多智能体深度强化学习及（量子）博弈分析

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Agenda

1. Introduction
2. Direct Interaction Modelling - Emergent Communication
3. Indirect Interaction Modelling
    a. Agent Modelling
    b. Implicit Interaction Modelling via Centralized Critic
    c. Explicit Policy Interaction Modelling via Policy-Space Analysis
4. Challenges and Perspectives X Quantum Theory

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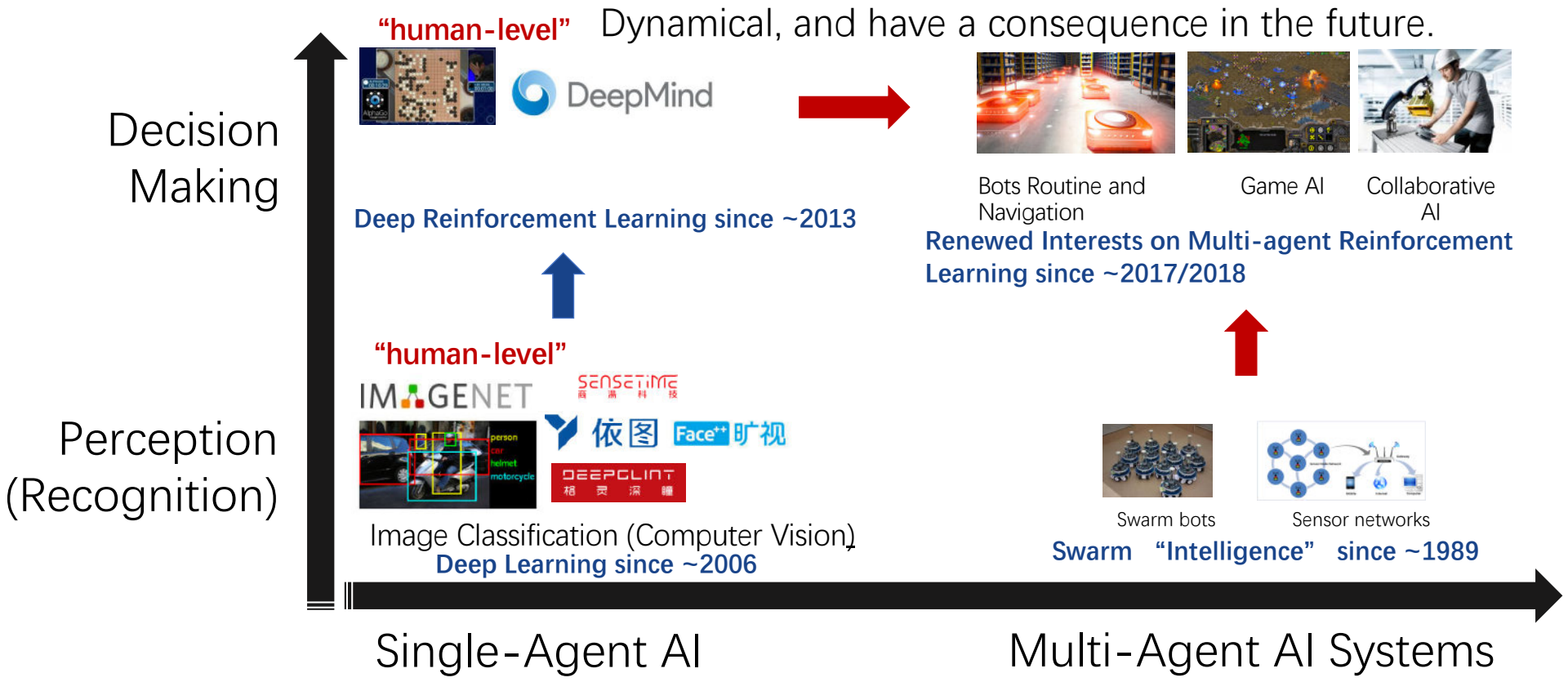
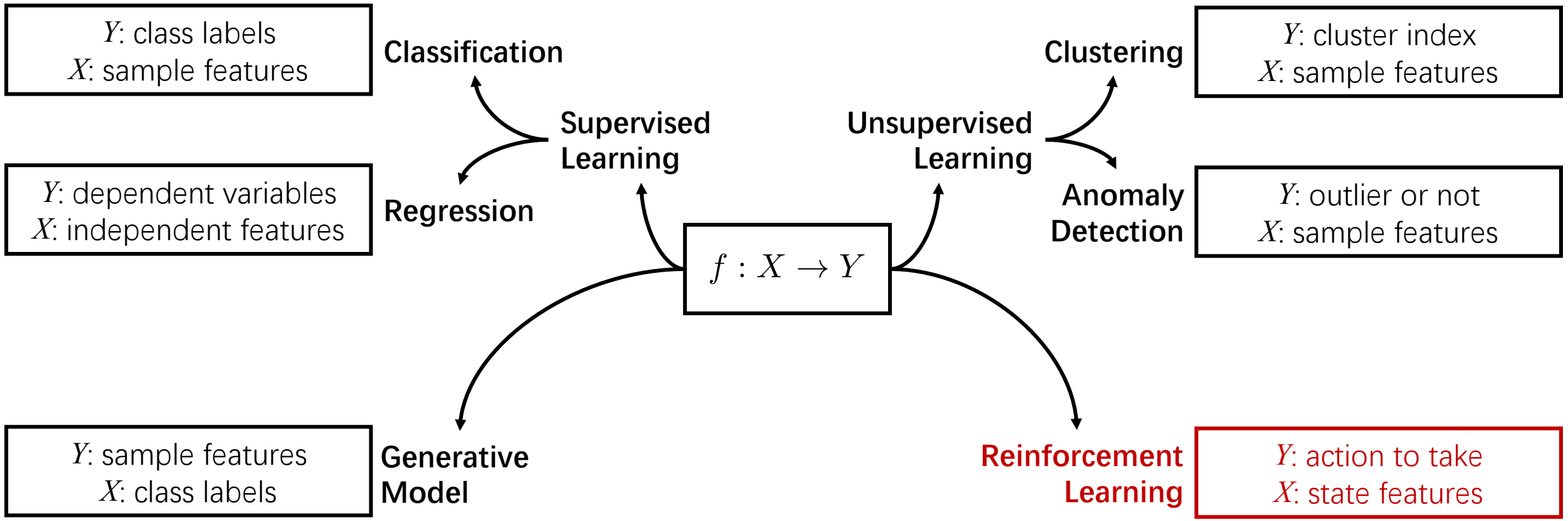
Some Observations on AI Progress



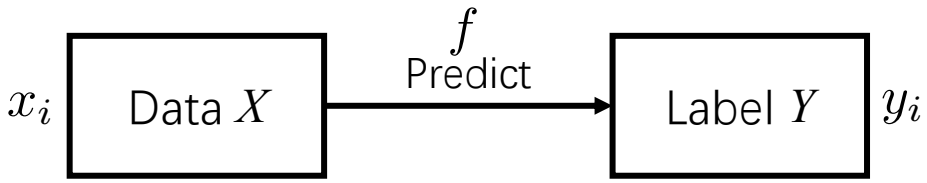
Machine Learning as Function Mapping

- Machine learning tasks are different in the experience available, the objective function, and the specific learning algorithms.



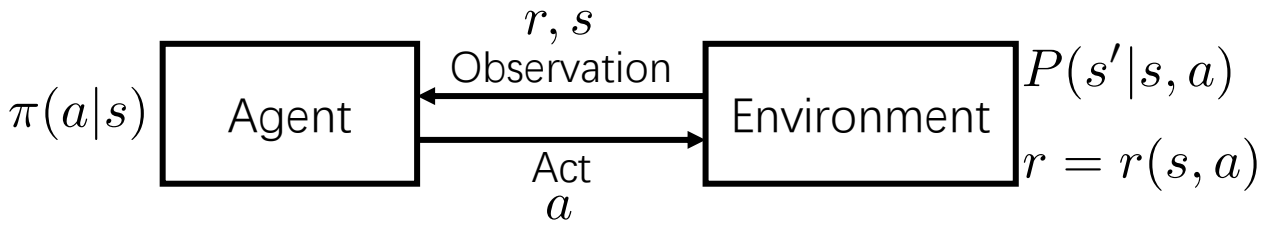
Typical Machine learning

Supervised Learning



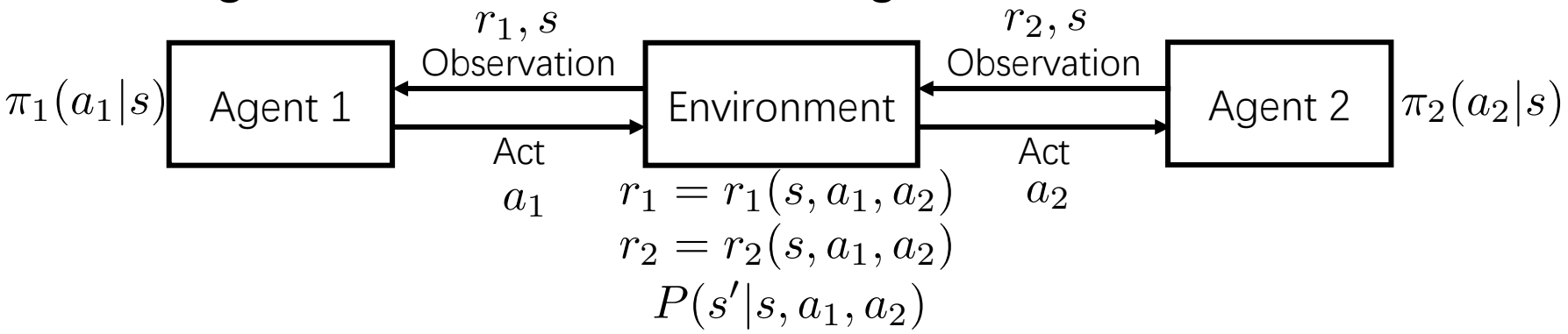
$$\arg\min_{\theta} \sum_{i=1}^{n} \text{Loss}\left(f_\theta\left(x_i\right), y_i\right)$$

Reinforcement Learning



$$\max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=1}^{\infty} \gamma^t r_t\right]$$

Multi-Agent Reinforcement Learning



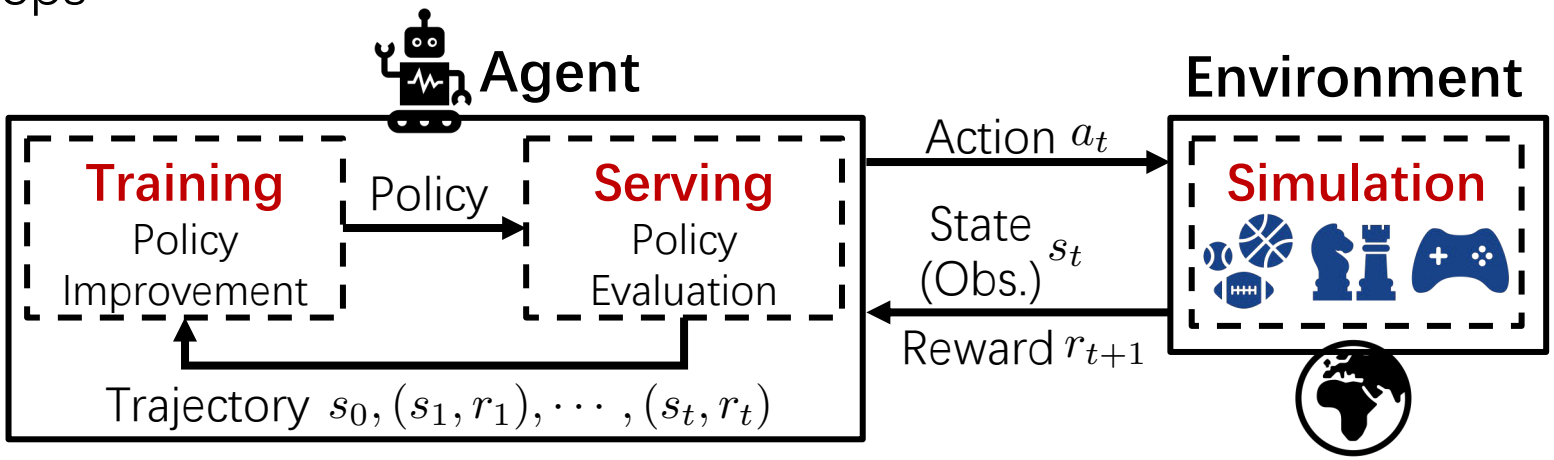
$$\max_{\pi_1} \mathbb{E}_{\pi_1,\pi_2}\left[\sum_{t=1}^{\infty} \gamma^t r_{1,t} \middle| P, \pi_2\right]$$

- Machine learning tasks are different in the experience available, the objective function, and the specific learning algorithms

Reinforcement Learning Settings

Agent interacts at discrete time steps

- ▶Observes state
- ▶Selects action.
- ▶Obtains immediate reward.
- ▶Observes resulting state.



- ▶Learning from interactions.
- ▶Learning how to map observations to actions.
- ▶Aim at maximize long-term discounted return.

