Informing the Public about a Pandemic

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This paper explores how governments may efficiently inform the public about an epidemic to induce compliance with their confinement measures. Using an information design framework, we find the government has an incentive to either downplay or exaggerate the severity of the epidemic if it heavily prioritizes the economy over population health or vice versa. Importantly, we find that the level of economic inequality in the population has an effect on these distortions. The more unequal the disease's economic impact on the population is, the less the government exaggerates and the more it downplays the severity of the epidemic. When the government weighs the economy and population health sufficiently equally, however, the government should always be fully transparent about the severity of the epidemic.

Key words: Public Health, Epidemic Control, Information Design, Strategic Behavior

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1 Introduction

Lockdowns and confinement measures constitute effective non-pharmaceutical interventions for slowing the spread of an epidemic (Anderson et al. 2020, Fowler et al. 2020). These measures enable the deployment of more time-consuming medical responses and may even bring the spread of the virus to a halt (Ferguson et al. 2020, Ji et al. 2020). The efficacy of these restrictions, however, relies heavily on individual compliance (Wright et al. 2020), which political leaders and governmental agencies influence by disseminating information on the epidemic's severity (Webster et al. 2020). However, several factors greatly complicate the proper choice of a communication strategy to influence the public compliance.

First, implementing confinement measures requires making trade-offs between health benefits and the economic costs that social distancing brings about (Hargreaves Heap et al. 2020, IMF 2020). Depending on whether health or the economy is of higher priority, a political leader might overstate or downplay the severity of the epidemic. For instance, political leaders in the United States appear to have belittled the risks associated with the COVID-19 epidemic (Paz 2020), while several European leaders have insisted on its severity (Bennhold and Eddy 2020).

These messages affect the population's perception of the risk associated with social interactions. However, compliance with confinement measures also depends on the socioeconomic status of individuals (Atchison et al. 2020, Van Rooij et al. 2020). In deciding to remain isolated, individuals weigh the benefits to their health against the income they might forgo (Wright et al. 2020, Gitmez et al. 2020). Thus, part of the population might have preferences that do not fully align with the government's priorities. The more heterogeneous a population is, the more individual preferences may differ (Bottan et al. 2020).

Furthermore, the choice of an individual to remain isolated (or not) creates externalities for the rest of the population. The more individuals isolate themselves, the less exposed the population is, which decreases the expected health risk for everyone (Bethune and Korinek 2020). This, however, decreases the incentive to remain confined and thus gives rise to strategic interactions within the population.

This paper explores how policymakers may efficiently inform the public about the severity of an epidemic to induce public compliance with their confinement measures. To address this question, we develop an information design framework, which accounts for the health-economy trade-off faced by both governments and individuals, the socioeconomic heterogeneity within the population and the externalities that social distancing brings about.

Our analysis reveals that governments should always fully disclose their information about the epidemic when they weigh the economy and population health relatively equally. Consistent with the existing literature and several pieces of anecdotal evidence (e.g., Alizamir et al. 2020, Paz 2020), we also find that governments sometimes have a rationale to misrepresent information on the epidemic. If the government's priorities are heavily biased toward the economy or the population's health, they may indeed downplay or exaggerate the severity of the epidemic, respectively. These distortions occur when the public incentives are in fact overall aligned with the government's priorities.

More specifically, consider a government that is heavily biased toward protecting the population's health over minimizing economic losses. When the symptoms of the disease are generally mild, individuals do not have strong incentives to isolate themselves, which does not fully align with the government's objective. Nevertheless, we find in our setup that the government always fully discloses its information about the severity of the epidemic.

In contrast, when symptoms are serious, individuals have a strong incentive to remain confined, which does align with the government's priorities. However, we find that the government sometimes *exaggerates* the severity of the epidemic. In fact, when symptoms are extremely serious, the government does not disclose any information. We obtain similar results in the opposite direction when the government is heavily biased toward minimizing economic costs. In this case, the government may *downplay* the severity of the epidemic when the symptoms are mild, which, again, does not align with the government's priorities.

Further, we find that the level of economic inequality in the population has an effect on these misrepresentations. The more heterogeneous the disease's economic impact is, the less the government exaggerates and the more it downplays the severity of the epidemic. In other words, the government of a population with significant levels of economic disparity has a greater tendency to downplay the severity of the epidemic than the government of a more economically equal society.

The general information design framework we consider in this paper was introduced by Kamenica and Gentzkow (2011), with growing applications in operations research and management science (see Candogan 2020, and the references therein; see also online Appendix E for a literature review). In particular, this approach has been especially fruitful for studying broad public health questions such as ours (e.g., Shi 2013, Alizamir et al. 2020). Our findings also complement the rapidly growing literature on global health and disaster management that has emerged since the COVID-19 outbreak, which analyzes different approaches to limit the spread of the disease (e.g., Birge et al. 2020, Housni et al. 2020 and Ramdas et al. 2020). In contrast to this literature, our study explores the role of information and public behavior in containing the epidemic and is related to agency problems and strategic behaviors in healthcare systems (e.g., Adida et al. 2017, Zorc et al. 2017).

2 Information Design Framework

We next develop an information design model in which a government (the sender) seeks to induce a certain level of social distancing in its population (the receivers), which is experiencing the spread of an infectious disease. The sender informs the public about the epidemic's severity in a way that may or may not reflect its proprietary information. Given the sender's message, each receiver decides whether to avoid social interactions. To make this choice, individuals need to weigh the economic losses that social isolation brings about against the risk of being infected. Each individual choice affects the probability of infection in the rest of the population, giving rise to a game within the population.

Individual choices to social distance

Each individual in the population chooses $\alpha \in \{0,1\}$, such that the individual remains isolated and avoids the disease when $\alpha = 1$ or engages in social interactions otherwise.[1] To make this choice, an individual needs to assess the risk of being infected when engaging in social interactions.

We assume that an infected individual incurs an expected *healthcare cost* $\kappa > 0$ due to the illness.[2] The health risk then corresponds to the perceived probability of being infected times cost κ. The perceived probability of infection for an individual engaging in social interactions is the product of the perceived infectiousness of the disease, denoted by μ, and the size of the socially active population, denoted by P (which yields μP).

Perceived infectiousness μ represents the subjective probability of becoming infected via contact with an individual in the public.[3] This probability is influenced by the information that the government disseminates about the epidemic. Population size P is the proportion of the population that engages in social interactions and is the equilibrium outcome of individual decisions. Thus, the product μP reflects the homogeneous mixing assumption typical of epidemic models (e.g., Anderson et al. 1992). Taken together, these values indicate that an individual faces a health risk equal to $(1-\alpha)\mu P\kappa$ given choice $\alpha \in \{0,1\}$.

Self-isolation, however, is economically costly, as individuals may forgo their source of income. We denote c as the *economic cost* that a confined individual incurs. This cost also accounts for the possibility to work from home, which is typically not the case for low-wage earners. We further consider a continuum of population heterogeneous in this economic cost. Specifically, we normalize the population size to 1 and assume that c follows a uniform distribution with mean θ, standard deviation σ and nonnegative support $\mathcal{C}$, i.e., $\mathcal{C} = [\theta - \sigma\sqrt{3}, \theta + \sigma\sqrt{3}]$ with $\theta \geq \sigma\sqrt{3} > 0$.[4] In particular, the larger σ is, the more unequal the economic impact of the epidemic on the society is.

[1] We restrict the individual decision to a binary choice for clarity, but our model easily accounts for settings where individuals decide on the proportion of time they remain confined (in which case α takes on real values with $\alpha \in [0,1]$).

[2] Cost κ captures the symptoms' seriousness and may include additional economic costs due to hospitalization.

[3] More precisely, μ corresponds to the product of (i) the (subjective) probability of disease transmission through contact between the individual and an infectious subject and (ii) the (perceived) proportion of infected individuals in the population engaging social interactions.

[4] This parametrization is purely for the the purpose of expressing the support of the uniform distribution in terms of its standard deviation.

Thus, given her perception of infectiousness μ and the socially active population size P, an individual decides whether to self-isolate, $\alpha \in \{0,1\}$, to minimize her total expected economic and health costs, namely $\alpha c + (1-\alpha)\mu P \kappa$.

Information structure and public information policies

Each individual is privately informed about her economic cost c but is uninformed about the severity of the epidemic. We refer to the infectiousness of the disease as ω,[5] which is a binary random variable $\omega \in \{\omega_\ell, \omega_h\}$ taking a high value $\omega_h \in (0,1)$ (representing a *highly severe* epidemic) with prior probability $\rho^\circ \in (0,1)$ and a low value $\omega_\ell \in (0, \omega_h)$ (representing a *less severe* epidemic) with the complement probability. Thus, the prior belief about the epidemic's infectiousness is given by $\mu^\circ = \rho^\circ \omega_h + (1-\rho^\circ)\omega_\ell$. We assume that $\omega_\ell \kappa \geq \inf \mathcal{C}$, i.e., the health risk in the absence of any self-isolation is not too small.[6]

In contrast, the government observes the actual realization of $\omega \in \{\omega_\ell, \omega_h\}$, and can further influence individual behavior by strategically disclosing this information. Formally, before ω is realized, the government commits to a public information policy $\Gamma = (\pi, \mathcal{M})$, which specifies (i) the space of all possible messages $\mathcal{M}$ that can be sent to the population and (ii) the probability $\pi(m \,|\, \omega)$ of sending each message $m \in \mathcal{M}$ given each realization of $\omega \in \{\omega_\ell, \omega_h\}$. Consistent with the literature, probability π can be thought of as an abstraction of the intensity with which messages are sent (see, e.g., Alizamir et al. 2020).[7]

We do not restrict the policy space a priori, but we show in Section 4 that the optimal information policy is of the following binary type. Consider the binary message space $\mathcal{M}_{\mathsf{B}} = \{m_\ell, m_h\}$ with $m_h \neq m_\ell$, where m_ℓ and m_h represent "low-severity" and "high-severity" alerts, respectively. An information policy is then given by two probabilities $\pi(m_i \,|\, \omega_i) \in [0,1]$ for $i \in \{\ell, h\}$, with $\pi(m_j \,|\, \omega_i) = 1 - \pi(m_i \,|\, \omega_i)$ for $j \neq i$ and $i, j \in \{\ell, h\}$. This class of policies includes the following two extreme policies as special cases.

Full disclosure. An information policy fully discloses the severity of the epidemic if $\pi(m_i \,|\, \omega_i) = 1$ for $i \in \{\ell, h\}$. Under this policy, message m_i precisely reveals the government's private information ω_i.

[5] Specifically, ω corresponds to the product of (i) the probability of disease transmission through contact between the individual and an infectious subject and (ii) the proportion of infected individuals in the population engaging in social interactions, which we assume to be known by the government.

[6] This condition ensures that a positive proportion of the population engages in social distancing at equilibrium, which simplifies the analysis. Note also that any positive κ satisfies this condition when $\inf \mathcal{C} = 0$.

[7] The more frequently and intensively the government communicates its messages, the more likely they will be received, i.e., the closer the value of $\pi(m \,|\, \omega)$ is to one.

No disclosure. An information policy discloses no information if $\pi(m_i \mid \omega_i) = \pi(m_i \mid \omega_j)$ for all $i, j \in \{\ell, h\}$, i.e., the probability of sending m_i is independent of ω. In this case, the message carries no inferential information about the government's private information.

There is a plethora of information policies between these two polar cases, which reveals some information about the government's private information, albeit with distortion. Of particular interest are the following two types of information distortions.

Downplay. An information policy is said to *downplay* the severity of the epidemic if $\pi(m_\ell \mid \omega_\ell) = 1$ and $\pi(m_\ell \mid \omega_h)$ is positive. In other words, the government may claim with positive probability that the epidemic is not severe (i.e., it sends message m_ℓ), even though its information indicates otherwise (i.e., it observes ω_h). The higher the probability $\pi(m_\ell \mid \omega_h)$, the more the government downplays the severity.

Exaggerate. An information policy is said to *exaggerate* if $\pi(m_h \mid \omega_h) = 1$ and $\pi(m_h \mid \omega_\ell)$ is positive. That is, the government may claim with positive probability that the epidemic is severe (i.e., it sends message m_h) even though its information indicates otherwise (i.e., it observes ω_ℓ). The higher the probability $\pi(m_h \mid \omega_\ell)$, the more the government exaggerates.

Equilibrium

Given the government's information policy Γ, the population makes Bayesian inferences about the government's knowledge of $\omega \in \{\omega_h, \omega_\ell\}$ upon receiving message m. That is, each individual updates her prior μ°, and we denote the corresponding posterior as $\mu_m \triangleq \mathbb{E}[\omega \mid m, \Gamma] \in [\omega_\ell, \omega_h]$ (see online Appendix B for its precise definition).

Given posterior μ_m, an individual with economic cost c determines α to minimize her total expected economic and healthcare costs, i.e.,

$$a(c, \mu_m) \in \underset{\alpha \in \{0,1\}}{\arg\min} \; \alpha c + (1-\alpha)\mu_m P_{a,\mu_m} \kappa, \tag{1}$$

where $a(c, \mu_m)$ is the individual optimal choice, and

$$P_{a,\mu_m} \triangleq \mathbb{E}_c[1 - a(c, \mu_m)] \tag{2}$$

is the size of the socially active population.[8]

The probability of an individual becoming infected, $(1-\alpha)\mu_m P_{a,\mu_m}$, depends on P_{a,μ_m}, which captures the *externalities* that engaging in social interactions creates. As fewer people self-isolate (i.e., as P_{a,μ_m} increases), a socially active individual is more likely to become

[8] In this paper, $\mathbb{E}_x[\cdot]$ represents the expectation with respect to the random variable x.

infected, raising her incentives to self-isolate, giving rise to a game among individuals. A strategy profile $a : \mathcal{C} \times [0,1] \to \{0,1\}$ (which maps the economic cost c and perceived infectiousness μ_m to the isolation choice) forms a Bayesian Nash equilibrium (BNE) if it satisfies (1)-(2).[9] We denote the set of BNEs induced by information policy Γ as $\mathcal{B}(\Gamma)$.

Government's problem

When managing the epidemic, the government needs to balance its population's health with the economy. The government assigns different levels of priority over these conflicting objectives, which may reflect different long- and short-term political goals. Specifically, we refer to C_h and C_e as the population's total expected health costs and economic costs, respectively, given the government's information policy and the population's corresponding response. The government balances these costs by assigning weights $\lambda \in [0,1]$ and $1-\lambda$ so that its objective function, C_λ, is equal to

$$C_\lambda(\Gamma, a) = \lambda\, C_e(\Gamma, a) + (1-\lambda)\, C_h(\Gamma, a), \quad \text{for } a \in \mathcal{B}(\Gamma)\,, \tag{3}$$

where

$$C_e(\Gamma, a) = \mathbb{E}_{c,m}\left[a(c, \mu_m)c \,|\, \Gamma\right], \quad \text{for } a \in \mathcal{B}(\Gamma)\,, \tag{4}$$

is the population's expected economic cost, and

$$C_h(\Gamma, a) = \mathbb{E}_{c,m}\left[(1 - a(c, \mu_m))\mu_m P_{a,\mu_m} \kappa \,|\, \Gamma\right], \quad \text{for } a \in \mathcal{B}(\Gamma)\,, \tag{5}$$

is the population's expected health cost. The government considers the total health and economic costs across the whole population, which the expectation over c in the expressions of C_e and C_h captures. Furthermore, the government may send its message with different levels of intensity as reflected by probability π, hence the expectation over m.

