Procurement with Cost and Non-Cost Attributes: Cost-Sharing Mechanisms

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A buyer faces a two-dimensional mechanism design problem for awarding a project to one among a set of contractors, each of whom is privately informed about his cost and his estimate of an a priori random non-cost attribute. The winning contractor realizes his non-cost attribute upon the project's completion and may "manipulate" it in a costless manner (if such a manipulation is beneficial to him). The non-cost attribute inflicts a disutility cost on the buyer. This procurement problem arises in situations such as highway construction projects, where completion times are a major concern. We establish the significance of incorporating the possibility of manipulation in two ways: (1) Using an optimal mechanism obtained by ignoring the possibility of manipulation can generate perverse incentives for the winning contractor to engage in manipulation. (2) The privacy of the non-cost estimates can generate information rent *only* due to the possibility of contractors' manipulation. We further study the family of *cost-sharing mechanisms* as a non-manipulable, easy-to-implement and near-optimal solution to the buyer's procurement problem. In a cost-sharing mechanism, the winning contractor is selected via a second-price auction and needs to reimburse a pre-specified fraction – referred to as the cost-sharing fraction – of the buyer's disutility cost upon completion of the project. We show that the cost-sharing fraction plays an unequivocal role in capturing the essential tradeoff between allocative inefficiency and information rent. We also characterize the optimal cost-sharing fraction and offer prescriptive guidelines on the choice of this fraction based on the second-moment information of the buyer's belief distribution. Finally, we establish the theoretical performance guarantees for the optimal cost-sharing mechanism.

Key words: procurement; non-cost attribute; multi-dimensional mechanism design; cost sharing

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1. Introduction

When a buyer (e.g., a firm or government agency) outsources a project, non-cost attributes such as completion time and quality are important considerations for the buyer in addition to cost. Motivated by such a context, we consider a buyer who wishes to award a project to one among a set of contractors, each of whom is privately informed about his cost and his estimate of the non-cost attribute. The non-cost attribute is subject to some exogenous randomness which is resolved after the project is completed; this attribute inflicts a disutility (or inconvenience) cost on the buyer. The buyer designs a procurement mechanism to minimize her expected *total cost*.

The highway construction industry offers an excellent example of the procurement problem described above. With a shift in focus from building new roads to 3R work (resurfacing, restoration, and rehabilitation) on existing ones, many projects require frequent road closures and traffic diversions, causing significant inconvenience (disutility) to road users (Lewis and Bajari 2011). As such, reducing the completion time of the project, thereby mitigating such social-welfare losses, has become a crucial objective in the awarding of contracts for many state agencies throughout the United States (Gupta et al. 2015). While contractors typically have estimates on their completion times, given their past experience and capabilities, the actual completion time may fluctuate due to a variety of reasons, including uncertain weather conditions and early/late completion of subcontractors' activities. Here, the chosen contractor's completion time for the project serves as an example of the non-cost attribute;[1] the buyer's (i.e., the government's) disutility cost arises from the inconvenience inflicted on road users until the project is completed.

Another prominent instance of multi-attribute procurement comes from the *Energy Service Company* (ESCO) industry in the United States (Stuart et al. 2016). Public institutions and private enterprises looking to improve the energy efficiency of their buildings and other infrastructure resort to ESCOs for identifying viable solutions. When selecting an ESCO, an important criterion, besides the up-front cost of installing the energy-efficient upgrades, is the potential energy savings delivered in the years following the installation. As Goldman et al. (2005) point out, energy-efficiency initiatives often generate significant savings; e.g., on a survey of 63 energy-saving projects, high-efficiency lighting reduced energy consumption to 53% of its baseline level. In practice, the energy savings are calculated based on a pre-specified equipment usage, termed as *stipulated operating hours*, in order to eliminate the facility owner's potential influence over the energy usage (U.S. Department of Energy 2015). ESCOs typically have the ability to forecast how an energy-efficient upgrade will perform (in terms of units of energy consumed per hour) according to the technology they employ (Vine 2005). However, the actual performance of an upgrade may deviate from its predicted amount due to exogenous random factors (e.g., weather condition). Thus, in this example, the performance of an energy-efficient upgrade corresponds to the non-cost attribute; a lower level of performance inflicts a corresponding disutility cost on the buyer (i.e., the owner of the facility).

In such settings, the buyer also faces the additional complication that the winning contractor may "manipulate" the realized non-cost attribute in a costless manner and thereby increase the disutility experienced by the buyer. For instance, the winning contractor can deliberately delay reporting the completion of a project if doing so benefits him. In the United States, the False Claims Act (FCA; 31 U.S.C. §3729-3733) specifically acknowledges the possibility of such manipulation; see Benarroche (2020) for potential violations of the FCA in the construction industry. Indeed, an optimal mechanism obtained by ignoring the possibility of manipulation can generate perverse incentives for the winning contractor to engage in manipulation (see Appendix B.1 for an illustrative example in which this situation emerges).

We find that the possibility of manipulation plays a pivotal role in generating information rent on the non-cost dimension of the contractors' private information (see Theorems 1-3 in §4). Specifically, the buyer would pay rent *only* on the contractor's cost information if either the *privacy of the non-cost estimates* or the *possibility of manipulation* were absent. It is the interplay between these two features that creates the rent on the non-cost dimension in addition to that on the cost dimension. Thus, the possibility of manipulation makes the buyer's mechanism design problem a genuinely multi-dimensional one. However, multi-dimensional mechanism design problems are known to lack analytical tractability in general and are notoriously challenging to solve. Consequently, we take a prescriptive perspective in this paper and aim to identify a non-manipulable, easy-to-implement, and near-optimal mechanism for the buyer.

Specifically, we focus on the family of *cost-sharing mechanisms*, indexed by a single parameter $\alpha \in [0, 1]$, which we refer to as the *cost-sharing fraction*. Under a cost-sharing mechanism with the fraction α, the winning contractor is selected through a standard second-price auction and reimburses an α fraction of the buyer's disutility cost upon the completion of the project; other contractors offer no services and receive no payment. Well-known examples of cost-sharing mechanisms include lane-rental mechanisms in highway construction and shared-savings contracts in ESCO markets. In highway construction, the *lane-rental* mechanism (Minnesota Department of Transportation 2017) awards a repair project to a contractor through an auction and charges the selected contractor a pre-specified daily lane-rental fee as a fraction of the actual road-user cost, until the project is completed. Bajari and Lewis (2009) estimate the potential impact of using lane-rental mechanisms with different lane-rental fractions and recommend using 10% as the lane rental rate. In energy-efficiency projects, an ESCO is typically selected through a bidding process and then enters into a *shared-savings contract* with the project owner. Under this contract, the actual energy savings realized upon the project's completion are divided between the owner and the ESCO according to an agreed-upon split percentage; e.g., 85% for the ESCO and 15% for the owner (Hawaii State Energy Office 2017).

Not only are cost-sharing mechanisms appealing in their implementation simplicity, they also capture the essential economic tradeoff that the informationally-disadvantaged buyer faces between allocating the project to the lowest-cost contractor and reducing her information rent (the winning contractor's markup over his cost). We demonstrate the unequivocal role of the cost-sharing fraction α in capturing this tradeoff (see Theorem 4 in §5). Consequently, an optimal cost-sharing arrangement is one that strikes a balance between these two countervailing forces.

Under bivariate normal beliefs on contractors' costs and non-cost estimates, we characterize the optimal cost-sharing fraction in closed-form (see Proposition 1 in §5). Notably, this fraction only relies on the second-moment information of the belief distribution and hence serves as a rule-of-thumb in practice (see §5.3). Furthermore, we provide prescriptive guidelines on how the optimal fraction should be set as a function of the relative informational asymmetry and correlation in cost and non-cost attributes, and the competitiveness of the supply base (see Proposition 2 in §5). Specifically, when the cost and non-cost attributes are negatively correlated, a buyer who faces a higher informational asymmetry in the contractors' non-cost estimates relative to their costs, should use a lower cost-sharing fraction. For the positive correlation case, as the informational asymmetry in the contractors' non-cost estimates increases relative to their costs, the optimal fraction should first increase and then decrease. As the correlation decreases or the supply base becomes more competitive, the buyer should use a larger cost-sharing fraction.

For arbitrary belief distributions on contractors' costs and non-cost estimates, we establish a series of theoretical performance guarantees offered by the optimal cost-sharing mechanism under varying degrees of distributional information about the beliefs (see Theorems 5-7 in §6). We further numerically assess the quality of these performance guarantees and demonstrate that the buyer's expected total cost under the optimal cost-sharing mechanism does not exceed the optimal cost by more than 10% on average (see Table 2 in §6).

Finally, we extend some of our results above to a moral-hazard setting, where contractors have intrinsic incentives to increase the project's non-cost attribute so as to reduce their costs. We find that the cost-sharing mechanism remains an effective solution for the buyer under this setting as well (see Proposition 3 in §7).

2. Literature Review

Our paper contributes to the well-established field of mechanism design that dates back to the seminal work by Mussa and Rosen (1978) and Myerson (1981). They modeled information asymmetry with a one-dimensional parameter and developed solution procedures that have since become standard in the literature for identifying optimal mechanisms. However, extending this analysis to multi-dimensional private information has proven to be difficult, in general. Here, the main technical challenge arises from resolving the complexity of the binding "non-local" incentive-compatibility constraints (Rochet and Choné 1998, Manelli and Vincent 2007, Belloni et al. 2010).

Because of such a challenge, a more fruitful endeavor has been to identify easy-to-implement and well-performing mechanisms rather than obtain an optimal mechanism. Many of these mechanisms are variations of the standard price-only auctions (Krishna 2009). A prominent example is the scoring-auction (Asker and Cantillon 2008), which specifies a scoring rule to convert a contractor's multi-dimensional bid to a scalar bid for evaluation. Chen et al. (2010) evaluate a scoring auction for procuring a product from suppliers differentiated by their costs and reliabilities. Chaturvedi and Martínez-de Albéniz (2011) find a sealed-bid mechanism for a setting similar to Chen et al. (2010). As a special type of scoring auctions, A+B auctions (Gupta et al. 2015, Tang et al. 2015) are particularly popular in awarding infrastructural projects, whereby contractors submit two-dimensional bids that are evaluated via a pre-determined scoring rule. Typically, these mechanisms do not entail any contingent payments and the focus of this literature is on characterizing the equilibrium bidding behavior under specific mechanisms.

In our setting, the non-cost attribute is determined by the contractor's type up to some exogenous randomness, and is only verifiable upon the completion of the project, calling for the design of mechanisms with contingent payments such as our cost-sharing mechanism. In response to Hansen (1985), Samuelson (1987) was the first to propose the notion of a cost-sharing mechanism as an example to illustrate the potential benefits of augmenting standard auctions with contingent payments. Since then, the study of auctions (or more general mechanisms) with contingent payments has been an active area of research; see Skrzypacz (2013) for a general survey. Nonetheless, the predominant focus in this stream of literature is to examine the equilibrium bidding behavior when bidders compete on the terms of the contingent payment that they offer to the auctioneer. This is distinct from our setting, in which it is the auctioneer who designs and pre-specifies the cost-sharing fraction. As another distinction, we consider bidders with multi-dimensional private information, whereas the current literature in this area typically works with single-dimensional private information. Cost-sharing arrangements are widely used as performance-based incentives to overcome the moral-hazard issue due to an agent's private actions (Laffont and Tirole 1993, Chu and Sappington 2007, Devalkar and Bala 2018). In contrast, our cost-sharing mechanism is used to overcome the adverse-selection friction due to contractors' private information in a multi-agent setting.

For the past decade, the Operations Management literature has witnessed a growing interest in auction design, and more broadly, mechanism design. Kostamis et al. (2009) study a procurement auction in a setting where the non-price dimension is known to the buyer, whereas, in our problem, it is unknown to the buyer and may also be subject to manipulation by the contractors. Wan and Beil (2009) and Chen et al. (2018) study the design of optimal auctions when firms employ qualification procedures to screen suppliers on non-cost attributes, in addition to their costs. Caldentey and Vulcano (2007) study a revenue management problem where a seller runs both a multi-unit auction and a list price channel to sell a product to consumers. Belloni et al. (2017) study the effect of rivalry among buyers (captured through a network diagram) on the firm's optimal mechanism for selling an indivisible good. Balseiro et al. (2015) study the design of repeated auctions for selling impressions on an ad-exchange when advertisers face a limited budget. Balseiro et al. (2019) study a dynamic mechanism design problem for allocating a resource to agents with private values in the absence of monetary transfers. Zhang et al. (2010) study a multi-period contract design problem for a supplier facing a retailer with private inventory information. Our work distinguishes itself from these studies due to the challenge of two-dimensional private information.

Taking a *prescriptive* perspective, we tackle this challenge by identifying the cost-sharing mechanism as an easy-to-implement, near-optimal and non-manipulable solution to the buyer's procurement problem. A similar approach is adopted by Chu and Sappington (2015), who consider a buyer procuring multiple units of a product from a supplier with private information about his production cost and capacity. The authors focus on the family of fixed-price-cost-reimbursement (FPCR) contracts, which are employed in practice due to their ease of implementation and have been analyzed by prior research. They demonstrate that FPCR contracts yield no incentive for the supplier to exaggerate his production capacity and can deliver near-optimal performance for the buyer by securing at least 75% of the surplus that she could secure under the optimal contract.

3. Model

Consider a buyer who wishes to award a single indivisible project to one among N contractors, indexed by $n = 1, 2, \ldots, N$.[2] If selected, contractor n incurs a cost x_n and delivers the project with a non-cost attribute t_n (e.g., completion time). Both the cost x_n and the non-cost attribute t_n depend on the contractor's privately-known innate characteristics (e.g., capability or technology). The attribute t_n is also affected by a priori uncertain exogenous factors (e.g., unpredictable weather conditions affecting project execution). As a convention in this paper, a higher value of the non-cost attribute is less desirable to the buyer. Before executing the project, each contractor n privately knows his cost x_n and his *estimate* y_n of the non-cost attribute t_n. The non-cost estimate y_n is basically an informative yet noisy signal of the a priori random non-cost attribute t_n; for instance, it can be the estimated mean of t_n. If contractor n is awarded the project, then the non-cost attribute t_n is realized upon the completion of the project.

A novel feature of our model is the possibility of a contractor's *costless upward manipulation* of the project's *ex post* non-cost attribute; that is, a contractor can privately inflate the non-cost attribute delivered to the buyer, denoted as $\widetilde{t}_n$, above the non-cost attribute t_n that the contractor has realized (i.e., $\widetilde{t}_n \geq t_n$) at no cost. For example, the winning contractor in a highway construction project can simply delay announcing the completion of an already completed project without incurring any cost.[3] If manipulation is not possible, then $\widetilde{t}_n = t_n$. The project with non-cost attribute $\widetilde{t}_n$ inflicts a disutility cost $V(\widetilde{t}_n)$ on the buyer, where V is a non-negative and increasing function.[4] As will become evident in §3.1, in our setting, a contractor may have an incentive to exercise such a manipulation if the buyer's procurement mechanism turns out to stipulate a higher payment to the contractor for delivering a higher non-cost attribute.[5] In §4, we investigate the role of manipulation, and show that such an incentive may indeed emerge if the buyer designs her procurement mechanism by ignoring the possibility of manipulation.

We also acknowledge scenarios where a contractor would have an intrinsic incentive to shirk *ex ante* (i.e., before the project's completion) and potentially increase the project's non-cost attribute, if doing so materially helps him lower his cost of executing the project. For example, a contractor may delay the completion time by utilizing fewer resources and thereby reducing his cost. Such scenarios are stereotypical examples of moral hazard, which we investigate in §7.

It is common knowledge that x_n and y_n are realizations of random variables X_n and Y_n. As is standard in the mechanism design literature, we also assume the pair (X_n, Y_n) to be independent and identically distributed across n to exclude full-rent extraction outcomes (Crémer and McLean 1985, 1988). The non-cost attribute t_n is a realization of a random variable T_n, which is, as described before, correlated with contractor n's non-cost estimate Y_n. For every n, we allow the cost X_n and non-cost estimate Y_n to be correlated, but let T_n be independent of X_n, conditional on Y_n; for example, $T_n = Y_n + \epsilon$ for some exogenous random variable ϵ that does not depend on X_n. This dependence structure captures the reality that a contractor's non-cost attribute typically depends on his innate characteristics represented by his non-cost estimate Y_n and is subject to exogenous randomness. Notationally, throughout the paper, we reserve capital letters to represent random variables and their corresponding lower-case letters to represent their realizations; bold-faced letters correspond to their vector counterparts. For instance, $\mathbf{X} := (X_1, X_2, \ldots, X_N)$ and, as a convention, $\mathbf{X}_{-n} := (X_1, \ldots, X_{n-1}, X_{n+1}, \ldots, X_N)$, whose realizations are denoted as $\mathbf{x}$ and $\mathbf{x}_{-n}$, respectively.[6]

3.1. Formulation of the buyer's problem.

The buyer's problem is to design a procurement mechanism that minimizes her expected *total cost*, i.e., the sum of the payments made to the contractors plus the disutility cost. In its most general form, a mechanism can be quite complex and the set of conceivable mechanisms that the buyer can consider is boundless. As is now standard in the literature, one typically invokes the Revelation Principle (e.g., Myerson 1982) to restrict attention to a simple class of mechanisms without loss of generality. However, the usual version of the Revelation Principle is not directly applicable to our setting due to the possibility of contractors' manipulation. Therefore, to formulate the buyer's problem, we need to first establish in Lemma 1 below a generalized version of the Revelation Principle, which allows us to restrict the search for the optimal mechanism within a set of truth-revealing and non-manipulable direct mechanisms, which we denote as $\{(\mathbf{Q},\mathbf{M})\}$.

Formally, under such a mechanism $(\mathbf{Q},\mathbf{M})$, contractors simply report their costs and non-cost estimates to the buyer. For any profile $(\mathbf{x},\mathbf{y})$ reported by the contractors, the mechanism $(\mathbf{Q},\mathbf{M})$ specifies (i) the probability $Q_n(\mathbf{x},\mathbf{y})$ of awarding the contract to contractor n such that $\sum_{m=1}^{N} Q_m(\mathbf{x},\mathbf{y})=1$, (ii) the payment $M_{nn}(x_n,y_n,\widetilde{t}_n)$ made to contractor n if he is selected and delivers to the buyer a non-cost attribute $\widetilde{t}_n$ upon the completion of the project, and (iii) the payment $M_{nm}(x_n,y_n)$ made to contractor n if contractor m is selected. The constraint $\sum_{m=1}^{N} Q_m(\mathbf{x},\mathbf{y})=1$ reflects the fact that the buyer has to award the project to one of the N contractors. As will be shown below in Lemma 1, it suffices to consider payments that are only contingent on a contractor's reported type and delivered non-cost attribute. Because of contractors' possible manipulation, contractor n can deliver $\widetilde{t}_n$ that is larger than the realized value t_n of T_n. Contractor n would have an incentive to do so if the payment scheme stipulates $M_{nn}(x_n,y_n,\widetilde{t}_n) > M_{nn}(x_n,y_n,t_n)$ for some $\widetilde{t}_n > t_n$.

Under $(\mathbf{Q},\mathbf{M})$, contractor n is expected to receive a payment of

$$\begin{aligned} M_n(x_n,y_n,\widetilde{t}_n) :=& \mathbb{E}\left[Q_n\left(x_n,\mathbf{X}_{-n},y_n,\mathbf{Y}_{-n}\right)\right] M_{nn}(x_n,y_n,\widetilde{t}_n) \\ &+ \sum_{\substack{m=1\\ m\neq n}}^{N} \mathbb{E}\left[Q_m\left(x_n,\mathbf{X}_{-n},y_n,\mathbf{Y}_{-n}\right)\right] M_{nm}(x_n,y_n), \end{aligned} \tag{1}$$

provided that all other contractors are truthful. We are now ready to state a version of the Revelation Principle tailored to our setting.[7]

Lemma 1 **(Revelation Principle)**. *For any procurement mechanism, there exists a direct mechanism* $(\mathbf{Q},\mathbf{M})$ *which is* *i) payoff-equivalent for all contractors and (weakly) cost-improving for the buyer;*

ii) non-manipulable, i.e., every contractor n finds it optimal to deliver the non-cost attribute as is (without any manipulation):

$$M_n\left(x_n, y_n, t_n\right) \geq M_n\left(x_n, y_n, \widetilde{t}_n\right), \quad \forall\ x_n, y_n, \widetilde{t}_n \geq t_n,\ \textit{and } n; \tag{NM}$$

iii) incentive compatible, i.e., all contractors find it optimal to truthfully report their private information (costs and non-cost estimates) to the buyer:

$$\begin{aligned} &\mathbb{E}\left[M_n\left(x_n, y_n, T_n\right) \mid Y_n = y_n\right] - x_n \mathbb{E}\left[Q_n\left(x_n, \mathbf{X}_{-n}, y_n, \mathbf{Y}_{-n}\right)\right] \\ \geq\ &\mathbb{E}\left[M_n\left(\widetilde{x}_n, \widetilde{y}_n, T_n\right) \mid Y_n = y_n\right] - x_n \mathbb{E}\left[Q_n\left(\widetilde{x}_n, \mathbf{X}_{-n}, \widetilde{y}_n, \mathbf{Y}_{-n}\right)\right], \quad \forall\ x_n, y_n, \widetilde{x}_n, \widetilde{y}_n,\ \textit{and } n; \end{aligned} \tag{IC}$$

iv) and individually rational, i.e., all contractors voluntarily participate in the mechanism:

$$\mathbb{E}\left[M_n\left(x_n, y_n, T_n\right) \mid Y_n = y_n\right] - x_n \mathbb{E}\left[Q_n\left(x_n, \mathbf{X}_{-n}, y_n, \mathbf{Y}_{-n}\right)\right] \geq 0, \quad \forall\ x_n, y_n,\ \textit{and } n. \tag{IR}$$

Compared to its usual version, the Revelation Principle established in Lemma 1 requires the set of direct mechanisms to also satisfy the non-manipulability (NM) constraints, in addition to the conventional incentive compatibility (IC) and individual rationality (IR) constraints. The (NM) constraints ensure that it is optimal for the buyer to not induce any manipulation. Since any manipulation can only take place *after* the project has been awarded to and completed by a contractor, the incentive for the selected contractor to manipulate, if any, must arise from the payment scheme $M_{nn}(x_n, y_n, \widetilde{t}_n)$. The (NM) constraints preclude such an incentive by essentially lowering the payment received by the selected contractor if he chose to manipulate, i.e., $M_n\left(x_n, y_n, t_n\right)$ is non-increasing in t_n, and the same is true for $M_{nn}(x_n, y_n, t_n)$ by the definition in (1). In particular, if manipulation is not possible (i.e., $\widetilde{t}_n = t_n$), then the (NM) constraints can be ignored. We note that, together, the (NM) and (IC) constraints prevent any contractor n from contemplating a double deviation (i.e., first lying on his cost and non-cost estimate (x_n, y_n), and then upon winning the contract, manipulating his non-cost attribute t_n).[8] For non-manipulable mechanisms, we can then formulate the usual (IC) and (IR) constraints by taking the expectation over the uncertain non-cost attribute T_n (conditional on its prior estimate y_n), as these two constraints are relevant *before* the project is awarded. Lemma 1 now allows us to formulate the buyer's problem as the following constrained optimization problem:

$$OPT := \min_{\{(\mathbf{Q},\mathbf{M})\}} \sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, Y_n, T_n\right) + Q_n\left(\mathbf{X}, \mathbf{Y}\right) V(T_n)\right] \quad \text{subject to (NM), (IC) and (IR),} \tag{$\mathcal{P}$}$$

where OPT is, in fact, the minimum expected cost to the buyer over all possible (not necessarily direct) procurement mechanisms.

3.2. Two lower bounds for OPT

Before concluding this section, we identify two lower bounds on the buyer's minimum cost OPT by relaxing some of the incentive compatibility constraints in $(\mathcal{P})$.

First-best cost. In the *first-best* benchmark, the buyer knows every contractor n's cost x_n and non-cost estimate y_n. Thus, the buyer awards the project to the most efficient contractor, who delivers the lowest expected total cost; that is, $\arg\min_n \{x_n + \mathbb{E}[V(T_n) \mid Y_n = y_n]\}$. The winning contractor is, in turn, compensated only for his cost; other contractors receive no payments. Since the winning contractor's payment is independent of his non-cost attribute, she has no incentive to manipulate it. As a result, the buyer incurs the following *first-best* cost:

$$FB = \mathbb{E}\left[\min_n \{X_n + \mathbb{E}[V(T_n) \mid Y_n]\}\right], \tag{2}$$

which can be easily verified to be the same as the optimal cost in $(\mathcal{P})$ when the (NM) and (IC) constraints are ignored, implying that $FB \leq OPT$.

Optimal cost when non-cost estimates are public information. Next, consider the case where the non-cost estimate vector, $\mathbf{y}$, is public information. In this case, the buyer's mechanism design problem essentially becomes a relaxation to $(\mathcal{P})$ in which the (IC) constraints with $\widetilde{y}_n \neq y_n$ are ignored. As will be verified later in Lemma 2, the *ex ante* optimal cost that the buyer can achieve in this case, denoted as OPT_{NC}, is given by

$$\begin{array}{c} OPT_{NC} = \min_{\{(\mathbf{Q},\mathbf{M})\}} \sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, Y_n, T_n\right) + Q_n\left(\mathbf{X}, \mathbf{Y}\right) V(T_n)\right] \\ \text{subject to (NM), (IC) with all } \widetilde{\mathbf{y}} = \mathbf{y}\text{, and (IR),} \end{array} \tag{$\mathcal{P}_{NC}$}$$

where the subscript "NC" stands for "non-cost" and signifies the public knowledge of $\mathbf{y}$. Since $(\mathcal{P}_{NC})$ is a relaxation to $(\mathcal{P})$, it immediately implies that $OPT_{NC} \leq OPT$.

Comparing the above two benchmarks, we note that the set of incentive compatibility constraints ignored in the first-best benchmark contains those ignored in $(\mathcal{P}_{NC})$. Thus, FB is a comparatively weaker lower bound on OPT than OPT_{NC}, which we formally state in the following lemma.

LEMMA 2. *Both FB and OPT_{NC} are lower bounds on the optimal cost OPT. Further, OPT_{NC} is a tighter lower bound on OPT compared to FB. That is, $FB \leq OPT_{NC} \leq OPT$.*

We note that, if the expected cost under a feasible mechanism is close to either of the two lower bounds, then it must be close to the optimal cost OPT. In §4, we use the lower bound OPT_{NC} to understand the role played by manipulation in the buyer's problem $(\mathcal{P})$. Later, in §6, we use both lower bounds, FB and OPT_{NC}, to assess the performance of the cost-sharing mechanism.

4. The Role of Manipulation in $(\mathcal{P})$

In this section, our goal is to understand the implications of the contractors' ability to manipulate the non-cost attribute. We find that it is the interplay between the *privacy* of the contractors' non-cost estimates and the *possibility of manipulation* that makes OPT strictly larger than OPT_{NC}. To this end, we consider the following discretized version of problem $(\mathcal{P})$: The random variables $X_n \in \mathcal{X}$, $Y_n \in \mathcal{Y}$, and $T_n \in \mathcal{T}$, follow bounded discrete probability distributions, where $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{T}$ denote the set of values these variables can take, respectively. Let $\mathcal{X} = \{\underline{x} + k\Delta_x : k = 1, 2, \ldots, l\}$ for some $\underline{x} \in \mathbb{R}$ and $\Delta_x > 0$. For any $t \in \mathcal{T}$ and $y \in \mathcal{Y}$, we denote the probability of T_n conditional on Y_n as $\pi(t, y) := \mathbb{P}[T_n = t \mid Y_n = y]$ and the cumulative probability as $\Pi(t, y) := \sum_{t' \in \mathcal{T}, t' \le t} \pi(t', y)$.

We first characterize the lower bound OPT_{NC} by solving $(\mathcal{P}_{NC})$, which we note is separable in the non-cost estimates $\mathbf{y}$; in other words, we can solve $(\mathcal{P}_{NC})$ for any fixed non-cost estimates $\mathbf{y}$, and subsequently take the ex ante expectation of the resulting optimal value with respect to $\mathbf{Y}$ to obtain OPT_{NC}. If the (NM) constraints are ignored, problem $(\mathcal{P}_{NC})$ reduces to a standard single-dimensional mechanism design problem. Given that T_n is independent of X_n conditional on Y_n, it can be solved by following the approach of Myerson (1981) (see Laffont and Martimort 2009, for the discrete-type version), provided that the virtual cost function

$$\psi(x, y) = x + \Delta_x \frac{\mathbb{P}(X_n < x, Y_n = y)}{\mathbb{P}(X_n = x, Y_n = y)}$$

is increasing in $x \in \mathcal{X}$ for any given $y \in \mathcal{Y}$, a standard assumption in the literature which we will also adopt in this section. Our result below confirms that the (NM) constraints are indeed inconsequential and can be ignored when solving $(\mathcal{P}_{NC})$.

