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Assignment 03
CS-3200
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Assignment 03 Report

In my matlab files, those are called programs are my source code. And the readme notes are in the matlab file too. I wrote those readme files as comments.

1.

$$.A = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ * & 1 & 0 \\ * & * & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

$$L=\begin{bmatrix} 1 & 0 & 0 \\ L21 & 1 & 0 \\ L31 & L32 & 1 \end{bmatrix} \quad U=\begin{bmatrix} 4 & 1 & -2 \\ 0 & \mathrm{U22} & \mathrm{U23} \\ 0 & 0 & U33 \end{bmatrix}$$

Multiplying out LU and setting the answer equal to A gives:

$$\begin{pmatrix} 4 & 1 & -2 \\ 4 * \mathrm{L21} & \mathrm{L21} + \mathrm{U22} & -2 * \mathrm{L21} + \mathrm{U23} \\ 4 * L31 & \mathrm{L31} + \mathrm{L32} * \mathrm{U22} & -2 * L31 + L32 * U23 + U33 \end{pmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix}$$

L21 = 1
U22 = 3
U23 = -1
L31 = 2
L32 = 2/3
U33 = -14/3

So an LU decomposition of A is

$$.A = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 14/3 \end{bmatrix}$$

2.

If A=LU, then Ax=LUx=b. Thus, first solve Ly=b, for y, then solve Ux=y for x.

Since $Ax=\begin{bmatrix} 0 \\ 3 \\ 16 \end{bmatrix}=b$

LY=b for the vector Y = $\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix}$

So LY = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix}\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix}$ = $\begin{bmatrix} 0 \\ 3 \\ 16 \end{bmatrix} = b$

y1 = 0
y1+y2 = 3 => y2=3
2*y1 + 2/3 * y2 + y3 = 16 => y3 = 14

Now that we have found Y we finish the procedure by solving UX = Y for X. That is we solve:

Ux = $\begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & -14/3 \end{bmatrix}\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix}$ = $\begin{pmatrix} 0 \\ 3 \\ 14 \end{pmatrix}$ = Y

x3 = 3
3*x2+(-1)*x3 = 3 => x2 = 2
4*x1+x2+(-2)*x3=0 => x1= 1

3.

B=$\begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 2 \end{bmatrix}$ C=$\begin{bmatrix} 2 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 4 \end{bmatrix}$

For B, we have where

L=$\begin{bmatrix} 1 & 0 & 0 \\ \mathrm{L21} & 1 & 0 \\ L31 & \mathrm{L32} & 1 \end{bmatrix}$ and U=$\begin{bmatrix} U11 & U12 & U13 \\ 0 & \mathrm{U22} & \mathrm{U23} \\ 0 & 0 & U33 \end{bmatrix}$

Multiplying out LU and setting the answer equal to B gives:

$$\begin{bmatrix} U11 & U12 & U13 \\ \mathrm{L21U11} & \mathrm{L21U12} + \mathrm{U22} & \mathrm{L21U13} + \mathrm{U23} \\ L31U11 & \mathrm{L31U12} + \mathrm{L32U22} & L31U13 + L32U23 + U33 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 2 \end{bmatrix}$$

U11=4
U12=1
U13=-2
L21=1
U22=3
U23=-1
L31=2
L32=2/3
U33=20/3
So an LU decomposition of B is

$$B = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 20/3 \end{bmatrix}$$

For C, we have where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \mathrm{L21} & 1 & 0 \\ L31 & \mathrm{L32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} U11 & U12 & U13 \\ 0 & \mathrm{U22} & \mathrm{U23} \\ 0 & 0 & U33 \end{bmatrix}$$

Multiplying out LU and setting the answer equal to B gives:

$$\begin{bmatrix} U11 & U12 & U13 \\ \mathrm{L21U11} & \mathrm{L21U12 + U22} & \mathrm{L21U13 + U23} \\ L31U11 & \mathrm{L31U12 + L32U22} & L31U13 + L32U23 + U33 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 4 \end{bmatrix}$$

U11=2

U12=1

U13=-2

L21=2

U22=2

U23=1

L31=4

L32=0

U33=12

So an LU decomposition of C is

$$C = \begin{bmatrix} 2 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 12 \end{bmatrix}$$

Since LU decomposition of A, B, and C are:

$$.A = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & -14/3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 & -2 \\ 0 & 3 & -1 \\ 0 & 0 & 20/3 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 1 & -2 \\ 4 & 4 & -3 \\ 8 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & 12 \end{bmatrix}$$

So for A and B, their LU decompositions are similar, but for U33 of A is -14/3, U33 for B is 20/3

For A and C, their LU decompositions are NOT similar because there are a lot differences.

