Designing Sustainable Products under Co-Production Technology

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Problem Definition. A manufacturer takes raw material with an exogenous quality distribution to make a traditional product and a co-product using material of quality above and below a well-established standard, respectively. The market consists of traditional consumers, who are only willing to pay for a product's consumption value, and some environmentally conscious (i.e., green) consumers, who additionally value the product's material conservation. In this context, we study the firm's optimal design of its co-product, i.e., its quality and price decisions.

Academic/Practical Relevance. Motivated by emerging conservation-oriented business practices exemplified by companies such as Taylor Guitars, our study informs resource-dependent firms whether and how to design their product line to leverage a co-product's environmental value. Our findings also yield important policy implications regarding the conservation of natural resources.

Methodology. We formulate and solve the firm's challenge as a constrained optimization problem, supplemented with extensive sensitivity analyses and robustness tests.

Results. When the material cost is intermediate and consumers are not sufficiently green, the firm should position the co-product without exploiting its environmental value. Otherwise, the firm should position the co-product by extracting its environmental value from green consumers, in which case the firm may strategically abandon some traditional consumers by leaving their demand unfulfilled.

Managerial Implications. Quotas and taxation on material supply in general act as policy substitutes. A greener market may inadvertently result in higher resource consumption and waste. Quotas can mitigate such adverse effects.

Key words: co-production; product line design; sustainability; scarce resource; taxation; quotas

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1. Introduction

In a variety of industries, companies use natural resources (e.g., wood, crude oil, agricultural produce, etc.) as raw material to manufacture their products. Such raw materials often vary in their physical characteristics (e.g., texture, color, density) that determine the final products' quality. Many products traditionally use only raw materials whose quality exceeds a certain conventional standard, while the remaining materials of inferior quality are simply discarded. Accelerated depletion of scarce natural resources, however, has rendered such practice economically unsustainable and environmentally irresponsible. Meanwhile, a growing number of consumers have become "green" in that they care about a product's environmental footprint and are willing to pay to reduce in the negative environmental impacts of their consumption. In fact, McKinsey found that more than 70% of consumers would pay an additional 5% for environmentally-friendly products (Miremadi et al. 2012) and Mintel Market Research (2010) reported that 35% of surveyed Americans would be willing to pay more for green products. *Co-production technology* that manufactures products of vertically differentiated quality from a common source of raw material has emerged as an innovative approach to address this challenge: It allows a firm to make co-products out of materials that would otherwise be discarded and, hence, to tap into the green consumer segment of the market. In this paper, we study a firm's product line strategy in the presence of co-production technology and green consumers.

The notion of green co-products is best exemplified by Taylor Guitars, a leading premier acoustic guitar manufacturer headquartered in El Cajon, CA. In the guitar industry, pure black ebony has been the most desirable material for making fingerboards (e.g., Beattie 2016, Dispa 2018). Unfortunately, only one out of ten ebony trees has a pure black, or "streakless," core (of the stem), while most ebony trees actually are variegated, i.e., they have a "streaked" core with a natural variation of tan-colored swirls (Arnseth 2013). The streak level of the ebony core varies along a continuous spectrum from tree to tree and can only be discerned after a tree is cut down. Traditionally, loggers only brought back streakless ebony. Due to transportation costs and the lack of a market, all harvested streaked trees were discarded on the forest floor. As a result, 90% of the logged ebony wood was simply wasted. Years of such extravagant logging practice has left Cameroon in Africa as the only legal source for high-quality ebony, forcing the country to impose a quota on the total ebony export.[1]

In 2011, Taylor Guitars acquired the largest ebony mill in Cameroon and internalized its raw material supply (Örsdemir et al. 2019), revealing this unsustainable logging practice for the first time (Taylor Guitars 2018). In response, Taylor Guitars started paying loggers an equal rate for all harvested ebony, whether streaked or streakless.[2] Such backward integration essentially provides the company with access to co-production capability, whereby ebony wood of various color purities can enter the production process and be used to make products of different quality levels. Indeed, in 2014, for the first time, Taylor Guitars introduced guitars made of streaked ebony along with

[1] Almost all ebony wood harvested in Cameroon is exported and its domestic demand is negligible.

[2] These comments were made by the supply chain director at Taylor Guitars in a private interview with the authors.

those made of traditional pure black ebony. To educate the market about the dire situation of ebony, the company also promotes the story of ebony through its company website, magazines and reseller training, "convincing guitar buyers that variations in wood color, often perceived as flaws, are actually signs of sustainably harvested ebony" (White 2012, Taylor Guitars 2018). The company's sustainable effort has subsequently been well-received by its customers and its streaked-ebony guitars were once even on back-order.

Leveraging co-products to achieve resource conservation has been gaining popularity in many other markets. For example, in the fishery industry, a sizeable amount of fish are so-called "bycatch," the incidental catch of non-target aquatic species, and are traditionally discarded due to being the wrong size, poor quality, or having low market value (Karp et al. 2011). Nowadays, many seafood restaurants take the initiative for more sustainable fish by broadening their menu selection to include the traditionally underappreciated or unrecognized bycatch (Gordinier 2015); examples include PJ Stoops, a fishmonger in Houston (Leschin-Hoar 2012) and Miya's Sushi, a Japanese restaurant in Connecticut.[3] Similarly, in the grocery industry, up to 20 percent of fruits and vegetables are tossed away purely due to their cosmetic imperfections (Godoy 2016). In 2014, Intermarche, the third-largest supermarket in France, launched an "inglorious fruits and vegetables" campaign by selling the cosmetically challenged produce at discounted prices. The campaign was a huge success and many major retailers in the U.S., such as Walmart, Whole Foods and Giant Eagle, have since joined this effort (Aubrey 2016, Godoy 2016). Similar trends can be found in the flooring industry, where engineered wood flooring that combines a top layer of solid wood with a base made out of wood scrap has emerged as a more sustainable substitute for traditional hardwood (Noriega 2010).

Motivated by the above emerging practices, we ask the following research questions: How should a firm design its product line strategy (i.e., its quality and price decisions) when the supply side is characterized by co-production technology and the demand side features green consumers? How should the cost characteristics of the co-production technology and consumers' environmental valuation affect the firm's strategy and its environmental impacts? For regulatory agencies, how do different policy levers, such as taxation and quotas, affect consumption of the raw material?

To address these questions, we consider a monopolistic firm that takes a raw material with exogenous quality distribution as its production input at a per-unit cost. The firm uses the portion of the material whose quality exceeds a conventional standard to manufacture a *traditional* product. At the same time, the firm also chooses a quality level below the conventional standard and uses material that is of quality between this lower level and the conventional standard to make a

[3] See http://www.miyassushi.com/mission/, accessed on April 23 2016.

co-product. A lower co-product's quality increases the material utilization (and hence lowers the production cost) but lowers consumers' willingness-to-pay for the product (and hence the firm's revenue), a tradeoff that the firm needs to balance when designing the co-product. The market consists of two consumer segments: traditional and green consumers.[4] Green consumers differ from traditional consumers in that green consumers value a product's resource conservation in addition to its consumption utility. Each consumer is free to purchase one of the two products or purchase nothing. The firm determines material quantity, the co-product's quality and both products' prices.

We identify two distinct strategies for the firm to position the co-product: (1) a *low-end* strategy, which prices the co-product only according to its consumption value, and (2) a *green* strategy, which charges an additional price premium to extract the co-product's environmental value from green consumers. The traditional product should be priced the same under both strategies according to its quality. Compared to the low-end strategy that makes the co-product appeal to both traditional and green consumers, the green strategy trades off the co-product's market size for its higher price by excluding the traditional consumers from purchasing the co-product. We find that the low-end strategy is optimal only when the material cost is intermediate; the firm should otherwise adopt the green strategy. Furthermore, the firm should lean toward the green strategy when consumers become *greener*, i.e., when the number of green consumers increases or existing green consumers exhibit a higher environmental valuation.

When the material cost is sufficiently high and, hence, the green strategy is adopted, the firm may *strategically abandon* some traditional consumers and leave their demand unfulfilled. In particular, we show that such strategic abandonment occurs even when the material cost is too high for the firm to remain profitable in the absence of co-production technology or the green strategy. Thus, strategic abandonment is a consequence of the interplay between co-production technology on the supply side and the presence of green consumers on the demand side.

Under the firm's optimal product line strategy, we find that having greener demand mostly leads to lower consumption and lower waste of raw material. However, exceptions arise when strategic abandonment occurs or when the firm switches from the low-end to the green strategy. In particular, expansion of the green consumer segment can inadvertently generate higher material consumption and waste. Nonetheless, greener demand always leads to a higher profit for the firm. These results challenge the conventional wisdom that a greener consumer base should always ameliorate the firm's environmental impacts, and the results highlight the need to explicitly take into account how the firm adjusts its product line strategy in response to public policies.

Last, we examine the effects of two widely used policy levers, taxation (Söderholm 2006) and quotas (Kirlin 2012) on total material consumption. While taxation is always effective at influencing

[4] In §6, we verify the robustness of our findings by extending our model to continuous demand heterogeneity.

the firm's strategy and environmental impacts, quotas are only effective when they become a binding material constraint for the firm. In that case, taxation and quotas act as policy substitutes in that a higher tax and a lower quota produce qualitatively similar effects on the firm's strategy, profit, environmental impacts and consumer surplus. Interestingly, in contrast to the conventional uni-product technology, taxation and quotas may, in fact, enhance consumer surplus, a result driven by the joint presence of co-production technology and green consumers. Finally, we find that imposing a quota can mitigate the adverse effect that a larger number of green consumers may generate greater material consumption and waste.

The remainder of the paper is organized as follows. In the next section, we first survey the pertinent literature. Section 3 presents our model. In §4, we formulate the firm's problem and characterize its optimal product line strategy. We devote §5 to evaluating the firm's environmental impacts and discuss their policy implications. In §6, we extend our analysis to the case of continuous heterogeneous demand and verify the robustness of our findings. We conclude the paper in §7. All the proofs and some detailed results are relegated to the online appendices.

2. Literature Review

Co-production technology, which features simultaneous and interdependent production of products at different quality levels, has been extensively studied in the operations management literature. Earlier studies in this area (e.g., Bitran and Gilbert 1994, Gerchak et al. 1996, Rao et al. 2004, and references therein) typically have taken product line decisions (i.e., quality grades and prices) as exogenously fixed and have focused on operational decisions (e.g., inventory, production schedule). More recently, Tomlin and Wang (2008) consider a two-product model and endogenize the pricing decisions by explicitly modeling customers' utility. Motivated by prevalent practices in industrial markets (e.g., semiconductor manufacturing), this literature typically assumes *downward substitution*—unmet demand for a low-quality product is fulfilled by a higher-quality product at the low-quality price. In a similar two-product setup, Bansal and Transchel (2014) relax this assumption by including a spill-up option. They examine the optimal amount of downward substitution and identify the optimal strategy of withholding low-quality inventory in order to generate upward substitution.

Within this strand of the literature, our work is closest to Chen et al. (2013), who examine a general product line design problem by endogenizing not only the quality and price decisions of each product but also the number of products in the line. Our model contrasts with theirs in two major aspects. First, the customers in their model only care about the utility derived from the product's consumption quality, whereas the consumers in our model are differentiated in their willingness-to-pay for the product's environmental benefits. Second, Chen et al. (2013) preclude their customers from purchasing the higher quality product when their most desired product is not available, whereas we allow consumers to freely choose among all available products.

Using lab or field experiments, many researchers (e.g., Krishna and Rajan 2009, Zheng et al. 2016, Kalkanci and Buell 2018) have provided empirical evidence for the existence of a green consumer segment, which constitutes a fundamental premise for many analytical models (e.g., Atasu et al. 2008). We base our model of green consumers on Kotchen's (2006) theory (see also e.g., Bagnoli and Watts 2003), which conceptualizes a green product as the joint provision of private goods that generate consumption value and (impure) public goods that produce social and environmental value. This theory has provided the underpinnings for numerous subsequent studies (e.g., Besley and Ghatak 2007, Pecorino 2013). More specifically, the green consumer in our model derives, in addition to the quality-dependent private consumption value, a public-good value that is embedded in the co-product and is measured relative to the high-quality traditional product. This notion that product "greenness" is a *relative* property has long been advocated by both academics as well as practitioners. For instance, according to Ottman (1998), "green is relative, describing products with less impact on the environment than their alternatives" (see also Jensen et al. 2004). Peattie (1995) makes a similar distinction: *relative green* products reduce the harm they cause to society or the environment, whereas *absolute green* products contribute to the improvement of society or the environment.

The way we model green consumers resembles the approach initiated by Chen (2001), who models the inherent trade-off between performance-based quality and environmental quality by assuming the sum of these two quality dimensions to be a constant. Focusing on similar tradeoffs, several subsequent studies have investigated green product design issues in various contexts ranging from product upgrading (Agrawal and Ulku 2013), servicizing (Agrawal and Bellos 2017), and sharing (Bellos et al. 2017), to extended producer responsibility legislation (Subramanian et al. 2009, Gui et al. 2018, Huang et al. 2018).

Finally, our work also speaks and contributes to the burgeoning literature on sustainable operations (Lee and Tang 2018). This line of work examines different means to mitigate firms' adverse environmental impact, such as discovering new economic values for the waste stream (Lee 2012) and adopting new technologies (Krass et al. 2013, Wang et al. 2017). Within this stream of research, our work is most closely related to Lee (2012), who considers using *by-product* synergy to render the waste stream into a profitable product. By-product technology is a manufacturing process where the waste generated from the production of the main product is used as the input to make products in a market category that does not directly compete with the main product. In contrast, we consider co-production technology where products are manufactured by using different quality segments of the *same* material, and those products do compete in the *same* market category.[5]

3. Model

We consider a monopolistic manufacturer (the firm) who takes a raw material (e.g., wood, fish) from nature to makes its products. The raw material contains an exogenous variation of a vertically differentiable physical attribute (e.g., wood color purity, fish meat texture), which we refer to as the *material quality*. The firm receives the material in its entirety and is not able to sort and selectively acquire the material according to its quality. This is either because the firm has vertically integrated the raw material supply (as in the case of Taylor Guitar) or/and because the harvesting method is inadequate to do so (as in the cases of ebony wood and fishery industry). While there exists a conventional quality standard (e.g., pure black ebony, usual fish species such as salmon, tuna and cod) for the product category, the material in all quality spectra can be used to make (possibly multiple) products in the same category. The material quality translates to *product quality* that determines the product's consumption value. These features on the supply side characteristically categorize the firm's manufacturing process as a *co-production technology* as commonly defined in the literature (e.g., Bitran and Gilbert 1994, Tomlin and Wang 2008, Chen et al. 2013, Bansal and Transchel 2014). First, the manufactured output consists of multiple products in the same market category. Second, these products are vertically differentiable according to a quality dimension. Third, the distribution of the product quality is exogenously given, rendering the product quality and quantity decisions no longer independent of each other.

More specifically, the firm acquires Q units of raw material at a per-unit cost c (e.g., logging or harvesting cost) to make its products. The units are normalized such that each product requires one unit of the raw material. For simplicity, material cost c is the only production cost. Material quality is measured by a scalar $q \in [0,1]$ with a higher value representing a more desirable variant of the physical attribute. The exogenous spectrum of the material quality follows a cumulative distribution function (c.d.f.) $F(q)$, whose probability density function is denoted as $f(q)$. We denote the complementary cumulative distribution function as $\bar{F}(q) := 1 - F(q)$. As is standard in the literature, we assume that $f(q)$ is log-concave, which captures a wide range of commonly used probability distributions and implies that both the failure rate $f(q)/\bar{F}(q)$ and the conditional failure rate $f(q)/\left(\bar{F}(q) - \bar{F}(q_t)\right)$ are increasing in $q \in [0,1]$ and $q \in [0, q_t]$, respectively (Bagnoli and Bergstrom 2005, Theorems 3 and 9).

[5] We note that co-production also differs from the "damaged goods" approach (e.g., Deneckere and McAfee 1996, Jones and Mendelson 2011), where low-end variants are derived from a high-end product by intentionally reducing the functionality or turning off some features in the high-end product primarily for the purpose of price discrimination.

Following the convention in the co-production literature (e.g., Chen et al. 2013, 2017, Lu et al. 2018), we say a product is of quality q if the product is made of raw material with quality in $[q, q']$ for some $q' > q$; namely, the product quality is determined by the minimum material quality that enters that product.[6] The industry has established a conventional quality standard, which we refer to as the *traditional quality* and denote as an exogenous parameter $q_t \in (0,1)$. The conventional production practice uses only raw materials with quality in $[q_t, 1]$ to make the *traditional product*, and simply discards the material with quality lower than q_t. The co-production technology allows the firm to make a lower-quality co-product using the otherwise discarded material in the quality spectrum $[q_o, q_t]$. Thus, the co-product's quality is given by $q_o \le q_t$. While both the traditional product and the product of quality q_o are referred to as co-products according to the literature, we hereafter reserve the term *co-product* to refer to the latter.

To maximize its profit, the firm chooses the material quantity Q, the co-product's quality q_o, and the prices for the traditional product and co-product, respectively denoted as p_t and p_o. Then, the co-production process simultaneously determines the quantities of these two products according to

$$Q_t := Q\bar{F}(q_t), \quad \text{and} \quad Q_o := Q\left[F\left(q_t\right) - F\left(q_o\right)\right], \text{ respectively.} \tag{1}$$

On the demand side, we consider a market characterized by the following three qualitative features. First, the market is heterogeneous and consists of two demand segments: *traditional* consumers, who derive only private-good consumption utility from a product, and *green* consumers, who additionally derive public-good utility from a product's contribution to resource conservation (Kotchen 2006, Besley and Ghatak 2007). Second, higher product quality generates higher private-good consumption value, and green consumers value the private-good consumption more than the public-good utility (Chen 2001, Kotchen 2006). Finally, the traditional product generates no public-good utility, whereas a co-product generates public-good utility, which increases in the co-product's environmental benefits measured relative to the traditional product (see §2). More specifically, green consumers perceive a co-product of lower quality as generating higher resource conservation and hence higher public-good utility. For empirical validation, we conducted experiments by recruiting human subjects using Amazon's Mechanical Turk. The data collected from our experiments indeed provide empirical evidence to support these three demand features (see Appendix A).

