MACHINE LEARNING AND PHYSICS

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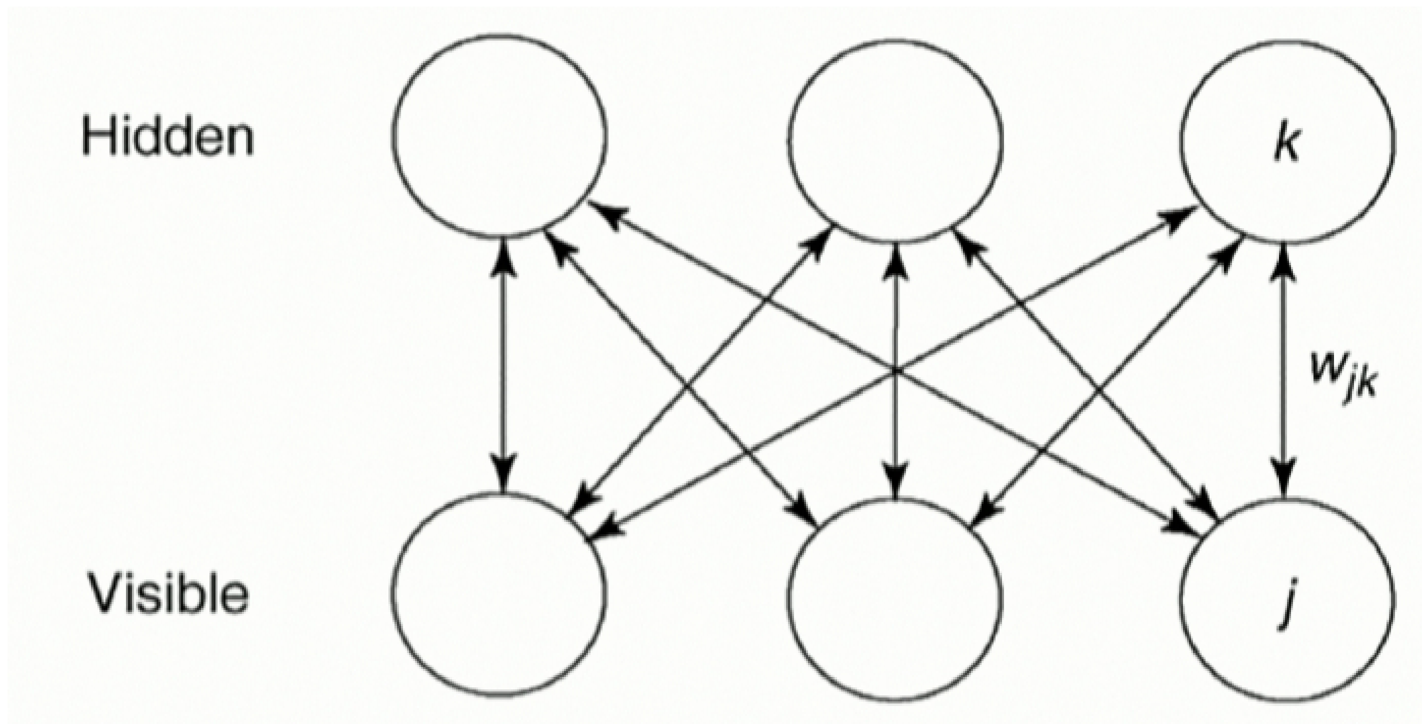
RESTRICTED BOLTZMANN MACHINE



Energy model: $$\mathbf{E}(\{v_i\},\{h_j\}) = \sum_i b_j h_j + \sum_{ij} v_i w_{ij} h_j + \sum_i c_i v_i$$

Joint probability: $$p_\lambda(\{v_i\},\{h_j\}) = \frac{e^{-\mathbf{E}(\{v_i\},\{h_j\})}}{\mathcal{Z}}$$

$$p_\lambda(\{v_i\}) = \sum_{\{h_j\}} p_\lambda(\{v_i\},\{h_j\}) = \frac{e^{-\mathbf{H}_\lambda^{RBM}[\{v_i\}]}}{\mathcal{Z}}$$

