Predator Prey Relations: Team 18

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I. Introduction

Predator and prey relationships are a common occurrence in ecology. When one population grows at a faster rate than the other, it spells the demise for both. When the predator population grows at a much faster rate than the prey population, prey do not have a chance to properly reproduce causing prey to see a sharp decline after a certain amount of time. When prey is uncontested, their rate of population growth increases exponentially causing a spike in the consumption of local flora that may disrupt the ecology of other species in that area. As the flora can be exhausted after a certain point, it is not ideal to have uncontested prey.

With the introduction of coyote populations into the southeastern United States, white-tailed deer population have seen a steady decline. Before the coyote, white-tailed deer were becoming overpopulated due to the extinction of their natural predator, the red wolf, in this region. With the introduction of the coyote, over the years this population control has quickly destabilized with an unforeseen increase in coyote population that has outstripped the previous deer population. With this current trend, the white-tailed deer population in this region may face further decline with significant environmental detriments in future years. The question becomes what is a feasible solution to create white-tailed deer to coyote stability in this region? The two solutions outlined in [1] include directly removing coyote through culling and altering the harvesting rate of white-tailed deer offspring.

As both players would like to see long term growth, a balance must be found between the two populations. To analyze such relations more closely we have chosen to observe the predator prey model for white-tailed deer and coyote of the southeastern United States using several distinct methods of game theory. We started with strategic form models to establish several baseline assumptions and to determine potential payoffs for particular strategies. We then chose to analyze an Evolutionary game in order to model competing strategies within the game's structure. Finally, we established a Lotka-Volterra model that we used to consider long term population fluctuations, and how the impact of one population would affect the other. These models allows us to view the effects of each player much more closely while also allowing players to take more nuanced strategies.

II. Models

White-tailed deer and coyote populations in America are the focus of the model we have chosen to represent. White-tailed deer populations (particularly female fawns) are commonly preyed upon by coyotes resulting in decreasing vital rates. Vital rates pertain to the speed of change in a specific population; by tracking the vital rates of the deer, it becomes possible to assume resulting shifts in deer population. Coyote predation of neonates and fawns reduces deer population directly. With the intention of stabilizing white-tailed deer populations, specific wildlife organizations have made attempts to improve deer vital rates through direct interference. In order to analyze this relationship, we have generated models consisting of coyote populations and wildlife management as players. The motivation of the coyote population is to increase in size by preying upon the white-tailed deer population, while (conversely) the motivation of the wildlife managers is to increase the vital rate of white-tailed deer through several methods. Coyote have the options to prey at differing rates. Wildlife managers have the options to leave everything as it is, reduce the harvest of female white tailed deer through hunting regulations, increase coyote harvest via trapping or hunting, or to do both active strategies. These strategies can be generalized into being less costly, less effective strategies or more costly, more effective strategies. Because these relations cannot be simplified into a single model we look at the models of two iterations of strategic form games, two iterations of evolutionary games, and a Lotka-Volterra model accompanied by a simulation.

III. Strategic Form Game

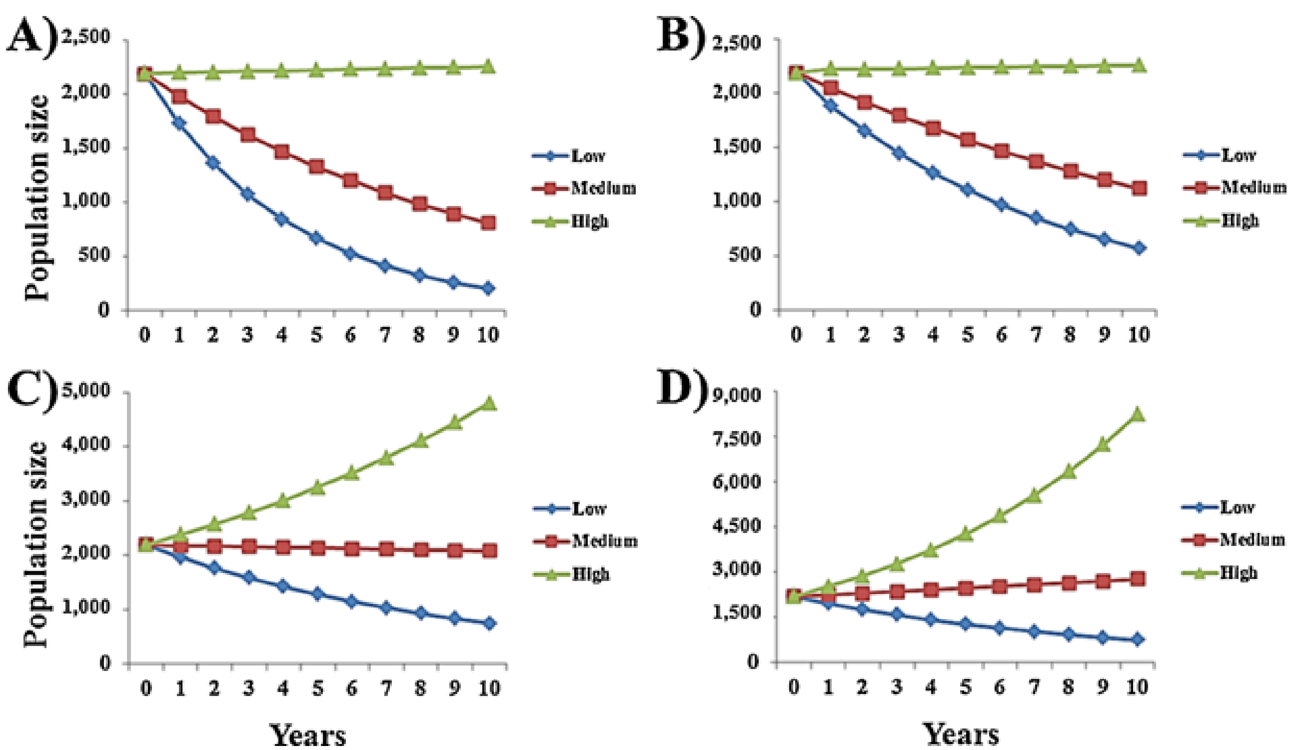

Fig. 1. "Predicted white-tailed deer population sizes after 10 years of A) current vital rates, B) reduced female harvest, C) coyote removal, and D) reduced female harvest combined with coyote removal at Fort Bragg Military Installation, North Carolina, USA. Predicted population sizes are based on low, medium, and high vital rates, where medium values represent mean predictions."[1]

A. Strategic Form Model

For the strategic form model the players of the system will be wildlife management and the coyote population.

Wildlife management has four primary strategies:

- Do Nothing [DN]
- Reduce female deer harvest [RH]
- Increase coyote harvest rates [RC]
- Both reduce female deer harvest and increase coyote harvest [B]

The coyote population has three primary strategies:

- Extreme Predation of deer [EP]
- Natural Predation of deer [NP]
- Relaxed Predation of deer [RP]

Wildlife management benefits most when using the combination of both strategies available while the coyote population benefits the most from when they are able to prey on white-tailed deer without constraint. These strategies are derived from Table I below.

	DN	**RH**	**RC**	**B**
EP	(**12**,1)	(**11**,2)	(**10**,3)	(**9**,**4**)
NP	(8,5)	(7,6)	(6,7)	(5,**8**)
RP	(4,9)	(3,10)	(2,11)	(1,**12**)

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TABLE I
PAYOFF OF COYOTE VS WILDLIFE MANAGEMENT

Table I represents the model for a strategic form game derived from Figure 1. The values included are not the payoffs for the individuals and are instead the priority of each strategy. The priority of each strategy is determined by associating the population difference in white-tailed deer after a projected 10 year span of implementing a specific strategy. As the payoff priority for both players relates directly to this population size, individual payoffs appear in inverse of each other.

B. Strategic Form Model Analysis

From Table I an obvious Nash equilibrium is found where both players choose the most intensive strategies. Wildlife management chooses to reduce female deer harvest and increase coyote removal. The coyote population chooses to prey on deer without constraint. In this scenario neither side obtains a relatively large or small payout. They compromise towards the median payout of their actions.

The cause of this relatively simplistic outcome is due to the fact that the payoff for each player has been simplified to a direct relationship based on the change in population determined by each strategy outlined in [1]. This outcome presupposes that the payoff of the wildlife management cannot be negatively impacted due to the expenditure of resources in a less prevalent scenario (for example, imposing more costly actions while coyote population hunts at a less than standard rate).