Without loss of generality, we normalize the total market size to one and let $n \in [0,1]$ denote the fraction of green consumers (and hence $1-n$ is the fraction of traditional consumers). Whether a consumer is traditional or green is her private information, whereas the firm only knows distribution

[6] This specification of product quality also complies with truth-in-advertising regulations (e.g., 15 U.S. Code §45 and the Federal Trade Commission Act of 1914). In fact, the qualitative nature of all our results remains intact if the product quality is defined as the average quality of material entering the product.

of consumer types. A product of quality $q \in \{q_t, q_o\}$ delivers the private-good utility uq enjoyed by both types of consumers, where the constant $u > 0$ represents their marginal willingness-to-pay (WTP) for the quality and is normalized to 1. The co-product of quality $q_o \leq q_t$ generates additional public-good utility $v\,(q_t - q_o)$ for a green consumer, where the constant $v \in (0,1)$ measures the green consumer's marginal WTP for resource conservation and the quality differential $q_t - q_o$ measures the magnitude of resource conservation: the greater the quality difference, the more resources are conserved. The fact that $v < 1$ indicates that the private-good value dominates the public-good value in contributing to a consumer's utility. In other words, a green consumer has higher total WTP for the traditional product than for the co-product. In essence, the size of the green demand segment n and the marginal WTP for resource conservation v capture the *volume* and *intensity* of the greenness in the market, respectively. In §6, we extend our base analysis to continuously heterogeneous consumers with v being a continuous random variable on $[0,1]$.

In summary, by purchasing a product with quality $q \in \{q_t, q_o\}$ at price $p \in \{p_t, p_o\}$, traditional and green consumers derive net utilities of

$$q - p, \quad \text{and} \quad q + v(q_t - q) - p, \text{ respectively.} \tag{2}$$

In other words, both the traditional and green consumers derive the utility $q_t - p_t$ from purchasing a traditional product. Traditional and green consumers, respectively, derive the utility $q_o - p_o$ and $q_o + v(q_t - q_o) - p_o$ from purchasing a co-product. Every consumer has a unit demand and purchases her next preferred product if her most preferred product is not available. A consumer can choose not to purchase anything and subsequently derive zero utility.

4. Formulation and Solution of Firm's Problem

In this section, we first formulate the firm's problem in §4.1, and characterize its solution in §4.2. Finally, we specialize our model and results to the case of uniform quality distribution in §4.3 to build the foundation for evaluating the firm's environmental impacts in the next section.

4.1. Firm's problem formulation

As the first step in the analysis, the following lemma reduces the firm's problem by expressing the firm's pricing decisions in terms of the products' qualities.

LEMMA 1. *For any given $q_o \leq q_t$, it is optimal for the firm to set the traditional product's price at $p_t^* = q_t$, and the co-product's price either at a* regular price *$p_o^* = q_o \leq p_t^*$, or a* premium price *$p_o^* = q_o + v(q_t - q_o) \leq p_t^*$.*

Lemma 1 implies that the firm always prices the traditional product at its quality level $p_t^* = q_t$ to extract the entire consumer surplus. However, there are two pricing strategies for the co-product, both of which yield a lower price than that of the traditional product. The *regular* price, $p_o^* = q_o$, only extracts the co-product's private-good value, and forfeits the opportunity to exploit the environmental value embedded in the co-product. Thus, the regular price essentially acts to position the co-product as a *low-end* product (relative to the traditional product as a high-end product). Indeed, when sold at the regular price, the co-product can appeal to all consumers, affording the traditional consumer zero surplus and the green consumer positive surplus. Alternatively, the firm can charge a higher *premium* price, $p_o^* = q_o + v(q_t - q_o)$, to extract the co-product's public-good value, and subsequently position the co-product as a *green* product, which can only appeal to green consumers.

Given the pricing strategy characterized in Lemma 1, a traditional (resp. green) consumer weakly prefers a traditional product (resp. co-product) to the other product.[7] Because consumers can purchase their next preferred products if their most preferred products are unavailable, there are four potential revenue sources for the firm. Let R_i^j denote the revenue from selling product $i \in \{t, o\}$ to consumer segment $j \in \{T, G\}$, where superscripts T and G refer to the traditional and green consumer segments, respectively. We provide these four revenue sources in Table 1 below, where $x^+ := \max(x, 0)$ and $\mathbb{1}[\cdot]$ is the indicator function taking a value of 1 (resp. 0) if its argument is true (resp. false).

Table 1 Firm's revenue sources

Demand segment	Purchase	Revenue
Traditional	traditional product	$R_t^T := q_t \min\{1-n, Q_t\}$
	co-product	$R_o^T := p_o \min\left\{(1-n-Q_t)^+, (Q_o - n)^+\right\} \mathbb{1}[p_o = q_o]$
Green	traditional product	$R_t^G := q_t \min\left\{(n - Q_o)^+,\ (Q_t - (1-n))^+\right\}$
	co-product	$R_o^G := p_o \min\{n,\ Q_o\}$

For example, the firm generates revenue from traditional consumers who purchase the co-product, i.e., $R_o^T > 0$, only when all the following three conditions hold: (1) the traditional product's supply does not suffice to satisfy all traditional consumers, $Q_t < 1 - n$, (2) the co-product's supply is in excess of the green demand, $Q_o > n$, and (3) the unfulfilled traditional consumers are willing to purchase a co-product, i.e., $q_o - p_o \geq 0$, suggesting the co-product must be sold at a regular price $p_o = q_o$. Therefore, $R_o^T := p_o \min\left\{(1-n-Q_t)^+,\ (Q_o - n)^+\right\} \mathbb{1}[p_o = q_o]$. All the other revenue

[7] Specifically, the conventional incentive compatibility constraints in the product line design literature hold: $q_t - p_t^* \geq q_o - p_o^*$ for the traditional consumer and $q_o + v(q_t - q_o) - p_o^* \geq q_t - p_t^*$ for the green consumer.

sources can be derived in a similar fashion. Accounting for the firm's material cost cQ, we can thus formulate the firm's problem as:

$$\begin{aligned}\Pi^* = \max_{q_o, p_o, Q} \quad & R_t^T + R_o^T + R_t^G + R_o^G - cQ \\ \text{subject to} \quad & 0 \le q_o \le q_t,\ p_o \in \{q_o,\ q_o + v(q_t - q_o)\},\ \text{and } Q \ge 0.\end{aligned} \tag{P}$$

4.2. Firm's optimal product line strategy

Our next lemma identifies three distinct product line strategies that can further simplify the firm's problem (P) to one with co-product quality q_o as the single decision variable.

LEMMA 2. *The firm enters the market (i.e., has positive production quantity) if and only if*

$$c \le \bar{c} := q_t \bar{F}(q_t) + \max_{q_o \in [0, q_t]} \left\{ [q_o + v(q_t - q_o)] \left[\bar{F}(q_o) - \bar{F}(q_t) \right] \right\}, \tag{3}$$

in which case the firm's optimal product line strategy must be one of the following three:

Green co-product, Full coverage (GF): $F(q_o) \ge (F(q_t) - n)/(1-n)$, $p_o = q_o + v(q_t - q_o)$ *and* $Q = 1/(1 - F(q_o))$. *All traditional consumers purchase a traditional product. If the first inequality strictly holds, some green consumers purchase a co-product and the rest of the green consumers purchase a traditional product; otherwise, all green consumers purchase a co-product.*

Green co-product, Partial coverage (GP): $F(q_o) < (F(q_t) - n)/(1-n)$, $p_o = q_o + v(q_t - q_o)$ *and* $Q = n/(F(q_t) - F(q_o))$. *All green consumers purchase a co-product. Some traditional consumers purchase a traditional product, and the rest of the traditional consumers are left unfulfilled.*

Low-end co-product, Full coverage (LF): $F(q_o) \le (F(q_t) - n)/(1-n)$, $p_o = q_o$ *and* $Q = 1/(1 - F(q_o))$. *All green consumers purchase a co-product. Some traditional consumers purchase a traditional product, and the rest of the traditional consumers purchase a co-product.*

The significance of Lemma 2 lies in showing that it is optimal for the firm to adopt one of the three mutually exclusive product line strategies. On one hand, both GF and GP strategies position the co-product as a green product (hence "G") by selling it exclusively to green consumers at the premium price, whereas LF strategy sells the co-product as a low-end product (hence "L") at the regular price, which can fulfill the demand from both segments. On the other hand, both GF and LF strategies achieve full market coverage (hence "F"), whereas GP strategy only partially covers the market (hence "P") by leaving some traditional consumers unfulfilled.

While both GF and LF strategies equate the total supply with the total market size (i.e., $Q(1 - F(q_o)) = 1$), they differ in that GF strategy uses the traditional product to fulfill some green demand whereas LF strategy does the opposite. As a result, GF strategy sets the green co-product's quality sufficiently high (i.e., $F(q_o) \ge (F(q_t) - n)/(1-n)$) to keep the green co-product's supply below the green demand (i.e., $Q_o \le n$). In particular, when the green co-product's supply is just enough to fulfill the entire green demand (i.e., $F(q_o) = (F(q_t) - n)/(1-n)$), the two demand segments are respectively fulfilled by the two products (i.e., $Q_t = 1 - n$ and $Q_o = n$). In contrast, LF strategy sets the co-product's quality low enough (i.e., $F(q_o) \le (F(q_t) - n)/(1-n)$) so that the low-end co-product can fulfill all green consumers (i.e., $Q_o \ge n$) and some traditional consumers.

Of particular interest is the possibility of GP strategy. It arises because the firm acquires just enough material to match the green product's supply with the green demand (i.e., $Q_o = n$) and chooses to limit the traditional product's supply below the traditional demand (i.e., $Q_t < 1 - n$). Subsequently, the co-product's quality needs to be set low enough (i.e., $F(q_o) < (F(q_t) - n)/(1-n)$). Notably, Lemma 2 rules out the possibility of a low-end co-product and partial coverage (i.e., LP strategy) that would position the co-product as a low-end product at the regular price and leave some consumers unfulfilled at the same time.[8] In fact, the firm is better off deviating from LP strategy to either LF or GP strategy.

The market entry condition (3) ensures that the unit material cost cannot exceed the maximum possible marginal revenue as represented by the right-hand side of (3). Indeed, given the co-product's quality q_o, one unit of material can produce $\bar{F}(q_t)$ and $\bar{F}(q_o) - \bar{F}(q_t)$ units of traditional products and co-products, respectively, which can, at most, be sold for q_t and $q_o + v(q_t - q_o)$, respectively (Lemma 1).

Lemma 2 reduces the firm's original problem (P) to three subproblems with a single decision variable q_o. The formulation and solutions of these subproblems are detailed in Lemmas C.2, C.3, and C.4 of Appendix C. The following theorem identifies which strategy is most profitable and, in turn, characterizes the firm's optimal product line design.

THEOREM 1. *Given p_t, v and n, there exist two thresholds $0 \le c_1^* \le c_2^* \le \bar{c}$ such that it is optimal for the firm to adopt (i) GF strategy for $c \le c_1^*$, (ii) GP or LF strategy for $c \in (c_1^*, c_2^*]$, and (iii) GP strategy for $c \in (c_2^*, \bar{c}]$. Moreover, if $F(q_t) \le n$, the optimal strategy is GF for all $c \in [0, \bar{c}]$; otherwise, $c_2^* < \bar{c}$ holds for sufficiently large v, and for each $c \in (c_1^*, c_2^*]$, there exists a threshold $\bar{v} \in [0,1]$ such that GP strategy is optimal when $v > \bar{v}$.*

Theorem 1 demonstrates that the co-product's position is not monotone in the material cost. For sufficiently low or high material cost, the firm should position the co-product as a green product, rationalizing the corporate environmentalism as a profit-maximizing strategy. The co-product should be positioned as a low-end product (i.e., LF strategy) only for intermediate material cost.

More specifically, when the material is sufficiently affordable (i.e., $c \le c_1^*$), the firm should adopt GF strategy to satisfy the entire market. Under GF strategy, the traditional product plays a

[8] LP strategy can emerge under quotas (see §5.2).

dominant role in fulfilling the demand (i.e., all traditional consumers and some green consumers purchase the traditional product). Intuitively, when the material is less costly, the firm can afford to set the co-product's quality higher to raise its price, thus limiting the co-product's supply and inducing some green consumers to purchase the pricier traditional product. Notably, the firm can only adopt GF strategy when the traditional product's quality is already sufficiently low (i.e., $F(q_t) \leq n$) so that the co-product's supply is insufficient to satisfy the green demand.

For an intermediate range of material costs (i.e., $c_1^* < c \leq c_2^*$), introducing the co-product as a low-end product also becomes profitable, as the co-product can also be used to fulfill the traditional demand and increase material utilization (i.e., reduce the effective cost for each product). Therefore, the market must be fully covered if the co-product is positioned as a low-end product, leading to LF strategy. Alternatively, the co-product can still be positioned as a green product at the higher premium price and hence exclusively fulfill the green demand, leading to GP strategy, under which the firm *strategically abandons* some traditional consumers by leaving them unfulfilled. As such, the firm faces the tradeoff between generating a larger demand via LF strategy and selling the co-product at a higher price via GP strategy. In particular, GP strategy becomes more profitable than LF strategy when the green consumer attaches a higher valuation to the co-product's environmental benefits (i.e., higher v), which raises the premium price.

In light of homogeneous product valuation within each segment and an unlimited supply of raw materials, the occurrence of strategic abandonment is perhaps unexpected, as we might expect the firm to satisfy either none or all of the demand from a consumer segment. More interestingly, it is the traditional consumers, who are willing to purchase the pricier traditional product, who are unfulfilled. It is worth noting that strategic abandonment is not an artifact of discrete demand segmentation. In §6 we show that strategic abandonment still emerges even when consumers have continuously heterogeneous valuation for a product's environmental benefits.

In fact, Theorem 1 shows that GP becomes the optimal strategy when the material becomes excessively costly (i.e., $c > c_2^*$) and positioning the co-product as a low-end product is not sufficient to recoup the high material cost. Indeed, the corollary below indicates that when the material becomes extremely expensive, GP strategy is the only option that enables the firm to enter the market (i.e., earn nonnegative profit).

COROLLARY 1. *Suppose $F(q_t) > n$ and let*

$$\underline{c} := q_t \bar{F}(q_t) + \max_{0 \leq q_o \leq F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)} \left\{ \left(\bar{F}(q_o) - \bar{F}(q_t) \right) q_o \right\} \in \left[q_t \bar{F}(q_t), \bar{c} \right]. \tag{4}$$

When no co-product is introduced (i.e., $q_o = q_t$), the firm enters the market if and only if $c \leq q_t \bar{F}(q_t)$. When the co-product is introduced but can only be positioned as a low-end product (i.e., $p_o = q_o$), *the firm enters the market if and only if* $c \le \underline{c}$*. For* $c \in (\underline{c}, \bar{c}]$*, the firm enters the market only when adopting GP strategy.*

In other words, if the co-production technology were unavailable or the green demand segment were absent (and hence the co-product can only be positioned as a low-end product), the firm would choose not to enter the market. Therefore, strategic abandonment is a consequence driven by the interplay between co-production technology on the supply side and the presence of green consumers on the demand side.

4.3. Uniform quality distribution

To illustrate and refine the characterization of the firm's optimal strategy in Theorem 1, we now specialize the material quality distribution to the standard uniform distribution (i.e., $F(q) = q$ for $q \in [0,1]$). For the rest of the paper, we will continue to work under this distribution and focus on $n < q_t$, because otherwise GF strategy is always optimal according to Theorem 1.

THEOREM 2. *Suppose that material quality is uniformly distributed on* $[0,1]$ *and* $n < q_t$*. The firm enters the market if and only if*

$$c \le \bar{c} = \begin{cases} q_t - q_t^2(3/4 - v)/(1-v), & \text{if } v < 0.5, \\ q_t - q_t^2(1-v), & \text{if } v \ge 0.5, \end{cases} \tag{5}$$

in which case its optimal strategy is given as follows:

(i) If $0 \le n \le \left(2 - 2\sqrt{1-q_t}\right)/q_t - 1$*, there exist two thresholds* $\hat{c} < \check{c} \le \bar{c}$ *such that the optimal strategy is GF for* $c \le \hat{c}$*, LF for* $\hat{c} < c \le \check{c}$*, and GP otherwise.*

(ii) If $\left(2 - 2\sqrt{1-q_t}\right)/q_t - 1 \le n < q_t/(2-q_t)$*, there exist two thresholds* $\hat{c}$ *and* $\bar{v}$ *such that the optimal strategy is GF for* $c \le \hat{c}$*, LF for* $\hat{c} < c \le \bar{c}$ *and* $v \le \bar{v}$*, and GP otherwise.*

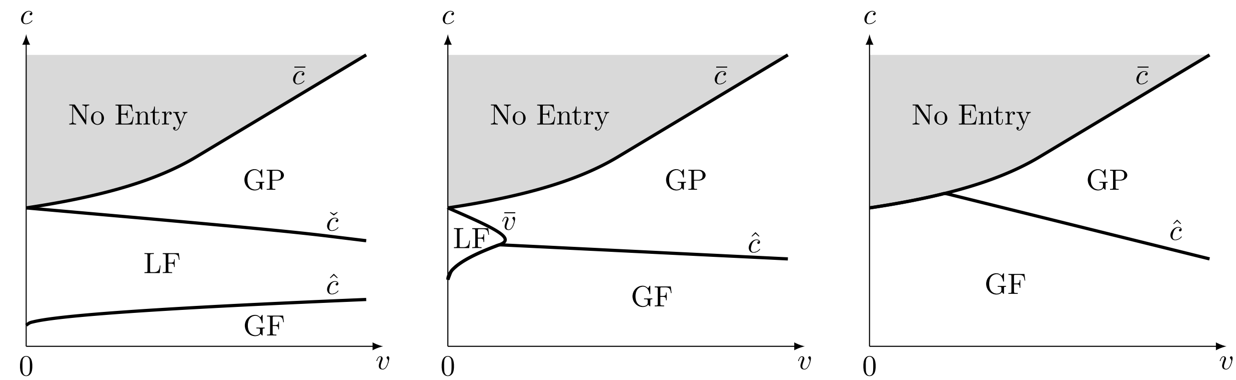
(iii) If $q_t/(2-q_t) \le n < q_t$*, there exists a threshold* $\hat{c}$ *such that the optimal strategy is GF for* $c \le \hat{c}$ *and GP otherwise.*

The optimal co-product's quality q_o^**, quantity* Q_o^**, and traditional product's quantity* Q_t^* *are given in Table 2.*

Table 2 Firm's optimal decisions.

Strategy	q_o^*	Q_t^*	Q_o^*
GP	$\left(q_t - \sqrt{\frac{c-(1-q_t)q_t}{1-v}}\right)^+$	$\frac{n(1-q_t)}{q_t - q_o^*} < 1-n$	n
GF	$\min\left\{1 - \sqrt{\frac{c}{1-v} + (1-q_t)^2}, \frac{q_t - n}{1-n}\right\}$	$\frac{1-q_t}{1-q_o^*} \ge 1-n$	$\frac{q_t - q_o^*}{1-q_o^*} \le n$
LF	$1 - \sqrt{c + (1-q_t)^2}$	$\frac{1-q_t}{1-q_o^*} < 1-n$	$\frac{q_t - q_o^*}{1-q_o^*} > n$

Figure 1 visualizes the three parametric ranges identified in Theorem 2. For narrative convenience, we will hereafter name the range of primitive parameters (i.e., c, n, and v) after the corresponding optimal strategy. For instance, *LF region* refers to the parametric range for which LF is the firm's optimal strategy. Similarly, we can define *GF* and *GP regions*.



