}\cdot 20 + \frac{1}{3}\cdot 40\right)$$

$$= p - 20 \geq 0$$

$$p \geq 20$$

$20 \leq p \leq 28$

$$g_b = \left(\frac{1}{3}\cdot b(L) + \frac{1}{3}\cdot b(M) + \frac{1}{3}\cdot b(H)\right) - p$$

$$= \left(\frac{1}{3}\cdot 14 + \frac{1}{3}\cdot 28 + \frac{1}{3}\cdot 42\right) - p$$

$$= 28 - p \geq 0$$

$$p \leq 28$$

(2) seller

Low vs not Low (M, L)

buyer

X L, M, H

$p \le 28$

exp. value.

$$g_s = p - \left(\frac{1/3}{2/3} \cdot s(M) + \frac{1/3}{2/3} \cdot s(L) \right)$$

$$= p - \left(\frac{1}{2} \cdot 20 + \frac{1}{2} \cdot 40 \right)$$

$$= p - 30 \geq 0$$

$$p \geq 30$$

① the seller will not sell M, H in the market

$p \geq 30 \longleftrightarrow p \le 28$

② the seller will sell L in the market

$s(L) = 0 \rightarrow p \geq 0$ —— $p \le 28$ ✓

buyer knows the seller only bring L in the market

$b(L) = 14 \quad p \le 14$

$0 \le p \le 14$

low

(3) seller buyer

✓ L, M, H — buyer knows ✗ L, M, H ✓

① the seller will not sell H ✗ $p \le 28$

$s(H) = 42$, $p \ge 42$

✓

② the seller will sell M, L

buyer knows M, L

$s(M) = 20$

$s(L) = 0$

$\Big\} \; p \ge 20$

$$g_b = \left(\frac{1/3}{2/3} b(M) + \frac{1/3}{2/3} b(L)\right) - p$$

$$= \left(\tfrac{1}{2}\cdot 28 + \tfrac{1}{2}\cdot 14\right) - p$$

$$= 21 - p \ge 0$$

$p \le 21$

$20 \le p \le 21$

M, L

QV Game Theory

(1)

Alan

not burn | burn

Not burn:

		Barbara C (p)	Barbara D ($1-p$)
Alan	C (q)	3,1	0,0
	D ($1-q$)	0,0	1,3

$e^C = e^D$

$\to q = \frac{1}{4}$

i) pure strategy Nash eqm.

① $(C, C) \longrightarrow (3,1)$

② $(D, D) \longrightarrow (1,3)$

ii) mixed strategy Nash eqm.

$$e^C = \underbrace{3 \cdot p + 0 \cdot (1-p)}_{} = e^D = 0 \cdot p + 1 \cdot (1-p)$$

$$3p = 1-p$$

$$4p = 1$$

$$p = \frac{1}{4} \sim 3 \cdot \frac{1}{4} + 0 \cdot \frac{3}{4} = \frac{3}{4}$$

Burn:

		Barbara C	Barbara D
Alan	C	2,1	-1,0
	D	-1,0	0,3

i) pure strategy Nash eqm.

① $(C, C) \longrightarrow (2,1)$

② $(D, D) \longrightarrow (0,3)$

ii) mixed strategy Nash eqm.

③ $\left(\left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{1}{4}, \frac{3}{4}\right) \right)$

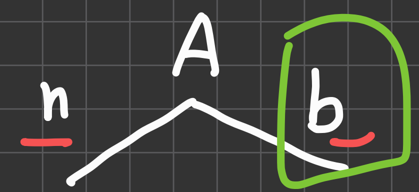
$\to \left(-\frac{1}{4}, \frac{3}{4}\right)$

$$\left(\left(\tfrac{1}{4},\tfrac{3}{4}\right),\left(\tfrac{1}{4},\tfrac{3}{4}\right)\right) \longrightarrow \left(\tfrac{3}{4},\tfrac{3}{4}\right)$$

③

There are 4 classes sub-game perfect Nash eqm:

1. Alan burns and both player then chooses C; ~~any~~ any subgame eqm. other than (C,C) is played after not burning



$(D,D) \to (1,3)$ $\qquad$ $(C,C) \to (2,1)$

mix $\to \left(\tfrac{3}{4},\tfrac{3}{4}\right)$

2. Alan does not burn and both players then choose C; and any subgame eqm. is played after burning

$(C,C) \to (3,1)$ $\quad >$ $\quad (C,C) \to (2,1)$

$(D,D) \to (0,3)$

mix $\to \left(-\tfrac{1}{4},\tfrac{3}{4}\right)$

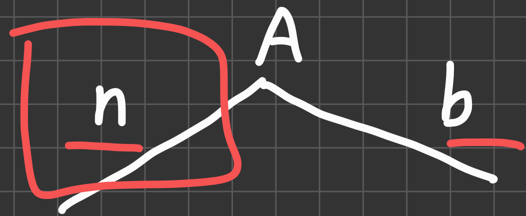
3. Alan does not burn and both players then choose D ; and any subgame eqm. other than (C,C) is played after burning

✓ $(D,D) \to (1,3)$

✓ $(D,D) \to (0,3)$

✓ $\text{mix} \to (-\frac{1}{4}, \frac{3}{4})$

4. Alan does not burn and both players then choose mix ; and any subgame eqm. other than (C,C) is played after burning.



$\text{mix} \to (\frac{3}{4}, \frac{3}{4})$ ✓

$(D,D) \to (0,3)$

$\text{mix} \to (-\frac{1}{4}, \frac{3}{4})$

(2)

Alan

not burn | burn

Not burn:

		Barbara C	Barbara D
Alan	C	3,1	0,0
	D	0,0	1,3

Burn:

		Barbara C	Barbara D
Alan	C	2,1	-1,0
	D	-1,0	0,3

~> extensive form game

-> normal form

Normal form of the game

Alan's strategy

nCD means "not burn", followed by C after n and b after D, etc.

Barbara's strategy

CD means C after n and b after D, etc.

Alan \ Barbara	CC	CD	DC	DD
nCC	(3,1)	...	...	...
nCD				
nDC				
nDD				
bCC				
bCD				
bDC				
bDD				

① nCC and nCD are "equivalent"

② nDC and nDD are "equivalent"

③ bCC and bDC are "equivalent"

④ bCD and bDD are "equivalent"

Barbara

Alan	CC	CD	DC	DD
nCC	(3,1)	(3,1)	(0,0)	(0,0)
~~nCD~~	~~(3,1)~~	~~(3,1)~~	~~(0,0)~~	~~(0,0)~~
nDC	(0,0)	(0,0)	(1,3)	(1,3)
~~nDD~~	~~(0,0)~~	~~(0,0)~~	~~(1,3)~~	~~(1,3)~~
bCC	(2,1)	(-1,0)	(2,1)	(-1,0)
bCD	(-1,0)	(0,3)	(-1,0)	(0,3)
~~bDC~~	~~(2,1)~~	~~(-1,0)~~	~~(2,1)~~	~~(-1,0)~~
~~bDD~~	~~(-1,0)~~	~~(0,3)~~	~~(-1,0)~~	~~(0,3)~~

① nC weakly dominates bD

② CC weakly dominates CD

DC weakly dominates DD

③ bC dominates nD

④ CC weakly dominates DC

⑤ nC dominates bC

Barbara

Alan	CC	CD	DC	DD
nC	(3,1)	(3,1)	(0,0)	(0,0)
nD	(0,0)	(0,0)	(1,3)	(1,3)
bC	(2,1)	(-1,0)	(2,1)	(-1,0)
bD	(-1,0)	(0,3)	(-1,0)	(0,3)

(nC, CC)