THEOREM 1. *The optimal cost of $(\mathcal{P}_{NC})$, with or without the* (NM) *constraints, is given by*

$$OPT_{NC} = \sum_{n=1}^{N} \mathbb{E}\left[\widetilde{u}_n(X_n, \mathbf{Y}) + (X_n + V(T_n))\, \widetilde{Q}_n(\mathbf{X}, \mathbf{Y})\right], \tag{3}$$

where the probability $\widetilde{Q}_n(\mathbf{x}, \mathbf{y})$ of selecting contractor n must satisfy

$$\left(\widetilde{Q}_1(\mathbf{x}, \mathbf{y}), \cdots, \widetilde{Q}_N(\mathbf{x}, \mathbf{y})\right) := \underset{\substack{q_1, \cdots, q_N \ge 0 \\ \sum_{n=1}^{N} q_n = 1}}{\arg\min} \quad \sum_{n=1}^{N} \left\{\psi(x_n, y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\right\} \cdot q_n \tag{4}$$

and the expected payoff of contractor n is given by

$$\widetilde{u}_n(x_n, \mathbf{y}) := \Delta_x \sum_{x' \in \mathcal{X}, x' > x_n} \mathbb{E}[\widetilde{Q}_n(\mathbf{X}, \mathbf{y}) \mid X_n = x']. \tag{5}$$

Theorem 1 shows that, when contractors' non-cost estimates are public information, the mere possibility of manipulation is *not* sufficient to generate any rent for the winning contractor in addition to the rent (i.e., $\widetilde{u}_n(x_n, \mathbf{y})$) generated by the privacy of costs. It is worth noting that the allocations in (4) and the payoffs in (5) can be implemented through a payment scheme that is independent of the ex post non-cost attribute $\widetilde{t}_n$. It is for this reason that $(\mathcal{P}_{NC})$ can be solved by ignoring the (NM) constraints.

Our next result shows that, in the absence of the (NM) constraints, the privacy of non-cost estimates alone cannot generate any additional rent on top of that generated by the cost-dimension (i.e., $\widetilde{u}_n(x_n, \mathbf{y})$) either.

THEOREM 2. *Suppose that the following full-rank condition holds:*

$$\pi(\cdot, y) \notin \text{ConvexHull}\{\pi(\cdot, y') \; : \; y' \in \mathcal{Y}, y' \neq y\} \quad \forall y \in \mathcal{Y}. \tag{6}$$

Then, the optimal cost of $(\mathcal{P})$ *without the* (NM) *constraints is equal to* OPT_{NC}.

According to Theorem 2, if the (NM) constraints are ignored and the full-rank condition (6) holds,[9] the buyer can achieve the same expected cost OPT_{NC} as in (3), even when the non-cost estimates are contractors' private information. This result is established by identifying a payment scheme $M_n(x_n, y_n, \widetilde{t}_n)$ that implements the same allocation $\widetilde{Q}_n(\mathbf{x}, \mathbf{y})$ as in (4) (i.e., the mechanism $(\widetilde{\mathbf{Q}}, \mathbf{M})$ satisfies the (IC) and (IR) constraints). In other words, if manipulation were not possible (i.e., $\widetilde{t}_n = t_n$ for all n) and thus the (NM) constraints can be ignored, the optimal solution of the two-dimensional mechanism design problem $(\mathcal{P})$ becomes effectively the same as that of the single-dimensional problem $(\mathcal{P}_{NC})$: the additional dimension of private information on contractors' non-cost estimates neither causes any distortion to the buyer's allocation nor increases her rent beyond what she obtained when only the cost dimension were private.

However, we note that the payment $M_n(x_n, y_n, \widetilde{t}_n)$ that the buyer uses to achieve the optimal outcome under problem $(\mathcal{P})$ (in the absence of the (NM) constraints) differs from the one used under problem $(\mathcal{P}_{NC})$; in particular, the former payment is indeed contingent on the ex post non-cost attribute $\widetilde{t}_n$, whereas, the latter payment, as mentioned earlier, is independent of $\widetilde{t}_n$. Indeed, in the absence of contractors' manipulation, the statistical relationship between T_n and Y_n remains intact and is governed by the conditional probability $\pi(\cdot, \cdot)$. The full-rank condition (6) essentially guarantees that T_n is a sufficiently informative statistical signal on the non-cost estimate Y_n. Thus, one can construct the payment $M_n(x_n, y_n, \widetilde{t}_n)$ to induce truthful revelation of contractors' non-cost estimates without distorting the allocation in (4) nor incurring additional rent beyond

OPT_{NC}. More specifically, the payment $M_n(x_n, y_n, \widetilde{t}_n)$ can be identified as a feasible solution to a linear system that equates the expected value of $M_n(x_n, y_n, T_n)$ prior to the realization of T_n with the corresponding expected payment in $(\mathcal{P}_{NC})$. As illustrated by the numerical example in Appendix B.1, the optimal payment $M_n(x_n, y_n, \widetilde{t}_n)$ obtained by ignoring the (NM) constraints may be increasing in $\widetilde{t}_n$, and hence, may induce the winning contractor to manipulate, that is, violate the (NM) constraints.

The presence of an ex post signal $\widetilde{t}_n$ and the capability of constructing a payment $M_n(x_n, y_n, \widetilde{t}_n)$ contingent on it distinguish our setting from a generic multi-dimensional mechanism design setting. Theorem 2 generalizes a similar finding obtained in Riordan and Sappington (1988) for the setting with single-dimensional private information, whereby the principal can achieve the first-best outcome. In our setting, given Y_n, since T_n and X_n are independent, the buyer still needs to pay the rent generated due to the privacy of contractors' costs, and, therefore, his expected cost equals OPT_{NC} instead of FB.

Taken together, Theorems 1 and 2 imply that if either the privacy of the non-cost estimates or the possibility of manipulation is absent, then the non-cost dimension is inconsequential in generating information rent. However, this is not necessarily the case if both these features are present, as established by the following result.

THEOREM 3. *Suppose that there exists $x \in \mathcal{X}$, $y \in \mathcal{Y}$ such that*

$$\sum_{\substack{y' \in \mathcal{Y} \\ y' \neq y}} \nu(y')\Pi(t, y') \geq \Pi(t, y) \quad \forall t \in \mathcal{T}, \quad \textit{and} \quad \sum_{\substack{y' \in \mathcal{Y} \\ y' \neq y}} \nu(y')\widetilde{U}(x, y') < \widetilde{U}(x, y) \tag{7}$$

for some $\nu(\cdot) \geq 0$ on $\mathcal{Y}$ with $\sum_{y' \in \mathcal{Y}, y' \neq y} \nu(y') = 1$, where $\widetilde{U}(x, y) := \mathbb{E}[\widetilde{u}_n(x, \mathbf{Y}) \mid Y_n = y]$ for all $x \in \mathcal{X}, y \in \mathcal{Y}$ with the function $\widetilde{u}_n(\cdot)$ as defined in (5).[10] *Then, the optimal cost of $(\mathcal{P})$ (with the* (NM) *constraints) is $OPT > OPT_{NC}$.*

As discussed before Theorem 3, the payment $M_n(x_n, y_n, \widetilde{t}_n)$ that implements the optimal outcome in Theorem 2 (and thus, achieves the buyer's cost as OPT_{NC}) may generate incentives for the winning contractor to engage in manipulation. That is, the optimal mechanism obtained for the buyer's problem $(\mathcal{P})$ by ignoring the (NM) constraints may be manipulable; see again Appendix B.1 for an example. Theorem 3 identifies a sufficient condition (7) under which such manipulation occurs and hence the buyer's optimal cost in $(\mathcal{P})$ cannot achieve OPT_{NC}. Since OPT_{NC} is a lower bound on OPT (Lemma 2), we must have $OPT > OPT_{NC}$. Intuitively, the observed non-cost attribute $\widetilde{t}_n$, which may now differ from t_n due to the contractor's manipulation, can no longer serve as a statistical signal for the contractor's non-cost estimate Y_n, thus preventing the buyer from exploiting $\widetilde{t}_n$ to avoid the rent on the non-cost dimension as in Theorem 2. For methodological interest, both conditions (6) and (7) are identified by applying Farkas' lemma on the buyer's mechanism design problem $(\mathcal{P})$, which is essentially a linear program.[11]

In summary, Theorems 1–3 show that both the privacy of non-cost estimates and the possibility of manipulation are *necessary* to generate rent in addition to that generated by contractors' private cost information, making the buyer's optimal cost OPT strictly higher than OPT_{NC}. In the absence of either of these two conditions, the buyer's optimal cost is equal to OPT_{NC}. As such, the (NM) constraints play a pivotal role in the buyer's problem $(\mathcal{P})$ by making it a genuinely two-dimensional mechanism design problem, which is known to be analytically intractable. In the next two sections, we study a family of non-manipulable mechanisms (with contingent payments) as easy-to-implement and near-optimal solutions to the buyer's procurement problem.

5. Cost-Sharing Mechanisms

Motivated by lane-rental mechanisms in highway construction and shared-savings contracts in ESCO markets (see §1) as well as by the previous literature on these mechanisms (see §2), we study in this section the family of cost-sharing mechanisms indexed simply by a single parameter. A *cost-sharing mechanism* with the *cost-sharing fraction* α, hereafter denoted by CS_α, selects a contractor through a second-price[12] sealed-bid auction, in which every contractor submits a sealed-bid to the buyer and the contractor with the lowest-bid wins. The selected contractor is paid the second-lowest bid and is, in addition, required to reimburse an α fraction of the buyer's disutility cost after the project is completed. Unselected contractors receive no payment.

The selected contractor, say n, incurs the cost x_n to execute the project and reimburses an α fraction of the buyer's disutility cost $V(\widetilde{t}_n)$. Since $V(\widetilde{t}_n)$ is an increasing function of $\widetilde{t}_n$, it is immediate that, under the cost-sharing mechanism CS_α, the contractor has no incentive to inflate his non-cost attribute ex post (i.e., $\widetilde{t}_n = t_n$). For brevity, we define a random variable $W_n := \mathbb{E}[V(T_n) \mid Y_n]$ to represent the buyer's estimated disutility cost from selecting contractor n with non-cost estimate Y_n; correspondingly, $w_n := \mathbb{E}[V(T_n) \mid Y_n = y_n]$ denotes its realization. Thus, under the cost-sharing mechanism, contractor n's total expected cost is $x_n + \alpha w_n$. Then, standard arguments for second-price sealed-bid auctions lead to the following result.

LEMMA 3. *For any $\alpha \in [0,1]$, CS_α is non-manipulable, and under this mechanism, it is a dominant strategy for every contractor n to bid $x_n + \alpha w_n$.*

5.1. Role of cost-sharing fraction α

Let $C(\alpha)$ denote the buyer's expected cost under CS_α. Let $l(\alpha,N)$ be the l-th lowest order statistic of $\{X_n+\alpha W_n : 1\le n\le N\}$; that is, $X_{n(\alpha,N)}+\alpha W_{n(\alpha,N)}\le X_{m(\alpha,N)}+\alpha W_{m(\alpha,N)}$ for all $n\le m$. Using the current notation, the first-best cost can be expressed as

$$FB=\mathbb{E}\left[X_{1(1,N)}+W_{1(1,N)}\right]. \tag{8}$$

Using Lemma 3, the buyer's expected cost under CS_α can be expressed as:

$$C(\alpha)=\mathbb{E}\left[X_{2(\alpha,N)}+\alpha W_{2(\alpha,N)}+(1-\alpha)W_{1(\alpha,N)}\right], \tag{9}$$

where the sum of the first two terms within the expectation operator is the payment made to the winning contractor and the third term is the portion of the disutility cost borne by the buyer. Thus, the deviation of $C(\alpha)$ from the first-best cost can be decomposed as follows:

$$\begin{aligned} C(\alpha)-FB \;=\; & \underbrace{\mathbb{E}\left[X_{1(\alpha,N)}+W_{1(\alpha,N)}-X_{1(1,N)}-W_{1(1,N)}\right]}_{\text{allocative inefficiency}} \\ & +\; \underbrace{\mathbb{E}\left[X_{2(\alpha,N)}+\alpha W_{2(\alpha,N)}-X_{1(\alpha,N)}-\alpha W_{1(\alpha,N)}\right]}_{\text{information rent}}. \end{aligned} \tag{10}$$

The above expression decomposes the buyer's cost increment over the first-best cost under the cost-sharing mechanism into two parts: The first term, referred to as the *allocative inefficiency*, represents the additional cost from awarding the project to contractor $1(\alpha,N)$ instead of the most efficient contractor $1(1,N)$. The second term represents the surplus given up by the buyer to the selected contractor $1(\alpha,N)$, whose total cost is $X_{1(\alpha,N)}+\alpha W_{1(\alpha,N)}$, but is paid $X_{2(\alpha,N)}+\alpha W_{2(\alpha,N)}$. This surplus is generated due to the buyer's inability to observe the winning contractor's total cost and hence is the *information rent* paid to the winning contractor. The following result explains how the choice of α affects the allocative inefficiency and the information rent.

THEOREM 4 **(Allocative Inefficiency vs. Information Rent)**. *Suppose that $X_n=\beta W_n+\xi_n$ for some constant coefficient $\beta\in(-\infty,\infty)$ and a random variable ξ_n, which has a log-concave probability density and is independent of $\mathbf{W}$ and ξ_m for $m\neq n$. Under the family of cost-sharing mechanisms $\{CS_\alpha : 0\le\alpha\le 1\}$, the allocative inefficiency always decreases in $\alpha\in[0,1]$, while the information rent*

- *increases in $\alpha\in[0,1]$ for $\beta\ge 0$;*
- *first decreases in $\alpha\in[0,-\beta)$ and then increases in $\alpha\in(-\beta,1]$, for $\beta\in(-1,0)$;*

- *decreases in* $\alpha \in [0,1]$ *for* $\beta \leq -1$.

To understand Theorem 4, we first consider the case when $\beta = 0$, i.e., X_n and W_n are independent for each n. In this case, Theorem 4 states that the allocative inefficiency decreases in α whereas the information rent increases in α. Under CS_α, we have argued that every contractor bids his *total* cost, which is a realization of the random variable $X_n + \alpha W_n$. As the cost-sharing fraction α goes up, $X_n + \alpha W_n$ becomes closer to $X_n + W_n$, the total cost based on which the first-best cost is established. As such, the allocative inefficiency reduces as the cost-sharing fraction α increases. In the extreme case, when $\alpha = 1$, the contractor with the lowest $X_n + W_n$ is selected, thus achieving a fully-efficient allocation. In contrast, the information rent increases with α; $\alpha \in [0,1]$. This is because a higher α renders the distribution of the contractor's total cost $X_n + \alpha W_n$ more dispersed, increasing the information asymmetry and hence the information rent faced by the buyer.

More generally, Theorem 4 allows for any correlations between a contractor's cost and non-cost estimate, as long as they follow a linear correlation structure $X_n = \beta W_n + \xi_n$.[13] It shows that the monotonicity of the allocative inefficiency with respect to the cost-sharing fraction α is preserved when costs and non-cost estimates become correlated. In contrast, the monotonicity of the information rent with respect to α is affected when they become negatively correlated (i.e., when $\beta < 0$). In this case, an increase in α makes $X_n + \alpha W_n$ more dispersed and hence generates higher information rent as long as $\alpha > -\beta$. Otherwise, the information rent decreases in α for $\alpha \leq -\beta$, and so does the allocative inefficiency; thus, the optimal cost-sharing fraction must be higher than $-\beta$. If $\beta \leq -1$, this immediately implies that the optimal cost-sharing fraction $\alpha^{opt} = 1$.

While the rent-efficiency tradeoff has been well-known in the standard single-dimensional mechanism-design setting, the significance of Theorem 4 lies in demonstrating the presence of such a tradeoff in a *multi-dimensional* setting, and more importantly, highlighting the unequivocal role of the cost-sharing fraction α in balancing this tradeoff.

The proof of Theorem 4 (see Appendix C) is also interesting in its own right – it neatly combines ideas from dispersive orderings (as defined in Definition 1 in §6) and order statistics. Specifically, since the information rent can be expressed as the spacing between two lowest order statistics, we leverage the fact that spacings between order statistics stochastically increase as the underlying distribution becomes larger in the dispersive order (Bartoszewicz 1986, Shaked and Shanthikumar 2007). The monotonicity of the allocative inefficiency is established using a sample-path argument.

5.2. Optimal cost-sharing fraction under normal beliefs

Having understood the tradeoff that the cost-sharing fraction aims to balance, we now turn to the characterization of the optimal fraction, which minimizes the buyer's expected cost over the family of cost-sharing mechanisms $\{\mathrm{CS}_\alpha : 0 \le \alpha \le 1\}$ and is formally defined as

$$\alpha^{opt} \;:=\; \arg\min_{\alpha} C(\alpha). \tag{11}$$

Characterizing the optimal cost-sharing fraction is analytically challenging, in general. In this section, we consider the case when (X_n, W_n) follows a bivariate normal distribution, characterized by the means (μ_X, μ_W), standard deviations (σ_X, σ_W), and correlation coefficient $\rho \in [-1, 1]$. One appealing feature of the bivariate normal distribution is the separability of the first-order moments (μ_X, μ_W) from the second-order moments $(\sigma_X, \sigma_W, \rho)$, which allows us to isolate and study their individual effects on the optimal cost-sharing fraction. This distributional specification is supported by the previous literature (see, e.g., Samuelson 1987, Bergemann and Morris 2009, Davis et al. 2014) as well as by the empirical evidence from highway construction projects (see Appendix F for more details). It is also consistent with the linear correlation structure assumed in Theorem 4, as the bivariate normal distribution of (X_n, W_n) lends itself to the representation $X_n = \beta W_n + \xi_n$, whereby W_n and ξ_n are normally-distributed, independent random variables and $\beta = \rho\sigma_X/\sigma_W$. In §5.3, we will numerically demonstrate that the optimal cost-sharing fraction obtained under the normal beliefs serves as an excellent heuristic for other non-normal belief distributions.

The bivariate normal distribution is analytically appealing as it allows us to explicitly quantify the *allocative inefficiency* and *information rent* identified in (10), and thus, compute the buyer's expected cost under the cost-sharing mechanism CS_α in closed-form. Let

$$\sigma_{X+\alpha W} \;:=\; \sigma_X \sqrt{(\alpha\sigma_W/\sigma_X + \rho)^2 + 1 - \rho^2}$$

denote the standard deviation of $X_n + \alpha W_n$. Then, the random variable $X_n + \alpha W_n$ has the same distribution as $\mu_X + \alpha\mu_W + \sigma_{X+\alpha W} \cdot Z_n$, where Z_n is the *standard* normal random variable. Thus, the order statistics, $X_{1(\alpha,N)} + \alpha W_{1(\alpha,N)}$ and $X_{2(\alpha,N)} + \alpha W_{2(\alpha,N)}$ can be expressed in terms of the lowest and second-lowest standard-normal order statistics $Z_{1:N}$ and $Z_{2:N}$, respectively. As a result, the information rent in (10) reduces to

$$\mathbb{E}\left[X_{2(\alpha,N)} + \alpha W_{2(\alpha,N)} - X_{1(\alpha,N)} - \alpha W_{1(\alpha,N)}\right] = \sigma_{X+\alpha W} \cdot \mathbb{E}[Z_{2:N} - Z_{1:N}]. \tag{12}$$

Further, given that contractors are ex ante symmetric from the buyer's perspective, we also have:[14]

$$\mathbb{E}\left[X_{1(\alpha,N)} + W_{1(\alpha,N)}\right] = N\mathbb{E}\Big[X_N + W_N;\; X_N + \alpha W_N \le X_{1(\alpha,N-1)} + \alpha W_{1(\alpha,N-1)}\Big]. \tag{13}$$

Leveraging (12) and explicitly computing (13) by using the fact that the conditional expectation of two normally-distributed random variables is also normally-distributed, we are able to derive a closed-form expression for the buyer's expected cost $C(\alpha)$ under normal beliefs, which is provided in Lemma C.2 of Appendix C. Notably, both (12) and (13) are shown to depend only on the second-moment information $(\sigma_X, \sigma_W, \rho)$ of the belief distribution on (X_n, W_n), and, so does $C(\alpha) - FB$. Using the closed-form expression for $C(\alpha)$ in Lemma C.2, we now characterize the optimal cost-sharing fraction in the following proposition.

PROPOSITION 1 **(Characterization of α^{opt})**. *Suppose that (X_n, W_n) follows a bivariate normal distribution with the second-order moments $(\sigma_X, \sigma_W, \rho)$. Then, the buyer's expected cost $C(\alpha)$ is quasi-convex in α and the optimal cost-sharing fraction α^{opt} is given as follows:*

- *If $\rho \le -\sigma_W/\sigma_X$, then $\alpha^{opt} = 1$.*
- *If $\rho/(1-\rho^2) > -(\sigma_W/\sigma_X)\,\mathbb{E}[Z_{1:N}]/\mathbb{E}[Z_{2:N} - Z_{1:N}]$, then $\alpha^{opt} = 0$.*
- *Otherwise, $\alpha^{opt} \in (0,1)$ is the unique solution to the cubic equation*

$$\left(\frac{\sigma_W}{\sigma_X}\alpha + \rho\right)^3 + r \cdot \left(\frac{\sigma_W}{\sigma_X}\alpha + \rho\right) + q \;=\; 0, \tag{14}$$

where $r = (1-\rho^2)\frac{\mathbb{E}[Z_{2:N} - 2Z_{1:N}]}{\mathbb{E}[Z_{2:N} - Z_{1:N}]} > 0$ and $q = (1-\rho^2)\left(\frac{\sigma_W}{\sigma_X} + \rho\right)\frac{\mathbb{E}[Z_{1:N}]}{\mathbb{E}[Z_{2:N} - Z_{1:N}]} < 0$.

Proposition 1 provides a complete characterization of the optimal cost-sharing fraction α^{opt} in terms of the model primitives, offering a specific prescription on using the cost-sharing mechanism in practice. In particular, it depends on the pair (X_n, W_n) only through its second moments $(\sigma_X, \sigma_W, \rho)$. Furthermore, the effect of N on α^{opt} works only through the expectations of the lowest and second-lowest standard normal order statistics $\mathbb{E}[Z_{1:N}]$ and $\mathbb{E}[Z_{2:N}]$.[15]

Notably, the optimal cost-sharing fraction should be set at 100% when the cost and the non-cost estimate have a strong negative correlation (i.e., $\rho \le -\sigma_W/\sigma_X < 0$), and set at 0% when the correlation becomes sufficiently positive (i.e., $\rho/(1-\rho^2) > -(\sigma_W/\sigma_X)\,\mathbb{E}[Z_{1:N}]/\mathbb{E}[Z_{2:N} - Z_{1:N}]$). Only for moderate levels of correlation (positive or negative) should the buyer strike a tradeoff between allocative inefficiency and information rent by choosing an interior fraction $\alpha^{opt} \in (0,1)$ as the solution to the cubic equation (14), whose closed-form expression is given by (C.8) in Appendix C.

The cost-sharing mechanism is also flexible in accommodating certain practical requirements; for example, limiting the contractors' financial obligations. In this case, the buyer can restrict the cost-sharing fraction α to be below a certain upper bound, say $\overline{\alpha} < 1$. Consequently, the optimal cost-sharing fraction will be $\min\{\alpha^{opt}, \overline{\alpha}\}$ by virtue of the quasi-convexity of $C(\alpha)$.

Our next result extracts the qualitative insights on the optimal cost-sharing fraction.

PROPOSITION 2 **(Comparative Statics on α^{opt}).** *Suppose that (X_n, W_n) follows a bivariate normal distribution with the second-order moments $(\sigma_X, \sigma_W, \rho)$. Then, for $\rho \leq 0$, the optimal cost-sharing fraction α^{opt} is decreasing in σ_W/σ_X. For $\rho \geq 0$, there exists a threshold $\overline{\sigma}(\rho) \in [0, \infty)$ with $\overline{\sigma}(0) = 0$, such that α^{opt} is increasing in $\sigma_W/\sigma_X \leq \overline{\sigma}(\rho)$ and decreasing otherwise. Moreover, α^{opt} is increasing in N and decreasing in ρ.*

Proposition 2 offers practical guidelines on prescribing the cost-sharing fraction α^{opt} according to (i) the relative uncertainty the buyer has regarding the contractors' non-cost and cost attributes as measured by σ_W/σ_X, (ii) the perceived correlation between these two attributes as measured by ρ, and (iii) the competitiveness of the supply base as measured by N.

Consider the negative-correlation case ($\rho \leq 0$) where, for example, a more expensive technology for executing a project can result in a superior non-cost attribute (e.g., faster completion, higher quality, lower failure rates, etc). Then, the buyer should let the selected contractor reimburse a larger fraction of her disutility cost if she is better informed about contractors' non-cost estimates (i.e., lower σ_W/σ_X), for instance, through a pre-qualification screening. In contrast, when the buyer faces contractors with fairly homogeneous costs but diverse non-cost aspects (i.e., higher σ_W/σ_X), say due to varied levels of expertise, a smaller cost-sharing fraction is recommended.

In contrast, when the correlation between cost and non-cost attributes is positive ($\rho \geq 0$), Proposition 2 suggests that the optimal cost-sharing fraction is not necessarily monotone in the ratio σ_W/σ_X. This is the case, for instance, when a contractor's cost is proportional to the time he takes to complete a project. The recommendation that the payment should be progressively more contingent on the realized non-cost attribute (through a larger α^{opt}) as this attribute becomes more heterogeneous (as long as $\sigma_W/\sigma_X \leq \overline{\sigma}(\rho)$) may appear counter-intuitive, in light of the fact that a larger α^{opt} would further increase the information rent generated by the non-cost dimension. The rationale here is that when the attributes are positively correlated, the buyer can use a higher cost-sharing fraction to discourage the selection of contractors with high non-cost estimates (and thereby high costs). Doing so helps gain efficiency and is cost-effective only when the information asymmetry in the non-cost dimension is low (i.e., $\sigma_W/\sigma_X \leq \overline{\sigma}(\rho)$) (and hence its contribution to the winning contractor's information rent is also low).

Overall, as the cost and non-cost attributes become more negatively correlated, a larger cost-sharing fraction is recommended so that the uncertainties in the cost and non-cost dimensions cancel each other and the overall information asymmetry faced by the buyer is reduced. This is akin to the notion of portfolio diversification, whereby one minimizes the overall risk by mixing investments with returns co-varying in opposite directions. Irrespective of the correlation, a more competitive supply-base (i.e., larger number of contractors, N) also allows the buyer to recoup a higher proportion of her inconvenience cost through a larger α^{opt}.

From a technical point of view, establishing the comparative statics on α^{opt} calls for a delicate analysis of the *marginal* effects of α on information rent and allocative inefficiency. The significance of Proposition 2 lies in successfully analyzing the dependence of these two effects on the model parameters (σ_X, σ_W, ρ and N). This is particularly challenging for the ratio σ_W/σ_X, as both marginal effects increase as σ_W/σ_X goes up, thus requiring us to compare their relative magnitudes. The effects of ρ and N on α^{opt} are comparatively more intuitive: According to the expression of $C(\alpha)$ in (C.4) of Appendix C, a larger value of N scales up the marginal gain in allocative efficiency (through $-\mathbb{E}[Z_{1:N}]$) and scales down the marginal increment in information rent (through $\mathbb{E}[Z_{2:N} - Z_{1:N}]$), thus leading to a higher α^{opt}. Similarly, the effect of ρ on the marginal rent dominates that on the marginal inefficiency, leading to the monotonicity of α^{opt} in ρ.

5.3. Optimal cost-sharing fraction under general belief distributions

In this section, we complement our theoretical findings from §5.2 on the optimal cost-sharing fraction under normal beliefs by numerically demonstrating their robustness for a much broader set of belief distributions on contractors' costs and non-cost attributes. Specifically, we demonstrate that (i) the comparative statics on α^{opt} under normal beliefs (Proposition 2) are robust with respect to distributional assumptions on (X_n, W_n), and also that (2) the cost-sharing fraction characterized by Proposition 1 (which is provably optimal under normal beliefs) offers an impressive rule of thumb to set the cost-sharing fraction in practice, irrespective of the buyer's beliefs on (X_n, W_n). To perform these tasks, we define our test-bed below, which is also used later in §6.