(a) $0 \le n \le \left(2 - 2\sqrt{1-q_t}\right)/q_t - 1$. (b) $\left(2 - 2\sqrt{1-q_t}\right)/q_t - 1 \le n < q_t/(2-q_t)$. (c) $q_t/(2-q_t) \le n < q_t$.

Figure 1 **Firm's optimal product line strategy with uniform material quality distribution for $q_t = 0.7$.**

Figures 1(a) and 1(b) confirm that the firm positions the co-product as a low-end product (i.e., adopts LF strategy) only for intermediate values of c. Moreover, the firm has a stronger incentive to position the co-product as the green product for either higher v or larger n. Indeed, LF region is only present for sufficiently small v and shrinks as n increases (i.e., from Figure 1(a) to 1(c)). Finally, for all three ranges of n (i.e., Figures 1(a)-1(c)), GP region expands as the green consumer's valuation for resource conservation v increases. In particular, when there is no green demand (i.e., $v = 0$), GP strategy is never optimal, showing again that strategic abandonment emerges from the joint force of co-production technology and the presence of green consumers.

5. Environmental Impacts

In this section, we examine how market "greenness" affects the firm's environmental impacts, its profit, and consumer surplus, and we subsequently compare two commonly used policy levers, taxation and quotas. We recall that the market's *greenness* can be measured either by its *volume*, namely the size of the green demand segment n, or by its *intensity*, namely the WTP for resource conservation v.

5.1. Effects of consumer's greenness (n and v)

Given the firm's optimal co-product quality q_o^* and material quantity Q^*, two metrics for the firm's environmental impacts are (1) the total resource *consumption* Q^*, and (2) the total resource *waste*

$Q^* q_o^*$. Alternatively, we measure the material consumption and waste *per unit of the product sold*.[9] Specifically, under the firm's optimal product line strategy, the total number of products sold is given by $S^* := Q^*(1 - q_o^*)$. Therefore, two additional metrics for the firm's environmental impacts are (3) *per-unit consumption* Q^*/S^* and (4) *per-unit waste* $Q^* q_o^*/S^*$. A lower value of any metric above indicates a more desirable (i.e., lower) environmental impact, which, interestingly, does not necessarily result from having greener consumers, as demonstrated by the following result.

Proposition 1. Ceteris paribus*, as n or v increases, the co-product is first positioned as a low-end product and then as a green product.*

(a) As long as the co-product remains in a low-end position, all environmental metrics are invariant in n or v.

(b) As long as the co-product remains in a green position, all environmental metrics are non-increasing in n or v, except that total consumption and total waste increase in n in GP region.

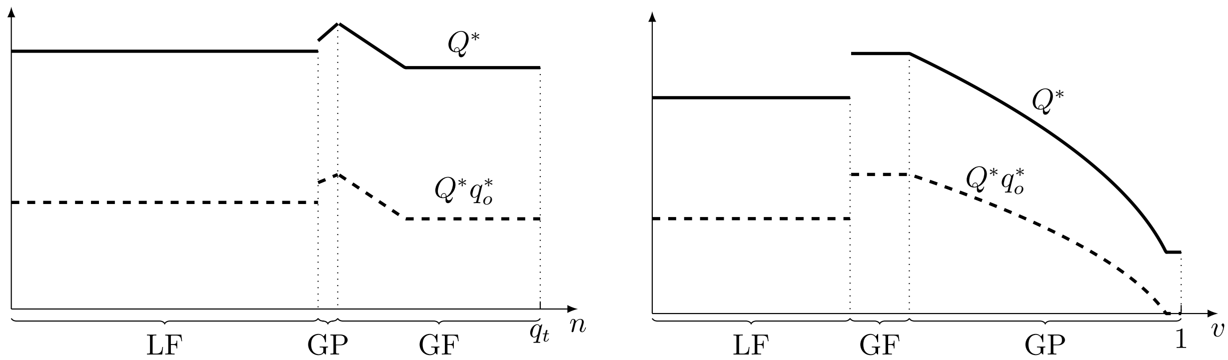
(c) All environmental metrics may demonstrate discontinuities in n or v, where the co-product switches from a low-end to a green product. In particular, such discontinuities are upward if c is sufficiently low.

As illustrated by Figure 1, the firm has a stronger incentive to position the co-product as a green product (i.e., adopt GF or GP strategy) when the demand becomes greener either in terms of volume (n) or intensity (v). On the other hand, if the co-product is positioned as a low-end product (i.e., in LF region), the consumer's greenness has no effect on the firm's material quantity and co-product quality decisions, rendering all environmental metrics invariant with respect to the volume (n) or intensity (v) of greenness, as indicated by Proposition 1(a).

When the co-product is positioned as a green product (i.e., in GF and GP regions), Proposition 1(b) shows that enlarging the volume of greenness (larger n) and strengthening its intensity (larger v) may have different effects on the firm's environmental impacts and thus may not act as strategic substitutes. More specifically, strengthening consumers' green intensity (e.g., through consumer education) *always* improves the firm's environmental impacts, whereas expanding the green segment (e.g., through advertisement) inadvertently results in greater resource consumption and waste in GP region (see Figure 2(a)).[10] This is because GP strategy abandons some traditional consumers and determines the firm's material quantity according to the green demand size (Lemma 2). Perhaps paradoxically, such an adverse effect must emerge when the material becomes so scarce and expensive that GP is the optimal strategy (Corollary 1).

[9] We thank one anonymous reviewer for suggesting these two metrics.

[10] Note that GP region can emerge when both n is high and low, as Figure 1 shows.



(a) Effects of n for $v = 0.16$ and $c = 0.254$.

(b) Effects of v for $n = 0.35$ and $c = 0.224$.

Figure 2 Resource consumption (Q^*) and waste ($Q^*q_o^*$) for $q_t = 0.7$.

The discontinuity of the firm's environmental impacts in the greenness parameters (n and v) documented by Proposition 1(c) reflects the qualitative distinction between positioning the co-product as a low-end product versus as a green one. In particular, such a switch in product line strategy may result in a negative environmental impact, as illustrated by Figure 2.[11] Indeed, when the raw material is not too costly, positioning the co-product as a green product allows the firm to choose a higher co-product quality level q_o^* than does positioning it as a low-end product, thus leading to higher material consumption and waste.

In addition to the firm's environmental impacts, a socially benevolent regulator may also care about consumer surplus and/or the firm's profit (Π^*). Following the convention, we compute *consumer surplus* by integrating all consumers' net utility under the firm's optimal product line strategy.

Proposition 2. *The firm's profit is non-decreasing in n and v. Consumer surplus is positive only in LF region, where it is increasing in n and v.*

According to Lemma 2, a positive utility, $nv(q_t - q_o^*)$, can be generated only when green consumers purchase the co-product as a low-end product (i.e., under LF strategy). Therefore, greener demand in terms of either volume (n) or intensity (v) generates higher consumer surplus. Interestingly, a greener market never compromises the firm's profitability, as greener demand allows the firm to better exploit the environmental value embedded in the co-product.

Taking together the findings in Propositions 1 and 2, we note that the firm's effort to expand the green demand segment (i.e., enlarging n) may as well be driven by its profit-maximization motive as by the claimed environmental benefits. Indeed, when the firm introduces the green co-product

[11] Specifically, the total consumption and total waste increase by up to 10% and 26% as the firm's strategy switches from LF to GP in Figure 2(a), and by up to 21% and 48% as the firm's strategy switches from LF to GF in Figure 2(b), respectively.

and abandons some traditional consumers, having more green consumers is surely profit-improving but can inadvertently exacerbate natural resource consumption and waste without improving the consumer surplus.

In contrast, Propositions 1 and 2 imply that in GP region, enhancing green consumers' WTP for resource conservation (i.e., raising v) improves the firm's profit and consumer surplus as well as ameliorates the firm's environmental impacts. Therefore, improving consumers' green volume (n) and intensity (v) are not necessarily strategy substitutes. For a social planner who aims to achieve a win-win-win outcome, strengthening green intensity by enhancing existing green consumers' WTP for the co-product's environmental benefits is perhaps more effective than recruiting new green consumers.

5.2. Policy levers

Taxation and quotas are two policy instruments widely applied by many countries to regulate the consumption of scarce natural resources for environmental reasons. For instance, taxes are levied on forestry in Croatia and Poland, on fisheries in Finland and Estonia, on natural gravel in Sweden, and on minerals in Bulgaria, Finland, Poland and the Netherlands (Söderholm 2006). As an example of quotas, the Cameroon government issues export permits that limit the total ebony export to 2,000 tons per year (Kirlin 2012). Understanding the implications of these two regulatory instruments on the firm's product line strategy and its environmental impacts offers policy makers valuable guidelines, which is the goal of this subsection.

Taxation on raw material is equivalent to increasing the per-unit material cost c. Therefore, the parametric dependence of the firm's optimal strategy characterized in Theorem 2 on the material cost would reveal the effects of taxation. Nonetheless, imposing a quota, which we denote as $\bar{Q}$, is more nuanced, as the firm's problem (P) needs to include an additional constraint $Q \leq \bar{Q}$. If the corresponding optimal material quantity Q^* is below $\bar{Q}$, the firm's optimal product line strategy characterized by Theorem 2 remains intact and the quota is inconsequential. Otherwise, the quota constraint becomes binding and plays the determining role in shaping the firm's product line strategy. Subsequently, the material cost $c\bar{Q}$ becomes a fixed cost and no longer affects the firm's strategy, its environmental impacts or consumer surplus.

Thus, we will focus on the case having the binding quota constraint $Q = \bar{Q} \leq 1/(1-q_t)$,[12] for which we follow a similar solution procedure as before and characterize the firm's optimal product line strategy in Proposition E.1 of Appendix E, as illustrated by Figure 3.

By comparing Figure 3 with Figure 1 (i.e., the firm's optimal strategy without quotas), we recognize that the quota $\bar{Q}$ plays a role similar to that of material cost c in our base model. When

[12] When $\bar{Q} > 1/(1-q_t)$, the material supply is sufficient to fulfill the entire market with the traditional product and hence is never subject to a quota constraint.

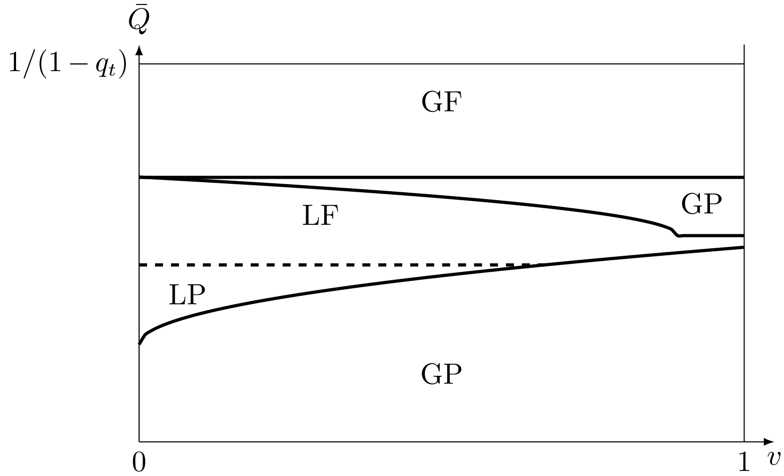


Figure 3 **Firm's optimal product line strategy under a binding quota with $q_t = 0.7$ and $n = 0.3$.**

the quota limit is high (corresponding to a small c), GF is the optimal strategy. When the quota limit is stringent (corresponding to a large c), GP is the optimal strategy.[13] LF strategy is optimal for intermediate $\bar{Q}$ (corresponding to an intermediate c). The only notable nuance is that with a binding quota, LP strategy (i.e., positioning the co-product as a low-end product and only fulfilling part of the market) can be optimal. Under LP strategy, the supply of traditional products is limited due to quotas and some traditional consumers are thus left unfulfilled, while all green consumers can be fulfilled by the regular-priced co-product and hence earn a positive surplus.

Next, we examine the effects of the two policy levers, taxation and quotas, on the firm's environmental impacts, profit and consumer surplus.

PROPOSITION 3. Ceteris paribus, *as c increases or $\bar{Q}$ decreases, the co-product is first positioned as a green product, then as a low-end product, and finally back as a green product.*

(a) As long as the co-product remains in the same (green or low-end) position, all environmental metrics are non-increasing in c or non-decreasing in $\bar{Q}$. Furthermore, when c increases or $\bar{Q}$ decreases from a low-end region (LF or LP) to GP region, all environmental metrics, except for total consumption, may demonstrate upward discontinuity.

(b) Consumer surplus is positive only in low-end regions (LF and LP), whereby it is increasing in c and non-increasing in $\bar{Q}$. In particular, consumer surplus is invariant in $\bar{Q}$ in the LP region.

(c) The firm's profit is always decreasing in c or increasing in $\bar{Q}$.

Consistent with our intuition, Proposition 3 states that taxation and quotas essentially act as policy substitutes and yield similar effects on the firm's environmental impacts, profit and consumer surplus. More specifically, Proposition 3(a) shows that the firm's environmental impacts can be

[13] Unlike our base model, GP strategy may leave some green consumers unfulfilled for sufficiently low quota limits.

ameliorated by either levying taxes (i.e., higher material cost c) or imposing a quota (i.e., lower material supply $\bar{Q}$), both of which induce the firm to consume less and better utilize the raw material.[14] Nonetheless, such positive environmental effects come at the expense of the firm's profitability, as indicated by Proposition 3(c).

Proposition 3(b) indicates that a positive consumer surplus can only result from the co-product being positioned as a low-end product, akin to Proposition 2. Since the firm positions the co-product as a low-end product only when the material cost is intermediate, a policy maker who is more concerned about consumer welfare may need to discreetly choose the tax or quota level so as to induce a low-end position for the co-product. Notably, as demonstrated by Proposition 3(b), a higher tax level or lower quota limit in fact raises the consumer surplus as long as the co-product is positioned as a low-end product. This stands in contrast to conventional unit-production technology, whereby higher production costs or tighter supply typically lowers consumer surplus. This distinct effect is unique to the co-production technology: Costlier material or more stringent supply conditions induce the firm to better utilize the raw material (through lower q_o^*), resulting in higher environmental benefits enjoyed by the green consumer.

Proposition 1 demonstrates that, absent quotas, higher green demand (i.e., n) can backfire and exacerbate the firm's material consumption and waste, especially under GP strategy. Fortunately, our next result suggests that such adverse effects can be averted by imposing quotas.

PROPOSITION 4. *Under a binding quota, raising n or v in GP region weakly reduces all environmental metrics and weakly increases the firm's profit.*

Under a binding quota, GP strategy can no longer fulfill larger green demand (i.e., larger n) by raising the firm's raw material consumption; rather, it needs to lower the co-product's quality q_o so as to increase the output of green co-products. As a result, raw material utilization increases and all environmental metrics are improved. Therefore, quotas are a better policy choice than taxation when it comes to curbing resource consumption or waste. Nonetheless, as suggested by Proposition 3(b), taxation is more effective than quotas for improving consumer surplus.

6. Continuous Demand Segment

In our base model, we assume that demand consists of two discrete segments. In this section, we consider a continuous demand segment and verify the robustness of the results established in our base model. Specifically, each consumer is characterized by her WTP (v) for environmental conservation; we refer to v as a consumer's *type* and assume that it follows a power distribution

[14] The upward discontinuities in some of the environmental metrics as the firm changes its product line strategy are an artifact of discrete demand segmentation.

with a cumulative distribution function $G^\alpha(v) := v^\alpha$ for $v \in [0,1]$. Here, $\alpha > 0$ is a constant, a larger value of which indicates a larger power distribution in the sense of *usual stochastic order* (e.g., Shaked and Shanthikumar 2007). In particular, $\alpha = 1$ corresponds to the uniform distribution.[15] For simplicity, we work with uniformly distributed material quality, i.e., $F(q) = q$.

In this setting, the firm still prices the traditional product at its quality level $p_t = q_t$ as in our base model (Lemma C.1). Thus, all consumers weakly prefer the traditional product to no purchase. Meanwhile, the firm chooses a cutoff type, denoted as $v_o \in [0,1]$, and prices the co-product at $p_o = q_o + v_o(q_t - q_o) \in [q_o, q_t]$. Subsequently, consumers with $v \geq v_o$ correspond to the green demand segment in our base model as they prefer the co-product to the traditional one, whereas consumers with $v < v_o$ correspond to the traditional demand segment in our base model as they prefer no purchase to the co-product. Because the firm can adjust v_o in a continuous fashion, it is always optimal for the firm to choose the cutoff type v_o (or equivalent p_o) so that the co-product's demand $1 - G^\alpha(v_o)$ matches its supply $Q_o = Q(q_t - q_o)$, i.e., $1 - v_o^\alpha = Q(q_t - q_o)$ (see Lemma C.1). Subsequently, the traditional product's demand is $G^\alpha(v_o) = v_o^\alpha$ and its supply is $Q_t = Q(1 - q_t)$. Therefore, the firm's problem can be formulated as

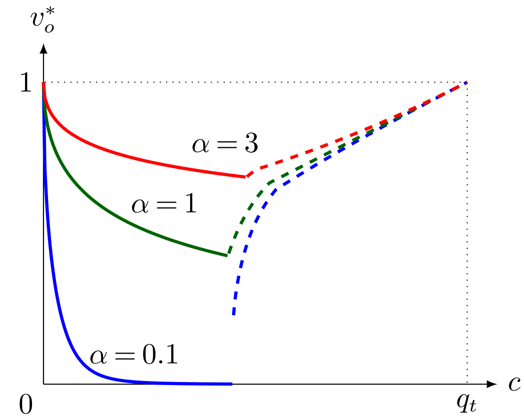
$$\begin{array}{cl} \max\limits_{v_o \in [0,1], q_o \in [0,q_t], p_o, Q \geq 0} & p_o Q_o + q_t \min\{Q_t, G^\alpha(v_o)\} - cQ \\ \text{subject to} & p_o = q_o + v_o(q_t - q_o), \quad \text{and} \quad Q_o = 1 - G^\alpha(v_o), \end{array} \tag{6}$$

whose solution $(v_o^*, q_o^*, p_o^*, Q^*)$ is characterized in Proposition B.1 of Appendix B.

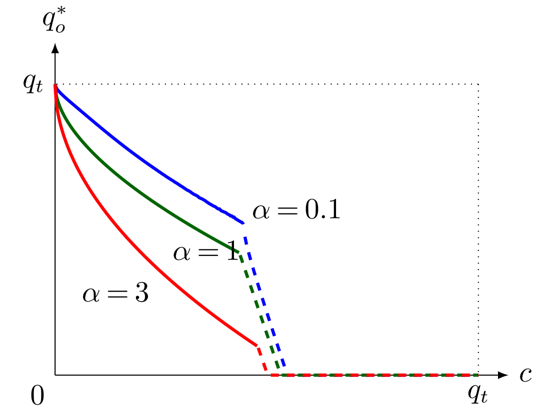
In a nutshell, the firm's product line strategy under continuous demand heterogeneity shares similar qualitative features with that under discrete demand segments. First, the firm only enters the market when the material is not too expensive (i.e., $c \leq q_t$). Second, for sufficiently affordable material (i.e., $c \leq q_t - q_t^2$), the firm fulfills all consumers and the entire market is covered; for extremely expensive material (i.e., $c > q_t - q_t^2(1 - q_t)^{1/\alpha}$), the market is not fully covered and the firm strategically abandons *some* consumers with $v \leq v_o^*$ (corresponding to strategic abandonment of traditional consumers in our base model). When the material cost is in the intermediate range, the firm strikes a tradeoff between these two strategies.