IV. Modified Strategic Form Game

A. Modified Strategic Form Model

Table II represents the changes to payoffs accounting for the resources spent by each player to perform those actions. In the coyote population, extreme predation saw a moderate resource cost due to how

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	DN	**RH**	**RC**	**B**
EP	(-2,0)	(-2,-1)	(-2,-1)	(-2,-2)
NP	(0,0)	(0,-2)	(0,-3)	(0,-5)
RP	(-1,0)	(-1,-3)	(-1,-5)	(-1,-8)

TABLE II
Changes from Resources Spent

overtime constant extreme predation leads to a much smaller population of prey both white-tailed deer and otherwise. Standard predation leads to no change in the current situation. Relaxed predation of white-tailed deer leads to a dependence on harder to catch prey that requires more resources for the same amount of food. For wildlife management doing nothing expends no resources to start with so there is no change in payoff. Reducing female deer harvest affects the payout in proportion to the deer saved. When coyote predate on deer less, there are more deer causing efforts for reducing female harvest being much more frequent when there are more deer in general. Removing coyote has a large cost when coyote are not active as this is wasted resources spent for unneeded removal. The combination of reducing female deer harvest and removing coyote is for now the sum of resources spent of each alone. Table III represents the modified table. If these tables were represented as a matrix it would shown as $I + II = III$. By eliminated strictly dominated strategies the strategy matrix becomes what is seen in Table IV.

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	DN	**RH**	**RC**	**B**
EP	(**10**,1)	(**9**,1)	(**8**,**2**)	(**7**,**2**)
NP	(8,**5**)	(7,4)	(6,4)	(5,3)
RP	(3,**9**)	(2,7)	(1,6)	(0,4)

TABLE III
MODIFIED PAYOFF OF COYOTE VS WILDLIFE MANAGEMENT

.

	DN	**RH**	**RC**	**B**
EP	(**10**,1)	(**9**,1)	(**8**,**2**)	(**7**,**2**)

TABLE IV
REDUCED MODIFIED PAYOFFS

B. Modified Strategic Form Model Analysis

In order to account for the use and over exertion of resources, we chose to represent the strategies with a conceptual diminishing return (shown in Table II). Strategies that required the use of resources would incur a penalty to that player. Counter-strategies that imposed scenarios in which the resources were put to poor use resulted in a further reduced payoff. Using these values to modify our original table results in a model that attempts to better represent the payoff for the particular strategies between wildlife management and the coyote population (Table III). Two Nash Equilibrium can be found based on the modified information: (Extreme Predation, Remove Coyote) and (Extreme Predation, Reduce Harvest & Remove). Removing Coyote weakly dominates Reduce Harvest & Remove. Despite the modified cost in resources for the coyote population to excessively hunt the white-tailed deer population, their far greater payoff outweighs the necessary expenditure of resources. However, due to the over exertion required from the wildlife management to implement both strategies, choosing to focus solely on removing coyotes becomes a viable option. The expected benefits from choosing to implement an additional strategy (Reduce Female Harvest) applies a minimal gain while incurring a greater loss due to the consumption of resources required to enact such an inclusive strategy. To demonstrate this point these concepts are applied to an evolutionary game.

V. Evolutionary Game

A. Evolutionary Model

For the evolutionary game model, players will be wildlife management and coyote population.
Wildlife management has two strategies:

- decrease female deer harvest rates [R]
- decrease female deer harvest and increase coyote harvest [B]

The coyote population has two strategies:

- predation of deer [D]
- predation of other [O]

For the evolutionary game model, we chose to reduce the player's actions to a more feasible state. Using the data from Figure 1, we observed that the payoff increase for wildlife management from choosing strategies DN and RH are strictly minimal as well as RC and B. For these reasons, we chose to combine the strategies. Additionally, we simplified coyote population players choices from levels of predation to the option of preying on deer or other based on the severity of predation. We chose to imply that a scenario that suggests a low level of predation on deer signifies the increase in predation of other species. We chose these strategies as to represent a more effective, costly choice and less effective, economical choice for each player.

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	R	B
D	(**150**, 100)	(130,**108**)
O	(140, **105**)	(**135**,99)

TABLE V

The mixed strategies for this initial evolutionary game come out to be $(\frac{3}{7}D + \frac{4}{7}O), (\frac{1}{3}R + \frac{2}{3}B)$ which means that coyote will choose to hunt other more often than deer and management will tend to do both strategies over only reducing female harvest to receive the highest payout over one iteration. Because these payoffs are repeated the evolutionary game looks towards replicator dynamics for a more nuanced solution. Replicator dynamics is a means to generalize the fitness of strategies for each play and relate them to the other player's actions.

$$m^t = \text{proportion of strategy R for wildlife management}$$

$$c^t = \text{proportion of strategy D for coyote population}$$

$$m^{t+1} = m^t \frac{20c^t + 130}{m^t(20c^t + 130) + (1 - m^t)(5c^t + 135)}$$

$$c^{t+1} = c^t \frac{-5m^t + 105}{c^t(-5m^t + 105) + (1 - c^t)(9m^t + 99)}$$

The starting proportions $m^0 = 0.9$ and $c^0 = 0.1$ represents the introduction and rise of the coyote population into environment. Wildlife Management is unconcerned so they center around the most efficient strategy of only reducing female deer harvest. The coyote population is not familiar with deer as a source of prey but, as time passes coyote learn deer is the most beneficial source of prey and management quickly

