Y-Intercept Coding Test:
A Parametric Portfolio Approach

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Overview

Overview

Given the limited stock information provided (low-dimensional stock characteristics), I borrow the parametric portfolio policy in Brandt, Santa-Clara, and Valkanov (2009) RFS to construct the portfolio.

A quick summary of my work:

1. Flexible: Portfolio weights are flexibly modeled by 5 trading-related characteristics, and parameters are determined by maximizing a utility function.
2. Time-varying: The model is estimated in a rolling way, and the properties of portfolios and their performance are shown.
3. Robust: Extensions include no-shart-sale constraint and transaction costs are imposed in the portfolio construction and performance is tested.

Overview for Code

There are three main parts in the submitted Python files:

1. *functions_parametric_portfolio*: this file has the functions related to the portfolio construction and backtest.
2. *functions_asset_pricing*: this file has the ancillary functions that are usually used in asset pricing studies.
3. *Main*: the main file to run.
    - Data clearing
    - Stock Characteristics construction
    - Portfolio construction and performance test
    - Extensions: no-short-sale constraint and transaction cost

Data

1. Impute Missing Data

There is no missing data.

2. Stock Returns

I assume that the provided variable *last* considers the dividends and shares split/merge.

This ensures the correctness of daily return ($r_{i,t}$ as the return for stock i at date t) calculations:

$$r_{i,t} = \frac{p_{i,t}}{p_{i,t-1}} - 1$$

The noise at daily frequency is large and transaction cots are high. Thus, this strategy is constructed at monthly return ($r_{i,\tau}$):

$$r_{i,\tau} = \prod_{t \in \tau} (1 + r_{i,t}) - 1$$

1. Firm Characteristics

There are five stock characteristics are used for portfolio construction:

1. CAPM Beta
2. CAPM variance
3. Dollar volume
4. Momentum: past 2-12 months cumulative returns
5. Reversal: last month's return

2. Ranking Firm Characteristics

Kelly, Pruitt, and Su (2019) JFE: Calculate stocks' ranks for each characteristic and then divide ranks by the number of non-missing observations.

This maps characteristics into the $[-1, 1]$ interval and focuses on their ordering as opposed to magnitude.

Portfolio Policy

3. Parametric Portfolio Policy I/II

Following Brandt et al. (2009) RFS, I parametrize the portfolio weight $w_{i,t}$ of each stock as a function of the firm's characteristics $x_{i,t}$ and estimate the coefficients θ of the portfolio policy by maximizing an utility μ:

$$w_{i,t} = f(x_{i,t}, \theta).$$

I apply a simple linear specification due to a smaller number of characteristics to be included.

$$w_{i,t} = \overline{w}_{i,t} + \frac{1}{N_t}\theta^T x_{i,t},$$

where $\overline{w}_{i,t}$ is the weight in a benchmark portfolio.

In the rest of this project, I take the equal-weight ($1/N_t$) market portfolio by DeMiguel et al. (2009) RFS as the benchmark.

3. Parametric Portfolio Policy II/II

The portfolio return is

$$r_{p,t+1} = \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} = \sum_{i=1}^{N_t} (\overline{w}_{i,t} + \theta^T x_{i,t}/N_t) r_{i,t+1}$$

I use the constant relative risk aversion (CRRA) preferences:

$$\mu\left(r_{p,t+1}\right) = \frac{\left(1 + r_{p,t+1}\right)^{1-\gamma}}{1-\gamma}$$

where γ is the risk-aversion level which is incorporated toward higher-order moments[1].

The estimator is

$$\hat{\theta} = \arg \max_{\{w_{i,t}\}_{i=1}^{N_t}} \frac{1}{T} \sum_{t=0}^{T-1} \mu\left(r_{p,t+1}\right).$$

[1] The higher value of γ, the higher level of risk aversion.

Portfolio Test

1. Model Training

- Estimation: Linear specification makes sure the optimization numerically robust and stable.
- To avoid extreme values in weights, I add constraints in the optimization on the characteristics exposures that $\theta \in [-10, 10]$.
- Out-of-sample design: A rolling 12-month period is as the training set for the model. Then it is used for the investments in the next month. So testing of strategy performance starts in the third year.
- The portfolios with different risk-aversion levels ($\gamma = 4, 6, 8$) are constructed.