Figure 4 illustrates the effects of material cost c and parameter α on the firm's optimal product line strategy characterized by Proposition B.1 and its subsequent environmental impacts. In the case of continuous demand heterogeneity, the firm can set *continuous* price p_o for the co-product by adjusting the cutoff type $v_o \in [0,1]$, which makes the co-product appeal only to consumers with sufficiently high environmental valuation (i.e., $v \geq v_o$). Thus, a larger optimal cutoff type v_o^* corresponds to positioning the co-product more as a green product in our base model, whereas a smaller v_o^* corresponds to positioning it more as a low-end product. Figure 4(a) shows the firm's

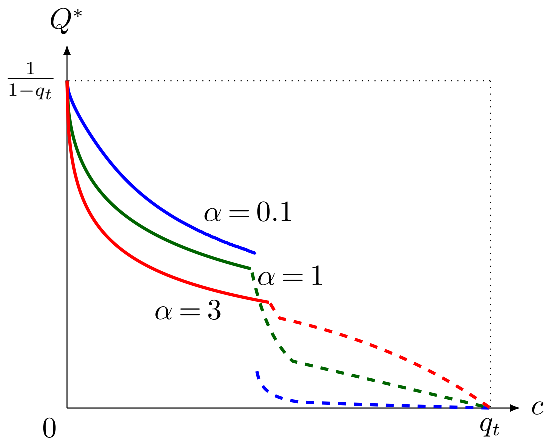
[15] We thank an anonymous reviewer for suggesting to rank the greenness of the market using stochastic orders.



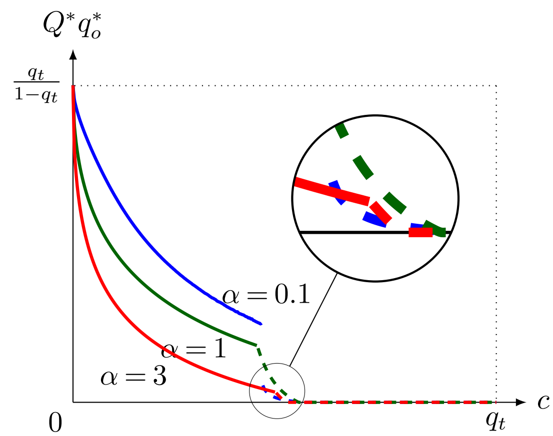
(a) Optimal cutoff type v_o^*.



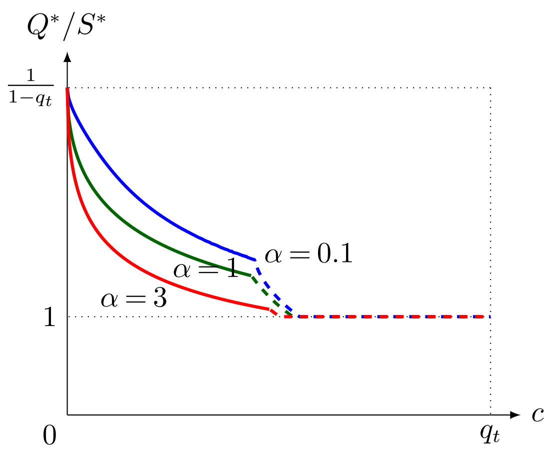
(b) Optimal green product's quality q_o^*.



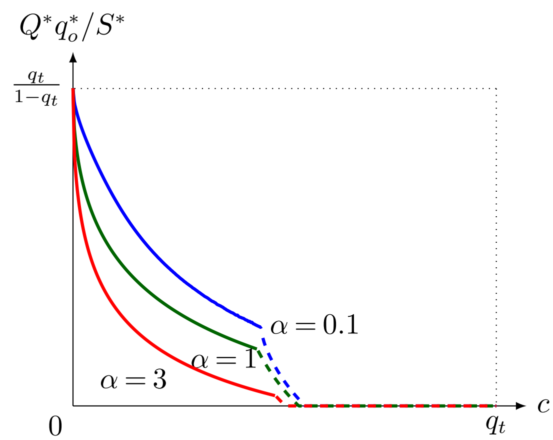
(c) Optimal material consumption Q^*.



(d) Material waste $Q^* q_o^*$.



(e) Material consumption per unit sale Q^*/S^*.



(f) Material waste per unit sale $Q^* q_o^*/S^*$.

Figure 4 Effects of c on firm's optimal strategy and its environmental impacts for $q_t = 0.7$. Solid lines represent full coverage of the market and dashed lines represent strategic abandonment of some consumers with $v \leq v_o^*$.

optimal cutoff type v_o^* as a function of the raw material cost c. It confirms two key features observed in our base model, as illustrated by Figure 1. First, the firm's product line strategy is non-monotonic in c: As c increases, v_o^* in Figure 4(a) first decreases (solid line), which corresponds to positioning the co-product more as a low-end product, and then increases (dashed line), which corresponds to positioning it more as a green product. Second, as c increases, the firm shifts from full coverage of the market (the solid line in Figure 4(a)) to strategically abandoning some consumers with low environmental valuation $v < v_o^*$ (the dashed line in Figure 4(a)).

The firm's environmental impacts under continuous demand heterogeneity is also consistent with those under discrete demand segments. As can be seen from Figure 4(b)-4(f), all metrics of environmental impacts defined in §5.1 (i.e., resource consumption, resource waste, and their per-unit counterparts) are improved as the material cost increases (see Proposition 3). Interestingly, as the consumer's WTP for environmental conservation becomes stochastically larger (i.e., higher α), overall material consumption and waste may increase, especially when strategic abandonment occurs (see Figures 4(c) and 4(d)). Such adverse effects of having more environmentally conscious consumers are consistent with the similar observation made concerning green demand size n in the case of discrete demand segments (see Proposition 1), because parameter α plays the same role as does n under continuously heterogeneous demand.

7. Conclusion

In this paper, we consider a firm that leverages co-production technology to make both a traditional product and a co-product from a common source of material with an exogenous quality distribution. On the demand side, consumers are heterogeneous in their valuation of the resource conservation enabled by the co-product. The firm maximizes its profit by making product line decisions, including the products' prices, the co-product's quality, and the raw material quantity.

We find that the firm should position the co-product as a green product that is priced to extract both its consumption and environmental value from consumers when material cost is sufficiently high or low. Such a green strategy is more profitable for the firm as demand becomes greener. Otherwise (i.e., when the raw material cost is intermediate), the firm should position the co-product as a low-end product that is priced to extract only its consumption value and to forfeit its environmental value.

Notably, when the material cost is sufficiently high and, hence, the green strategy is adopted, the firm may strategically abandon some traditional consumers, who only value a product's consumption utility and are, in fact, willing to purchase the pricier traditional product. We find that such strategic abandonment emerges as a result of the interplay between co-production technology on the supply side and the presence of green consumers on the demand side. Furthermore, under strategic abandonment, expansion of the green consumer segment improves the firm's profit but exacerbates resource consumption and waste. In other words, the firm's incentive to expand the green market may sometimes backfire, causing negative environmental consequences.

Our analytic results shed some light on the potential environmental impacts of green co-products offered by companies such as Taylor Guitars, the firm whose business model initially motivated this research. Taylor Guitars does not appear to strategically abandon their traditional consumers, according to our conversation with the company's supply chain director. As such, our analysis suggests increasing the number of green consumers both improves the firm's profitability as well as lowers ebony consumption. Hence, our finding supports Taylor Guitars' recent efforts to promote guitars made of streaked ebony. Nevertheless, our results call for careful scrutiny of the general practice of green co-production in various industries.

Finally, we speak to policy makers by examining commonly used policy instruments such as taxation and quotas. We find that raising taxes and lowering quotas on the material supply produce qualitatively similar effects on the firm's profit, consumer surplus and environmental impacts. However, quotas can help mitigate the adverse effect that a larger green consumer segment may result in higher resource consumption and waste.

All research has its limits, and ours is no exception. Future research that extends our model to incorporate other features from practice may lead to additional insights. On the supply side, we allow the firm to freely choose any co-product quality below the traditional product's quality, whereas regulatory mandate or technological barriers may give rise to situations where the co-product quality is constrained. Our work focuses on the co-product design with the co-production in place, but we have kept oblivious to how the firm acquired such co-production capability. Endogenizing the formation of the firm's co-production capability would enrich our results, albeit beyond the scope of the current paper.

On the demand side, our model takes the green demand characteristics (n, v) as exogenously given and it could be interesting to explicitly model the firm's effort in developing the green demand segment. As in much of the literature on product line design, we model a product's private-good utility simply proportional to the product's quality. Admittedly, other specifications of the private-good utility may be appropriate depending on the application contexts. Our model setup also features one homogenous segment of traditional consumers, who are flexible in choosing between the traditional product or co-product. In situations where the quality standards are rigid, there may be *inflexible* traditional consumers (e.g., diehard ebony fans), who are not willing to purchase a lower-quality co-product even if the traditional products are sold out. Another potential generalization is to make the green consumer's marginal WTP for resource conservation dependent on the material quality used in the co-product (e.g., a nonnegative function $v(q)$ of the material quality $q \in [q_t, q_o]$, which implies that the total public-good utility becomes an integration $\int_{q_o}^{q_t} v_e(q)dq$. We expect that these generalizations will not qualitatively impact our key findings.

Our work also suggests a number of directions for future research. We focus in this paper on a monopolistic firm's product line strategy with the traditional product's quality exogenously given. Our model is particularly applicable to markets, such as the musical instrument industry, where brand substitutability is very limited and the quality standard for the traditional product is persistent. Future studies can build on the present work by extending the analysis to competitive markets with substitutable products or to markets where firms may possess the monopolistic power to redefine the traditional product's quality. Finally, it may be interesting to consider co-products that are not purely ranked by their vertically differentiated quality levels but are also horizontally differentiated by consumers' personal tastes.

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Appendix A: Experiment to Support the Demand Model in §3

Based on our demand model in §3, we propose the following four hypotheses:

H1. Some consumers derive additional WTP for the co-product after being informed of the co-product's resource conservation, i.e., green consumers exist.

H2. Larger magnitude of the co-product's resource conservation leads to higher increment of WTP in H1, i.e., the co-product's public-good utility is positively correlated with its environmental benefits.

H3. The consumers in H1 still prescribe higher WTP to the traditional product than to the co-product, i.e., green consumers value the private-good consumption more than the public-good utility.

H4. Some consumers do not derive additional WTP for the co-product even after being informed of the co-product's resource conservation, i.e., the green demand size $n \in (0,1)$.

To test these hypothesis, we recruit 294 subjects from Amazon's Mturk[16] to participate in the following experiment[17]. We first inform subjects that pure black ebony wood is the conventional and highest-quality material to make guitar fretboard and such a guitar sells for \$500. Then, we inquire their WTP, denoted as WTP_1, for the guitar made of streaked ebony (i.e., the co-product), which establishes the private-good consumption value for the co-product. Next, we present the visual information that the co-product acts to conserve the ebony forest by making use of streaked ebony that would be otherwise discarded. The subjects are inquired again for their WTP, denoted as WTP_2, for the co-product. The 294 subjects are randomly assigned to two groups: one group is presented with the information that 90% of ebony are streaked and can be utilized by the co-product, whereas, for the other group, we replace 90% with 50%. Table A.1 summarize the results from these two groups.

[16] Lee et al. (2018) verify the validity of using Amazon's Mturk for behavioral experiments in operations management.

[17] The actual wordings and the photos used in the experiment are available upon request.

Table A.1 Results from the experiment.

	Group 1 (90% streaked ebony)	Group 2 (50% streaked ebony)
Sample size	98	196
Average WTP_1	\$369	\$376
Average WTP_2	\$397	\$388

Based on the results from the experiment, we find statistical evidence to support all four hypotheses:

1. Paired t-test within each group shows that \$397 is statistically higher than \$369 ($p < .001$) for Group 1 and that \$388 is higher than \$376 ($p < .05$) for Group 2. H1 is supported.

2. Using t-test between the two groups, we find that there is no statistical difference between \$376 and \$369 ($p = 0.36$), whereas the increment in WTP \$397 − \$369 = \$28 for Gourp 1 is statistically higher than \$388 − \$376 = \$12 for Group 2 ($p < .05$). H2 is supported.

3. For both groups, WTP_2 is significantly lower than the traditional product's price \$500 ($p < 0.000$), supporting H3.

4. Finally, 48% of subjects across the two groups report higher WTP after being presented with the environmental benefits of the co-product, supporting H4.

Appendix B: Supplementary Result to §6

PROPOSITION B.1. *For $v_o \in [0,1]$, we define*

$$\psi_1(v_o) := q_t - (1-q_t)\left[\frac{(1-v_o)(1-v_o^\alpha)^2}{v_o^\alpha} + \frac{c}{(1-q_t)^2}v_o^\alpha\right], \tag{B.1}$$

$$\psi_2(v_o) := (1-v_o^\alpha)\left[q_t - 2\sqrt{(c-q_t+q_t^2)(1-v_o)}\right], \tag{B.2}$$

$$\psi_3(v_o) := q_t(1-v_o^\alpha)\left[v_o - \left(c-q_t+q_t^2\right)/q_t^2\right]. \tag{B.3}$$

Then, the firm only enters the production for $c \le q_t$, in which case $q_o^* = q_t - (1-(v_o^*)^\alpha)/Q^*$, $p_o^* = q_o^* + v_o^*(q_t - q_o^*)$, and v_o^* and Q^* are characterized as follows:

- *For $c \le q_t - q_t^2$, we have $v_o^* = \arg\max_{v_o \in [(1-q_t)^{1/\alpha},1]} \psi_1(v_o)$ and $Q^* = (v_o^*)^\alpha/(1-q_t)$, in which case all consumers are fulfilled.*
- *For $q_t - q_t^2 < c \le q_t - q_t^2(1-q_t)^{1/\alpha}$, we have $v_o^* = \arg\max_{v_o\in[0,1]} \psi_1(v_o)\mathbb{1}\left[(1-q_t)^{1/\alpha} \le v_o \le \bar{v}_o\right] + \psi_2(v_o)\mathbb{1}\left[\bar{v}_o \le v_o \le (q_t-c)/q_t^2\right] + \psi_3(v_o)\mathbb{1}\left[v_o \ge (q_t-c)/q_t^2\right]$, where $\bar{v}_o \in \left[(1-q_t)^{1/\alpha}, (q_t-c)/q_t^2\right]$ uniquely satisfies*

$$\frac{(1-\bar{v}_o^\alpha)\sqrt{1-\bar{v}_o}}{\bar{v}_o^\alpha} = \frac{\sqrt{c-q_t+q_t^2}}{1-q_t}. \tag{B.4}$$

  — If $v_o^* \in \left[(1-q_t)^{1/\alpha}, \bar{v}_o\right]$, then $Q^* = (v_o^*)^\alpha/(1-q_t)$ and all consumers are fulfilled.

  — If $v_o^* \in [\bar{v}_o, (q_t-c)/q_t^2]$, then $Q^* = (1-(v_o^*)^\alpha)\sqrt{\frac{1-v_o^*}{c-q_t+q_t^2}}$ and all but some consumers with $v < v_o^*$ are fulfilled.

  — If $v_o^* \in [(q_t-c)/q_t^2, 1]$, then $Q^* = (1-(v_o^*)^\alpha)/q_t$ and all but some consumers with $v < v_o^*$ are fulfilled.
- *For $q_t - q_t^2(1-q_t)^{1/\alpha} < c \le q_t$, we have $v_o^* = \arg\max_{v_o \in [(1-q_t)^{1/\alpha},1]} \psi_3(v_o)$ and $Q^* = (1-(v_o^*)^\alpha)/q_t$, in which case all but some consumers with $v < v_o^*$ are fulfilled.*

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Designing Sustainable Products under Co-Production Technology

Appendix C: Proofs in Section 4

Lemma C.1. *In the firm's optimal product line, we must have $q_o \le p_o \le p_t = q_t$.*

1. *In the base model of section 3, $p_o \in \{q_o, q_o + v(q_t - q_o)\}$.*

2. *In the extension of section 6, there exists a threshold $v_o \in [0,1]$ such that $p_o = q_o + v_o(q_t - q_o)$ and $Q_o = 1 - G^{\alpha}(v_o) = 1 - v_o^{\alpha}$.*

Proof of Lemma C.1. We first show that $q_o \le p_o \le p_t \le q_t$ in the optimum. We first note that $p_t \le q_t$ follows from (2), as it would otherwise imply negative utility $q_t - p_t < 0$ for both traditional and green consumers and no sales of traditional products. To see that $p_o \le p_t$, we note that $p_o > p_t$ implies that $(q_t - p_t) - [q_o + v_e(q_t - q_o) - p_o] = (1 - v_e)(q_t - q_o) + (p_o - p_t) > 0$, for any $v_e \in [0,1]$, i.e., the traditional product is always preferred to the co-product by all consumers. Thus, the firm can increase its profit by raising p_t to $p_t' = p_o$ while still keeping $(q_t - p_t') - [q_o + v_e(q_t - q_o) - p_o] = (1 - v_e)(q_t - q_o) \ge 0$.

We now demonstrate that $p_o \ge q_o$. Suppose the opposite. Then, $q_o + v_e(q_t - q_o) - p_o > 0$ for all $v_e \in [0,1]$, i.e., the consumers would rather purchase the co-products, if available, than purchasing nothing. Let $\hat{v} \in [0,1]$ be such that $q_o + v(q_t - q_o) - p_o \ge (\le) q_t - p_t$for $v \ge (\le)\hat{v}$. That is, consumers with $v \ge \hat{v}$ prefer the co-product, whereas consumers with $v \le \hat{v}$ prefer the traditional product. Then, the firm can in fact increase the revenue by raising both p_o and p_t by the same amount to $p_o' = q_o$ and $p_t' = p_t + (q_o - p_o) \le q_t$, respectively, without altering the consumers' purchase decision, because $q_o + v(q_t - q_o) - p_o' = v(q_t - q_o) \ge 0$ for all $v \in [0,1]$ and $q_o + v(q_t - q_o) - p_o' \ge (\le) q_t - p_t'$, for $v \ge (\le)\hat{v}$.Thus, we must have $p_o \ge q_o$.

