Problem set 4 Information in Economics*

Last updated: November 20, 2022†

Directions: PS4 covers Lecture 7 and 8. Please try your best to answer each question as completely as possible. Part of the solutions will be presented the on the seminar of Week 12 (time: December 4, 2022; venue: PB 205). Solutions that are not presented on the seminar will be posted on Moodle.

I. **(JR 4.8)** In the Cournot oligopoly of Section 4.2.1, suppose that $J = 2$. Let each duopolist have constant average and marginal costs, as before, but suppose that $0 \leq c^1 < c^2$. Show that firm 1 will have greater profits and produce a greater share of market output than firm 2 in the Nash equilibrium.

II. **(JR 4.13)** Duopolists producing substitute goods q_1 and q_2 face inverse demand schedules:

$$p_1 = 20 + \frac{1}{2}p_2 - q_1 \text{ and } p_2 = 20 + \frac{1}{2}p_1 - q_2,$$

respectively. Each firm has constant marginal costs of 20 and no fixed costs. Each firm is a Cournot competitor in price, not quantity. Compute the Cournot equilibrium in this market, giving equilibrium price and output for each good.

III. **(JR 4.22)** A monopolist faces linear demand $p = \alpha - \beta q$ and has cost $C = cq + F$, where all parameters are positive, $\alpha > c$, and $(\alpha - c)^2 > 4\beta F$.
(1) Solve for the monopolists output, price, and profits.
(2) Calculate the deadweight loss and show that it is positive.

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†Do not hesitate to email me when you find errors like typos, *etc*. Any updates of this problem set will be notified via Moodle announcements.

(3) If the government requires this firm to set the price that maximises the sum of consumer and producer surplus, and to serve all buyers at that price, what is the price the firm must charge? Show that the firm's profits are negative under this regulation, so that this form of regulation is not sustainable in the long run.

IV. Consider two firms who compete by simultaneously choosing prices (a Bertrand game). If Firm 1 and 2 choose prices p_1 and p_2, respectively, the quantity that consumers demand from each firm is:

$$q_1(p_1, p_2) = ap_1 + bp_2$$
$$q_2(p_1, p_2) = ap_2 + bp_1$$

Firm 2 has constant marginal cost c. With probability θ Firm 1 has constant marginal cost c_H while with probability $(1-\theta)$ Firm 1 has constant marginal cost c_L. Assume that there are no fixed costs. Prices must be nonnegative ($p_1 \geq 0$, $p_2 \geq 0$) and firms wish to maximize profit. Important note: Even though it is a Bertrand game, the demand functions are differentiable and continuous for both firms. Let $\theta = \frac{1}{2}$, $a = 108$, $b = \frac{1}{2}$, $c_H = 16$, $c_L = 8$, and $c = 12$.

(1) Find the best response functions for Firms 1 and 2.

(2) Find a pure strategy Bayes-Nash equilibrium to this game.

V. It is typical for the government to allocate construction contracts, such as repaving a highway, by holding an auction for the contract. The auction rules are as follows. Each bidder is to submit a sealed bid. The lowest bidder will win the contract, and the winning bidder will be paid an amount equal to the second lowest bid. Suppose that each bidder draws a cost, ci, of completing the job from the uniform distribution over the interval $[\underline{c}, \bar{c}]$. The cost draws are made independently of each other. All bidders are aware of the common distribution of costs as well as the fact that cost draws are made independently of one another.

(1) Suppose that the bidders are risk-neutral. Find a Bayes-Nash equilibrium for this auction.

(2) Suppose that the bidders are risk-neutral. Suppose we changed the format so that the winning bidder, who is still the lowest bidder, now receives a payment equal to his bid if he wins (instead of a payment equal to the 2nd lowest bid as before). Find a Bayes-Nash equilibrium for this new set of auction procedures.