Numerical test-bed. To efficiently capture lower-order moments (e.g., means and variances) for different distributions, we generate the pair (X_n, W_n) as

$$X_n = \mu_X + \rho\sigma_X \left(\frac{\widetilde{W}_n - \widetilde{\mu}_W}{\widetilde{\sigma}_W}\right) + \sigma_X\sqrt{1-\rho^2}\left(\frac{\widetilde{X}_n - \widetilde{\mu}_X}{\widetilde{\sigma}_X}\right) \quad \text{and} \quad W_n = \mu_W + \sigma_W\left(\frac{\widetilde{W}_n - \widetilde{\mu}_W}{\widetilde{\sigma}_W}\right), \quad (15)$$

where $\widetilde{X}_n$ and $\widetilde{W}_n$ are two *independent* random variables with means $\widetilde{\mu}_X$ and $\widetilde{\mu}_W$ and standard deviations $\widetilde{\sigma}_X$ and $\widetilde{\sigma}_W$, respectively. The construction in (15) ensures that, in line with the case of normal distributions, the first-order and second-order moments of (X_n, W_n) are given by (μ_X, μ_W) and $(\sigma_X, \sigma_W, \rho)$, respectively, regardless of the *distribution* of (X_n, W_n).

Let $\mathcal{D}$ denote the set of the following 8 distributions: (i) beta distribution, Beta(a,b), with shape parameters $(a,b)=(1,1),(2,2),(2,5)$ and $(5,2)$, and (ii) gamma distribution, Gamma(a,b), with shape and scale parameters $(a,b)=(1,1),(3,1),(2,0.5)$ and $(4,0.5)$. Note that Beta$(1,1)$ and Gamma$(1,1)$ correspond to the standard uniform and exponential distributions, respectively.

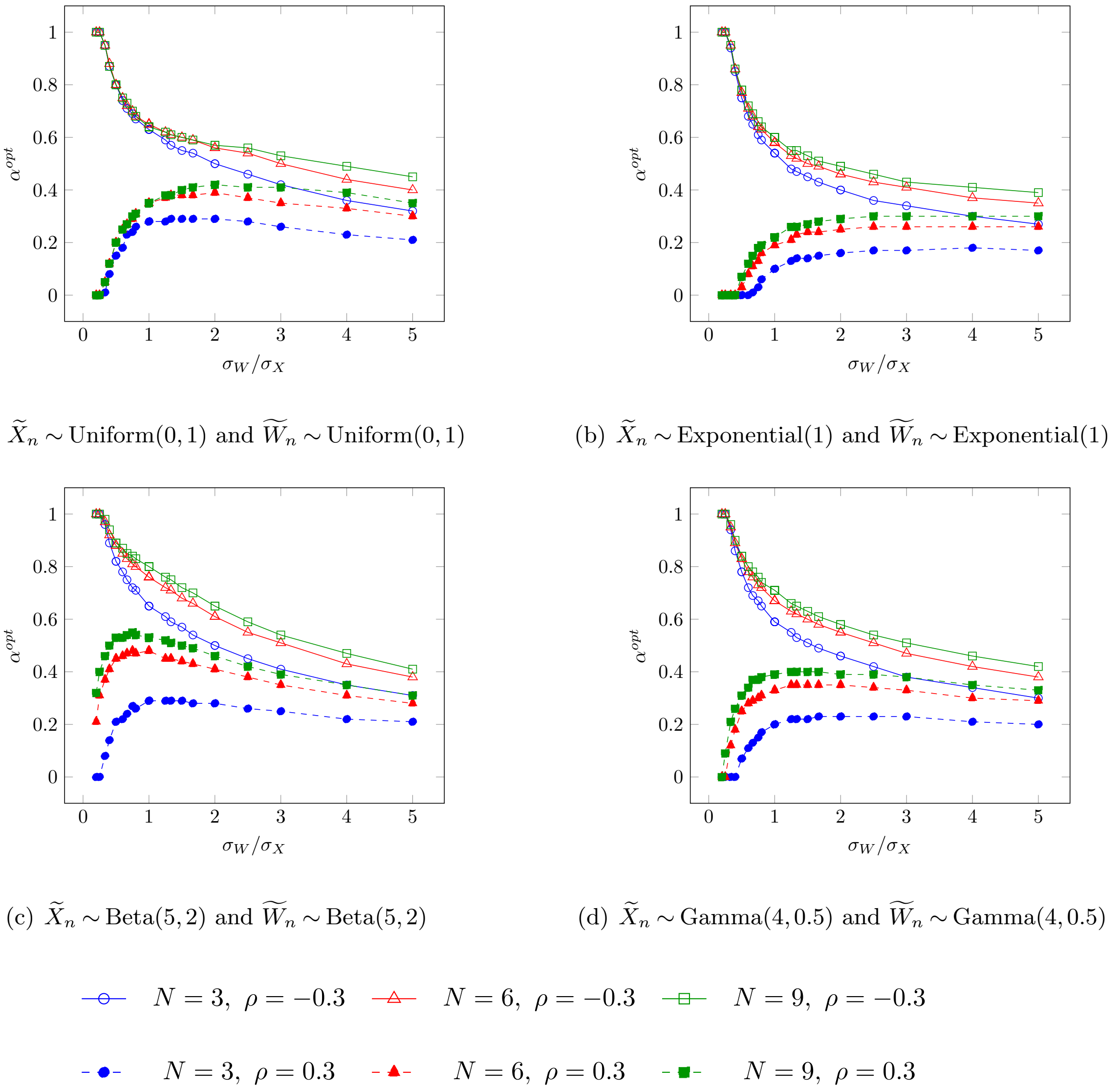
We conduct our numerical analysis for 24 possible distributions of (X_n,W_n) by choosing different combinations of distributions of $(\widetilde{X}_n,\widetilde{W}_n)$ as follows: (1) In the first set of 8 instances, $\widetilde{X}_n$ and $\widetilde{W}_n$ are identically distributed according to one of the distributions in the set $\mathcal{D}$. (2) In the second set of 8 instances, $\widetilde{X}_n$ is standard normal and the distribution of $\widetilde{W}_n$ is drawn from the set $\mathcal{D}$. (3) In the third and final set of 8 instances, the distribution of $\widetilde{X}_n$ is drawn from the set $\mathcal{D}$ and $\widetilde{W}_n$ is standard normal. For every distribution of (X_n,W_n), we fix $\mu_X=30$ and $\mu_W=30$, and allow $\sigma_X\in\{1,2,3,4,5\}$ and $\sigma_W\in\{1,2,3,4,5\}$. We let the number of contractors N vary from 2 to 10. The correlation coefficient $\rho\in\{-0.9,-0.6,-0.3,0,0.3,0.6,0.9\}$. In total, we thus have $24\times5\times5\times9\times7=37,800$ instances in our test-bed.

Comparative statics on the optimal cost-sharing fraction. Recall from Proposition 2 that, when (X_n,W_n) follows a bivariate normal distribution, (a) α^{opt} is decreasing in σ_W/σ_X if $\rho\le0$, (b) α^{opt} is initially increasing and then decreasing in σ_W/σ_X if $\rho>0$, (c) α^{opt} is increasing in N and decreasing in ρ. Our numerical results confirm the robustness of these findings with respect to the distributional assumption on (X_n,W_n); see Figure 1 for an illustration.

Optimal fraction from normal beliefs as a heuristic. Since the cost-sharing fraction characterized in Proposition 1 *only* requires the second-order moment information of (X_n,W_n) and the size N of the supply base, it can be used as a heuristic cost-sharing fraction for *any* (not necessarily normal) belief distribution of (X_n,W_n). Specifically, for any (X_n,W_n) defined according to (15) with second-order moments (σ_X,σ_W,ρ), let α_H^{opt} be the cost-sharing fraction obtained from Proposition 1, where the subscript H stands for "heuristic." It is a heuristic because while α_H^{opt} is provably optimal under normal beliefs, it may be sub-optimal under other belief distributions. This heuristic notably does not require specific distributional information of $(\widetilde{X}_n,\widetilde{W}_n)$ from which (X_n,W_n) is generated according to (15).

For every instance in our test-bed, we compute (i) the absolute gap between the heuristic fraction α_H^{opt} and the optimal fraction α^{opt}, that is, $|\alpha^{opt}-\alpha_H^{opt}|$, as well as (ii) the percentage increment in the buyer's cost from using the cost-sharing fraction α_H^{opt} as opposed to the optimal fraction α^{opt}:

$$\Delta_H=\frac{C(\alpha_H^{opt})-C(\alpha^{opt})}{C(\alpha^{opt})}.$$



(a) $\widetilde{X}_n \sim \text{Uniform}(0,1)$ and $\widetilde{W}_n \sim \text{Uniform}(0,1)$

(b) $\widetilde{X}_n \sim \text{Exponential}(1)$ and $\widetilde{W}_n \sim \text{Exponential}(1)$

(c) $\widetilde{X}_n \sim \text{Beta}(5,2)$ and $\widetilde{W}_n \sim \text{Beta}(5,2)$

(d) $\widetilde{X}_n \sim \text{Gamma}(4,0.5)$ and $\widetilde{W}_n \sim \text{Gamma}(4,0.5)$

Figure 1 The optimal cost-sharing fraction α^{opt} as a function of σ_W/σ_X for different values of N and ρ under 4 different distributions of $(\widetilde{X}_n, \widetilde{W}_n)$, $n=1,2,\ldots,N$.

A small value of $|\alpha^{opt} - \alpha_H^{opt}|$ or Δ_H indicates that α_H^{opt} is a good heuristic for α^{opt}. Indeed, as shown by Table 1, the heuristic α_H^{opt} offers an impressive rule of thumb to prescribe the cost-sharing fraction in practice, for beliefs including but not limited to normal beliefs.[16]

6. Performance Guarantees for the Optimal Cost-Sharing Mechanism

We now study the performance of the cost-sharing mechanism. Recall from §3 that the optimal solution to the buyer's problem $(\mathcal{P})$ is unknown and so is her optimal cost OPT. Thus, we benchmark the buyer's cost $C(\alpha^{opt})$ with two lower bounds on OPT: (i) the first-best cost FB, and (ii) the

Table 1 For each value of N, summary statistics (average, standard deviation and maximum) of Δ_H and of $|\alpha^{opt} - \alpha_H^{opt}|$ across $24 \times 5 \times 5 \times 7 = 4,200$ instances of the test-bed defined in §5.3, which includes 24 families of distributions of $(\widetilde{X}_n, \widetilde{W}_n)$, and, for each family, $5 \times 5 \times 7 = 175$ instances of $(\sigma_X, \sigma_W, \rho)$.

	N	2	3	4	5	6	7	8	9	10
Δ_H	Avg (%)	0.00	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.04
	Stdev (%)	0.01	0.02	0.04	0.04	0.05	0.06	0.06	0.07	0.07
	Max (%)	0.06	0.27	0.43	0.53	0.62	0.68	0.71	0.76	0.81
$\lvert\alpha^{opt} - \alpha_H^{opt}\rvert$	Avg	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06	0.07
	Stdev	0.01	0.03	0.04	0.05	0.06	0.06	0.07	0.07	0.08
	Max	0.08	0.20	0.27	0.33	0.38	0.40	0.43	0.45	0.47

optimal cost OPT_{NC} when the contractors' non-cost estimates are publicly-known (see Lemma 2). The metrics we use are the relative gaps $[C(\alpha^{opt}) - FB]/FB$ and $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$, both serving as upper bounds on the optimality gap $[C(\alpha^{opt}) - OPT]/OPT$ with the latter being the tighter bound because $FB \le OPT_{NC}$. Note that the relative gap $[C(\alpha^{opt}) - FB]/FB$ also quantifies the "loss of efficiency" under the optimal cost-sharing mechanism.

The goal of this section is to establish theoretical performance guarantees for the optimal cost-sharing mechanism, i.e., obtain upper bounds on the relative gaps $[C(\alpha^{opt}) - FB]/FB$ and $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$, for a broad range of belief distributions on (X_n, W_n) including but not limited to the bivariate normal distribution. We will also numerically assess the quality of our performance guarantees using the same test-bed defined in §5.3, and in doing so, illustrate the near-optimality of the cost-sharing mechanism.

Our first set of results (Theorems 5-6, and Corollary 1) establishes theoretical upper bounds on the relative gap $[C(\alpha^{opt}) - FB]/FB$ under two distinct families of belief distributions on (X_n, W_n). We present these results in an ascending order of the amount of distributional information that is needed to specify these distributional families. The family of distributions considered in Theorem 5 below requires the first- and second-order moment information on (X_n, W_n).[17]

Theorem 5 **(Performance Guarantee with respect to FB).** *Suppose that (X_n, W_n) belongs to a family of distributions such that $X_n + W_n$ has a log-concave cumulative distribution function*[18] *with its mean lower bounded by μ and its standard deviation upper bounded by σ. Then, the relative gap*

$$\frac{C(\alpha^{opt}) - FB}{FB} \ \le\ \Delta_1 \ :=\ \frac{\sigma \cdot 2/\sqrt{3}}{\mu - \sigma\left(\frac{N-1}{\sqrt{2N-1}}\right)}, \tag{16}$$

for all N such that $\mu - \sigma\left(\frac{N-1}{\sqrt{2N-1}}\right) > 0$. Further, the inequality in (16) becomes binding when $N = 2$, X_n is uniformly distributed over the interval $[\mu_X - \sqrt{3}\sigma_X, \mu_X + \sqrt{3}\sigma_X]$ and $W_n = \mu_W$ for $n = 1, 2$, $\mu = \mu_X + \mu_W$ and $\sigma = \sigma_X$.

To derive this result, it suffices to bound FB from below and $C(\alpha^{opt}) - FB$ from above. Since FB is equal to the expected lowest-order statistic of $X_n + W_n$ (see (8)), the lower bound on FB is obtained by leveraging existing results on bounds on expected order statistics (see, e.g., §4.2 of David and Nagaraja 2004). Since $C(\alpha^{opt}) \leq C(1)$, we can upper bound $C(\alpha^{opt}) - FB$ by $C(1) - FB$, which, according to (10), is equal to the expected spacing between the second-lowest and lowest order statistics of $X_n + W_n$. The expected spacing is decreasing in the number of contractors N (provided that $X_n + W_n$ has a log-concave cumulative distribution function) and admits an upper bound at $N = 2$ that only depends on σ.[19] Theorem 5 also shows that Δ_1 is tight, when the asymmetric information on the contractors' non-cost attributes is absent (i.e., $W_n = \mu_W$ for all n), in addition to the requirement that there are two contractors and their costs are uniformly distributed.

To assess the quality of the bound Δ_1, we compute its value across all instances of our test-bed specified in §5.3. For computing the bound, we only require the lower bound μ on the mean and the upper bound σ on the standard deviation of $X_n + W_n$ from the test-bed; specifically, we set $\mu = \mu_X + \mu_W = 60$, and $\sigma = \sqrt{\sigma_X^2 + \sigma_W^2 + 2\rho\sigma_X\sigma_W}$, where $\sigma_X \in \{1,2,3,4,5\}$, $\sigma_W \in \{1,2,3,4,5\}$ and $\rho = \{-0.9, -0.6, -0.3, 0, 0.3, 0.6, 0.9\}$. For different values of N, Table 2 presents the summary statistics (average, standard deviation and maximum) of Δ_1 across $5 \times 5 \times 7 = 175$ instances of (μ, σ). It is evident that the upper bound on the relative gap between $C(\alpha^{opt})$ and FB is close to 10% on average, illustrating that the cost-sharing mechanism is indeed an attractive and near-optimal solution to the buyer's mechanism design problem $(\mathcal{P})$.[20]

Table 2 The summary statistics of Δ_1, for different values of N, when $\mu = \mu_X + \mu_W = 60$, and $\sigma = \sqrt{\sigma_X^2 + \sigma_W^2 + 2\rho\sigma_X\sigma_W}$, where $\sigma_X \in \{1,2,3,4,5\}$, $\sigma_W \in \{1,2,3,4,5\}$ and $\rho = \{-0.9, -0.6, -0.3, 0, 0.3, 0.6, 0.9\}$.

N	2	3	4	5	6	7	8	9	10
Avg (%)	8.70	8.96	9.17	9.35	9.52	9.67	9.82	9.96	10.10
Stdev (%)	3.98	4.20	4.38	4.54	4.69	4.84	4.98	5.11	5.25
Max (%)	20.70	21.95	22.99	23.94	24.84	25.71	26.55	27.39	28.22

Finally, from (16), we note that Δ_1 increases in σ, which is an upper bound on the standard deviation of $X_n + W_n$. That is, the performance guarantee Δ_1 deteriorates as the buyer becomes more uncertain regarding $X_n + W_n$. Inspired by this observation, we further improve our performance guarantee relative to Δ_1 by establishing another performance guarantee via the notion of dispersive ordering, which also implies ordering of random variables with respect to their standard deviations (see Shaked and Shanthikumar 2007, Equation 3.B.25). First, we present the formal definition of the dispersive order, and then, we present our result in Theorem 6.

Definition 1 (Shaked and Shanthikumar 2007). Let X and Y be two random variables with distribution functions F and G, respectively. Let F^{-1} and G^{-1} be the right continuous inverses of F and G, respectively. The random variable X is said to be *smaller in dispersive order* than Y, denoted by $X \leq_{disp} Y$, if $F^{-1}(b) - F^{-1}(a) \leq G^{-1}(b) - G^{-1}(a)$ for all $0 < a \leq b < 1$.

The dispersive order, as defined above, ranks two random variables according to the distance between *any* two quantiles of each variable. Thus, it requires more distributional knowledge about these random variables than just the second moment information, and hence is a stronger notion than the standard deviation to compare the uncertainties of belief distributions. In particular, while two distributions can always be ranked according to their standard deviations, they may not be ranked according to the dispersive order, i.e., the dispersive order is a partial order.

Theorem 6 **(Performance Guarantee with respect to FB via Dispersive Order)**. *Suppose that, for $n = 1, 2, \ldots, N$, (X_n, W_n) belongs to a family of distributions such that $X_n + W_n \leq_{disp} \overline{Z}_n$ with $\mathbb{E}[\overline{Z}_n] \leq \mathbb{E}[X_n + W_n]$ for some random variable $\overline{Z}_n$, and that all $\overline{Z}_n$ are independent and identically distributed. Then, the relative gap*

$$\frac{C(\alpha^{opt}) - FB}{FB} \ \leq \ \Delta_2 \ := \ \frac{\mathbb{E}[\overline{Z}_{2:N} - \overline{Z}_{1:N}]}{\mathbb{E}[\overline{Z}_{1:N}]}, \tag{17}$$

for all N such that $\mathbb{E}[\overline{Z}_{1:N}] > 0$.

Relative to the performance guarantee Δ_1, the performance guarantee Δ_2 obtained in (17) requires the existence of an upper-bound random variable $\overline{Z}_n$ that dominates $X_n + W_n$ in the dispersive order. Since the dispersive order is only a partial order, the existence of $\overline{Z}_n$ is not always guaranteed. However, if such $\overline{Z}_n$ exists, Δ_2 can yield a potentially tighter upper bound on the relative gap $[C(\alpha^{opt}) - FB]/FB$ than Δ_1, as one can use $\overline{Z}_n$ to identify a more precise lower bound on FB and upper bound on $C(1) - FB$ (which further upper bounds $C(\alpha^{opt}) - FB$ as before) than the ones identified in Theorem 5. More specifically, to establish a lower bound on FB, we show that the expected value of the lowest order statistic of a random variable decreases as it becomes more dispersed (provided that its mean does not increase). Thus, our condition $X_n + W_n \leq_{disp} \overline{Z}_n$ along with $\mathbb{E}[\overline{Z}_n] \leq \mathbb{E}[X_n + W_n]$ immediately implies $FB \geq \mathbb{E}[\overline{Z}_{1:N}]$. To establish an upper bound on $C(1) - FB$, which is, as pointed earlier, the expected spacing between the second-lowest and lowest order statistics of $X_n + W_n$, we leverage the fact that the expected spacing increases as $X_n + W_n$ becomes larger in the dispersive order and hence obtain $C(1) - FB \leq \mathbb{E}[\overline{Z}_{2:N} - \overline{Z}_{1:N}]$.

To illustrate the strength of the performance guarantee Δ_2, our next corollary shows that Δ_2 immediately yields the performance guarantee for the bivariate normal family as examined in §5.2.

Here, we note that, for this family of distributions, $X_n + W_n$ follows normal distributions, which are ranked in dispersive order by their standard deviations.

COROLLARY 1 **(Normal Family)**. *Let (X_n, W_n) belong to a family of bivariate normal distributions such that $X_n + W_n$ has its mean lower bounded by μ and its standard deviation upper bounded by σ. Then, $X_n + W_n \leq_{disp} \overline{Z}_n$, where $\overline{Z}_n$ is the univariate normal random variable with mean μ and standard deviation σ. Thus, the performance guarantee Δ_2 for this normal family takes the form*

$$\frac{C(\alpha^{opt}) - FB}{FB} \;\leq\; \Delta_2 \;=\; \frac{\sigma \cdot \mathbb{E}[Z_{2:N} - Z_{1:N}]}{\mu + \sigma \cdot \mathbb{E}[Z_{1:N}]}, \tag{18}$$

for all N such that $\mu + \sigma \cdot \mathbb{E}[Z_{1:N}] > 0$, where Z_n is the standard normal random variable.

To assess the quality of the performance guarantee established by Theorem 6, we numerically evaluate Δ_2 given in Corollary 1 for the parameters in the test-bed specified in §5.3, which amount to $5 \times 5 \times 7 = 175$ instances of (μ, σ). For different values of N, the summary statistics of Δ_2 are listed in Table 3. As is evident from this table (and, for comparison purpose, also Table 2), the bound Δ_2, which leverages finer distributional information of the normal family, provides a *sharper* performance guarantee than Δ_1, which uses only the information on the first two moments.

Table 3 The summary statistics of Δ_2, for different values of N, when (X_n, W_n) belongs to the normal family defined in Corollary 1 with $\mu = \mu_X + \mu_W = 60$ and $\sigma = \sqrt{\sigma_X^2 + \sigma_W^2 + 2\rho\sigma_X\sigma_W}$, where $\sigma_X \in \{1,2,3,4,5\}$, $\sigma_W \in \{1,2,3,4,5\}$ and $\rho = \{-0.9, -0.6, -0.3, 0, 0.3, 0.6, 0.9\}$.

N	2	3	4	5	6	7	8	9	10
Avg (%)	8.49	6.54	5.75	5.34	5.02	4.86	4.66	4.56	4.43
Stdev (%)	3.88	3.05	2.73	2.56	2.43	2.36	2.28	2.24	2.19
Max (%)	20.16	15.94	14.27	13.43	12.77	12.46	12.04	11.86	11.59

Finally, we establish below a theoretical upper bound on the relative gap $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$, which provides a better performance guarantee for cost-sharing mechanisms via a tighter lower bound OPT_{NC} (than FB) for OPT.

THEOREM 7 **(Performance Guarantee with respect to OPT_{NC})**. *Suppose that (X_n, W_n) belongs to a family of distributions such that X_n and W_n are independent with log-concave probability densities, the probability density of X_n is upper bounded by a constant f_m, and $X_n + W_n$ has its mean lower bounded by μ and its standard deviation upper bounded by σ. Then, the relative gap*

$$\frac{C(\alpha^{opt}) - OPT_{NC}}{OPT_{NC}} \;\leq\; \Delta_3 \;:=\; \frac{\sigma \cdot \frac{2}{\sqrt{3}} - \frac{1}{(N+1)\cdot f_m}}{\mu - \sigma\left(\frac{N-1}{\sqrt{2N-1}}\right) + \frac{1}{(N+1)\cdot f_m}}, \tag{19}$$

for all N such that $\mu - \sigma\left(\frac{N-1}{\sqrt{2N-1}}\right) + \frac{1}{(N+1)\cdot f_m} > 0$.

Similar to the performance guarantee Δ_1 established in Theorem 5, the performance guarantee Δ_3 obtained in (19) also requires only coarse information about the belief distribution of (X_n, W_n). In addition to the first two moments as required by Δ_1, the bound Δ_3 also requires one more piece of distributional information, namely the maximum probability density f_m of the belief distribution on the contractor's cost X_n. We note that the existence of f_m is typically guaranteed as almost all commonly-used probability distributions have upper-bounded densities. While existing results on expected order statistics pave a way for a bound on FB, similar characterizations of OPT_{NC} are lacking due to the non-linearity of the virtual-cost function associated with a contractor's cost. In the proof of Theorem 7, we overcome this challenge by using the independence assumption[21] (between X_n and W_n) which allows the virtual-cost to be written as a function of only the contractor's cost (X_n), and subsequently, by using the bounded density assumption (on X_n) to lower bound the virtual-cost. As a result, we establish that $OPT_{NC} \geq FB + 1/(f_m \cdot (N+1))$, which, together with the lower bound on FB (as obtained in Theorem 5), offers a lower bound on OPT_{NC}.

In essence, the performance guarantee Δ_3 improves upon Δ_1 through a lower bound on OPT_{NC} that is strictly tighter than FB by an amount $1/(f_m \cdot (N+1))$, which increases as N or f_m decrease. As such, Δ_3 will demonstrate its value (as a superior performance guarantee relative to Δ_1), when the number of contractors is small in the supply market (which is often the case when the firm is outsourcing a complex project) or when the maximum probability density of X_n is small (which is often the case when the contractors' costs are fairly diverse).[22]

7. Contractors' Intrinsic Incentives to Increase the Non-Cost Attribute

Recall that in our base model, the winning contractor can only manipulate the project's non-cost attribute *ex post* (i.e., after the non-cost attribute's realization), and the incentive of doing so stems exclusively from the buyer's payment scheme. In this section, we examine situations where the winning contractor can shift the distribution of the project's non-cost attribute upwards *ex ante* (i.e., before the non-cost attribute's realization), because of his intrinsic incentive to reduce his cost of executing the project. For example, the winning contractor can potentially delay the project's completion by *privately* utilizing fewer resources (e.g., workers) and thereby possibly lowering his cost.[23] In the presence of such intrinsic incentives, the buyer not only faces the adverse selection issue as in the base model (due to the privacy of contractors' costs and non-cost estimates), but also faces a moral hazard issue due to the winning contractor's hidden action. Nonetheless, we will demonstrate that the cost-sharing mechanism remains an effective solution for the buyer.

To this end, we extend our base model to incorporate the contractors' intrinsic incentives to increase the project's non-cost attribute, or equivalently to increase the buyer's disutility cost. A buyer needs to award a project to one among N contractors and aims to minimize her expected total cost. Each contractor $n = 1, 2, \ldots, N$ is endowed with a two-dimensional private type (x_n, w_n), which is a realization of the random variable pair (X_n, W_n). Here, as in our base model, x_n denotes contractor n's cost of executing the project and w_n denotes the buyer's expected disutility cost from working with contractor n. If selected, contractor n, while executing the project, can shirk (e.g., utilize fewer resources) by an amount $s_n \in [0, \overline{s}]$ for some $\overline{s} > 0$, where s_n is unobservable to the buyer. As a result, the contractor's cost reduces from x_n to $x_n - g \cdot s_n$, whereas the buyer's expected disutility cost increases from w_n to $w_n + h \cdot s_n^2/2$. Here, $g \geq 0$ and $h \geq 0$ are publicly-known parameters that measure the marginal effects of the contractor's shirking on his own cost and on the buyer's disutility cost, respectively. When $g = h = 0$, this setting reduces to the base model studied in §3.[24] Since every contractor n's cost and non-cost estimate are now endogenously correlated through his action s_n, we assume that X_n and W_n are independent (mutually as well as across n). Following §5, we further assume (X_n, W_n) to be normally distributed with means (μ_X, μ_W) and standard deviations (σ_X, σ_W).