The government is more biased toward the economy when $\lambda > 1/2$ and toward the population's health when $\lambda < 1/2$. At the extremes, the government's priority lies solely in reducing either economic costs C_e (when $\lambda = 1$) or health costs C_h (when $\lambda = 0$).

The government's problem is then to design an information policy that minimizes the total expected costs. Although we show in Section 3 that the optimal population equilibrium is unique, an information policy may induce multiple BNEs. As is standard in the

[9] This is consistent with the equilibrium definition of nonatomic games (Schmeidler 1973, Mas-Colell 1984).

information design literature (e.g., Kamenica and Gentzkow 2011), the sender optimizes over Γ by focusing on the equilibrium in $\mathcal{B}(\Gamma)$, which minimizes the sender's objective function (also referred to as the sender-preferred equilibrium). Thus, the government chooses (Γ, a) with $a \in \mathcal{B}(\Gamma)$ to minimize the total expected cost, namely,

$$C^\star \triangleq \min_{\Gamma, a \in \mathcal{B}(\Gamma)} C_\lambda(\Gamma, a)\,. \tag{6}$$

In the following, we refer to $(\Gamma^\star, a^\star)$ as the corresponding optimal policy and equilibrium.

3 Social Distancing Equilibrium

We next show that for any perceived infectiousness μ, a unique social distancing equilibrium exists in the population. This equilibrium is characterized by a threshold in the individual's economic cost. (We provide the closed-form expressions for this and the following results as well as all the proofs in online Appendix A.)

Proposition 1 **(Equilibrium Characterization).** *Given $\mu \in [\omega_\ell, \omega_h]$, a unique BNE exists, which is given by threshold $c^\star(\mu)$ such that*

$$a^\star(c, \mu) = \begin{cases} 1 & c \le c^\star(\mu), \\ 0 & c \ge c^\star(\mu), \end{cases} \tag{7}$$

and the size of the socially active population in equilibrium is

$$P_{a^\star,\mu} = \frac{c^\star(\mu)}{\mu\kappa}\,. \tag{8}$$

Furthermore, threshold $c^\star(\mu)$ increases concavely and size $P_{a^\star,\mu}$ decreases in μ.

In other words, the equilibrium behavior in the population is uniquely characterized by a single threshold $c^\star$ in the economic cost of isolation, such that only individuals who suffer an economic cost less than this threshold stay in confinement. The rest of the population, i.e., individuals with higher economic costs, engages in social interactions.

Proposition 1 also sheds light on the role of the externalities that individual choices create in the population. Indeed, an individual whose economic cost is exactly at threshold $c^\star$ is indifferent between avoiding and engaging in social interactions, i.e., $c^\star(\mu) = \mu\kappa P_{a^\star,\mu}$ per (8). Thus, two countervailing forces shape the behavior of $c^\star$ as a function of μ. On the one hand, the higher the perceived infectiousness μ is, the higher the value of $c^\star$ for a fixed size

$P_{a^\star,\mu}$, i.e., the more individuals have a strong enough incentive to isolate themselves. On the other hand, a higher μ lowers the equilibrium size of the socially active population $P_{a^\star,\mu}$ because fewer individuals engage in social interactions. The net effect is that $c^\star$ increases in μ, albeit at a diminishing rate. In other words, because of the externalities that individual choices create (via size $P_{a^\star,\mu}$), the additional number of confined individuals decreases when the epidemic appears to be more severe.

Proposition 1 enables reformulating the government's problem in terms of threshold $c^\star$. Specifically, for any given perceived infectiousness μ, the population's economic and healthcare costs as well as the government's total cost in equilibrium can be written as

$$K_e(c^\star(\mu)) \triangleq \mathbb{E}_c\left[a^\star(c,\mu)c\right] = \mathbb{E}_c\left[c\mathbb{1}\{c \le c^\star(\mu)\}\right], \tag{9}$$

$$K_h(c^\star(\mu)) \triangleq \mathbb{E}_c\left[(1-a^\star(c,\mu))\mu P_{a^\star,\mu}\kappa\right] = c^\star(\mu)\mathbb{E}_c\left[\mathbb{1}\{c \ge c^\star(\mu)\}\right] \quad \text{and} \tag{10}$$

$$K_\lambda(c^\star(\mu)) \triangleq \lambda K_e(c^\star(\mu)) + (1-\lambda)K_h(c^\star(\mu)). \tag{11}$$

In particular, $C_e = \mathbb{E}_m\left[K_e\left(c^\star(\mu_m)\right) \mid \Gamma\right]$ and $C_h = \mathbb{E}_m\left[K_h\left(c^\star(\mu_m)\right) \mid \Gamma\right]$. The government's problem becomes then,

$$C^\star = \min_\Gamma \mathbb{E}_m\left[K_\lambda\left(c^\star(\mu_m)\right) \mid \Gamma\right]. \tag{12}$$

The optimal policy that solves (12) is fully determined by the lower convex envelope of total cost $K_\lambda\big(c^\star(\cdot)\big)$ (see Kamenica and Gentzkow 2011 for why this is the case and online Appendix C for additional intuition regarding our setup). However, cost $K_\lambda(\cdot)$ is in general neither concave nor convex, and threshold $c^\star(\cdot)$ is nonlinear due to negative externalities within the population (per Proposition 1), which makes the analysis highly nontrivial.

4 Optimal Information Policy

We are now ready to present the main findings of our analysis.

Governments may misinform the public.

The next result determines when the government fully discloses its private information, when it exaggerates or downplays the severity of the epidemic, and when it refrains from disclosing any information (where these different information policies are as formally defined in Section 2).

Theorem 1 **(Optimal Information Policy)**. *Given $\sigma > 0$ and $\kappa > 0$, the optimal information policy that solves (12) is characterized by four thresholds $\lambda_1, \lambda_2, \lambda_3$, and λ_4 in λ, with $\lambda_1 \le \lambda_2 \le 1/2 \le \lambda_3 \le \lambda_4$, and two thresholds μ_λ^{EX} and μ_λ^{DP} in μ°, such that if*

- $\lambda \le \lambda_1$ *or* $\lambda \ge \lambda_4$, *no disclosure is optimal.*
- $\lambda_2 \le \lambda \le \lambda_3$, *full disclosure is optimal.*
- $\lambda_1 < \lambda < \lambda_2$, *exaggerating is optimal if* $\mu^\circ < \mu_\lambda^{\mathsf{EX}}$, *and no disclosure is optimal otherwise.*
- $\lambda_3 < \lambda < \lambda_4$, *downplaying is optimal if* $\mu^\circ > \mu_\lambda^{\mathsf{DP}}$, *and no disclosure is optimal otherwise.*

Furthermore, the thresholds $\lambda_1, \lambda_2, \lambda_3$ *and* λ_4 *are all nondecreasing in* κ, *with* $\lambda_1(\overline{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$ *and* $\lambda_3(\overline{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$ *for positive cutoffs* $\overline{\kappa}_0 > \underline{\kappa}_0 > \overline{\kappa}_1 > \underline{\kappa}_1$.

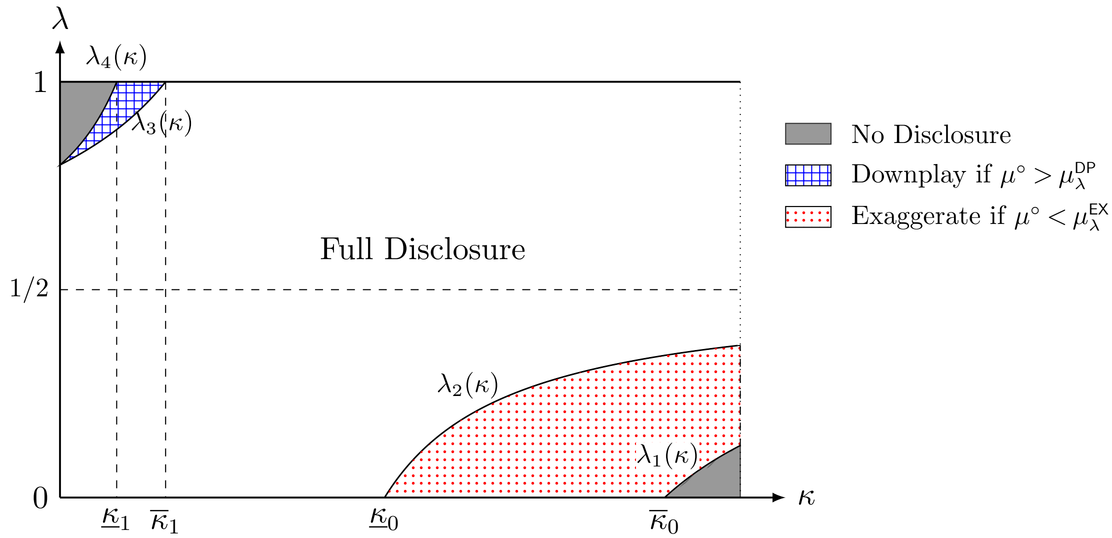


Figure 1 **Optimal policy in** (κ, λ)**-space for** $\omega_\ell = 0.3$, $\omega_h = 0.8$, $\theta = 10$ **and** $\sigma = 10/\sqrt{3}$.

Theorem 1 reveals that the government sometimes informs the public in a way that is not consistent with its own information about the epidemic. The incentives to do so depend on both the government's priorities (λ) and the health cost that individuals are facing (κ). Figure 1 depicts the distortions characterized by the theorem as a function of λ and κ.

When the government weighs the economy and population health relatively equally, i.e., when λ is sufficiently close to 1/2, the government fully discloses its information about the severity of the epidemic. In fact, the government is always fully transparent when $\lambda \in [\lambda_2, \lambda_3]$ (with $\lambda_2 \le 1/2 \le \lambda_3$). However, if the government has more polarized preferences (i.e., λ is sufficiently far from 1/2), the government may misrepresent its information. This happens when health costs also take on extreme values (the dotted and hatched areas in Figure 1). Specifically, when λ is small and κ is large, the government exaggerates the epidemic's severity if the population's prior perception of the risk is low ($\mu^\circ < \mu_\lambda^{\mathsf{EX}}$). And when λ is large and κ is small, the government downplays the epidemic's severity if the population's prior perception of the risk is high ($\mu^\circ > \mu_\lambda^{\mathsf{DP}}$). If both λ and κ take on very extreme values, the government prefers not to disclose any information (the shaded areas in Figure 1).

These distortions are perhaps best illustrated by the polar (or extreme) cases in which the government is concerned solely with either the economy ($\lambda = 1$) or with population health ($\lambda = 0$).

COROLLARY 1 **(Economy-biased Government)**. *If the government's objective is solely to minimize economic costs ($\lambda = 1$), it is optimal to*

- *fully disclose if $\kappa \geq \overline{\kappa}_1$,*
- *downplay if $\underline{\kappa}_1 < \kappa < \overline{\kappa}_1$ and $\mu^\circ > \mu_1^{\mathsf{DP}}$, and*
- *not disclose any information otherwise.*

Downplaying the severity of the epidemic should always promote public behaviors that favor the economy, and a government that focuses solely on this objective may be prone to systematically misrepresenting its information in this way. However, Corollary 1 indicates that such a government refrains from misinforming the public when health costs are sufficiently high ($\kappa \geq \overline{\kappa}_1$). This is despite the fact that high health costs incentivize individuals to remain confined, undermining the government's economy-biased objective. In contrast, the government may downplay the severity of the epidemic when health costs are small, which is precisely when individuals have stronger incentives to participate in the economy. When the health costs are very small, the government does not disclose any information.

Opposite results hold when the government focuses solely on the population's health.

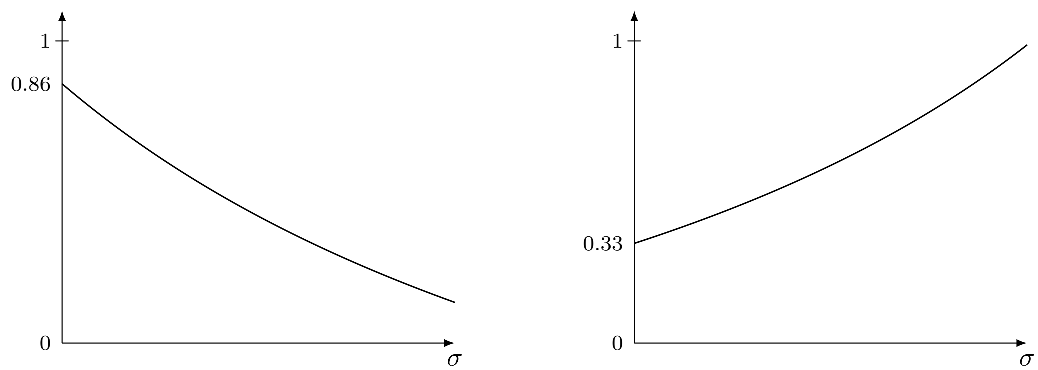
COROLLARY 2 **(Healthcare-biased Government)**. *If the government's objective is solely to minimize healthcare costs ($\lambda = 0$), it is optimal to*

- *fully disclose if $\kappa \leq \underline{\kappa}_0$,*
- *exaggerate if $\underline{\kappa}_0 < \kappa < \overline{\kappa}_0$ and $\mu^\circ < \mu_0^{\mathsf{EX}}$, and*
- *not disclose any information otherwise.*

Thus, when the government focuses solely on health, full disclosure is optimal when the health costs are small enough, which in fact incentivizes more individuals to engage in social interactions. The government may exaggerate the epidemic's severity precisely when the health costs are large, giving the population strong incentives to remain confined.

Governments exaggerate less but downplay more as economic inequalities increase. This paper's main finding is a link between the unequal economic impact of the epidemic and the misinformation identified in Theorem 1. In our setup, the unequal impact of the epidemic is captured by σ, the effect of which is characterized next.

THEOREM 2 **(Effect of economic inequality)**. *Thresholds $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are non-increasing in σ. Further, the optimal exaggeration probability $\pi^\star(m_h \,|\, \omega_\ell)$ is decreasing and the optimal downplaying probability $\pi^\star(m_\ell \,|\, \omega_h))$ is increasing in σ.*



(a) $\pi^\star(m_h \,|\, \omega_\ell)$ for $\lambda = 0$ and $\kappa = 75 \in [\underline{\kappa}_0, \bar{\kappa}_0]$ (b) $\pi^\star(m_\ell \,|\, \omega_h)$ for $\lambda = 1$ and $\kappa = 16 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

Figure 2 Optimal distortion probabilities as a function of σ for $\omega_\ell = 0.3$, $\omega_h = 0.8$ and $\theta = 10$.