Objective: $\max_{\pi} \mathbb{E}_{\pi} \left[\sum_{t=1}^{\infty} \gamma^t r_t \right]$

- ▶Key Features:
  - Learner is not told which action to take.
  - Delayed reward.
  - Exploration and exploitation.
  - Possibility of delayed reward
  - In between supervised and unsupervised learning

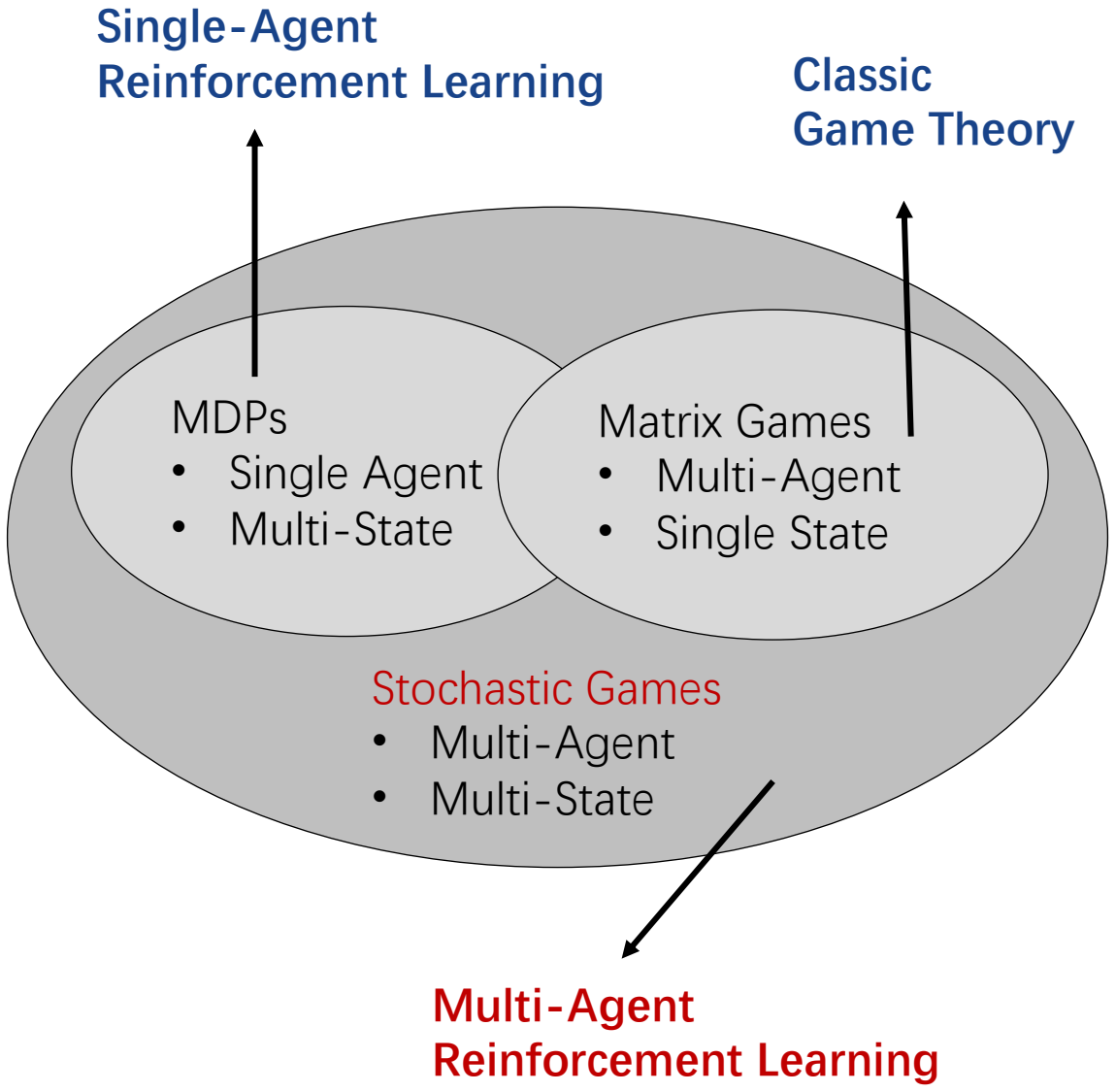
Multi-Agent in the Wild

- ►Games (Poker, Go, Shogi, StarCraft)
- ►Auctions (ad auctions, spectrum auctions)
- ►Online platforms (sharing economy, crowdsourcing)
- ►Cooperative interaction (language, advice, decision support)
- ►Resource allocation (packet routing, server allocation)

Multi-Agent System: **Agents** acting towards its **objectives** while all interacting in a shared **environment**.

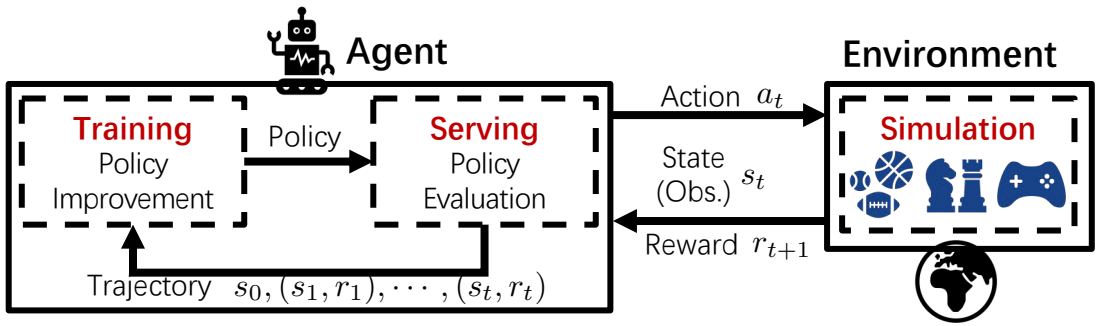
What Are We Studying?

- ►Markov decision processes
  - one decision maker
  - multiple states
- ►Matrix games / Repeated games
  - multiple decision makers
  - one state (e.g., one normal form game)
- ►Stochastic games (Markov games)
  - multiple decision makers
  - multiple states (e.g., multiple normal form games)



From Single-Agent to Multi-Agent RL

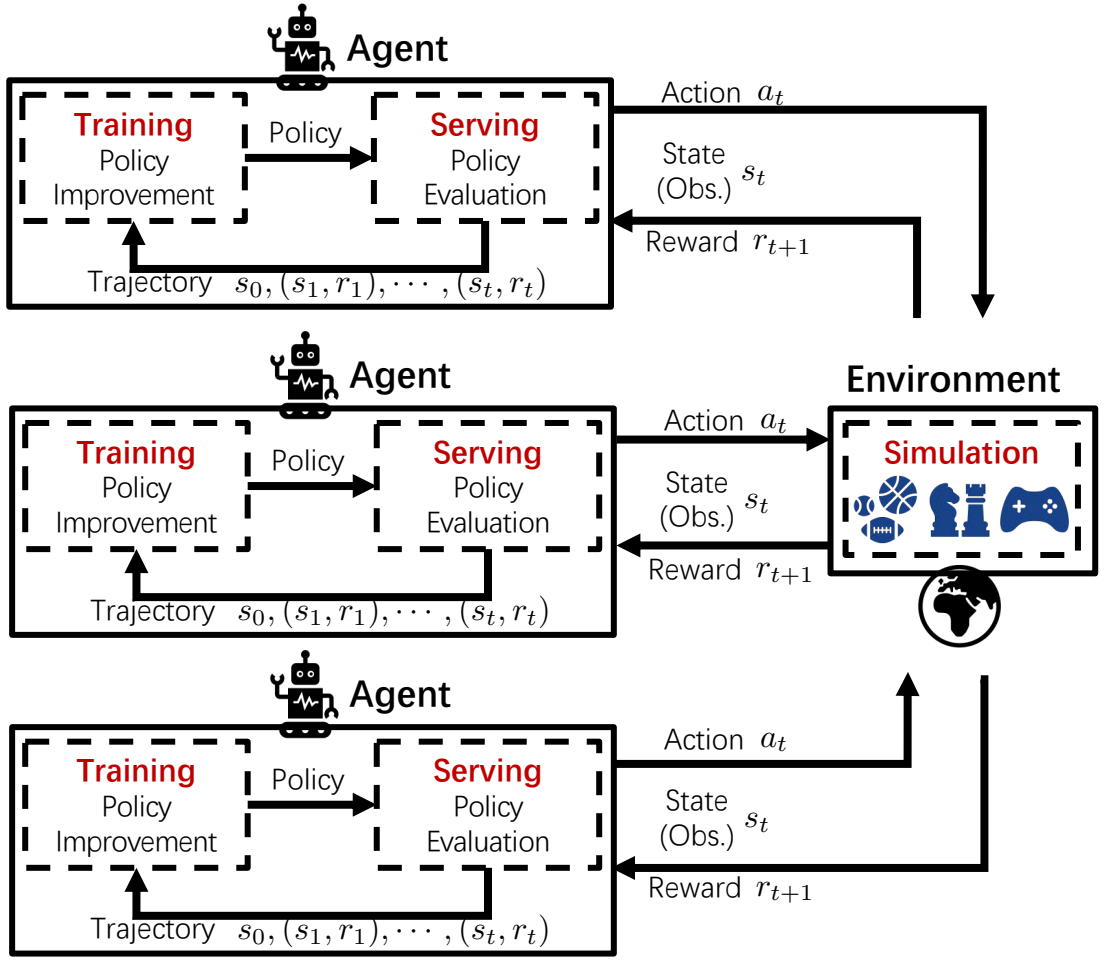
- Combination of state and joint action space: curse of dimensionality => Scalability
- Agents **interact** with the environment and each other
  - => Multi-objectives, Game Theory, Communication
  - Learning is simultaneous => Non-stationary



$$s' \sim P(s'|s, a)$$

Single-Agent

Usually formalized as Stochastic Game (SG), where $s' \sim P(s'|s, a_1, a_2, a_3 \ldots)$.

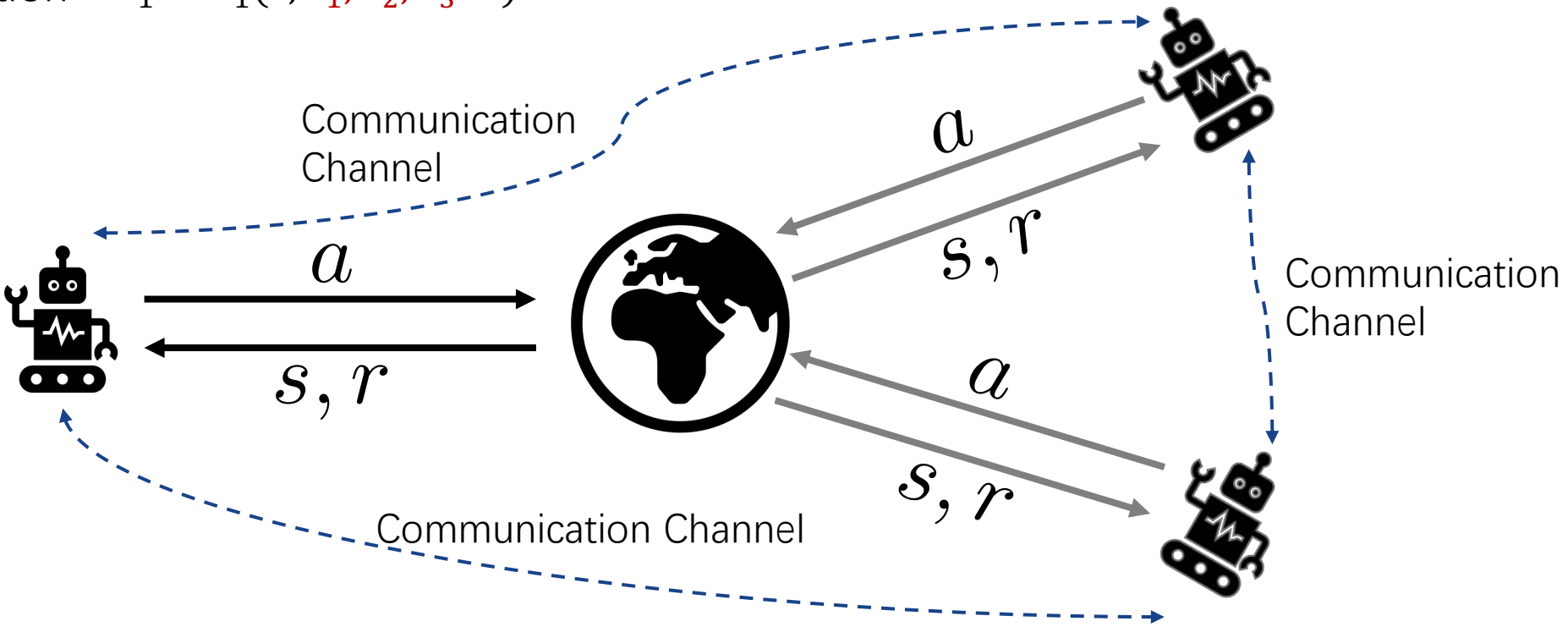


Multi-Agent

Direct and Indirect Interactions

► Direct Interaction
  - Communication

► Indirect Interaction via Environment
  - State joint action transition $s' \sim P(s'|s, a_1, a_2, a_3 \ldots)$
  - Reward function $r_1 = R_1(s, a_1, a_2, a_3 \ldots)$

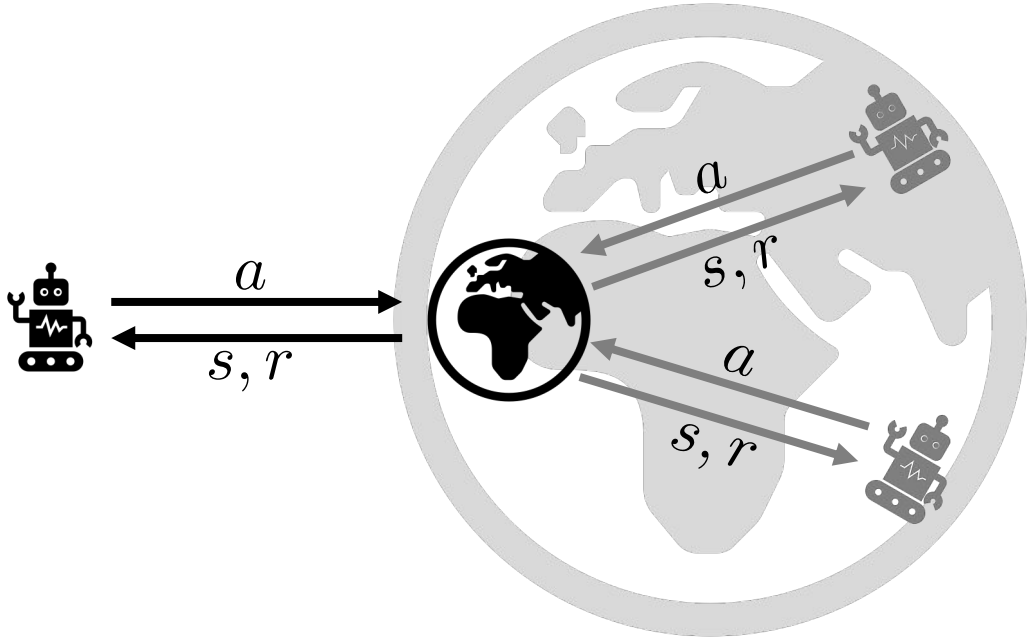


Interactions in a multi-agent system

Interaction Example: Agents in Traffic

Naïve Solution: Independent Learner

- ►Naive extension to multi-agent setting.
- ►Agents mutually ignore each other.
- ►Treat the interactions with other agents as noise.
- ►Downgrade to a **MDP**.