In the setting described above, the buyer faces a mixed mechanism-design problem, featuring two-dimensional private information represented by (x_n, w_n), and a hidden action represented by s_n. Therefore, under the cost-sharing mechanism CS_α (described in §5), the cost-sharing fraction α plays a dual role: it not only balances the tradeoff between allocative inefficiency and information rent as in (10), but also acts to mitigate the moral hazard friction by influencing contractors' shirking decisions. More specifically, under the cost-sharing mechanism CS_α, contractor n, if selected, chooses $s_n \in [0, \overline{s}]$ to minimize his total cost $x_n^{s_n} + \alpha w_n^{s_n}$, where $x_n^{s_n} = x_n - g \cdot s_n$ and $w_n^{s_n} = w_n + h \cdot s_n^2/2$. Thus, it is optimal for contractor n to choose $s(\alpha) = \min\{\overline{s}, g/(\alpha h)\}$. In the limit, if the buyer sets the cost-sharing fraction α to 0, the cost-sharing mechanism CS_α reduces to a price-only auction, in which case, the winning contractor finds it optimal to shirk by the maximum amount $\overline{s}$. As α increases, the incentive to shirk diminishes (i.e., $s(\alpha)$ is non-increasing in α). We assume that $g/h < \overline{s}$ to avoid the uninteresting case in which $s(\alpha) \equiv \overline{s}$ for all $\alpha \in [0, 1]$. Following similar notation as in our base model, we use $l(\alpha, N)$ to denote the l-th lowest order statistic of $\left\{X_n^{s(\alpha)} + \alpha W_n^{s(\alpha)} : n = 1, \cdots, N\right\}$. The buyer's expected cost $C_s(\alpha)$ under the cost-sharing mechanism CS_α is:

$$C_s(\alpha) := \mathbb{E}\left[X_{2(\alpha,N)}^{s(\alpha)} + \alpha W_{2(\alpha,N)}^{s(\alpha)} + (1-\alpha)\, W_{1(\alpha,N)}^{s(\alpha)}\right]. \tag{20}$$

We denote the corresponding optimal cost-sharing fraction by $\alpha_s^{opt} := \arg\min_{0 \leq \alpha \leq 1} C_s(\alpha)$. As in §6, we will benchmark the performance of the optimal cost-sharing mechanism $C_s(\alpha_s^{opt})$ against that of the first-best scenario, in which both contractor's type (x_n, w_n) and action s_n are publicly observable. Under the first-best scenario, the buyer would select and dictate contractor $1(1,N)$ to shirk by $s(1)$, resulting in the first-best cost:

$$FB_s := \mathbb{E}\left[X^{s(1)}_{1(1,N)} + W^{s(1)}_{1(1,N)}\right]. \tag{21}$$

Proposition 3 below characterizes the optimal cost-sharing mechanism in the presence of contractors' intrinsic incentives to increase the project's non-cost attribute.

Proposition 3 **(Characterization of α_s^{opt} and Performance Guarantee on $C_s(\alpha_s^{opt})$).** *In the presence of contractors' intrinsic incentives to increase the project's non-cost attribute, (i) the optimal cost-sharing fraction α_s^{opt} decreases in σ_W and h, and increases in N and g, ceteris paribus; and (ii) the relative gap*

$$\frac{C_s(\alpha_s^{opt}) - FB_s}{FB_s} \;\leq\; \frac{\sigma \cdot \mathbb{E}[Z_{2:N} - Z_{1:N}]}{\mu + \sigma \cdot \mathbb{E}[Z_{1:N}] - g^2/(2h)}, \tag{22}$$

where $\mu = \mu_X + \mu_W$, $\sigma = \sqrt{\sigma_X^2 + \sigma_W^2}$ and N is such that $\mu + \sigma \cdot \mathbb{E}[Z_{1:N}] - g^2/(2h) > 0$.

In line with Proposition 2, the first result in Proposition 3 shows that the optimal fraction decreases as the informational asymmetry on W_n becomes more pronounced, and increases as the supply-base becomes more competitive. In addition, when the marginal benefit of reducing his cost (g) is large or the marginal increase in the buyer's disutility cost (h) is small, the winning contractor has a stronger incentive to shirk, which is curtailed via a larger α_s^{opt}.

The second result in Proposition 3 generalizes the performance guarantee given in (18) for the base model, to that given in (22) under our current setting. As expected, the performance of the optimal cost-sharing mechanism deteriorates in the presence of contractors' intrinsic incentives to increase the project's non-cost attribute. However, as long as the value of $g^2/(2h)$ is dominated by that of $\mu + \sigma\mathbb{E}[Z_{1:N}]$, the performance guarantee in (22) is not significantly weak relative to (18).

8. Conclusion

In this paper, we studied the design of procurement mechanisms for a buyer who wishes to award a project among contractors with multi-dimensional private information (namely, cost and an estimate of an *a priori* random non-cost attribute). Upon the project's completion, the buyer incurs a disutility cost inflicted by the winning contractor's non-cost attribute, which is subject to possible costless upward manipulation by the contractor. We find that the possibility of manipulation plays a pivotal role in generating information rent from the non-cost estimates in addition to that generated by the cost dimension, thus making the mechanism design problem a genuine two-dimensional one.

We further study the *cost-sharing mechanism* as a non-manipulable, easy-to-implement and near-optimal solution for the buyer. A cost-sharing mechanism augments the standard second-price auction by requiring the winning contractor to reimburse a pre-specified fraction of the buyer's disutility cost upon completion of the project. For implementation, we offer prescriptive recommendations on the choice of the cost-sharing fraction based on the correlation and relative degree of information asymmetry between the cost and non-cost attributes. Furthermore, we also demonstrate the near-optimal cost performance of the cost-sharing mechanism by establishing its theoretical performance guarantees. Finally, we show that the cost-sharing mechanism remains robust in the presence of contractors' intrinsic incentives to increase the project's non-cost attributes.

In the paper, we assumed that the buyer does not have any outside option to execute the project and must award the project to one among the N contractors. If the buyer does have an outside option with some finite total cost, we can augment the cost-sharing mechanism with a reserve price, which together with the cost-sharing fraction, can be optimized. Our preliminary exploration suggests that the optimal augmented cost-sharing mechanism still performs very well, and that the optimal cost-sharing fraction in the presence of an outside option (and hence with a reserve price) tends to be *larger* than that in the absence of an outside option (and hence without a reserve price). Intuitively, relative to our current setting without an outside option, the presence of an outside option can reduce the information rent borne by the buyer (through a reserve price), subsequently allowing her to improve allocative efficiency by employing a larger cost-sharing fraction.

There are several other settings with multi-dimensional information asymmetry for which cost-sharing mechanisms can be investigated in future research. For instance, in settings with multiple ex-post observable non-cost attributes, one can extend our cost-sharing mechanism to attach (possibly different) cost-sharing fractions to each non-cost attribute. More generally, one can consider a broader class of cost-sharing mechanisms in which the winning contractor's reimbursement is not necessarily a constant proportion of the buyer's disutility cost as examined in this paper. Further, one can also envision other types of contingent clauses. For instance, manufacturers typically include in their supply contracts contingent clauses for future business or supplier-development investment based on the quality (a non-cost attribute) delivered by their suppliers in the past.

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Endnotes

[1]Quality is another prominent non-cost attribute and affects the buyer's maintenance cost.

[2]We assume that the buyer is incapable of completing the project in-house (e.g., due to lack of expertise or technology). This assumption is consistent with our motivating examples, where state agencies must rely on private contractors to repair highways (e.g., California Department of Transportation does not use any reserve price in its public procurement auctions; see Krasnokutskaya and Seim 2011). In our conclusion (§8), we comment on the consequence of having an outside option with a finite total cost to execute the project.

[3]Downward manipulation of the ex post non-cost attribute is typically *not* costless in practice as, for example, the winning contractor needs to exert costly effort to expedite the completion of a project.

[4]The function $V(\cdot)$ only acts to translate the contractor's non-cost attribute to the buyer's disutility cost, which is what matters in our subsequent analysis. Further, V is common knowledge. For instance, the Texas Department of Transportation publishes the road user costs regularly (Texas Department of Transportation 2018); similarly, facility owners and ESCOs agree on how energy savings will be calculated (U.S. Department of Energy 2015).

[5]In a similar spirit, Chu and Sappington (2015) consider a setting in which a supplier can understate his production capacity in a costless manner. They also point out that the incentives to do so can arise in practice where, for example, the supplier of a commodity good is paid at the price that equates the supply and demand (see the references therein).

[6]For notational simplicity, we do not specify the support of the random variables X_n, Y_n and T_n wherever it is not necessary to do so (e.g., in Lemma 1 and in the formulation of the buyer's problem ($\mathcal{P}$)), and specify it wherever necessary (e.g., when the random variables are discrete as in Section 4 or when they are continuous as in Section 5).

[7]In the paper, the expectations are taken over *all* random variables (in capital letters) inside the expectation operator $\mathbb{E}[\cdot]$. Variables excluded from expectation are kept in lower case.

[8]The sequential realization of contractors' private information in our setting resembles the information structure of the sequential screening problem in Courty and Li (2000).

[9]The full-rank condition (6) is relatively easy to satisfy in the sense that the set of conditional probabilities $\pi(\cdot,\cdot)$ violating (6) has measure zero in the space of matrices representing all conditional probabilities.

[10]In our definition of $\widetilde{U}(\cdot,\cdot)$, we drop the subscript n since the contractors are ex ante symmetric.

[11]For the discretized version, the buyer's problem ($\mathcal{P}$) is a finite-dimensional linear program, which allows us to leverage existing LP theory to establish Theorems 2 and 3. We note that Theorem 1 holds in general, even when the distributions of (X_n, Y_n, T_n) are unbounded or continuous. In Appendix B.3, we establish a slightly weaker version of Theorem 2 in a setting with continuous distributions. However, Theorem 3, which is established as a *negative* result by identifying a sufficient condition for the *non-existence* of a non-manipulable payment scheme for the buyer to achieve OPT_{NC}, seems difficult to generalize to continuous distributions.

[12]The Revenue Equivalence Theorem (see e.g., Krishna 2009) applies to our setting, and therefore, the cost-sharing mechanism can be implemented using the first-price auction format (or any other auction format) that results in the same allocation as that under the second-price auction format.

[13]This linear correlation structure has been widely adopted in the literature (see, e.g., Kostamis et al. 2009). Also, the log-concavity assumption on ξ_n is now standard in the literature and includes most of the commonly-used distributions (see, e.g., Bagnoli and Bergstrom 2005, Li and Wan 2017).

[14]Here, we adopt the notational convention that, for a random variable X and an event A, $\mathbb{E}[X;A] = \mathbb{E}[X \mid A] \cdot \mathbb{P}[A]$.

[15]$\mathbb{E}[Z_{1:N}]$ and $\mathbb{E}[Z_{2:N}]$ can be computed in closed form for $N = 2, 3$ (see Exercise 3.2.5 of David and Nagaraja 2004) and through Monte Carlo simulation for larger values of N.

[16]Since Δ_H remains consistently close to 0, the larger values of $|\alpha^{opt} - \alpha_H^{opt}|$ for a few instances in the test-bed must be due to the buyer's cost $C(\alpha)$ being less sensitive to α.

[17]It is a common approach in the operations management literature (e.g., Gallego and Moon 1993) to define a family of probability distributions that have pre-specified first two moments.

[18]A sufficient condition to ensure the log-concavity of the cumulative distribution function of $X_n + W_n$ is the log-concavity of its probability density function (Bagnoli and Bergstrom 2005).

[19]All the performance guarantees obtained for the cost-sharing mechanism in this section leverage the relaxation $C(\alpha^{opt}) \leq C(1)$, and hence, are also applicable on the total-cost auction, i.e., on the cost-sharing mechanism that mandates the winning contractor to reimburse 100% of the buyer's inconvenience cost.

[20]From Table 2, we also note that, on average, the bound Δ_1 becomes loose for large N. This is because our upper bound on $C(\alpha^{opt}) - FB$ does not depend on N, whereas, our lower bound on FB, which is, to the best of our knowledge, the best bound on FB (David and Nagaraja 2004), becomes loose for large N. In practice, N is likely to be small (see, e.g., Lewis and Bajari 2011 and the empirical evidence presented in Appendix F) and thus we assess Δ_1 and our subsequent performance guarantees for $N \leq 10$. For large values of N, Proposition D.1 in Appendix D shows that the relative gap $\left[C(\alpha^{opt}) - FB\right]/FB$ converges to zero under some mild conditions.

[21] In Table D.1 of Appendix D, using our test-bed defined in §5.3, we numerically demonstrate that when X_n and W_n are *correlated*, the relative gap $\left[C(\alpha^{opt}) - OPT_{NC}\right]/OPT_{NC}$ is below 0.6%, on average and 4.39%, at most.

[22]For the family of distributions of (X_n, W_n) defined in Theorem 7, since $\Delta_3 \leq \Delta_1$, and also since Δ_1 is close to 10% on average (see Table 2), we omit illustrating Δ_3 numerically.

[23]We thank an anonymous referee for suggesting this possibility.

[24]As in the base model, we allow contractors' costless upward manipulation of the non-cost attribute. The cost-sharing mechanism is again non-manipulable (as in §5), but may generate an incentive for the winning contractor to shirk (i.e., choose $s_n > 0$).

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Electronic Companion

Procurement with Cost and Non-Cost Attributes: Cost-Sharing Mechanisms

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Appendix A: Proofs for §3

Proof of Lemma 1. The buyer and contractors play a game with communication: The buyer first announces and commits to an (arbitrary) mechanism $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$, which lets each contractor n send his a message η_n from an ambient message space $\mathcal{M}$, and then maps $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_N)$ to the probability $\widehat{Q}_n(\boldsymbol{\eta})$ of awarding the project to contractor n and the payment $\widehat{M}_{mn}(\boldsymbol{\eta}, \widetilde{t}_n)$ made to each contractor m when contractor n is awarded the project. Here, we allow $\widehat{M}_{mn}$ to be also contingent upon the ex post non-cost attribute $\widetilde{t}_n$ delivered (and potentially inflated) by the winning contractor n to the buyer. Accordingly, we let the random variable $\widehat{A}(\boldsymbol{\eta}) \in \{1, \cdots, N\}$ to denote the identity of the winning contractor, i.e. $\widehat{A}(\boldsymbol{\eta}) = n$ with probability $\widehat{Q}_n(\boldsymbol{\eta})$.

Given the buyer's mechanism $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$, each contractor (i) chooses his message and, if awarded the project, (ii) decides the amount to inflate the project's ex post non-cost attribute to maximize his expected total payoff. Under $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$, a Bayesian Nash equilibrium (BNE) among all contractors thus consists of (i) a reporting strategy $\eta_n^*(x_n, y_n) \in \mathcal{M}$, which maps each contractor n's cost x_n and non-cost estimate y_n to a message in $\mathcal{M}$, and (ii) an inflation strategy $\gamma_n^*(\eta_n, t_n) \geq 0$, which inflates the project's ex post non-cost attribute $T_n = t_n$ to $t_n + \gamma_n^*(\eta_n, t_n)$, provided that contractor n has sent a message η_n (not necessarily an equilibrium one) and won the project. (As can be seen below, the notation indicates that contractor n's inflation strategy only depends on his message η_n and the project's non-cost attribute t_n.)

We now describe the equilibrium condition. In order for γ_n^* to be part of BNE, each contractor n, if awarded the project, finds it optimal not to unilaterally deviate from the inflation amount

$$\gamma_n^*(\eta_n, t_n) = \underset{\widetilde{\gamma} \geq 0}{\arg\max}\ \mathbb{E}\left[\widehat{M}_{nn}\left(\eta_n, \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n}, \mathbf{Y}_{-n}), t_n + \widetilde{\gamma}\right) \,\middle|\, \widehat{A}\left(\eta_n, \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n}, \mathbf{Y}_{-n})\right) = n\right], \quad \text{(A.1)}$$

where $\boldsymbol{\eta}_{-n}^*(\mathbf{x}_{-n}, \mathbf{y}_{-n})$ is the vector $\boldsymbol{\eta}^*(\mathbf{x}, \mathbf{y}) = (\eta_1^*(x_1, y_1), \cdots, \eta_N^*(x_N, y_N))$ with $\eta_n^*(x_n, y_n)$ dropped, and the conditional expectation with respect to $(\mathbf{X}_{-n}, \mathbf{Y}_{-n})$ signifies that at the time of making inflation decision, the contractor n only knows his message η_n, the project's non-cost attribute t_n, and the fact that he has won the project (i.e., $\widehat{A}\left(\eta_n, \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n}, \mathbf{Y}_{-n})\right) = n$) but does not know other contractors' messages.

In order for η_n^* to be part of BNE, each contractor n finds it optimal not to unilaterally deviate from the message

$$\eta_n^*(x_n,y_n)=\underset{\widetilde{\eta}\in\mathcal{M}}{\arg\max}\ \mathbb{E}\Big[\widehat{M}_n\Big(\widetilde{\eta},\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),y_n,\mathbf{Y}_{-n}\Big)-\ x_n\widehat{Q}_n\Big(\widetilde{\eta},\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\Big)\Big], \quad (A.2)$$

where

$$\widehat{M}_n(\boldsymbol{\eta},\mathbf{y}):=\sum_{m=1}^{N}\widehat{Q}_m(\boldsymbol{\eta})\cdot\mathbb{E}\left[\widehat{M}_{nm}(\boldsymbol{\eta},T_m+\gamma_m^*(\eta_m,T_m))\,\Big|\,Y_m=y_m\right]. \quad (A.3)$$

Subsequently, under the mechanism $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$, the expected equilibrium payoff for contractor n is given by

$$\mathbb{E}\Big[\widehat{M}_n\Big(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),y_n,\mathbf{Y}_{-n}\Big)-\ x_n\widehat{Q}_n\Big(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\Big)\Big], \quad (A.4)$$

and the expected total cost for the buyer is

$$\sum_{n=1}^{N}\mathbb{E}\left[\widehat{M}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y}),\mathbf{Y}\right)+\widehat{Q}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y})\right)V\left(T_n+\gamma_n^*(\eta_n^*(X_n,Y_n),T_n)\right)\right]. \quad (A.5)$$

In particular, contractor n is willing to participate in the mechanism $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$ if and only if his expected payoff in (A.4) is nonnegative.

From $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$, we construct a new direct mechanism $(\mathbf{Q},\mathbf{M})$, which, based on contractors' reported type profile $(\mathbf{x},\mathbf{y})$,

- awards the project to contractor n with probability

$$Q_n(\mathbf{x},\mathbf{y}):=\widehat{Q}_n\left(\boldsymbol{\eta}^*(\mathbf{x},\mathbf{y})\right), \quad (A.6)$$

- makes the payment

$$\begin{aligned}M_{nn}(x_n,y_n,t_n):=\mathbb{E}\Big[&\widehat{M}_{nn}\left(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),t_n+\gamma_n^*(\eta_n^*(x_n,y_n),t_n)\right)\\ &\Big|\ \widehat{A}\left(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=n\Big]\end{aligned} \quad (A.7)$$

  to contractor n if he is awarded the project and delivers an ex post non-cost attribute t_n, and

- makes the payment

$$\begin{aligned}M_{nm}(x_n,y_n):=\mathbb{E}\Big[&\widehat{M}_{nm}\left(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),T_m+\gamma_m^*(\eta_m^*(X_m,Y_m),T_m)\right)\\ &\Big|\ \widehat{A}\left(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=m\Big]\end{aligned} \quad (A.8)$$

  to contractor n if contractor $m\neq n$ is awarded the project.

We now demonstrate that under the mechanism $(\mathbf{Q},\mathbf{M})$, non-manipulation (i.e., no inflation of the project's ex post non-cost attribute) and truth-telling (i.e., truthfully report of their costs and non-cost estimates) form a BNE among contractors, subsequently establishing (NM) and (IC).

Suppose all contractors $m\neq n$ report their types truthfully and do not manipulate; contractor n with type (x_n,y_n) reports $(\widetilde{x}_n,\widetilde{y}_n)$. Then, according to (A.6), each contractor m expects to win the project with probability

$$\mathbb{E}\left[Q_m\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\right]=\mathbb{E}\left[\widehat{Q}_m\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)\right]. \tag{A.9}$$

According to (A.7), if he indeed wins and delivers a non-cost attribute $t_n+\tilde{\gamma}$ with $\tilde{\gamma}\geq 0$, he expects to receive a payment of

$$\begin{aligned}M_{nn}(\widetilde{x}_n,\widetilde{y}_n,t_n+\tilde{\gamma})=&\mathbb{E}\left[\widehat{M}_{nn}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),t_n+\tilde{\gamma}+\gamma_n^*(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),t_n+\tilde{\gamma})\right)\right.\\&\qquad\qquad\qquad\left.\Big|\ \widehat{A}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=n\right]\\\leq&\mathbb{E}\left[\widehat{M}_{nn}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),t_n+\gamma_n^*(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),t_n)\right)\right.\\&\qquad\qquad\qquad\left.\Big|\ \widehat{A}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=n\right]\\=&M_{nn}(\widetilde{x}_n,\widetilde{y}_n,t_n),\end{aligned}$$

where the inequality follows from the optimality of $\gamma_n^*(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),t_n)$ when the realized non-cost attribute is t_n as in (A.1). Since $(\widetilde{x}_n,\widetilde{y}_n)$ is arbitrary, we show that contractor n has no unilateral incentive to inflate t_n and establish that $M_{nn}(x_n,y_n,t_n)$ is non-increasing in t_n, which is equivalent to (NM) by definition (1).

Subsequently, using (1), contractor n's expected payment is given as

$$\begin{aligned}&\mathbb{E}\left[M_n\left(\widetilde{x}_n,\widetilde{y}_n,T_n\right)\mid Y_n=y_n\right]\\=&\mathbb{E}\left[Q_n\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\right]\mathbb{E}\left[M_{nn}(\widetilde{x}_n,\widetilde{y}_n,T_n)\mid Y_n=y_n\right]+\sum_{\substack{m=1\\m\neq n}}^{N}\mathbb{E}\left[Q_m\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\right]M_{nm}(\widetilde{x}_n,\widetilde{y}_n)\\=&\mathbb{E}\left[\widehat{Q}_n\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)\right]\\&\cdot\mathbb{E}\left[\widehat{M}_{nn}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),T_n+\gamma_n^*(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),T_n)\right)\ \Big|\ \widehat{A}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=n,Y_n=y_n\right]\\&+\sum_{\substack{m=1\\m\neq n}}^{N}\mathbb{E}\left[\widehat{Q}_m\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)\right]\\&\quad\cdot\mathbb{E}\left[\widehat{M}_{nm}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),T_m+\gamma_m^*(\eta_m^*(X_m,Y_m),T_m)\right)\ \Big|\ \widehat{A}\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)=m\right]\\=&\mathbb{E}\Big[\widehat{M}_n\Big(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),y_n,\mathbf{Y}_{-n}\Big)\Big],\quad\forall\widetilde{x}_n,\widetilde{y}_n,\end{aligned}\tag{A.10}$$

where the second equality follows from (A.7)-(A.9) and the third equality follows from the definition of conditional expectation and the definition in (A.3). Contractor n's expected cost is given by

$$x_n\mathbb{E}\left[Q_n\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\right]=x_n\mathbb{E}\left[\widehat{Q}_n\left(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\right)\right],\ \text{according to (A.9).}$$

Therefore, the optimality of $\eta_n^*(x_n,y_n)$ in (A.2) immediately implies that

$$\begin{aligned}
&\mathbb{E}\left[M_n\left(\widetilde{x}_n,\widetilde{y}_n,T_n\right)\mid Y_n=y_n\right]-x_n\mathbb{E}\left[Q_n\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\right]\\
=&\mathbb{E}\Big[\widehat{M}_n\Big(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),y_n,\mathbf{Y}_{-n}\Big)-\ x_n\widehat{Q}_n\Big(\eta_n^*(\widetilde{x}_n,\widetilde{y}_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\Big)\Big]\\
\le&\mathbb{E}\Big[\widehat{M}_n\Big(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n}),y_n,\mathbf{Y}_{-n}\Big)-\ x_n\widehat{Q}_n\Big(\eta_n^*(x_n,y_n),\ \boldsymbol{\eta}_{-n}^*(\mathbf{X}_{-n},\mathbf{Y}_{-n})\Big)\Big]\\
=&\mathbb{E}\left[M_n\left(x_n,y_n,T_n\right)\mid Y_n=y_n\right]-x_n\mathbb{E}\left[Q_n\left(x_n,\mathbf{X}_{-n},y_n,\mathbf{Y}_{-n}\right)\right],\quad\forall x_n,y_n,\widetilde{x}_n,\widetilde{y}_n,
\end{aligned}$$

establishing (IC), i.e., contractor n has no unilateral incentive to deviate from truth-telling.

In particular, the last equality above, together with (A.4), indicates that mechanism $(\mathbf{Q},\mathbf{M})$ is payoff-equivalent to mechanism $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$ for all contractors, who thus will participate in $(\mathbf{Q},\mathbf{M})$ if and only if (IR) holds.

Finally, the buyer's expected cost under $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$ given by (A.5) satisfies

$$\begin{aligned}
&\sum_{n=1}^{N}\mathbb{E}\left[\widehat{M}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y}),\mathbf{Y}\right)+\widehat{Q}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y})\right)V\left(T_n+\gamma_n^*(\eta_n^*(X_n,Y_n),T_n)\right)\right]\\
\text{(as } V(\cdot)\text{ is increasing and } \gamma_n^*\ge 0)\quad &\ge\sum_{n=1}^{N}\mathbb{E}\left[\widehat{M}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y}),\mathbf{Y}\right)+\widehat{Q}_n\left(\boldsymbol{\eta}^*(\mathbf{X},\mathbf{Y})\right)V\left(T_n\right)\right]\\
\text{(by (A.6) and (A.10))}\quad &=\sum_{n=1}^{N}\mathbb{E}\left[M_n\left(X_n,Y_n,T_n\right)+Q_n\left(\mathbf{X},\mathbf{Y}\right)V(T_n)\right],
\end{aligned}$$

which is the buyer's expected cost under $(\mathbf{Q},\mathbf{M})$. That is, the mechanism $(\mathbf{Q},\mathbf{M})$ is cost-improving over the mechanism $(\widehat{\mathbf{Q}},\widehat{\mathbf{M}})$ for the buyer. ■

Proof of Lemma 2. Lemma 2 is established by verifying the following two statements.

1. FB given in (2) is the optimal cost of the relaxed problem of $(\mathcal{P})$ ignoring (NM) and (IC).

   Proof. Once (NM) and (IC) are ignored, (IR) must be binding in order to minimize the objective function in $(\mathcal{P})$, i.e., we can substitute $\mathbb{E}\left[M_n\left(X_n,Y_n,T_n\right)\right]=\mathbb{E}\left[X_nQ_n\left(\mathbf{X},\mathbf{Y}\right)\right]$ in the objective function. Thus, the optimal cost of the relaxed problem is given by FB in (2).

2. OPT_{NC} given in $(\mathcal{P}_{NC})$ is the buyer's *ex ante* optimal cost when $\mathbf{y}$ is public knowledge.

Proof. Since the constraints in $(\mathcal{P}_{NC})$ are imposed for each $\mathbf{y}$, the optimization problem $(\mathcal{P}_{NC})$ becomes separable in $\mathbf{y}$ and hence $OPT_{NC} = \mathbb{E}[OPT_{NC}(\mathbf{Y})]$, where

$$OPT_{NC}(\mathbf{y}) = \min_{\{(\mathbf{Q},\mathbf{M})\}} \sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, y_n, T_n\right) + Q_n\left(\mathbf{X}, \mathbf{y}\right) V(T_n) \mid Y_n = y_n\right] \qquad (\mathcal{P}_{NC}(\mathbf{y}))$$

$$\text{subject to } M_n\left(x_n, y_n, t_n\right) \geq M_n\left(x_n, y_n, \widetilde{t}_n\right) \quad \forall\, x_n, t_n, \widetilde{t}_n \geq t_n \text{ and } n, \qquad \text{(A.11)}$$

$$\mathbb{E}\left[M_n\left(x_n, y_n, T_n\right) - x_n Q_n\left(x_n, \mathbf{X}_{-n}, \mathbf{y}\right) \mid Y_n = y_n\right]$$

$$\geq \mathbb{E}\left[M_n\left(\widetilde{x}_n, y_n, T_n\right) - x_n Q_n\left(\widetilde{x}_n, \mathbf{X}_{-n}, \mathbf{y}\right) \mid Y_n = y_n\right], \quad \forall\, x_n, \widetilde{x}_n, \text{ and } n, \qquad \text{(A.12)}$$

$$\mathbb{E}\left[M_n\left(x_n, y_n, T_n\right) - x_n Q_n\left(x_n, \mathbf{X}_{-n}, \mathbf{y}\right) \mid Y_n = y_n\right] \geq 0, \quad \forall\, x_n, \text{ and } n. \qquad \text{(A.13)}$$

Now, we recognize, by definition, that the above optimization problem $(\mathcal{P}_{NC}(\mathbf{y}))$ is nothing but the buyer's mechanism-design problem with public knowledge of $\mathbf{y}$ and hence that $OPT_{NC}(\mathbf{y})$ is its optimal cost. ∎

Appendix B: An illustrative Example and Proofs for §4

In this appendix, we suppress the index for the contractor's identity because of symmetry. We reserve the notation x_k, y_i and t_j to denote the k^{th} value in the ordered set $\mathcal{X} = \{x_1, x_2, \ldots, x_l\}$, i^{th} value in the ordered set $\mathcal{Y} = \{y_1, y_2, \ldots, y_p\}$, and j^{th} value in the ordered set $\mathcal{T} = \{t_1, t_2, \ldots, t_m\}$, respectively. Accordingly, the conditional probability $\pi(t_j, y_i)$ and cumulative probability $\Pi(t_j, y_i)$ are denoted by π_{ij} and Π_{ij}, respectively. Let $\boldsymbol{\pi}_i := (\pi_{i1}, \cdots, \pi_{im})$ and $\boldsymbol{\Pi}_i := (\Pi_{i1}, \cdots, \Pi_{im})$ for all i. Consider an arbitrary contractor who wins the project with probability Q_i^k when his cost is x_k and non-cost estimate is y_i, and, upon completion of the project, receives an expected payment M_{ij}^k if the non-cost experienced by the buyer is t_j. The constraints (NM), (IC) and (IR) for this contractor specialize in the discrete setting as follows:

$$M_{ij}^k \geq M_{i(j+1)}^k \quad \forall i, j, k, \qquad \text{(B.1)}$$

$$\sum_{j=1}^{m} \pi_{ij} M_{ij}^k - x_k Q_i^k \geq \sum_{j=1}^{m} \pi_{ij} M_{i'j}^{k'} - x_k Q_{i'}^{k'}, \quad \forall i, k, i', k', \qquad \text{(B.2)}$$

$$\sum_{j=1}^{m} \pi_{ij} M_{ij}^k - x_k Q_i^k \geq 0, \quad \forall i, k. \qquad \text{(B.3)}$$

B.1. An illustrative example for §4

In this section, we present a simple numerical example to illustrate that, if the buyer ignores the possibility of manipulation (that is, ignores the non-manipulability constraints (B.1)), then the optimal mechanism obtained thereby (as identified in Theorem 2) can generate incentives for the winning contractor to engage in manipulation.