In other words, the more unequal the economic impact of an epidemic is, the less the government may exaggerate, but the more it may downplay the epidemic's severity. First, as heterogeneity σ increases, the parametric regions in which the government may exaggerate and downplay (as depicted in Figure 1) shrink and expand, respectively. Second, when these misrepresentations occur, the probabilities of exaggerating and downplaying decrease and increase in σ, respectively. Figure 2 illustrates this point for the limit cases. Figure 2a and Figure 2b depict the probabilities of exaggerating and downplaying the severity of the epidemic for $\lambda = 0$ and $\lambda = 1$, respectively, as heterogeneity σ increases.

5 Discussion

Informing a population about an epidemic is challenging because different individuals trade off between their wealth and health differently, which further creates externalities within the rest of the population. The more unequal the epidemic's economic impact is in the population, the more heterogeneous the public reaction to governmental messages can be.

To tackle this problem, we have followed an analytic approach based on an information design framework. This approach yields three main findings: i) governments should always fully disclose their information about the epidemic when they weigh the economy and population health relatively equally, ii) but governments have an incentive to misrepresent their information about the epidemic when they have polarized objectives, and iii) the more unequal the epidemic's economic impact is in the population, the less governments exaggerate but the more they are tempted to downplay the severity of the epidemic. The latter result is especially relevant given the rise of economic inequality in many societies (Piketty 2013).

Formally establishing these results requires several simplifications. In particular, we assume that the economic impact of the epidemic is uniformly distributed within the population. This assumption is not fundamental to our results, and our insights hold overall, at least numerically, with a beta distribution (see online Appendix D). In addition, all individuals in our setup expect similar levels of symptom severity from infection. Nevertheless, our main insights should hold as long as the economic costs of remaining isolated are not too strongly correlated with the severity of the symptoms.

Our work primarily focuses on the trade-off between the economy and the population's health that governments typically face when dealing with a pandemic. Governments, however, face many additional sources of friction when disclosing their information about a disease. For instance, such information is often plagued by uncertainty (e.g., due to data quality, volatility, model (mis)specification and assumptions, to name a few), which is hard to communicate and prone to misinterpretation. This may further harm the credibility of the government and hinders its ability to elicit proper responses, especially in dynamic settings where past messages influence future perceptions (as in Alizamir et al. 2020). Accounting for these effects constitutes an important research direction. A government may further worry about irrational and panic behaviors (e.g., hospital run, grocery hoarding). To capture this concern, the population's strategic interaction in our model could be replaced by a coordination/global game, where individuals may panic if they believe others would react similarly (see Basak and Zhou 2020, for such a model, albeit in a finance context).

Empirical and survey-based studies (e.g., Webster et al. 2020, Sabat et al. 2020) have recently established the paramount importance of the government's role in providing information about the COVID-19 outbreak, and called for policymakers to adopt effective communication strategies to contain the epidemic. What constitutes such strategies, however, is difficult to study empirically, especially when accounting for different governmental priorities and population characteristics, and largely remains an open question. Nonetheless, our analytical approach allows uncovering important mechanisms to consider when inducing public compliance.

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Online Appendix

A Proofs of Results

Before providing proofs, we first provide some definitions used in the statements of the results in the paper and the proofs. First, let $\gamma \triangleq \sigma 2\sqrt{3}$ (which also corresponds to $\gamma = \sup \mathcal{C} - \inf \mathcal{C}$).

The closed-form expression of the equilibrium threshold $c^\star(\mu)$ is

$$c^\star(\mu) = \frac{\mu\kappa(\theta + \sigma\sqrt{3})}{\sigma 2\sqrt{3} + \mu\kappa}.$$

The thresholds over μ are given by

$$\mu_\lambda^{\mathsf{DP}} \triangleq \rho_\lambda^{\mathsf{DP}}(\omega_h - \omega_\ell) + \omega_\ell \tag{13}$$

$$\mu_\lambda^{\mathsf{EX}} \triangleq \rho_\lambda^{\mathsf{EX}}(\omega_h - \omega_\ell) + \omega_\ell \tag{14}$$

where

$$\rho_\lambda^{\mathsf{DP}} \triangleq \frac{\gamma^2(5\lambda - 4) - 2\kappa^2\omega_\ell\omega_h(2\lambda - 1) - \gamma\kappa(2(1-\lambda)\omega_h + \lambda\omega_\ell)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_h(2\lambda - 1) + \gamma\lambda)} \tag{15}$$

$$\rho_\lambda^{\mathsf{EX}} \triangleq \frac{\Big(2\kappa\omega_\ell(1 - 2\lambda) - \gamma(4 - 5\lambda)\Big)(\kappa\omega_\ell + \gamma)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_\ell(2\lambda - 1) + \gamma\lambda)}. \tag{16}$$

Then, the optimal downplaying and exaggerating probabilities (we use subscripts to highlight the differences) are in order respectively:

$$\pi^\star_{\mathsf{DP}}(m_h \,|\, \omega_h) = \frac{\omega_h - \omega_\ell}{\mu^\circ - \omega_\ell}\left(\frac{\mu^\circ - \mu_\lambda^{\mathsf{DP}}}{\omega_h - \mu_\lambda^{\mathsf{DP}}}\right) \text{ and } \pi^\star_{\mathsf{DP}}(m_\ell \,|\, \omega_\ell) = 1, \tag{17}$$

$$\pi^\star_{\mathsf{EX}}(m_h \,|\, \omega_h) = 1 \text{ and } \pi^\star_{\mathsf{EX}}(m_\ell \,|\, \omega_\ell) = \frac{(\mu_\lambda^{\mathsf{EX}} - \mu^\circ)(\omega_h - \omega_\ell)}{(\omega_h - \mu^\circ)(\mu_\lambda^{\mathsf{EX}} - \omega_\ell)}. \tag{18}$$

Furthermore, we define for a given γ (and hence σ),

$$\lambda_1(\kappa) \triangleq \max\left(-\epsilon, G_\ell^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{19}$$

$$\lambda_2(\kappa) \triangleq \max\left(-\epsilon, G_2^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{20}$$

$$\lambda_3(\kappa) \triangleq \min\left(1 + \epsilon, G_1^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{21}$$

$$\lambda_4(\kappa) \triangleq \min\left(1 + \epsilon, G_h^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{22}$$

for $\epsilon > 0$ and $Y^{-1}(\cdot)$ represents the inverse of a function Y. Note here that we project functions λ_i, $i = 1, \ldots, 4$ in order to extend their domains to positive real numbers because $\kappa/\gamma \geq 0$. Since $\lambda \in [0,1]$, the extended parts ($1+\epsilon$ and $-\epsilon$) do not affect our analysis.

$$G_h(\lambda) \triangleq \frac{1-\frac{5\lambda}{4}}{\omega_h\left(\frac{1}{2}-\lambda\right)} \text{ and } G_\ell(\lambda) \triangleq \frac{1-\frac{5\lambda}{4}}{\omega_\ell\left(\frac{1}{2}-\lambda\right)} \text{ for } \lambda \in (-\infty, 1/2) \cup (1/2, \infty)\,, \tag{23}$$

$$G_1(\lambda) \triangleq \sqrt{\frac{5\lambda-4}{2(2\lambda-1)\omega_h\omega_\ell} + \left(\frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2} - \frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell} \text{ for } \lambda \in (4/5, \infty)\,, \tag{24}$$

$$G_2(\lambda) \triangleq \sqrt{\frac{4-5\lambda}{2(1-2\lambda)\omega_h\omega_\ell} + \left(\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2} + \frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell} \text{ for } \lambda \in (-\infty, 1/2)\,. \tag{25}$$

We also define

$$\begin{aligned}
\overline{\kappa}_0 &\triangleq \gamma G_\ell(0) \\
\underline{\kappa}_0 &\triangleq \gamma G_2(0) \\
\overline{\kappa}_1 &\triangleq \gamma G_1(1) \\
\underline{\kappa}_1 &\triangleq \gamma G_h(1)\,.
\end{aligned} \tag{26}$$

In the proofs of results, we denote the first and second derivatives of an arbitrary function $Y(x)$ with respect to x by $Y'(x)$ and, respectively $Y''(x)$.

Proof of Proposition 1. Fix μ. It is straightforward to see from (1) and (2), the equilibrium action $a^\star(c,\mu)$ in (7) of an individual with cost c is the unique solution to

$$\min_{\alpha \in \{0,1\}} \alpha c + (1-\alpha)\mu P_{a^\star,\mu}\kappa \tag{27}$$

for $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$. We next show that $P_{a^\star,\mu}$ is uniquely pinned down by $a^\star(c,\mu)$ for $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$ and (2). Let $F(\cdot)$ denote the c.d.f. of the uniform distribution with support $\mathcal{C}$.

$$P_{a^\star,\mu} = 1 - F\left(\mu P_{a^\star,\mu}\kappa\right)\,. \tag{28}$$

Let $h(x) \triangleq x - 1 + F(\mu x\kappa)$ for all $x \in [0,1]$. The function h is continuous and strictly increasing. Furthermore, $h(0) = -1 < 0$ since $\theta - \gamma/2 \geq 0$ and $h(1) = F(\mu\kappa) \geq 0$. Thus, we conclude that $P_{a^\star,\mu}$ is the unique point such that $P_{a^\star,\mu} = h^{-1}(0)$. Finally, solving the unique $P_{a^\star,\mu}$ of $P_{a^\star,\mu} = h^{-1}(0)$ for a uniform distribution over $[\theta-\gamma/2, \theta+\gamma/2]$ leads to $P_{a^\star,\mu} = (\theta+\sigma\sqrt{3})/(2\sigma\sqrt{3}+\mu\kappa)$. Using the fact that $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$, we obtain $c^\star(\mu) = \dfrac{\mu\kappa(\theta+\sigma\sqrt{3})}{\sigma 2\sqrt{3}+\mu\kappa}$. It is straightforward to show the monotonicity and concavity properties by taking first and second derivatives of $P_{a^\star,\mu}$ and $c^\star(\mu)$ with respect to μ. Q.E.D.

Proof of Theorem 1. Before proving Theorem 1, we first provide four lemmas and some new notation. We delegate the proofs of those lemmas to the end. For simplicity, we define $\tilde{K}_i(\mu) \triangleq K_i(c^\star(\mu))$ for $i \in \{e,h\}$ and so $\tilde{K}_\lambda(\mu) \triangleq K_\lambda(c^\star(\mu))$ for $\lambda \in [0,1]$.

LEMMA 1. *Functions G_h, G_ℓ, G_1 and G_2 defined respectively in* (23)-(25) *are strictly increasing. Furthermore, $G_1(\lambda) > G_h(\lambda)$, $G_\ell(\lambda) > G_1(\lambda)$ for $\lambda \in (4/5,1]$, and $G_\ell(\lambda) > G_2(\lambda)$, $G_2(\lambda) > G_h(\lambda)$ for $\lambda \in [0,1/2)$.*

LEMMA 2. *Consider λ_i, for $i=1,\dots,4$ defined in* (19)-(22)*, then $\lambda_1 \le \lambda_2 \le 1/2 \le 4/5 \le \lambda_3 \le \lambda_4$. Further, thresholds $\lambda_1,\lambda_2,\lambda_3$ and λ_4 are all nondecreasing in κ with $\lambda_1(\overline{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$ and $\lambda_3(\overline{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$ for positive cutoffs $\overline{\kappa}_0 > \underline{\kappa}_0 > \overline{\kappa}_1 > \underline{\kappa}_1$ defined in* (26).

LEMMA 3. *We have the following.*

A. The function $\tilde{K}_\lambda(\mu)$ is concave if either one of the following conditions holds.
   1. $\lambda \in [1/2,4/5]$.
   2. $\lambda \in (4/5,1] \wedge \kappa/\gamma \ge G_\ell(\lambda)$.
   3. $\lambda \in [0,1/2) \wedge G_h(\lambda) \ge \kappa/\gamma$.

B. The function $\tilde{K}_\lambda(\mu)$ is convex if either one of the following conditions holds.
   1. $\lambda \in [0,1/2) \wedge \kappa/\gamma \ge G_\ell(\lambda)$,
   2. $\lambda \in (4/5,1] \wedge G_h(\lambda) \ge \kappa/\gamma$.

C. If $\lambda \in (4/5,1] \wedge G_\ell(\lambda) > \kappa/\gamma \ge G_1(\lambda)$, then $\tilde{K}_\lambda(\mu)$ is first convex and then concave with inflection point $\mu_{in} \in (\omega_\ell,\omega_h)$ and it satisfies $\tilde{K}'_\lambda(\omega_\ell)(\omega_h-\omega_\ell) \ge \tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)$.

D. If $\lambda \in [0,1/2) \wedge G_2(\lambda) \ge \kappa/\gamma > G_h(\lambda)$, then $\tilde{K}_\lambda(\mu)$ is first concave and then convex with inflection point $\tilde{\mu}_{in} \in (\omega_\ell,\omega_h)$ and it satisfies $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h-\omega_\ell) \ge \tilde{K}_\lambda(\omega_\ell)$.

E. If $\lambda \in (4/5,1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$, then $\tilde{K}_\lambda(\mu)$ is first convex and then concave with inflection point $\mu_{in} \in (\omega_\ell,\omega_h)$ such that $\mu_\lambda^{DP} \le \mu_{in}$. Moreover, $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{DP}) = \tilde{K}'(\mu_\lambda^{DP})(\omega_h - \mu_\lambda^{DP})$.

F. If $\lambda \in [0,1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda)$, then $\tilde{K}_\lambda(\mu)$ is first concave and then convex with inflection point $\tilde{\mu}_{in} \in (\omega_\ell,\omega_h)$ such that $\mu_\lambda^{EX} \ge \tilde{\mu}_{in}$. Moreover, $\tilde{K}_\lambda(\mu_\lambda^{EX}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}'_\lambda(\mu_\lambda^{EX})(\mu_\lambda^{EX} - \omega_\ell)$.

We further define the following conditions.

$$\mathrm{FD}_1 : \lambda \in [1/2,4/5] \tag{29}$$

$$\mathrm{FD}_2 : \lambda \in (4/5,1] \wedge \kappa/\gamma \ge G_1(\lambda) \tag{30}$$

$$\mathrm{FD}_3 : \lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma \tag{31}$$

$$\mathrm{ND}_1 : \lambda \in [0, 1/2) \wedge \kappa/\gamma \geq G_\ell(\lambda) \tag{32}$$

$$\mathrm{ND}_2 : \lambda \in (4/5, 1] \wedge G_h(\lambda) \geq \kappa/\gamma \tag{33}$$

$$\mathrm{DP} : \lambda \in (4/5, 1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda) \tag{34}$$

$$\mathrm{EX} : \lambda \in [0, 1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda) \tag{35}$$

LEMMA 4. *The lower convex envelope $k_\lambda(\mu)$ of $\tilde{K}_\lambda(\mu)$ is*

$$k_\lambda(\mu) \triangleq \begin{cases} \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] & \text{if } \mathrm{FD}_1 \vee \mathrm{FD}_2 \vee \mathrm{FD}_3\,, \\ \tilde{K}_\lambda(\mu) & \text{if } \mathrm{ND}_1 \vee \mathrm{ND}_2\,, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \leq \mu_\lambda^{DP}\}} + \left[\tilde{K}_\lambda(\mu_\lambda^{DP}) + (\mu - \mu_\lambda^{DP}) \frac{\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{DP})}{(\omega_h - \mu_\lambda^{DP})}\right] 1_{\{\mu > \mu_\lambda^{DP}\}} & \text{if } \mathrm{DP}\,, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \geq \mu_\lambda^{EX}\}} + \left[\tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell) \frac{\tilde{K}_\lambda(\mu_\lambda^{EX}) - \tilde{K}_\lambda(\omega_\ell)}{(\mu_\lambda^{EX} - \omega_\ell)}\right] 1_{\{\mu < \mu_\lambda^{EX}\}} & \text{if } \mathrm{EX}\,. \end{cases} \tag{36}$$

where $1_{\{\cdot\}}$ represents the indicator function.

