An Introduction to Inverse Optimization

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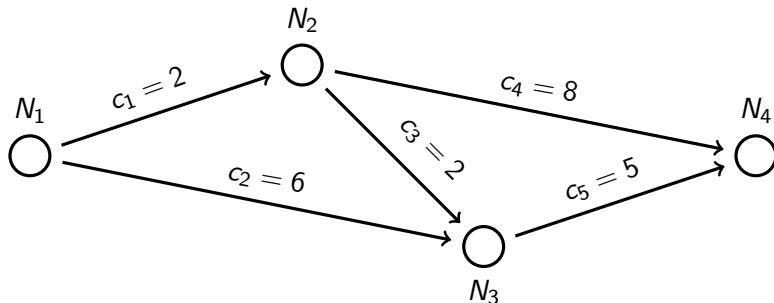Concordia University

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Combinatorial Optimization Problems
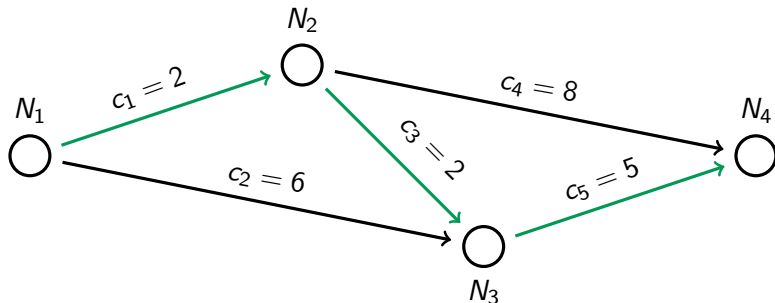
Routing Problem

GIVEN   a network   · · · · ·   CONSTRAINTS

Combinatorial Optimization Problems

Routing Problem

GIVEN   a network   ⋯⋯   CONSTRAINTS
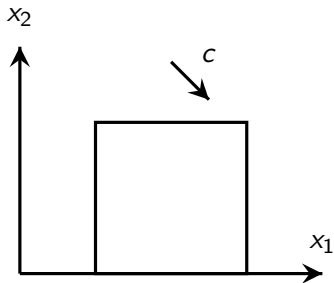 FIND   a path with the least Cost   ⋯⋯   OBJECTIVE

Modelling Optimization Problems

(Forward) Optimization Problem

$$(FOP) \quad \underset{\mathbf{x}}{\text{minimize}} \quad c^{'}x \qquad \text{Objective}$$

$$\text{subject to} \quad Ax \leq b \qquad \text{Constraints}$$
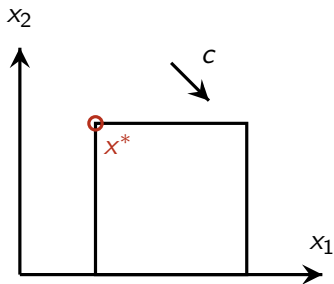
GEOMETRIC INTERPRETATION

Forward Optimization Problem

$$(FOP) \quad \underset{\mathbf{x}}{\text{minimize}} \quad x_1 - x_2$$
$$\text{subject to} \quad 0 \leq x_1 \leq 1$$
$$0 \leq x_2 \leq 1$$

GEOMETRIC INTERPRETATION

Forward Optimization Problem

$$(FOP) \quad \underset{\mathbf{x}}{\text{minimize}} \quad x_1 - x_2$$
$$\text{subject to} \quad 0 \leq x_1 \leq 1$$
$$0 \leq x_2 \leq 1$$

What is Inverse Optimization (IO)?

Forward Optimization Problem

$$(FOP) \quad \underset{\mathbf{x}}{\text{minimize}} \quad c'x$$
$$\text{subject to} \quad Ax \leq b,$$

An Example of IO
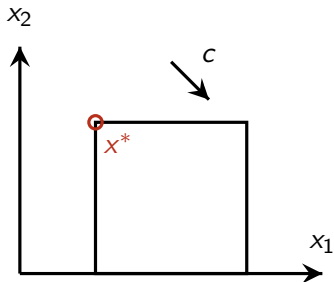
Given   Constraints *i.e.*, **A**, **b**

Find   Cost Vector c

WHAT IS INVERSE OPTIMIZATION (IO)?