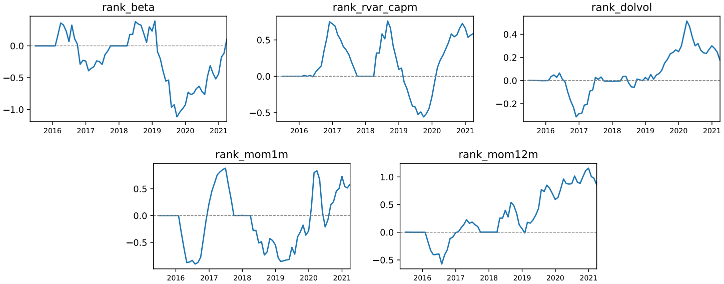
2. Portfolio Properties I/II

The following table shows the characteristic exposures on weights (θ) and properties of portfolio weights. The stock has a lower beta, higher idiosyncratic risk, lower reversal, and higher momentum is more selected by this portfolio policy.

	gamma=4	gamma=6	gamma=8	1/N
theta rank_dolvol	0.000	0.000	0.000	NaN
theta rank_beta	-2.202	-2.205	-2.258	NaN
theta rank_rvar_capm	2.537	2.248	1.584	NaN
theta rank_mom1m	-1.129	-1.507	-1.519	NaN
theta rank_mom12m	3.506	3.318	2.778	NaN
|w|*100	1.190	1.139	1.028	0.527
max w*100	3.505	3.302	2.778	NaN
min w*100	-2.347	-2.178	-1.785	NaN

2. Portfolio Properties II/II

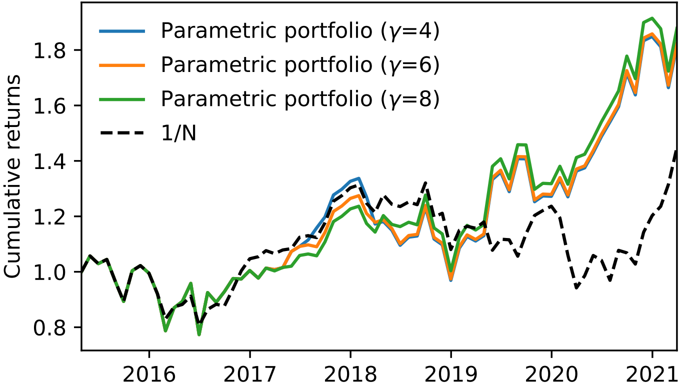
These figures demonstrate the time-varying portfolio characteristics ($\sum_i^{N_t} w_{i,t}x_{i,t}$).[2]



[2] The higher the value, the characteristics is preferred more by this policy.

3. Portfolio Performance I/III

This figure demonstrates the cumulative returns of my portfolios with different risk aversion levels as well as the benchmark ($1/N$).



3. Portfolio Performance II/III

For portfolio performance evaluation, I consider the following metrics[3]:

1. Average returns
2. Standard deviations of returns
3. Sharpe ratio
4. Alpha of CAPM ($1/N$)
5. Beta of CAPM ($1/N$)
6. Standard deviation of CAPM ($1/N$) residuals
7. Information ratio

[3] Due the limitation of provided data, the risk-free rate is assumed to be 0.

3. Portfolio Performance III/III

All of these metrics are calculated monthly. The risk aversion level balances the return and risk. The significant *alpha* but the *beta* that is larger than 1 indicates that this is not a good risk-neutral strategy.

	gamma=4	gamma=6	gamma=8	1/N
Average returns	0.011	0.011	0.012	0.007
Std. returns	0.069	0.068	0.068	0.057
Sharpe ratio	0.162	0.164	0.171	0.128
Alpha	0.007	0.007	0.007	NaN
Beta	0.600	0.598	0.608	NaN
Std. CAPM residuals	0.059	0.058	0.058	NaN
Information ratio	0.060	0.061	0.069	NaN
Observations	72.000	72.000	72.000	72.000

Extensions

1. No-short-sale Constraint I/III

No-short-sale constraint in long-only equity portfolios:

$$w_{i,t} = \frac{max\left[0, w_{i,t}\right]}{\sum_{j=1}^{N_t} max\left[0, w_{i,t}\right]}$$