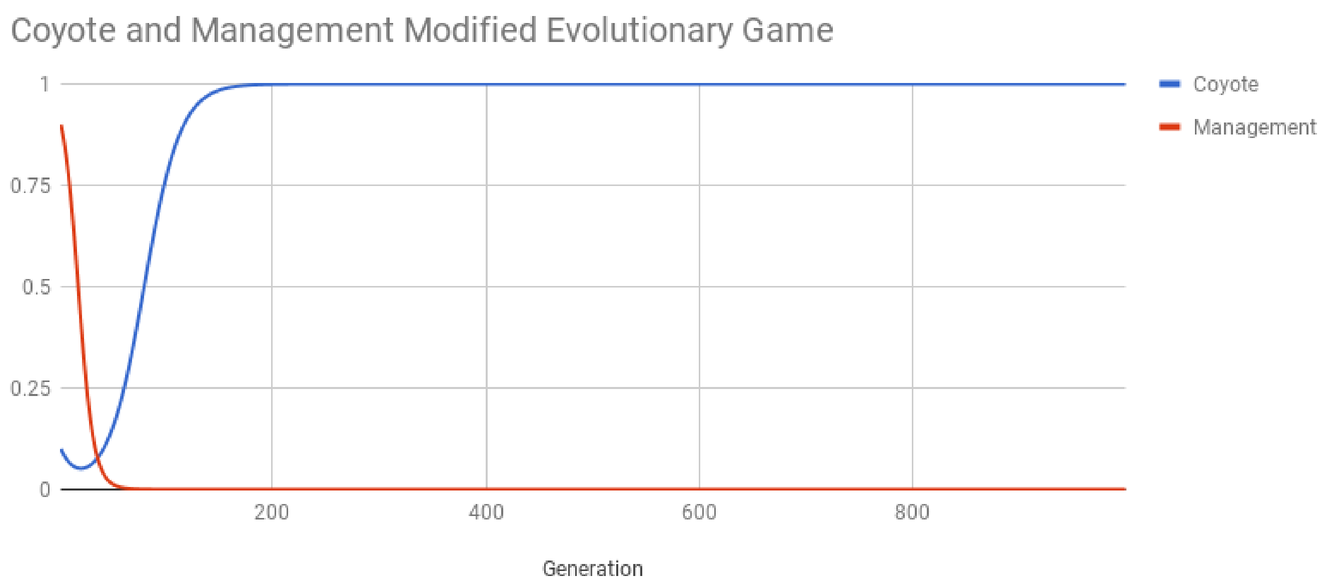

Fig. 2. Initial Scenario with $m^0 = 0.9$ and $c^0 = 0.1$

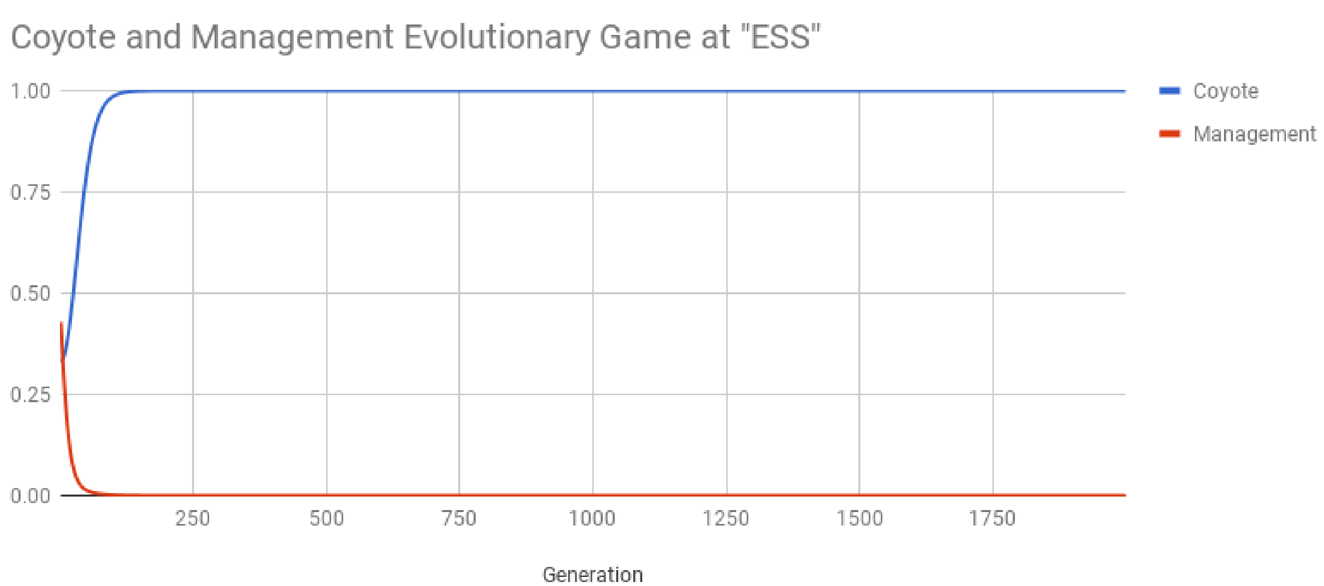

Fig. 3. Initial Scenario with $m^0 = \frac{3}{7}$ and $c^0 = \frac{1}{3}$

attempts to remove coyote realizing the harm that could potentially be done. Because the benefits of deer heavily outweigh the downside of being hunted more heavily, coyote always hunt deer causing wildlife management to always resort to doing both their initial strategy and their additional strategy.

B. Evolutionary Model Analysis

Solving for the replicator dynamics we find the starting proportions for a supposed evolutionary stable strategy to be where $m^0 = \frac{3}{7}$ and $c^0 = \frac{1}{3}$. Applying these values into the model gives us what is seen in Figure 3. As a strategy for both players has completely invaded another strategy from the initial proportion of strategies, this is not an ESS most likely due to the (O,B) payoff for coyote being relatively low in comparison to (D,R). To find a reachable ESS, the coyote population must have a payoff in (O,B) that is greater than or equal to (D,R). Realistically this would never happen as deer are simply too common and beneficial for coyote to hunt other prey.

VI. Modified Evolutionary Game

A. Modified Evolutionary Model

In order to establish a oscillating replicator dynamic, we determined that a change to the coyote population's payoff was necessary. By increasing the payoff for the coyote population for strategy set (O,B) to a point that exceeds the payoff of strategy sets (D,B) and (O,R), we observed that the desired replicator dynamic was achievable. Increasing this payoff represents coyote coming to the conclusion that hunting prey other than deer is more beneficial during periods where wildlife management is employing both strategies. The new payoffs are represented in Table VI below.

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	R	B
D	(**150**, 100)	(130,**108**)
O	(140, **105**)	(<u>**150**</u>,99)

TABLE VI

The mixed strategies for this modified game come out to be $(\frac{3}{7}D + \frac{4}{7}O), (\frac{2}{3}R + \frac{1}{3}B)$ meaning the alteration to the coyote's payoff has influenced wildlife management to favor reducing harvest more. This makes sense as coyote now tend to hunt prey besides deer as is expected of them in an ideal world. Additionally, below Figure (Figure 6) is the phase diagram representing the choice given a particular quadrant of the graph. Each quadrant signifies a particular strategy available to a player, where the arrows indicate a shift to the other possible strategy.

$$m^t = \text{proportion of strategy R for wildlife management}$$

$$c^t = \text{proportion of strategy D for coyote population}$$

$$m^{t+1} = m^t \frac{20c^t + 130}{m^t(20c^t + 130) + (1 - m^t)(-10c^t + 150)}$$

$$c^{t+1} = c^t \frac{-5m^t + 105}{c^t(-5m^t + 105) + (1 - c^t)(9m^t + 99)}$$

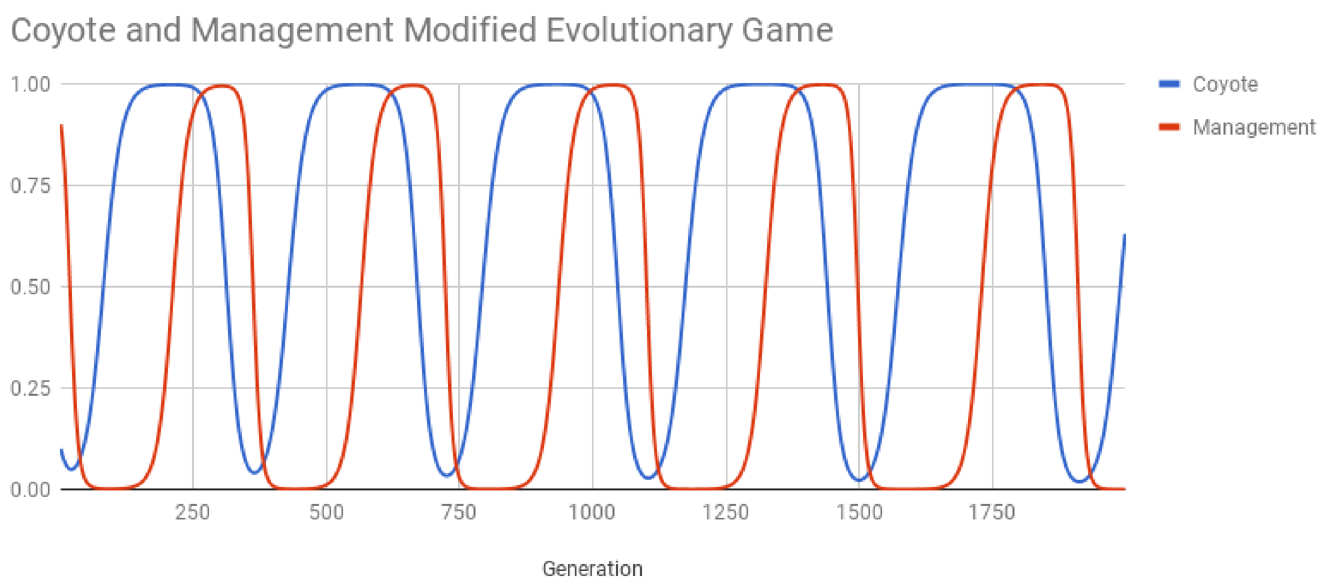

Fig. 4. Modified Scenario with $m^0 = 0.9$ and $c^0 = 0.1$

B. Modified Evolutionary Model Analysis

In order to account for the new payoffs for both players, it became necessary to imply specific assertions that could be reconcilable within the given system. For example, in order to justify the increase in payoff for coyotes to choose strategy O during a period in which wildlife management chooses strategy B, we considered the concept that wildlife management would have a difficult time removing coyotes as they avert their natural hunting patterns. Pragmatically, this scenario could represent the wildlife management's inability to inact their strategy based on the action of the coyotes.