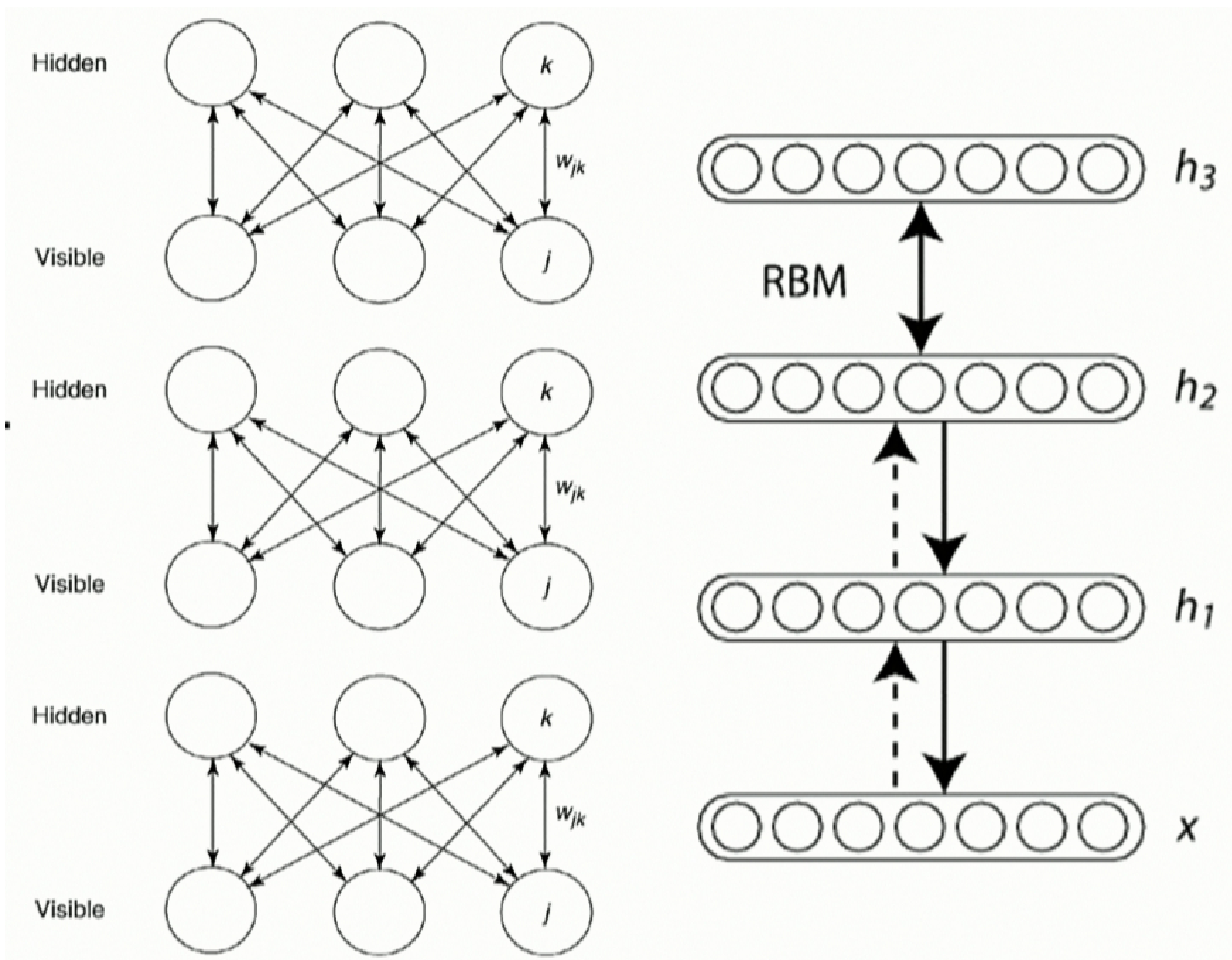
STATISTICAL MECHANICS

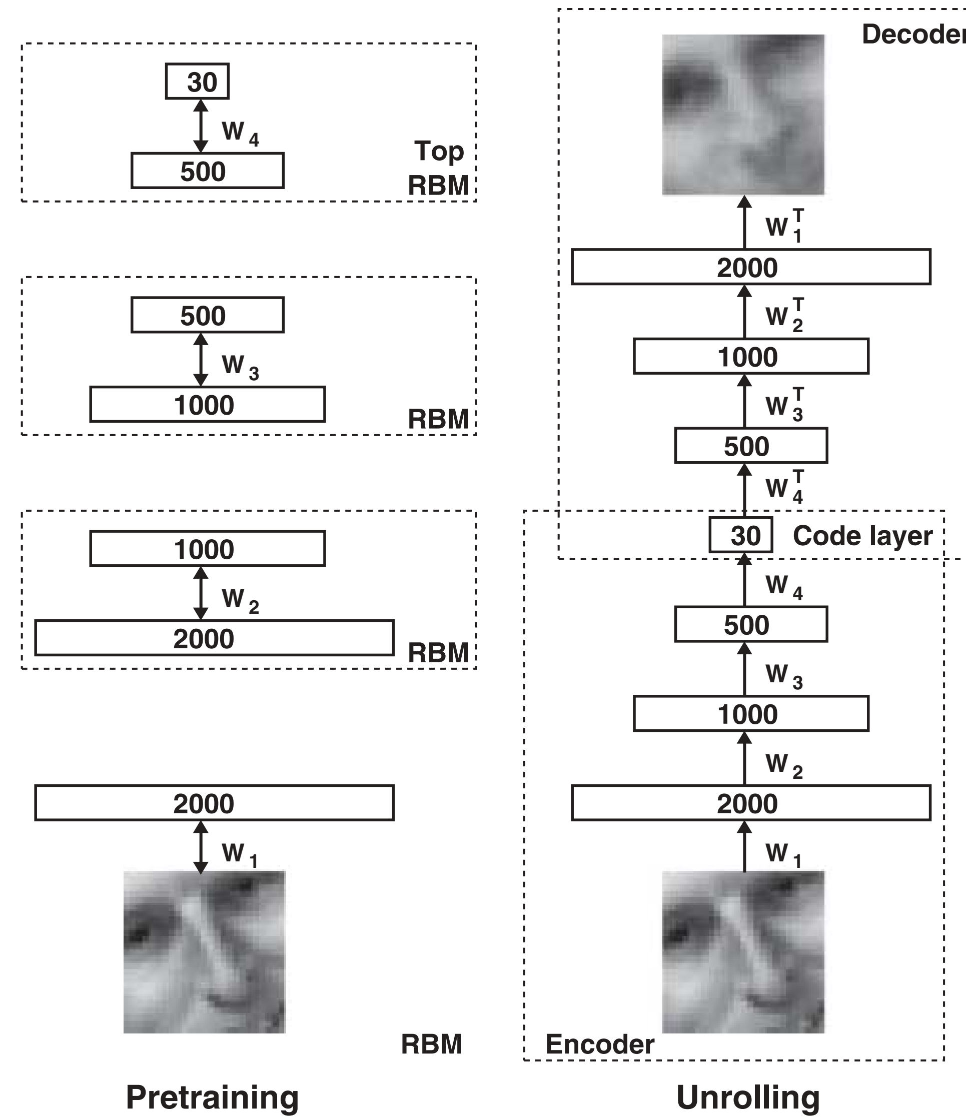
Boltzmann Distribution $$P(\{v_i\}) = \frac{e^{-\mathbf{H}(\{v_i\})}}{Z}$$

$$Z = \mathrm{T}r_{v_i} e^{-\mathbf{H}(\{v_i\})} \equiv \sum_{v_1, \ldots v_N = \pm 1} e^{-\mathbf{H}(\{v_i\})}.$$

$$\mathbf{H}[\{v_i\}] = -\sum_i K_i v_i - \sum_{ij} K_{ij} v_i v_j - \sum_{ijk} K_{ijk} v_i v_j v_k + \ldots.$$