Combining these lemmas, we are now ready to prove the theorem.

Following Corollary 2 of Kamenica and Gentzkow (2011), we know that the optimal cost corresponding to any prior mean μ is given by $k_\lambda(\mu)$ because $k_\lambda(\mu)$ is the lower convex envelope of $\tilde{K}_\lambda(\mu)$ (see Lemma 4). We first provide a set of conditions, and characterize the optimal information disclosure policy for each bullet point.

- If FD_1 or FD_2 or FD_3, full disclosure is optimal. This item follows because we have $k_\lambda(\lambda) = \tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell)[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]/(\omega_h - \omega_\ell)$ so the optimal value can be achieved by the perfect information policy which induces posterior means ω_h and ω_ℓ. Thus, the binary message space $\mathcal{M}_\mathsf{B} = \{m_\ell, m_h\}$ with $\pi^\star(m_i \,|\, \omega_i) = 1$ for $i \in \{\ell, h\}$ achieves the optimal cost and hence constitute the optimal policy.

  Next, we show that if $\lambda_2 \leq \lambda \leq \lambda_3$, then FD_1 or FD_2 or FD_3 and hence full disclosure is optimal.

  — When $4/5 < \lambda \leq \lambda_3$, ($4/5 \leq \lambda_3$, see Lemma 2), we have $\lambda \leq 1$ and $\lambda \leq G_1^{-1}(\kappa/\gamma)$ from $\lambda \leq \lambda_3$. Thus, it follows that $\lambda \in (4/5, 1]$ and $\kappa/\gamma \geq G_1(\lambda)$ and hence FD_2.

  — When $1/2 \leq \lambda \leq 4/5$, we directly obtain FD_1.

  — When $\lambda_2 \leq \lambda < 1/2$, ($\lambda_2 \leq 1/2$, see Lemma 2), we have $\lambda \geq 0$ and $\lambda \geq G_2^{-1}(\kappa/\gamma)$ from $\lambda \geq \lambda_2$. Thus, it follows that $\lambda \in [0, 1/2)$ and $G_2(\lambda) \geq \kappa/\gamma$ and hence FD_3.

- If ND_1 or ND_2, no disclosure is optimal. The lower convex envelope $k_\lambda(\mu)$ is equal to the function $\tilde{K}_\lambda(\mu)$. Thus, no disclosure policy is the uniquely optimal. Note that $\lambda \le \lambda_1$ implies ND_1, and $\lambda \ge \lambda_4$ implies ND_2 from the definitions of λ_1 and λ_4. Hence, we prove the second bullet point of the theorem.
- If DP and $\mu^\circ > \mu_\lambda^{\mathsf{DP}}$, it is optimal to downplay the risk. The optimal cost can be achieved by no disclosure policy if $\mu^\circ \le \mu_\lambda^{\mathsf{DP}}$ because $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ in that case. Otherwise, the optimal cost can be achieved by inducing posterior means ω_h with probability $(\mu^\circ - \mu_\lambda^{\mathsf{DP}})/(\omega_h - \mu_\lambda^{\mathsf{DP}})$ and $\mu_\lambda^{\mathsf{DP}}$ with the remaining probability. Specifically, the following probability distribution provided in (17) with a binary message space $\mathcal{M}_{\mathsf{B}}$ achieves this posterior distribution. We next show that $\lambda_3 < \lambda < \lambda_4$ implies DP. Since, $\lambda_3 \ge 4/5$, we obtain that $\lambda \in (4/5, 1]$. Definitions of λ_3 and λ_4 (see (19)-(22)) imply $G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$.
- If EX and $\mu^\circ < \mu_\lambda^{\mathsf{EX}}$, it is optimal to exaggerate the risk. The optimal cost can be achieved by no disclosure policy if $\mu^\circ \ge \mu_\lambda^{\mathsf{EX}}$. Otherwise, the optimal cost can be achieved by inducing posterior means ω_ℓ with probability $(\mu_\lambda^{\mathsf{EX}} - \mu^\circ)/(\mu_\lambda^{\mathsf{EX}} - \omega_\ell)$ and $\mu_\lambda^{\mathsf{EX}}$ with the remaining probability. Specifically, the following probability distribution provided in (18) with a binary message space $\mathcal{M}_{\mathsf{B}}$ achieves this posterior distribution. We conclude the proof by showing that $\lambda_1 < \lambda < \lambda_2$ implies EX. Since $\lambda_2 \le 1/2$, we obtain that $\lambda \in [0, 1/2)$. Definitions of λ_3 and λ_4 (see (19)-(22)) $G_2(\lambda) < \kappa/\gamma < G_\ell(\lambda)$.

Proof of Lemma 1. To prove this result, we first provide the derivatives of G_ℓ and G_h with respect to λ.

$$G_h'(\lambda) = \frac{3}{2\omega_h(1-2\lambda)^2} \text{ and } G_\ell'(\lambda) = \frac{3}{2\omega_\ell(1-2\lambda)^2} \tag{37}$$

Since these terms above are positive, the claim follows. Now consider, G_2. The terms $\frac{4-5\lambda}{2(1-2\lambda)\omega_h\omega_\ell}$ and $\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$ are increasing functions of λ. Hence, G_2 is also increasing. In order to prove, G_1 is increasing we define the following notation for simplicity. Let $A_1(\lambda) \triangleq \frac{5\lambda-4}{2(2\lambda-1)\omega_h\omega_\ell}$, $A_2(\lambda) \triangleq \frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell}$, i.e., $G_1(\lambda) = \sqrt{A_1(\lambda) + (A_2(\lambda))^2} - A_2(\lambda)$. Taking derivative of A_1 and A_2, it can be shown that A_1 is increasing and A_2 is decreasing. Furthermore, it follows that

$$G_1'(\lambda) = \frac{A_1'(\lambda)}{2\sqrt{A_1(\lambda) + (A_2(\lambda)^2)}} - A_2'(\lambda)\left[1 - \frac{1}{\sqrt{\frac{A_1(\lambda)}{(A_2(\lambda))^2} + 1}}\right]. \tag{38}$$

This expression reveals that $G_1'(\lambda) > 0$ because $A_1'(\lambda)$ is positive, and $A_2'(\lambda)$ is negative.

Next, we prove $G_1(\lambda) > G_h(\lambda)$ and $G_\ell(\lambda) \geq G_1(\lambda)$ for $\lambda \in (4/5, 1]$. Note that we can write $G_h(\lambda) = A_1(\lambda)\omega_\ell$ and $G_\ell(\lambda) = A_1(\lambda)\omega_h$. First, we consider $G_1(\lambda) \geq G_h(\lambda)$, we need to show that

$$\sqrt{A_1(\lambda) + [A_2(\lambda)]^2} > A_1(\lambda)\omega_\ell + A_2(\lambda)$$

Using simple algebraic operations, we equivalently represent the inequality above as

$$1 > \frac{(1-\lambda)\omega_h + (3\lambda - 2)\omega_\ell}{\omega_h(2\lambda - 1)} \iff \omega_h > \omega_\ell\,.$$

For the last inequality we use the fact that $\lambda \in (4/5, 1]$ so $(3\lambda - 2) > 0$. Now, we consider $G_\ell(\lambda) > G_1(\lambda)$, we need to show that

$$A_1(\lambda)\omega_h + A_2(\lambda) > \sqrt{A_1(\lambda) + [A_2(\lambda)]^2}$$

Algebraic operations yield that the above inequality can be written as follows:

$$\frac{(3\omega_h + \omega_\ell)\lambda - 2\omega_h}{4\lambda\omega_\ell - 2\omega_\ell} > 1 \iff \omega_h > \omega_\ell$$

Here, we use the fact that $\lambda \in (4/5, 1]$ so $(3\lambda - 2) > 0$.

We next prove $G_\ell(\lambda) > G_2(\lambda)$, $G_2(\lambda) > G_h(\lambda)$ for $\lambda \in [0, 1/2)$. For simplicity we define $B_1(\lambda) = G_\ell(\lambda)/\omega_h$ and $B_2(\lambda) \triangleq \frac{\lambda\omega_h + 2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$. Using this notation, we first need to show that

$$\omega_h B_1(\lambda) > \sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda)$$

Simplifying this inequality we obtain that

$$\frac{((3\omega_h - \omega_\ell)\lambda - 2\omega_h + \omega_\ell)}{(2\lambda - 1)\omega_\ell} > 1 \iff \omega_h > \omega_\ell\,.$$

To obtain the last inequality, we use the fact that $\lambda \in [0, 1/2)$ to show $2 - 3\lambda > 0$. Thus, $G_\ell(\lambda) > G_2(\lambda)$ follows.

Finally, $G_2(\lambda) > G_h(\lambda)$ which can equivalently represented as follows:

$$\sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda) > \omega_\ell B_1(\lambda)$$

Using straightforward algebraic operations, we get the equivalent inequality.

$$1 > \frac{\omega_\ell(2 - 3\lambda) - \lambda\omega_h}{2\omega_h(1 - 2\lambda)} \iff \omega_h > \omega_\ell$$

As in the previous steps, we the fact that $\lambda \in [0, 1/2)$. Q.E.D.

Proof of Lemma 2 Since we know G_h, G_ℓ, G_1 and G_2 (see (23)-(25)) are strictly increasing in their domains from Lemma 1, their inverse are also strictly increasing (see, Binmore 1982, p. 111). Since κ/γ is strictly increasing in κ, thresholds λ_i are all nondecreasing in κ.

Because $G_1(4/5) = G_h(4/5) = 0$, it follows that $\lambda_3 \geq 4/5$ and $\lambda_4 \geq 4/5$. Furthermore, $G_1(\lambda) > G_h(\lambda)$ for $\lambda \in (4/5, 1]$ (see Lemma 1). Therefore, $\lambda_4 \geq \lambda_3 \geq 4/5$. Note that $\lim_{\lambda\to 1/2} G_\ell(\lambda) = \lim_{\lambda\to 1/2} G_2(\lambda) = \infty$, and G_ℓ and G_2 are increasing so we get $1/2 \geq \lambda_2$ and $1/2 \geq \lambda_1$. Since, $G_\ell(\lambda) > G_2(\lambda)$ for $\lambda \in [0, 1/2)$ it follows that $1/2 \geq \lambda_2 \geq \lambda_1$.

Using the definitions of G_h, G_ℓ, G_1 and G_2, we compute $\overline{\kappa}_0 = \gamma G_\ell(0) = 2\gamma/\omega_\ell$, $\underline{\kappa}_0 = \gamma G_2(0) = \gamma(\sqrt{2/(\omega_h\omega_\ell) + 1/\omega_h} - (1/\omega_h)^2)$, $\overline{\kappa}_1 = \gamma G_1(1) = \gamma\left(\sqrt{1/(2\omega_h\omega_\ell) + (1/(4\omega_h))^2} - 1/(4\omega_h)\right)$ and $\underline{\kappa}_1 = \gamma G_h(1) = \gamma/(2\omega_h)$. Then, we obtain $\overline{\kappa}_0 > \underline{\kappa}_0 > \overline{\kappa}_1 > \underline{\kappa}_1$.

Finally, the definitions of $\overline{\kappa}_i, \underline{\kappa}_i$ for $i \in \{0, 1\}$ imply that $\lambda_1(\overline{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$ and $\lambda_3(\overline{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$. Q.E.D.

Proof of Lemma 3. Using the derived expression of $c^\star(\mu)$ from Proposition 1 and the definition of $\tilde{K}_\lambda(\mu)$, we obtain that

$$\tilde{K}_\lambda(\mu) = \frac{(3\lambda - 2)}{2\gamma}\left(\frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa}\right)^2 + (1 - \lambda)\left(\frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa}\right)\frac{\theta + \gamma/2}{\gamma} - \frac{\lambda(\theta - \gamma/2)^2}{2\gamma}. \quad (39)$$

The second derivative of $\tilde{K}_\lambda$ with respect to μ is given by

$$\tilde{K}''_\lambda(\mu) = -\frac{4(\theta + \gamma/2)^2\kappa^2\gamma\left[\frac{\kappa}{\gamma}\mu\left(\lambda - \frac{1}{2}\right) + \left(1 - \frac{5\lambda}{4}\right)\right]}{(\kappa\mu + \gamma)^4}. \quad (40)$$

Part A. Note that $\tilde{K}''_\lambda(\mu)$ in (40) is negative when $\lambda \in [1/2, 4/5]$. Thus, the first item follows. Recalling the definition of G_h and G_ℓ from (23), one can show that the second and third items imply negative $\tilde{K}''_\lambda(\mu)$ because $\frac{\kappa}{\gamma}\mu\left(\lambda - \frac{1}{2}\right) + \left(1 - \frac{5\lambda}{4}\right)$ is positive under those conditions.

Part B. To prove this part, we again use (40). Simple algebra yields that $\frac{\kappa}{\gamma}\mu\left(\lambda - \frac{1}{2}\right) + \left(1 - \frac{5\lambda}{4}\right)$ is negative and hence $\tilde{K}''_\lambda(\mu)$ is positive when the first and second conditions hold.

Part C. First note that $G_1(\lambda) > G_h(\lambda)$ (see Lemma 1). Thus, we know the parameter regime is such that neither the second item of Part A nor the second item of Part B holds. This implies there exists an inflection point μ_{in} such that $\tilde{K}_\lambda(\mu)$ is first concave and then convex because $\tilde{K}''_\lambda(\mu)$ is first negative and then positive. We next derive $\tilde{K}_\lambda(\omega_h)-\tilde{K}_\lambda(\omega_\ell)-\tilde{K}'_\lambda(\omega_\ell)(\omega_h-\omega_\ell)$ and show that it is nonpositive when $\lambda\in(4/5,1]\wedge G_\ell(\lambda)>\kappa/\gamma\geq G_1(\lambda)$.

$$-\frac{2[(\omega_h-\omega_\ell)(\theta+\gamma/2)\kappa\gamma]^2(\lambda-\frac{1}{2})\left[\omega_\ell\omega_h\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)}\right]}{(\kappa\omega_h+\gamma)^2(\kappa\omega_\ell+\gamma)^3} \tag{41}$$

In order to prove $\tilde{K}_\lambda(\omega_h)-\tilde{K}_\lambda(\omega_\ell)-\tilde{K}'_\lambda(\omega_\ell)(\omega_h-\omega_\ell)\leq 0$, it is sufficient to show the following inequality holds.

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell}\geq 0 \tag{42}$$

We can rewrite the left-hand side of the above inequality as

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell} \tag{43}$$

$$=\left(\frac{\kappa}{\gamma}+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2-\left(\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell}. \tag{44}$$

This expression above is nonnegative whenever $\kappa/\gamma\geq G_1(\lambda)$ and we conclude the proof of this part.