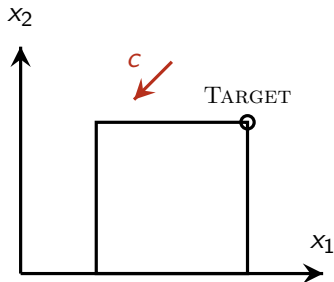Forward Optimization Problem

(FOP) $\quad \underset{\mathbf{x}}{\text{minimize}} \quad c'x$

$\quad$ subject to $\quad Ax \leq b,$

An Example of IO

Given $\quad$ Constraints $i.e.,$ $\mathbf{A}, \mathbf{b}$

Find $\quad$ Cost Vector c

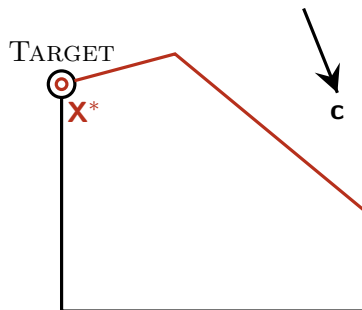# Variants of IO Tasks

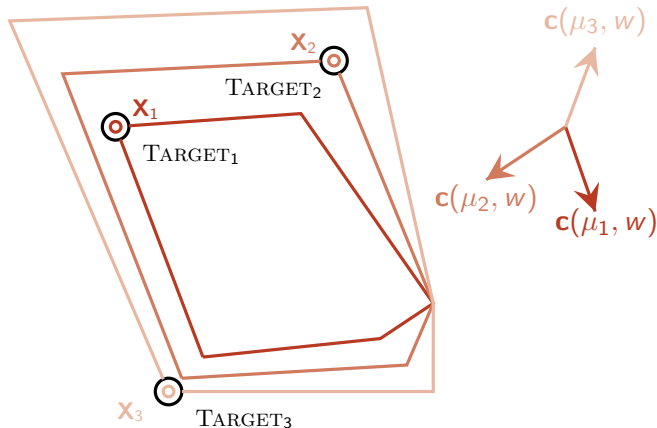Learn Cost Vector **c**
consistent with Target
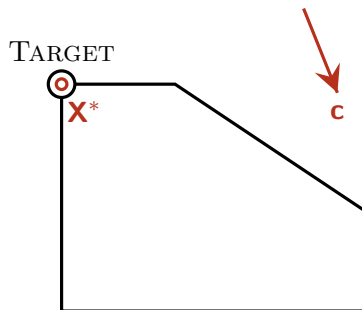
Variants of IO Tasks

Learn Constraints **A**, **b**
consistent with Target

VARIANTS OF IO TASKS

LEARN WEIGHTS w OF PARAMETRIC PROBLEMS
$\mathbf{c}(\mu, w)$, $\mathbf{A}(\mu, w)$, $\mathbf{b}(\mu, w)$
CONSISTENT WITH TARGET

FURTHER READING I

To learn more about Inverse Optimization, please see the following papers:

- OVERVIEW OF INVERSE OPTIMIZATION

  Ravindra K Ahuja and James B Orlin. "Inverse optimization". In: *Operations Research* 49.5 (2001), pp. 771–783

  T. C. Y. Chan, T Lee, and D. Terekhov. "Goodness of Fit in Inverse Optimization". In: *Management Science* (2018)

- SINGLE OBSERVATIONS INVERSE OPTIMIZATION

  M. D. Troutt et al. "Linear programming system identification: The general nonnegative parameters case". In: *European Journal of Operational Research* 185.1 (2008), pp. 63–75

  M. D. Troutt et al. "Linear programming system identification: The general nonnegative parameters case". In: *European Journal of Operational Research* 185.1 (2008), pp. 63–75

  Timothy C Y Chan et al. "Multiple Observations and Goodness of Fit in Generalized Inverse Optimization". In: *arXiv preprint arXiv:1804.04576* (2018)

FURTHER READING II

Timothy C Y Chan and Neal Kaw. "Inverse optimization for the recovery of constraint parameters". In: *arXiv preprint arXiv:1811.00726* (2018)

- PARAMETRIC INVERSE OPTIMIZATION

A. Keshavarz, Y. Wang, and S. Boyd. "Imputing a convex objective function". In: *2011 IEEE International Symposium on Intelligent Control*. IEEE. 2011, pp. 613–619

Javier Saez-Gallego and Juan Miguel Morales. "Short-term forecasting of price-responsive loads using inverse optimization". In: *IEEE Transactions on Smart Grid* (2017)

Anil Aswani, Zuo-Jun Shen, and Auyon Siddiq. "Inverse optimization with noisy data". In: *Operations Research* 63.3 (2018)

THANK YOU !