In this illustrative example, there are $N = 2$ contractors, whose X_n and Y_n $(n = 1, 2)$ are independently and identically distributed as follows:

$$\mathbb{P}[X_n = 10] = \mathbb{P}[Y_n = 10] = 0.2, \quad \mathbb{P}[X_n = 20] = \mathbb{P}[Y_n = 20] = 0.3, \quad \mathbb{P}[X_n = 30] = \mathbb{P}[Y_n = 30] = 0.5.$$

The probability of T_n conditional on Y_n is given below:

π_{ij}		t_j 15	25	35
y_i	10	0.5	0.3	0.2
	20	0.3	0.2	0.5
	30	0.1	0.2	0.7

The buyer's disutility cost $V(\cdot)$ is linear.

For this illustrative example, we first solve $(\mathcal{P}_{NC})$ and identify the corresponding payment scheme. We then solve the relaxation of $(\mathcal{P})$ obtained by ignoring constraints (B.1), and identify the payment scheme that implements the optimal allocation of $(\mathcal{P}_{NC})$ and achieves OPT_{NC}. Finally, we note that such a payment scheme may not be monotone in the winning contractor's ex post non-cost attribute, generating incentive for contractors' manipulation.

Optimal payment $\{\widetilde{M}_i^k\}$ for $(\mathcal{P}_{NC})$. From Theorem 1, the optimal mechanism for $(\mathcal{P}_{NC})$ awards the project to a contractor with cost x_k and non-cost estimate y_i with probability[25]

$$\widetilde{Q}_i^k := \mathbb{E}\left[\widetilde{Q}_n(\mathbf{X}, \mathbf{Y}) \,\middle|\, X_n = x_k, Y_n = y_i\right] = \mathbb{E}\left[\widetilde{Q}_n(x_k, \mathbf{X}_{-n}, y_i, \mathbf{Y}_{-n})\right], \tag{B.4}$$

with $\widetilde{Q}_n(\cdot)$ given by (4), and receives the expected payment $\widetilde{M}_i^k = \widetilde{U}_i^k + x_k \widetilde{Q}_i^k$, where

$$\widetilde{U}_i^k := \widetilde{U}(x_k, y_i) \tag{B.5}$$

with $\widetilde{U}(x, y)$ given in the statement of Theorem 3. Table B.1 below computes $\{\widetilde{M}_i^k\}$ for our illustrative example.

Notably, the winning contractor's expected payment $\widetilde{M}_i^k$ only depends on his cost x_k and non-cost estimate y_i, but not on the ex post non-cost attribute t_j.

Table B.1 The optimal payment scheme $\{\widetilde{M}_i^k\}$ for problem $(\mathcal{P}_{NC})$.

$\widetilde{M}_i^k$		x_k 10	20	30
y_i	10	22	19.9	13.5
	20	19.5	17.15	9.75
	30	15.5	12.75	3.75

Optimal payment $\{\widehat{M}_{ij}^k\}$ for the relaxed problem of $(\mathcal{P})$ with (B.1) ignored. Since π_{ij} specified above satisfies the full rank condition (6), Theorem 2 implies the existence of a payment scheme that implements the same allocation $\widetilde{Q}_i^k$ (i.e., satisfies (B.2) and (B.3)) and achieves the cost OPT_{NC}, the optimal cost of problem $(\mathcal{P}_{NC})$. Therefore, to identify the optimal mechanism for the relaxed problem of $(\mathcal{P})$ with (B.1) ignored, the buyer needs to design the payment scheme to incentivize the contractors to report their costs and non-cost estimates truthfully in a way such that no extra rent for the contractors is generated in addition to that already generated in $(\mathcal{P}_{NC})$. Thus, the supporting payment scheme must entail contingent payment based on the ex post observation of the winning contractor's non-cost attribute to ensure that (i) constraints (B.2) and (B.3) hold, and (ii) the *expected* contingent payment (averaged over the realizations of the non-cost attribute) must be the same as the optimal payment $\{\widetilde{M}_i^k\}$ for $(\mathcal{P}_{NC})$, which, as mentioned earlier, is not contingent on the ex post non-cost attribute. These two requirements gives rise to a system of linear inequalities on the payment $\widehat{M}_{ij}^k$ received by a contractor with cost x_k, non-cost estimate y_i and ex post non-cost attribute t_j. Solving such a linear system yields the supporting payment scheme $\{\widehat{M}_{ij}^k\}$.

In particular, we can represent $\widehat{M}_{ij}^k$ by decomposing it into two components as

$$\widehat{M}_{ij}^k = \widetilde{M}_i^k + f_{ij}^k, \quad \forall (i,j,k),$$

where $\{f_{ij}^k\}$ is the *rewards/penalties* contingent on t_j in addition to $\widetilde{M}_i^k$ such that $\sum_j \pi_{ij} f_{ij}^k = 0$. For our illustrative example, $\{\widetilde{M}_i^k\}$ is already given in Table B.1 and $\{f_{ij}^k\}$ is computed in Table B.2.

Table B.2 The values of $\{f_{ij}^k\}$ for problem $(\mathcal{P})$ without the non-manipulability constraints (B.1). Note that f_{ij}^k is non-monotone in t_j and hence violates (B.1).

		$x_k = 10$			$x_k = 20$			$x_k = 30$		
f_{ij}^k		t_j 15	25	35	t_j 15	25	35	t_j 15	25	35
y_i	10	14.84	−22	−4.11	13.63	−19.9	−4.22	9.13	−13.5	−2.58
	20	14.88	−19.5	−1.13	15.1	−20.4	−0.9	11	−19	1
	30	17.88	−15.5	1.88	11.94	−12.75	1.94	0.94	−3.75	0.94

Consider, for example, a contractor who reported his cost as $x_k = 10$ and non-cost estimate as $y_i = 10$. From Table B.1, the buyer offers him the payment of $\widetilde{M}_i^k = 22$ (that only depends on his reported cost and non-cost estimate). In addition, from Table B.2, she also offers a reward of $f_{ij}^k = 14.84$ if he delivers the non-cost attribute as $t_j = 15$, and a penalty of $f_{ij}^k = -22$ and $f_{ij}^k = -4.11$ if he delivers the non-cost attributes as $t_j = 25$ and $t_j = 35$, respectively. Indeed, the expected value of rewards/penalties is $\sum_{j=1}^3 \pi_{ij} f_{ij}^k = 0.5 * 14.84 + 0.3 * (-22) + 0.2 * (-4.11) \approx 0$.

Payment $\{\widehat{M}_{ij}^k\}$ may be non-monotone in t_j. However, for this particular instance, the optimal penalty/reward f_{ij}^k is not monotone in the non-cost attribute t_j, and the same is true for $\{\widehat{M}_{ij}^k\}$, generating an incentive for manipulation: if the contractor realizes the non-cost attribute as $t_j = 25$, he has the incentive to inflate it (at no cost) to $t_j = 35$ in order to gain a higher payment (in the form of a lower penalty).

As illustrated by this example, if the buyer ignores the non-manipulability constraints (B.1) when solving $(\mathcal{P})$, the optimal mechanism may not automatically satisfy (B.1) and is therefore subject to contractors' manipulation. In fact, it is straightforward to verify that condition (7) holds for this illustrative example: for $(i,k) = (2,1)$ and $(\nu_1, \nu_2, \nu_3) = (0.5, 0.0, 0.5)$, we have

$$\sum_{i' \neq i} \nu_{i'} = 1, \quad \sum_{i' \neq i} \nu_{i'} \widetilde{U}_{i'}^k = 0.5 \times 12.2 + 0.5 \times 7.0 = 9.6 < 10.2 = \widetilde{U}_i^k, \text{ and}$$

$$\sum_{i' \neq i} \nu_{i'} \mathbf{\Pi}_{i'} = 0.5 \times [0.5, 0.8, 1.0] + 0.5 \times [0.1, 0.3, 1.0] = [0.3, 0.55, 1.0] \geq [0.3, 0.5, 1.0] = \mathbf{\Pi}_i.$$

Then, by Theorem 3, there does not exist any non-manipulable payment scheme that can implement $\widetilde{Q}_i^k$ and achieve OPT_{NC}; hence, $OPT > OPT_{NC}$.

B.2. Proofs for §4.

Lemma B.1. *For any given i, k, k', we have*

$$x_{k'} \widetilde{Q}_i^k + \widetilde{U}_i^{k'} \geq x_k \widetilde{Q}_i^k + \widetilde{U}_i^k. \tag{B.6}$$

Proof of Lemma B.1. From the definitions of $\widetilde{Q}_i^k$ and $\widetilde{U}_i^k$, we have:

$$x_{k'} \widetilde{Q}_i^k + \widetilde{U}_i^{k'} - \left(x_k \widetilde{Q}_i^k + \widetilde{U}_i^k \right) = \Delta_x \left\{ (k'-k) \widetilde{Q}_i^k - \sum_{\tilde{k}=k+1}^{k'} \widetilde{Q}_i^{\tilde{k}} \right\} = \Delta_x \sum_{\tilde{k}=k+1}^{k'} \underbrace{\left(\widetilde{Q}_i^k - \widetilde{Q}_i^{\tilde{k}} \right)}_{\geq 0} \geq 0, \text{ if } k' > k.$$

$$x_{k'} \widetilde{Q}_i^k + \widetilde{U}_i^{k'} - \left(x_k \widetilde{Q}_i^k + \widetilde{U}_i^k \right) = \Delta_x \left\{ \sum_{\tilde{k}=k'+1}^{k} \widetilde{Q}_i^{\tilde{k}} - (k-k') \widetilde{Q}_i^k \right\} = \Delta_x \sum_{\tilde{k}=k'+1}^{k} \underbrace{\left(\widetilde{Q}_i^{\tilde{k}} - \widetilde{Q}_i^k \right)}_{\geq 0} \geq 0, \text{ if } k' < k.$$

This completes the proof of Lemma B.1. ■

Proof of Theorem 1. As shown in the proof of Lemma 2, problem $(\mathcal{P}_{NC})$ can be decomposed into problems $(\mathcal{P}_{NC}(\mathbf{y}))$ that are separable in $\mathbf{y}$, and $OPT_{NC} = \mathbb{E}[OPT_{NC}(\mathbf{Y})]$. Given Y_n, since T_n is independent of X_n, following Myerson (1981) (see Laffont and Martimort 2009 for the discrete-type version), we first recognize that (A.12) and (A.13) hold if and only if, for every n and x_n,

$$\mathbb{E}\left[M_n(x_n, y_n, T_n) - x_n Q_n(x_n, \mathbf{X}_{-n}, \mathbf{y}) \mid Y_n = y_n\right] = \Delta_x \sum_{\substack{x' \in \mathcal{X} \\ x' > x_n}} \mathbb{E}[Q_n(x', \mathbf{X}_{-n}, \mathbf{y})], \tag{B.7}$$

$$\text{and} \quad \mathbb{E}\left[Q_n(x_n, \mathbf{X}_{-n}, \mathbf{y})\right] \text{ is decreasing in } x_n. \tag{B.8}$$

Substituting (B.7) into the objective function of $(\mathcal{P}_{NC}(\mathbf{y}))$ yields

$$\min_{\{(\mathbf{Q},\mathbf{M})\}} \quad \sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, y_n, T_n\right) + V(T_n) Q_n\left(\mathbf{X}, \mathbf{y}\right) \mid Y_n = y_n\right] \quad \text{subject to (A.11), (A.12) and (A.13)}$$

$$= \min_{\{(\mathbf{Q},\mathbf{M})\}} \quad \sum_{n=1}^{N} \mathbb{E}\left\{\mathbb{E}\left[\left\{X_n + \mathbb{E}[V(T_n) \mid Y_n = y_n]\right\} Q_n(X_n, \mathbf{X}_{-n}, \mathbf{y}) + \Delta_x \sum_{x' \in \mathcal{X}, x' > X_n} Q_n(x', \mathbf{X}_{-n}, \mathbf{y}) \,\middle|\, X_n\right]\right\}$$

$$\text{subject to (A.11), (B.7) and (B.8)}$$

$$= \min_{\{(\mathbf{Q},\mathbf{M})\}} \quad \sum_{n=1}^{N} \mathbb{E}\left[\left\{\psi(X_n, y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\right\} Q_n(X_n, \mathbf{X}_{-n}, \mathbf{y})\right] \quad \text{subject to (A.11), (B.7) and (B.8)},$$

where the last equality follows from the standard interchange of the order of summations (see Laffont and Martimort 2009).

We claim that any optimal mechanism for $\mathcal{P}_{NC}(\mathbf{y})$ *must* use the allocation rule $(\widetilde{Q}_1(\mathbf{x}, \mathbf{y}), \cdots, \widetilde{Q}_N(\mathbf{x}, \mathbf{y}))$ given by (4). First, consider the mechanism $\{\widetilde{\mathbf{Q}}, \widetilde{\mathbf{M}}\}$, where $\widetilde{\mathbf{Q}}$ is as defined in (4) and the payment scheme $\widetilde{\mathbf{M}}$ satisfies:

$$\widetilde{M}_n(x_n, y_n, t_n) = x_n \mathbb{E}[\widetilde{Q}_n(x_n, \mathbf{X}_{-n}, \mathbf{y})] + \Delta_x \sum_{x' \in \mathcal{X}, x' > x_n} \mathbb{E}[\widetilde{Q}_n(x', \mathbf{X}_{-n}, \mathbf{y})]. \tag{B.9}$$

Then, mechanism $\{\widetilde{\mathbf{Q}}, \widetilde{\mathbf{M}}\}$ is feasible to $\mathcal{P}_{NC}(\mathbf{y})$. Indeed, it satisfies (B.7) and (B.8) (since $\psi(x_n, y_n)$ is increasing in x_n). Since $\widetilde{\mathbf{M}}$ does not depend on t_n, it is non-manipulable, and thus, the constraint (A.11) is also satisfied.

Therefore, for any feasible mechanism $\{\mathbf{Q}, \mathbf{M}\}$ for $(\mathcal{P}_{NC}(\mathbf{y}))$ such that

$$\sum_{n=1}^{N} \left\{\psi(x_n, y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\right\} Q_n(\mathbf{x}, \mathbf{y}) > \sum_{n=1}^{N} \left\{\psi(x_n, y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\right\} \widetilde{Q}_n(\mathbf{x}, \mathbf{y}),$$

for some $(\mathbf{x},\mathbf{y}) \in \mathcal{X} \times \mathcal{Y}$, we must have

$$
\begin{aligned}
&\sum_{n=1}^{N} \mathbb{E}\left[\Big\{\psi(X_n,y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\Big\} Q_n(\mathbf{X},\mathbf{y})\right] \\
>&\sum_{n=1}^{N} \mathbb{E}\left[\Big\{\psi(X_n,y_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\Big\} \widetilde{Q}_n(\mathbf{X},\mathbf{y})\right],
\end{aligned}
$$

implying that $\{\mathbf{Q},\mathbf{M}\}$ cannot be optimal. The result follows. ■

Proof of Theorem 2. By Theorem 1, it suffices to show that there exists a payment scheme $\{\widehat{M}_{ij}^k\}$ that implements $\widetilde{Q}_i^k$ and $\widetilde{U}_i^k$, which, by (B.2) and (B.3), translates to

$$
\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k \geq \sum_{j=1}^{m} \pi_{ij} \widehat{M}_{i'j}^{k'} - x_k \widetilde{Q}_{i'}^{k'}, \quad \forall i,k,i',k', \tag{B.10}
$$

$$
\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k = \widetilde{U}_i^k, \quad \forall i,k. \tag{B.11}
$$

Substituting (B.11) into (B.10) and then swapping the indices (i,k) with (i',k'), we obtain that (B.10) and (B.11) are equivalent to

$$
\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k \geq \widetilde{U}_i^k, \quad \forall i,k, \tag{B.12}
$$

$$
\sum_{j=1}^{m} \pi_{i'j} \widehat{M}_{ij}^k - x_{k'} \widetilde{Q}_i^k \leq \widetilde{U}_{i'}^{k'}, \quad \forall i,k,i',k'. \tag{B.13}
$$

For an arbitrary (i,k), the system (B.12) and (B.13) can be rewritten in matrix form as

$$
pl+1\ \text{dimension}\left\{
\underbrace{\begin{bmatrix}
\text{l-tuple}\left\{\begin{matrix} -\boldsymbol{\pi}_1 \\ \vdots \\ -\boldsymbol{\pi}_1 \end{matrix}\right. \\
\vdots \\
\text{l-tuple}\left\{\begin{matrix} -\boldsymbol{\pi}_{i'} \\ \vdots \\ -\boldsymbol{\pi}_{i'} \end{matrix}\right. \\
\vdots \\
\text{l-tuple}\left\{\begin{matrix} -\boldsymbol{\pi}_p \\ \vdots \\ -\boldsymbol{\pi}_p \end{matrix}\right. \\
\text{single-tuple}\quad \boldsymbol{\pi}_i
\end{bmatrix}}_{m\ \text{dimension}}
\right.
\begin{bmatrix} \widehat{M}_{i1}^k \\ \vdots \\ \widehat{M}_{im}^k \end{bmatrix}
\geq
\begin{bmatrix}
\left.\begin{matrix} -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_1^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_1^l\right) \end{matrix}\right\} \text{l-tuple} \\
\vdots \\
\left.\begin{matrix} -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_{i'}^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_{i'}^l\right) \end{matrix}\right\} \text{l-tuple} \\
\vdots \\
\left.\begin{matrix} -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_p^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_p^l\right) \end{matrix}\right\} \text{l-tuple} \\
x_k \widetilde{Q}_i^k + \widetilde{U}_i^k \qquad \text{single-tuple}
\end{bmatrix}. \tag{B.14}
$$

Suppose there exists a $(pl+1)$-dimensional non-negative vector

$$\boldsymbol{\lambda} := (\lambda_1^1, \cdots, \lambda_1^l, \cdots, \lambda_{i'}^1, \cdots, \lambda_{i'}^l, \cdots, \lambda_p^1, \cdots, \lambda_p^l, \lambda_0) \geq \mathbf{0}$$

such that

$$\boldsymbol{\lambda} \begin{bmatrix} -\boldsymbol{\pi}_1 \\ \vdots \\ -\boldsymbol{\pi}_1 \\ \vdots \\ -\boldsymbol{\pi}_{i'} \\ \vdots \\ -\boldsymbol{\pi}_{i'} \\ \vdots \\ -\boldsymbol{\pi}_p \\ \vdots \\ -\boldsymbol{\pi}_p \\ \boldsymbol{\pi}_i \end{bmatrix} = \mathbf{0} \quad \text{and} \quad \boldsymbol{\lambda} \begin{bmatrix} -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_1^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_1^l\right) \\ \vdots \\ -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_{i'}^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_{i'}^l\right) \\ \vdots \\ -\left(x_1 \widetilde{Q}_i^k + \widetilde{U}_p^1\right) \\ \vdots \\ -\left(x_l \widetilde{Q}_i^k + \widetilde{U}_p^l\right) \\ x_k \widetilde{Q}_i^k + \widetilde{U}_i^k \end{bmatrix} > 0. \tag{B.15}$$

Then, left-multiplying $\boldsymbol{\lambda}$ on both sides of (B.14) would lead to a contradiction, proving the non-existence of a payment scheme $\{\widehat{M}_{ij}^k\}$ satisfying (B.10) and (B.11). Farkas' lemma asserts that the reverse also holds: the non-existence of a non-negative $\boldsymbol{\lambda}$ satisfying (B.15) must imply the existence of $\{\widehat{M}_{ij}^k : j = 1, \ldots, m\}$ satisfying (B.14). Since (i,k) is arbitrary, this, in turn, will show the existence of the payment scheme $\{\widehat{M}_{ij}^k\}$ satisfying (B.10) and (B.11).

Notice that the equality in (B.15) can be re-written as

$$\lambda_0 \pi_{ij} - \sum_{i'=1}^{p} \left(\sum_{k'=1}^{l} \lambda_{i'}^{k'} \right) \pi_{i'j} = 0, \quad \Leftrightarrow \quad \left(\lambda_0 - \sum_{k'=1}^{l} \lambda_i^{k'} \right) \pi_{ij} = \sum_{\substack{i'=1 \\ i' \neq i}}^{p} \left(\sum_{k'=1}^{l} \lambda_{i'}^{k'} \right) \pi_{i'j}, \quad \forall j = 1, \ldots, m.$$

Therefore, (6) immediately implies that

$$\lambda_0 = \sum_{k'=1}^{l} \lambda_i^{k'}, \quad \text{and} \quad \lambda_{i'}^{k'} \equiv 0 \text{ for all } i' \neq i, k'. \tag{B.16}$$

Using (B.16), the left-hand side of the inequality in (B.15) reduces to

$$\sum_{k'=1}^{l} \lambda_i^{k'} \underbrace{\left[x_k \widetilde{Q}_i^k + \widetilde{U}_i^k - \left(x_{k'} \widetilde{Q}_i^k + \widetilde{U}_i^{k'} \right) \right]}_{\leq 0} \leq 0,$$

where the inequality follows from (B.6). Thus, there does not exist a non-negative $\boldsymbol{\lambda}$ that satisfies (B.15), which establishes the feasibility of the system of linear inequalities represented by (B.12) and (B.13), or equivalently, (B.10) and (B.11). ■

Proof of Theorem 3. Consider an optimal mechanism $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$ to $(\mathcal{P})$ under which the buyer's expected cost equals OPT_{NC}. Then, $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$ must also solve $(\mathcal{P}_{NC})$, because $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$ is also feasible to $(\mathcal{P}_{NC})$ (which is a relaxation of $(\mathcal{P})$) and yields the optimal cost OPT_{NC} for $(\mathcal{P}_{NC})$. Then, by Theorem 1, there must exist $\widetilde{\mathbf{Q}}$ satisfying (4) such that $\widehat{\mathbf{Q}} = \widetilde{\mathbf{Q}}$ and $\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k = \widetilde{U}(x_k, y_i) = \widetilde{U}_i^k$ for all i, k. To prove Theorem 3, we now show that such a mechanism $(\widehat{\mathbf{Q}}, \widehat{\mathbf{M}})$ does not exist. That is, the following system of inequalities, derived from (B.1)–(B.3), is infeasible.

$$\widehat{M}_{ij}^k \geq \widehat{M}_{i(j+1)}^k \quad \forall i, j, k, \tag{B.17}$$

$$\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k \geq \sum_{j=1}^{m} \pi_{ij} \widehat{M}_{i'j}^{k'} - x_k \widetilde{Q}_{i'}^{k'}, \quad \forall i, k, i', k', \tag{B.10}$$

$$\sum_{j=1}^{m} \pi_{ij} \widehat{M}_{ij}^k - x_k \widetilde{Q}_i^k = \widetilde{U}_i^k, \quad \forall i, k, \tag{B.11}$$

Let

$$D := \left.\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}}_{m\text{-dimension}}\right\} (m-1)\text{-dimension}$$

Consider a fixed pair (i, k). For this pair, (B.17) can be rewritten as

$$D \begin{bmatrix} \widehat{M}_{i1}^k \\ \vdots \\ \widehat{M}_{im}^k \end{bmatrix} \geq \mathbf{0}. \tag{B.18}$$

From (B.14), Farkas' Lemma implies that the system defined by (B.17), (B.10), and (B.11), is infeasible if and only if there exists a $(m+pl)$-dimensional non-negative vector

$$\boldsymbol{\lambda} := (\bar{\lambda}_1, \cdots, \bar{\lambda}_{m-1}, \lambda_1^1, \cdots, \lambda_1^l, \cdots, \lambda_{i'}^1, \cdots, \lambda_{i'}^l, \cdots, \lambda_p^1, \cdots, \lambda_p^l, \lambda_0) \geq \mathbf{0}$$

such that

$$\boldsymbol{\lambda}\begin{bmatrix} D \\ -\boldsymbol{\pi}_1 \\ \vdots \\ -\boldsymbol{\pi}_1 \\ \vdots \\ -\boldsymbol{\pi}_{i'} \\ \vdots \\ -\boldsymbol{\pi}_{i'} \\ \vdots \\ -\boldsymbol{\pi}_p \\ \vdots \\ -\boldsymbol{\pi}_p \\ \boldsymbol{\pi}_i \end{bmatrix} = \mathbf{0} \quad \text{and} \quad \boldsymbol{\lambda}\begin{bmatrix} \left.\begin{matrix} 0 \\ \vdots \\ 0 \end{matrix}\right\} (m-1)\text{-tuple} \\ -\left(x_1\widetilde{Q}_i^k + \widetilde{U}_1^1\right) \\ \vdots \\ -\left(x_l\widetilde{Q}_i^k + \widetilde{U}_1^l\right) \\ \vdots \\ -\left(x_1\widetilde{Q}_i^k + \widetilde{U}_{i'}^1\right) \\ \vdots \\ -\left(x_l\widetilde{Q}_i^k + \widetilde{U}_{i'}^l\right) \\ \vdots \\ -\left(x_1\widetilde{Q}_i^k + \widetilde{U}_p^1\right) \\ \vdots \\ -\left(x_l\widetilde{Q}_i^k + \widetilde{U}_p^l\right) \\ x_k\widetilde{Q}_i^k + \widetilde{U}_i^k \end{bmatrix} > 0. \tag{B.19}$$

The equalities in (B.19) can be rewritten as

$$\begin{aligned} \bar{\lambda}_1 =& \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)\pi_{i'1} - \lambda_0\pi_{i1}, \\ \bar{\lambda}_j =& \bar{\lambda}_{j-1} + \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)\pi_{i'j} - \lambda_0\pi_{ij}, \quad j = 2, \cdots, m-1, \\ 0 =& \bar{\lambda}_{m-1} + \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)\pi_{i'm} - \lambda_0\pi_{im}, \end{aligned}$$

which are equivalent to

$$\lambda_0 = \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right), \quad \text{and} \quad \bar{\lambda}_j = \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)\Pi_{i'j} - \lambda_0\Pi_{ij}, \quad j = 1, \cdots, m-1, \tag{B.20}$$

or alternatively,

$$\lambda_0 = \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right), \quad \text{and} \quad \bar{\boldsymbol{\lambda}} = \sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)(\boldsymbol{\Pi}_{i'} - \boldsymbol{\Pi}_i), \tag{B.21}$$

where $\bar{\boldsymbol{\lambda}} = \left(\bar{\lambda}_1, \cdots, \bar{\lambda}_m\right)$ and $\bar{\lambda}_m = 0$ because $\Pi_{i'm} = \Pi_{im} = 1$ for all i' and i. Using the expression of λ_0 above, the inequality in (B.19) can be written as

$$\sum_{i'=1}^{p}\sum_{k'=1}^{l}\lambda_{i'}^{k'}\left[x_k\widetilde{Q}_i^k + \widetilde{U}_i^k - \left(x_{k'}\widetilde{Q}_i^k + \widetilde{U}_{i'}^{k'}\right)\right] > 0. \tag{B.22}$$

Now let $\lambda_{i'}^{k'}=0$ for all i' and $k'\neq k$, $\lambda_{i'}^{k}=\nu_{i'}$ for all $i'\neq i$, and $\lambda_{i}^{k}=0$. Accordingly, we define

$$\lambda_0=\sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)=\sum_{\substack{i'=1\\ i'\neq i}}^{p}\nu_{i'}=1>0, \quad \text{and}$$

$$\bar{\boldsymbol{\lambda}}=\sum_{i'=1}^{p}\left(\sum_{k'=1}^{l}\lambda_{i'}^{k'}\right)(\boldsymbol{\Pi}_{i'}-\boldsymbol{\Pi}_{i})=\left(\sum_{\substack{i'=1\\ i'\neq i}}^{p}\nu_{i'}\boldsymbol{\Pi}_{i'}\right)-\boldsymbol{\Pi}_{i}\geq 0,$$

where we have used the definition of $\{\nu_i\}_{\forall i}$ and (7). Thus, $\boldsymbol{\lambda}$ specified above is non-negative and obviously satisfies (B.21). It also immediately follows from (7) that $\boldsymbol{\lambda}$ specified above satisfies (B.22):

$$\sum_{i'=1}^{p}\sum_{k'=1}^{l}\lambda_{i'}^{k'}\left[x_k\widetilde{Q}_i^k+\widetilde{U}_i^k-\left(x_{k'}\widetilde{Q}_i^k+\widetilde{U}_{i'}^{k'}\right)\right]=\widetilde{U}_i^k-\sum_{i'\neq i}\nu_{i'}\widetilde{U}_{i'}^k>0.$$

Therefore, we have identified a non-negative vector $\boldsymbol{\lambda}$ satisfying (B.19) for some (i,k), proving the non-existence of the payment scheme $\{\widehat{M}_{ij}^k\}$ satisfying (B.17), (B.10) and (B.11). ∎

B.3. Extension of Theorem 2 for Continuous $(\mathbf{X},\mathbf{Y},\mathbf{T})$

In this section, we extend Theorem 2 in §4 to a setting where $(\mathbf{X},\mathbf{Y},\mathbf{T})$ follow continuous distributions. For finite discrete distributions of $(\mathbf{X},\mathbf{Y},\mathbf{T})$, Theorem 2 establishes the sufficiency of the full-rank condition (6) for the optimal value of problem $(\mathcal{P})$ without the (NM) constraints to equal OPT_{NC}; that is, there exists a mechanism satisfying (IC) and (IR), under which the buyer fully extracts the rent on the contractors' non-cost estimates $\mathbf{Y}$. Previous research (see Lopomo et al. 2019, and the references therein) suggests that, in general, the (continuous version of) full-rank condition (6) does not suffice for establishing such a full-rent-extraction type result for continuously distributed $(\mathbf{X},\mathbf{Y},\mathbf{T})$.[26] Therefore, rather than extending Theorem 2 to its full generality, we establish a slightly weaker version of Theorem 2 for a specific, albeit realistic, setting with continuously distributed $(\mathbf{X},\mathbf{Y},\mathbf{T})$.