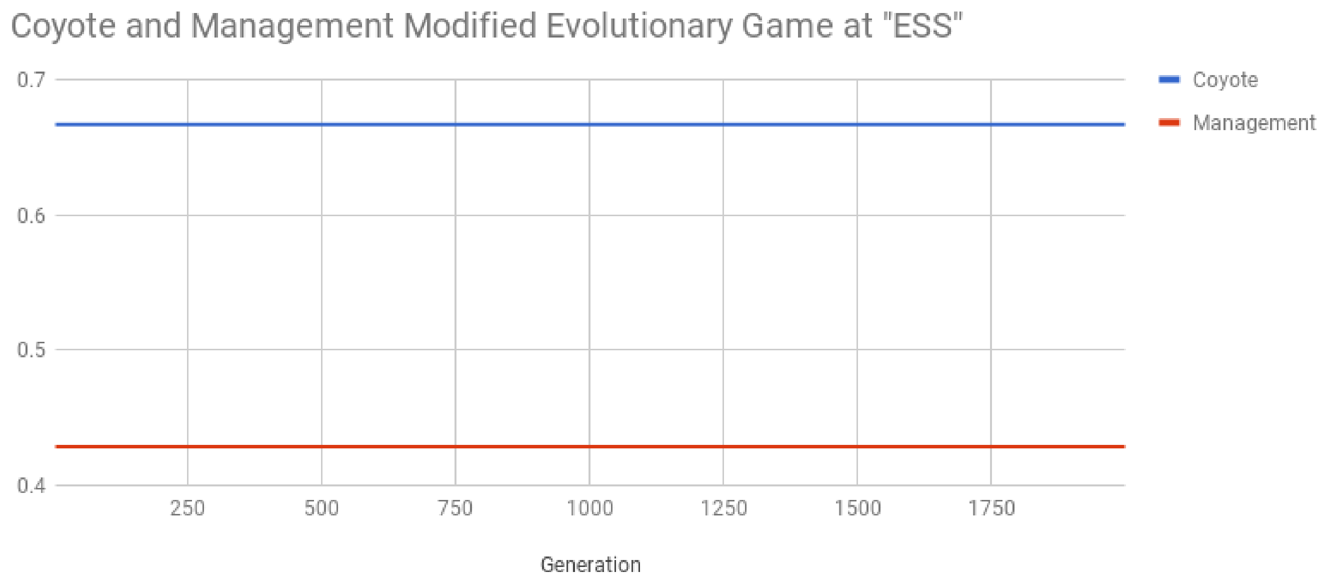

Fig. 5. Modified Scenario with $m^0 = \frac{3}{7}$ and $c^0 = \frac{2}{3}$

Solving for these replicator dynamics and using those values arrives at Figure 5. This is certainly an ESS as the proportion of each population of players that chooses each strategy does not change. The ESS in this case aligns with the mixed Nash equilibrium. As an equilibrium is only found starting at this point. Because the original evolutionary game is a better representation of the system where the payoff in (D,R) for the coyote will naturally never be smaller than (B,O), wildlife management either needs to come up with a more extreme strategy or come up with a complete different strategy to achieve equilibrium. Lotka-Volterra approaches this problem from a different perspective with a model that is similar to how evolutionary games function but is fundamentally different in what is being shown.

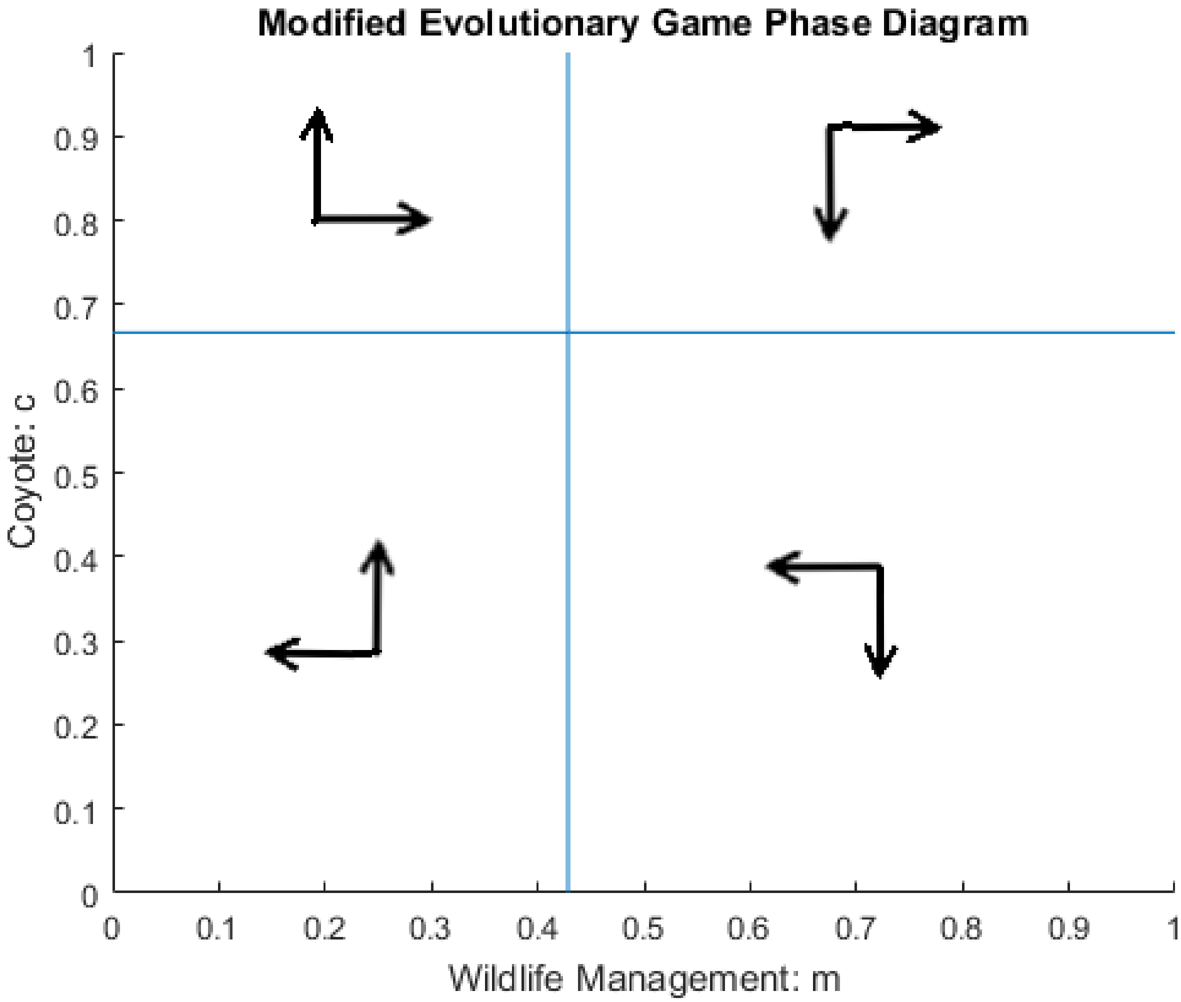

Fig. 6. Phase Diagram of Modified Evolutionary Game

VII. Lotka-Volterra Game

A. Lotka-Volterra Model

The Lotka-Volterra model is a pair of equations used to describe the relationship between predator and prey. We analyze white-tailed deer and coyote, but we will be using the generic terms. Outlined below is the traditional Lotka-Volterra equation [2].

$$\frac{dx}{dt} = \alpha x - \beta xy \qquad \text{and} \qquad \frac{dy}{dt} = \delta xy - \gamma y$$

- x represents the number of prey
- y represents the number of predator
- t represents time
- $\frac{dx}{dt}$ represents the growth of prey over time

- $\frac{dy}{dt}$ represents the growth of predators over time
- α represents the coefficient of prey growth
- β represents the coefficient for how often prey and predator meet
- δ represents the coefficient of predator growth for a given prey
- γ represents the coefficient for the loss of predators due to death or emigration