Why Multi-Agent Learning is Hard?

The agents are in a shared environment, they are mutually affected during the interactions, cannot only optimize an agent independently.

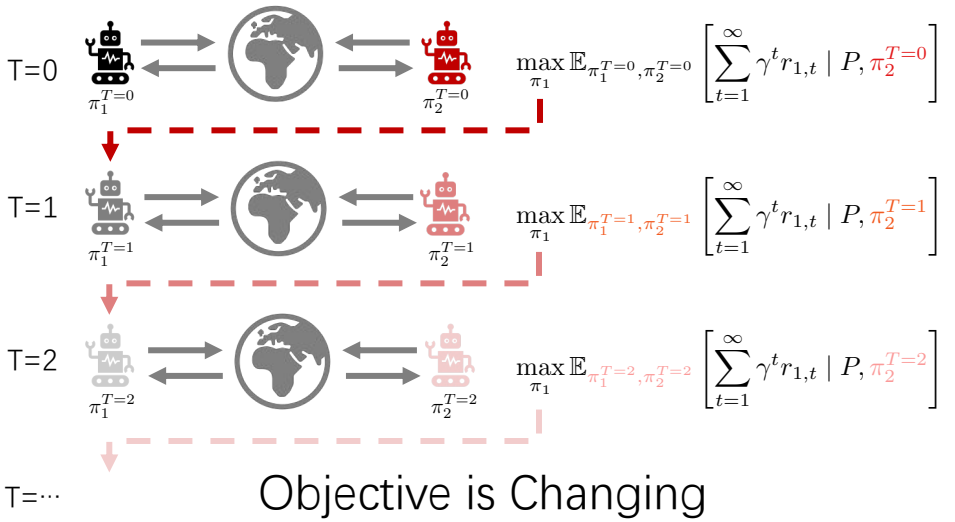
- The system is a moving target: during the learning, agents' policies are changing simultaneously.
- Need for coordination/compromise: agents behaviors have impacts on other agents (like merge).

Can we take the mutual influence or interactions into account to shape the learning?

The other agents are learning and updating its policy over time: $P^t(a_{-i}|s) \neq P^{t+\delta t}(a_{-i}|s)$

Therefore, for a single naive agent i, the environment it perceives is non-stationary:

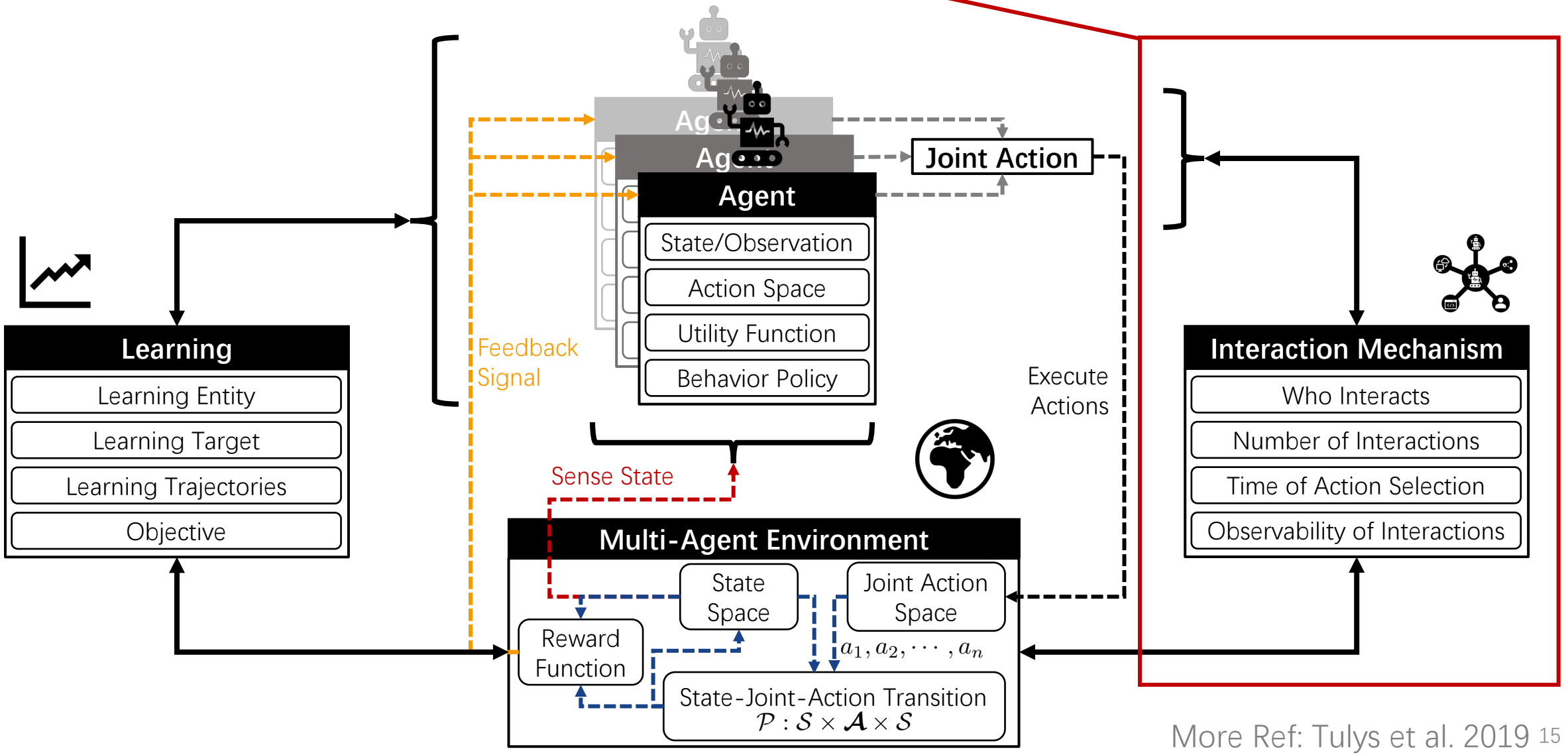
$$
\begin{aligned}
P_t(s'|s,a_i) &= \sum_{a_{-i}} P(s'|s,a_i,a_{-i}) P_t(a_{-i}|s) \\
&\neq \sum_{a_{-i}} P(s'|s,a_i,a_{-i}) P^{t+\delta t}(a_{-i}|s) \\
&= P^{t+\delta t}(s'|s,a_i).
\end{aligned}
$$



Objective is Changing

Why Multi-Agent Learning is Hard?

►How to manage additional interactions in MARL?



More Ref: Tulys et al. 2019

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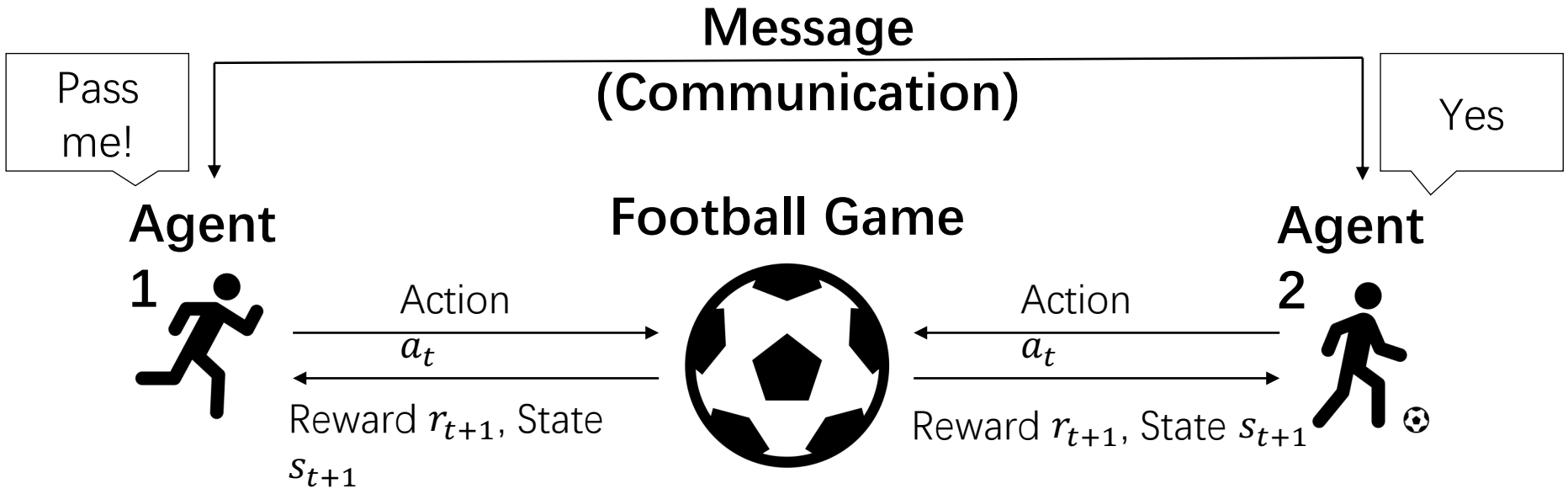
Direct Interaction Modelling – Emergent Communication

- AI requires the collaboration of multiple agents
- The communication between agents is vital to coordinate the behavior of each individual



Cooperate with Communication

Bidirectional-Coordinated Nets (BiCNet)

- ►Bi-directional recurrent networks
  - Means of communication
  - Connect each individual agent's policy and and Q networks.
- ►Multi-agent deterministic actor-critic.