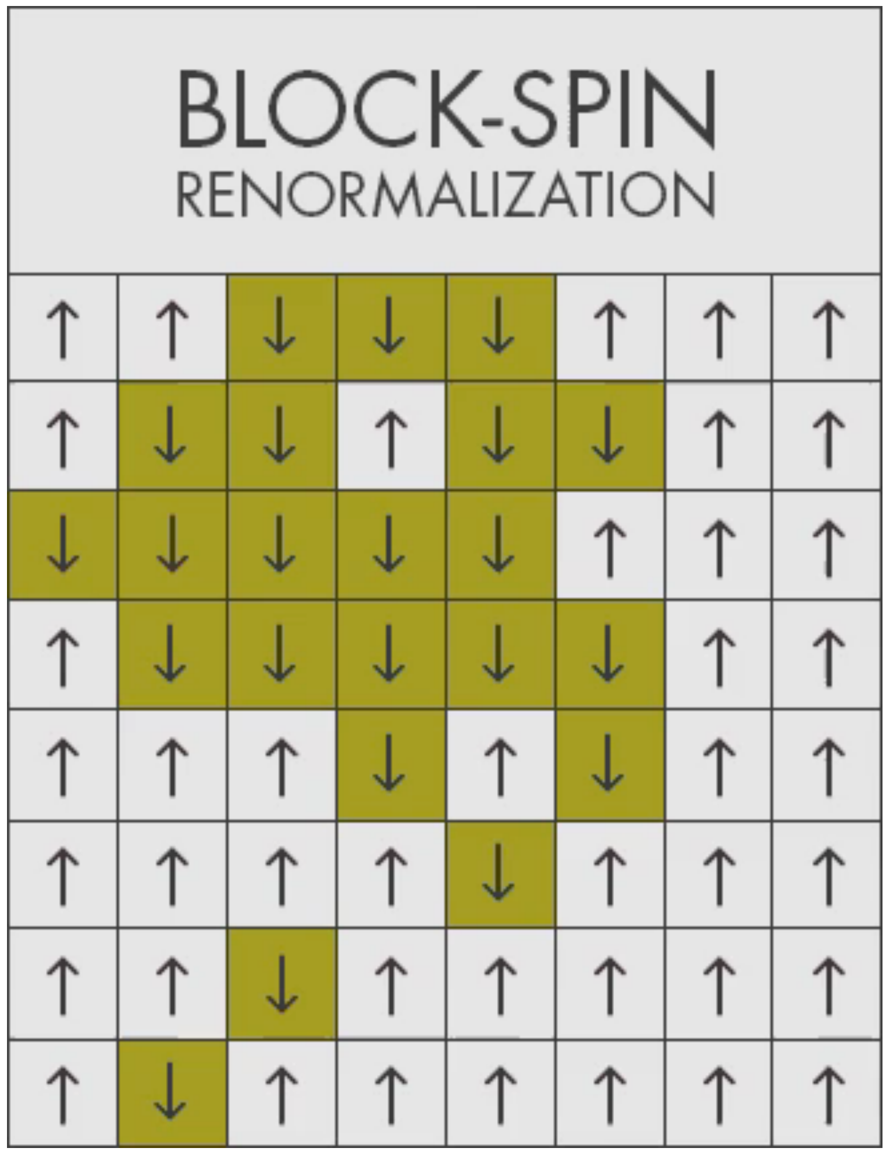
DEEP LEARNING

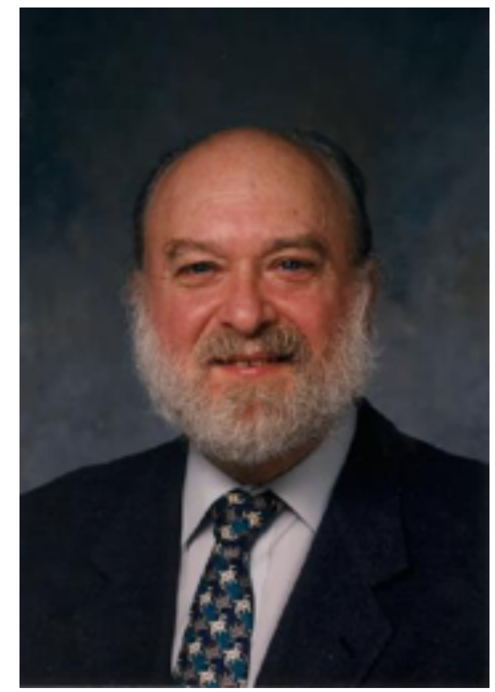




G. E. Hinton and R. R. Salakhutdinov, Science 313, 504 (2006).

RENORMALIZATION GROUP





Leo Kadanoff

$$\mathbf{H}^{RG}[\{h_j\}] = -\sum_i \tilde{K}_i h_i - \sum_{ij} \tilde{K}_{ij} h_i h_j - \sum_{ijk} \tilde{K}_{ijk} h_i h_j h_k + \ldots$$

DEEP LEARNING = RENORMALIZATION GROUP

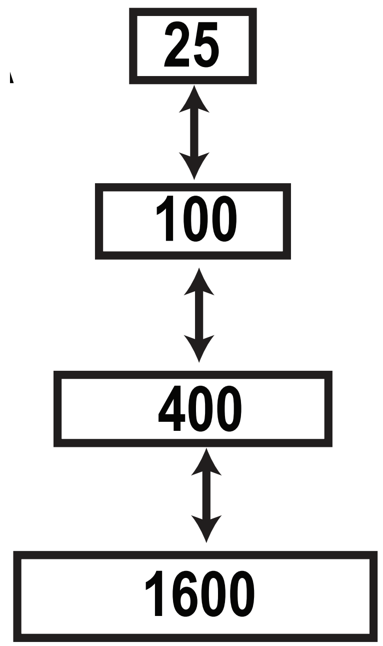
Pankaj Mehta

David Schwab

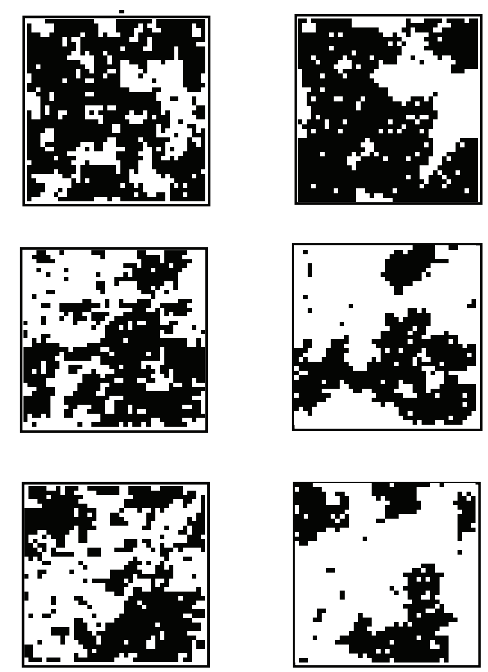
$$\mathbf{H}^{RG}_{\lambda}[\{h_j\}] = \mathbf{H}^{RBM}_{\lambda}[\{h_j\}]$$