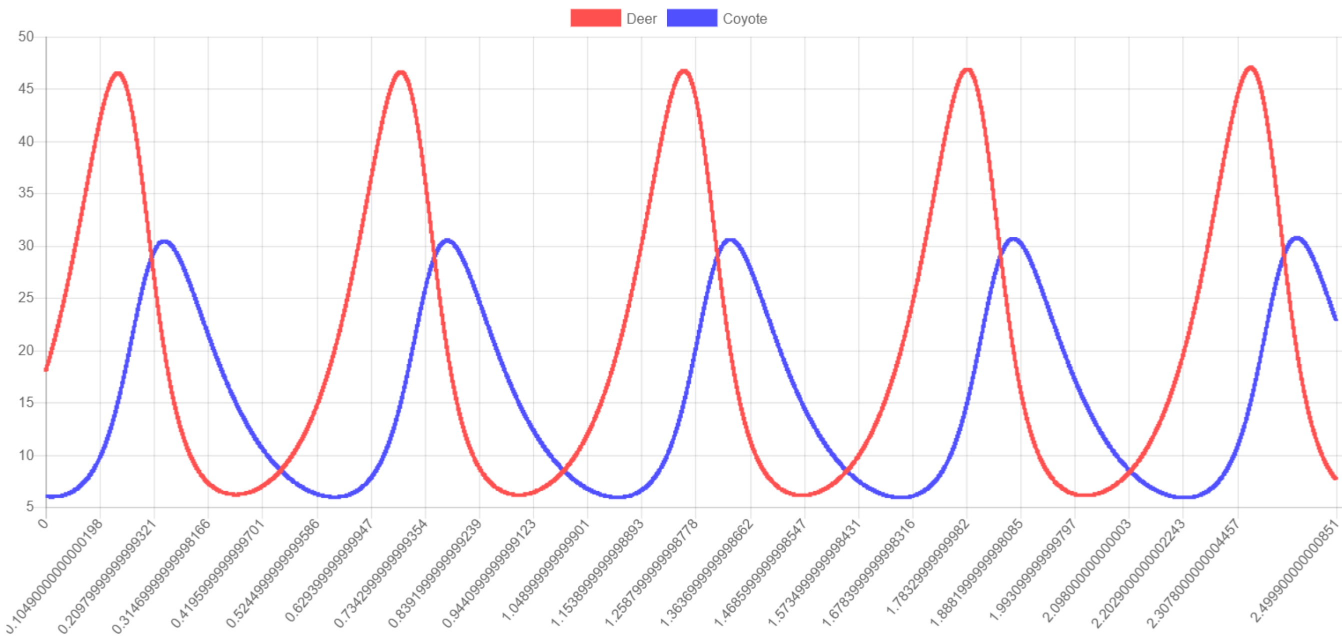

Fig. 7. Lotka-Volterra model with $x_0 = 18$ $y_0 = 6$ $\alpha = 15$ $\beta = 1$ $\gamma = 10$ $\delta = .5$

B. Lotka-Volterra Analysis

We want to reduce fluctuations because volatility can invoke a sudden extinction of a species. We are able to remove the fluctuations by finding an equilibrium achieved by setting the growth of predators and prey equal to zero. These are also known as the steady states of predator and prey. They are calculated as follows:

$$\frac{dx}{dt} = \alpha x - \beta xy = 0 \qquad \frac{dy}{dt} = dxy - cy = 0 SS_y = \frac{\alpha}{\beta} \qquad SS_x = \frac{\gamma}{\delta}$$

If initial predator and prey values are set to the steady state values, we will have an equilibrium at all times.

Furthermore, since the coefficients do not change, changing the values of predator and prey to the steady state values at any time will still result in equilibrium.

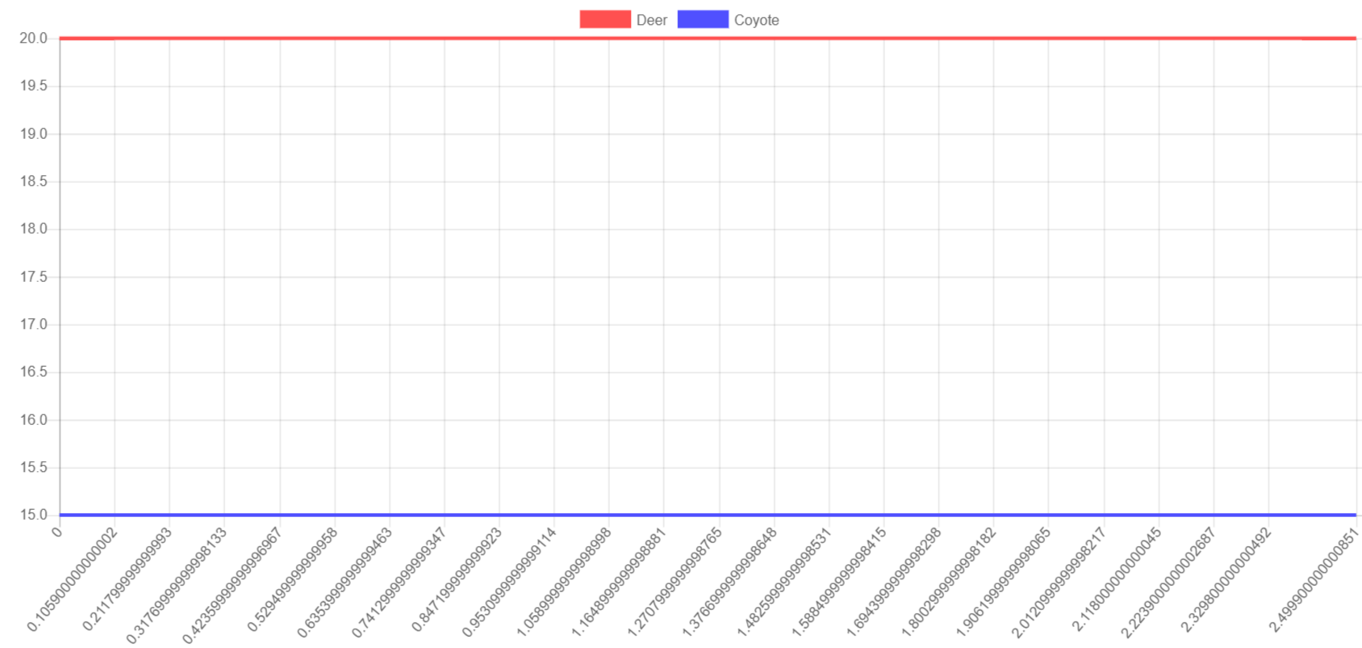

Fig. 8. Lotka-Volterra with initial steady state values

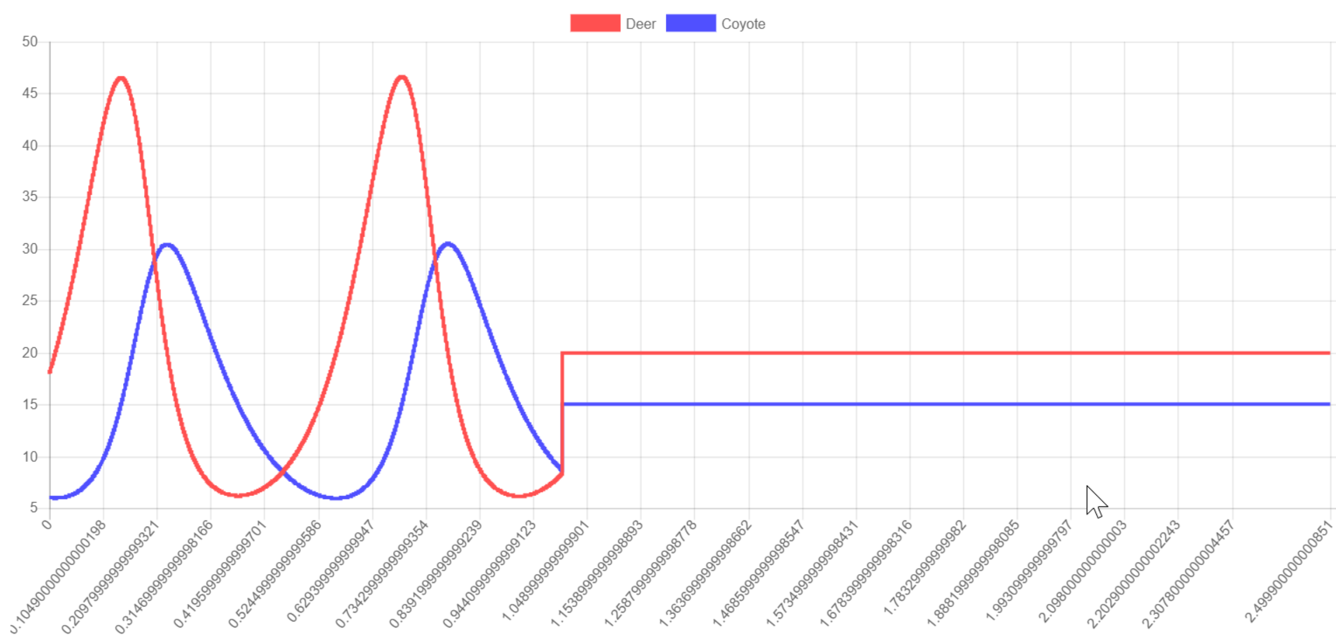

Fig. 9. Lotka-Volterra with steady state values set later

C. Modified Lotka-Volterra

In order to include the second player, the harvest rate of the prey and the rate of predator removal must be added to the model. They are represented as the following:

$$\frac{dx}{dt} = (\alpha - \epsilon)x - \beta xy \qquad \text{and} \qquad \frac{dy}{dt} = \delta xy - (\gamma + \zeta)y$$

- ϵ represents the coefficient for the altered harvest rate of the prey population.
- ζ represents the coefficient for the altered harvest rate of the predator population.

