Problem set 2 Uncertainty, and production*

October 25, 2022†

Directions: PS2 covers Lecture 3 and 4. Please try your best to answer each question as completely as possible. Part of the solutions will be presented the on the seminar of Week 8 (time: Novenver 4, 2022; venue: PB 205). Solutions that are not presented on the seminar will be posted on Moodle.

I. An individual has wealth w. Her von Neumann-Morgenstern utility function over non-negative levels of wealth is

$$u(w) = w^{\rho},$$

where $0 < \rho < 1$. The individual is offered the following bet. If she pays x, with probability $\frac{1}{2}$ she receives nothing, and with probability $\frac{1}{2}$ she receives $x(1+s)$, where $s > 1$. How much will she bet (as a function of s)?

II. **(JR 2.25)** Consider the quadratic Bernoulli utility function $u(w) = a + bw + cw^2$.

(1) What restrictions, if any, must be placed on parameters a, b, and c for this function to display risk aversion?

(2) Over what domain of wealth is the quadratic Bernoulli utility function defined?

(3) Given the gamble

$$g = \left(\frac{1}{2} \circ (w+h), \frac{1}{2} \circ (w-h)\right),$$

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†Do not hesitate to email me when you find errors like typos, *etc*. Any updates of this problem set will be notified via Moodle announcements.

show that the certainty equivalent, CE, is strictly smaller than the expected value of the gamble, $E(g)$; and that $P > 0$.

(4) Show that this function, satisfying the restrictions in part (1), cannot represent preferences that display *decreasing* absolute risk aversion.

III. Consider a firm that uses only one factor of production, x, with a production technology

$$y = 70\sqrt{x}.$$

Let w denote the price of input x. Compute the marginal cost and the average cost of producing y. Verify that the average cost is less than the marginal cost for all values of y. Explain why this is so.

IV. **(JR 3.54)** Consider a firm with the cost function

$$c(y, w_1, w_2) = y^2(w_1 + w_2)$$

where w_i denotes the price of input i for $i = 1, 2$. Let p denote the output price. Derive the output supply function $y(p, w_1, w_2)$, and the input demand functions $x_i(p, w_1, w_2)$ for $i = 1, 2$.

V. Consider a firm with production function

$$y = (x_1^\rho + x_2^\rho)^\alpha,$$

where $0 < \rho < 1$ and $\alpha > 0$.

(1) For what value of ρ and α are there
  (i) increasing returns to scale;
  (ii) constant return to scale;
  (iii) decreasing returns to scale?

(2) Suppose that there are decreasing return to scale.[1]
  (i) Find the long-run cost function.
  (ii) Derive the output supply function and the input demand functions for the long-run cost function in part (2) - (i).[2]

[1] Hint: Decreasing return to scale means $\rho\alpha < 1$.

[2] Hint: The long-run cost function in part (2)-(i) is $c(w, y) = y^{\frac{1}{\rho\alpha}} \left(w_1^{\frac{\rho}{\rho-1}} + w_2^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}$.