http://arxiv.org/abs/1410.3831

DEEP LEARNING NETWORK AS RG

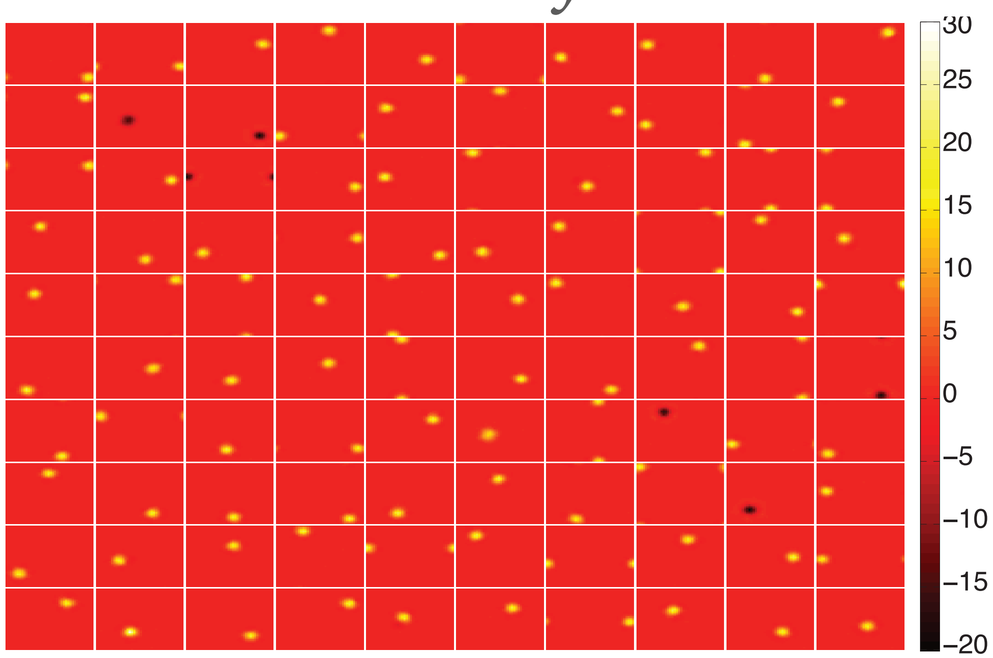


Sample

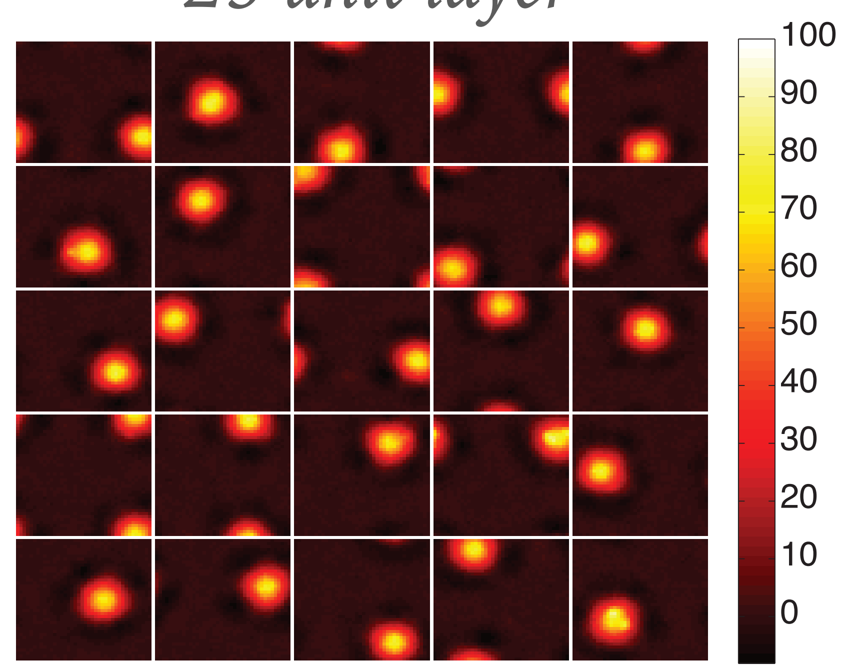


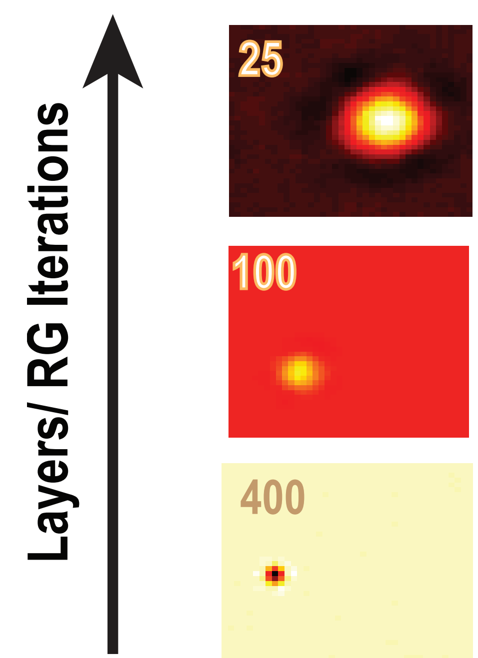
Reconstruction

100 unit layer



25 unit layer

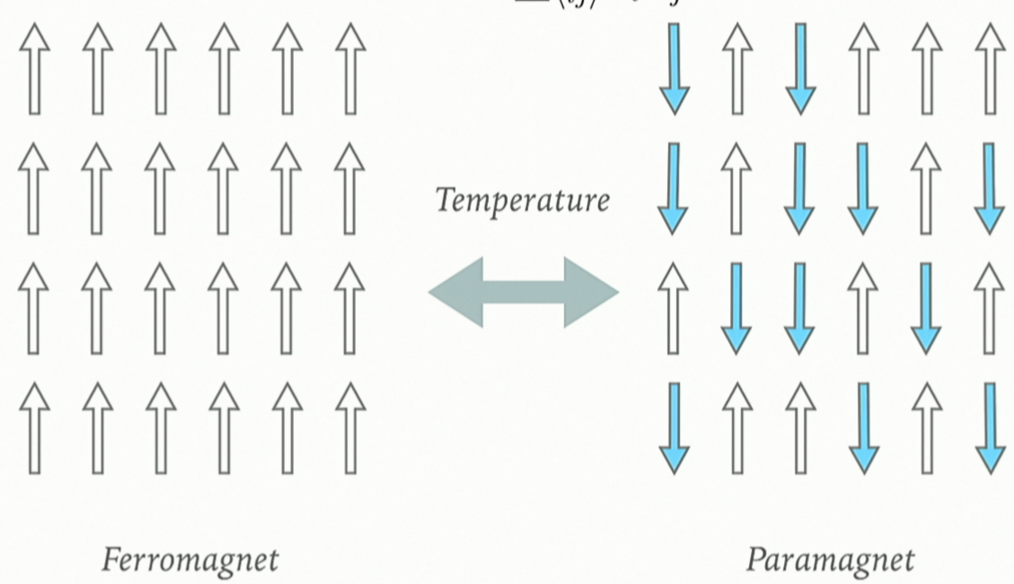