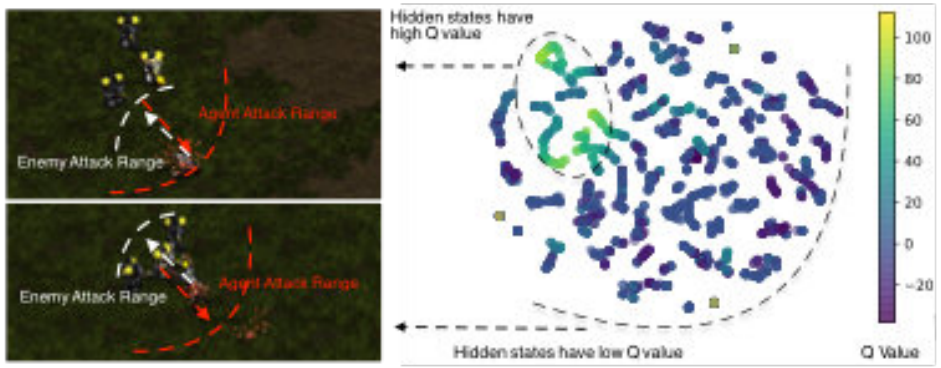
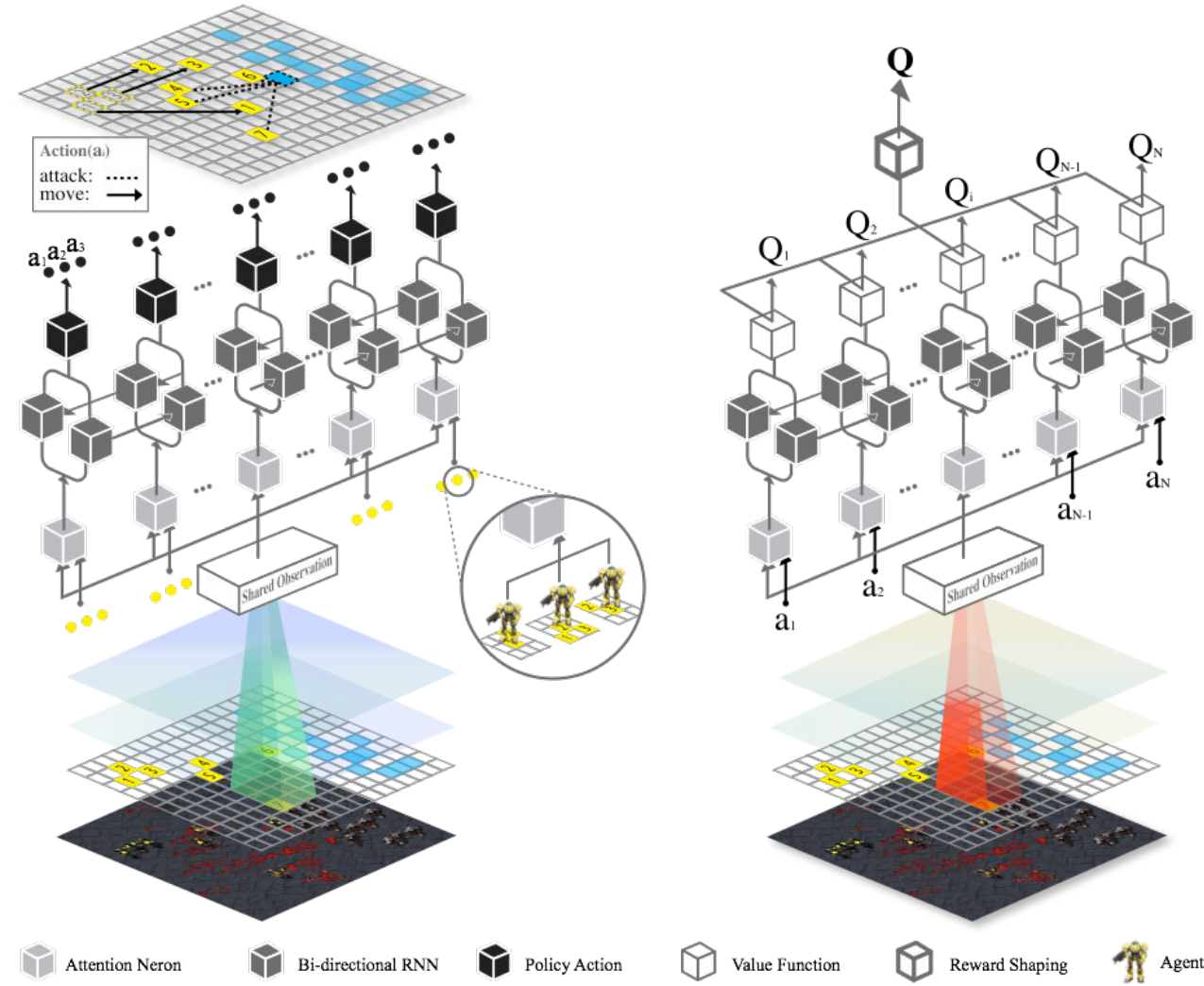


Figure 4: Visualisation for 3 Marines vs. 1 Super Zergling combat. **Upper Left**: State with high Q value; **Lower Left**: State with low Q value; **Right**: Visualisation of hidden layer outputs for each step using TSNE, coloured by Q values.

High Q-value steps are aggregated in the same area.



(a) Multiagent policy networks with grouping (b) Multiagent Q networks with reward shapin

[Peng, Peng*, and Wen, Ying*. 2017.]

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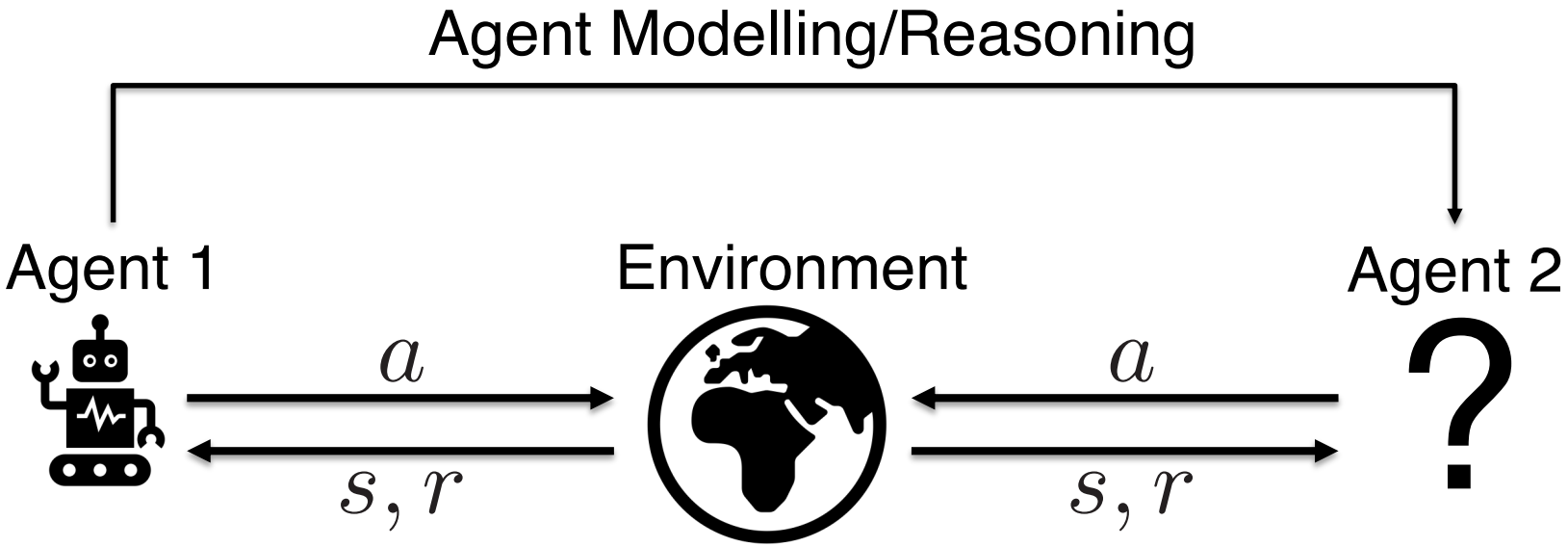
Indirect Interaction Modelling - Agent Modelling

Act based on the other agents' movement?

- But the problem is the other agents wouldn't do everything as you wish!
- Besides, the agent i's objective $\eta_i(\pi) = \mathbb{E}_{a_i, a_{-i} \sim \pi_i, \pi_{-i}}[r_i]$ doesn't explicitly include the other agents's policies:

Build a model for the other agents and include it in the learning objective!



Opponent-Aware Models

- Maintain a belief over environment state and the other agents' models (e.g., learning algorithms, observation, actions, their beliefs over other agents, etc.)

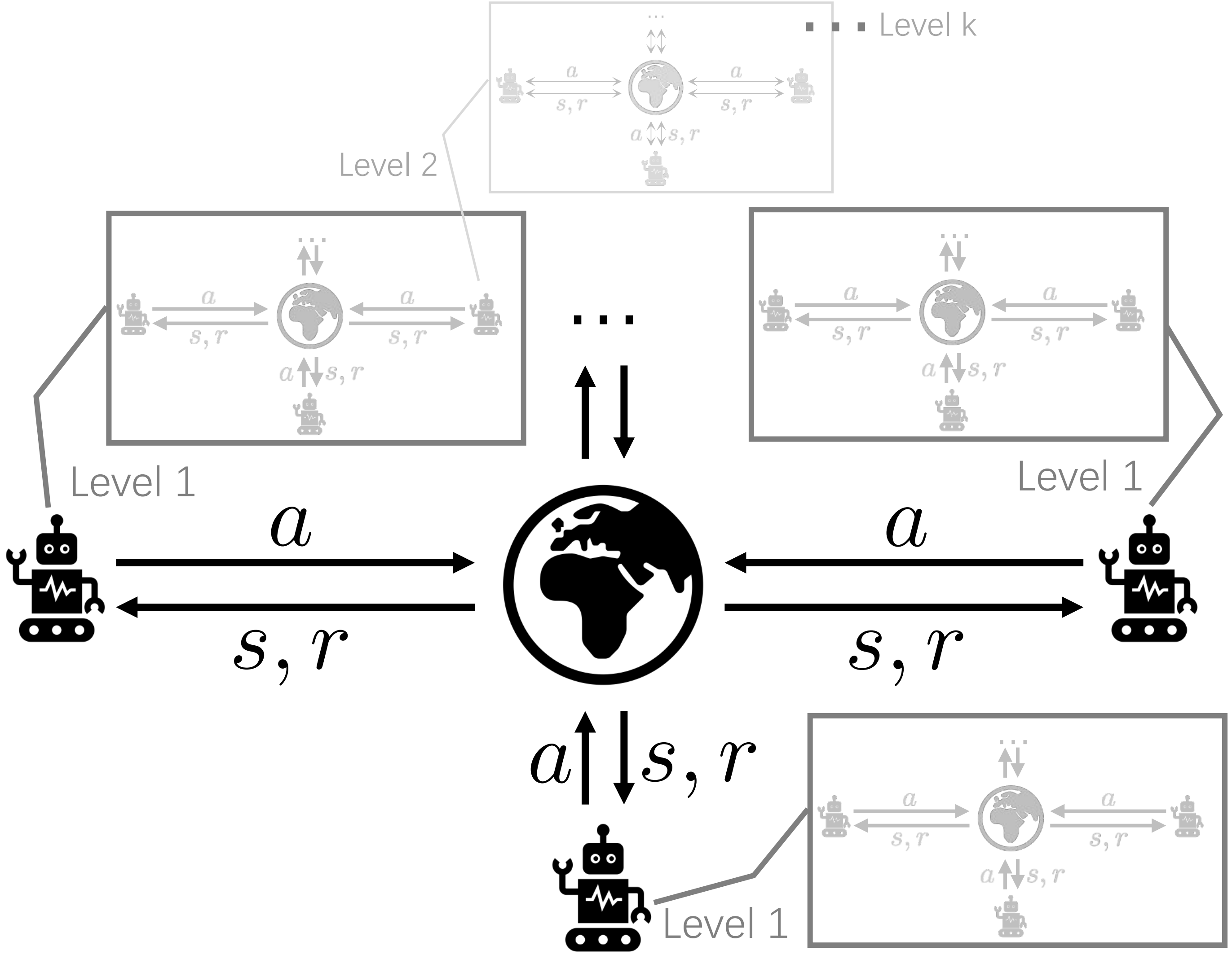
Observed interaction history (past actions, states, ...)

↓

Agent / Opponent Model

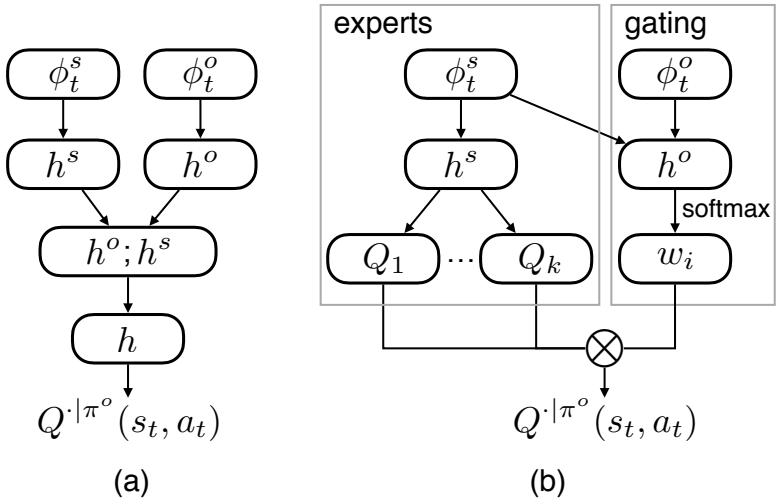
↓

Predicted property of interest (actions, class, goal, ...)



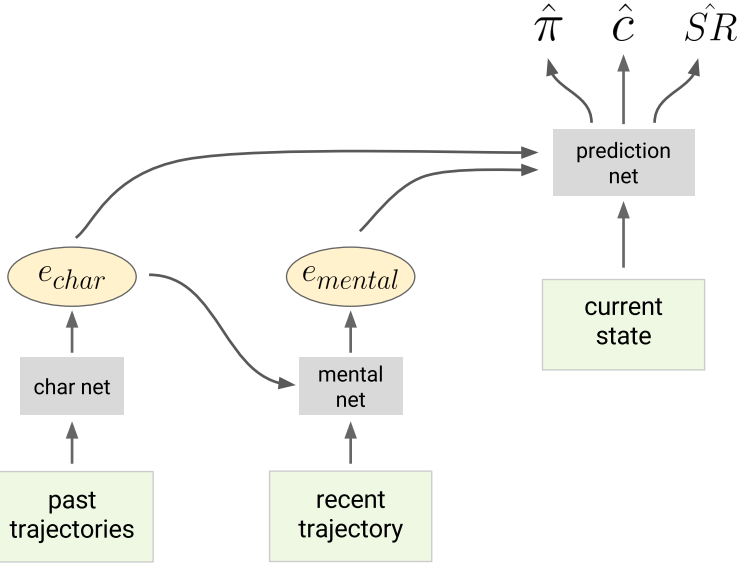
Idea: account for beliefs, models, and/or learning algorithms of other agents

Deep Reinforcement Opponent Networks



(a) DRON-concat: opponent representation is concatenated with the state representation.

(b) (b) DRON-MoE: Q-values predicted by K experts are combined linearly by weights from the gating network.

