Hints:

3C #3:

Let $\{u_1, \cdots, u_m\}$ be a basis of $\ker(T)$. Extend the list $\{u_1, \cdots, u_m\}$ to get a basis $\{v_1, \cdots, v_n, u_1, \cdots, u_m\}$ of V.

Let $w_i = T(v_i)$, for $i = 1, \cdots, n$. You will verify that $\{w_1, \cdots, w_n\}$ is a basis of the range of T.

Extend the list $\{w_1, \cdots, w_n\}$ to get a basis $\{w_1, \cdots, w_n, w_{n+1}, \cdots, w_r\}$ of W.

You will show that with respect to the bases $\{v_1, \cdots, v_n, u_1, \cdots, u_m\}$ of V and $\{w_1, \cdots, w_n, w_{n+1}, \cdots, w_r\}$ of W, the matrix $\mathcal{M}(T)$ is in the desired form.

3D #7:

a) You want to verify that E contains the zero map and E is closed under addition and scalar multiplication.

b) Fix $v \neq 0$. Define a linear map $f_v : \mathcal{L}(V, W) \to W$ satisfying $f_v(T) = T(v)$. Verify that this is a linear map and E is the null space of f_v. Use (3.5) to show that f_v is surjective. Finally, you will use the fundamental theorem of linear map to compute the dimension of E.

3D #10:

Note that $ST = I$ implies that S is surjective. And hence S is invertible in $\mathcal{L}(V)$ by (3.69).