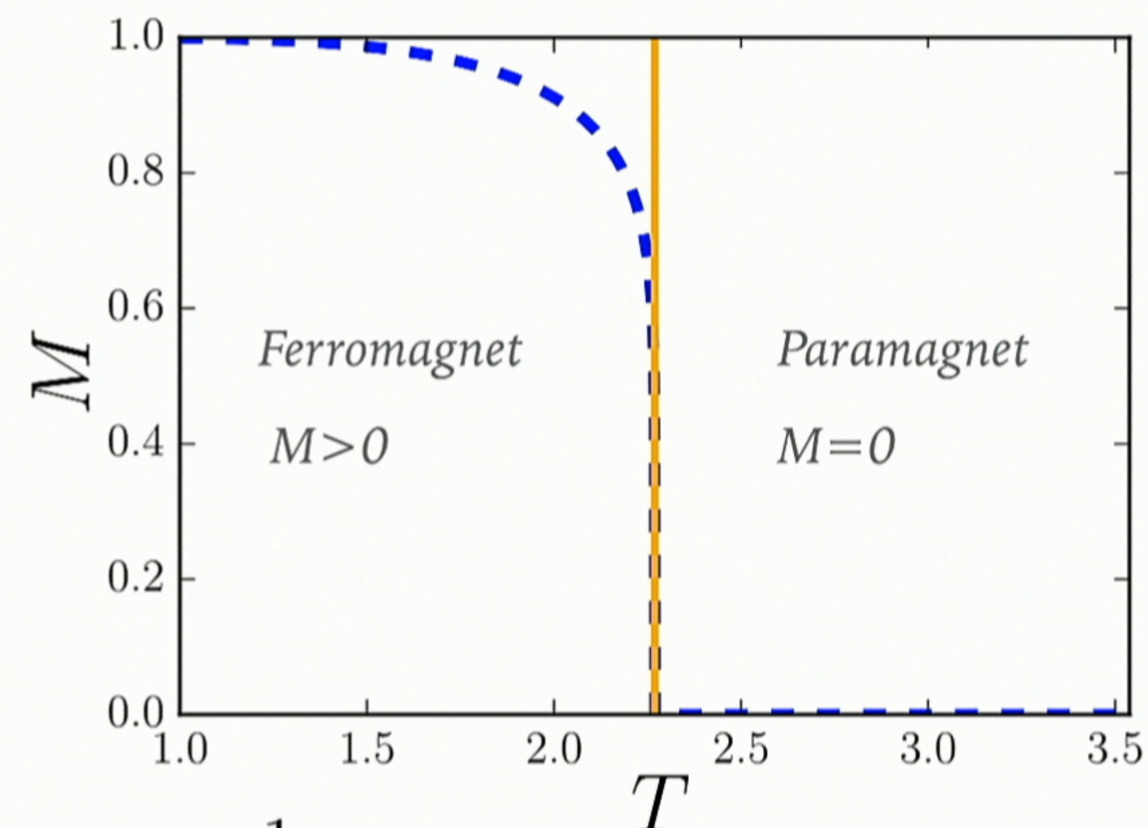
http://arxiv.org/abs/1410.3831

LEARNING PHYSICS

$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$



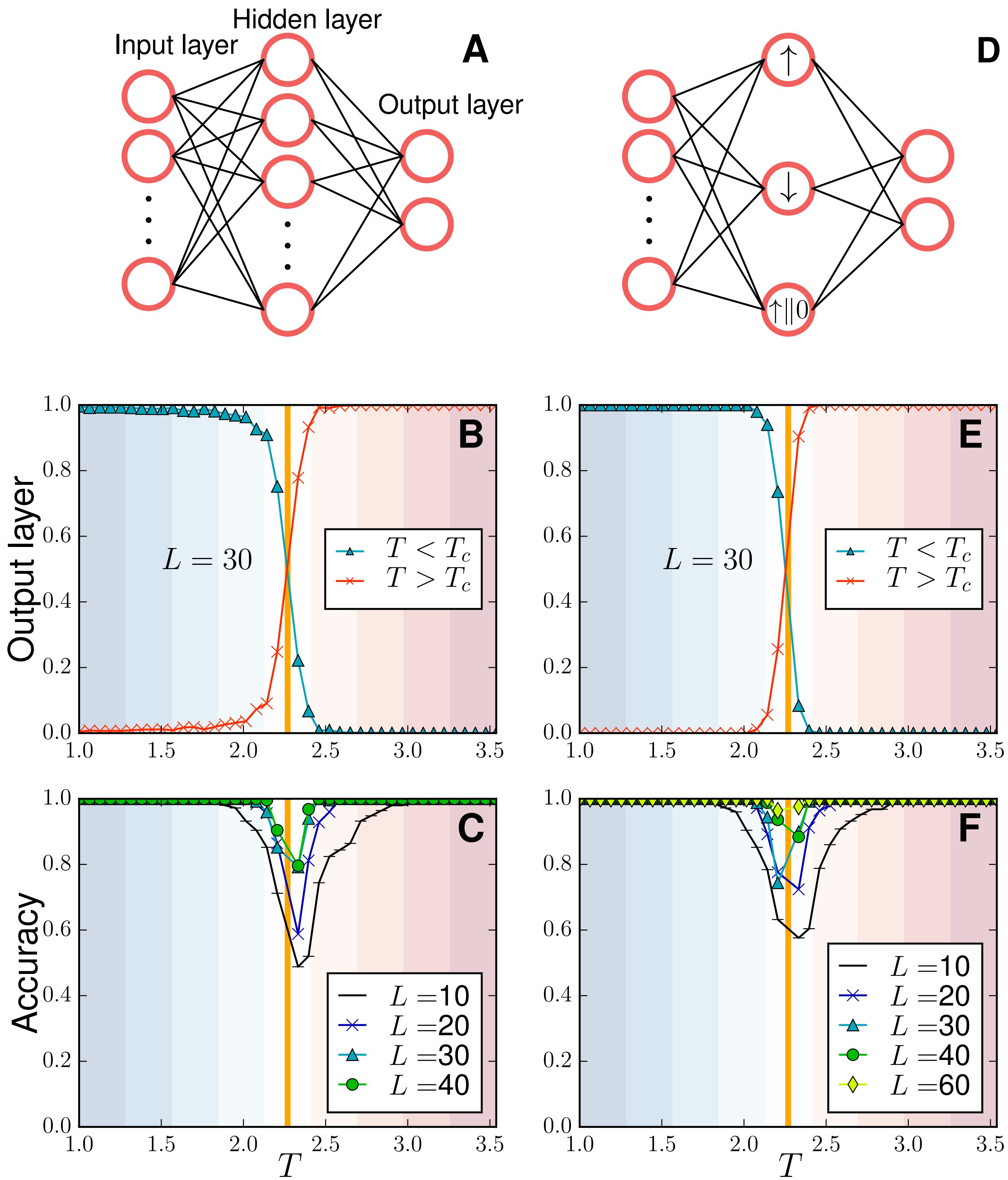
Phase transition



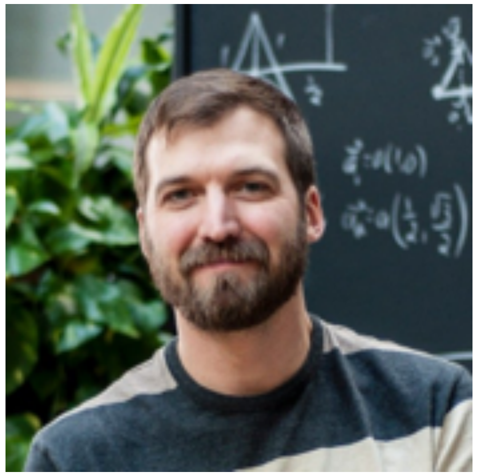
It is a measure of the degree of order in the system

