8B

#3 The multiplicity of an eigenvalue λ of an operator T is defined to be the dimension of $G(\lambda, T)$ (see (8.24) in your book.).

The part that asks you to show T and $S^{-1}TS$ have the same set of eigenvalues was one of the homework problems in Chapter 5.

One way of showing that given an eigenvalue λ, two generalized eigenspaces $G(\lambda, T)$ and $G(\lambda, S^{-1}TS)$ have the same dimensions is to directly establish an isomorphism between these two spaces. To do so, you may use the following observation:

$$S^{-1}(T - \lambda I)S = S^{-1}TS - \lambda I.$$