1. No-short-sale Constraint II/III

The following table shows the properties of portfolio weights. With the no-short-sale constraint, the distribution of weights shrinks.

	gamma=4	gamma=8	gamma=4 (nonshort)	gamma=8 (nonshort)
theta rank_dolvol	0.000	0.000	0.000	0.000
theta rank_beta	-2.202	-2.258	-2.202	-2.258
theta rank_rvar_capm	2.537	1.584	2.537	1.584
theta rank_mom1m	-1.129	-1.519	-1.129	-1.519
theta rank_mom12m	3.506	2.778	3.506	2.778
|w|*100	1.190	1.028	0.452	0.452
max w*100	3.505	2.778	1.740	1.433
min w*100	-2.347	-1.785	0.106	0.182

1. No-short-sale Constraint III/III

The following table demonstrates the portfolio performance metrics. With the no-short-sale constraint, the portfolio realizes both lower return and lower risk. On the other hand, the *beta* is more close to 1 without short positions.

	gamma=4	gamma=8	gamma=4 (nonshort)	gamma=8 (nonshort)
Average returns	0.011	0.012	0.008	0.008
Std. returns	0.069	0.068	0.054	0.054
Sharpe ratio	0.162	0.171	0.145	0.143
Alpha	0.007	0.007	0.001	0.001
Beta	0.600	0.608	0.902	0.906
Std. CAPM residuals	0.059	0.058	0.017	0.016
Information ratio	0.060	0.069	0.032	0.027
Observations	72.000	72.000	72.000	72.000

2. Transaction Cost I/II

The portfolio turnover is the sum of all the absolute changes in portfolio weights

$$T_t = \sum_{t=1}^{N_t} \|w_{i,t} - w_{i,t-1}\| .$$

The return to the portfolio net of trading costs is

$$r_{p,t+1} = \sum_{t=1}^{N_t} w_i r_{i,t+1} - c_{i,t} \|w_{i,t} - w_{i,t-1}\| .$$

$c_{i,t}$ is the proportional transaction cost, which can be estimated from market liquidity measures or characteristics. Here, it is a linear function of the normalized market equity (from 0 to 1), $c_{i,t} = (0.002 - 0.001 \times me_{i,t}) \times T_t$.[4]

[4] The constants indicate that trading smaller stocks has a higher transaction cost. Specifically, the smallest stock has a transaction cost of 0.2%, and that of the largest is 0.1%. *me* is approximated by the log of *dolvol* in this case.

2. Transaction Cost II/II

The following table shows the portfolio performance when taking transaction costs into consideration. With transaction costs, the realized returns are obviously lower and the *alphe* is negative. γ also controls the level of turnover. The portfolio with higher risk aversion ($\gamma = 8$) has a higher monthly SR of 0.084 than the lower risk aversion case with an SR of 0.054.

	gamma=4	gamma=8	gamma=4 (with tc.)	gamma=8 (with tc.)
Average returns	0.011	0.012	0.004	0.006
Std. returns	0.069	0.068	0.068	0.067
Sharpe ratio	0.162	0.171	0.054	0.084
Alpha	0.007	0.007	-0.001	0.001
Beta	0.600	0.608	0.611	0.619
Std. CAPM residuals	0.059	0.058	0.058	0.057
Information ratio	0.060	0.069	-0.058	-0.026
Observations	72.000	72.000	72.000	72.000

Future Works

1. The model can be estimated for various industries separately.

2. Macro variables z_t are also able to be instrumented in the weights like:

$$w_{i,t} = \overline{w}_{i,t} + \frac{1}{N_t}\theta^T \left(z_t \otimes x_{i,t}\right).$$

3. High-dimensional characteristics: LASSO, Ridge, deep learning, IPCA, etc.

Brandt, M. W., P. Santa-Clara, and R. Valkanov (2009). Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns. *Review of Financial Studies 22*(9), 3411–3447.

DeMiguel, V., L. Garlappi, and R. Uppal (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? *Review of Financial Studies 22*(5), 1915–1953.

Kelly, B. T., S. Pruitt, and Y. Su (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics 134*(3), 501–524.