The modified Lotka-Volterra model will look essentially the same because ϵ and ζ directly adds to α and γ respectively.

D. Modified Lotka-Volterra Analysis

Finding the steady state values of the modified Lotka-Volterra model is done the same way as the regular model. The modified Lotka-Volterra model's steady state values are calculated as follows:

$SS_y = (\alpha - \epsilon)/\beta \qquad SS_x = (\gamma + \zeta)/\delta$

If we set the current amount of predator and prey to the steady state values, then we can find the ϵ and ζ that will create an equilibrium.

The equilibrium points for these models are as follows:

$$y = (\alpha - \epsilon)/\beta \qquad x = (\gamma + \zeta)/\delta SS_\epsilon = \alpha - y\beta \qquad SS_\zeta = x\delta - \gamma$$

Since it is difficult to change the amount of predator and prey at a set amount of time, changing the harvest and removal rates may be a more practical means of creating equilibrium.

Take note that we can only enact ϵ and ζ values when $\frac{dx}{dt} > 0$ and $\frac{dy}{dt} > 0$ because we cannot have negative ϵ and ζ values.

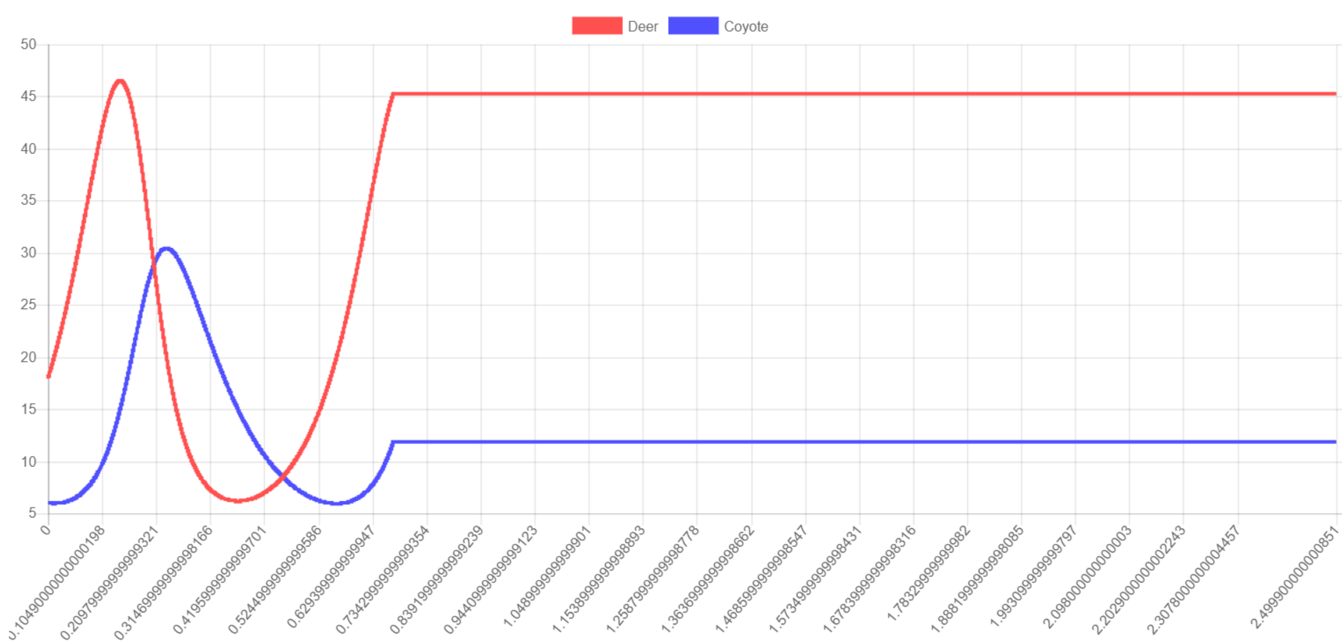

Fig. 10. Enacting SS_ϵ and SS_ζ at $t = .67$

The drawback to this method is that we must know exactly how much deer and coyote population there is at the time of harvest and removal. If we miscount the amount of predator and prey, then it can greatly skew the results.

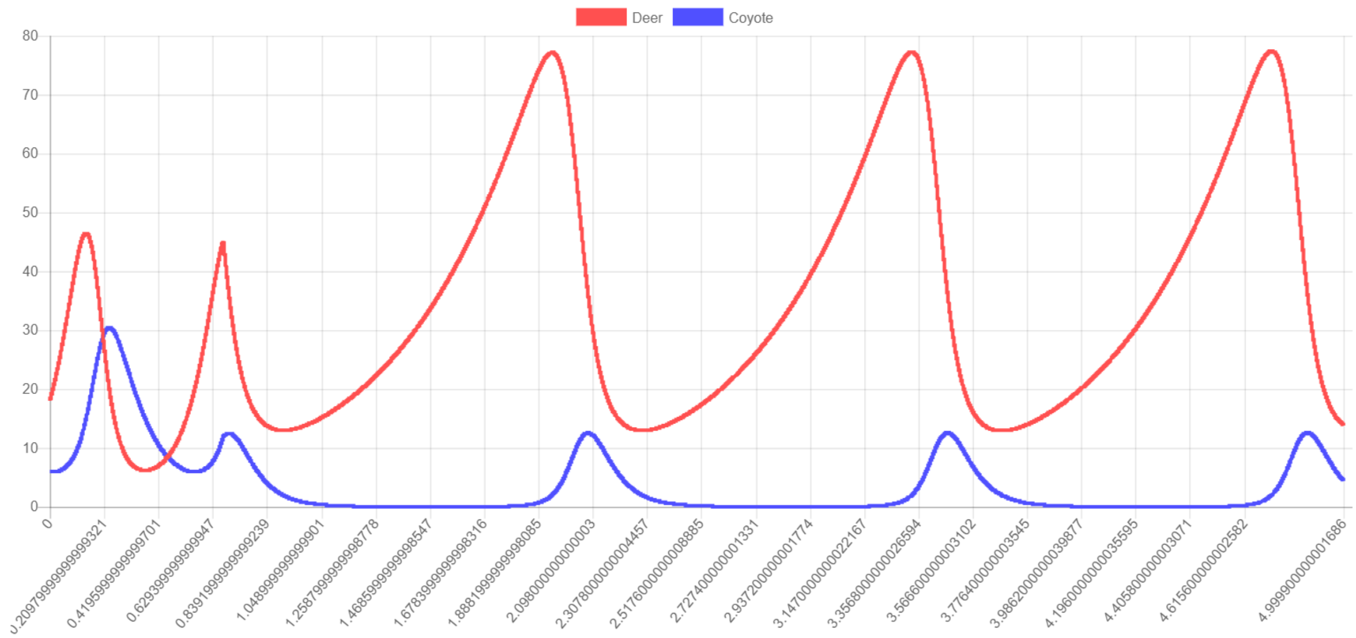

Fig. 11. Enacting the wrong SS_ϵ and SS_ζ at $t = .67$

Given this drawback, it may be easier to change the amount of prey and predator population to match the steady state values rather than enacting ϵ and ζ.

Generally speaking, if a group can determine the exact amount of prey and predator, then harvesting rates should be preferred. If a group cannot, then trying to set prey and predator values to match steady state values may be an option.

VIII. Predator Prey Simulation

A. Inspiration

We created a computer simulation based on the Lotka-Volterra model in order to determine the practicality of our solutions in a controlled environment.

B. Semantics

- Predator and prey are represented by circles
- Predator and prey move in a random direction
- Predator and prey change to a new random direction every step

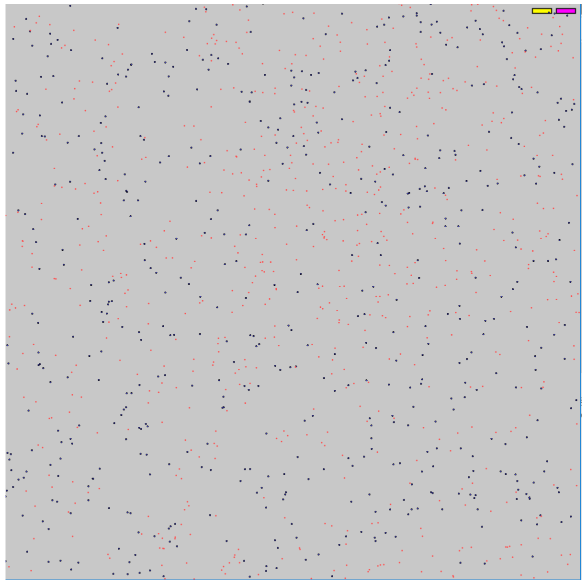

Fig. 12. Predator Prey Simulation: Deer represented by red dots. Coyote represented by blue dots. The buttons at the top right toggle the chart and simulation visuals.

