Contents

iso

$\mathcal{L}(V, W) \xrightarrow{\mathcal{M}} \mathbb{F}^{mn}$

$\dim(V) = \dim(\text{Range } T) + \dim(\ker(T))$

$T: V \to W$

T is invertible

T is Iso.

$\Updownarrow$

$\det(\mathcal{M}(T))^{[1, B]} \neq 0.$

T, U is invariant subspace.

$T_U: V/U \to V/U.$

V. $V^* = \mathcal{L}(V, F)$

$P(z)=0$ HAS SOLUTION IN $\mathbf{C}$

$(x-1)(x-\lambda)(x-\bar{\lambda})$

$\boxed{\text{chapter 9 A}}$

$x^3 - 1 = (x-1)(x^2+x+1)$

$A = \begin{pmatrix} -1 & 4 & 4 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{pmatrix}$ $\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = 4$

$\det(zI - A) = \det\begin{pmatrix} z+1 & & 4 \\ & z-3 & \\ 0 & & z-4 \end{pmatrix}$

$= (z+1)(z-3)(z-4)$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$\boxed{5C,8}$ $T \in \mathcal{L}(V)$ (complex).

$\boxed{5C}$ Eigenvector space

$$E(T,\lambda_i) = \{ v \in V \mid Tv = \lambda_i v \}$$
$$= \{ v \in V \mid (T-\lambda_i)v = 0 \}$$

$\boxed{8}$

$$G(T,\lambda_i) = \{ v \in V \mid (T-\lambda_i)^j v = 0 \text{ for some } j \}$$

$$E(T,\lambda_i) \subseteq G(T,\lambda_i)$$

$G(T,2) = \operatorname{span}\{u_1 \cdot u_3\}$

$$\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \begin{pmatrix} 2 & 1 & & \\ & 2 & & \\ & & 2 & \\ & & & \boxed{5} \end{pmatrix}$$

$G(T,5)$

$\lambda_1 = 2 \qquad \lambda_2 = 5$

$E(T,2) = \operatorname{span}\{u_1, u_3\}$

$\cap$

$G(T,2)$

$E(T,5) = \operatorname{span}\{u_4\}$

$\boxed{18}$

$$V = G(T,\lambda_1) \oplus \cdots \oplus G(T,\lambda_m)$$

$$\cup\!\!| \qquad\qquad\qquad \cup\!\!|$$

$$V \neq E(T,\lambda_1) \oplus \cdots \oplus E(T,\lambda_m)$$

$\boxed{5C}$ IF $V = E(T,\lambda_1) \oplus \cdots \oplus E(T,\lambda_m)$

then T is diagonalizable

(The Jordan normal form of T is diagonal)

$\iff$ $G(T,\lambda_i) = E(T,\lambda_i)$ FOR $\forall \lambda_i$

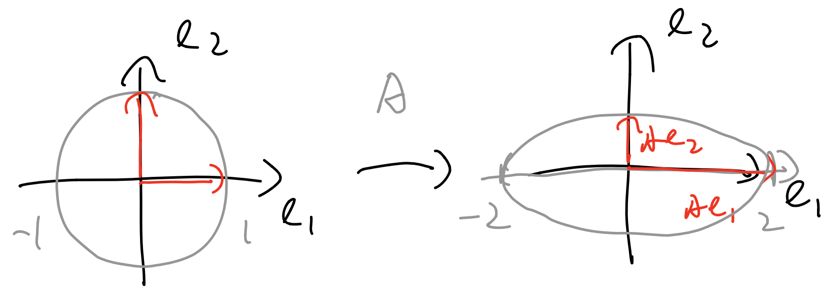
$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \overset{u_2}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 2 & & \\ & 2 & \\ & & 2 \end{pmatrix}$$

$$G(T,2) \neq E(T,2) \qquad \begin{pmatrix} \lambda_1 & & \\ & \boxed{\begin{matrix} 1 & \\ \ddots & \ddots \\ & \lambda_1 \end{matrix}} \end{pmatrix}$$

?

u_2 is not an eigen vector.

$\underline{\text{Ex.}}$ $\begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$



$\underline{\text{Ex}}$ $\begin{pmatrix} 2 & 1 \\ 0 & 2^{-1} \end{pmatrix} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$\lambda_1 = 2$ v_1

$\lambda_2 = 2^{-1}$ v_2

$\begin{pmatrix} 2 & 0 \\ 0 & 2^{-1} \end{pmatrix}$ wrt v_1 v_2