Since $q_o \le p_q \le p_t \le q_t$, there exist two thresholds $0 \le v_o \le v^g \le 1$ such that i) the co-product yields nonnegative utility $q_o + v(q_t - q_o) - p_o \ge 0$ if and only if $v \ge v_o$, and ii) the co-product is preferred to the traditional product $q_o + v(q_t - q_o) - p_o \ge p_t - q_t$ if and only if $v \ge v^g$. Hence,

$$p_o = q_o + v_o(q_t - q_o), \quad \text{and} \quad p_t = q_t - (v^g - v_o)(q_t - q_o). \tag{C.1}$$

- When $v_e \in \{0, v\}$, the thresholds $v_o, v^g \in \{0, v\}$, which yields three possibilities: (1) $v_o = v^g = 0$ corresponding to $p_o = q_o$ and $p_t = q_t$, (2) $v_o = 0$ and $v^g = v$ corresponding to $p_o = q_o$ and $p_t = q_t - v(q_t - q_o)$, and (3) $v_o = v^g = v$ corresponding to $p_o = q_o + v(q_t - q_o)$ and $p_t = q_t$. We claim that the second scenario will be dominated by the first scenario. In fact, the second scenario makes the consumers with $v_e = v$ indifferent between the green and traditional products, and consumers with $v_e = 0$ strictly prefer the traditional product to the co-product. In this case, the firm can in fact increase the revenue by keeping p_o unchanged (hence $v_o = 0$) raising p_t to q_t (hence $v^g = 0$), making consumers of type $v_e = v$ strictly prefer co-products. Thus, we further have $p_t = q_t$ and the first property in the lemma hold.
- When $v_e \in [0,1]$, we now show that $p_t = q_t$, which, by (C.1), establishes $p_o = q_o + v_o(q_t - q_o)$ in the second property. Suppose otherwise that $p_t < q_t$ and hence that $v_o < v^g$. Then, the traditional product always yields nonnegative utility for all consumer types and is preferred to the co-product by consumers with $v \le v^g$; the co-product yields nonnegative utility for all consumers with $v \ge v_o$ and is preferred to the traditional product by consumers with $v \ge v^g$. Hence, for any fixed supply of traditional and co-products, we have three fulfillment patterns to consider:

1. If all consumers with $v_e \le v^g$ and some consumers with $v_e \ge v^g$ are fulfilled by the traditional products, then consider raising p_t to q_t to make $v^g = v_o$. As the supply of co-products is fixed and the traditional products yields nonnegative utility for all consumer types, all consumers who are originally fulfilled by the traditional products will still purchase the traditional products, in creasing the firm's profit.

2. If all consumers with $v_e \le v_o$ are fulfilled by the traditional products, all consumers with $v_e \ge v^g$ are fulfilled by the co-products, and consumers with $v_e \in [v_o, v^g]$ are fulfilled partly by the traditional products and partly by the co-products, then consider raising p_t to q_t to make $v^g = v_o$. Similar to the previous case, all consumers who are originally fulfilled by the green and traditional products will still be fulfilled in the same pattern, increasing the firm's profit.

3. Finally, if only (some) consumers with $v_e \le v_o$ are fulfilled by the traditional products and all consumers with $v_e \ge v_o$ are either fulfilled by the co-products or left unfulfilled, then the firm can raise p_t to q_t and maintain the same supply quantities increasing its profit.

Finally, we show that $Q_o = 1 - G^\alpha(v_o)$, where $Q_o = Q\left[F(q_t) - F(q_o)\right]$ is the supply of co-products $1 - G^\alpha(v_o)$ is the potential demand for the co-product. If $1 - G^\alpha(v_o) < Q_o$, then the firm can keep Q, v_o and p_t fixed and only increase $q_o \le q_t$ (and hence reduce Q_o) until $1 - G^\alpha(v_o) = Q_o$, which in term increases price $p_o = q_o + v_o(q_t - q_o) \le q_t$ and the firm's profit. Therefore, we must have $1 - G^\alpha(v_o) \ge Q_o$. On the other hand, suppose $1 - G^\alpha(v_o) > Q_o$. The firm can increase v_o until $1 - G^\alpha(v_o) = Q_o$, leading to higher $p_o = q_o + v_o(q_t - q_o)$ and profit. Overall, we thus have $1 - G^\alpha(v_o) = Q_o$. □

Proof of Lemma 1. The lemma is immediate from the first two properties of Lemma C.1. □

Proof of Lemma 2. For any given q_o, each unit of raw material can generate $1 - F(q_t) = \bar{F}(q_t)$ traditional products that can be sold at price q_t and $F(q_t) - F(q_o) = \bar{F}(q_o) - \bar{F}(q_t)$ co-products that can be sold *at most* for a price $q_o + v(q_t - q_o)$, by virtue of Lemma 1. On the other hand, the per-unit material cost is c. Therefore, the maximum possible revenue generated by each unit of raw material is $q_t\bar{F}(q_t) + [q_o + v(q_t - q_o)]\left[\bar{F}(q_o) - \bar{F}(q_t)\right]$. Therefore, the firm enters the market and acquire positive quantity of raw material as long as $c \le q_t\bar{F}(q_t) + [q_o + v(q_t - q_o)]\left[\bar{F}(q_o) - \bar{F}(q_t)\right]$, which is equivalent to condition (3).

When the firm enters the market, the material quantity that exactly matches the supply of traditional products with the size of traditional segment (i.e., $Q_t = 1 - n$) is given by $Q = (1-n)/\left[1 - F(q_t)\right]$ by (1). Similarly, the material quantity that exactly matches the supply of co-products with the size of green segment (i.e., $Q_o = n$) is given by $Q = n/\left[F(q_t) - F(q_o)\right]$. We claim that the optimal product line entails

$$\min\left\{\frac{1-n}{1-F(q_t)}, \frac{n}{F(q_t)-F(q_o)}\right\} \le Q \le \max\left\{\frac{1-n}{1-F(q_t)}, \frac{n}{F(q_t)-F(q_o)}\right\}. \tag{C.2}$$

In fact, since the material is costly, the upper bound on Q in (C.2) immediately follows. To obtain the lower bound in (C.2), suppose it does not hold. Then, neither green nor traditional segment is completely fulfilled, and the firm can sell additional co-products at price $q_o + v(q_t - q_o)$ to green consumers and additional traditional products at price q_t to traditional consumers at a marginal cost of c. Under condition (3), doing so would increase the firm's profit, thus establishing the lower bound.

If $F(q_o) \ge \frac{F(q_t)-n}{1-n}$, then (C.2) reduces to $\frac{1-n}{1-F(q_t)} \le Q \le \frac{n}{F(q_t)-F(q_o)}$, implying that the entire traditional segment can be satisfied by traditional products (i.e., $Q_t = Q[1 - F(q_t)] \ge 1 - n$) while the green segment's demand may not be fully satisfied by co-products (i.e., $Q_o = Q[F(q_t) - F(q_o)] \leq n$). Since the green consumers are indifferent (both receive zero net utility) between a co-product priced at $q_o + v(q_t - q_o)$ and a traditional product priced at $q_t \geq q_o + v(q_t - q_o)$, it is thus most profitable for the firm to produce just enough excess traditional products to satisfy the unfulfilled green consumers, i.e., $Q_t + Q_o = Q[1 - F(q_t)] + Q[F(q_t) - F(q_o)] = 1$, while maintaining the price of co-products at $q_o + v(q_t - q_o)$. This precisely corresponds to GF strategy.

If $F(q_o) \leq \frac{F(q_t)-n}{1-n}$, then (C.2) reduces to $\frac{n}{F(q_t)-F(q_o)} \leq Q \leq \frac{1-n}{1-F(q_t)}$, implying that all the green segment's demand can be satisfied by co-products (i.e., $Q_o = Q[F(q_t) - F(q_o)] \geq n$) while the traditional segment's demand may not be fully satisfied by traditional products (i.e., $Q_t = Q[1 - F(q_t)] \leq 1 - n$). Therefore, the firm can consider the following two pricing strategies:

- If the co-product is priced at $p_o = q_o + v(q_t - q_o)$, the traditional consumer will not purchase the co-product, suggesting that either $Q = n/[F(q_t) - F(q_o)]$, i.e., the firm only produces enough co-products to fulfill the entire green segment and leaves some traditional consumers unsatisfied, or $Q = (1-n)/[1-F(q_t)]$, i.e., the firm acquires sufficient raw material to produce enough traditional products to fulfill the entire traditional segment. The former case corresponds to GP strategy. In the latter case, however, the firm can be better off by increasing the co-product's quality q_o so that no excess co-products are produced (i.e., $Q_o = Q[F(q_t) - F(q_o)] = n$) and the price $p_o = q_o + v(q_t - q_o)$ also becomes higher. This strategy in fact coincides with GF strategy.
- If the co-product is priced at $p_o = q_o$, the traditional consumers become indifferent between the traditional and co-products. Therefore, the firm can either acquires sufficient raw material so as to satisfy all the traditional consumers with traditional products (i.e., $Q_t = Q[1 - F(q_t)] = 1 - n$), or acquires just enough raw material and produce excess co-products to satisfy the traditional demand that is not fulfilled by traditional products, i.e., $Q_t + Q_o = Q[1 - F(q_t)] + Q[F(q_t) - F(q_o)] = 1$. However, the former strategy is never optimal because the firm can then increase the co-product's price to $q_o + v(q_t - q_o)$, which corresponds to GF strategy. This leaves the latter strategy as only viable option, which corresponds to LF strategy. $\square$

LEMMA C.2 **(GF Strategy).** *Under GF strategy of Lemma 2, the firm's problem (P) reduces to*

$$\pi_{\mathrm{GF}}(c, v, n, q_t) := \max_{F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right) \leq q_o \leq q_t} \Pi_{\mathrm{GF}}(q_o) := \frac{1}{\bar{F}(q_o)} \left\{ q_t \bar{F}(q_t) - c + \left(\bar{F}(q_o) - \bar{F}(q_t)\right)[q_o + v(q_t - q_o)] \right\}. \tag{GF}$$

The co-product's quality solving (GF) is given by

$$q_o^{\mathrm{GF}} := \begin{cases} q_{\mathrm{GF}}^*, & \text{if } c < c_{\mathrm{GF}}, \\ F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right), & \text{if } c \geq c_{\mathrm{GF}}, \end{cases} \tag{C.3}$$

where $q_{\mathrm{GF}}^* \in \left[F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right), q_t\right)$ uniquely satisfies

$$\bar{F}(q_{\mathrm{GF}}^*) - \bar{F}(q_t) + \left[\bar{F}(q_t)(q_t - q_{\mathrm{GF}}^*) - \frac{c}{1-v}\right] f(q_{\mathrm{GF}}^*)/\bar{F}(q_{\mathrm{GF}}^*) = 0, \tag{C.4}$$

and

$$c_{\mathrm{GF}} := (1-v)\left\{\bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right)\right] + \frac{\left[F(q_t) - \frac{(F(q_t)-n)^+}{1-n}\right]\left[1 - \frac{(F(q_t)-n)^+}{1-n}\right]}{f\left(\frac{(F(q_t)-n)^+}{1-n}\right)}\right\}. \tag{C.5}$$

Accordingly, the co-product's price and the material quantity are given by $p_o^{\mathrm{GF}} = q_o^{\mathrm{GF}} + v\left(q_t - q_o^{\mathrm{GF}}\right)$ *and* $Q^{\mathrm{GF}} = 1/(1-F(q_o^{\mathrm{GF}}))$, *respectively.*

Proof of Lemma C.2. By substituting $p_o = q_o + v(q_t - q_o)$ and $Q = 1/(1-F(q_o))$ into (P), we immediately obtain (GF). Direct calculation reveals that

$$\Pi'_{\mathrm{GF}}(q_o) = \frac{1}{\bar{F}(q_o)}\left\{(1-v)\left[\bar{F}(q_o) - \bar{F}(q_t)\right] + \left[(1-v)\bar{F}(q_t)(q_t - q_o) - c\right] f(q_o)/\bar{F}(q_o)\right\}.$$

Therefore, we have

- For $c \le (1-v)\bar{F}(q_t)(q_t - q_o)$ or equivalently $q_o \le \left[q_t - \frac{c}{(1-v)}/\bar{F}(q_t)\right]^+$, we have $\Pi'_{\mathrm{GF}}(q_o) \ge 0$.
- For $c > (1-v)\bar{F}(q_t)(q_t - q_o)$ or equivalently $q_o > \left[q_t - \frac{c}{(1-v)}/\bar{F}(q_t)\right]^+$, The log-concavity of $F(\cdot)$ implies that $\bar{F}(q_o)\Pi'_{\mathrm{GF}}(q_o) = (1-v)\left[\bar{F}(q_o) - \bar{F}(q_t)\right] + \left[(1-v)\bar{F}(q_t)(q_t - q_o) - c\right] f(q_o)/\bar{F}(q_o)$ is decreasing in q_o and takes negative value at $q_o = q_t$.

Thus, $\Pi_{\mathrm{GF}}(q_o)$ is quasi-concave in $q_o \in [0, q_t]$ and its maximum is determined by the following two cases:

1. If $\Pi'_{\mathrm{GF}}(q_o) \le 0$ at $q_o = F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right)$ or equivalently $c \ge c_{\mathrm{GF}}$, then $\Pi_{\mathrm{GF}}(q_o)$ is maximized at $q_o^{\mathrm{GF}} = F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right)$, yielding (C.3).
2. Otherwise, $\Pi'_{\mathrm{GF}}(q_o) > 0$ at $q_o = F^{-1}\left(\frac{(F(q_t)-n)^+}{1-n}\right)$. Then, $\Pi_{\mathrm{GF}}(q_o)$ is maximized where $\Pi'_{\mathrm{GF}}(q_o) = 0$, i.e., at $q_o^{\mathrm{GF}} = q^*_{\mathrm{GF}}$ given by (C.4), yielding (C.3). □

Lemma C.3 **(GP Strategy)**. *GP strategy of Lemma 2 is only feasible for* $n \le F(q_t)$, *under which the firm's problem* (P) *reduces to*

$$\pi_{\mathrm{GP}}(c, v, n, q_t) := \max_{0 \le q_o \le F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)} \Pi_{\mathrm{GP}}(q_o) := n\left\{\frac{q_t\bar{F}(q_t) - c}{\bar{F}(q_o) - \bar{F}(q_t)} + q_o + v\left(q_t - q_o\right)\right\}. \tag{GP}$$

The co-product's quality solving (GP) *is given by*

$$q_o^{\mathrm{GP}} := \begin{cases} F^{-1}\left(\frac{F(q_t)-n}{1-n}\right), & \text{if } c \le c_{\mathrm{GP}}, \\ q^*_{\mathrm{GP}}, & \text{if } c > c_{\mathrm{GP}}, \end{cases} \tag{C.6}$$

where $q^*_{\mathrm{GP}} \in \left[0, F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right)$ *is given by*

$$q^*_{\mathrm{GP}} = \inf\left\{q \ge 0 : (1-v)\left(\bar{F}(q) - \bar{F}(q_t)\right)^2 - \left(c - q_t\bar{F}(q_t)\right) f(q) \le 0\right\}, \tag{C.7}$$

and

$$c_{\mathrm{GP}} := q_t\bar{F}(q_t) + (1-v)\frac{n^2\bar{F}^2(q_t)}{(1-n)^2 f\left(F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right)}. \tag{C.8}$$

Accordingly, the co-product's price and the material quantity are given by $p_o^{\mathrm{GP}} = q_o^{\mathrm{GP}} + v\left(q_t - q_o^{\mathrm{GP}}\right)$ *and* $Q^{\mathrm{GP}} = n/\left(F(q_t) - F(q_o^{\mathrm{GP}})\right)$, *respectively.*

Proof of Lemma C.3. The feasibility of GP strategy necessitates $F(q_o) \le (F(q_t) - n)/(1-n)$, which requires $n \le F(q_t)$. By straightforward verification, substituting $p_o = q_o + v(q_t - q_o)$ and $Q = n/(F(q_t) - F(q_o))$ reduces (P) to (GP). Direct calculation reveals that

$$\Pi'_{\mathrm{GP}}(q_o) = \frac{n}{\bar{F}(q_o) - \bar{F}(q_t)}\left\{\frac{\left(q_t\bar{F}(q_t) - c\right) f(q_o)}{\bar{F}(q_o) - \bar{F}(q_t)} + (1-v)\left(\bar{F}(q_o) - \bar{F}(q_t)\right)\right\}$$

Therefore, we have

- For $c \leq q_t \bar{F}(q_t)$, we have $\Pi'_{\mathrm{GP}}(q_o) > 0$ and hence the maximum of $\Pi_{\mathrm{GP}}(q_o)$ is achieved at $q_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$.
- For $c > q_t \bar{F}(q_t)$, the log-concavity of $F(\cdot)$ implies that $\frac{(q_t \bar{F}(q_t)-c) f(q_o)}{\bar{F}(q_o)-\bar{F}(q_t)} + (1-v)\left(\bar{F}(q_o) - \bar{F}(q_t)\right)$ is decreasing in q_o and takes

1. nonnegative value for all $q_o \in \left[0, F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right]$ if $q_t \bar{F}(q_t) < c \leq c_{\mathrm{GP}}$, implying that $\Pi_{\mathrm{GP}}(q_o)$ is maximized at $q_o^{\mathrm{GP}} = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$ and yielding (C.6);
2. positive value at $q_o = 0$ but negative value at $q_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$ if $c_{\mathrm{GP}} < c < q_t \bar{F}(q_t) + (1-v) F^2(q_t)/f(0)$, implying that$\Pi_{\mathrm{GP}}(q_o)$ is maximized where $\Pi'_{\mathrm{GP}}(q_o) = 0$, i.e., $q_o^{\mathrm{GP}} = q^*_{\mathrm{GP}}$ given by (C.7) and yielding (C.6);
3. non-positive value for all $q_o \in \left[0, F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right]$ if $c \geq q_t \bar{F}(q_t) + (1-v) F^2(q_t)/f(0)$, implying that $\Pi_{\mathrm{GP}}(q_o)$ is maximized at $q_o^{\mathrm{GP}} = 0 = q^*_{\mathrm{GP}}$, again given by (C.7), yielding (C.6). □

Lemma C.4 **(LF Strategy)**. *LF strategy of Lemma 2 is only feasible for $n \leq F(q_t)$, under which the firm's problem (P) reduces to*

$$\pi_{\mathrm{LF}}(c, n, q_t) := \max_{0 \leq q_o \leq F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)} \Pi_{\mathrm{LF}}(q_o) := \left\{ q_t \bar{F}(q_t) - c + \left(\bar{F}(q_o) - \bar{F}(q_t)\right) q_o \right\} / \bar{F}(q_o), \tag{LF}$$

independent of v. The co-product's quality solving (LF) is given by

$$q_o^{\mathrm{LF}} := \begin{cases} F^{-1}\left(\frac{F(q_t)-n}{1-n}\right), & \text{if } c \leq c_{\mathrm{LF}}, \\ q^*_{\mathrm{LF}}, & \text{if } c > c_{\mathrm{LF}}, \end{cases} \tag{C.9}$$

where $q^*_{\mathrm{LF}} \in \left(0, F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right)$ uniquely satisfies

$$\bar{F}(q^*_{\mathrm{LF}}) - \bar{F}(q_t) + \left[\bar{F}(q_t)(q_t - q^*_{\mathrm{LF}}) - c\right] f(q^*_{\mathrm{LF}}) / \bar{F}(q^*_{\mathrm{LF}}) = 0, \tag{C.10}$$

and

$$c_{\mathrm{LF}} := \bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right] + \frac{n \bar{F}^2(q_t)}{(1-n)^2 f\left(F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right)}. \tag{C.11}$$

Accordingly, the co-product's price and the material quantity are given by $p_o^{\mathrm{LF}} = q_o^{\mathrm{LF}}$ and $Q^{\mathrm{LF}} = 1/\left(1 - F(q_o^{\mathrm{LF}})\right)$, respectively.