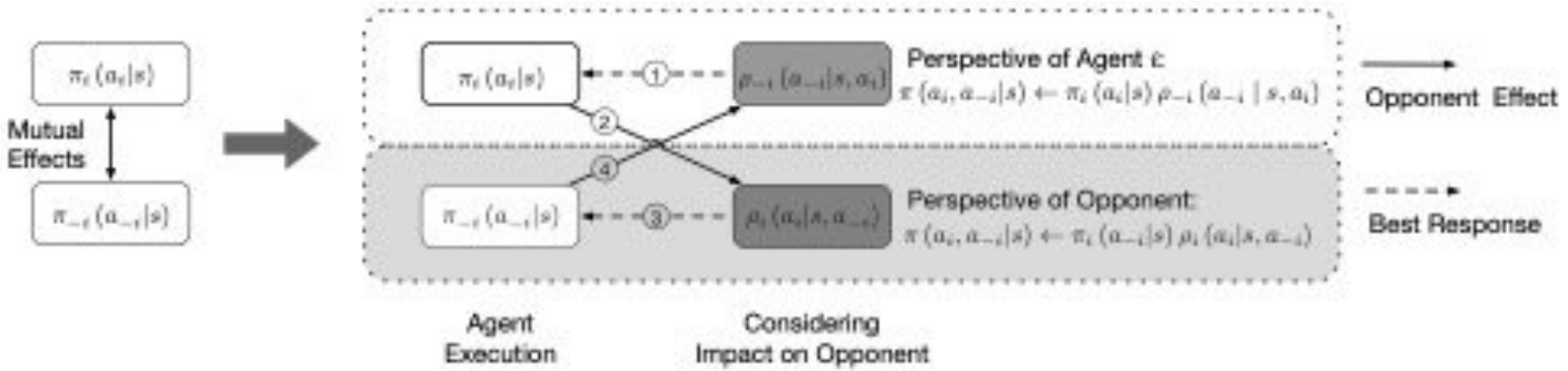


Theory of Mind neural network (*ToMnet*)

- The character net: parses trajectories to a character embedding, $\mathbf{e}_{\text{char}}$.
- The mental state net parses the trajectory on the current episode, to mental state, $\mathbf{e}_{\text{mental}}$.
- These embeddings are fed into the prediction net and outputs
  - next-step action probabilities,
  - probabilities of whether certain objects will be consumed
  - predicted successor representations

[He, He, et al. 2016, Raileanu, Roberta, et al. 2018. Rabinowitz, Neil C., et al. 2018,]

Mutual Influence as Recursive Reasoning



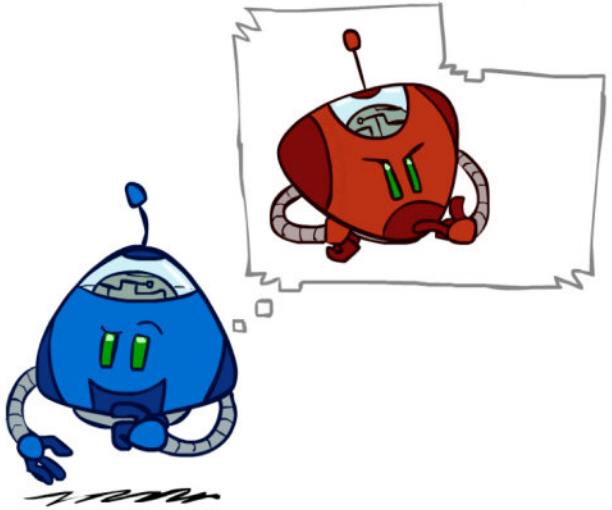
Probabilistic recursive reasoning framework. Recursive reasoning decouples the connections between agents. ①: agent i takes the best response after considering all the potential consequences of opponents' actions given its own action a_i. ②: how agent i behaves in the environment serves as the prior for the opponents to learn how their actions would affect a_i. ③: similar to ①, opponents take the best response to agent i. ④: similar to ②, opponents' actions are the prior knowledge to agent i on estimating how a_i will affect the opponents. Looping from step 1 to 4 forms recursive reasoning.

$$\pi(a_i, a_{-i}|s) = \underbrace{\pi_{-i}(a_{-i}|s, a_i)\pi_i(a_i|s)}_{\text{Perspective of Agent}i} = \underbrace{\pi_i(a_i|s, a_{-i})\pi_{-i}(a_{-i}|s)}_{\text{Perspective of Agent}-i}.$$

$$\begin{aligned}
\pi\left(a_i, a_{-i} \mid s\right) &= \pi_{-i}\left(a_{-i} \mid s, a_i\right) \pi_i\left(a_i \mid s\right) \rightarrow \text{ Correlated, Perspective of Agent } i \\
&= \pi_i\left(a_i \mid s, a_{-i}\right) \pi_{-i}\left(a_{-i} \mid s\right) \rightarrow \text{ Correlated, Perspective of Agent } -i \\
&\approx \pi_i\left(a_i \mid s\right) \pi_{-i}\left(a_{-i} \mid s\right) \rightarrow \text{ Non-correlated}
\end{aligned}$$

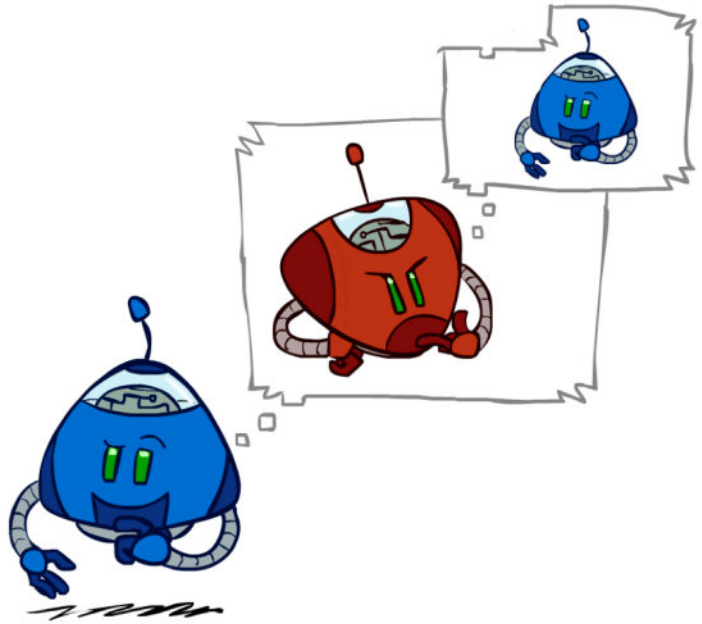
[Wen, Ying*, Yang, Yaodong*. et al. 2019.]

Recursive Reasoning Levels



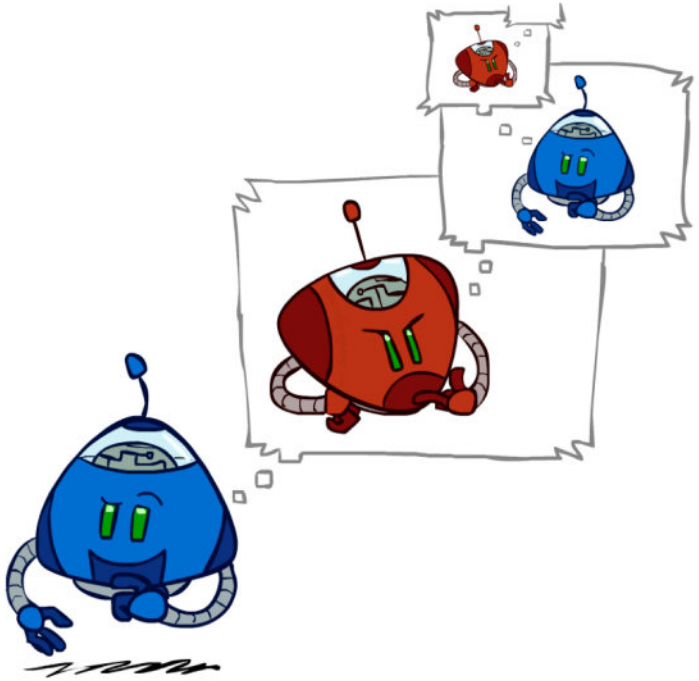
Source: CS 188, Berkeley.

Level 1



Source: CS 188, Berkeley.

Level 2

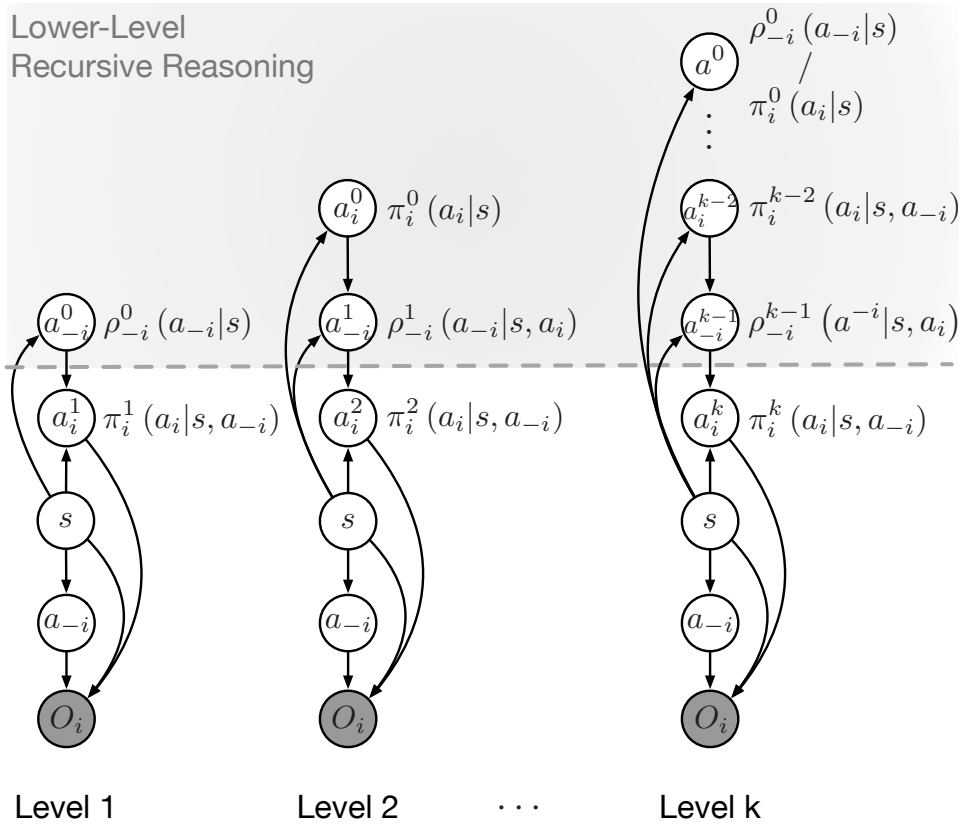


Source: CS 188, Berkeley.

Level 3

Level-k Recursive Reasoning

Agent i rolls out the recursive reasoning about opponents in its mind (grey area). In the recursion, agents with higher-level beliefs take the best response to the lower-level thinkers' actions. Higher-level models would conduct all the computations that the lower-level models have done, e.g. level-2 model contains level-1 model by integrating out $\pi_{i,0}(a_i|s)$.



Graphical model of the level-k reasoning model.

[Wen, Ying*, Yang, Yaodong*. et al. 2020.]

More Agent Modeling Work

►More comprehensive survey about opponent modeling:

- Albrecht, Stefano V., and Peter Stone. "**Autonomous agents modelling other agents: A comprehensive survey and open problems**." Artificial Intelligence 258 (2018): 66-95.

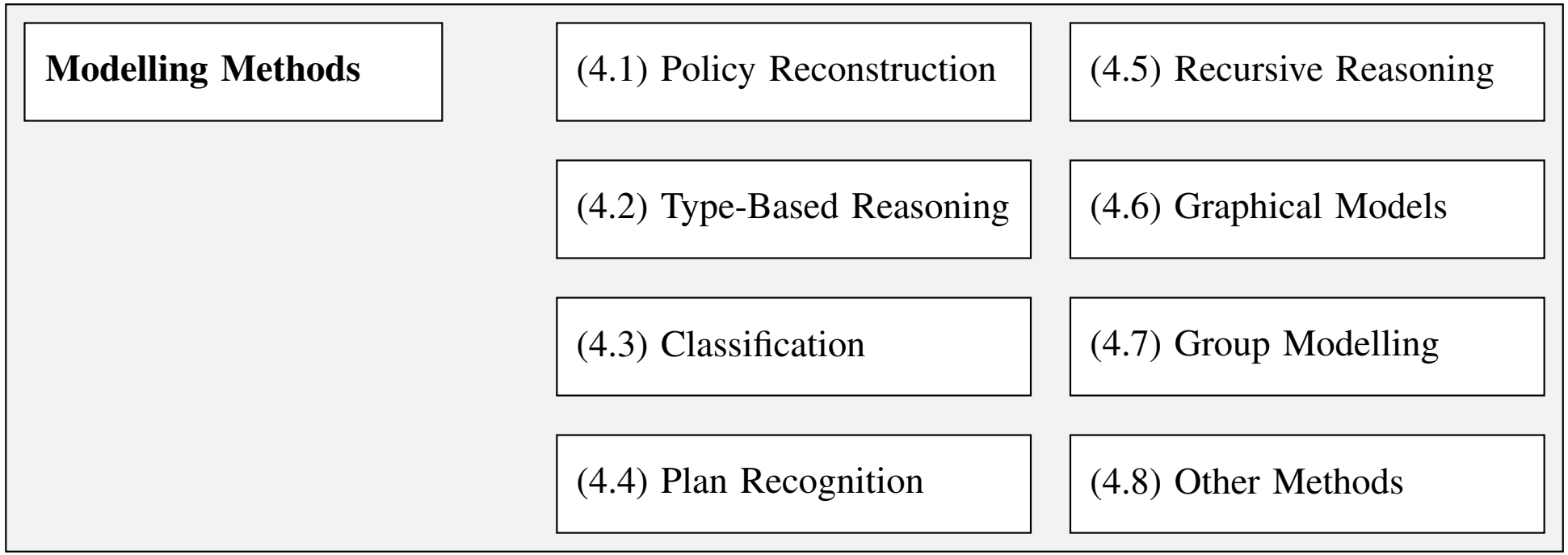


Figure 2: Surveyed modelling methods. Brackets show linked section numbers.