Part D. Note that $G_\ell(\lambda)>G_2(\lambda)$ (see Lemma 1) so the parameter regime in this part is such that neither the third item of Part A nor the first item of Part B holds. Thus, there exists an inflection point $\tilde{\mu}_{in}\in(\omega_\ell,\omega_h)$ such that $\tilde{K}_\lambda(\mu)$ is first convex and then concave because $\tilde{K}''_\lambda(\mu)$ is first positive and then negative. Next, we show that $\tilde{K}_\lambda(\omega_h)-\tilde{K}'_\lambda(\omega_h)(\omega_h-\omega_\ell)-\tilde{K}_\lambda(\omega_\ell)$ is nonnegative when $\lambda\in[0,1/2)\wedge G_2(\lambda)\geq\kappa/\gamma>G_h(\lambda)$. The expression is given by

$$-\frac{2[(\omega_h-\omega_\ell)(\theta+\gamma/2)\kappa\gamma]^2\left(\frac{1}{2}-\lambda\right)\left[\omega_\ell\omega_h\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{2(1-2\lambda)}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(1-2\lambda)}\right]}{(\kappa\omega_h+\gamma)^3(\kappa\omega_\ell+\gamma)^2}.$$

To show $\tilde{K}_\lambda(\omega_h)-\tilde{K}'_\lambda(\omega_h)(\omega_h-\omega_\ell)-\tilde{K}_\lambda(\omega_\ell)\geq 0$, we can directly focus on the following inequality.

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell}\leq 0 \tag{45}$$

Rewriting this expression, we obtain the following.

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \tag{46}$$

$$=\left(\frac{\kappa}{\gamma}+\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2-\left(\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2-\frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell}\leq 0 \tag{47}$$

The last inequality holds because $\kappa/\gamma\leq G_2(\lambda)$.

Part E. Since $G_\ell(\lambda) > G_1(\lambda)$ (see Lemma 1), there exists an inflection point μ_{in} as in Part C. Differently from Part C, this time $G_1(\lambda) > \kappa/\gamma$ so $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell) - \tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) > 0$. Moreover, we know $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$ is an increasing function of $\mu \in (\omega_\ell, \mu_{in})$ due to convexity and $\tilde{K}_\lambda(\mu_{in}) + \tilde{K}'_\lambda(\mu_{in})(\omega_h - \mu_{in}) > \tilde{K}_\lambda(\omega_h)$ due to concavity for $\mu \in (\mu_{in}, \omega_h)$. Thus, $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$ crosses $\tilde{K}_\lambda(\omega_h)$ for some $\mu = \mu_\lambda^{\mathsf{DP}}$ such that $\mu_\lambda^{\mathsf{DP}} \le \mu_{in}$.

Part F. Note that $G_2(\lambda) > G_h(\lambda)$ (see Lemma 1) therefore there exists an inflection point $\tilde{\mu}_{in}$ as in Part D. Unlike Part D, $\kappa/\gamma > G_2(\lambda)$ thus $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell) < 0$. Due to concavity in $(\omega_\ell, \tilde{\mu}_{in})$, we know $\tilde{K}_\lambda(\omega_\ell) < \tilde{K}_\lambda(\tilde{\mu}_{in}) - \tilde{K}'_\lambda(\tilde{\mu}_{in})(\tilde{\mu}_{in} - \omega_\ell)$. Besides, $\tilde{K}_\lambda(\mu) - \tilde{K}'_\lambda(\mu)(\mu - \omega_\ell)$ is a decreasing function of $\mu \in (\tilde{\mu}_{in}, \omega_h]$ due to convexity after the inflection point $\tilde{\mu}_{in}$. Thus, there must be a point $\mu_\lambda^{\mathsf{EX}}$ in $(\tilde{\mu}_{in}, \omega_h)$ such that $\tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) - \tilde{K}'_\lambda(\mu_\lambda^{\mathsf{EX}})(\mu_\lambda^{\mathsf{EX}} - \omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$. Q.E.D.

Proof of Lemma 4. In order to check if the proposed function k_λ is the lower convex envelope of $\tilde{K}_\lambda$ or not, we use the verification approach provided in Oberman (2007). In particular, we show that the function k_λ satisfies (Ob) of Oberman (2007) equation for $\tilde{K}_\lambda$. This equation is

$$\max\{k_\lambda(\mu) - \tilde{K}_\lambda(\mu), -k''_\lambda(\mu)\} = 0\,. \tag{Ob}$$

- $\mathrm{FD}_1 \vee \mathrm{FD}_2 \vee \mathrm{FD}_3$: We need to show $\tilde{K}_\lambda(\mu) \ge v(\mu)$ because $k''_\lambda(\mu) = 0$. Since $k_\lambda(\mu)$ is in fact the line connecting $\tilde{K}_\lambda(\omega_\ell)$ and $\tilde{K}_\lambda(\omega_h)$ in this case, the result immediately follows when $\tilde{K}_\lambda$ is concave. Note that Part A of Lemma 3 shows that $\tilde{K}_\lambda$ is concave when FD_1. For FD_2 and FD_3, we need to complement that the same part of the lemma by considering $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \ge G_1(\lambda)$ and $\lambda \in [0, 1/2) \wedge G_2(\lambda) \ge \kappa/\gamma > G_h(\lambda)$, respectively.

  Assume that $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \ge G_1(\lambda)$. For $\mu \le \mu_{in}$, Taylor's theorem implies that $\tilde{K}_\lambda(\omega_\ell) + \tilde{K}'_\lambda(\omega_\ell)(\mu - \omega_\ell) < \tilde{K}_\lambda(\mu)$ because $\tilde{K}_\lambda$ is convex in $[\omega_\ell, \mu_{in}]$. Using Part C of Lemma 3 to replace $\tilde{K}'_\lambda(\omega_\ell)$ with its lower bound, we obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$. When $\mu \ge \mu_{in}$, we know $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_{in})x + \tilde{K}_\lambda(\omega_h)(1 - x)$ for any $x \in (0, 1)$ due to concavity. Furthermore, we know that $\tilde{K}_\lambda(\mu_{in}) \ge \tilde{K}_\lambda(\omega_\ell) + \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ from the previous analysis. Thus, we obtain that

  $$\tilde{K}_\lambda(\mu) > \left(1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}\right) x\tilde{K}_\lambda(\omega_\ell) + \left(x\frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell} + 1 - x\right)\tilde{K}_\lambda(\omega_h)$$

  Here, we can set $\left(1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}\right) x = \left(1 - \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}\right)$ because $0 \le \frac{1 - \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}}{1 - \frac{\mu_{in} - \omega_\ell}{\omega_h - \omega_\ell}} \le 1$ since $\mu \ge \mu_{in}$. Thus, we obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ for $\mu \ge \mu_{in}$, too.

Next, assume that $\lambda \in [0,1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$. Taylor's theorem implies that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_h) + \tilde{K}'_\lambda(\omega_h)(\mu - \omega_h)$ for $\mu > \tilde{\mu}_{in}$ due to convexity. Since $\mu - \omega_h$ is negative, we replace $\tilde{K}'_\lambda(\omega_h)$ with its upper bound obtained in Part D of Lemma 3. Thus, it follows that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ for $\mu > \tilde{\mu}_{in}$. When $\mu \leq \tilde{\mu}_{in}$, we know that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\tilde{\mu}_{in})x + (1-x)\tilde{K}_\lambda(\omega_\ell)$ for any $x \in [0,1]$ because $\tilde{K}_\lambda$ is concave in $[\omega_\ell, \tilde{\mu}_{in}]$. Moreover, we know that $\tilde{K}_\lambda(\tilde{\mu}_{in}) \geq \tilde{K}_\lambda(\omega_\ell) + \frac{\tilde{\mu}-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ from the previous analysis. Combining these observations, we obtain

$$\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell)x + \frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]x + (1-x)\tilde{K}_\lambda(\omega_\ell) \tag{48}$$

$$= \tilde{K}_\lambda(\omega_\ell) + x\frac{\tilde{\mu}_{in} - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] \tag{49}$$

Here, we can set $x = \left(\frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}\right) / \left(\frac{\tilde{\mu}_{in}-\omega_\ell}{\omega_h-\omega_\ell}\right)$ because $\mu \leq \tilde{\mu}_{in}$, and obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$. This completes the proof of this item of the lemma.

- $\text{ND}_1 \vee \text{ND}_2$: Part B of Lemma 3 shows that $\tilde{K}_\lambda$ is convex, thus its lower convex envelope is equal to $\tilde{K}_\lambda(\mu)$ itself (see Boyd and Vandenberghe 2004, pp. 94).
- DP: We consider two cases separately. First, assume that $\mu \leq \mu_\lambda^{\mathsf{DP}}$, then $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ and $\tilde{K}_\lambda(\mu)$ is convex because $\mu_\lambda^{\mathsf{DP}} \leq \mu_{in}$ (see Part E of Lemma 3). Next, assume that $\mu \geq \mu_\lambda^{\mathsf{DP}}$, then $k''_\lambda(\mu) = 0$ by its definition. Thus, we need to show that $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu \geq \mu_\lambda^{\mathsf{DP}}$ to prove that $k_\lambda(\mu)$ satisfies (Ob). Taylor's theorem implies that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_\lambda^{\mathsf{DP}}) + (\mu - \mu_\lambda^{\mathsf{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{DP}})$ for $\mu_{in} \geq \mu$ due to convexity. From Part E of Lemma 3, we also know $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\mathsf{DP}}) + (\mu - \mu_\lambda^{\mathsf{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{DP}})$. Thus, $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu_{in} \geq \mu \geq \mu_\lambda^{\mathsf{DP}}$.

  Next, we consider $\mu \geq \mu_{in}$. When we look at the boundary $\mu = \mu_{in}$, the former observation implies $\tilde{K}_\lambda(\mu_{in}) \geq k_\lambda(\mu_{in})$. Because $\tilde{K}_\lambda$ is concave when $\mu \geq \mu_{in}$, it follows that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_{in}) + (\mu - \mu_{in})[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_{in})]/(\omega_h - \mu_{in})$ which constitutes the line connecting $\tilde{K}_\lambda(\mu_{in})$ to $\tilde{K}_\lambda(\omega_h)$. Because $k_\lambda(\mu)$ is in fact the line connecting $k_\lambda(\mu_{in})$ to $k_\lambda(\omega_h) = \tilde{K}_\lambda(\omega_h)$ by definition, it follows that $\tilde{K}_\lambda(\mu) \geq k_\lambda(\mu)$ for $\mu \geq \mu_{in}$, too.

  Combining these, we conclude that $k_\lambda(\mu)$ satisfies (Ob).
- EX: We consider two cases separately. If $\mu \geq \mu_\lambda^{\mathsf{EX}}$, then $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ and $\tilde{K}_\lambda(\mu)$ is convex from Part F of Lemma 3 because $\mu_\lambda^{\mathsf{EX}} \geq \tilde{\mu}_{in}$. We next consider $\mu_\lambda^{\mathsf{EX}} \geq \mu$. By its definition, $k''_\lambda(\mu) = 0$ when $\mu_\lambda^{\mathsf{EX}} \geq \mu$. Thus, we need to show that $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu_\lambda^{\mathsf{EX}} \geq \mu$ to prove that $k_\lambda(\mu)$ satisfies (Ob).

When $\mu_\lambda^{\mathsf{EX}} \geq \mu \geq \tilde{\mu}_{in}$, Taylor's theorem implies that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) + (\mu - \mu_\lambda^{\mathsf{EX}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{EX}})$ because $\tilde{K}_\lambda(\mu)$ is convex for $\mu \geq \tilde{\mu}_{in}$. Furthermore, we know $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) + (\mu - \mu_\lambda^{\mathsf{EX}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{EX}})$ because $\tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}'_\lambda(\mu_\lambda^{\mathsf{EX}})(\mu_\lambda^{\mathsf{EX}} - \omega_\ell)$ (see Part F of Lemma 3). Thus, $k_\lambda(\mu) \leq \tilde{K}_\lambda(\mu)$ for $\mu_\lambda^{\mathsf{EX}} \geq \mu \geq \tilde{\mu}_{in}$.

Next, we consider $\tilde{\mu}_{in} \geq \mu$. By definition $k_\lambda(\omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$, and $k_\lambda(\mu)$ is the line connecting $k_\lambda(\omega_\ell)$ and $k_\lambda(\tilde{\mu}_{in})$ for $\tilde{\mu}_{in} \geq \mu$. We also know $\tilde{K}_\lambda(\mu)$ is above the line connecting $\tilde{K}_\lambda(\omega_\ell)$ and $\tilde{K}_\lambda(\tilde{\mu}_{in})$ because $\tilde{K}_\lambda$ is concave for $\tilde{\mu}_{in} \geq \mu$. Hence, we need to show $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$ to complete the proof. The previous observation also implies that $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$ so the claim follows. Q.E.D.

Proof of Theorem 2. We first prove the monotonicity of downplaying and exaggerating probabilities in γ. Because $\gamma = 2\sigma\sqrt{3}$, we also obtain the same monotonicity behavior in σ.

Note that the optimal downplaying probability, see (17), is increasing in $\mu_\lambda^{\mathsf{DP}}$ when $\lambda_1 < \lambda < \lambda_2$, and exaggerating probability, see (18), is decreasing $\mu_\lambda^{\mathsf{EX}}$ when $\lambda_3 < \lambda < \lambda_4$. Thus, we focus on the derivatives of $\mu_\lambda^{\mathsf{DP}}$ and $\mu_\lambda^{\mathsf{EX}}$ with respect to γ.