To this end, we introduce the notion of δ-approximate incentive compatibility (δ-IC) constraints (see Azevedo and Budish 2019, and references therein), defined as follows for some $\delta\geq 0$:

$$\begin{aligned}&\mathbb{E}\left[M_n\left(x_n,y_n,T_n\right)-x_nQ_n\left(x_n,\mathbf{X}_{-n},y_n,\mathbf{Y}_{-n}\right)\mid Y_n=y_n\right]\\ \geq\ &\mathbb{E}\left[M_n\left(\widetilde{x}_n,\widetilde{y}_n,T_n\right)-x_nQ_n\left(\widetilde{x}_n,\mathbf{X}_{-n},\widetilde{y}_n,\mathbf{Y}_{-n}\right)\mid Y_n=y_n\right]\ -\ \delta,\quad \forall x_n,\widetilde{x}_n,\widetilde{y}_n,y_n,\ \text{and } n.\end{aligned} \tag{δ-IC}$$

In essence, the (δ-IC) constraint relaxes the *exact* (IC) in $(\mathcal{P})$ by an amount δ. In particular, (δ-IC) reduces to (IC) when $\delta=0$. We describe the setting of our focus below.

Assumption B.1. *Function $V(\cdot)$ is differentiable and increasing, with its derivative bounded from above by a positive constant B_1, i.e., $0 < V'(\cdot) \le B_1$. Let contractor n's cost $X_n \in [\underline{x}, \overline{x}]$ be independent of his non-cost estimate $Y_n \in [\underline{y}, \overline{y}]$. Let $F_X(\cdot)$ and $f_X(\cdot)$, respectively, denote the cumulative distribution and probability density functions of X_n, and define $\psi(x) := x + \frac{F_X(x)}{f_X(x)}$ as the virtual-cost function increasing in x such that $\frac{d}{dx}\psi(x) \ge B_2 > 0$ for some positive constant B_2.*[27] *Finally, contractor n's non-cost attribute $T_n = Y_n \epsilon_1 + \epsilon_0$, where ϵ_0 and $\epsilon_1 \ge 0$ are mutually independent random variables with means 0 and 1, respectively.*

Our main result in this section is the following theorem.

Theorem B.1. *Suppose that Assumption B.1 holds. Then, for any $\delta > 0$, there exists a mechanism $(\mathbf{Q}, \mathbf{M})$ satisfying (δ-IC) and (IR), under which the value of the objective function in ($\mathcal{P}$) equals OPT_{NC}, i.e.,*

$$\sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, Y_n, T_n\right) + Q_n\left(\mathbf{X}, \mathbf{Y}\right) V\left(T_n\right)\right] = OPT_{NC}.$$

Remark B.1. Theorem B.1 is slightly weaker than Theorem 2, which requires establishing Theorem B.1 for $\delta = 0$. One could potentially prove this stronger result by showing that the objective function of ($\mathcal{P}$) subject to (δ-IC) and (IR) is continuous at $\delta = 0$. However, this would require the application of the Berge Maximum Theorem (Aliprantis and Border 2006, Theorem 17.31) in infinite-dimensional mechanism space equipped with an appropriate topology (see, e.g., Page 1992). However, this treatment requires a significant amount of technical preparation and notational burden, and is hence beyond the scope of this paper.

Proof of Theorem B.1: We define a mechanism $(\mathbf{Q}, \mathbf{M})$ in (B.23)–(B.25) below:

$$\left(Q_1(\mathbf{x},\mathbf{y}), \cdots, Q_N(\mathbf{x},\mathbf{y})\right) := \underset{\substack{q_1, \cdots, q_N \in \{0,1\} \\ \sum_{n=1}^N q_n = 1}}{\arg\min} \quad \sum_{n=1}^{N} \Big\{\psi(x_n) + \mathbb{E}[V(T_n) \mid Y_n = y_n]\Big\} \cdot q_n, \tag{B.23}$$

and $M_n\left(x_n, y_n, t_n\right) := \mathbb{E}\left[m_n\left(x_n, \mathbf{X}_{-n}, y_n, \mathbf{Y}_{-n}, t_n\right)\right]$ for $n = 1, 2, \ldots, N$, where

$$m_n(\mathbf{x}, \mathbf{y}, t_n) := x_n Q_n(\mathbf{x}, \mathbf{y}) + \int_{x_n}^{\overline{x}} Q_n(z, \mathbf{x}_{-n}, \mathbf{y}) \, dz \; - \; A \cdot b(y_n, t_n) \cdot Q_n(\mathbf{x}, \mathbf{y}), \tag{B.24}$$

with

$$A := \frac{B_1^2}{4\delta B_2^2} \qquad \text{and} \qquad b(y,t) := (y-t)^2 - \frac{(t^2 - E[\epsilon_0^2])E[(1-\epsilon_1)^2]}{E[\epsilon_1^2]} - E[\epsilon_0^2]. \tag{B.25}$$

We show that the mechanism $(\mathbf{Q}, \mathbf{M})$ defined above satisfies (δ-IC) and (IR) and the value of the objective function in ($\mathcal{P}$) under this mechanism is OPT_{NC}.

Let

$$U_n(\widetilde{x}_n,\widetilde{y}_n;\mathbf{x},\mathbf{y}) := \mathbb{E}\left[m_n\left(\widetilde{x}_n,\mathbf{x}_{-n},\widetilde{y}_n,\mathbf{y}_{-n},T_n\right) \mid Y_n = y_n\right] - x_n Q_n\left(\widetilde{x}_n,\mathbf{x}_{-n},\widetilde{y}_n,\mathbf{y}_{-n}\right).$$

Then, the constraints (δ-IC) and (IR) are implied by the following constraints

$$U_n(x_n,y_n;\mathbf{x},\mathbf{y}) \geq U_n(\widetilde{x}_n,\widetilde{y}_n;\mathbf{x},\mathbf{y}) - \delta, \quad \text{and,} \quad U_n(x_n,y_n;\mathbf{x},\mathbf{y}) \geq 0 \quad \forall \widetilde{x}_n,\widetilde{y}_n,\mathbf{x},\mathbf{y},n. \tag{B.26}$$

Now, we show that *the mechanism* $(\mathbf{Q},\mathbf{M})$ *specified in* (B.23)–(B.25) *satisfies* (B.26), *which will also imply that it satisfies* (δ-IC) *and* (IR).

First, we note that it is straightforward to verify the following properties of $b(\cdot,\cdot)$:

LEMMA B.2. *For a given $y \in [\underline{y},\bar{y}]$, the function $B(\widetilde{y},y) := \mathbb{E}[b(\widetilde{y}, y\epsilon_1 + \epsilon_0)]$ is a convex function of $\widetilde{y}$ and attains a minimum value of zero at $\widetilde{y} = y$; the function $k(y-\widetilde{y}) - A \cdot B(\widetilde{y},y)$ is concave in $\widetilde{y}$ and attains a maximum value of $k^2/(4A)$ at $\widetilde{y} = y - k/(2A)$.*

We note that the mechanism in (B.23)–(B.25) satisfies $U_n(x_n,y_n;\mathbf{x},\mathbf{y}) \geq 0$ for all $\mathbf{x},\mathbf{y},n$. This follows directly from (B.24) and Lemma B.2 which establishes that $B(y_n,y_n) = 0$. That is,

$$\begin{aligned}
U_n(x_n,y_n;\mathbf{x},\mathbf{y}) &= \mathbb{E}\left[m_n\left(\mathbf{x},\mathbf{y},T_n\right) \mid Y_n = y_n\right] - x_n Q_n\left(\mathbf{x},\mathbf{y}\right) \\
&= \int_{x_n}^{\overline{x}} Q_n(z,\mathbf{x}_{-n},\mathbf{y})\, dz \ - \ A \cdot \mathbb{E}[b(y_n, y_n\epsilon_1 + \epsilon_0)] \cdot Q_n(\mathbf{x},\mathbf{y}) \\
&= \int_{x_n}^{\overline{x}} Q_n(z,\mathbf{x}_{-n},\mathbf{y})\, dz \geq 0.
\end{aligned}$$

Thus, the second inequality in (B.26) is satisfied. Now we need to establish the first inequality in (B.26). For the remainder of our proof, we define

$$\ell\left(\mathbf{x}_{-n},\mathbf{y}\right) := \max\left\{z : \psi(z) + \mathbb{E}[V(T_n)|Y_n = y_n] \leq \psi(x_k) + \mathbb{E}[V(T_k)|Y_k = y_k] \quad \forall k \neq n\right\}. \tag{B.27}$$

Following the mechanism design literature (Krishna 2009, Chapter 5), we use the following interpretation of the mechanism $(\mathbf{Q},\mathbf{M})$: (a) If $x_n > \ell\left(\mathbf{x}_{-n},\mathbf{y}\right)$, we have $Q_n(\mathbf{x},\mathbf{y}) = 0$ and $m_n(\mathbf{x},\mathbf{y},t_n) = 0$. This follows from (B.23), (B.24) and (B.27), along with the fact that $\psi(z)$ is increasing in z. (b) If $x_n \leq \ell\left(\mathbf{x}_{-n},\mathbf{y}\right)$, we have $Q_n(\mathbf{x},\mathbf{y}) = 1$ and $m_n(\mathbf{x},\mathbf{y},t_n)$ given as follows[28]:

$$\begin{aligned}
m_n(\mathbf{x},\mathbf{y},t_n) &= x_n Q_n(\mathbf{x},\mathbf{y}) + \int_{x_n}^{\overline{x}} Q_n(z,\mathbf{x}_{-n},\mathbf{y})\, dz \ - \ A \cdot b(y_n,t_n) \cdot Q_n(\mathbf{x},\mathbf{y}) \\
&= x_n + \int_{x_n}^{\ell\left(\mathbf{x}_{-n},\mathbf{y}\right)} 1\, dz + \int_{\ell\left(\mathbf{x}_{-n},\mathbf{y}\right)}^{\overline{x}} 0\, dz \ - \ A \cdot b(y_n,t_n) \\
&= \ell\left(\mathbf{x}_{-n},\mathbf{y}\right) \ - \ A \cdot b(y_n,t_n).
\end{aligned} \tag{B.28}$$

Since $0 < V'(\cdot) \leq B_1$, we also have, for all n

$$\frac{d}{dy}\mathbb{E}[V(T_n)|Y_n = y] = \mathbb{E}[\epsilon_1 V'(y\epsilon_1 + \epsilon_0)] \in [0, B_1], \tag{B.29}$$

by using the fact that $\epsilon_1 \geq 0$ and $\mathbb{E}[\epsilon_1] = 1$. Fix an arbitrary $\widetilde{x}_n, \widetilde{y}_n, \mathbf{x}, \mathbf{y}, n$ and $\delta > 0$. We consider the following cases:

- Case 1: $Q_n(\widetilde{x}_n, \mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) = 0$
  From (B.24), we have $U_n(\widetilde{x}_n, \widetilde{y}_n; \mathbf{x}, \mathbf{y}) = 0 \leq U_n(x_n, y_n; \mathbf{x}, \mathbf{y}) + \delta$, thus, satisfying (B.26).

- Case 2: $Q_n(\widetilde{x}_n, \mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) = 1$.
  Then, $\widetilde{x}_n \leq \ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n})$. We have two sub-cases.

  - Assume $\widetilde{y}_n > y_n$. We have:

$$\begin{aligned} U_n(\widetilde{x}_n, \widetilde{y}_n; \mathbf{x}, \mathbf{y}) &= \mathbb{E}\left[m_n(\widetilde{x}_n, \mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}, T_n) \mid Y_n = y_n\right] - x_n Q_n(\widetilde{x}_n, \mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) \\ &= \ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) - A \cdot B(\widetilde{y}_n, y_n) - x_n \quad \text{(using (B.28))} \\ &\leq \ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) - x_n \qquad (\because \;\; B(\cdot,\cdot) \geq 0;\ \text{see Lemma B.2}) \\ &\leq \ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n. \quad (\because \;\; \ell(\mathbf{x}_{-n}, \mathbf{y}) \text{ is decreasing in } y_n \text{ by (B.29)}) \end{aligned}$$

    If $x_n > \ell(\mathbf{x}_{-n}, \mathbf{y})$, then we have $Q_n(\mathbf{x}, \mathbf{y}) = 0$ and $m_n(\mathbf{x}, \mathbf{y}, t_n) = 0$ and consequently, $U_n(x_n, y_n; \mathbf{x}, \mathbf{y}) = 0$, implying that $\ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n < 0 = U_n(x_n, y_n; \mathbf{x}, \mathbf{y})$. If $x_n \leq \ell(\mathbf{x}_{-n}, \mathbf{y})$, then using (B.28) and the fact that $B(y_n, y_n) = 0$ from Lemma B.2, $U_n(x_n, y_n; \mathbf{x}, \mathbf{y}) = \ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n$.

    Both cases with respect to x_n imply that $\ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n \leq U_n(x_n, y_n; \mathbf{x}, \mathbf{y})$. This, together with $U_n(\widetilde{x}_n, \widetilde{y}_n; \mathbf{x}, \mathbf{y}) \leq \ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n$ imply that $U_n(\widetilde{x}_n, \widetilde{y}_n; \mathbf{x}, \mathbf{y}) \leq U_n(x_n, y_n; \mathbf{x}, \mathbf{y})$, proving (B.26).

  - Assume $\widetilde{y}_n \leq y_n$. Using (B.27),

$$\psi(\ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n})) + \mathbb{E}[V(T_n)|Y_n = \widetilde{y}_n] = \min_{k \neq n}\{\psi(x_k) + \mathbb{E}[V(T_k)|Y_k = y_k]\}.$$

    Differentiating both sides of this equality with respect to $\widetilde{y}_n$, we have

$$\psi'(\ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}))\frac{\partial}{\partial \widetilde{y}_n}\ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) + \frac{d}{d\widetilde{y}_n}\mathbb{E}[V(T_n)|Y_n = \widetilde{y}_n] = 0.$$

    By (B.29), we have

$$\frac{\partial}{\partial \widetilde{y}_n}\ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) \geq -B_1/B_2.$$

Therefore, we obtain

$$\ell(\mathbf{x}_{-n}, \mathbf{y}) - \ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) \geq -(B_1/B_2)(y_n - \widetilde{y}_n). \tag{B.30}$$

Therefore,

$$\begin{aligned} U_n(\widetilde{x}_n, \widetilde{y}_n; \mathbf{x}, \mathbf{y}) &= \ell(\mathbf{x}_{-n}, \widetilde{y}_n, \mathbf{y}_{-n}) - x_n - A \cdot B(\widetilde{y}_n, y_n) \\ &\leq \ell(\mathbf{x}_{-n}, \mathbf{y}) + (B_1/B_2)(y_n - \widetilde{y}_n) - x_n - A \cdot B(\widetilde{y}_n, y_n) \\ &\leq (B_1/B_2)^2/(4A) + \ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n \\ &= \delta + \ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n, \end{aligned} \tag{B.31}$$

where the first inequality follows from (B.30), the second inequality from Lemma B.2, and the last equality from the definition of A in (B.25). Using the same steps as in the previous case, namely $\widetilde{y}_n > y_n$, we have $\ell(\mathbf{x}_{-n}, \mathbf{y}) - x_n \leq U_n(x_n, y_n; \mathbf{x}, \mathbf{y})$, which together with (B.31) lead to (B.26).

Thus, (B.26) holds for Case 2.

To summarize cases 1–2, we have established that the mechanism $(\mathbf{Q}, \mathbf{M})$ specified in (B.23)–(B.25) satisfies (B.26).

To conclude the proof of Theorem B.1, it remains to show that *the objective function of* $(\mathcal{P})$ *is* OPT_{NC} *under the mechanism* $(\mathbf{Q}, \mathbf{M})$ *specified in* (B.23)–(B.25). To that end, we prove the lemma below.

Lemma B.3. *The following equality holds:*

$$\mathbb{E}\left[X_n Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y}) + \int_{X_n}^{\overline{x}} Q_n(z, \mathbf{x}_{-n}, \mathbf{y})\ dz\right] = \mathbb{E}[\psi(X_n) \cdot Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y})].$$

Proof of Lemma B.3: The equality in Lemma B.3 follows from the standard exchange of integrals (Myerson 1981) described below.

$$\begin{aligned} &\mathbb{E}\left[X_n Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y}) + \int_{X_n}^{\overline{x}} Q_n(z, \mathbf{x}_{-n}, \mathbf{y})\ dz\right] \\ =&\mathbb{E}\left[X_n Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y})\right] + \int_{\underline{x}}^{\overline{x}} \int_{x}^{\overline{x}} Q_n(z, \mathbf{x}_{-n}, \mathbf{y}) f_X(x)\ dz\ dx \\ =&\mathbb{E}\left[X_n Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y})\right] + \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{z} Q_n(z, \mathbf{x}_{-n}, \mathbf{y}) f_X(x)\ dx\ dz \\ =&\mathbb{E}\left[X_n Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y})\right] + \int_{\underline{x}}^{\overline{x}} Q_n(z, \mathbf{x}_{-n}, \mathbf{y}) \frac{F_X(z)}{f_X(z)} f_X(z)\ dz = \mathbb{E}[\psi(X_n) \cdot Q_n(X_n, \mathbf{x}_{-n}, \mathbf{y})]. \end{aligned}$$

This completes the proof of Lemma B.3. □

The objective function of $(\mathcal{P})$ under the mechanism $(\mathbf{Q},\mathbf{M})$ specified in (B.23)–(B.25) is obtained as:

$$\sum_{n=1}^{N} \mathbb{E}\left[M_n\left(X_n, Y_n, T_n\right) + V(T_n) Q_n\left(\mathbf{X},\mathbf{Y}\right)\right]$$

$$= \sum_{n=1}^{N} \mathbb{E}\Big[X_n Q_n(\mathbf{X},\mathbf{Y}) + \int_{X_n}^{\overline{x}} Q_n(z,\mathbf{X}_{-n},\mathbf{Y})\ dz\ -\ A\cdot b(Y_n, Y_n\epsilon_1+\epsilon_0)\cdot Q_n(\mathbf{X},\mathbf{Y}) + V(T_n)Q_n\left(\mathbf{X},\mathbf{Y}\right)\Big]$$

$$= \sum_{n=1}^{N} \mathbb{E}\Big[\Big(\psi(X_n) + \mathbb{E}[V(T_n)|Y_n]\Big)Q_n(\mathbf{X},\mathbf{Y}) - A\cdot B(Y_n,Y_n)Q_n(\mathbf{X},\mathbf{Y})\Big] \quad \text{(using Lemma B.3)}$$

$$= \sum_{n=1}^{N} \mathbb{E}\Big[\Big(\psi(X_n) + \mathbb{E}[V(T_n)|Y_n]\Big)Q_n(\mathbf{X},\mathbf{Y})\Big] \quad \text{(since } B(y_n,y_n)=0;\text{see Lemma B.2)}$$

$$= \mathbb{E}\Big[\min_n \left\{\psi(X_n) + \mathbb{E}[V(T_n)|Y_n]\right\}\Big] \quad \text{(using the definition of } Q_n(\cdot,\cdot) \text{ in (B.23))}$$

$$= OPT_{NC} \quad \text{(see the proof of Theorem 1).}$$

This completes the proof of Theorem B.1. ■

Appendix C: Proofs for §5

Proof of Lemma 3. Fix an arbitrary $\alpha \in [0,1]$. First, we argue that CS_α is non-manipulable. The selected contractor, say n, incurs the cost x_n to execute the project, gets paid at the second lowest bid among all the bids $\{b_1,\cdots,b_N\}$ submitted by N contractors, and reimburses an α fraction of the buyer's disutility cost $V(\widetilde{t}_n)$, where $\widetilde{t}_n \geq t_n$. Therefore, contractor n's total ex post payoff is $\min_{m\neq n} b_m - \alpha V(\widetilde{t}_n) - x_n$. Since $V(\widetilde{t}_n)$ is an increasing function of $\widetilde{t}_n$, the contractor's payoff is decreasing in $\widetilde{t}_n$. Thus, under CS_α, it is optimal for the contractor to not inflate his non-cost attribute ex post; i.e., $\widetilde{t}_n = t_n$.

In the remainder of this proof, we show that it is a dominant strategy for every contractor n to bid $x_n + \alpha w_n$. The proof of this part mimics the standard analysis of second-price sealed-bid auctions. Consider an arbitrary contractor n and a bid vector $\mathbf{b}_{-n}$ for all contractors $m \neq n$. We consider the following two cases:

Case 1: $x_n + \alpha w_n \leq \min_{m\neq n} b_m$. If contractor n bids truthfully (i.e., submits a bid that is equal to his true total cost), then he obtains a positive utility given by $\min_{m\neq n} b_m - x_n - \alpha w_n$. We now argue that, for contractor n, bidding truthfully weakly dominates all other strategies. If the contractor bids $b_n \leq x_n + w_n$, then he still wins. However, his utility remains unchanged from that obtained under truthful bidding. If the contractor bids $b_n > x_n + w_n$, then there are two possibilities: In the first scenario, the contractor still wins (i.e., $b_n \leq b_m$ for all m), but his utility does not change from that obtained under truthful bidding. In the second scenario, the contractor does not win (i.e., $b_n > b_m$ for some $m \neq n$) and his utility is 0, which is lower than that obtained under truthful bidding.

Case 2: $x_n + \alpha w_n > \min_{m\neq n} b_m$. If contractor n bids truthfully, then his utility is 0. We now argue that, for contractor n, bidding truthfully weakly dominates all other strategies. If the contractor bids $b_n \geq x_n + w_n$, then he still loses, and gets a zero payoff. If the contractor bids $b_n < x_n + w_n$, then there are two possibilities: In the first scenario, the contractor still loses (i.e., $b_n > b_m$ for some m), and gets a zero payoff. In the second scenario, the contractor wins (i.e., $b_n \leq b_m$ for all m). However, his expected utility is $\min_{m\neq n} b_m - x_n - \alpha w_n$, which is negative and hence, lower than that obtained under truthful bidding.

The above two cases complete the proof of Lemma 3. ■

Proof of Theorem 4. We first show that the allocative inefficiency is decreasing in α. Since Y_n is identically distributed across all n, so is $W_n = \mathbb{E}[V(T_n) \mid Y_n]$. We need to show that

$$\mathbb{E}[X_{1(\alpha_1,N)} + W_{1(\alpha_1,N)}] \geq \mathbb{E}[X_{1(\alpha_2,N)} + W_{1(\alpha_2,N)}], \quad \forall\ 0 \leq \alpha_1 \leq \alpha_2 \leq 1. \tag{C.1}$$

We will show this by proving the following stronger claim: Consider an arbitrary realization of the sequence $\{(X_n, W_n) : n = 1, 2, \ldots, N\}$, which we denote by $\{(x_n, w_n) : n = 1, 2, \ldots, N\}$. We claim that

$$x_{1(\alpha_1,N)} + w_{1(\alpha_1,N)} \geq x_{1(\alpha_2,N)} + w_{1(\alpha_2,N)}. \tag{C.2}$$

By definition, we know that

$$x_{1(\alpha_1,N)} + \alpha_1 w_{1(\alpha_1,N)} \leq x_{1(\alpha_2,N)} + \alpha_1 w_{1(\alpha_2,N)} \quad \text{and} \quad x_{1(\alpha_2,N)} + \alpha_2 w_{1(\alpha_2,N)} \leq x_{1(\alpha_1,N)} + \alpha_2 w_{1(\alpha_1,N)}.$$

These two statements imply that

$$\alpha_2[w_{1(\alpha_2,N)} - w_{1(\alpha_1,N)}] \leq x_{1(\alpha_1,N)} - x_{1(\alpha_2,N)} \leq \alpha_1[w_{1(\alpha_2,N)} - w_{1(\alpha_1,N)}].$$

Since $\alpha_2 \geq \alpha_1$, we obtain

$$w_{1(\alpha_2,N)} - w_{1(\alpha_1,N)} \leq 0. \tag{C.3}$$

Using these inequalities, we obtain:

$$\begin{aligned}
&[x_{1(\alpha_1,N)} + w_{1(\alpha_1,N)}] - [x_{1(\alpha_2,N)} + w_{1(\alpha_2,N)}] = [x_{1(\alpha_1,N)} - x_{1(\alpha_2,N)}] + [w_{1(\alpha_1,N)} - w_{1(\alpha_2,N)}] \\
&\geq \alpha_2[w_{1(\alpha_2,N)} - w_{1(\alpha_1,N)}] + [w_{1(\alpha_1,N)} - w_{1(\alpha_2,N)}] = (1-\alpha_2)[w_{1(\alpha_1,N)} - w_{1(\alpha_2,N)}] \geq 0.
\end{aligned}$$

This proves the claimed inequality in (C.2), which in turn implies the desired result (C.1).

We now prove the result concerning information rent. First, we recall several definitions and results from the stochastic ordering literature (see §3.B.2 in Shaked and Shanthikumar 2007 for details):

DEFINITION C.1. Let X and Y be two random variables such that $\mathbb{P}(X > x) \le \mathbb{P}(Y > x)$ for all $x \in (-\infty, \infty)$. Then, X is said to be *smaller than Y in the usual stochastic order* (denoted by $X \le_{st} Y$).

DEFINITION C.2. A random variable Z is said to be *dispersive* if $X + Z \le_{disp} Y + Z$ whenever $X \le_{disp} Y$, and Z is independent of X and Y.