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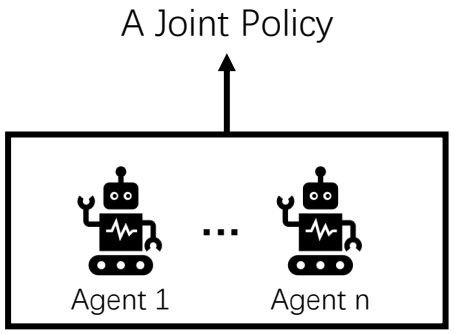
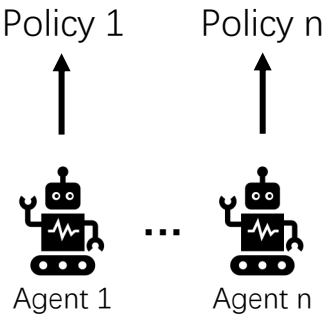
Implicit Interaction Modelling via Centralized Critic



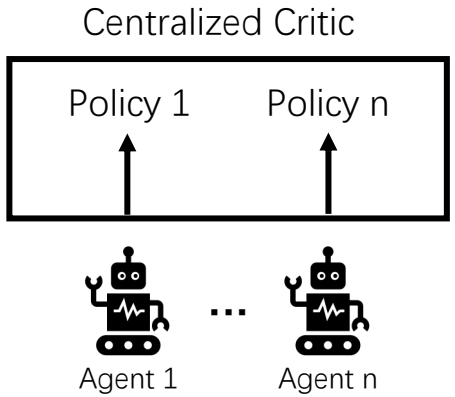
Fully Centralized Controller

Scalability ✗



Fully Decentralized Controllers

Non-stationarity
Credit assignment ✗



Centralized Critic with Decentralized Policy

Centralized Training with Decentralized Execution ✓

How to learn joint policy $\boldsymbol{\pi}$ or value function Q ?

Multi-Agent Deep Deterministic Policy Gradient

- If consider N continuous policies μ_i w.r.t. parameters θ_i, the gradient can be written as

$$\nabla_{\theta_i} J(\boldsymbol{\mu}_i) = \mathbb{E}_{\mathbf{x},a\sim\mathcal{D}}[\nabla_{\theta_i}\boldsymbol{\mu}_i(a_i|o_i)\nabla_{a_i} Q_i^{\boldsymbol{\mu}}(\mathbf{x}, a_1, ..., a_N)|_{a_i=\boldsymbol{\mu}_i(o_i)}]$$

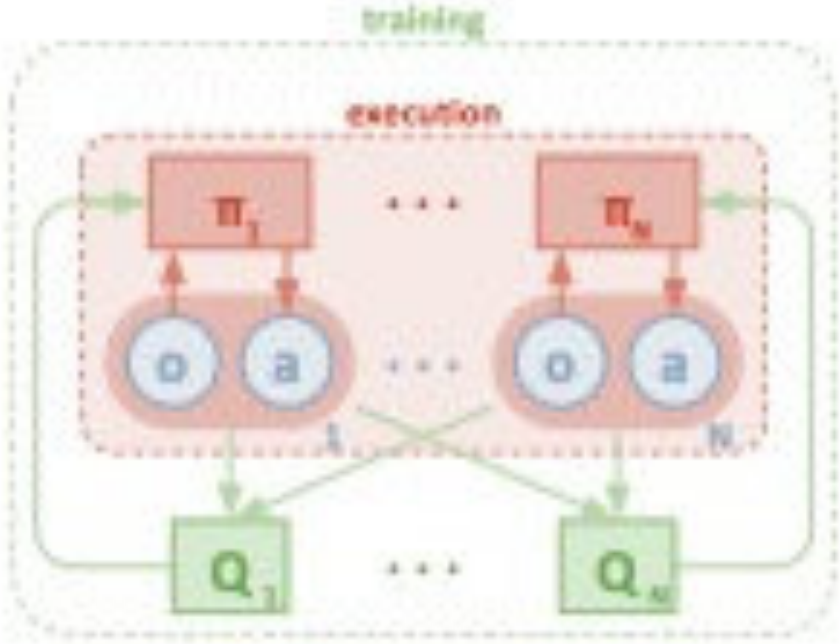
- The Q-function can be optimized by loss function:

$$\mathcal{L}(\varphi_i) = \mathbb{E}_{\mathbf{x},a,r,\mathbf{x}'}[(Q_i^{\boldsymbol{\mu}}(\mathbf{x}, a_1, \ldots, a_N) - y)^2], \quad y = r_i + \gamma\, Q_i^{\boldsymbol{\mu}'}(\mathbf{x}', a'$$

- Decentralized actors (policies, π) to keep the self-play framework.
- Centralized critic (baseline, Q) to ease learning and reduce variance

- Robust Extension : MiniMax Multi-agent Deep Deterministic Policy Gradient (M3DDPG)
  - Minimax objective: introduce minimax concept into deep MARL
  - Key idea: replace the inner-loop minimization by a one-step gradient descent

$$\nabla_{\theta_i} J(\theta_i) = \left[\begin{array}{c} \nabla_{\theta_i}\mu_i(o_i)\,\nabla_{a_i} Q^{\mu}_{\mathrm{M},i}(\mathrm{x}, a_1^\star, \ldots, a_i, \ldots a_N^\star) \mid \\ a_i = \mu_i(o_i) \\ a_j^\star = a_j + \hat{\epsilon}_j, \quad \forall j \neq i \\ \hat{\epsilon}_j = -\alpha_j \nabla_{a_j} Q^{\mu}_{\mathrm{M},i}(\mathrm{x}, a_1, \ldots, a_N) \end{array}\right]$$



Centralized training and decentralized execution

[Lowe, Ryan, et al. 2017. Li, Shihui, et al. 2019.]

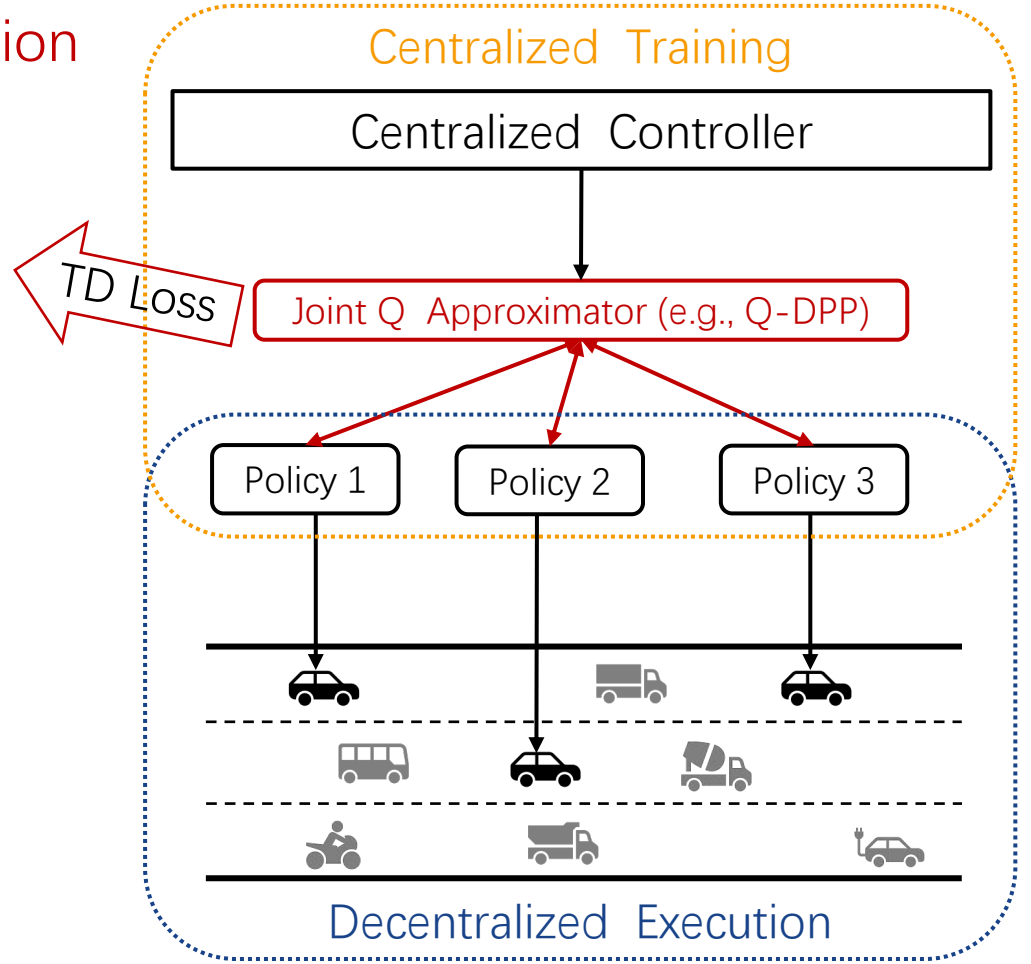
Centralized Critic Decomposition Methods

- Setting: Dec-POMDP
- Paradigm: Centralized training with decentralized execution

$$\mathcal{L}(\boldsymbol{\theta}) = \mathbb{E}_{(\boldsymbol{\tau},\boldsymbol{a},r,\boldsymbol{\tau}')\in D}\left[\left(r + \gamma V\left(\tau';\boldsymbol{\theta}^{-}\right) - Q(\boldsymbol{\tau},\boldsymbol{a};\boldsymbol{\theta})\right)^2\right]$$

$$V\left(\tau';\theta^{-}\right) = \max_{a'} Q\left(\tau',a';\theta^{-}\right)$$

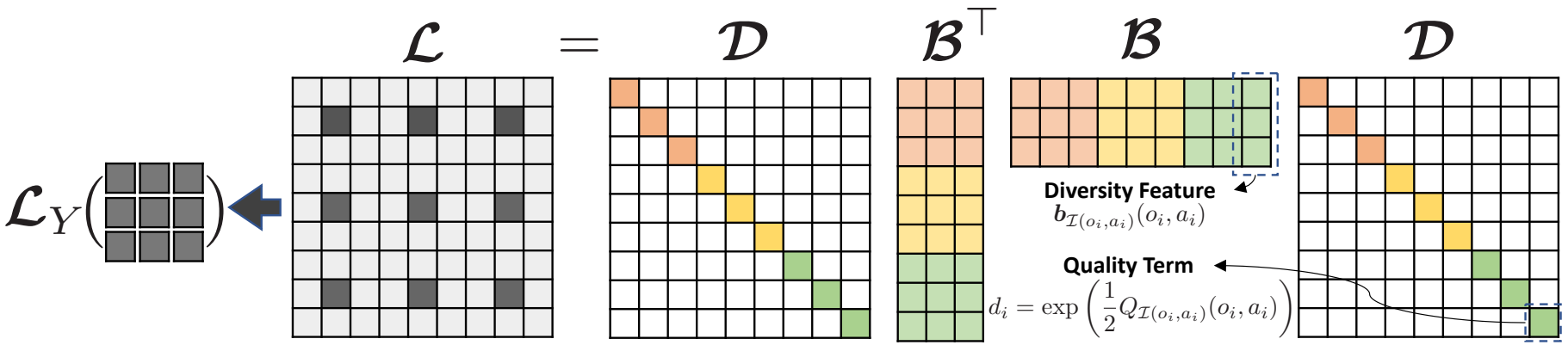
- Individual-Global Maximization (IGM) Principle
  - Consistent action selection between joint and individuals
  - $$\arg\max_{\boldsymbol{a}} Q^{\pi}(s,\boldsymbol{a}) = \left\{\arg\max_{a_i} Q_i\left(s,a_i\right)\right\}_{i\in\mathcal{N}}$$
- Cooperative multi-agent RL shares a single joint reward signal
  - Need to learn to decompose the team value function into agent-wise value functions.
- Although learning requires some centralization, the learned agents can be deployed independently.