Proof of Lemma C.4. The feasibility of GP strategy necessitates $F(q_o) \leq (F(q_t) - n)/(1-n)$, which requires $n \leq F(q_t)$. By straightforward verification, substituting $p_o = q_o$ and $Q = 1/(1 - F(q_o))$ reduces (P) to (LF). Direct calculation reveals that

$$\Pi'_{\mathrm{LF}}(q_o) = \frac{1}{\bar{F}(q_o)} \left\{ \bar{F}(q_o) - \bar{F}(q_t) + \left[\bar{F}(q_t)(q_t - q_o) - c\right] f(q_o)/\bar{F}(q_o) \right\}.$$

Therefore, we have

- For $\bar{F}(q_t)(q_t - q_o) \geq c$ or equivalently $q_o \leq \left[q_t - \frac{c}{\bar{F}(q_t)}\right]^+$, we have $\Pi'_{\mathrm{LF}}(q_o) \geq 0$.
- For $\bar{F}(q_t)(q_t - q_o) < c$ or equivalently $q_o > \left[q_t - \frac{c}{\bar{F}(q_t)}\right]^+$, the log-concavity of $F(\cdot)$ implies that $\bar{F}(q_o)\Pi'_{\mathrm{LF}}(q_o) = \bar{F}(q_o) - \bar{F}(q_t) + \left[\bar{F}(q_t)(q_t - q_o) - c\right] f(q_o)/\bar{F}(q_o)$ is decreasing in q_o and takes nonnegative value at $q_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$ if and only if $c < \bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right] + \frac{n\bar{F}^2(q_t)}{(1-n)^2 f\left(F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right)}$.

Thus, $\Pi_{\mathrm{LF}}(q_o)$ is quasi-concave in $q_o \in [0, q_t]$ and its maximum can be of the following cases:

1. For $c \le \bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right] \le c_{\mathrm{LF}}$ or equivalently $F^{-1}\left(\frac{F(q_t)-n}{1-n}\right) \le q_t - \frac{c}{\bar{F}(q_t)}$, the maximum of $\Pi_{\mathrm{LF}}(q_o)$ is achieved at $q_o^{\mathrm{LF}} = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$, yielding (C.9).

2. For $\bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right] < c \le c_{\mathrm{LF}}\bar{F}(q_t)\left[q_t - F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)\right]$, we have $F^{-1}\left(\frac{F(q_t)-n}{1-n}\right) > \left[q_t - \frac{c}{\bar{F}(q_t)}\right]^+$ and $\Pi'_{\mathrm{LF}}(q_o) \ge 0$ at $q_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$, where $\Pi_{\mathrm{LF}}(q_o)$ is maximized. Thus, we obtain (C.9).

3. For $c > c_{\mathrm{LF}}$, we have $F^{-1}\left(\frac{F(q_t)-n}{1-n}\right) > \left[q_t - \frac{c}{\bar{F}(q_t)}\right]^+$ and $\Pi'_{\mathrm{LF}}(q_o) < 0$ at $q_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$. Hence, $\Pi_{\mathrm{LF}}(q_o)$ is maximized at $p_o^{\mathrm{LF}} = p^*_{\mathrm{LF}}$ given by (C.10), again yielding (C.9). □

Proof of Theorem 1. We compare the optimal profits of the three subproblems (GF), (GP) and (LF), the largest of which determines the global optimal solution to the firm's problem (P). To that end, we first make the following claims, which follow from straightforward verification.

Claim 1. $\pi_{\mathrm{GF}}(c,v,n,q_t) \ge 0$ or $\pi_{\mathrm{GP}}(c,v,n,q_t) \ge 0$ if and only if condition (3) holds (i.e., $c \le \bar{c}$), while $\pi_{\mathrm{LF}}(c,n,q_t) \ge 0$ if and only if $c \le \underline{c} < \bar{c}$, where $\underline{c}$ is defined by (4) and clearly smaller than $\bar{c}$ in contrast with the definition of $\bar{c}$ in (3). This claim follows from the fact that $\Pi_{\mathrm{GF}}(q_o)$ and $\Pi_{\mathrm{GP}}(q_o)$ are of the same sign as $q_t\bar{F}(q_t) + \left(\bar{F}(q_o) - \bar{F}(q_t)\right)[q_o + v\,(q_t - q_o)] - c$, which is nonnegative if and only if condition (3) holds. Instead, $\Pi_{\mathrm{LF}}(q_o)$ is of the same sign as $q_t\bar{F}(q_t) + \left(\bar{F}(q_o) - \bar{F}(q_t)\right) q_o - c$, which is nonnegative if and only if condition (4) holds.

Claim 2. When $n > F(q_t)$, only subproblem (GF) is feasible and hence the solution to (P) is given by (C.3). In this case, we let $c_1^* = c_{\mathrm{GF}} \wedge \bar{c}$ and $c_2^* = \bar{c}$. For the rest of the proof, we will focus on the case $n \le F(q_t)$.

Claim 3. When $n \le F(q_t)$, we have $c_{\mathrm{GF}} = (1-v)c_{\mathrm{LF}} \le c_{\mathrm{LF}}$, which allows us to divide the proof into three different parametric scenarios. In particular, if we will denote $q_o^\sharp := F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$, then

$$\Pi_{\mathrm{GF}}(q_o^\sharp) = \Pi_{\mathrm{GP}}(q_o^\sharp), \quad \text{and} \quad \Pi_{\mathrm{GF}}(q_o) \ge \Pi_{\mathrm{LF}}(q_o) \text{ for all } q_o. \tag{C.12}$$

Scenario 1: $c_{\mathrm{GF}} \le c_{\mathrm{LF}} \le c_{\mathrm{GP}}$.

- For $c \le c_{\mathrm{GF}}$, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q^*_{\mathrm{GF}}) \ge \Pi_{\mathrm{GF}}(q_o^\sharp) \begin{cases} = \Pi_{\mathrm{GP}}(q_o^\sharp) = \pi_{\mathrm{GP}}(c,v,n,q_t), \\ \ge \Pi_{\mathrm{LF}}(q_o^\sharp) = \pi_{\mathrm{LF}}(c,n,q_t), \end{cases}$$

where we use the optimality of q^*_{GF} to obtain the first inequality and (C.12) to obtain the other (in)equalities. Thus, GF strategy is optimal and the optimal co-product's quality is q^*_{GF}.

- For $c_{\mathrm{GF}} < c \le c_{\mathrm{LF}}$, similarly, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q_o^\sharp) \begin{cases} = \Pi_{\mathrm{GP}}(q_o^\sharp) = \pi_{\mathrm{GP}}(c,v,n,q_t) \\ \ge \Pi_{\mathrm{LF}}(q_o^\sharp) = \pi_{\mathrm{LF}}(c,n,q_t). \end{cases}$$

Therefore, GF strategy is optimal and the optimal co-product's quality is $q_o^\sharp$.

- For $c_{\mathrm{LF}} < c \le c_{\mathrm{GP}}$, Lemmas C.2, C.3 and C.4 imply that the larger profit between $\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q_o^\sharp) = \Pi_{\mathrm{GP}}(q_o^\sharp) = \pi_{\mathrm{GP}}(c,v,n,q_t)$ and $\pi_{\mathrm{LF}}(c,n,q_t) = \Pi_{\mathrm{LF}}(q^*_{\mathrm{LF}})$ determines the solution. By the Envelope Theorem,

$$\frac{\partial \pi_{\mathrm{GF}}(c,v,n,q_t)}{\partial c} = \frac{\partial \pi_{\mathrm{GP}}(c,v,n,q_t)}{\partial c} = -1/\bar{F}\left(q_o^\sharp\right) \le -1/\bar{F}\left(q^*_{\mathrm{LF}}\right) = \frac{\partial \pi_{\mathrm{LF}}(c,n,q_t)}{\partial c}.$$

That is, $\pi_{\mathrm{GP}}(c,v,n,q_t) - \pi_{\mathrm{LF}}(c,n,q_t)$ is decreasing in $c \in [c_{\mathrm{LF}}, c_{\mathrm{GP}}]$. Let

$$\widehat{c} := \max\left\{c \in [c_{\mathrm{LF}}, c_{\mathrm{GP}}] : \pi_{\mathrm{GP}}(c,v,n,q_t) \ge \pi_{\mathrm{LF}}(c,n,q_t)\right\}.$$

Then, for $c_{\mathrm{LF}} < c \le \widehat{c}$, the optimal co-product's quality is $q_o^\sharp$ and GF strategy is optimal. For $\widehat{c} < c \le c_{\mathrm{GP}}$, subproblem (LF) dominates the other two subproblems; thus, LF strategy is optimal and the optimal co-product's quality is q^*_{LF}.

- For $c > c_{\mathrm{GP}}$, Lemmas C.2, C.3 and C.4 imply that $\pi_{\mathrm{LF}}(c,n,q_t) = \Pi(q^*_{\mathrm{LF}})$ and $\pi_{\mathrm{GP}}(c,v,n,q_t) = \Pi_{\mathrm{GP}}(q^*_{\mathrm{GP}}) \geq \Pi_{\mathrm{GP}}(q^\sharp_o) = \Pi_{\mathrm{GF}}(q^\sharp_o) = \pi_{\mathrm{GF}}(c,v,n,q_t)$. Thus, if $\pi_{\mathrm{GP}}(c,v,n,q_t) \geq \pi_{\mathrm{LF}}(c,n,q_t)$, then GP strategy is optimal with q^*_{GP} being the optimal co-product's quality; otherwise, LF strategy is optimal with q^*_{LF} being the optimal co-product's quality. In particular, according to Claim 1, the former case must be true for $c \geq \underline{c}$.

In sum, we obtain the theorem by letting $c^*_1 = \widehat{c} \wedge \bar{c}$ and $c^*_2 = (\widehat{c} \vee \underline{c}) \wedge \bar{c}$. In particular, for sufficiently large $v \leq 1$, we have $\widehat{c} < \bar{c}$ and hence $c^*_2 < \bar{c}$, because $\widehat{c} \leq c_{\mathrm{GP}} \to q_t \bar{F}(q_t) < q_t$ while $\bar{c} \to q_t$ as $v \to 1$.

$\boxed{\textit{Scenario 2: } c_{\mathrm{GF}} \leq c_{\mathrm{GP}} \leq c_{\mathrm{LF}}.}$ • For $c \leq c_{\mathrm{GF}}$, following arguments similar to the first scenario, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q^*_{\mathrm{GF}}) \geq \Pi_{\mathrm{GF}}(q^\sharp_o) \begin{cases} = \Pi_{\mathrm{GP}}(q^\sharp_o) = \pi_{\mathrm{GP}}(c,v,n,q_t), \\ \geq \Pi_{\mathrm{LF}}(q^\sharp_o) = \pi_{\mathrm{LF}}(c,n,q_t). \end{cases}$$

Thus, GF strategy is optimal and the optimal co-product's quality is q^*_{GF}.

- For $c_{\mathrm{GF}} < c \leq c_{\mathrm{GP}}$, Lemmas C.2, C.3 and C.4 again imply that

$$\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q^\sharp_o) \begin{cases} = \Pi_{\mathrm{GP}}(q^\sharp_o) = \pi_{\mathrm{GP}}(c,v,n,q_t) \\ \geq \Pi_{\mathrm{LF}}(q^\sharp_o) = \pi_{\mathrm{LF}}(c,n,q_t). \end{cases}$$

Therefore, GF strategy is optimal and the optimal co-product's quality is $q^\sharp_o$.

- For $c_{\mathrm{GP}} < c \leq c_{\mathrm{LF}}$, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\mathrm{GP}}(c,v,n,q_t) = \Pi_{\mathrm{GP}}(q^*_{\mathrm{GP}}) \geq \Pi_{\mathrm{GP}}(q^\sharp_o) = \Pi_{\mathrm{GF}}(q^\sharp_o) = \pi_{\mathrm{GF}}(c,v,n,q_t) \geq \Pi_{\mathrm{LF}}(q^\sharp_o) = \pi_{\mathrm{LF}}(c,n,q_t).$$

Therefore, GP strategy is optimal and the optimal co-product's quality is q^*_{GP}.

- For $c > c_{\mathrm{LF}}$, Lemmas C.2, C.3 and C.4 imply that $\pi_{\mathrm{LF}}(c,n,q_t) = \Pi_{\mathrm{LF}}(q^*_{\mathrm{LF}})$ and $\pi_{\mathrm{GP}}(c,v,n,q_t) = \Pi_{\mathrm{GP}}(q^*_{\mathrm{GP}}) \geq \Pi_{\mathrm{GP}}(q^\sharp_o) = \Pi_{\mathrm{GF}}(q^\sharp_o) = \pi_{\mathrm{GF}}(c,v,n,q_t)$. Thus, if $\pi_{\mathrm{GP}}(c,v,n,q_t) \geq \pi_{\mathrm{LF}}(c,n,q_t)$, then GP strategy is optimal with q^*_{GP} being the optimal co-product's quality; otherwise, FL strategy is optimal with q^*_{LF} being the optimal co-product's quality. In particular, according to Claim 1, the former case must be true for $c \geq \underline{c}$.

In sum, we obtain the theorem by letting $c^*_1 = c_{\mathrm{GP}} \wedge \bar{c}$ and $c^*_2 = (c_{\mathrm{GP}} \vee \underline{c}) \wedge \bar{c}$. (In fact, if $c_{\mathrm{LF}} \geq \underline{c}$, we can set $c^*_2 = c_{\mathrm{GP}} \wedge \bar{c}$.) In particular, for large enough $v \leq 1$, we have $c_{\mathrm{GP}} < \bar{c}$ and hence $c^*_2 < \bar{c}$, because $c_{\mathrm{GP}} \to q_t \bar{F}(q_t) < q_t$ while $\bar{c} \to q_t$ as $v \to 1$.

$\boxed{\textit{Scenario 3: } c_{\mathrm{GP}} \leq c_{\mathrm{GF}} \leq c_{\mathrm{LF}}.}$ • For $c \leq c_{\mathrm{GP}}$, following arguments similar to the first scenario, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q^*_{\mathrm{GF}}) \geq \Pi_{\mathrm{GF}}(q^\sharp_o) \begin{cases} = \Pi_{\mathrm{GP}}(q^\sharp_o) = \pi_{\mathrm{GP}}(c,v,n,q_t), \\ \geq \Pi_{\mathrm{LF}}(q^\sharp_o) = \pi_{\mathrm{LF}}(c,n,q_t). \end{cases}$$

Thus, GF strategy is optimal with q^*_{GF} being the optimal co-product's quality.

- For $c_{\mathrm{GP}} < c \leq c_{\mathrm{GF}}$, Lemmas C.2, C.3 and C.4 imply that that $\pi_{\mathrm{GP}}(c,v,n,q_t) = \Pi_{\mathrm{GP}}(q^*_{\mathrm{GP}})$ and $\pi_{\mathrm{GF}}(c,v,n,q_t) = \Pi_{\mathrm{GF}}(q^*_{\mathrm{GF}}) \geq \Pi_{\mathrm{GF}}(q^\sharp_o) \geq \Pi_{\mathrm{LF}}(q^\sharp_o) = \pi_{\mathrm{LF}}(c,n,q_t)$. Therefore, we need to compare $\pi_{\mathrm{GF}}(c,v,n,q_t)$ and $\pi_{\mathrm{GP}}(c,v,n,q_t)$. By the Envelope Theorem,

$$\frac{\partial \pi_{\mathrm{GF}}(c,v,n,q_t)}{\partial c} = -1/\bar{F}(q^*_{\mathrm{GF}}) < -1/\bar{F}(q^*_{\mathrm{GP}}) \leq -n/\left(\bar{F}(q^*_{\mathrm{GP}}) - \bar{F}(q_t)\right) = \frac{\partial \pi_{\mathrm{GP}}(c,v,n,q_t)}{\partial c}$$

where the first inequality follows from $q^*_{\mathrm{GF}} > q^*_{\mathrm{GP}}$ and the second inequality from the fact that $q^*_{\mathrm{GP}} \leq q^\sharp_o = F^{-1}\left(\frac{F(q_t)-n}{1-n}\right)$ or equivalently $F(q^*_{\mathrm{GP}}) \leq \frac{F(q_t)-n}{1-n}$. Namely, $\pi_{\mathrm{GF}}(c,v,n,q_t) - \pi_{\mathrm{GP}}(c,v,n,q_t)$ is decreasing in

$c \in [c_{\rm GP}, c_{\rm GF}]$. On the other hand, as shown above, $\pi_{\rm GF}(c_{\rm GP}, v, n, q_t) - \pi_{\rm GP}(c_{\rm GP}, v, n, q_t) \geq 0$, whereas as shown below $\pi_{\rm GF}(c_{\rm GF}, v, n, q_t) - \pi_{\rm GP}(c_{\rm GF}, v, n, q_t) \leq 0$. Therefore, there exists a unique $\widehat{c} \in [c_{\rm GP}, c_{\rm GF}]$ such that GF strategy is optimal for $c_{\rm GP} \leq c \leq \widehat{c}$ with $q^*_{\rm GF}$ being optimal co-product's quality, and GP strategy is optimal for $\widehat{c} \leq c \leq c_{\rm GF}$ with $q^*_{\rm GP}$ being optimal co-product's quality.

- For $c_{\rm GF} < c \leq c_{\rm LF}$, Lemmas C.2, C.3 and C.4 imply that

$$\pi_{\rm GP}(c, v, n, q_t) = \Pi_{\rm GP}(q^*_{\rm GP}) \geq \Pi_{\rm GP}(q^\sharp_o) = \Pi_{\rm GF}(q^\sharp_o) = \pi_{\rm GF}(c, v, n, q_t) \geq \Pi_{\rm LF}(q^\sharp_o) = \pi_{\rm LF}(c, n, q_t).$$

Therefore, GP strategy is optimal with $q^*_{\rm GP}$ being optimal co-product's quality.