$$M = \frac{1}{N} \sum_i \langle \sigma_i \rangle, \qquad \sigma_i = \pm 1$$

Lars Onsager Phys. Rev. 65, 117



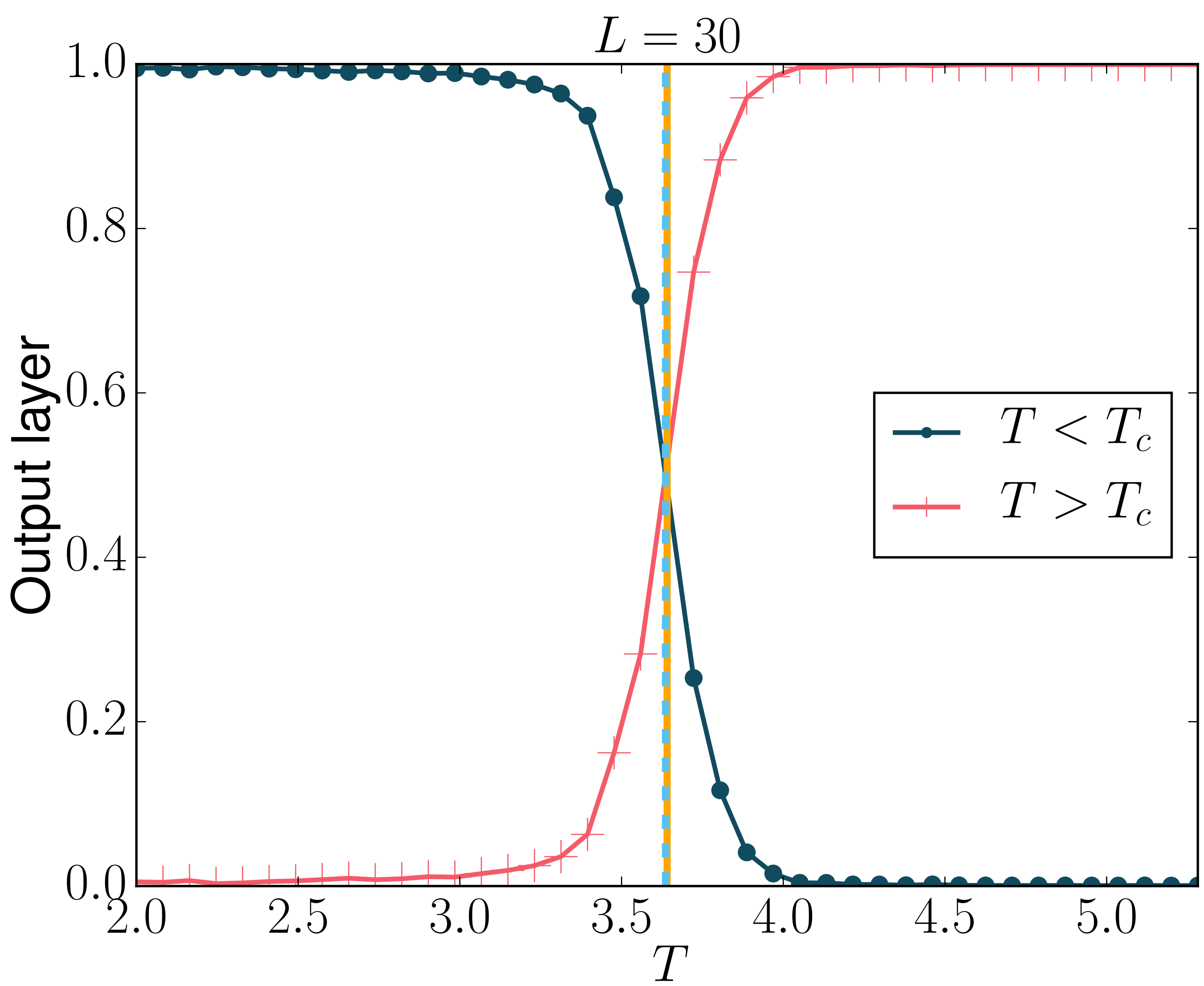
Juan Carrasquilla

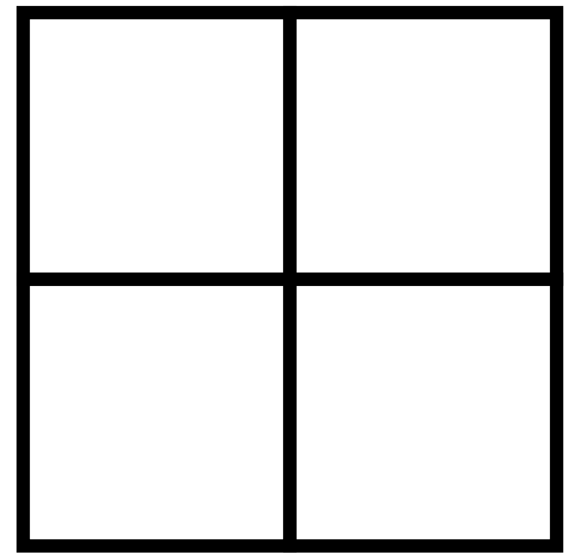


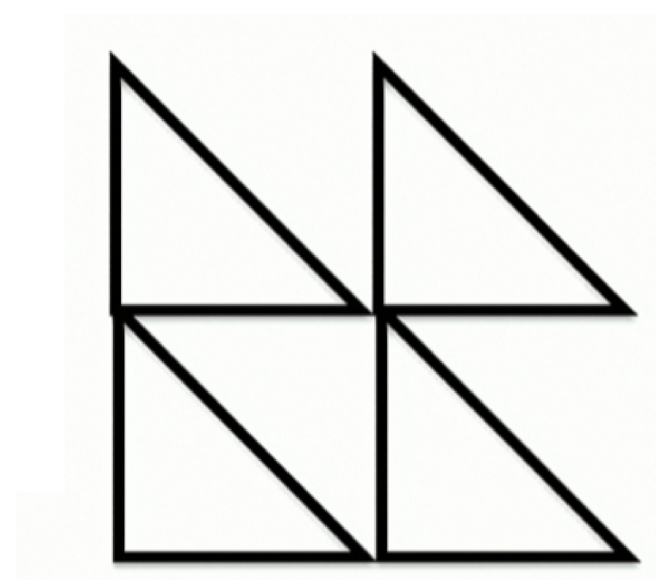
Roger Melko

http://arxiv.org/abs/1605.01735

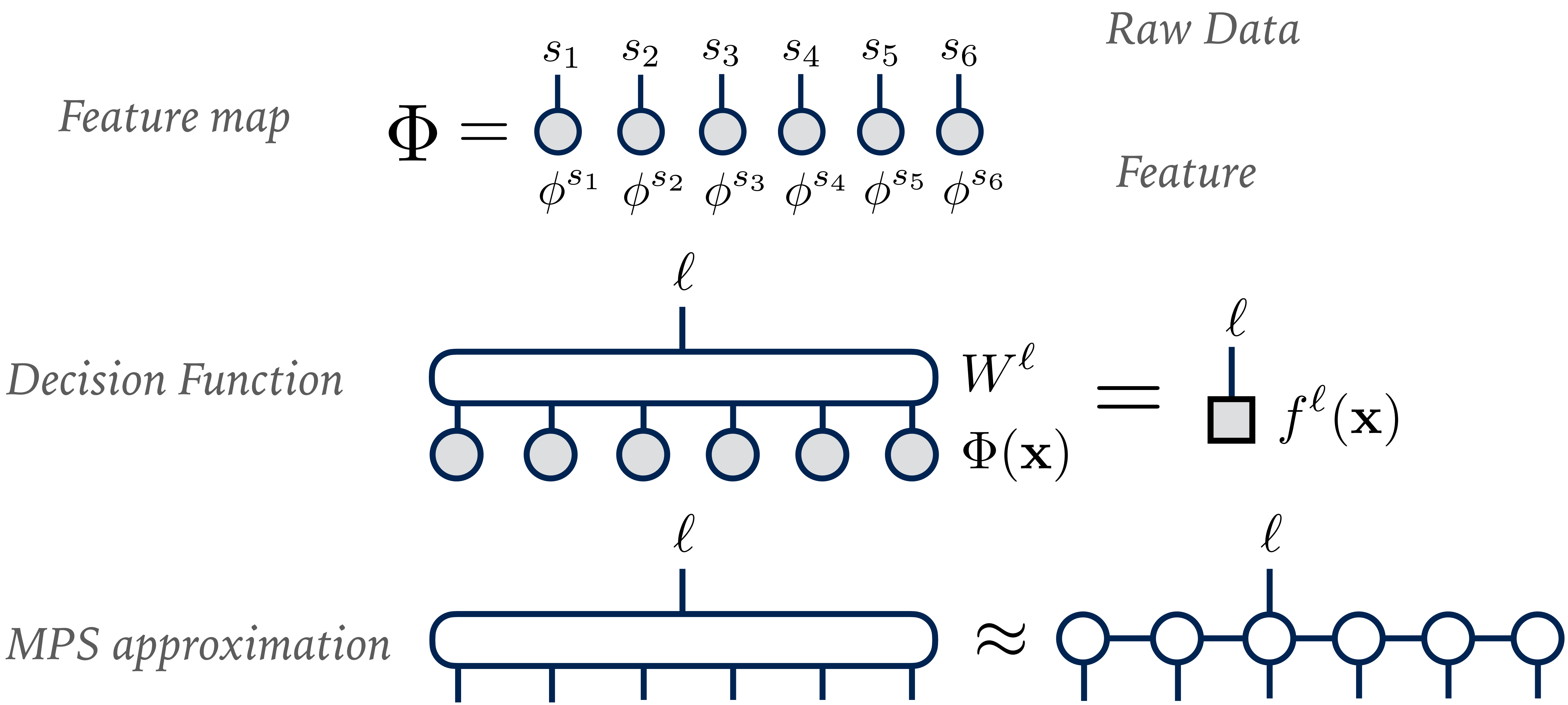