The derivative of $\mu_\lambda^{\mathsf{DP}}$ with respect to γ is given by

$$\frac{\gamma^2(5\lambda-4)\left[-8(1-\lambda)\frac{\left(\lambda-\frac{1}{2}\right)}{(5\lambda-4)}\frac{\kappa^2\omega_h^2}{\gamma^2}+8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}+\lambda\right]}{16\left(\kappa\omega_h\left(\lambda-\frac{1}{2}\right)+\frac{\gamma\lambda}{4}\right)^2\kappa}. \tag{50}$$

Since λ is inside $(4/5,1]$ when downplaying, the derivative is positive if and only if the term inside square brackets is positive. Thus, we focus on that term.

$$-8(1-\lambda)\frac{\left(\lambda-\frac{1}{2}\right)}{(5\lambda-4)}\frac{\kappa^2\omega_h^2}{\gamma^2}+8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}+\lambda = 8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}\left(1-\frac{(1-\lambda)}{(5\lambda-4)}\frac{\kappa\omega_h}{\gamma}\right)+\lambda \tag{51}$$

Next, we show that $\left(1-\frac{(1-\lambda)}{(5\lambda-4)}\frac{\kappa\omega_h}{\gamma}\right)$ is positive because the remaining terms are positive. To do so, we use the upper bound $G_1(\lambda)$ on κ/γ. In particular, we prove that $\frac{5\lambda-4}{1-\lambda} \geq \omega_h G_1(\lambda)$. First note that

$$\begin{aligned}&\frac{5\lambda-4}{1-\lambda}-\omega_h G_1(\lambda)\\&=\frac{5\lambda-4}{1-\lambda}-\sqrt{\frac{(5\lambda-4)\omega_h}{2\left(2\lambda-1\right)\omega_\ell}+\left(\frac{2\left(1-\lambda\right)\omega_h+\lambda\omega_\ell}{4\left(2\lambda-1\right)\omega_\ell}\right)^2}+\frac{2\left(1-\lambda\right)\omega_h+\lambda\omega_\ell}{4\left(2\lambda-1\right)\omega_\ell}.\end{aligned} \tag{52}$$

In the following, we show that the sum of positive terms above is larger than the negative term.

$$\left[\frac{5\lambda-4}{1-\lambda}+\frac{2\left(1-\lambda\right)\omega_h+\lambda\omega_\ell}{4\left(2\lambda-1\right)\omega_\ell}\right]^2-\frac{(5\lambda-4)\omega_h}{2\left(2\lambda-1\right)\omega_\ell}-\left(\frac{2\left(1-\lambda\right)\omega_h+\lambda\omega_\ell}{4\left(2\lambda-1\right)\omega_\ell}\right)^2 \tag{53}$$

$$= \frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}\left\{\frac{2(2\lambda-1)\omega_\ell}{(1-\lambda)\omega_h}\left[\frac{5\lambda-4}{1-\lambda}+\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{2\,(2\lambda-1)\,\omega_\ell}\right]-1\right\} \tag{54}$$

$$= \frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}\left\{\frac{2\,(2\lambda-1)\,\omega_\ell(5\lambda-4)}{\omega_h(1-\lambda)^2}+\frac{\lambda\omega_\ell}{\omega_h(1-\lambda)}+1\right\}>0 \tag{55}$$

The last inequality follows because all terms are positive. Combining these, we conclude that $\mu_\lambda^{\mathsf{DP}}$ is increasing in γ.

The derivative of $\mu_\lambda^{\mathsf{EX}}$ with respect to γ is given by

$$\frac{\gamma^2(4-5\lambda)\left[8(1-\lambda)\frac{\kappa^2\omega_\ell^2}{\gamma^2}\frac{\left(\frac{1}{2}-\lambda\right)}{(4-5\lambda)}+8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)-\lambda\right]}{16\left(\kappa\omega_\ell\left(\lambda-\frac{1}{2}\right)+\frac{\gamma\lambda}{4}\right)^2\kappa}. \tag{56}$$

Recall that λ is inside $[0,1/2)$ when exaggerating, so the derivative is positive if and only if the term inside square brackets is positive. Hence, we focus on that term.

$$8(1-\lambda)\frac{\kappa^2\omega_\ell^2}{\gamma^2}\frac{\left(\frac{1}{2}-\lambda\right)}{(4-5\lambda)}+8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)-\lambda=8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)\left(\frac{\kappa\omega_\ell}{\gamma}\frac{(1-\lambda)}{4-5\lambda}+1\right)-\lambda \tag{57}$$

Here, note that $\left(\frac{\kappa\omega_\ell}{\gamma}\frac{(1-\lambda)}{4-5\lambda}+1\right)$ is larger than 1, thus we next show that $8\frac{\kappa\omega_\ell}{\gamma}(1/2-\lambda)\geq\lambda$ to complete the proof. To do so, we use the lower bound on κ/γ.

$$\frac{\kappa}{\gamma}8\omega_\ell(1/2-\lambda)\geq G_2(\lambda)8\omega_\ell(1/2-\lambda)=\lambda+\frac{2(1-\lambda)\omega_\ell}{\omega_h}\geq\lambda\,. \tag{58}$$

Thus, we conclude that $\mu_\lambda^{\mathsf{EX}}$ is increasing in γ.

The remaining part of the theorem follows because functions G_2 and G_ℓ on $[0,1/2)$ and G_1, G_h on $(4/5,1]$ are increasing functions and κ/γ is a decreasing function of σ. Figure 3 provides an illustration for this result. Q.E.D.

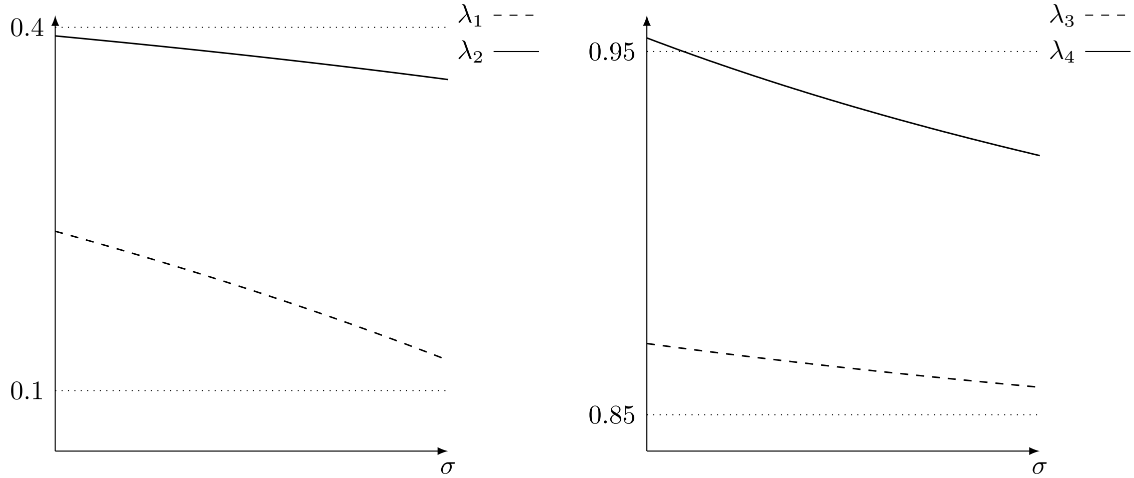
B Belief Update using Bayes' Rule

The prior belief of the population is ρ°, and suppose that the sender commits to an information disclosure policy $\Gamma=(\pi,\mathcal{M})$. After receiving message m, receivers update their beliefs from ρ° to ρ_m according Bayes' rule as follows.

$$\rho_m\triangleq\mathbb{P}(\omega=\omega_h\,|\,m)=\frac{\pi(m\,|\,\omega_h)\rho^\circ}{\pi(m\,|\,\omega_h)\rho^\circ+\pi(m\,|\,\omega_\ell)(1-\rho^\circ)}\,. \tag{59}$$

Accordingly, the posterior mean corresponding to the posterior belief is

$$\mu_m\triangleq\rho_m\omega_h+(1-\rho_m)\omega_\ell\,. \tag{60}$$



(a) λ_1 and λ_2 and $\kappa = 150$

(b) λ_3 and λ_4 and $\kappa = 9$

Figure 3 **Threshold functions** λ_i, $i = 1, \ldots, 4$ **as a function of** σ **for** $\omega_\ell = 0.3$, $\omega_h = 0.8$ **and** $\theta = 10$.

C Technical Intuition of Our Results

In this section of the appendix, we provide further intuition regarding Theorem 1. To do so, we introduce the notation $c^\star(\mu\kappa)$ such that $c^\star(\mu\kappa) = c^\star(\mu)$ in Proposition 1 with some abuse of notation.

As discussed in Section 3, the shape of $K_\lambda(c^\star(\mu\kappa))$ is jointly determined by that of $K_i(\cdot)$ ($i \in \{e, h\}$) and that of the threshold $c^\star(\mu\kappa)$, which we recall is an increasing concave function of μ due to negative externalities among the receivers (as implied by Proposition 1). Therefore, we next explore the shape of $K_i(\cdot)$ ($i \in \{e, h\}$).

Proposition 2. *For any $x \in \mathcal{C}$, the function $K_e(x)$ is increasing convex, whereas the function $K_h(x)$ is concave and unimodal with the peak point at $x = (\theta + \sigma\sqrt{3})/2$.*

Proof of Proposition 2. We first derive the expressions for $K_i(x)$ for $i \in \{e, h\}$ for uniform distribution with mean θ and support $\mathcal{C}$. Note that we assume $x \in \mathcal{C}$.

$$K_e(x) = \mathbb{E}_c[1_{\{c \le x\}} c] = \int_{\theta - \gamma/2}^{x} \frac{x}{\gamma} \mathrm{d}x = \frac{1}{2\gamma} \left[(x)^2 - \left(\theta - \frac{\gamma}{2} \right)^2 \right] \tag{61}$$

$$K_h(x) = x(1 - F(x)) = x \left(1 - \frac{x - \theta + \gamma/2}{\gamma} \right) \tag{62}$$

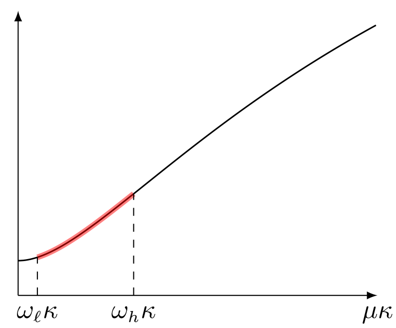
Using these, we get $K_e'(x) = x/\gamma$ and $K_e''(x) = 1/\gamma$ and prove the first bullet point. Furthermore, it follows that $K_h'(x) = [(\theta + \gamma/2) - 2x]/\gamma$ and $K_h''(x) = -2/\gamma$. Because the second derivative is negative, it follows that $K_h(x)$ is concave. Since $K_h'(x) \ge 0$ when $x \le (\theta + \gamma/2)/2$, and $K_h'(x) \le 0$ when $x \ge (\theta + \gamma/2)/2$, the result follows. Q.E.D.

Thus, as equilibrium threshold $c^\star$ increases, more individuals incurring increasingly higher economic cost remain in confinement. Hence, the overall economic cost increases at an increasing rate (i.e., $K_e(\cdot)$ is increasing convex). In contrast, total healthcare cost $K_h(\cdot)$ first increases as $c^\star$ increases, because the expected healthcare cost per individual is given by $\mu\kappa P_{a^\star,\mu} = c^\star$. However, as threshold $c^\star$ keeps increasing, fewer individuals choose to engage in social interaction, eventually lowering the total healthcare cost (i.e., $K_h(\cdot)$ then decreases).

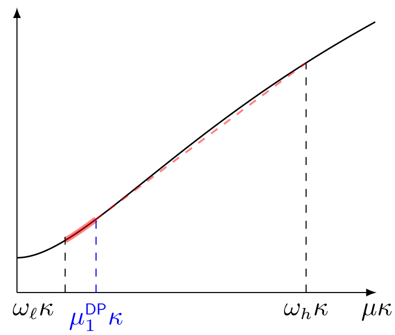
As the optimal policy is obtained through the lower convex envelop of $K_\lambda\big(c^\star(\mu\kappa)\big)$ as a function of μ, the nature of the optimal information policy characterized above is fundamentally driven by the second-order behavior of function $K_\lambda\big(c^\star(\cdot)\big)$. Specifically, as illustrated by Figure 4, $K_e\big(c^\star_\gamma(\cdot)\big)$ is first convex and then concave, whereas $K_h\big(c^\star_\gamma(\cdot)\big)$ is the opposite, i.e, first concave and then convex. The domain of these functions are $[\omega_\ell\kappa, \omega_h\kappa]$ because $\mu \in [\omega_\ell, \omega_h]$. In effect, κ acts to control the active domain of function $K_\lambda\big(c^\star(\cdot)\big)$, over which the lower convex envelope will be constructed.

Take the economy-biased government ($\lambda = 1$) as an example. When the healthcare cost is low ($\kappa \le \underline{\kappa}_1$), function $K_e\big(c^\star(\cdot)\big)$ is entirely convex over $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope is itself (see Figure 4a), suggesting that the optimal martingale split of μ° is simply not to split and hence no disclosure is optimal. When the healthcare cost moves into the intermediate range ($\underline{\kappa}_1 < \kappa < \overline{\kappa}_1$), the inflection point of function $K_\lambda\big(c^\star(\cdot)\big)$ falls within $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope consists of itself over $[\omega_\ell\kappa, \mu_1^{\mathsf{DP}}\kappa]$ and the straight line connecting $\big(\mu_1^{\mathsf{DP}}\kappa, K_e\big(c^\star(\mu_1^{\mathsf{DP}}\kappa)\big)\big)$ and $\big(\omega_h\kappa, K_e\big(c^\star(\omega_h\kappa)\big)\big)$ (see Figure 4b), where the straight line and function $K_e\big(c^\star(\cdot)\big)$ are tangent with each other at $\big(\mu_1^{\mathsf{DP}}\kappa, K_e\big(c^\star(\mu_1^{\mathsf{DP}}\kappa)\big)\big)$ (see (13) for the closed-form expression of μ_1^{DP}). Thus, as in the previous case, no disclosure is optimal if $\mu^\circ \le \mu_1^{\mathsf{DP}}$; otherwise, the optimal martingale split of μ° would induce posterior beliefs at μ_1^{DP} and ω_h, which can be implemented by downplaying (see (17) in for exact expression of $\pi^\star(m_\ell \,|\, \omega_h)$). When the healthcare cost is sufficiently large ($\kappa \ge \overline{\kappa}_1$), function $K_e\big(c^\star(\cdot)\big)$ is entirely concave over $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope is the straight line connecting $\big(\omega_\ell\kappa, K_e\big(c^\star_\gamma(\omega_\ell\kappa)\big)\big)$ and $\big(\omega_h\kappa, K_e\big(c^\star_\gamma(\omega_h\kappa)\big)\big)$ (see Figure 4c), suggesting that the optimal martingale split of μ° would induce posterior beliefs at ω_ℓ and ω_h and hence full disclosure is optimal.