- Every step occurs half a second
- Each prey has α (0-1) chance to reproduce every step
- Each predator has γ (0-1) chance to die every step
- Each predator has β (0-1) chance to kill each prey that is touching it every step
- Each predator has δ (0-1) chance to reproduce after killing a prey
- Every step records the current count of prey and predator allowing a visual line chart to be generated

Notice when the coyote population starts to dominate deer population, the oscillations become uncontrollable.

C. Drawbacks

The simulation assumes that deer and coyote walk in random straight lines with no knowledge of where others deer and coyote are. The complex nature of creatures are generalized into probabilities. All of the Lotka-Volterra's assumptions also apply to the drawbacks of this simulation.

All simulation coefficients are percentages rather than being any real number like the Lotka-Volterra model. This means that Lotka-Volterra coefficients cannot be directly translated to the simulation, and the steady state formulas will be invalid.

The β coefficient in the Lotka Volterra model would be affected by how often predator and prey change direction, the adjusted window size, the predator and prey size, and the β value in the simulation. This implies that the β variable will be significantly harder to determine than the other variables.

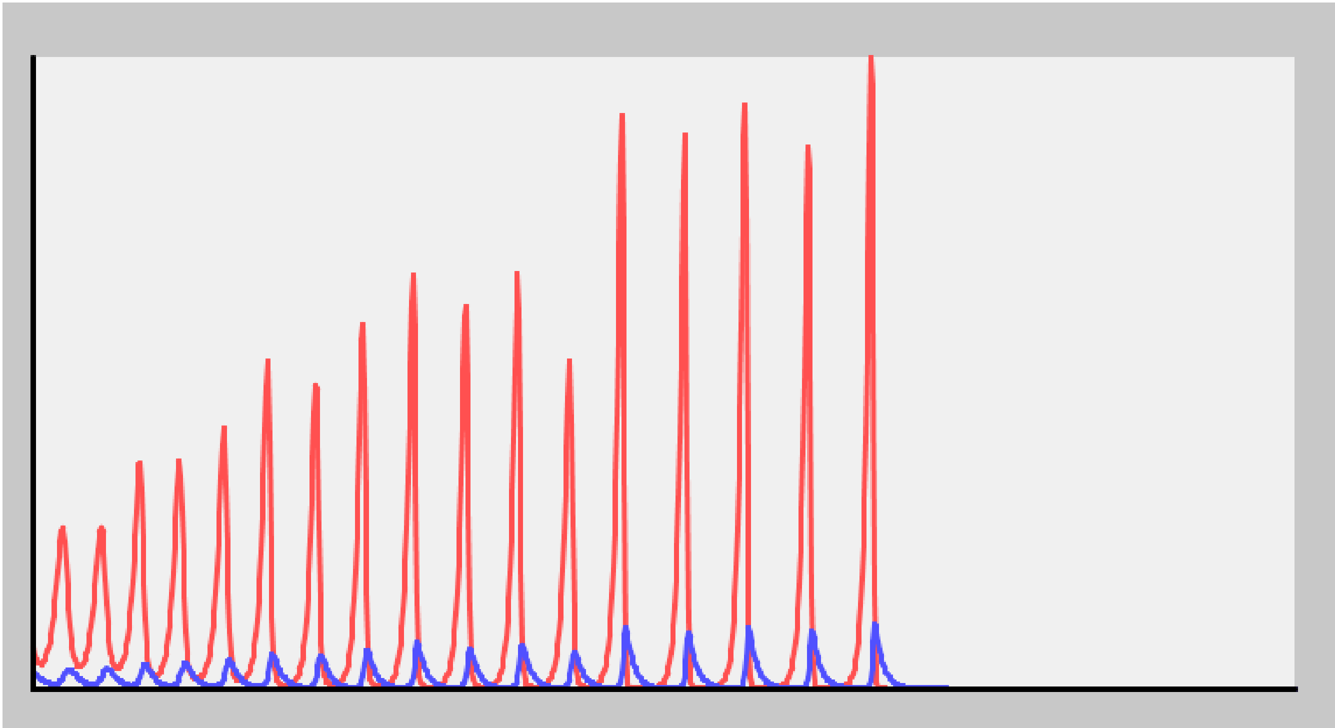

Fig. 13. Predator Prey Experiment 1: Deer population is represented in red. Coyote population is represented in blue. The oscillation becomes more violent until it becomes unsustainable.

Applying harvest and removal rates that are calculated from coefficients will be pointless since the coefficients are not directly linked to the Lotka-Volterra Model

D. Experiments

We run the simulation many times to see if certain variable changes are consistent with the graph generated.

We used values of $\alpha = .25$, $\beta = .5$, $\gamma = .15$, and $\delta = .1$ because they are relatively realistic values. Extreme values like $\alpha = .9$ and $\delta = .9$ result in equally extreme results where populations die off really quickly.

Generally, deer are three times as populous as coyote, so we began with the simulation with 1800 deer and 600 coyote. The simulation reveals that the periods become exponentially amplified until both the predator and prey die off (See figure 13). A large majority of the time, the deer died off which led to coyote dying off, but there were a few times where the coyote died off and deer levels became infinitely large.

For our second experiment, we wanted to know how much less time it would take for a population to die out when coyotes began with higher numbers than deer. We used 500 deer and 1000 coyote for the simulations. The deer and coyote survived an average of 4 periods (mostly in the range of 1-6) before dying out which is significantly less than the previous experiment (See figure 14. This reveals that we must be vigilant to take action before a species dies off.

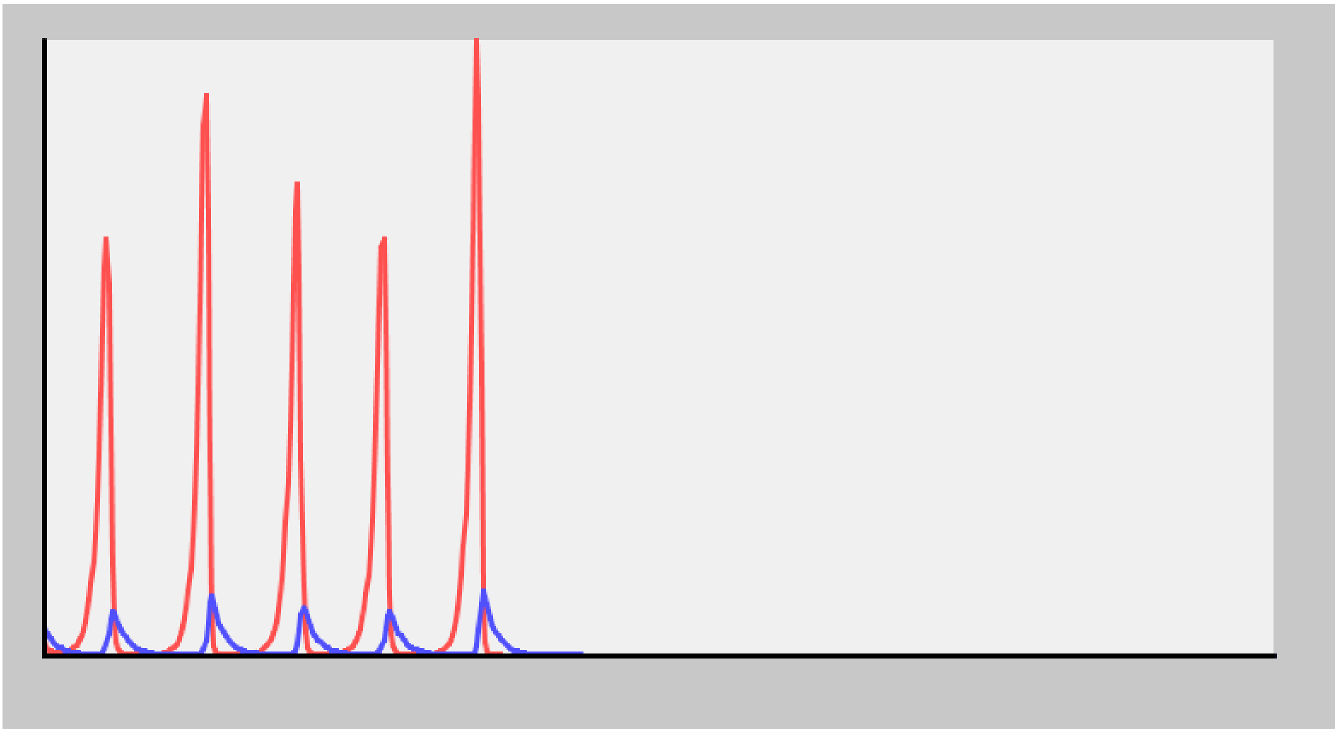

Fig. 14. Predator Prey Experiment 2: A coyote dominant population leads to quick end to both species

Like before, when the coyote population dominates the deer population, the demise of both populations soon follows.