Q-DPP: A Function Approximator for Cooperative MARL

The DPP kernel $\boldsymbol{\mathcal{L}}$ can represent the joint Q-function $Q^{\boldsymbol{\pi}}(\boldsymbol{o}, \boldsymbol{a})$ with both quality (maximizing reward) & diversity (different behaviors).

$$
\begin{aligned}
Q^{\boldsymbol{\pi}}(\boldsymbol{o}, \boldsymbol{a}) :&= \operatorname{logdet}\left(\boldsymbol{\mathcal{L}}_{Y=\left\{\left(o_1^t, a_1\right), \ldots, \left(o_N^t, a_N\right)\right\} \in \mathcal{C}\left(\boldsymbol{o}^t\right)} = \boldsymbol{\mathcal{W}}_Y \boldsymbol{\mathcal{W}}_Y^{\top}\right) \\
&= \log\left(\operatorname{tr}\left(\boldsymbol{\mathcal{D}}_Y^{\top} \boldsymbol{\mathcal{D}}_Y\right) \det\left(\boldsymbol{\mathcal{B}}_Y^{\top} \boldsymbol{\mathcal{B}}_Y\right)\right) \qquad \text{Quality-Diversity Decomposition} \\
&= \sum_{i=1}^{N} \underbrace{Q_{\mathcal{I}\left(o_i^t, a_i\right)}\left(o_i, a_i\right)}_{\text{Quality}} + \underbrace{\operatorname{logdet}\left(\boldsymbol{\mathcal{B}}_Y^{\top} \boldsymbol{\mathcal{B}}_Y\right)}_{\text{Diversity}}
\end{aligned}
\tag{3}
$$



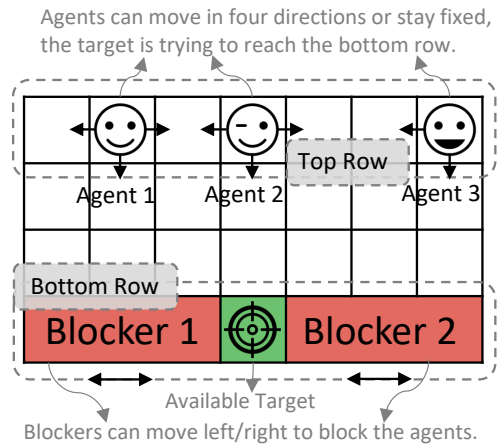
Example of Q-DPP with quality-diversity kernel decomposition in a single-state three-player learning task.

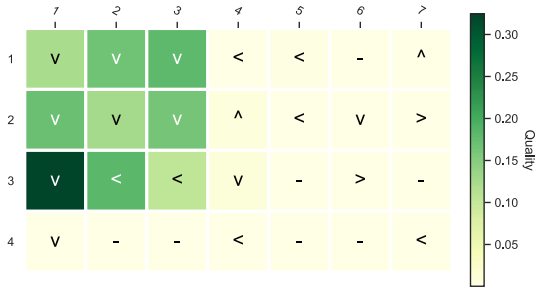
[Yang, Yaodong*; Wen, Ying*, et al. 2020.]

Q-DPP Performance

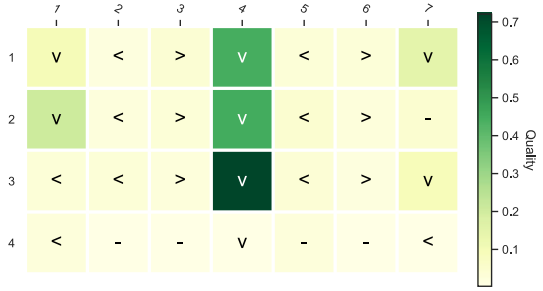
The acquired diverse behaviors on Blocker Game.

- The optimal action of one agent doesn't depend on the others.
- The behaviors acquired by different agents are orthogonal.
- Q-DPP meets the decentralizability assumption naturally.

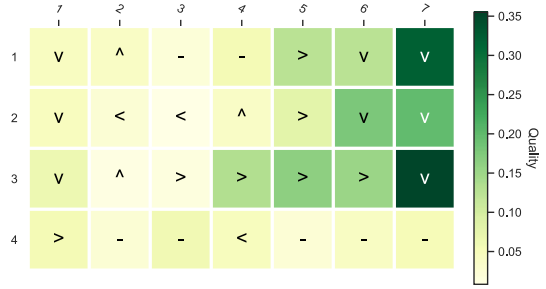




Agent 1



Agent 2

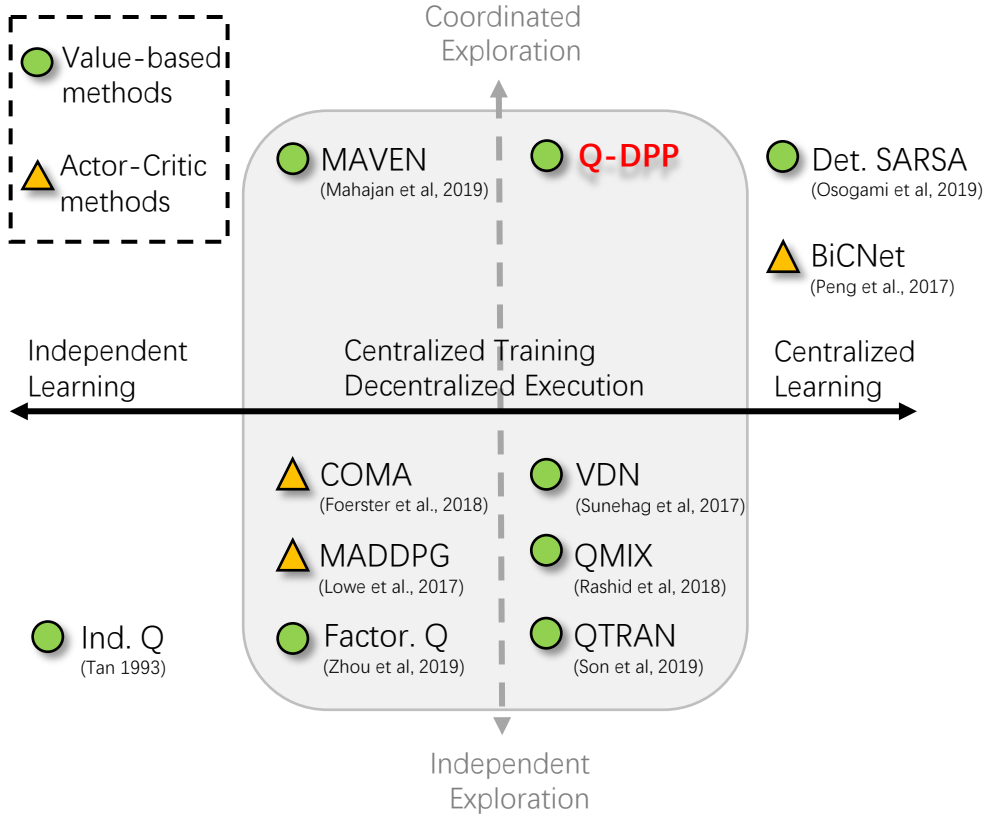


Agent 3

Learned state values ($\text{argmax}_{a_i} Q(s, a_i)$) for each agent.

Centralized Critic Decomposition Methods

- ►Value-Based Methods
  - Paradigm: centralized training with decentralized execution.
  - Methods: VDN, QMIX, QTRAN, MAVEN, QPLEX, NDQ, ROMA...
- ►Continuous actions extensions: COVDN and COMIX. VDN and QMIX respectively.
- ►Actor-Critic Methods
  - Paradigm: centralized critic and decentralized actors.
  - DOP: off-policy decomposed multi-agent policy gradient.
- ►Question:
  - Can we exploit the implicit structures in the centralized critic to boost the learning?

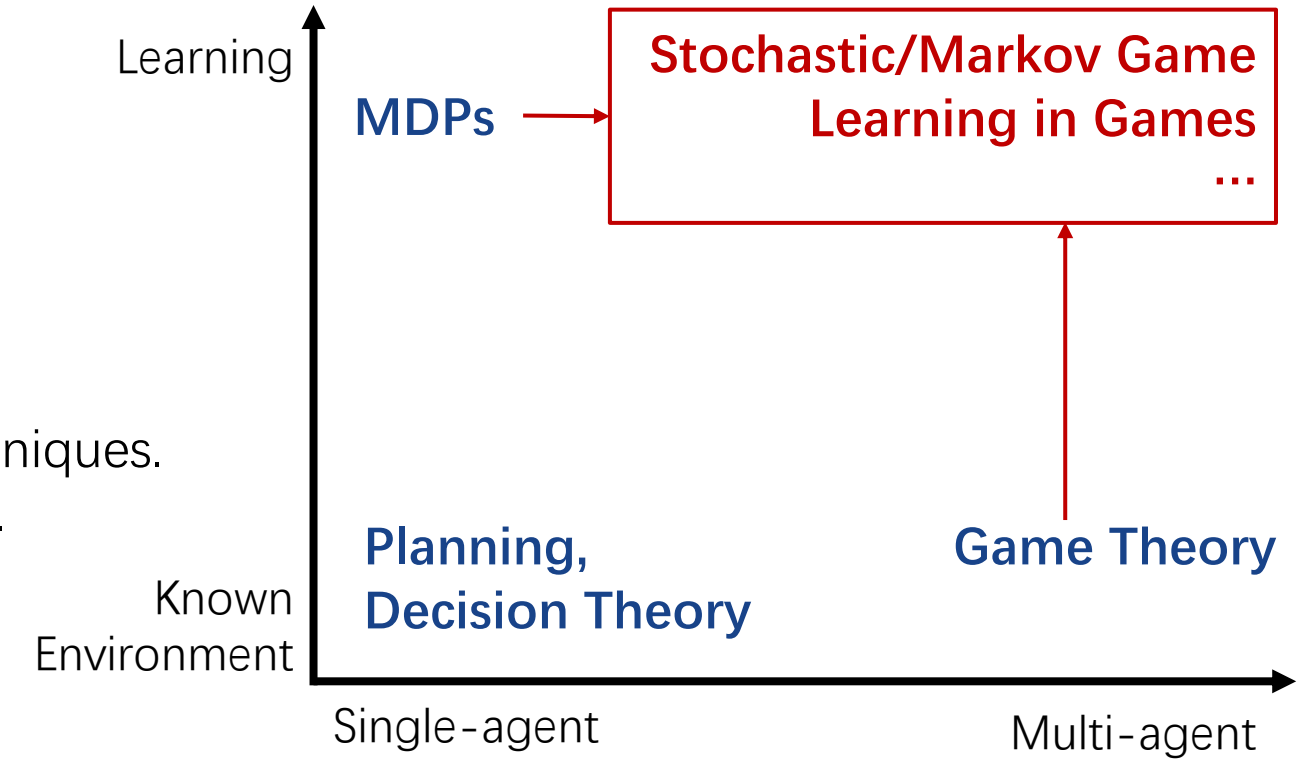


Agenda

1. Introduction
2. Direct Interaction Modelling - Emergent Communication
3. Indirect Interaction Modelling
   a. Agent Modelling
   b. Implicit Interaction Modelling via Centralized Critic
   c. **Explicit Policy Interaction Modelling via Policy-Space Analysis**
4. Challenges and Perspectives X Quantum Theory

Where is Game Theory – Explicit Policy Analysis

- ►Games Theory
  - Modeling the interactions.
- ►Reinforcement Learning
  - Learning decision making.
- ►Some successful combinations:
  - Empirical game theoretic techniques.
  - Policy space response oracles.
  - …



Can we use the game theory to guide
the multi-agent reinforcement learning?

Changing Policies' Relationship

Improvement Against Fixed Opponent Policy

$$\eta_i(\pi_i, \pi_{-i}) \overset{\leq}{\longrightarrow} \eta_i(\pi'_i, \pi_{-i}) \quad \eta_{-i}(\pi_i, \pi'_{-i}) \overset{\geq}{\longleftarrow} \eta_{-i}(\pi_i, \pi_{-i})$$

Unknown Improvement Against Opponent's Simultaneous Learning

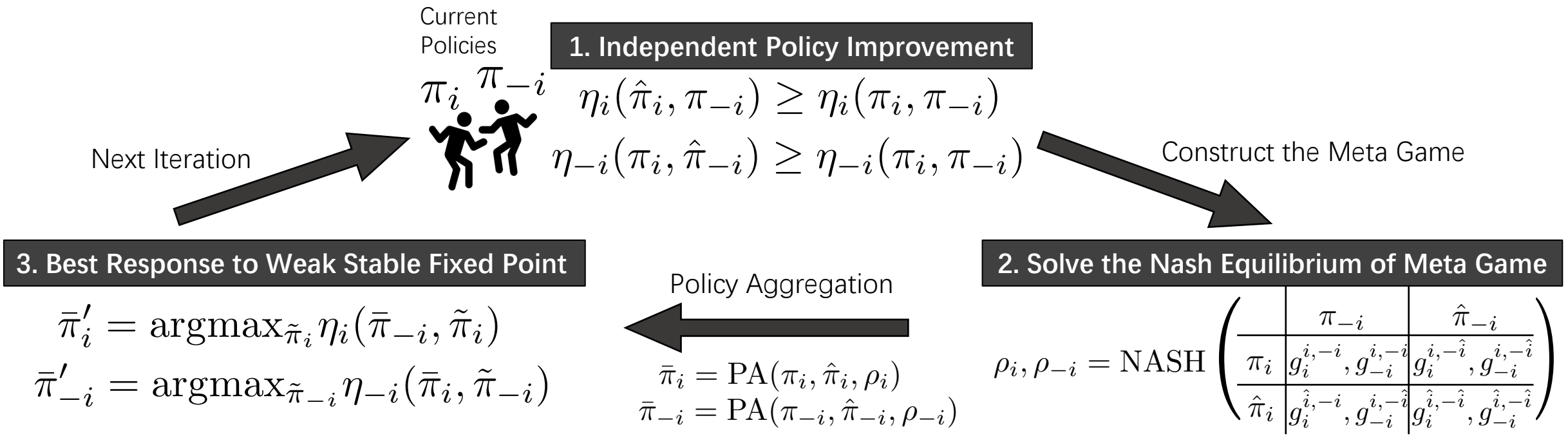
$$? \quad \eta_i(\pi'_i, \pi'_{-i}) \quad \eta_{-i}(\pi'_i, \pi'_{-i}) \quad ?$$

The relationship of discounted returns η_i for an agent i given the different joint policy pairs, where π_i is the current policy, π'_i is the simultaneously updated policy.