PROPOSITION C.1. *(Shaked and Shanthikumar 2007) If $X \le_{st} Y$, then $\mathbb{E}[g(X)] \le \mathbb{E}[g(Y)]$ for any non-decreasing function $g(\cdot)$.*

PROPOSITION C.2. *(Shaked and Shanthikumar 2007) Let X be a random variable. Then $X \le_{disp} aX$ for all $a \ge 1$.*

PROPOSITION C.3. *(Shaked and Shanthikumar 2007) The random variable X is dispersive if and only if X has a log-concave density.*

PROPOSITION C.4. *(Bartoszewicz 1986) Let X and Y be two random variables with distribution functions F and G, respectively. Let $X_{1:N}, X_{2:N}, \ldots, X_{N:N}$ and $Y_{1:N}, Y_{2:N}, \ldots, Y_{N:N}$ denote order statistics of random samples of size N from distributions F and G, respectively. If $X \le_{disp} Y$, then $X_{n:N} - X_{(n-1):N} \le_{st} Y_{n:N} - Y_{(n-1):N}$ for all $n = 1, 2, \ldots, N$, where $X_{0:N} = \inf\{x : F(x) > 0\}$ and $Y_{0:N} = \inf\{y : G(y) > 0\}$.*

We now use the above definitions and results to complete the proof of Theorem 4. Let $I(\alpha) = \mathbb{E}\left[X_{2(\alpha,N)} + \alpha W_{2(\alpha,N)} - X_{1(\alpha,N)} - \alpha W_{1(\alpha,N)}\right]$ denote the information rent. Since $X_n = \beta W_n + \xi_n$ for any n, we have $I(\alpha) = \mathbb{E}\left[\xi_{2(\alpha,N)} + (\beta+\alpha) W_{2(\alpha,N)} - \xi_{1(\alpha,N)} - (\beta+\alpha) W_{1(\alpha,N)}\right]$. We study $I(\alpha)$ for the cases $\alpha < -\beta$ and $\alpha > -\beta$ separately.

- For any $\alpha_1 \le \alpha_2 < -\beta$, we have $\beta + \alpha_1 \le \beta + \alpha_2 < 0$ and $(\beta+\alpha_1)/(\beta+\alpha_2) \ge 1$. From Proposition C.2, we have

$$\left(\frac{\beta+\alpha_1}{\beta+\alpha_2}\right)(\beta+\alpha_2)W_n \ge_{disp} (\beta+\alpha_2)W_n \Longleftrightarrow (\beta+\alpha_1)W_n \ge_{disp} (\beta+\alpha_2)W_n, \quad \text{for all } n.$$

  Given that ξ_n has a log-concave probability density and is independent of W_n for all n, using Proposition C.3 and Definition C.2, we obtain

$$\xi_n + (\beta+\alpha_1)W_n \ge_{disp} \xi_n + (\beta+\alpha_2)W_n \quad \forall\ n.$$

This, together with Proposition C.4 implies that

$$\begin{aligned}&\xi_{2(\alpha_1,N)}+(\beta+\alpha_1)W_{2(\alpha_1,N)}-\xi_{1(\alpha_1,N)}-(\beta+\alpha_1)W_{1(\alpha_1,N)}\\ \geq_{st}\ &\xi_{2(\alpha_2,N)}+(\beta+\alpha_2)W_{2(\alpha_2,N)}-\xi_{1(\alpha_2,N)}-(\beta+\alpha_2)W_{1(\alpha_2,N)}.\end{aligned}$$

Applying Proposition C.1 to this result yields $I(\alpha_1)\geq I(\alpha_2)$, i.e., $I(\alpha)$ is decreasing in $\alpha\in[0,-\beta)$.

- For $-\beta<\alpha_1\leq\alpha_2$, we have $\beta+\alpha_2\geq\beta+\alpha_1>0$ and $(\beta+\alpha_2)/(\beta+\alpha_1)\geq 1$. From Proposition C.2, we recognize that

$$\left(\frac{\beta+\alpha_2}{\beta+\alpha_1}\right)(\beta+\alpha_1)W_n\geq_{disp}(\beta+\alpha_1)W_n\Longleftrightarrow(\beta+\alpha_2)W_n\geq_{disp}(\beta+\alpha_1)W_n,\quad\text{for all } n.$$

Following identical argument as in the previous case, we can show that $I(\alpha_1)\leq I(\alpha_2)$, i.e., $I(\alpha)$ is increasing in $\alpha\in(-\beta,1]$. ■

Remark: For the remaining proofs in this appendix, let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal probability density function and cumulative distribution function, respectively. For notational brevity, we denote $\sigma_r:=\sigma_W/\sigma_X$, $A(N):=\mathbb{E}[Z_{2:N}-Z_{1:N}]>0$ and $B(N):=-\mathbb{E}[Z_{1:N}]\geq 0$ in the proofs.

LEMMA C.1. *For any N, we have $N\mathbb{E}[\phi(Z_{1:N-1})]=-\mathbb{E}[Z_{1:N}]$ and $\mathbb{E}[Z_{2:N}-Z_{1:N}]=\mathbb{E}\left[\frac{\Phi(Z_{1:N})}{\phi(Z_{1:N})}\right]$.*

Proof of Lemma C.1. The density of $Z_{1:N}$ is given by $\phi_{1:N}(z)=N\phi(z)[1-\Phi(z)]^{N-1}$. Differentiating this with respect to z, we obtain:

$$N(N-1)\phi^2(z)[1-\Phi(z)]^{N-2}=N\phi'(z)[1-\Phi(z)]^{N-1}-\phi'_{1:N}(z),$$

where $\phi'(z)=-z\phi(z)$. Thus,

$$\begin{aligned}N\mathbb{E}[\phi(Z_{1:N-1})]&=\int_{-\infty}^{\infty}N(N-1)\phi(z)[1-\Phi(z)]^{N-2}\phi(z)\ dz\\&=-\int_{-\infty}^{\infty}Nz\phi(z)[1-\Phi(z)]^{N-1}\ dz-\int_{-\infty}^{\infty}\phi'_{1:N}(z)\ dz\\&=-\int_{-\infty}^{\infty}z\phi_{1:N}(z)\ dz-\int_{-\infty}^{\infty}d\phi_{1:N}(z)=-\int_{-\infty}^{\infty}z\phi_{1:N}(z)\ dz=-\mathbb{E}[Z_{1:N}].\end{aligned}$$

The second result follows from the fact that

$$\mathbb{E}\left[\frac{\Phi(Z_{1:N})}{\phi(Z_{1:N})}\right]=\int_{-\infty}^{\infty}\frac{\Phi(x)}{\phi(x)}N\phi(x)[1-\Phi(x)]^{N-1}dx=\int_{-\infty}^{\infty}N\Phi(x)[1-\Phi(x)]^{N-1}dx,$$

which is equal to $\mathbb{E}[Z_{2:N}-Z_{1:N}]$ according to Exercise 3.1.1 of David and Nagaraja (2004). ■

LEMMA C.2. *Under the cost-sharing mechanism CS_α, the buyer's expected cost $C(\alpha)$ is:*

$$C(\alpha) = FB + \underbrace{\left[\sigma_{X+W} - \sigma_{X+\alpha W} - \frac{(1-\alpha)\left(\alpha\sigma_W^2 + \rho\sigma_X\sigma_W\right)}{\sigma_{X+\alpha W}}\right]\mathbb{E}[-Z_{1:N}]}_{\textit{allocative inefficiency}} + \underbrace{\sigma_{X+\alpha W}\mathbb{E}[Z_{2:N} - Z_{1:N}]}_{\textit{information rent}}, \tag{C.4}$$

where the buyer's first-best cost is:

$$FB = \mathbb{E}\left[X_{1(1,N)} + W_{1(1,N)}\right] = \mu_X + \mu_W + \sigma_X\sqrt{\left(\sigma_W/\sigma_X + \rho\right)^2 + 1 - \rho^2}\ \mathbb{E}[Z_{1:N}]. \tag{C.5}$$

Proof of Lemma C.2. Define $X'_n = X_n - \mu_X$ and $W'_n = W_n - \mu_W$. Following (12) and (13) in §5, we can rewrite

$$C(\alpha) = \mu_X + \mu_W + \sigma_{X+\alpha W}\mathbb{E}[Z_{2:N} - Z_{1:N}] + N\mathbb{E}\left[X'_N + W'_N; X'_N + \alpha W'_N \le X'_{1(\alpha,N-1)} + \alpha W'_{1(\alpha,N-1)}\right].$$

The bivariate normal distribution implies that

$$\begin{aligned}\mathbb{E}\left[X'_N + W'_N \mid X'_N + \alpha W'_N\right] &= \frac{\mathrm{Cov}\left(X'_N + W'_N, X'_N + \alpha W'_N\right)}{\sigma^2_{X+\alpha W}}\left(X'_N + \alpha W'_N\right) \\ &= \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma^2_{X+\alpha W}}\left(X'_N + \alpha W'_N\right).\end{aligned}$$

Thus,

$$\begin{aligned}\mathbb{E}\left[X'_N + W'_N; X'_N + \alpha W'_N \le z\right] &= \int_{-\infty}^{z}\mathbb{E}\left[X'_N + W'_N; X'_N + \alpha W'_N \in [y, y+dy]\right] \\ &= \int_{-\infty}^{z}\underbrace{\mathbb{E}\left[X'_N + W'_N | X'_N + \alpha W'_N \in [y, y+dy]\right]}_{\frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma^2_{X+\alpha W}}y}\underbrace{\mathbb{P}\left[X'_N + \alpha W'_N \in [y, y+dy]\right]}_{\frac{1}{\sigma_{X+\alpha W}}\phi\left(\frac{y}{\sigma_{X+\alpha W}}\right)dy} \\ &= \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma^2_{X+\alpha W}}\int_{-\infty}^{z}\frac{y}{\sigma_{X+\alpha W}}\phi\left(\frac{y}{\sigma_{X+\alpha W}}\right)dy \\ &= \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma_{X+\alpha W}}\int_{-\infty}^{z}d\left[-\phi\left(\frac{y}{\sigma_{X+\alpha W}}\right)\right] \\ &= -\frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma_{X+\alpha W}}\phi\left(\frac{z}{\sigma_{X+\alpha W}}\right).\end{aligned}$$

Using the above expression, we obtain

$$\begin{aligned}C(\alpha) &= \mu_X + \mu_W + \sigma_{X+\alpha W}\mathbb{E}[Z_{2:N} - Z_{1:N}] - \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma_{X+\alpha W}}N\mathbb{E}\left[\phi\left(\frac{X'_{1(\alpha,N-1)} + \alpha W'_{1(\alpha,N-1)}}{\sigma_{X+\alpha W}}\right)\right] \\ &= \mu_X + \mu_W + \sigma_{X+\alpha W}\mathbb{E}[Z_{2:N} - Z_{1:N}] - \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma_{X+\alpha W}}N\mathbb{E}\left[\phi(Z_{1:N-1})\right] \\ &= \mu_X + \mu_W + \sigma_{X+\alpha W}\mathbb{E}[Z_{2:N} - Z_{1:N}] + \frac{\sigma_X^2 + (1+\alpha)\rho\sigma_X\sigma_W + \alpha\sigma_W^2}{\sigma_{X+\alpha W}}\mathbb{E}[Z_{1:N}],\end{aligned}$$

where the second equality follows from standardization of the random variable $X_n' + \alpha W_n'$ and the last equality follows from the first result of Lemma C.1. This completes the proof of Lemma C.2.■

Proof of Proposition 1. Recall that $\sigma_r := \sigma_W/\sigma_X$, $A(N) := \mathbb{E}[Z_{2:N} - Z_{1:N}] > 0$ and $B(N) := -\mathbb{E}[Z_{1:N}] \geq 0$. We first recognize that $\sigma_{X+\alpha W} = \sigma_X\sqrt{(\alpha\sigma_r+\rho)^2+1-\rho^2}$. Differentiating the buyer's expected cost $C(\alpha)$ in (C.4), we obtain:

$$\frac{d}{d\alpha}C(\alpha) = \frac{\sigma_W\sigma_X^3}{\sigma_{X+\alpha W}^3}c(\alpha), \tag{C.6}$$

where

$$c(\alpha) = A(N)\,(\alpha\sigma_r+\rho)\left[(\alpha\sigma_r+\rho)^2+1-\rho^2\right] - B(N)\left[(1-\alpha)(1-\rho^2)\sigma_r\right]. \tag{C.7}$$

- Assume $\rho > 0$. Since $c(\alpha)$ is increasing in α, we have that $C(\alpha)$ is quasi-convex in α. From (C.7), $c(0) = \rho A(N) - \sigma_r(1-\rho^2)B(N)$. Define $\underline{\sigma}(\rho) = (\rho/(1-\rho^2))A(N)/B(N)$. For $\sigma_r \in (0, \underline{\sigma}(\rho))$, given that $c(\alpha)$ is increasing in α and $c(0) \geq 0$, by (C.6), $C'(\alpha) \geq 0$ for all α; thus, $\alpha^{opt} = 0$. For $\sigma_r \geq \underline{\sigma}(\rho)$, since $c(\alpha)$ is increasing in α with $c(0) \leq 0$ and $c(1) \geq 0$ by (C.7), we have $c(\alpha^{opt}) = 0$.

- Assume $\rho \leq 0$. The quasi-convexity of $C(\alpha)$ follows by considering the two cases below:

  (1) If $\sigma_r \leq -\rho$, we must have $\alpha\sigma_r + \rho \leq 0$ for all $\alpha \in [0,1]$, which implies that $c(\alpha) \leq 0, \forall\alpha$. Thus, $\alpha^{opt} = 1$.

  (2) If $\sigma_r \geq -\rho$, we have $\alpha\sigma_r + \rho \leq 0$ for any $0 \leq \alpha \leq -\rho/\sigma_r \leq 1$, for which, again by (C.7), we have $c(\alpha) \leq 0$. For $-\rho/\sigma_r \leq \alpha \leq 1$, we have $\alpha\sigma_r + \rho \geq 0$ and hence $c(\alpha)$ is increasing in α by (C.7). Further, $c(1) \geq 0$. Therefore, $c(\alpha^{opt}) = 0$ and $\alpha^{opt} \geq -\rho/\sigma_r$.

We complete the proof by noting that, the equation $c(\alpha) = 0$ can be re-written as a cubic equation in $(\alpha\sigma_W/\sigma_X + \rho)$ expressed in (14). From §3.8 of Abramowitz and Stegun (1948), this equation has a unique real solution (since $(q/2)^2 + (r/3)^3 > 0$), which is given by

$$-\frac{\rho\sigma_X}{\sigma_W} + \frac{\sigma_X}{\sigma_W}\left[\left(-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{r}{3}\right)^3}\right)^{1/3} + \left(-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{r}{3}\right)^3}\right)^{1/3}\right], \tag{C.8}$$

with q and r as defined in the statement of the theorem. ■

LEMMA C.3. *For any ρ, we have $\widetilde{\alpha}^{opt} := \sigma_r\alpha^{opt} + \rho$ increasing in σ_r. In particular, $\widetilde{\alpha}^{opt} \nearrow \infty$ as $\sigma_r \to \infty$.*

Proof of Lemma C.3. Let $\widetilde{\alpha} := \sigma_r\alpha + \rho$ be a monotone increasing transformation of α. Then, using the relationship $\alpha = (1/\sigma_r)(\widetilde{\alpha} - \rho)$, we can rewrite $c(\alpha)$ in (C.7) as a function of $\widetilde{\alpha}$ as:

$$\widetilde{c}(\widetilde{\alpha}) = \underbrace{\mathbb{E}[Z_{2:N} - Z_{1:N}]\left[\widetilde{\alpha}^3 + (1-\rho^2)\widetilde{\alpha}\right] + (-\mathbb{E}[Z_{1:N}])\,(1-\rho^2)\widetilde{\alpha}}_{\text{increasing in } \widetilde{\alpha}} - \underbrace{(\sigma_r+\rho)(1-\rho^2)\,(-\mathbb{E}[Z_{1:N}])}_{\text{increasing in } \sigma_r}, \tag{C.9}$$

from which the result follows using Topkis (1998). For a sufficiently large σ_r, Proposition 1 implies that $c(\alpha^{opt}) = 0$, or equivalently $\widetilde{c}(\widetilde{\alpha}^{opt}) = 0$. Thus, (C.9) immediately implies that $\widetilde{\alpha}^{opt} \nearrow \infty$ as $\sigma_r \to \infty$. ■

Proof of Proposition 2. We divide the proof into three parts to show the monotonicity of α^{opt} in N, ρ and σ_W/σ_X, respectively.

Monotonicity in N. Let $N_2 \geq N_1 \geq 2$. Let $C(\alpha, N_1)$ and $C(\alpha, N_2)$ represent the buyer's expected costs under the cost-sharing mechanism when there are N_1 and N_2 contractors, respectively. From (C.6), we have:

$$\frac{\partial}{\partial\alpha}C(\alpha,N_2) - \frac{\partial}{\partial\alpha}C(\alpha,N_1) = \frac{\sigma_W\sigma_X^3}{\sigma_{X+\alpha W}^3}\Big\{ (\alpha\sigma_r+\rho)\left[(\alpha\sigma_r+\rho)^2+1-\rho^2\right]\mathbb{E}[Z_{2:N_2}-Z_{1:N_2}-Z_{2:N_1}+Z_{1:N_1}] - \left[(1-\alpha)(1-\rho^2)\sigma_r\right]\mathbb{E}[Z_{1:N_1}-Z_{1:N_2}]\Big\}.$$

By the second result of Lemma C.1,

$$\mathbb{E}[Z_{2:N_2}-Z_{1:N_2}-Z_{2:N_1}+Z_{1:N_1}] = \mathbb{E}\left[\frac{\Phi(Z_{1:N_2})}{\phi(Z_{1:N_2})}\right] - \mathbb{E}\left[\frac{\Phi(Z_{1:N_1})}{\phi(Z_{1:N_1})}\right] \leq 0, \tag{C.10}$$

where the inequality follows from Proposition C.1 by noting that, for standard normal distribution, $\Phi(z)/\phi(z)$ is increasing in z (see Bagnoli and Bergstrom 2005) and that $Z_{1:N_2} \leq_{st} Z_{1:N_1}$ (see Theorem 1.B.1 and Theorem 1.B.28 of Shaked and Shanthikumar 2007). By the same argument, since $\mathbb{E}[Z_{1:N}]$ is decreasing in N, we have

$$\mathbb{E}[Z_{1:N_1}-Z_{1:N_2}] \geq 0. \tag{C.11}$$

We now consider the following cases:

- If $\rho > 0$, then $\alpha\sigma_r + \rho > 0$ and hence (C.10) and (C.11) imply that $\frac{\partial}{\partial\alpha}C(\alpha,N_2) - \frac{\partial}{\partial\alpha}C(\alpha,N_1) \leq 0$. That is, $C(\alpha,N)$ has decreasing differences in (α,N).
- If $\rho \leq 0$, then $\alpha^{opt} = 1$ for $\sigma_r \leq -\rho$, and $\alpha^{opt} \in [-\rho/\sigma_r, 1]$ with $c(\alpha^{opt}) = 0$ for $\sigma_r \geq -\rho$ (see Proposition 1). Therefore, we focus on the case when $\sigma_r \geq -\rho$ and consider $\alpha \in [-\rho/\sigma_r, 1]$. Since $\alpha\sigma_r + \rho \geq 0$, again (C.10) and (C.11) imply that $C(\alpha,N)$ has decreasing differences in (α,N).

In both cases, using Topkis (1998), we have that α^{opt} is increasing in N.

Monotonicity in ρ. First, it is straightforward to see that there exists a unique $\rho^* \in [0,1]$ that solves

$$\frac{\sigma_W}{\sigma_X} = -\frac{\rho}{1-\rho^2}\frac{\mathbb{E}[Z_{2:N}-Z_{1:N}]}{\mathbb{E}[Z_{1:N}]},$$

because the function $\rho/(1-\rho^2)$ increases from 0 to ∞ as ρ increases from 0 to 1. For $\rho \in [-1, -\sigma_W/\sigma_X]$, from Proposition 1, $\alpha^{opt} = 1$, and thus, is a constant with respect to ρ. Similarly, for $\rho \in [\rho^*, 1]$, $\alpha^{opt} = 0$, which is again a constant and lower than the optimal fraction from the previous case. From Proposition 1, we know that for $\rho \in (-\sigma_W/\sigma_X, \rho^*)$, the optimal fraction belongs in $(0,1)$. Therefore, it remains to show that, for $\rho \in (-\sigma_W/\sigma_X, \rho^*)$, the optimal fraction is decreasing in ρ. By definition, α^{opt} satisfies $c(\alpha^{opt}) = 0$, where $c(\alpha)$ is given in (C.7). Dividing this identity with $(1-\rho^2)$ gives:

$$\left\{ \underbrace{A(N)\left[\frac{(\alpha\sigma_r+\rho)^3}{1-\rho^2} + (\alpha\sigma_r+\rho)\right]}_{\text{increasing in } \rho} - B(N)\left[(1-\alpha)\sigma_r\right] \right\}_{\alpha=\alpha^{opt}} = 0.$$

The fact that the first term is increasing in ρ even when $\rho \le 0$ follows from $\alpha^{opt}\sigma_r + \rho \ge 0$ (see the proof of Proposition 1). This, along with the fact that $c(\alpha)$ is increasing in α, completes the proof that α^{opt} is decreasing in ρ.

Monotonicity in $\sigma_r = \sigma_W/\sigma_X$. Whenever $c(\alpha^{opt}) = 0$, (C.7) implies that

$$A(N)\left[(\sigma_r\alpha^{opt}+\rho)^3 + (1-\rho^2)(\sigma_r\alpha^{opt}+\rho)\right] - B(N)(1-\rho^2)(1-\alpha^{opt})\sigma_r = 0. \tag{C.12}$$

Differentiating (C.12) with respect to σ_r yields

$$\begin{aligned}
&\underbrace{\left[3A(N)(\alpha^{opt}\sigma_r+\rho)^2 + (1-\rho^2)(A(N)+B(N))\right]\sigma_r^2}_{\ge 0} \frac{d}{d\sigma_r}\alpha^{opt} \\
&= B(N)(1-\rho^2)(1-\alpha^{opt})\sigma_r - A(N)\left[3\left(\widetilde{\alpha}^{opt}\right)^2 + 1 - \rho^2\right]\alpha^{opt}\sigma_r \\
\text{(by (C.12))} \quad &= A(N)\left[\left(\widetilde{\alpha}^{opt}\right)^3 + (1-\rho^2)\widetilde{\alpha}^{opt}\right] - A(N)\left[3\left(\widetilde{\alpha}^{opt}\right)^2 + 1 - \rho^2\right](\widetilde{\alpha}^{opt} - \rho) \\
&= -A(N)H(\widetilde{\alpha}^{opt}),
\end{aligned} \tag{C.13}$$

where $H(x) := 2x^3 - 3\rho x^2 - \rho(1-\rho^2)$ for any x, and $\widetilde{\alpha}^{opt} := \sigma_r\alpha^{opt} + \rho$. By (C.13), the sign of $\frac{d}{d\sigma_r}\alpha^{opt}$ is opposite to the sign of $H(\widetilde{\alpha}^{opt})$.

- Assume $\rho \ge 0$. By Proposition 1, $\alpha^{opt} = 0$ for $\sigma_r \in (0, \underline{\sigma}(\rho))$. Therefore, we focus on $\sigma_r \ge \underline{\sigma}(\rho)$, where $c(\alpha^{opt}) = 0$. Clearly, $\widetilde{\alpha}^{opt} \ge 0$. Then, $H'(\widetilde{\alpha}^{opt}) = 6(\widetilde{\alpha}^{opt} - \rho)\widetilde{\alpha}^{opt} = 6\sigma_r\alpha^{opt}\widetilde{\alpha}^{opt} \ge 0$, which along with Lemma C.3 implies that $H(\widetilde{\alpha}^{opt})$ is increasing in σ_r. At $\sigma_r = \underline{\sigma}(\rho)$, since $\alpha^{opt} = 0$, we have $H(\widetilde{\alpha}^{opt}) = -\rho \le 0$; by Lemma C.3, we have $\widetilde{\alpha}^{opt} \to \infty$ and hence $H(\widetilde{\alpha}^{opt}) > 0$ as $\sigma_r \to \infty$. Define $\overline{\sigma}(\rho) = \max\{\sigma_r : H(\widetilde{\alpha}^{opt}) \le 0\}$. Given the facts that $H(\widetilde{\alpha}^{opt}) \le 0$ for $\sigma_r = \underline{\sigma}(\rho)$, $H(\widetilde{\alpha}^{opt}) > 0$ for $\sigma_r \to \infty$ and that $H(\widetilde{\alpha}^{opt})$ is increasing in σ_r, we have $\overline{\sigma}(\rho) \in [0, \infty)$. Also, at $\rho=0$, $H(\widetilde{\alpha}^{opt})=2(\widetilde{\alpha}^{opt})^3\geq 0$; thus, $\overline{\sigma}(0)=0$. Then, by definition, $H(\widetilde{\alpha}^{opt})$ is negative for $\sigma_r\in(0,\overline{\sigma}(\rho)]$ and positive otherwise. By (C.13), we have α^{opt} is increasing in $\sigma_r\in(0,\overline{\sigma}(\rho)]$ and decreasing otherwise.

- Assume $\rho\leq 0$. By Proposition 1, $\alpha^{opt}=1$ for $\sigma_r\leq-\rho$. Therefore, we focus on $\sigma_r\geq-\rho$, where $\alpha^{opt}\in[-\rho/\sigma_r,1]$ with $c(\alpha^{opt})=0$. Clearly, $\widetilde{\alpha}^{opt}\geq 0$. Then, as in the case above, $H(\widetilde{\alpha}^{opt})$ is increasing in σ_r. At $\sigma_r=-\rho$, $\alpha^{opt}=1$ and $\widetilde{\alpha}^{opt}=0$; thus $H(\widetilde{\alpha}^{opt})=-\rho(1-\rho^2)\geq 0$. This along with the fact that $H(\widetilde{\alpha}^{opt})$ is increasing in σ_r imply that $H(\widetilde{\alpha}^{opt})\geq 0$ for all $\sigma_r\geq-\rho$. By (C.13), α^{opt} is decreasing in σ_r. ■

Appendix D: Proofs and Additional Numerical Studies for §6

First, we establish that the relative gap between $C(\alpha^{opt})$, the buyer's expected cost under the optimal cost-sharing mechanism, and the first-best cost FB goes to 0 as the number of contractors approaches to ∞.

Proposition D.1. *Suppose that (X_n,W_n) belongs to a family of distributions such that X_n+W_n has a strictly positive support and its probability density function is continuous and lower bounded by a positive number over its support. Then,*

$$\lim_{N\to\infty}\frac{C(\alpha^{opt})-FB}{FB}\ =\ 0.$$

Proof of Proposition D.1: We need the following lemma:

Lemma D.1. *Consider a set of independent and identically distributed random variables $\{X_n,n=1,2,\ldots,N\}$, the minimum of whose support is given by $\underline{x}$. Then, $X_{1:N}\xrightarrow{d}\underline{x}$ as $N\to\infty$.*

Proof of Lemma D.1: Let $F(\cdot)$ denote the cumulative distribution function of X_n. Then,

$$\lim_{N\to\infty}\mathbb{P}\left[X_{1:N}\leq x\right]=\lim_{N\to\infty}1-(1-F(x))^N=\begin{cases}0, & \text{for } x\leq\underline{x},\\ 1, & \text{for } x>\underline{x},\end{cases}$$

because $F(x)=0$ for $x\leq\underline{x}$ and $F(x)>0$ for $x>\underline{x}$. That is, $\mathbb{P}\left[X_{1:N}\leq x\right]$ converges to the cumulative distribution function of the constant random variable $\underline{x}$, wherever continuous, as $N\to\infty$. By definition, $X_{1:N}\xrightarrow{\mathrm{d}}\underline{x}$ as $N\to\infty$. □

Using the expression for $C(\alpha)$ in (9), we obtain:

$$\frac{C(\alpha^{opt})-FB}{FB}\leq\frac{C(1)-FB}{FB}=\frac{\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right]}{\mathbb{E}[X_{1(1,N)}+W_{1(1,N)}]},\tag{D.1}$$

where the inequality follows from the optimality of α^{opt}.

Let $F_{X+W}(\cdot)$ and $f_{X+W}(\cdot)$ denote the cumulative distribution and probability density functions of the random variable X_n+W_n, respectively. Since, by assumption, $f_{X+W}(\cdot)$ is lower bounded by a positive number over its support, the support must be finite, whose minimum and maximum we denote as $\underline{x}+\underline{w}>0$ and $\bar{x}+\bar{w}>0$, respectively. Then,

$$\mathbb{E}\left[\frac{F_{X+W}(X_{1(1,N)}+W_{1(1,N)})}{f_{X+W}(X_{1(1,N)}+W_{1(1,N)})}\right]=\int_{\underline{x}+\underline{w}}^{\bar{x}+\bar{w}}\frac{F_{X+W}(z)}{f_{X+W}(z)}Nf_{X+W}(z)[1-F_{X+W}(z)]^{N-1}dz$$

$$=\int_{\underline{x}+\underline{w}}^{\bar{x}+\bar{w}}NF_{X+W}(z)[1-F_{X+W}(z)]^{N-1}dz=\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right], \quad \text{(D.2)}$$

where the last equality follows from Exercise 3.1.1 of David and Nagaraja (2004). Thus,

$$\lim_{N\to\infty}\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right]=\lim_{N\to\infty}\mathbb{E}\left[\frac{F_{X+W}(X_{1(1,N)}+W_{1(1,N)})}{f_{X+W}(X_{1(1,N)}+W_{1(1,N)})}\right]$$

$$=\mathbb{E}\left[\frac{F_{X+W}(\underline{x}+\underline{w})}{f_{X+W}(\underline{x}+\underline{w})}\right]=0, \quad \text{(D.3)}$$

where the second equality follows from the Portmanteau Lemma of weak convergence, because $X_{1(1,N)}+W_{1(1,N)}\xrightarrow{\text{d}}\underline{x}+\underline{w}$ by Lemma D.1 and $F_{X+W}(\cdot)/f_{X+W}(\cdot)$ is bounded and continuous over $[\underline{x}+\underline{w},\bar{x}+\bar{w}]$ under the assumption that $f_{X+W}(\cdot)$ is continuous and lower bounded by a positive number.

