UNIVERSITY OF NOTTINGHAM NINGBO
SCHOOL OF ECONOMICS

Problem set 3 General equilibrium, and game theory*

November 8, 2022†

Directions: PS3 covers Lecture 5 and 6. Please try your best to answer each question as completely as possible. Part of the solutions will be presented the on the seminar of Week 10 (time: November 18, 2022; venue: PB 205). Solutions that are not presented on the seminar will be posted on Moodle.

I. Consider an exchange economy with two agents, 1 and 2, and two goods, x_1 and x_2. Suppose $u^1(x_1^1, x_2^1) = x_1^1 x_2^1$, and $u^2(x_1^2, x_2^2) = x_1^2 x_2^2$. Let the endowments of the two agents be $e^1 \equiv (e_1^1, e_2^1) = (3, 2)$ and $e^2 \equiv (e_1^2, e_2^2) = (2, 3)$. Draw an Edgeworth box representation of this economy, illustrating
   (1) the endowment point $e \equiv (e^1, e^2)$,
   (2) the indifference curves passing through e for both agents,
   (3) the set of contract curve, and
   (4) the set of barter equilibria.

II. **(JR 5.18)** In a two-good, two-consumer economy, utility functions are

$$u^1(x_1, x_2) = x_1(x_2)^2 \text{ and } u^2(x_1, x_2) = (x_1)^2 x_2.$$

   Total endowments are (10, 20).
   (1) A social planner wants to allocate goods to maximise consumer 1's utility while holding consumer 2's utility at $u^2 = 8000/27$. Find the assignment of goods to consumers that solves the planners problem and show that the solution is Pareto efficient.

*Instructor: Ying Chen. Email: *ying.chen2@nottingham.edu.cn* (or ask Ying anything anonymously). Office hour: Fridays 4.30-5.30 p.m. (w6-12); Trent 133 the staff lounge (next to Arabica).

†Do not hesitate to email me when you find errors like typos, *etc*. Any updates of this problem set will be notified via Moodle announcements.

(2) Suppose, instead, that the planner just divides the endowments so that $e^1 = (10, 0)$ and $e^2 = (0, 20)$ and then lets the consumers transact through perfectly competitive markets. Find the Walrasian equilibrium and show that the WEAs are the same as the solution in part (1).

III. Consider the following Robinson Crusoe economy. Robinson the consumer is endowed with zero units of coconuts, x, and 24 hours of time, h, so that $e = (0, 24)$. His preferences are defined over $\mathbb{R}^2_+$ and represented by $u(x, h) = xh$. Robinson the producer uses the consumer's labor services, l, to produce coconuts, y, according to the production function $y = \sqrt{l}$. The producer sells the coconuts to the consumer. All profits from the production and sale of coconuts are distributed to the consumer.

(1) Let p denote the price of coconuts, and normalize $p = 1$. Let w denote the price of Robinsons time. Find the Walrasian equilibrium prices and allocation of this economy.

(2) Now suppose that Robinson does not think about a market, but simply chooses to enjoy h hours of leisure and spend $24 - h$ hours collecting coconuts. What is his optimal choice of h? How many coconuts does he get? Compare your answer to the answer to part (1).

IV. **(JR 7.10)** Calculate the set of Nash equilibria in the following games.

(1) Game 1:

	L	R
U	1,1	0,0
D	0,0	0,0

Also show that there are two Nash equilibria, but only one in which neither player plays a weakly dominated strategy.

(2) Game 2:

	L	R
U	1,1	0, 1
D	1,0	-1,-1

Also show that there are infinitely many Nash equilibria, only one of which has neither player playing a weakly dominated strategy.

(3) Game 3:

	L	l	m	M
U	1,1	1,2	0,0	0,0
C	1,1	1,1	10,10	-10,-10
D	1,1	-10,-10	10,-10	1,-10

Also show that there is a unique strategy determined by iteratively eliminating weakly dominated strategies.

V. Suppose that there are 2 hunters, Fred and Barney. The hunters can go after big game or small game. If a hunter goes after small game then he catches small game for a payoff of 1. If he goes after big game and he hunts alone he fails to catch anything, for a payoff of 0. However, if a hunter hunts for big game and both hunters are hunting big game, then they have a hunting party and catch the big game for a payoff of 3 each.

(1) Write down the normal form version of this 2-player game.

(2) What is (are) the pure strategy Nash equilibria (PSNE) of this game?

(3) Is there a mixed strategy Nash equilibrium to this game? If so find it.

VI. **(JR 7.18)** Reconsider the two countries from the previous exercise, but now suppose that country 1 can be one of two types, 'aggressive' or 'non-aggressive'. Country 1 knows its own type. Country 2 does not know country 1's type, but believes that country 1 is aggressive with probability $\varepsilon > 0$. The aggressive type places great importance on keeping its weapons. If it does so and country 2 spies on the aggressive type this leads to war, which the aggressive type wins and justifies because of the spying, but which is very costly for country 2. When country 1 is non-aggressive, the payoffs are as before (*i.e.*, as in the previous exercise). The payoff matrices associated with each of the two possible types of country 1 are given below.

Country 1 is 'aggressive'
Probablility ε

	Spy	Don't Spy
Keep	10,-9	5,-1
Destroy	0,2	0,2

Country 1 is 'non-aggressive'
Probablility $1 - \varepsilon$

	Spy	Don't Spy
Keep	-1,1	1,-1
Destroy	0,2	0,2

(1) What action must the aggressive type of country 1 take in any Bayesian-Nash equilibrium?

(2) Assuming that $\varepsilon < \frac{1}{5}$, find the unique Bayes-Nash equilibrium. (Can you prove that it is unique?)