3E:

#4: The isomorphism T between $\mathcal{L}(V_1 \times \cdots \times V_m, W)$ to $\mathcal{L}(V_1, W) \times \cdots \times \mathcal{L}(V_1, W)$ is defined to be $T(\varphi) = (\varphi_1, \cdots, \varphi_n)$ with φ_i in $\mathcal{L}(V_i, W)$ sending a vector v_i in V_i to $\varphi(0, \cdots, v_i, \cdots, 0)$.

You will verify that this is a linear map. Then you need to show that this is invertible by either defining the inverse map or show it is both surjective and injective.

#5: Let $p_i : W_1 \times \cdots \times W_n \to W_i$ be the linear map sending $(w_1, \cdots, w_n)$ to w_i. Then the isomorphism T between $\mathcal{L}(V, W_1 \times \cdots \times W_n)$ to $\mathcal{L}(V, W_1) \times \mathcal{L}(V, W_n)$ is defined to be $T(\varphi) = (\varphi_1, \cdots, \varphi_n)$ with φ_i in $\mathcal{L}(V, W_i)$ sending a vector v in V to $p_i \circ \varphi(v)$ in W_i.

You will verify that this is a linear map. Then you need to show that this is invertible by either defining the inverse map or show it is both surjective and injective.

#12: By assumption V/U is finite dimensional. Let $v_1 + U, \cdots, v_n + U$ be a basis of V/U. Let $\pi : V \to V/U$ be the quotient map. Given any $v \in V$, there exists a_i such that $\pi(v) = \sum_i a_i(v_i + U)$. You will show that $v - \sum_i a_i v_i$ is in U, and prove that $v \mapsto (v - \sum_i a_i v_i, \pi(v))$ is an isomorphism.