- For $c > c_{\rm LF}$, Lemmas C.2, C.3 and C.4 imply that $\pi_{\rm GP}(c, v, n, q_t) = \Pi_{\rm GP}(q^*_{\rm GP}) \geq \Pi_{\rm GP}(q^\sharp_o) = \Pi_{\rm GF}(q^\sharp_o) = \pi_{\rm GF}(c, v, n, q_t)$ and $\pi_{\rm LF}(c, n, q_t) = \Pi_{\rm LF}(q^*_{\rm LF})$. Therefore, if $\pi_{\rm GP}(c, v, n, q_t) \geq \pi_{\rm LF}(c, n, q_t)$, then GP strategy is optimal with $q^*_{\rm GP}$ being the optimal co-product's quality; otherwise, LF strategy is optimal with $q^*_{\rm LF}$ being the optimal co-product's quality. In particular, according to Claim 1, the former case must be true for $c \geq \underline{c}$.

In sum, we obtain the theorem by letting $c^*_1 = \widehat{c} \wedge \bar{c}$ and $c^*_2 = (\widehat{c} \vee \underline{c}) \wedge \bar{c}$. (In fact, if $c_{\rm LF} \geq \underline{c}$, we can set $c^*_1 = c^*_2 = \widehat{c} \wedge \bar{c}$.) In particular, for sufficiently large $v \leq 1$, we have $\widehat{c} < \bar{c}$ and hence $c^*_2 < \bar{c}$, because $\widehat{c} \leq c_{\rm GF} \to 0$ while $\bar{c} \to q_t$ as $v \to 1$.

In all the three scenarios above, the existence of $\bar{v}$ follows from the fact that $c_{\rm LF}$ and $\pi_{\rm LF}(c, n, q_t)$ are independent of v while $c_{\rm GP}$ is decreasing in v and $\pi_{\rm GP}(c, v, n, q_t)$ is increasing in v. $\square$

Proof of Corollary 1. If $q_o = q_t$, then the marginal revenue from one unit of material is $q_t \bar{F}(q_t) \leq \underline{c}$ and hence the firm enters the market if and only if $c \leq q_t \bar{F}(q_t)$. If the co-product is introduced as a low-end product (i.e., $p_o = q_o$), then the marginal revenue is given by $\underline{c}$ in (4) and hence the firm enters the market if and only if $c \leq \underline{c}$. Therefore, Theorem 1 implies that GP strategy is optimal for $c \in (\underline{c}, \bar{c}]$. $\square$

Proof of Theorem 2. We specialize Lemmas C.2, C.3, and C.4 with $F(q) = q$ to obtain

$$\pi_{\rm GF}(c, v, n, q_t) = \max_{q_o \in \left[\frac{q_t - n}{1-n}, q_t\right]} \Pi_{\rm GF}(q_o) \equiv \frac{1-q_t}{1-q_o} q_t + \frac{q_t - q_o}{1-q_o}[q_o + v(q_t - q_o)] - \frac{c}{1-q_o}, \tag{C.13}$$

$$\pi_{\rm GP}(c, v, n, q_t) = \max_{q_o \in \left[0, \frac{q_t - n}{1-n}\right]} \Pi_{\rm GP}(q_o) \equiv (1-q_t)(\frac{n}{q_t - q_o}) q_t + n[q_o + v(q_t - q_o)] - \frac{nc}{q_t - q_o}, \tag{C.14}$$

$$\pi_{\rm LF}(c, n, q_t) = \max_{q_o \in \left[0, \frac{q_t - n}{1-n}\right]} \Pi_{LF}(q_o) \equiv \frac{1-q_t}{1-q_o} q_t + \frac{q_t - q_o}{1-q_o} q_o - \frac{c}{1-q_o}. \tag{C.15}$$

Solution to (C.13). Straightforward calculation yields $\frac{\partial^2 \Pi_{GF}}{\partial q_o^2} = -\frac{2(c + (1-q_t)^2(1-v))}{(1-q_o)^3} < 0$, so the unconstrained optimal solution can be obtained below by the first-order-condition, which leads to $q_o = 1 - \sqrt{(c + (1-q_t)^2(1-v))/(1-v)}$. With this solution, the constraint $q_o \geq \frac{q_t - n}{1-n}$ is equivalent to $c \leq c_{\rm GF} \equiv \frac{n(2-n)(1-q_t)^2(1-v)}{(1-n)^2}$. Therefore, we have $q_o \geq \frac{q_t - n}{1-n}$.

$$q_o^{\rm GF} = \begin{cases} 1 - \sqrt{(c + (1-q_t)^2(1-v))/(1-v)}, & \text{if } c \leq c_{\rm GF}, \\ (q_t - n)/(1-n), & \text{if } c > c_{\rm GF}. \end{cases} \tag{C.16}$$

Solution to (C.14). Straightforward calculation yields $\frac{\partial^2 \Pi_{\rm GP}}{\partial q_o^2} = -\frac{2n(c - (1-q_t)q_t)}{(q_t - q_o)^3}$. Thus, when $c - (1-q_t)q_t > 0$, thereby $\frac{\partial^2 \Pi_{\rm GP}}{\partial q_o^2} < 0$, the unconstrained optimal solution can be obtained by the first-order-condition, which leads to $q_o = q_t - \sqrt{(c - (1-q_t)q_t)/(1-v)}$. With this solution, the constraint $0 \leq q_o \leq \frac{q_t - n}{1-n}$ is equivalent to $\underline{c}_{\rm GP} \leq c \leq \bar{c}_{\rm GP}$, where $\underline{c}_{\rm GP} \equiv \frac{(1-q_t)(q_t(1-2n) + n^2(1-(1-q_t)v))}{(1-n)^2}$ and $\bar{c}_{\rm GP} \equiv q_t(1 - vq_t)$. It is also straightforward to show that the requirement $c-(1-q_t)q_t>0$ holds when $c_{\mathrm{GF}}\le c\le \underline{c}_{\mathrm{GP}}$. Hence the concavity of Π_{GP} also implies that the optimal solution is given by

$$q_o^{\mathrm{GP}}=\begin{cases}(q_t-n)/(1-n), & \text{if } c<\underline{c}_{\mathrm{GP}},\\ q_t-\sqrt{(c-(1-q_t)q_t)/(1-v)}, & \text{if } \underline{c}_{\mathrm{GP}}\le c\le \bar{c}_{\mathrm{GP}},\\ 0, & \text{if } c>\bar{c}_{\mathrm{GP}}.\end{cases} \tag{C.17}$$

Solution to (C.15). Straightforward calculation yields $\frac{\partial^2\Pi_{\mathrm{LF}}}{\partial q_o^2}=-\frac{2(c+(1-q_t)^2)}{(1-q_o)^3}<0$. So the unconstrained optimal solution can be obtained by the first-order-condition, which leads to $q_o^*=1-\sqrt{c+(1-q_t)^2}$. With this solution, the constraint $0\le q_o^*\le\frac{q_t-n}{1-n}$ is equivalent to $\underline{c}_{\mathrm{LF}}\le c\le\bar{c}_{\mathrm{LF}}$, where $\underline{c}_{\mathrm{LF}}\equiv\frac{(2-n)n(1-q_t)^2}{(1-n)^2}$ and $\bar{c}_{\mathrm{LF}}\equiv q_t(2-q_t)$. Thus, by the concavity of Π_{LF}, we obtain the following optimal solution for the LF subproblem:

$$q_o^{\mathrm{LF}}=\begin{cases}(q_t-n)/(1-n), & \text{if } c\le\underline{c}_{\mathrm{LF}},\\ 1-\sqrt{c+(1-q_t)^2}, & \text{if } \underline{c}_{\mathrm{LF}}<c\le\bar{c}_{\mathrm{LF}},\\ 0, & \text{if } c>\bar{c}_{\mathrm{LF}}.\end{cases} \tag{C.18}$$

Combining (C.16) and (C.17), we obtain the following joint optimal solution for GF and GP strategies:

$$q_o^{\mathrm{GF-GP}}=\begin{cases}1-\sqrt{(c+(1-q_t)^2(1-v))/(1-v)}, & \text{if } c\le c_{\mathrm{GF}},\\ (q_t-n)/(1-n), & \text{if } c_{\mathrm{GF}}<c\le\underline{c}_{\mathrm{GP}},\\ q_t-\sqrt{(c-(1-q_t)q_t)/(1-v)}, & \text{if } \underline{c}_{\mathrm{GP}}<c\le\bar{c}_{\mathrm{GP}},\\ 0, & \text{if } c>\bar{c}_{\mathrm{GP}}.\end{cases} \tag{C.19}$$

Using (C.18) and (C.19), we compute and compare $\max\{\pi_{\mathrm{GF}}(c,v,n,q_t),\pi_{\mathrm{GP}}(c,v,n,q_t)\}$ with $\pi_{\mathrm{LF}}(c,n,q_t)$. Specifically, solving $\Pi_{\mathrm{LF}}(1-\sqrt{c+(1-q_t)^2})>\Pi_{\mathrm{GF}}((q_t-n)/(1-n))$ yields $c>c_{\mathrm{GF-LF}}\equiv\frac{n(1-q_t)^2(2-n(1-\sqrt{v}))(1+\sqrt{v})}{(1-n)^2}$; solving $\Pi_{\mathrm{LF}}(1-\sqrt{c+(1-q_t)^2})>\Pi_{\mathrm{GP}}(q_t-\sqrt{(c-(1-q_t)q_t)/(1-v)})$ yields $v<\bar{v}\equiv\frac{(4c(1-n^2)-8(1-A)-4(n^2-3-n(1-A)+A)q_t-(5+2n-3n^2)q_t^2)}{4n^2(c-(1-q_t)q_t)}$ where $A\equiv\sqrt{c+(1-q_t)^2}$. It can be shown that $\bar{v}$ intersects with $\bar{c}$ at $(v,c)=(0,q_t-\frac{3q_t^2}{4})$. Therefore LF region exists only if $\frac{\partial\bar{v}}{\partial c}<0$ at $c=q_t-\frac{3q_t^2}{4}$, which is equivalent to $n<\frac{q_t}{2-q_t}$. Similarly, the boundary $n=(2-2\sqrt{1-q_t})/q_t-1$ is obtained by solving for n such that $\underline{c}_{\mathrm{GP}}$ and $c_{\mathrm{GF-LF}}$ intersect at $v=1$. These lead to the following threshold values in the corollary:

(i) For $0\le n\le(2-2\sqrt{1-q_t})/q_t-1$, $\hat{c}=\underline{c}_{\mathrm{GP}}$ and $\check{c}$ is the relevant solution to $\Pi_{GP}((q_t-\sqrt{(c-(1-q_t)q_t)/(1-v)})^+)=\Pi_{LF}(1-\sqrt{c+(1-q_t)^2})$, which is implicitly defined by $v=\bar{v}$.

(ii) For $(2-2\sqrt{1-q_t})/q_t-1\le n<q_t/(2-q_t)$, $\hat{c}=\min\{\underline{c}_{\mathrm{GP}},c_{\mathrm{GF-LF}}\}$ and $\bar{v}$ is given above.

(iii) For $q_t/(2-q_t)\le n<q_t$, $\hat{c}=\underline{c}_{\mathrm{GP}}$.

Subsequently, using the optimal strategy identified above, we can compute the marginal revenue, which leads to (5). Finally, the expressions for q_o^* in Table 2 follow from (C.16)-(C.18); the inequalities on Q_t^* and Q_o^* in Table 2 follow from Lemma 2. □

Appendix D: Proofs in Section 5.1

Proof of Proposition 1. Recall that $\bar{v}$, given in proof of Theorem 2, is the boundary between LF and GP regions and $\hat{c}$ is the boundary between LF and GF regions. Then the co-product positioning can only moves from low-end to green because it can be shown that (1) $\bar{v}$ is concave in c and decreasing in n, and (2) $\hat{c}$ increases in v and n.

For part (a), Table 2 implies that in LF region, $q_o^*=1-\sqrt{c+(1-q_t)^2}$, $Q^*=\frac{1}{1-q_o^*}$ and $S^*=1$, which in turn imply that $\frac{\partial Q^*}{\partial n}=\frac{\partial Q^*q_o^*}{\partial n}=\frac{\partial Q^*/S^*}{\partial n}=\frac{\partial Q^*q_o^*/S^*}{\partial n}=0$ and $\frac{\partial Q^*}{\partial v}=\frac{\partial Q^*q_o^*}{\partial v}=\frac{\partial Q^*/S^*}{\partial v}=\frac{\partial Q^*q_o^*/S^*}{\partial v}=0$.

For part (b), Table 2 implies that in GP region, $q_o^* = \left(q_t - \sqrt{\frac{c-(1-q_t)q_t}{1-v}}\right)^+$, $Q^* = \frac{n}{q_t - q_o^*}$ and $S^* = n + Q^*(1-q_t)$, which in turn imply that $\frac{\partial Q^*}{\partial n} = \max\left\{\frac{1}{q_t}, \sqrt{\frac{1-v}{c-(1-q_t)q_t}}\right\} > 0$ and $\frac{\partial q_o^*}{\partial n} = 0$, suggesting $\frac{\partial Q^* q_o^*}{\partial n} > 0$. Also, it can be shown that $\frac{\partial Q^*}{\partial v} = \frac{n}{(q_t - q_o^*)^2}\frac{\partial q_o^*}{\partial v}$, and $\frac{\partial q_o^*}{\partial v} \le 0$ implies that $\frac{\partial Q^*}{\partial v} \le 0$, suggesting $\frac{\partial Q^* q_o^*}{\partial v} \le 0$. Also, we have $Q^*/S^* = \frac{1}{1-q_o^*}$, which implies that that $\frac{\partial Q^*/S^*}{\partial n} = 0$, suggesting $\frac{\partial Q^* q_g^*/S^*}{\partial n} = 0$. Furthermore it can be shown that $\frac{\partial Q^*/S^*}{\partial v} = \frac{1}{(1-q_o^*)^2}\frac{\partial q_o^*}{\partial v} \le 0$ and $\frac{\partial Q^* q_o^*/S^*}{\partial v} = \frac{\partial q_o^*}{\partial v}\frac{1}{1-q_o^*} + q_o^* \frac{\partial Q^*/S^*}{\partial v} \le 0$. Following similar approach, we can demonstrate the monotonicity of all environmental metrics in GF region.

For part (c), we hereby only identify the condition for the upward discontinuity of the total consumption Q^* in v; all other properties can be established by following a similar argument. We note that the total consumption is $Q_{LF} = 1/\sqrt{c + (1-q_t)^2}$ under LF strategy, $Q_{GF} = \frac{1}{1-q_o^*}$ under GF strategy, and $Q_{GP} = \frac{n}{q_t - q_o^*}$ under GP strategy, where q_o^* is provided correspondingly in Table 2. First, we consider the comparison between Q_{LF} and Q_{GF} along the boundary $\hat{c}$ between LF and GF regions. It can be shown that (1) the boundary $\hat{c}$ between LF and GF increases in v, (2) when $c = \hat{c}|_{v=0} = \frac{(2-n)n(1-q_t)^2}{(1-n)^2}$ we have $Q_{GF} > Q_{LF}$, and (3) $\frac{\partial(Q_{GF} - Q_{LF})}{\partial c} > 0$. Thus, we must have $Q_{GF} > Q_{LF}$ along the entire LF-GF boundary. Second, we consider the comparison between Q_{LF} and Q_{GP}. By Theorem 2, for $n \le \left(2 - 2\sqrt{1-q_t}\right)/q_t - 1$, LF and GP regions share boundary $\check{c}$, along which it can be shown that $Q_{LF} > Q_{GP}$. For $\left(2 - 2\sqrt{1-q_t}\right)/q_t - 1 \le n < q_t/(2-q_t)$, LF and GP regions share boundary $\bar{v}$, which, as a function of c, reaches its maximum at $c = \theta_v(q_t, n) \equiv \frac{(1+n)(1-q_t)^2 q_t (4 - q_t - n q_t)}{(2 - q_t - n q_t)^2}$. It can be verified that $Q_{LF} < Q_{GF}$ along the boundary $\bar{v}$ if $c < \theta_v(q_t, n)$. $\square$

Proof of Proposition 2. Consumer surplus is zero in GF and GP regions because $p_t^* = q_t$ which is equal to the utility a traditional consumer would earn and $p_o^* = q_o + v(q_t - q_o) =$ which is equal to the utility a green consumer would earn in those cases. In LF region, we have $p_t^* = q_t$ so the traditional consumers earn zero utility, but $p_o^* = q_o^*$ which implies that a green consumer earns a utility $q_o^* + v(q_t - q_o^*) - p_o^* = v(q_t - q_o^*) > 0$, hence the total consumer surplus is $nv(q_t - q_o^*) > 0$. Notice that $q_o^* = 1 - \sqrt{c + (1-q_t)^2}$ which is independent in v and n. Also since $q_t - q_o^* > 0$ by definition, it proceeds that $nv(q_t - q_o^*)$ increases in both n and v. With the optimal decision given in Theorem 2, it is straightforward to show that $\frac{\partial \Pi^*}{\partial n} \ge 0$ and $\frac{\partial \Pi^*}{\partial v} \ge 0$. $\square$

Appendix E: Proofs in Section 5.2

In this subsection, we first characterize the firm's optimal product line strategy under binding quota constraint, i.e., material quantity is given by $\bar{Q}$. The pricing decision characterized by Lemma C.1 still holds. Thus, given the the co-product's quality $\bar{q}_o$ and price $\bar{p}_o$, the firm's profit function is given by

$$\pi(\bar{q}_o, \bar{p}_o) = \bar{Q}\left[(1-q_t)q_t + (q_t - \bar{q}_o)\bar{p}_o - c\right]. \tag{E.1}$$

Following similar argument as in Lemma 2, the lemma below derives three subproblems for the firm.

LEMMA E.1. *Under binding quota $\bar{Q}$, the firm enters the market if and only if $c \le (1-q_t)q_t + (q_t - \bar{q}_o)\bar{p}_o$, whereby the optimal $(\bar{q}_o, \bar{p}_o)$ must solve one of the following three subproblems that yields the highest profit:*

$$\text{If } \bar{Q} \ge \tfrac{1-n}{1-q_t}, \text{ then } \quad \max \pi(\bar{q}_o, \bar{p}_o), \text{ subject to } \left(1 - 1/\bar{Q}\right)^+ \le \bar{q}_o \le q_t \text{ and } \bar{p}_o = \bar{q}_o + v(q_t - \bar{q}_o); \tag{$\bar{S}_1$}$$

$$\text{If } \bar{Q} \le \tfrac{1-n}{1-q_t}, \text{ then } \quad \max \pi(\bar{q}_o, \bar{p}_o), \text{ subject to } \left(q_t - n/\bar{Q}\right)^+ \le \bar{q}_o \le q_t \text{ and } \bar{p}_o = \bar{q}_o + v(q_t - \bar{q}_o); \tag{$\bar{S}_2$}$$

$$\text{If } \bar{Q} \le \tfrac{1-n}{1-q_t}, \text{ then } \quad \max \pi(\bar{q}_o, \bar{p}_o), \text{ subject to } \left(1 - 1/\bar{Q}\right)^+ \le \bar{q}_o \le \left(q_t - n/\bar{Q}\right)^+ \text{ and } \bar{p}_o = \bar{q}_o. \tag{$\bar{S}_3$}$$

Clearly, each subproblem reduces to a constrained optimization problem with a single decision variable $\bar{q}_o$ and hence can be solved following standard procedure. The lemma below documents their solutions.