Can we allows agents to consider the policy-space mutual influence and shape the learning direction which gives stable improvement guarantee?

Wen, Ying; Chen, Hui; Yang, Yaodong; Tian, Zheng; Li, Minne; Chen, Xu; Wang, Jun, Multi-Agent Trust Region Learning.

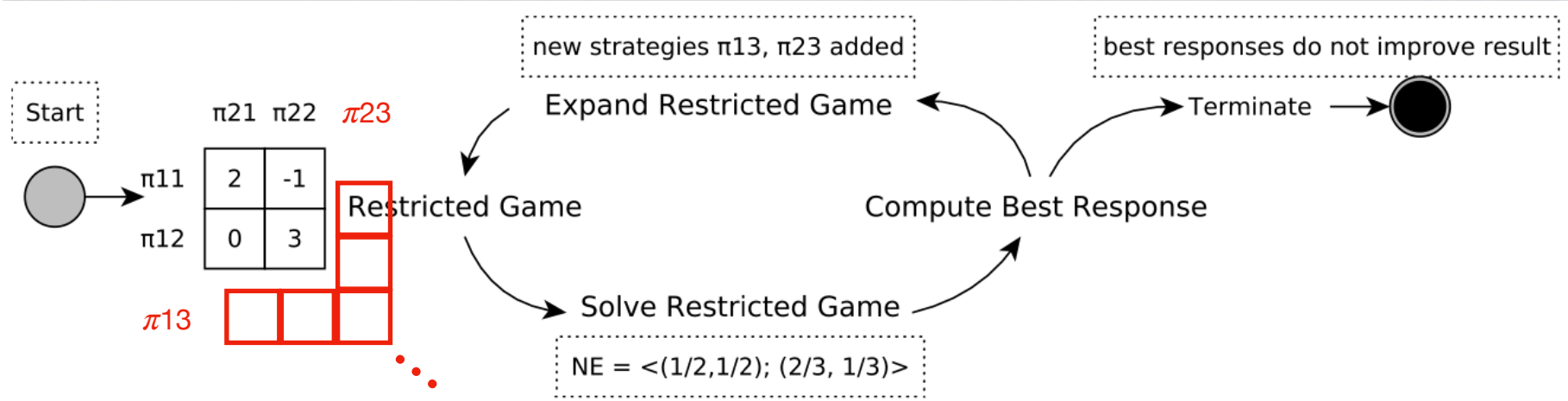
Multi-Agent Trust Region Learning



Overview of the multi-agent trust region learning phases in two-agent games. It can be easily extended to n-agent case by solving the n-agent two-action matrix form meta game.

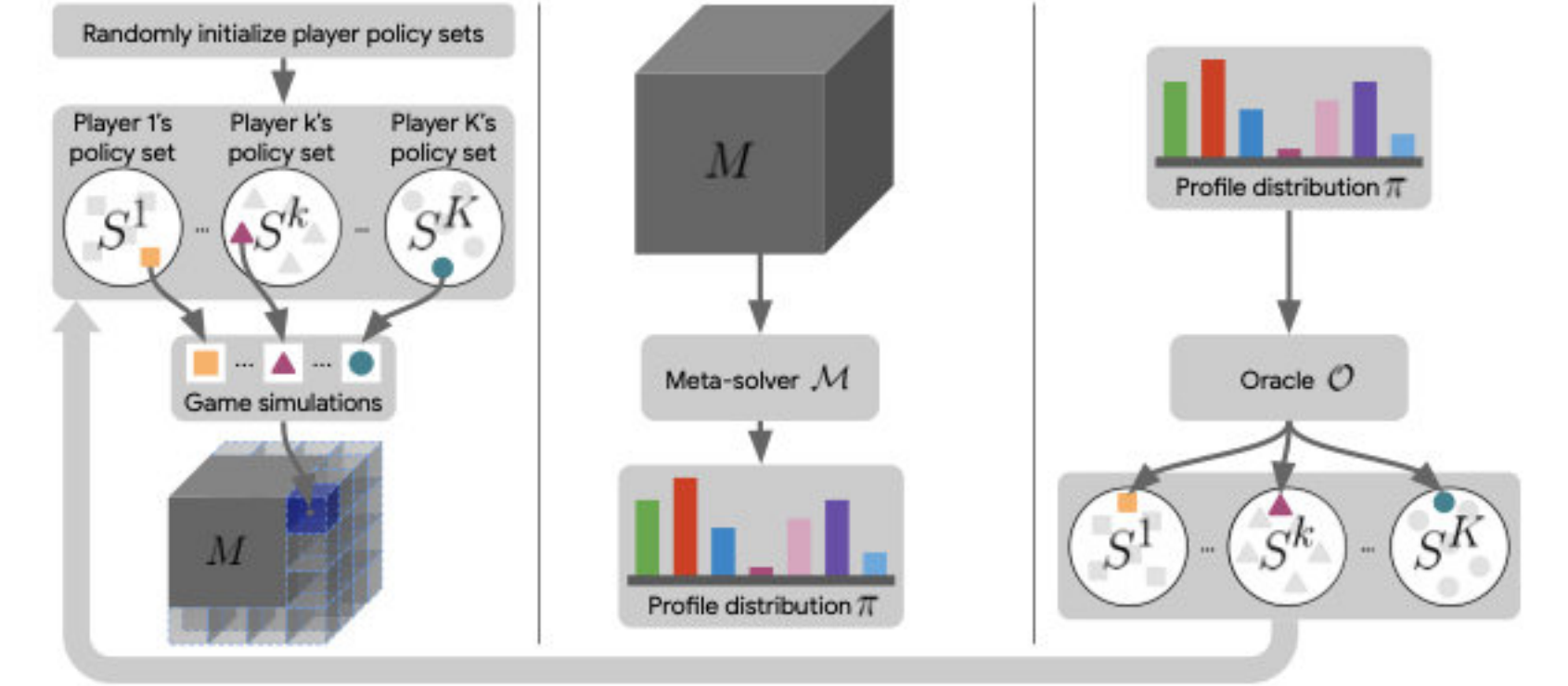
Double Oracle

- Double Oracle is also an iterated best response method, the difference is that, it best responds to the opponent’ s Nash equilibrium at each iteration.
- If the newly added best response is already in the strategy pool, then terminate. Why “oracle” : the search for a best response is guided by an oracle. It guarantees to converge to minimax equilibrium in finite games.



[Lanctot, Marc, et al. 2017.]

PSRO Algorithm Phases



(a) Complete: compute missing payoff tensor M entries via game simulations.

(b) Solve: given the updated payoff tensor M, calculate meta-strategy π via meta-solver $\mathcal{M}$.

(c) Expand: append a new policy to each player's policy space using the oracle $\mathcal{O}$.

[Muller, Paul, et al. 2019.]

Agenda

1. Introduction
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    a. Opponent Modelling
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4. Challenges and Perspectives X Quantum Theory

From Fixed-Play to Open-Ended-Play

Policy Space Distribution

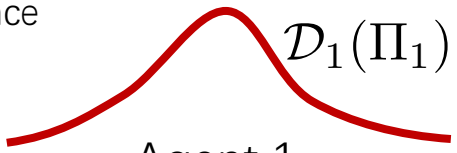
Π_1 Policy Space of Agent i

$\pi_1 \in \Pi_1$ A Policy Instance

$\pi_1 \sim \mathcal{D}_1(\Pi_1)$

Sample a Policy According to the Policy Space Distribution

$\mathcal{D}_i(\Pi_i)$

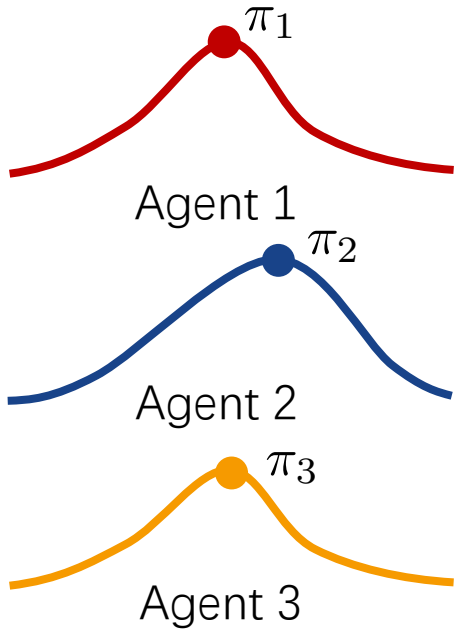


$\pi_2 \in \Pi_2$

$\pi_2 \sim \mathcal{D}_2(\Pi_2)$

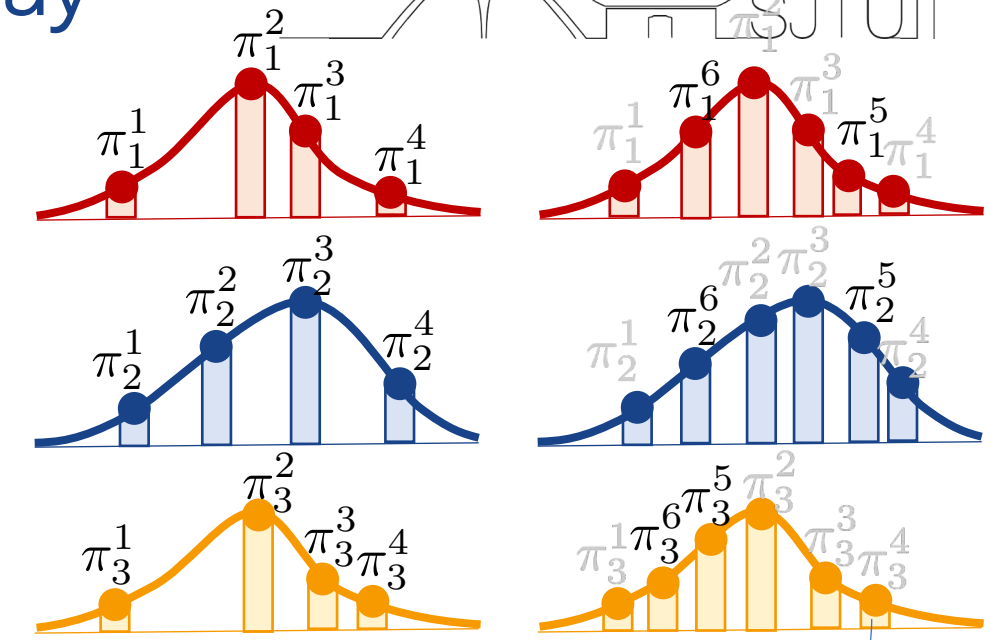
$\pi_3 \in \Pi_3$

$\pi_3 \sim \mathcal{D}_3(\Pi_3)$



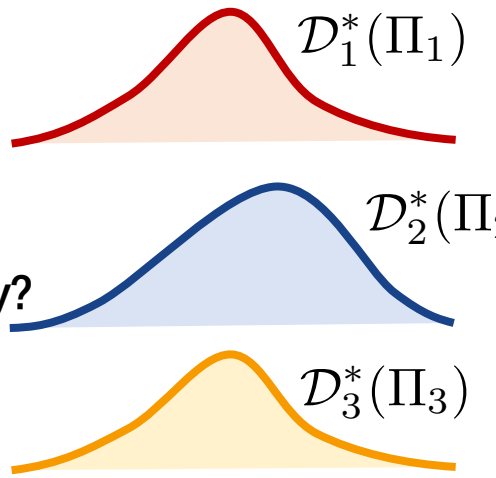
Co-Play/Fixed-Play

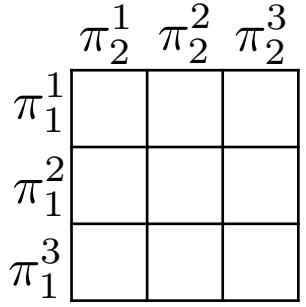
Deterministic distribution



Open-Ended-Play: Eval + Expand

Discrete distribution





Meta-game Analysis

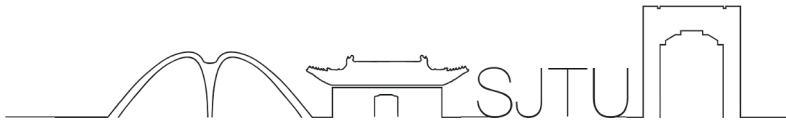
Mixed Policy-Space Strategy

Continuous distribution

Problems:

1. **It is hard to evaluate the policy pool.**
2. **Can we have a continuous distribution to sample the policy?**
3. **Better policy representation?**
4. **How to introduce the prior to select the policy?**

量子力学与多智能体学习



►直接引入，量子博弈，扩张策略集，助力博弈分析

►连续策略空间的建模和优化
- 问题：策略空间过于巨大，无法直接优化
- 方案：通过量子/贝叶斯神经网络构建策略网络，使直接优化策略空间策略的学习变成可能。

►策略空间博弈分析：可以根据经验元博弈构建一个策略交互图的概率转移矩阵
- 问题：一般随机游走求解策略的平稳分布随着策略池大小，求解难度呈指数被增长
- 方案：通过量子游走直接并行线性求解策略交互图的平稳分布

►一般现实博弈中，尤其是竞争博弈，一个纯策略总是有反制策略
- 问题：纯策略在现实中不够鲁棒
- 方案：能否构建量子混合态策略，根据对手的行为进行观测（引入prior？），坍缩成一个纯策略。
- 例如：比如王者荣耀，不同英雄组合，根据阵容及对方行为，采取不同的策略。

Thanks for Listening!

Q&A

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This slides is available at: github.com/ying-wen/dai-slides

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