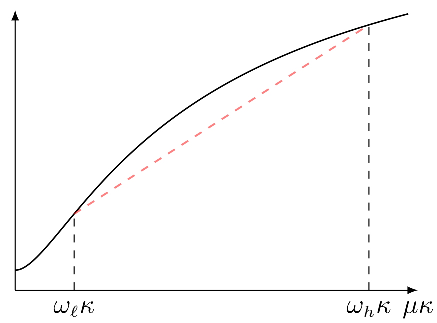
Similarly, the healthcare-biased government's optimal policy can be constructed from the lower convex envelope of $K_h\big(c^\star(\cdot)\big)$ (see Figures 4d-4f), albeit in the direction opposite



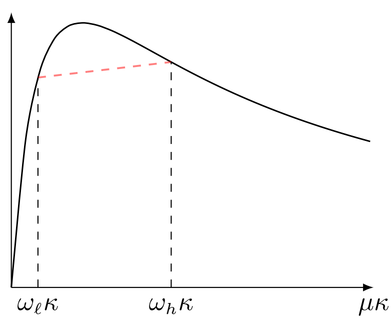
(a) $K_e(c^\star(\cdot))$ for $\kappa \le \underline{\kappa}_1$



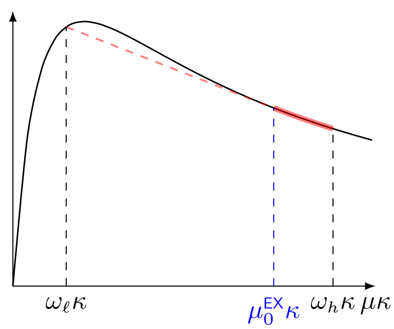
(b) $K_e(c^\star(\cdot))$ for $\underline{\kappa}_1 < \kappa < \overline{\kappa}_1$



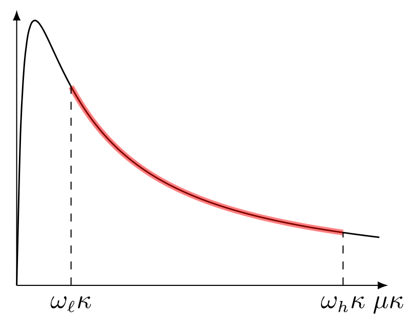
(c) $K_e(c^\star(\cdot))$ for $\kappa \ge \overline{\kappa}_1$



(d) $K_h(c^\star(\cdot))$ for $\kappa \le \underline{\kappa}_0$



(e) $K_h(c^\star(\cdot))$ for $\underline{\kappa}_0 < \kappa < \overline{\kappa}_0$



(f) $K_h(c^\star(\cdot))$ for $\kappa \ge \overline{\kappa}_0$

Figure 4 **Convexification of $K_\lambda\big(c^\star(\mu\kappa)\big)$ as a function of μ for $\lambda \in \{1,0\}$, $\omega_\ell = 0.15$, $\omega_h = 0.9$, $\gamma = 20$ and $\theta = 10$. Solid red lines represent that $K_\lambda\big(c^\star(\mu\kappa)\big)$ coincides with its lower convex envelope, whereas dashed red lines represent $K_\lambda\big(c^\star(\mu\kappa)\big)$ is strictly above its lower convex envelope.**

to that of the economy-biased policy. This is because $K_h\big(c^\star(\cdot)\big)$ demonstrates the second-order behavior in contrast to that of $K_e\big(c^\star(\cdot)\big)$. Therefore, when the government needs to make a trade off between the economic and healthcare costs by minimizing $K_\lambda(c^\star(\mu\kappa))$ for some $\lambda \in [0,1]$, the opposing behaviors of $K_e\big(c^\star(\cdot)\big)$ and $K_h\big(c^\star(\cdot)\big)$ may offset each other and subsequently damp down the information distortion in the optimal policy.

D Numerical Study

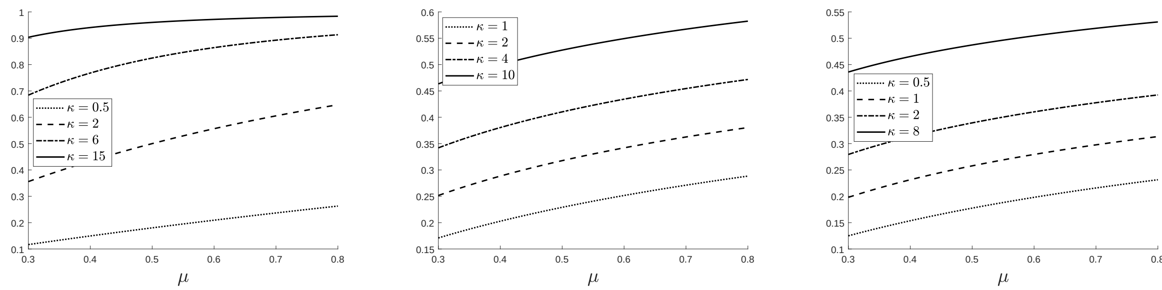
In this section of the appendix, we provide a numerical study where we consider different probability distributions for the economic cost c. For any given value of λ and κ we first numerically characterize the objective function of the sender $K_\lambda(c^\star(\mu))$ using the equilibrium threshold $c^\star(\mu)$. Next, we analyze the lower convex envelope of $K_\lambda(c^\star(\mu))$ to characterize the optimal information policy. Moreover, we numerically illustrate the effect of economic inequality on downplaying and exaggerating probabilities.

D.1 Optimal Policy

In order to derive the sender's objective function $K_\lambda(c^\star(\mu))$, we first need to compute the equilibrium threshold as a function of μ using the unique solution of the following equation.

$$\frac{c^\star(\mu)}{\mu\kappa} = 1 - F\left(c^\star(\mu)\right) \tag{63}$$

In Proposition 1, we solve $c^\star(\mu)$ in closed form for uniform distribution. Due to nega-



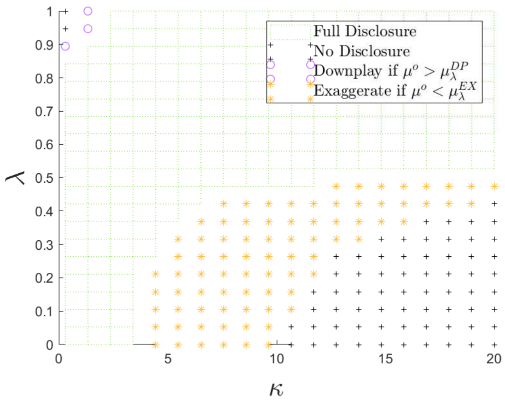
(a) $a = 0.5$ and $b = 0.5$ (b) $a = 1$ and $b = 3$ (c) $a = 2$ and $b = 5$

Figure 5 Equilibrium threshold $c^\star(\mu)$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and beta distributions with different a and b

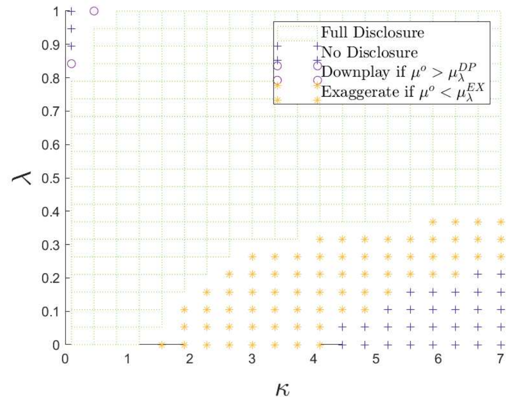
tive externalities (captured by $P_{a^\star,\mu}$), the equilibrium threshold increases albeit with a diminishing rate for uniform distribution. Although the same properties (monotonicity and concavity) hold for distributions with an increasing density function, it is not straightforward to see whether negative externalities affect the equilibrium threshold in the same way for different distributions. To analyze this, we consider three beta distributions whose density functions are decreasing (with parameters $a = 1$ and $b = 3$), unimodal with single peak point (with parameters $a = 2$ and $b = 5$), and nonmonotone with single lowest point (with parameters $a = 0.5$ and $b = 0.5$). Figure 5 illustrates that the same properties of $c^\star(\mu)$, and hence the effect of negative externalities on the equilibrium, are indeed extended to those cases.

Next, we numerically analyze the optimal information policy in the (κ, λ)-space.

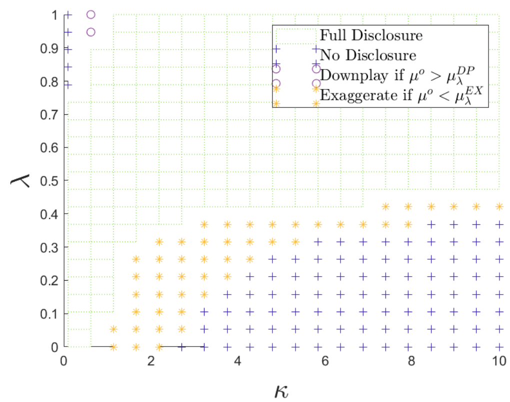
For the same three beta distributions, Figure 6 plots the government's optimal policy in the (κ, λ)-space, which demonstrates qualitatively the same patterns as those in Figure 1 of the paper. Hence, the main insights of Theorem 1 are also present for different beta distributions. More specifically, when the government weighs the economy and population health sufficiently equally, i.e., when λ is sufficiently close to 1/2, the government fully discloses its information about the severity of the epidemic. However, a more polarized



(a) $a = 0.5$ and $b = 0.5$



(b) $a = 1$ and $b = 3$



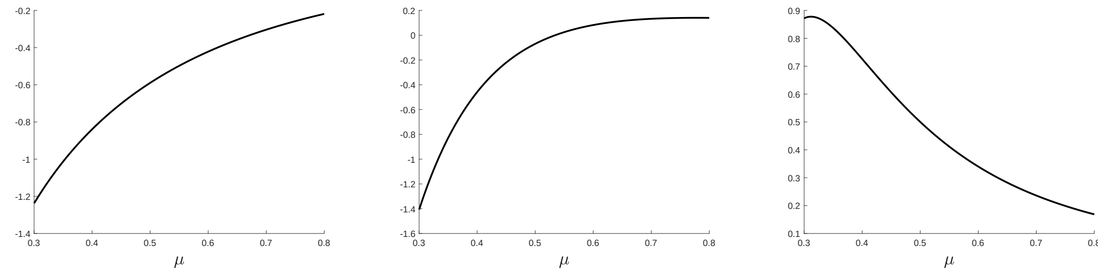
(c) $a = 2$ and $b = 5$

Figure 6 Optimal Policy in (κ, λ)-space for $\omega_\ell = 0.3$, $\omega_h = 0.8$ and beta distributions with parameters a and b

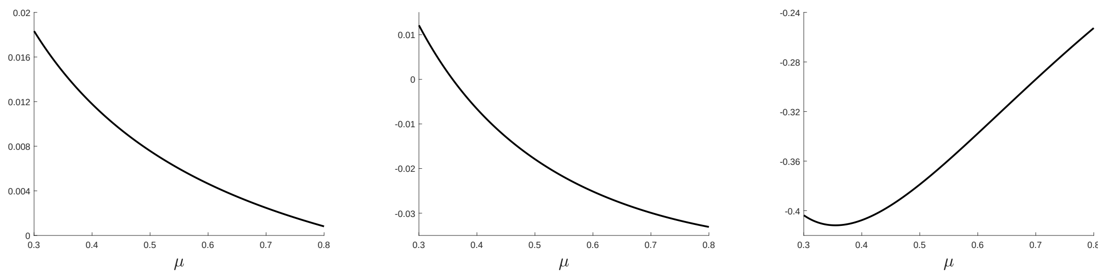
government (i.e., λ is sufficiently far from 1/2), may misrepresent its information. As in the case of uniform distribution, this happens when health costs also take on extreme values. When λ is small and κ is large, the government exaggerates the epidemic's severity if the population's perception of the risk is low ($\mu^\circ < \mu_\lambda^{\mathsf{EX}}$). On the other hand, when λ is large and κ small, the government downplays the epidemic's severity if the population's perception of the risk is high ($\mu^\circ > \mu_\lambda^{\mathsf{DP}}$). If both λ and κ take on very extreme values, the government prefers not to disclose any information.

The technical intuition behind the results in Figure 6 is driven by the second order behavior of $K_\lambda(c^\star(\mu))$ as discussed in Appendix C (see Figure 4). Therefore, to conclude this section, we numerically compute and illustrate the second derivative of objective function $K_\lambda(c^\star(\mu))$.

In Figures 7-9, we provide examples which correspond to no disclosure (positive second derivative as in Figures 7c, 7d,8c, 8d, 9c, 9d), full disclosure (negative second derivative as

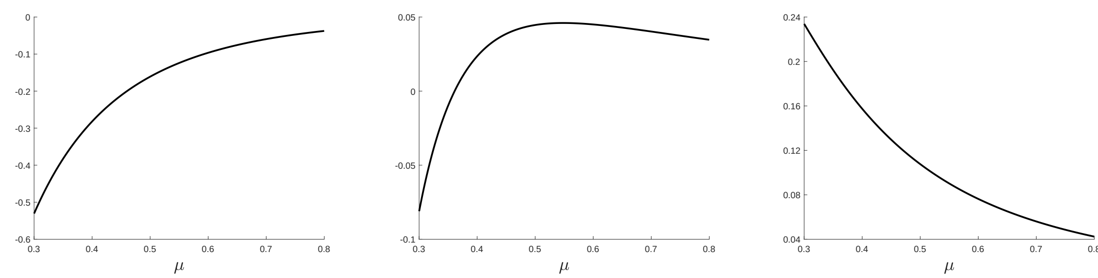


(a) $\lambda = 0.1$ and $\kappa = 2$ (b) $\lambda = 0.1$ and $\kappa = 6$ (c) $\lambda = 0.1$ and $\kappa = 15$

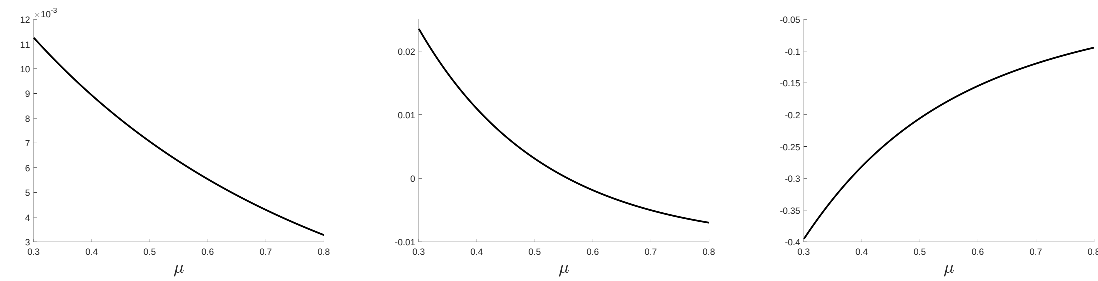


(d) $\lambda = 0.95$ and $\kappa = 0.5$ (e) $\lambda = 0.95$ and $\kappa = 1.2$ (f) $\lambda = 0.95$ and $\kappa = 4$

Figure 7 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = b = 0.5$**



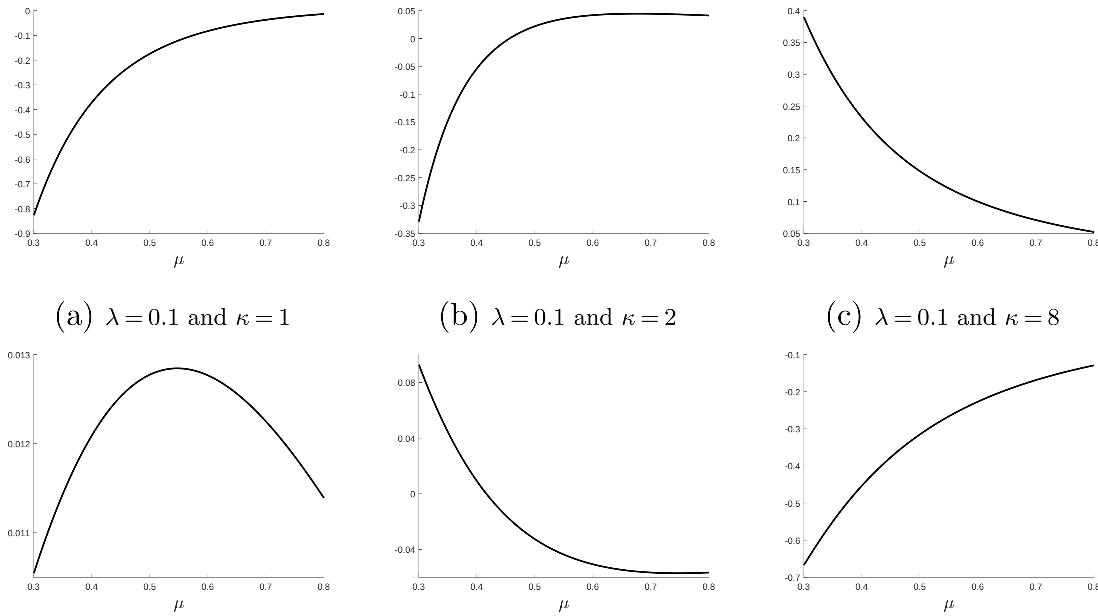
(a) $\lambda = 0.1$ and $\kappa = 1$ (b) $\lambda = 0.1$ and $\kappa = 4$ (c) $\lambda = 0.1$ and $\kappa = 10$



(d) $\lambda = 0.95$ and $\kappa = 0.1$ (e) $\lambda = 0.95$ and $\kappa = 0.25$ (f) $\lambda = 0.95$ and $\kappa = 2$

Figure 8 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = 1$ and $b = 3$**

in Figures 7a, 7f, 8a, 8f, 9a, 9f), downplaying (first positive then negative second derivative as in Figures 7e, 8e, 9e), and exaggerating (first negative then positive second derivative as in Figures 7b, 8b, 9b) information policies.