E. Resolution

Since it is very difficult to find the steady state values, we must take a pragmatic approach. Based on the two main experiments shown prior, there is strong evidence that when coyote populations are greater than deer populations there will soon be an extinction. To combat this, we increased removal rates of coyote (which have the similar effect as reducing the rates of deer harvesting) when there are greater numbers of coyote than deer. To do this, we increased the decay rate γ by .3 when coyote numbers are greater than deer, and changed γ back to the original value when the deer overtake the coyote again. This had powerful results (see figure 15).

It appeared to remove the increases in amplitude by minimizing the time coyote populations were dominant. Every single simulation we ran had the result of both species surviving at least 30 periods

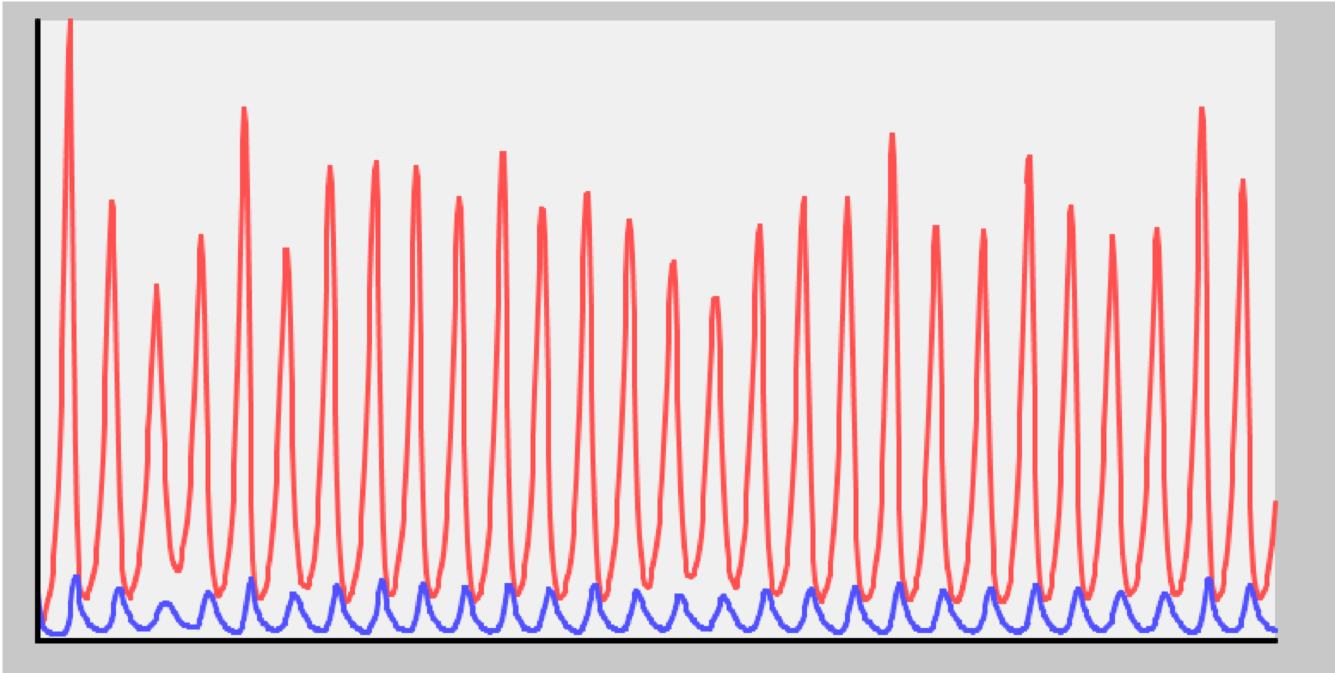

Fig. 15. Predator Prey Resolution: Controlling coyote populations when they are overtaking the deer results in consistent oscillation

(we ended the simulations at around 30). The first period is always has the strongest amplitude which corresponds with our hypothesis.

IX. Discussion

Finding a realistic answer to stabilizing the coyote and deer population is found to be a very difficult subject due to how coyote benefit from predation in a disproportionate manner from how wildlife management benefit from influencing the coyote population in hopes of reverting the decline of white-tailed deer. By the Nash equilibrium obtained from the strategic form game, it appears the best response from players is to expend as much effort as possible, and choose strategies that maximize payout. While this may be true for the game, the best response in reality does not follow this logic. Maximum effort strategies lead to diminishing returns over long periods of time, and can result in poor consumption of resources. The strategic form game proposed would only be seen as the initial time frame. For a more accurate picture of the situation, a strategic form game that accounts for a long stretch of time would be greatly beneficial.

Pragmatically, the modified standard form game that accounts for the use of resources better represents this particular scenario. Adjusting the model for the required effort of each strategy results in a more realistic representation of the wildlife management's choices; choosing all available options may seem to have the most impact on protecting the white-tailed deer population, but due to the intrinsic cost of each strategy this becomes a less reliable option. Accounting for these intrinsic costs would then allow wildlife management to favor removing coyote from the population as this seems to be the deciding factor of the chosen strategy.

While the modified strategy nails down the most effective means to deal with the shrinking white-tailed deer strategy it also demonstrated examples in which coyote solely favor hunting deer to the maximum allowed value. To illustrate the sustainability of this equilibrium an evolutionary game was introduced showing once again coyote favor hunting deer and wildlife management favors removing coyote. This was not a Nash equilibrium as reaching this point required a drift from the initial state. As this does not garner any new information, a modification to the evolutionary game was made in which a payoff for the coyote population was increased past a threshold in order to incentivize the hunting of other prey during specific scenarios. This can be alternatively viewed as coyote hunting less deer while still allowing for growth in their population.

The Lotka-Volterra game does account for some of these concerns but requires many unknown variables to be rather precise for it to function for our purposes. Unfortunately, due to a lack of environmental studies providing the specific data we required, we had no other option but to postulate several of these values based on scientific estimations. Instead of finding a concrete solution we show the potential of Lotka-Volterra through a demonstration of what it is capable of accomplishing. Lotka-Volterra inherently model populations over long stretches of time therefore the intented solution from Lotka-Volterra may not be feasible from a realistic perspective. With a working Lotka-Volterra system an equilibrium can be achieved, although this requires both the prey population and the predator population to be in a state of growth as deer cannot be added to the population. As this intrinsically requires the prey population to further decline, the equilibrium found with the modified Lotka-Volterra is not a feasible solution to the current problem.

The predator prey simulation has a real world solution such that its policy is easy to implement and understand, and one does not need to figure out the coefficients of models. It also complements the results of the form games as it agrees that coyote removal reduced deer harvest can potentially prevent either population from dropping to dangerous levels. Although the growth of the deer and coyote may not be at equilibrium, they are fluctuating safely according simulations.

X. Conclusion

The current state of the ecosystem of white-tailed deer and coyote is currently in danger of an uncontrolled increase of the coyote population with a similar sudden decrease in the deer population. While it is unlikely that the white-tailed deer population will become extirpated in this region, sudden shifts in their population may cause unforeseen consequences in other parts of the ecosystem as their relationship is not a closed system. If wildlife management acts to prevent these patterns, it is possible to reduce these effects or completely alleviate any worries. Our models have shown that reducing the coyote population heavily will allow for the stabilization of the deer population. The degree by which the coyote population is required to be reduced is not practical so the best effort is put forth in this matter.

References

[1] Chitwood, Michael C., Marcus A. Lashley, John C. Kilgo, Christopher E. Moorman, and Christopher S. Deperno. "White-tailed deer population dynamics and adult female survival in the presence of a novel predator." *The Journal of Wildlife Management* 79, no. 2 (2015): 211-19. doi:10.1002/jwmg.835.

[2] Robert D. Holt , and Gary A. Polis ,"A Theoretical Framework for Intraguild Predation," The American Naturalist 149, no. 4 (Apr., 1997): 745-764. https://doi.org/10.1086/286018