Combining (D.3) and the fact that $\mathbb{E}\left[X_{1(1,N)}+W_{1(1,N)}\right]\geq\underline{x}+\underline{w}>0$, (D.1) implies that $\lim_{N\to\infty}\frac{C(\alpha^{opt})-FB}{FB}\leq 0$, which leads to the the desired result by noting that $C(\alpha^{opt})\geq FB$. ■

Proof of Theorem 5: Let μ_{X+W} and σ_{X+W}, respectively denote the mean and standard deviation of X_n+W_n. From §4.2 in David and Nagaraja (2004), we first note that

$$FB=\mathbb{E}[X_{1(1,N)}+W_{1(1,N)}]\geq\mu_{X+W}-\sigma_{X+W}\cdot\left(\frac{N-1}{\sqrt{2N-1}}\right). \quad \text{(D.4)}$$

We now make three observations: (1) Recall from (D.2) that

$$\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right]=\mathbb{E}\left[\frac{F_{X+W}(X_{1(1,N)}+W_{1(1,N)})}{f_{X+W}(X_{1(1,N)}+W_{1(1,N)})}\right].$$

(2) Since $F_{X+W}(\cdot)$ is log-concave, $F_{X+W}(\cdot)/f_{X+W}(\cdot)$ is increasing. (3) Also, for any $N_2\geq N_1$, we have $X_{1(1,N_1)}+W_{1(1,N_1)}\geq_{st}X_{1(1,N_2)}+W_{1(1,N_2)}$ (see Theorem 1.B.1 and Theorem 1.B.28 of Shaked and Shanthikumar 2007). These three observations, together with Proposition C.1, imply that $\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right]$ is decreasing in N. Then, we have

$$\mathbb{E}\left[\left(X_{2(1,N)}+W_{2(1,N)}\right)-\left(X_{1(1,N)}+W_{1(1,N)}\right)\right]\leq\mathbb{E}\left[\left(X_{2(1,2)}+W_{2(1,2)}\right)-\left(X_{1(1,2)}+W_{1(1,2)}\right)\right]$$

$$=2\mu_{X+W}-2\mathbb{E}\left[\left(X_{1(1,2)}+W_{1(1,2)}\right)\right]$$

$$\leq 2\mu_{X+W}-2\cdot\left[\mu_{X+W}-\sigma_{X+W}\cdot\left(\frac{1}{\sqrt{3}}\right)\right]=\sigma_{X+W}\cdot\frac{2}{\sqrt{3}}, \quad \text{(D.5)}$$

where the first equality follows from the fact that $\mathbb{E}[X_{2(1,2)} + W_{2(1,2)} + X_{1(1,2)} + W_{1(1,2)}] = 2\mathbb{E}[X_1 + W_1] = 2\mu_{X+W}$ and the last inequality follows from (D.4).

Applying (D.4) and (D.5) in (D.1) yields

$$\frac{C(\alpha^{opt}) - FB}{FB} \leq \frac{\sigma_{X+W} \cdot 2/\sqrt{3}}{\mu_{X+W} - \sigma_{X+W} \cdot \left(\frac{N-1}{\sqrt{2N-1}}\right)} \leq \frac{\sigma \cdot 2/\sqrt{3}}{\mu - \sigma \cdot \left(\frac{N-1}{\sqrt{2N-1}}\right)},$$

where the last inequality follows from the assumptions that $\mu_{X+W} \geq \mu$ and $\sigma_{X+W} \leq \sigma$. Thus, (16) is established.

Finally, we verify that (16) is tight when there are two contractors (i.e., $N = 2$), X_n follows a uniform distribution over the interval $[\mu_X - \sqrt{3}\sigma_X, \mu_X + \sqrt{3}\sigma_X]$ and $W_n = \mu_W$ for $n = 1, 2$, $\mu = \mu_X + \mu_W$ and $\sigma = \sigma_X$. In this case, $X_n + W_n$ follows a uniform distribution (which has a log-concave cumulative distribution function), and its mean and standard deviation are given by $\mu_X + \mu_W$ and σ_X, respectively; thus, the mean of $X_n + W_n$ is at least μ and the standard deviation of $X_n + W_n$ is at most σ. Let $\{U_n, n = 1, 2\}$ be the set of two independent standard uniform random variables. Then, we have $X_n = \mu_X - \sqrt{3}\sigma_X + 2\sqrt{3}\sigma_X U_n$. Further, using (9), we have $C(\alpha) = \mu_W + \mu_X - \sqrt{3}\sigma_X + 2\sqrt{3}\sigma_X \mathbb{E}[U_{2:2}]$ for any α, and using (8), we have $FB = \mu_W + \mu_X - \sqrt{3}\sigma_X + 2\sqrt{3}\sigma_X \mathbb{E}[U_{1:2}]$. Using the fact that $\mathbb{E}[U_{1:N}] = 1/(N+1)$ and $\mathbb{E}[U_{2:N}] = 2/(N+1)$ for any $N \geq 2$, we obtain the following:

$$\frac{C(\alpha^{opt}) - FB}{FB} = \frac{\sigma_X \cdot 2/\sqrt{3}}{\mu_X + \mu_W - \sigma_X/\sqrt{3}} = \frac{\sigma \cdot 2/\sqrt{3}}{\mu - \sigma/\sqrt{3}}.$$

This completes the proof of Theorem 5. ■

Proof of Theorem 6: To prove this result, we first establish the following lemma.

LEMMA D.2. *Let $\{X_n, n = 1, 2, \ldots, N\}$ and $\{Y_n, n = 1, 2, \ldots, N\}$ be two sets of independent and identically distributed random variables. If $X_n \leq_{disp} Y_n$ and $\mathbb{E}[X_n] \geq \mathbb{E}[Y_n]$, then $\mathbb{E}[X_{1:N}] \geq \mathbb{E}[Y_{1:N}]$.*

Proof of Lemma D.2: Using the recursive relation between expected values of order statistics (see §3.4 in David and Nagaraja 2004), we have

$$\mathbb{E}[X_{1:k} - X_{1:k-1}] = -\frac{1}{k}\mathbb{E}[X_{2:k} - X_{1:k}], \text{ for any } k = 2, 3, \ldots, N,$$

which, by summing over $k = 2, 3, \ldots, N$, results in

$$\mathbb{E}[X_{1:N}] = \mathbb{E}[X_n] - \sum_{k=2}^{N} \frac{1}{k}\mathbb{E}[X_{2:k} - X_{1:k}].$$

Similarly, we have

$$\mathbb{E}[Y_{1:N}] = \mathbb{E}[Y_n] - \sum_{k=2}^{N} \frac{1}{k}\mathbb{E}[Y_{2:k} - Y_{1:k}].$$

Since $X_n \leq_{disp} Y_n$, by Proposition C.4, we have $\mathbb{E}[X_{2:k} - X_{1:k}] \leq \mathbb{E}[Y_{2:k} - Y_{1:k}]$ for any $k = 2, 3, \ldots, N$. This, together with $\mathbb{E}[X_n] \geq \mathbb{E}[Y_n]$, leads to the desired result. □

Since $X_n + W_n \leq_{disp} \overline{Z}_n$, from Proposition C.4, we have

$$\mathbb{E}\left[\left(X_{2(1,N)} + W_{2(1,N)}\right) - \left(X_{1(1,N)} + W_{1(1,N)}\right)\right] \leq \mathbb{E}[\overline{Z}_{2:N} - \overline{Z}_{1:N}].$$

Since $X_n + W_n \leq_{disp} \overline{Z}_n$ and $\mathbb{E}[\overline{Z}_n] \leq \mathbb{E}[X_n + W_n]$ by assumption, Lemma D.2 then implies that

$$\mathbb{E}\left[\left(X_{1(1,N)} + W_{1(1,N)}\right)\right] \geq \mathbb{E}[\overline{Z}_{1:N}] > 0.$$

The above two inequalities, together with (D.1), yields

$$\frac{C(\alpha^{opt}) - FB}{FB} \leq \frac{\mathbb{E}[\overline{Z}_{2:N} - \overline{Z}_{1:N}]}{\mathbb{E}[\overline{Z}_{1:N}]},$$

and completes the proof. ■

Proof of Theorem 7: Let $F_X(\cdot)$ and $f_X(\cdot)$ denote the cumulative distribution and probability density functions of X_n, respectively. Since $f_X(\cdot)$ is log-concave, $F_X(\cdot)$ is also log-concave (Bagnoli and Bergstrom 2005). Following the same argument as in the proof of Theorem 1, we can show that

$$OPT_{NC} = \mathbb{E}\left[\min_n \left\{X_n + \frac{F_X(X_n)}{f_X(X_n)} + W_n\right\}\right]. \tag{D.6}$$

Since $F_X(\cdot)$ is log-concave, $F_X(\cdot)/f_X(\cdot)$ is increasing, implying

$$\begin{aligned} OPT_{NC} &\geq \mathbb{E}\left[\min_n \left\{X_n + \frac{F_X(X_{1:N})}{f_X(X_{1:N})} + W_n\right\}\right] = \mathbb{E}\left[\frac{F_X(X_{1:N})}{f_X(X_{1:N})} + \min_n \{X_n + W_n\}\right] \\ &\geq \frac{\mathbb{E}\left[F_X(X_{1:N})\right]}{f_m} + \mathbb{E}\left[\min_n \{X_n + W_n\}\right] = \frac{1}{(N+1)\cdot f_m} + FB, \end{aligned} \tag{D.7}$$

where the last inequality uses the assumption that $f_X(\cdot) \leq f_m$ and the last equality follows from the facts that $FB = \mathbb{E}[\min_n \{X_n + W_n\}]$ and $\mathbb{E}[F_X(X_{1:N})] = \mathbb{E}[U_{1:N}] = 1/(N+1)$ with U_n being independent standard uniform random variables. Using (D.7), we have

$$\frac{C(\alpha^{opt}) - OPT_{NC}}{OPT_{NC}} \leq \frac{C(1) - OPT_{NC}}{OPT_{NC}} \leq \frac{C(1) - FB - \frac{1}{(N+1)\cdot f_m}}{FB + \frac{1}{(N+1)\cdot f_m}} \leq \frac{\sigma_{X+W} \cdot \frac{2}{\sqrt{3}} - \frac{1}{(N+1)\cdot f_m}}{\mu_{X+W} - \sigma_{X+W} \cdot \left(\frac{N-1}{\sqrt{2N-1}}\right) + \frac{1}{(N+1)\cdot f_m}},$$

where the last inequality follows from the fact that

$$FB = \mathbb{E}[X_{1(1,N)} + W_{1(1,N)}] \geq \mu_{X+W} - \sigma_{X+W} \cdot \left(\frac{N-1}{\sqrt{2N-1}}\right), \text{ by (D.4), and}$$

$$C(1) - FB = \mathbb{E}\left[\left(X_{2(1,N)} + W_{2(1,N)}\right) - \left(X_{1(1,N)} + W_{1(1,N)}\right)\right] \leq \sigma_{X+W} \cdot \frac{2}{\sqrt{3}}, \text{ by (D.5),}$$

and the fact that $X_n + W_n$ has a log-concave probability density (and thus, log-concave cumulative distribution function) because X_n and W_n are independent and both have log-concave densities (see §18 in Marshall et al. 1979). Since $\mu_{X+W} \geq \mu$ and $\sigma_{X+W} \leq \sigma$ by assumption, we obtain (19), completing the proof of Theorem 7. ■

Numerical study on the relative gap $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$:

Below, in Table D.1, we complement Theorem 7, which characterizes a performance guarantee for cost-sharing mechanisms with OPT_{NC} as the benchmark when X_n and W_n are independent, to now include cases when X_n and W_n are correlated. Specifically, we numerically compute the relative gap $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$ across all instances of our test-bed, which, as defined in §5.3, allows X_n and W_n to be correlated.

Table D.1 For each value of N, summary statistics (average, standard deviation and maximum) of the relative gap $[C(\alpha^{opt}) - OPT_{NC}]/OPT_{NC}$ across the test-bed defined in §5.3.

N	2	3	4	5	6	7	8	9	10
Avg (%)	0.58	0.34	0.25	0.20	0.17	0.16	0.14	0.13	0.12
Stdev (%)	0.86	0.51	0.40	0.34	0.31	0.29	0.27	0.26	0.25
Max (%)	4.39	3.17	3.23	3.32	3.39	3.43	3.47	3.50	3.52

Appendix E: Proof of Proposition 3

As discussed in §7, the winning contractor's optimal shirking is given by $s(\alpha) = \min\{\overline{s}, g/(\alpha h)\}$. Thus, following arguments similar to the proof of Lemma C.2, we obtain

$$C_s(\alpha) = \mu_X - g \cdot s(\alpha) + \mu_W + h \cdot \frac{(s(\alpha))^2}{2} + \sqrt{\sigma_X^2 + \alpha^2\sigma_W^2}\,\mathbb{E}[Z_{2:N} - Z_{1:N}] + \frac{\sigma_X^2 + \alpha\sigma_W^2}{\sqrt{\sigma_X^2 + \alpha^2\sigma_W^2}}\mathbb{E}\left[Z_{1:N}\right],$$

and

$$FB_s = \mu_X - gs(1) + \mu_W + h \cdot \frac{(s(1))^2}{2} + \sqrt{\sigma_X^2 + \sigma_W^2}\,\mathbb{E}[Z_{1:N}].$$

Define $\underline{\alpha} = g/(\bar{s}h)$, which, by assumption lies in $(0,1)$. Suppose that $\alpha_s^{opt} \in (\underline{\alpha}, 1)$. Here, $g/(\alpha_s^{opt} h) < \bar{s}$ and thus, $s(\alpha_s^{opt}) = g/(\alpha_s^{opt} h)$. Differentiating $C_s(\alpha)$ with respect to α, we obtain

$$\frac{d}{d\alpha} C_s(\alpha) = \sigma_X \frac{\sigma_r^2(1-\alpha)}{(1+\alpha^2\sigma_r^2)^{3/2}} \mathbb{E}[Z_{1:N}] + \sigma_X \frac{\alpha\sigma_r^2}{\sqrt{1+\alpha^2\sigma_r^2}} \mathbb{E}[Z_{2:N} - Z_{1:N}] + \frac{g^2}{\alpha^2 h}\left(1 - \frac{1}{\alpha}\right), \tag{E.1}$$

$$\frac{d^2}{d\alpha^2} C_s(\alpha) = \sigma_X \left\{ \frac{\sigma_r^2}{(1+\alpha^2\sigma_r^2)^{3/2}} \mathbb{E}[Z_{2:N} - Z_{1:N}] - \mathbb{E}[Z_{1:N}] \frac{\sigma_r^2}{(1+\alpha^2\sigma_r^2)^{5/2}} \left[2\alpha\sigma_r^2(1-\alpha) + 1 + \alpha\sigma_r^2\right] \right\} + \frac{g^2}{\alpha^3 h}\left(-2 + \frac{3}{\alpha}\right) > 0. \tag{E.2}$$

In proving that α_s^{opt} is decreasing in σ_W, we explicitly consider the dependence of $C_s(\alpha)$ on σ_r and denote it as $C_s(\alpha, \sigma_r)$. In light of (E.2), using the first-order condition, we have $\frac{\partial}{\partial\alpha} C_s(\alpha_s^{opt}, \sigma_r) = 0$. Rearranging terms in (E.1), we obtain

$$\sigma_X \frac{\alpha_s^{opt}\sigma_r^2}{\sqrt{1+(\alpha_s^{opt})^2\sigma_r^2}} \mathbb{E}[Z_{2:N} - Z_{1:N}] = -\sigma_X \frac{\sigma_r^2(1-\alpha_s^{opt})}{(1+(\alpha_s^{opt})^2\sigma_r^2)^{3/2}} \mathbb{E}[Z_{1:N}] - \frac{g^2}{(\alpha_s^{opt})^2 h}\left(1 - \frac{1}{\alpha_s^{opt}}\right). \tag{E.3}$$

Keeping σ_X fixed, differentiating $\frac{\partial}{\partial\alpha} C_s(\alpha, \sigma_r)$ with respect to σ_r and evaluating the resulting expression at α_s^{opt} yields

$$\begin{aligned}
&\frac{\partial^2}{\partial\alpha\partial\sigma_r} C_s(\alpha_s^{opt}, \sigma_r) \\
=&\frac{\sigma_X\sigma_r}{\left[1+(\alpha_s^{opt})^2\sigma_r^2\right]^{3/2}} \left\{ \alpha_s^{opt}\left[2+(\alpha_s^{opt})^2\sigma_r^2\right] \mathbb{E}[Z_{2:N} - Z_{1:N}] + \frac{(1-\alpha_s^{opt})\left[2-(\alpha_s^{opt})^2\sigma_r^2\right]}{\left[1+(\alpha_s^{opt})^2\sigma_r^2\right]} \mathbb{E}[Z_{1:N}] \right\} \\
=&\frac{\sigma_X\sigma_r}{\left[1+(\alpha_s^{opt})^2\sigma_r^2\right]^{3/2}} \left\{ \left(\frac{(2+(\alpha_s^{opt})^2\sigma_r^2)\sqrt{1+(\alpha_s^{opt})^2\sigma_r^2}}{\sigma_X\sigma_r^2} \right) \cdot \left(\frac{g^2(1-\alpha_s^{opt})}{(\alpha_s^{opt})^3 h} \right) \right. \\
&\left. - \frac{2(\alpha_s^{opt})^2\sigma_r^2(1-\alpha_s^{opt})}{1+(\alpha_s^{opt})^2\sigma_r^2} \mathbb{E}[Z_{1:N}] \right\} \geq 0,
\end{aligned}$$

where the second equality follows by (E.3).

Since $\frac{\partial}{\partial\alpha} C_s(\alpha_s^{opt}, \sigma_r) = 0$, the Implicit Function Theorem immediately implies that

$$\frac{d}{d\sigma_r}\alpha_s^{opt} = -\frac{\partial^2}{\partial\alpha\partial\sigma_r} C_s(\alpha_s^{opt}, \sigma_r) \Big/ \frac{\partial^2}{\partial\alpha^2} C_s(\alpha_s^{opt}, \sigma_r) \leq 0.$$

That is, α_s^{opt} is decreasing in σ_r and hence also in $\sigma_W = \sigma_X\sigma_r$.

Examining (E.1), we observe that $\frac{d}{d\alpha} C_s(\alpha)$ is decreasing in $g \geq 0$ and increasing in h for any $\alpha \in (0,1)$; thus α_s^{opt} is increasing in g and decreasing in h. Further, since $\mathbb{E}[Z_{2:N} - Z_{1:N}]$ and $\mathbb{E}[Z_{1:N}]$ are both decreasing in N, $\frac{d}{d\alpha} C_s(\alpha)$ is decreasing in N for any $\alpha \in (0,1)$, which immediately implies that α_s^{opt} is increasing in N.

Suppose that $\alpha_s^{opt} \in (0, \underline{\alpha})$. Here, $g/(\alpha_s^{opt} h) > \overline{s}$ and thus, $s(\alpha_s^{opt}) = \overline{s}$. Using the same steps as in the previous case, it can be shown that α_s^{opt} is decreasing in σ_W and increasing in N, and remains constant for small changes in g and h. We skip the details for brevity.

The upper bound on the relative gap between $C_s(\alpha_s^{opt})$ and FB_s is directly established by the optimality of α_s^{opt} as follows:

$$\frac{C_s(\alpha_s^{opt}) - FB_s}{FB_s} \leq \frac{C_s(1) - FB_s}{FB_s} = \frac{\sqrt{\sigma_X^2 + \sigma_W^2}\,\mathbb{E}[Z_{2:N} - Z_{1:N}]}{\mu_X + \mu_W + \sqrt{\sigma_X^2 + \sigma_W^2}\,\mathbb{E}[Z_{1:N}] - g^2/(2h)}.$$

This completes the proof of Proposition 3. ■

Appendix F: Empirical Evidence from Utah Department of Transportation

In this appendix, we provide empirical evidence to support the bivariate normal assumption on the belief distributions of each contractor n's cost and disutility cost (X_n, W_n). We construct our data set using publicly available data on highway construction and maintenance projects let by Utah Department of Transportation (UDOT) over the time period Jan 15, 2019 to Feb 12, 2020; the data are accessible at UDOT's website (`https://app.udot.utah.gov/apex/pdbs/f?p=232:19:`).[29]

This dataset contains a total of 152 projects, out of which 5 projects were awarded on the basis of only the cost and are therefore excluded from our study. The remaining 147 projects were awarded on the basis of both the cost and completion time. For each of these 147 projects, UDOT provides the engineer estimates on the cost (with a cost breakdown for line items such as reconstructing manhole, roadway excavation, lighting system etc.) and the expected completion time (typically expressed in days); the contractors were asked to submit bids on their costs (with a breakdown for all line items) and expected completion times. The completion times were further converted into the buyer's disutility cost using UDOT-specified (daily) rates. Subsequently, for each bidding contractor, his *total cost* was calculated as the sum of his cost-bid and the dollar value of his corresponding time-bid. These projects were then awarded to the contractor with the lowest total cost. We further narrow the scope of our data by dropping 8 projects that ended up with only one bidding contractor, which yields a set of 139 projects as the focus of our empirical analysis. Table F.1 summarizes the key project characteristics for this data set.

As the contractors' costs and disutility costs are purely their private information, we use contractors' bids as a proxy to estimate the buyer's belief distribution of their costs and disutility costs. However, as shown by Table F.1, each project only has a small number of bids and the projects are quite heterogeneous in nature. To overcome these limitations, we pool and standardize the bids

Table F.1 Summary statistics (minimum, median and maximum) of the number of contractors, and UDOT's engineer estimates on the cost and disutility cost across 139 projects.

	Min	Median	Max
Number of bidding contractors	2	3	8
UDOT's estimated cost	$15,500	$1,665,045.57	$119,040,615.60
UDOT's estimated disutility cost	$4,480	$120,000	$10,280,000

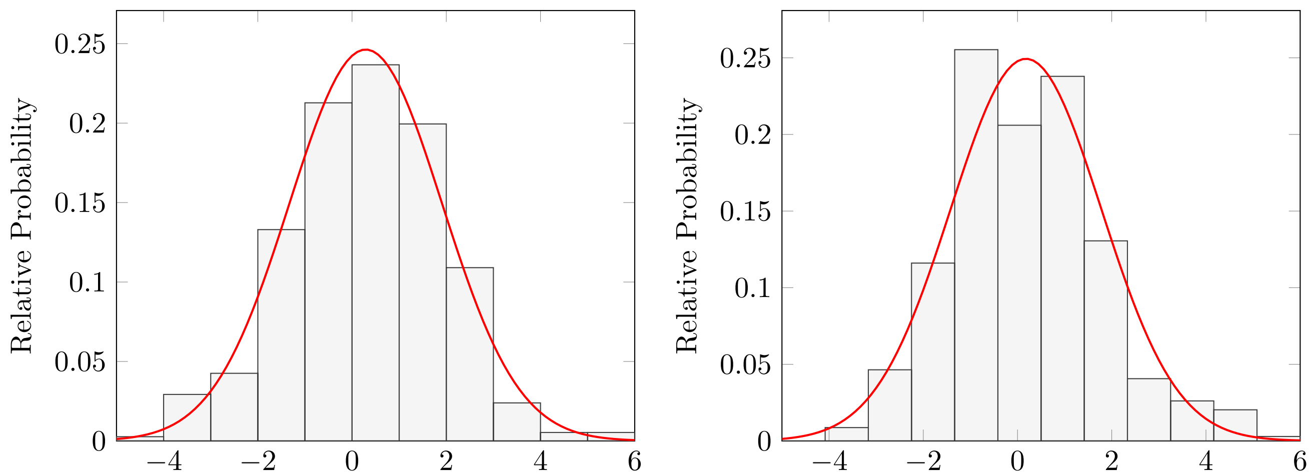
across all 139 projects by leveraging the UDOT's engineer estimates on the cost and disutility cost. Specifically, we standardize the contractors' cost bids as follows: For every project, (i) we center each bidding contractor's cost bid by subtracting the UDOT's estimated cost from the contractor's cost bid, (ii) obtain the standard deviation of all the centered cost bids (received for that project) calculated in part (i), and (iii) finally, divide each centered cost bid calculated in part (i) by the standard deviation calculated in part (ii). The same standardization procedure is applied to the contractors' bids on their disutility costs for every project.

Take the project F-0007(22)7 as an example. The UDOT's estimated cost and disutility cost were $\$1,146,206.20$ and $\$89,490$, respectively. Two contractors, Geneva Rock Products, Inc. (GRP) and Intermountain Slurry Seal, INC (ISS) submitted bids on this project: GRP's bids on the cost and disutility cost were $\$1,186,038.35$ and $\$78,500$, respectively; the corresponding bids from ISS were $\$1,454,000$ and $\$98,910$, respectively. Following the standardization procedure described above, (i) we first calculate two centered cost bids as $1,186,038.35 - 1,146,206.20 = 39,832.15$ and $1,454,000 - 1,146,206.20 = 307,793.80$. (ii) Then, we calculate the standard deviation of these two centered bids as $189,477.50$. (iii) Finally, we divide each centered bid by the standard deviation to obtain two standardized cost bids for this project: $39,832.15/189,477.50 = 0.21$ and $307,793.80/189,477.50 = 1.62$. Following the same procedure, the two standardized bids on the disutility cost for this project are given by -0.76 and 0.65.

Table F.2 Summary statistics (average, standard deviation, minimum, median and maximum) of the standardized bids on cost and disutility cost across 139 projects (excluding the outliers).

	Avg	Stdev	Min	Median	Max
Standardized bids on cost	0.29	1.62	−4.24	0.25	5.42
Standardized bids on disutility-cost	0.18	1.59	−3.90	0	5.36

After standardizing the bids across 139 projects, we further eliminate the outliers that fall outside of the Box-Whisker maximum reach (i.e., 1.5 times the interquartile range below the first quartile and above the third quartile), resulting in a subsample of 376 pairs of standardized bids on cost and disutility cost. Table F.2 provides the summary statistics for this subsample, whose marginal distributions are plotted in Figure F.1 and demonstrate a relatively normal distribution. Finally, Figure F.2 shows the scatter plot of the standardized bids on cost and disutility cost for this subsample, again illustrating that the bivariate normal distribution seems to fit the data quite well.



(a) Cost (b) Disutility Cost

Figure F.1 The marginal distributions of the standardized bids on cost and disutility cost across 139 projects (excluding the outliers).

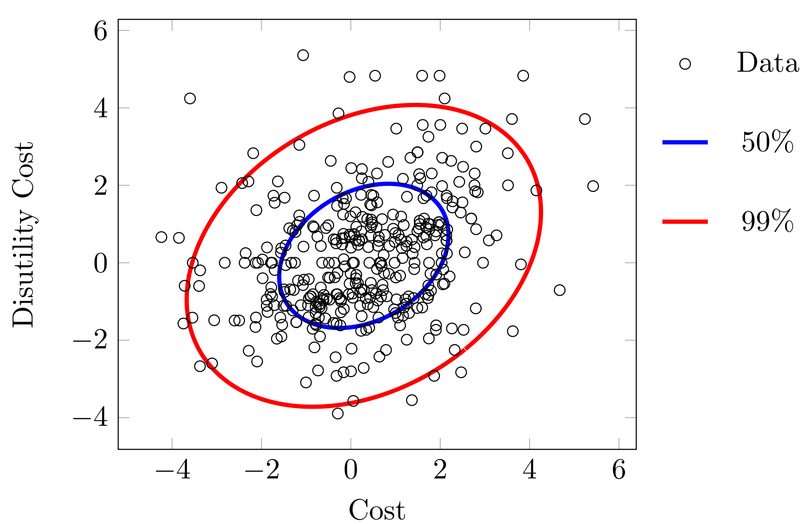


Figure F.2 Scatter plot of the standardized bids on cost and disutility cost across 139 projects (excluding the outliers), in comparison with the 50% and 99% iso-quantile curves corresponding to the bivariate normal distribution with first- and second-order moments estimated from the data.

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