(a) $\lambda = 0.1$ and $\kappa = 1$ (b) $\lambda = 0.1$ and $\kappa = 2$ (c) $\lambda = 0.1$ and $\kappa = 8$

(d) $\lambda = 0.95$ and $\kappa = 0.1$ (e) $\lambda = 0.95$ and $\kappa = 0.5$ (f) $\lambda = 0.95$ and $\kappa = 2$

Figure 9 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = 2$ and $b = 5$**

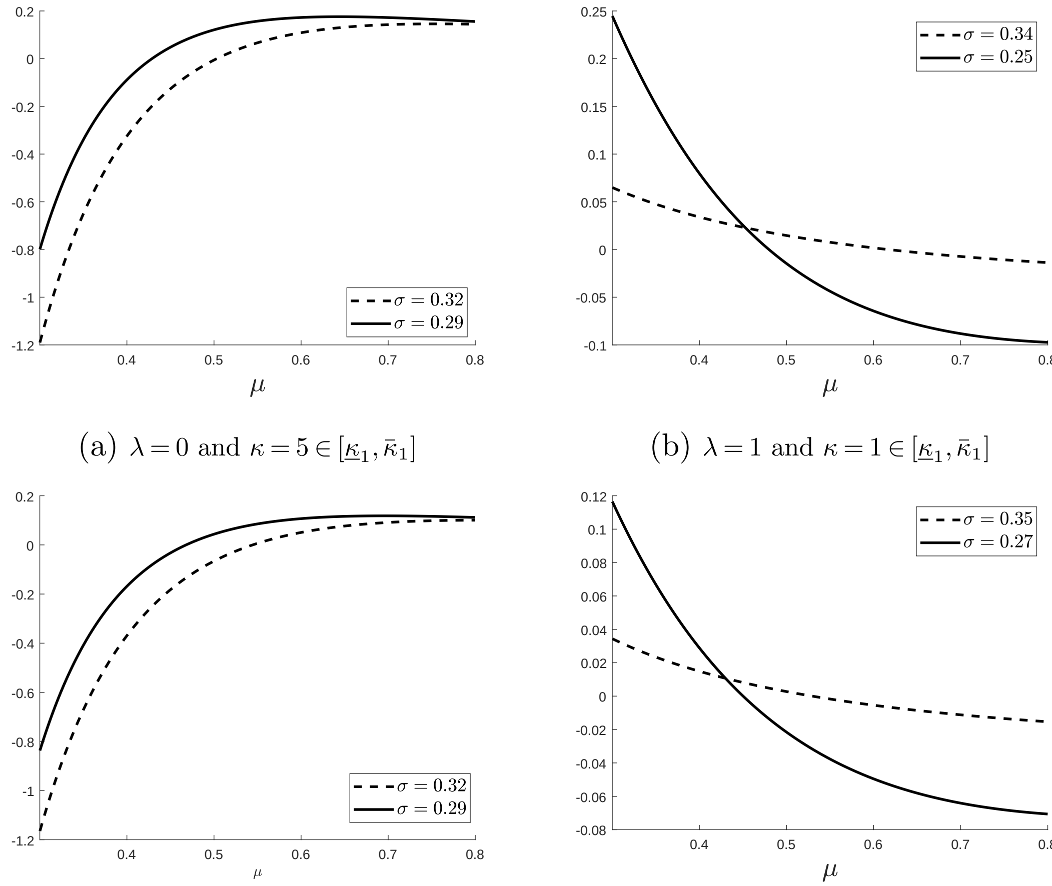
D.2 Effect of Economic Inequality

We next analyze with our numerical study how probabilities of downplaying and exaggerating change when the standard deviation of the economic cost distribution increases and its mean remains the same, see Figure 11.

Probability distributions in the beta distribution family are characterized by two parameters a and b. If these parameters are equal to each other, i.e., $a = b = x$ for some x, the mean of the distribution equals to $a/(a+b) = x/(2x) = 1/2$, and its standard deviation equals to $\sqrt{1/(8x+4)}$. Therefore, we parameterize the beta distributions by taking $a = b = x$ and varying x below, so that we only vary the standard deviation of a beta distribution but keep its mean fixed at 1/2.

Moreover, we identify ranges of σ values for each (κ, λ) pair such that optimal policy is either downplaying or exaggerating. Figure 10 illustrates the second derivative of $K_\lambda(c^\star(\mu))$ for different (κ, λ) pairs and σ values. Take for example Figure 10a. In that figure, the objective function $K_\lambda(c^\star(\mu))$ is first concave and then convex for $\sigma \in [0.29, 0.32]$. Therefore, we could use $[0.29, 0.32]$ as the range of σ when we analyze how $\pi^\star_{\mathsf{EX}}(m_h \,|\, \omega_\ell)$ changes in σ for $\lambda = 0$ and $\kappa = 5$.

Finally, Figure 11 exemplifies that the main insights in Theorem 2 (as in Figure 2 of our paper) can be extended to distributions other than uniform.



(a) $\lambda = 0$ and $\kappa = 5 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

(b) $\lambda = 1$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

(c) $\lambda = 0.1$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$

(d) $\lambda = 0.97$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

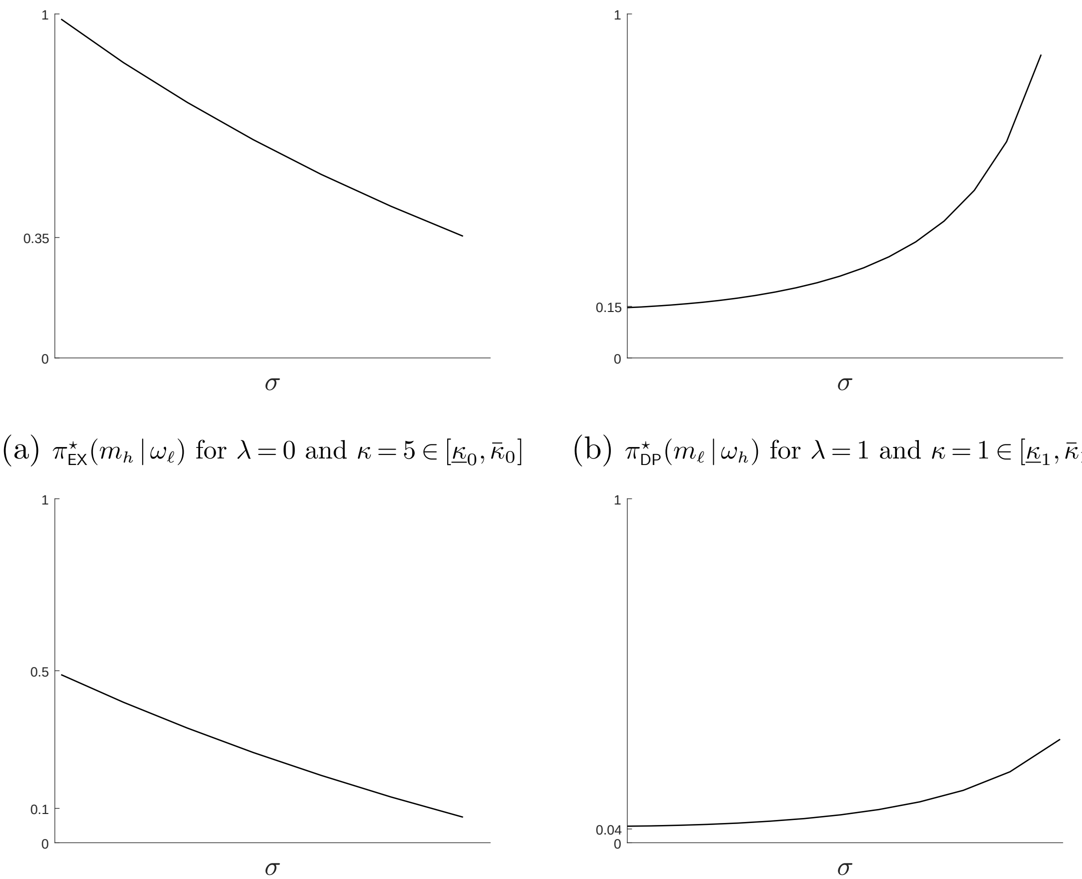
Figure 10 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$, $\omega_h = 0.8$, and the beta distributions with mean $1/2$.**

E Extensive Literature Review

In this section of the appendix, we provide an extensive review of the studies which are omitted in the main body of our paper, and elaborate on the differences between them and ours.

Kamenica and Gentzkow (2011) were among the first to establish the Bayesian persuasion framework and developed a general solution approach based on the notion of concavification. In a generic model setting, Kamenica and Gentzkow (2011) consider a single receiver and simplify her response in a *reduced form*, i.e., as an *exogenous* function of the posterior belief (induced by the sender's message and information policy). Subsequently, the sender's payoff is rendered as a function of the posterior belief, and the optimal policy is then identified through the concavification of that sender's reduced-form payoff function.[10]

[10] Of course, the concavification-based approach is not the unique solution method for Bayesian persuasion. Depending on the nature of problems, some other equivalent methods are available, such as using Rothschild and Stiglitz's (1970)



(a) $\pi^{\star}_{\mathsf{EX}}(m_h \,|\, \omega_\ell)$ for $\lambda = 0$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$ (b) $\pi^{\star}_{\mathsf{DP}}(m_\ell \,|\, \omega_h)$ for $\lambda = 1$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

(c) $\pi^{\star}_{\mathsf{EX}}(m_h \,|\, \omega_\ell)$ for $\lambda = 0.1$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$ (d) $\pi^{\star}_{\mathsf{DP}}(m_\ell \,|\, \omega_h)$ for $\lambda = 0.97$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

Figure 11 Optimal distortion probabilities as a function of σ for $\omega_\ell = 0.3$, $\omega_h = 0.8$, and the beta distributions with mean $1/2$.

Much of following research applies this general framework to various contexts by specifically modeling the micro-foundation for the (possibly multiple) receivers' responses and strategic interactions in order to derive the receivers' response functions (rather than assuming them in a reduced form). As such, the specific features of these contexts translate, through the receiver's response functions, to specific curvatures of the sender's payoff function, which in turn determines different types of optimal policies.

The main feature that our model captures is the *externality* that engaging in social interactions creates due to the nature of an epidemic disease. Accordingly, our analysis derives specific properties of the population's equilibrium response in a nonatomic game

mean-preserving spread (Gentzkow and Kamenica 2016), formulation as linear programs (Lingenbrink and Iyer 2019, Küçükgül et al. 2019).

(see Proposition 1) and subsequently the curvature of the government's payoff function (see Appendix C), based on which our main results (Theorems 1 and 2) hinge.

More precisely, our results stem from the joint consideration of several components: sender's bias (captured by λ), public information policy for multiple receivers (all receivers get the same information), a heterogeneous population (captured by c), and negative externalities (captured by P). Without any of those components, our results will disappear.

Some representative papers applying the information design framework in other (mostly operations) contexts include the following.

- As one of the pioneers in this area, Rayo and Segal (2010) study how a platform/seller should recommend its ads/products based on its proprietary information about the profitability and value-to-consumer of its offerings. In that model, there is a single receiver, while we analyze a multiple receiver setting with heterogeneous economic costs and negative externalities.
- Lingenbrink and Iyer (2019) study how a designer can reveal informative signals on the queue length (in the long-run equilibrium) to influence customers' joining decisions. Their setting is de facto a single receiver one because, each receiver is short-lived and gets a different signal upon arrival depending on the queue length whereas a continuum of receivers are simultaneously present in our setup and get the same information.
- Candogan and Drakopoulos (2020) add a Bayesian persuasion game onto a network game (with quadratic payoffs à la Ballester et al. 2006) to study a social network platform's information policy to influence receivers' beliefs about signal quality and subsequently their engagement decisions. The externalities studied in that paper are *positive*. Specifically, receivers derive higher utilities from a certain action if their connections also do the same. However, we analyze a setting with *negative* externalities (specifically driven by the nature of an epidemic setting) where taking a certain action becomes less favorable for a receiver if more receivers do the same.
- Drakopoulos et al. (2020) and Lingenbrink and Iyer (2018) demonstrate that obfuscating inventory and demand information can create stockout risk which motivates buyers to make early purchases. The nature of the strategic interaction among receivers in those two papers is characteristically different from the strategic externalities introduced by the homogeneous mixing nature of an epidemic in our setting. More specifically, Drakopoulos et al. (2020) focus on private information policies under which each

receiver gets a different signal/information. On the other hand, we focus on public information policies which represents better how governments inform their populations because all receivers get the same information. This is a fundamentally different setup. Moreover, public information policies are degenerate for Drakopoulos et al. (2020) while the optimal public information policy takes different forms in our setup based on the model parameters (λ and κ). Despite studying public information policies, Lingenbrink and Iyer (2018) focus on a homogeneous population and they numerically show in their appendix that their results are not extended to heterogeneous population. We however study a heterogeneous population consisting of receivers with different economic cost c's. Moreover, we characterize the effect of this inequality on the optimal information policy.

The settings studied by the above examples are essentially *static*; recent development in this area has extended the information design framework to *dynamic* settings, where methodological innovation may be needed. Examples include:

- In a two-arm bandit setting, Papanastasiou et al. (2018) (also see Bimpikis and Papanastasiou 2019, for a review of other works in similar model settings) study how to strategically disclose previous agents' outcomes to induce higher level of exploration (as opposed to exploitation) among self-interested agents (who have the right to choose between the arms).
- In a two-period setting, Alizamir et al. (2020) study a much related topic in the context of WHO's declaration of a Public Health Emergency of International Concern. However, their focus is on the WHO's reputation concern when trading off elicitation of today's response with tomorrow's reputation (so as to elicit proper responses tomorrow) due to the possibility of false alarms or missed alerts.
- In a revenue management setting, Küçükgül et al. (2019) design the information disclosure of previous customers' purchase decisions in order to influence upcoming customers when each customer has a piece of private information about the underlying product. In other words, the sender uses information instrument to engineer agents' social learning.