APPLY TO DIFFERENT SYSTEM







LEARNING THROUGH QUANTUM INSPIRED NETWORK



- Tensor networks approximate exponentially quantum states
- Apply tensor networks to learning

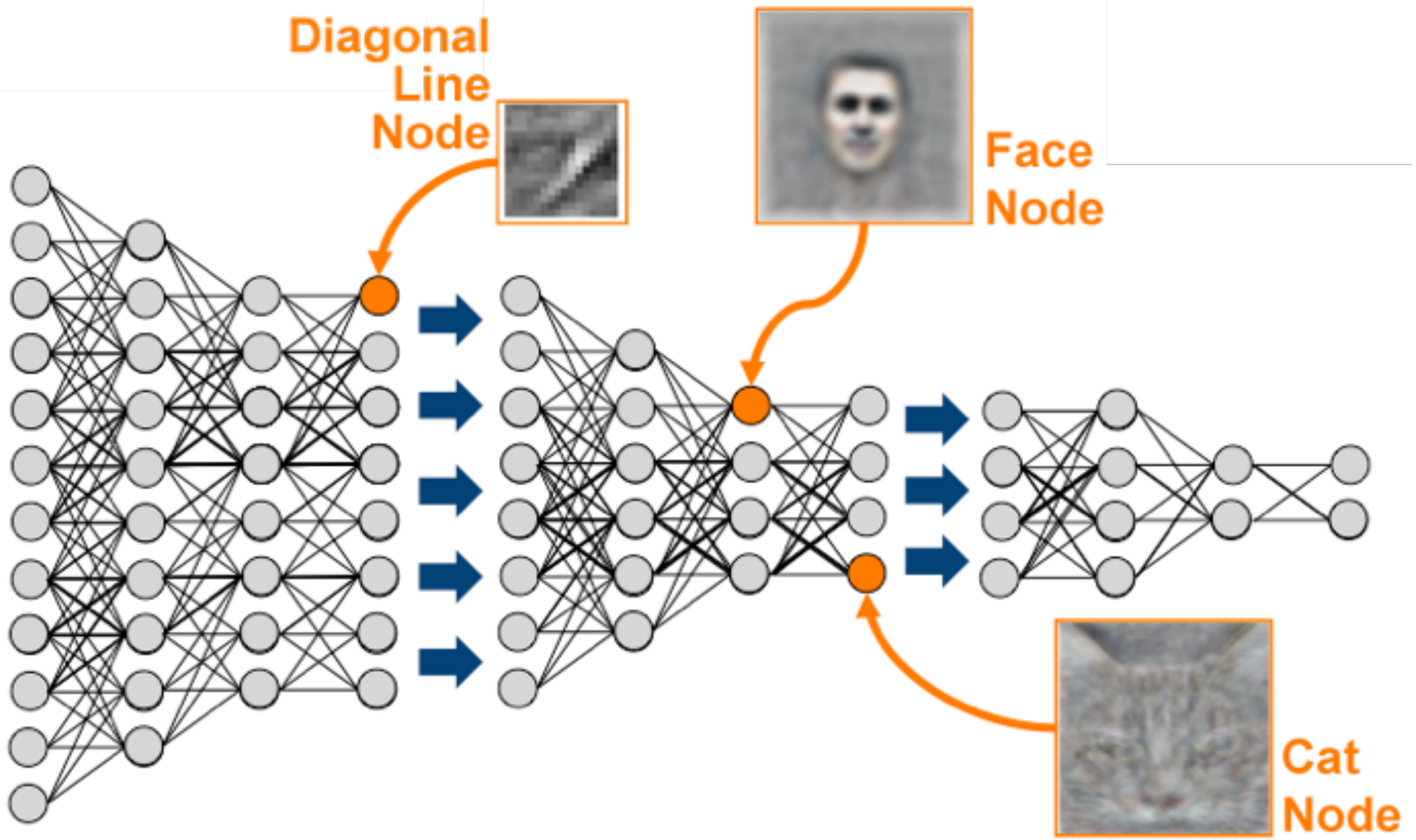
MNIST

Bond dimension	Test Set Error
m = 10	~5% (500/10,000 incorrect)
m = 20	~2% (200/10,000 incorrect)
m = 120	0.97% (97/10,000 incorrect)