Lemma E.2. *The optimal co-product's quality $\bar{q}_o^{(i)}$ and price $\bar{p}_o^{(i)}$ of subproblem ($\bar{S}_i$) is given by*

$$\bar{q}_o^{(1)} = \begin{cases} 1-1/\bar{Q}, & \text{if } \bar{Q} \geq \max\left\{\frac{2(1-v)}{2(1-v)-q_t(1-2v)}, \frac{1-n}{1-q_t}\right\}, \\ \frac{q_t(1-2v)}{2(1-v)}, & \text{if } \frac{1-n}{1-q_t} \leq \bar{Q} \leq \frac{2(1-v)}{2(1-v)-q_t(1-2v)}, \end{cases} \quad \text{and} \quad \bar{p}_o^{(1)} = \bar{q}_o^{(1)} + v(q_t - \bar{q}_o^{(1)}); \tag{E.2}$$

$$\bar{q}_o^{(2)} = \begin{cases} q_t - n/\bar{Q}, & \text{if } \frac{2n(1-v)}{q_t} \leq \bar{Q} \leq \frac{1-n}{1-q_t}, \\ \frac{q_t(1-2v)}{2(1-v)}, & \text{if } 0 \leq \bar{Q} \leq \frac{2n(1-v)}{q_t}, \end{cases} \quad \text{and} \quad \bar{p}_o^{(2)} = \bar{q}_o^{(2)} + v(q_t - \bar{q}_o^{(2)}) \tag{E.3}$$

$$\bar{q}_o^{(3)} = \begin{cases} 1-1/\bar{Q}, & \text{if } \frac{2}{2-q_t} \leq \bar{Q} \leq \frac{1-n}{1-q_t}, \\ q_t/2, & \text{if } \frac{2n}{q_t} \leq \bar{Q} \leq \frac{2}{2-q_t}, \\ q_t - n/\bar{Q}, & \text{if } \frac{n}{q_t} \leq \bar{Q} \leq \min\left\{\frac{2n}{q_t}, \frac{1-n}{1-q_t}\right\}, \\ 0, & \text{if } 0 \leq \bar{Q} \leq \frac{n}{q_t}, \end{cases} \quad \text{and} \quad \bar{p}_o^{(3)} = \bar{q}_o^{(3)}. \tag{E.4}$$

By comparing the optimal profits of the three subproblems, we obtain the global optimal co-product's quality and price in the next proposition.

Proposition E.1. *If $\bar{Q} \geq \frac{1-n}{1-q_t}$, the optimal co-product is given by (E.2), which corresponds to full coverage of the market if $\bar{Q} \geq \max\left\{\frac{2(1-v)}{2(1-v)-q_t(1-2v)}, \frac{1-n}{1-q_t}\right\}$ and to partial coverage if $\frac{1-n}{1-q_t} \leq \bar{Q} \leq \frac{2(1-v)}{2(1-v)-q_t(1-2v)}$. If $\bar{Q} < \frac{1-n}{1-q_t}$ and $\frac{1-n}{1-q_t} \leq \frac{2n}{q_t}$, the optimal co-product is given by (E.3), corresponding to partial coverage. If $\bar{Q} < \frac{1-n}{1-q_t}$ and $\frac{1-n}{1-q_t} > \frac{2n}{q_t}$, the optimal co-product is given by (E.4) for $\min\left\{\frac{2n(1+\sqrt{v})}{q_t}, \frac{B-\sqrt{B^2-4AC}}{2A}\right\} \leq \bar{Q} \leq \frac{B+\sqrt{B^2-4AC}}{2A}$ and otherwise by (E.3), where $A \equiv 1-q_t$, $B \equiv 2-(1+n)q_t$ and $C \equiv 1-n^2(1-v)$. For $\bar{Q} \leq 2/(2-q_t)$, (E.4) corresponds to partial coverage; otherwise, (E.4) corresponds to full coverage.*

Proof of Proposition E.1. When $\bar{Q} \geq \frac{1-n}{1-q_t}$, only ($\bar{S}_1$) is feasible and hence the optimal co-product is given by (E.2). For $\bar{Q} \leq \frac{1-n}{1-q_t}$, both ($\bar{S}_2$) and ($\bar{S}_3$) are feasible and the optimal co-product is determined by the one that yields higher profit. Given $\bar{Q}$, to compare the profits between ($\bar{S}_2$) and ($\bar{S}_3$), it suffices to only compare the revenues generated by the co-product, which we respectively denote as $\Pi_o^2(\bar{q}_o) = \bar{Q}(q_t - \bar{q}_o)[\bar{q}_o + v(q_t - \bar{q}_o)]$ and $\Pi_o^3(\bar{q}_o) = \bar{Q}(q_t - \bar{q}_o)\bar{q}_o$. In particular, we note that

$$\Pi_o^2(\bar{q}_o) \geq \Pi_o^3(\bar{q}_o) \text{ for all } \bar{q}_o. \tag{E.5}$$

- When $\frac{1-n}{1-q_t} \leq \frac{2n}{q_t}$, we have, by (E.4), $\bar{q}_o^{(3)} = \left(q_t - n/\bar{Q}\right)^+$. Hence, $\Pi_o^2\left(\bar{q}_o^{(2)}\right) \geq \Pi_o^2\left(\left(q_t - n/\bar{Q}\right)^+\right) \geq \Pi_o^3\left(\left(q_t - n/\bar{Q}\right)^+\right)$, where the first inequality follows from the optimality of (E.4) and the second one from (E.5). Therefore, the optimal co-product is given by (E.3).
- When $\frac{1-n}{1-q_t} > \frac{2n}{q_t}$, similar to the last case, the optimal co-product is given by (E.3) for $\bar{Q} \in \left[0, \frac{2n}{q_t}\right]$. For $\bar{Q} \in \left[\frac{2n}{q_t}, \frac{1-n}{1-q_t}\right]$, (E.3) implies that $\Pi_o^2(\bar{q}_o^{(2)}) = n\left[q_t - (1-v)n/\bar{Q}\right]$, while (E.4) implies that

$$\Pi_o^3(\bar{q}_o^{(3)}) = \begin{cases} \bar{Q}q_t^2/4, & \text{if } \bar{Q} \in \left[\frac{2n}{q_t}, \frac{2}{2-q_t}\right], \\ \left(\bar{Q}-1\right)\left(q_t - 1 + 1/\bar{Q}\right) & \text{if } \bar{Q} \in \left[\frac{2}{2-q_t}, \frac{1-n}{1-q_t}\right]. \end{cases}$$

A direct comparison reveals that $\Pi_o^2(\bar{q}_o^{(2)}) \leq \Pi_o^3(\bar{q}_o^{(3)})$ if and only if $\min\left\{\frac{2n(1+\sqrt{v})}{q_t}, \frac{B-\sqrt{B^2-4AC}}{2A}\right\} \leq \bar{Q} \leq \frac{B+\sqrt{B^2-4AC}}{2A}$, hence concluding the proof.

The demand fulfillment patterns under (E.2)-(E.4) follow from straightforward verification. □

Proof of Proposition 3. (a) We first demonstrate the monotonicity of each environmental metric $P \in \{Q^*, Q^* q_o^*, Q^*/S^*, Q^* q_o^*/S^*\}$ in c, which follows by showing $\frac{\partial P}{\partial c} \le 0$ within each strategy region. For instance, in LF region we have $\frac{\partial Q^*}{\partial c} = -\frac{1}{2(\sqrt{c+(1-q_t)^2})^3} < 0$. The discontinuity of $P \in \{Q^*, Q^* q_o^*, Q^*/S^*, Q^* q_o^*/S^*\}$ between LF and GP regions follows from Proposition 1. To demonstrate the monotonicity of each P in $\bar{Q}$, we first note that total material consumption is simply $\bar{Q}$, which is increasing in $\bar{Q}$. By (E.2)-(E.4), the total waste $\bar{Q} q_o^{(i)}$ is also increasing in $\bar{Q}$ within each strategic region. Since the total units sold is given by $\bar{S}^* = \bar{Q}(1 - \bar{q}_o^{(i)})$, the per-unit consumption and waste are given by $\bar{Q}/\bar{S}^* = 1/(1 - \bar{q}_o^{(i)})$ and $\bar{Q} q_o^{(i)}/\bar{S}^* = \bar{q}_o^{(i)}/(1 - \bar{q}_o^{(i)})$, respectively. By (E.2)-(E.4), it is then straightforward to verify that they are increasing in $\bar{Q}$ within each strategic region. As $\bar{Q}$ decreases, the optimal strategy changes from (E.2) to (E.3), then to (E.4), and finally back to (E.3). The corresponding optimal co-product's quality decreases in all switching places except for the last one, i.e., from (E.4) to (E.3), which occurs when $\bar{Q} \in [2n/q_t, 2/(2-q_t)]$. At this switch, the co-product's quality changes from $\bar{q}_o^{(3)} = q_t/2$ to $\bar{q}_o^{(2)} = q_t - n/\bar{Q}$, and it is straightforward to show that $q_t - n/\bar{Q} \ge q_t/2$ (i.e., an upward jump) for $\bar{Q} \ge 2n/q_t$.

(b) In the absence of quota, Proposition 2 implies positive consumer surplus in LF region, which is given by $nv(q_t - q_o^*) = nv\left(\sqrt{c + (1-q_t)^2} - (1-q_t)\right)$ and is increasing in c. Under quota, only when the firm prices the co-product as a low-end product (i.e., $\bar{p}_o^{(3)} = \bar{q}_o^{(3)}$), the consumer surplus is positive and is given by $nv\left(q_t - \bar{q}_o^{(3)}\right)$. By Proposition E.1, when (E.4) corresponds to full coverage, the optimal co-product's quality is $1 - 1/\bar{Q}$ and hence the consumer surplus is $nv(q_t - 1 + 1/\bar{Q})$, increasing in $\bar{Q}$; when (E.4) corresponds to partial coverage, the optimal co-product's quality is $q_t/2$ and hence the consumer surplus is $nvq_t/2$, independent of $\bar{Q}$.

(c) The monotonicity of firm's profit in c and $\bar{Q}$ is straightforward. □

Proof of Proposition 4. By Proposition E.1, the optimal co-product's quality in GP region is given by $\bar{q}_o^{(2)}$ in (E.3). In GP region, the total material consumption is $\bar{Q}$, the traditional product's revenue is $\bar{Q}\,(1-q_t)\,q_t$, and the total material cost is $c\bar{Q}$, which are all independent of n or v. Thus, to show the monotonicity of the firm's profit, we just need to examine the co-product's revenue, which is given by

$$\Pi_o^2\left(\bar{q}_o^{(2)}\right) = \bar{Q}\left(q_t - \bar{q}_o^{(2)}\right)\left[\bar{q}_o^{(2)} + v\left(q_t - \bar{q}_o^{(2)}\right)\right] = \begin{cases} n\left[q_t - (1-v)n/\bar{Q}\right], & \text{if } \frac{2n(1-v)}{q_t} \le \bar{Q} \le \frac{1-n}{1-q_t}, \\ \frac{q_t^2 \bar{Q}}{4(1-v)}, & \text{if } 0 \le \bar{Q} \le \frac{2n(1-v)}{q_t}, \end{cases}$$

which can be verified to be weakly increasing in n or v. Next, we compute the per-unit consumption

$$1/(1-\bar{q}_o^{(2)}) = \begin{cases} \frac{\bar{Q}}{(1-q_t)\bar{Q}+n}, & \text{if } \frac{2n(1-v)}{q_t} \le \bar{Q} \le \frac{1-n}{1-q_t}, \\ \frac{2(1-v)}{2(1-v)-(1-2v)q_t}, & \text{if } 0 \le \bar{Q} \le \frac{2n(1-v)}{q_t}, \end{cases}$$

which can be verified to be weakly decreasing in n or v. Similarly, we compute the total waste

$$\bar{Q}\bar{q}_o^{(2)} = \begin{cases} \bar{Q}q_t - n, & \text{if } \frac{2n(1-v)}{q_t} \le \bar{Q} \le \frac{1-n}{1-q_t}, \\ \frac{\bar{Q}q_t(1-2v)}{2(1-v)}, & \text{if } 0 \le \bar{Q} \le \frac{2n(1-v)}{q_t}, \end{cases}$$

which again can be verified to be weakly decreasing in n or v. Finally, we compute the per-unit waste

$$\bar{Q}\bar{q}_o^{(2)}/(1-\bar{q}_o^{(2)}) = \begin{cases} \frac{\bar{Q}q_t - n}{(1-q_t)\bar{Q}+n}, & \text{if } \frac{2n(1-v)}{q_t} \le \bar{Q} \le \frac{1-n}{1-q_t}, \\ \frac{q_t(1-2v)}{2(1-v)-(1-2v)q_t}, & \text{if } 0 \le \bar{Q} \le \frac{2n(1-v)}{q_t}, \end{cases}$$

which can also be verified to be weakly decreasing in n or v. □

Appendix F: Proofs in Section B

Proof of Proposition B.1. We first note that the constraint $Q_o = 1 - G^\alpha(v_o) = 1 - v_o^\alpha$ is equivalent to

$$q_o = q_t - (1 - v_o^\alpha)/Q \geq 0, \tag{F.1}$$

which implies that $Q \geq (1 - v_o^\alpha)/q_t$ and

$$p_o = q_o + v_o(q_t - q_o) = q_t - (1 - v_o)(1 - v_o^\alpha)/Q. \tag{F.2}$$

Substituting (F.1) and (F.2) into the objective function of (6) reduces it to

$$[q_t - (1 - v_o)(1 - v_o^\alpha)/Q](1 - v_o^\alpha) + q_t \min\{(1 - q_t)Q, v_o^\alpha\} - cQ,$$

which further reduces to $q_t - (1 - v_o)(1 - v_o^\alpha)/Q - cQ$ and is obviously increasing in v_o for $v_o^\alpha \leq (1 - q_t)Q$. Hence, the optimal solution to (6) must lie within $v_o^\alpha \geq (1 - q_t)Q$, which reduces the seller's problem (6) to

$$\max_{v_o \in \left[(1-q_t)^{1/\alpha}, 1\right], (1-v_o^\alpha)/q_t \leq Q \leq v_o^\alpha/(1-q_t)} \Psi(v_o, Q) = q_t(1 - v_o^\alpha) - (1 - v_o)(1 - v_o^\alpha)^2/Q + [q_t(1 - q_t) - c]Q. \tag{F.3}$$

Here, in order to (F.3) to be feasible, we require $v_o \geq (1 - q_t)^{1/\alpha}$ so that $(1 - v_o^\alpha)/q_t \leq v_o^\alpha/(1 - q_t)$.

To solve (F.3), we examine the following cases:

- Suppose $c \leq q_t - q_t^2$. Then, $\Psi(v_o, Q)$ is increasing in Q, which implies that $\Psi(v_o, Q)$ is maximized at $Q = v_o^\alpha/(1 - q_t)$ to $\Psi(v_o, v_o^\alpha/(1 - q_t)) = \psi_1(v_o)$, whose maximum must be nonnegative because $\psi_1(1) = q_t - c/(1 - q_t) \geq 0$. Therefore, we obtain the first result in the proposition.
- Suppose $c > q_t - q_t^2$. For any given v_o, it is straightforward to verify that $\Psi(v_o, Q)$ is increasing in $Q \leq (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}$ and is decreasing in $Q \geq (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}$. Thus, we further divide this case into the two parameter ranges.

1. For $q_t - q_t^2 < c \leq q_t - q_t^2(1 - q_t)^{1/\alpha}$, we note that function $(1 - v_o^\alpha)/v_o^\alpha\sqrt{1 - v_o}$ is monotonically decreasing in v_o, taking value $q_t/(1 - q_t)\sqrt{1 - (1 - q_t)^{1/\alpha}} \geq \sqrt{c - q_t + q_t^2}/(1 - q_t)$ at $v_o = (1 - q_t)^{1/\alpha}$ and taking value smaller than $\sqrt{c - q_t + q_t^2}/(1 - q_t)$ at $v_o = (q_t - c)/q_t^2$. Thus, there exists a unique solution $\bar{v}_o \in \left[(1 - q_t)^{1/\alpha}, (q_t - c)/q_t^2\right]$ to (B.4) such that

$$v_o \leq \bar{v}_o \quad \Leftrightarrow \quad v_o^\alpha/(1 - q_t) \leq (1 - v_o^\alpha)\sqrt{\frac{1 - v_o}{c - q_t + q_t^2}}.$$

Hence,

—for $v_o \in \left[(1 - q_t)^{1/\alpha}, \bar{v}_o\right]$, we have $\Psi(v_o, Q)$ increasing in $Q \in [(1 - v_o^\alpha)/q_t, v_o^\alpha/(1 - q_t)]$, which implies that $\Psi(v_o, Q)$ is maximized at $Q = v_o^\alpha/(1 - q_t)$ to $\Psi(v_o, v_o^\alpha/(1 - q_t)) = \psi_1(v_o)$;

—for $v_o \in [\bar{v}_o, (q_t - c)/q_t^2]$, we have $(1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}} \in [(1 - v_o^\alpha)/q_t, v_o^\alpha/(1 - q_t)]$, and hence $\Psi(v_o, Q)$ is maximized at $Q = (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}$ to $\Psi\left(v_o, (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}\right) = \psi_2(v_o)$;

—and for $v_o \in [(q_t - c)/q_t^2, 1]$, we must have $(1 - v_o^\alpha)/q_t \geq (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}$, which implies that $\Psi(v_o, Q)$ is maximized at $Q = (1 - v_o^\alpha)/q_t$ to $\Psi(v_o, (1 - v_o^\alpha)/q_t) = \psi_3(v_o)$,whose maximum must be nonnegative because $\psi_3(1) = 0$.

Therefore, we obtain the second result in the proposition. In particular, we note that

2. Suppose $c > q_t - q_t^2(1 - q_t)^{1/\alpha}$. It is straightforward to verify that $(1 - v_o^\alpha)/q_t > (1 - v_o^\alpha)\sqrt{\frac{1-v_o}{c-q_t+q_t^2}}$ for all $v_o \in \left[(1 - q_t)^{1/\alpha}, 1\right]$. Therefore, $\Psi(v_o, Q)$ is maximized at $Q = (1 - v_o^\alpha)/q_t$ to $\Psi(v_o, (1 - v_o^\alpha)/q_t) = \psi_3(v_o)$, whose maximum again must be nonnegative because $\psi_3(1) = 0$. In particular, if $c \geq q_t$, we must have $\psi_3(v_o) < 0$ for all $v_o < 1$, because $1 - v_o^\alpha > 0$ and $v_o - (c - q_t + q_t^2)/q_t^2 < 0$. Therefore, for $c \geq q_t$, the optimal $v_o^* = 1$ and $Q^* = 0$, i.e., the firm stops entering production. $\square$