Miles Stoudenmire

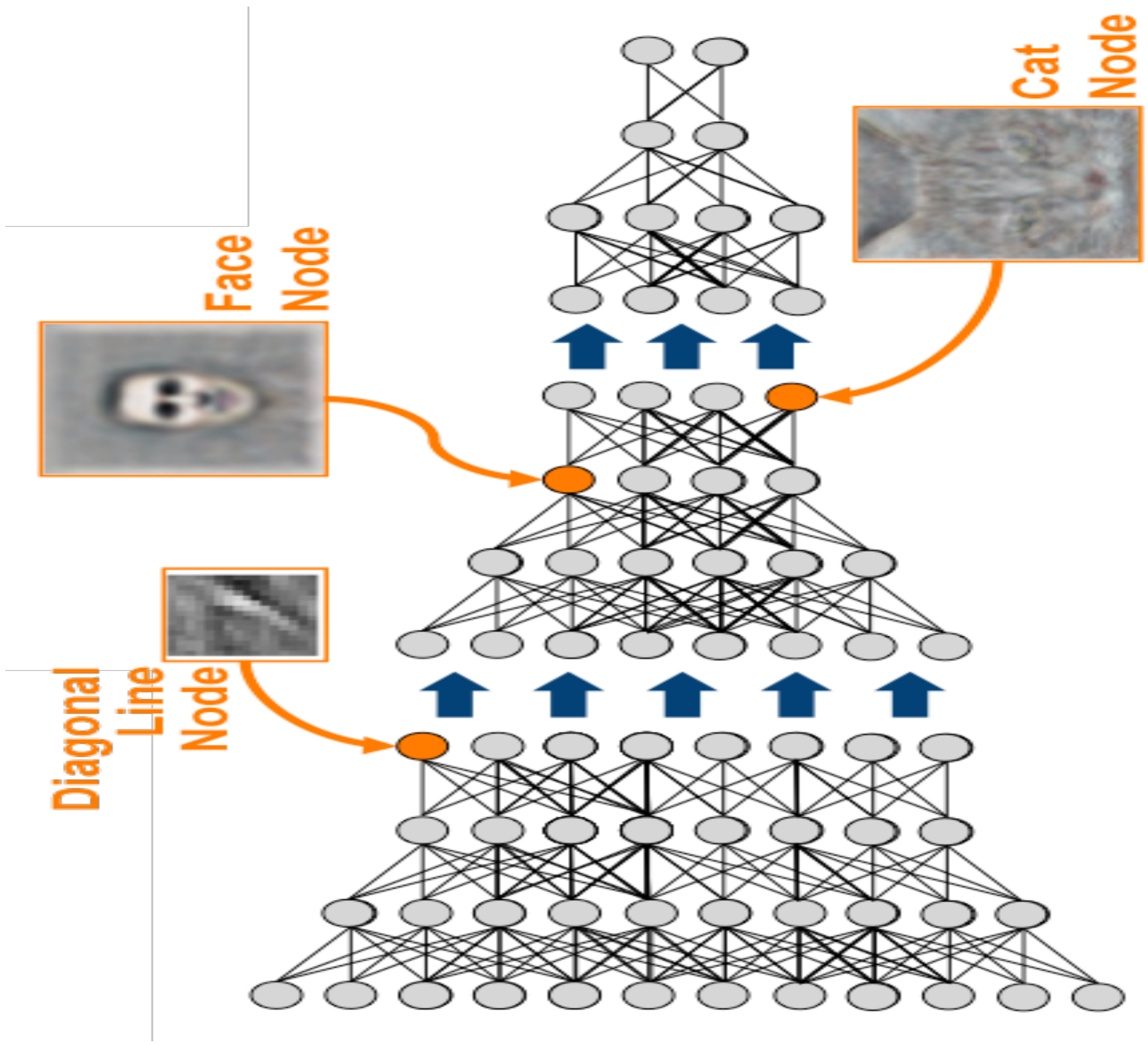
David Schwab

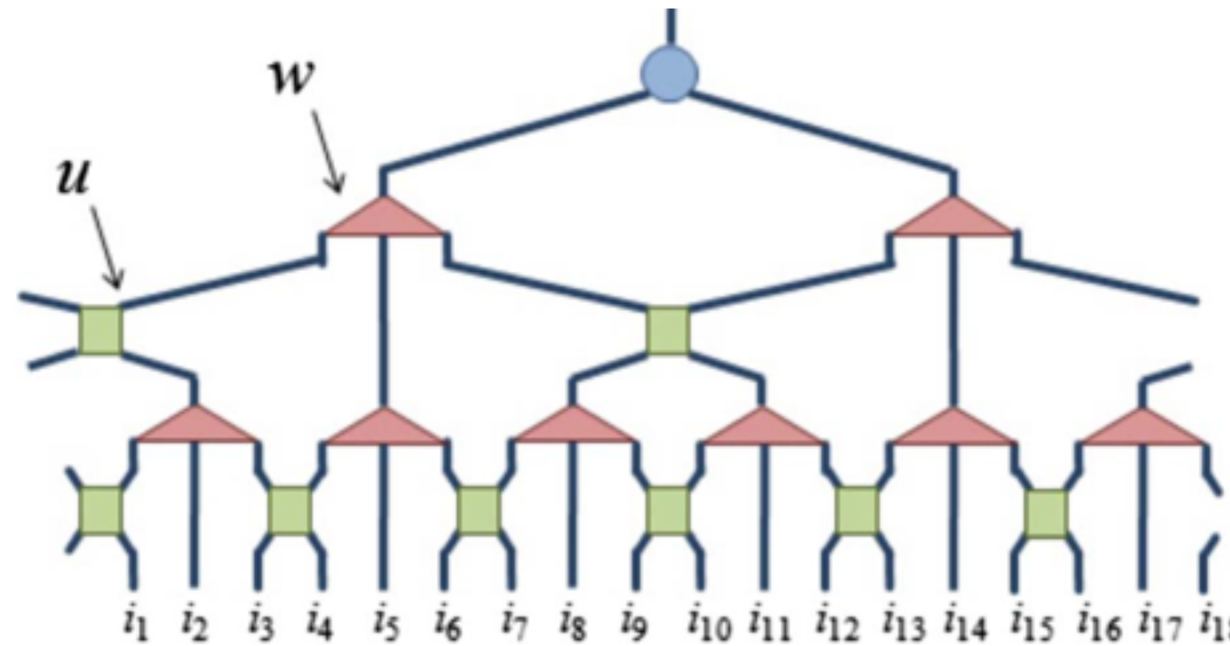
DNN AND TENSOR NETWORK



http://theanalyticsstore.ie/deep-learning/

TENSOR NETWORK





Deep learning and the renormalization group
http://arxiv.org/abs/1301.3124

An exact mapping between the Variational Renormalization Group and Deep Learning
http://arxiv.org/abs/1410.3831

GOALS

- Develop Quantum algorithms to speed up ML
  - Quantum ML
- Use ML to study physics
  - Phase transition, Fermion Sign Problem, etc.
- Develop new quantum-inspired algorithm for classical ML
  - Use tensor networks (MPS, PEPS, MERA) in ML
  - Critical Dataset (Power-law correlation)
- Understanding why DNN/CNN/RNN works using physics