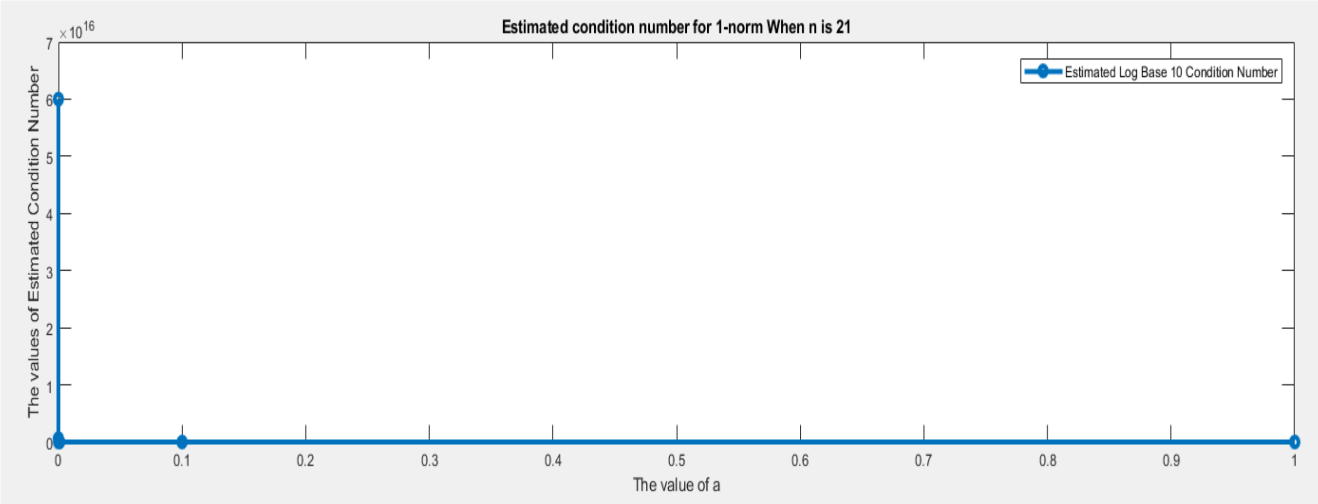
For L31 of A is 2, L31 of C is 4; L32 of A is 2/3, L32 of C is 0; U11 of A is 4, U11 of C is 2; U22 of A is 3, U22 of C is 2; U23 of A is -1, U23 of C is 1; U33 of A is -14/3, U33 of C is 12.

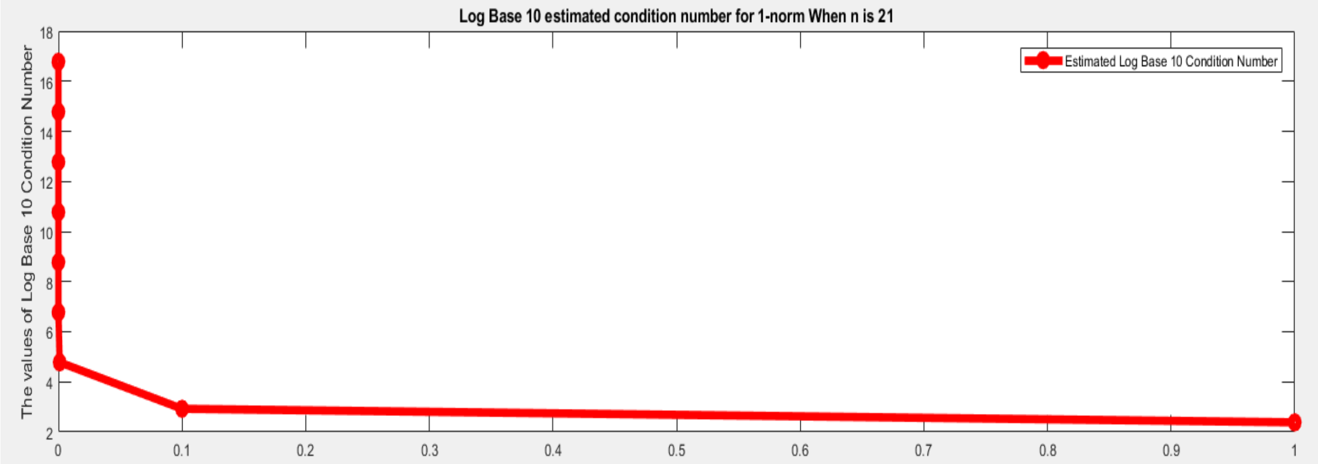
4.

I used the 1-norm to compute the estimated condition number. In matlab, I used the cond() function to compute the condition number.

Value of a / Value of n	1.0	0.1	1.0e-3	1.0e-5	1.0e-7
21	242	840	6.0264e+04	6.0003e+06	6.0000e+08
41	882	3.0764e+03	2.2092e+05	2.2001e+07	2.2000e+09
81	3.3620e+03	1.1738e+04	8.4344e+05	8.4003e+07	8.4000e+09
161	1.3122e+04	4.5818e+04	3.2933e+06	3.2801e+08	3.2800e+10

Value of a / Value of n	1.0e-9	1.0e-11	1.0e-13	1.0e-15
21	6.0000e+10	6.0000e+12	6.0000e+14	6.0000e+16
41	2.2000e+11	2.2000e+13	2.2000e+15	2.2000e+17
81	8.4000e+11	8.4000e+13	8.4000e+15	8.4000e+17
161	3.2800e+12	3.2800e+14	3.2800e+16	3.2800e+18





So here is the graph the shows how does the condition number vary with the value of a when n = 21. The first graph is the actual estimated condition number. Since it grows too fast when a is getting smaller, so on the second graph, I used the log base 10 condition number instead of the actual estimated condition number. So this way, we can see how fast it grows on the estimated condition number when a is getting smaller.

If $H1 = 8$ and $Hr = 4$ Solve the system of equations for n= 161

When **a =1.0,**
h1 = -71.5247, h2= -151.0494, h160= -154.9506, and h161= -75.4753

When **a =1.0e-5,**

h1= -111.9868, h2=-231.9737, h160=-7.8026e+06, and h161= -3.9013e+06

When **a = 1.0e-15,**
h1 = -111.9877, h2=-231.9753, h160= -7.8025e+16, and h161 = -3.9012e+16

Iterative refinement:

When **a =1.0,** I got these improved Residuals when I run my Matlab code IterativeRefinement(1).

```
   1.0e+06 *

  Columns 1 through 11

    0.1765    0.3530    0.5293    0.7053    0.8811    1.0565    1.2314
1.4058    1.5796    1.7527    1.9250

  Columns 12 through 22

    2.0966    2.2672    2.4369    2.6055    2.7731    2.9395    3.1047
3.2685    3.4311    3.5922    3.7518

  Columns 23 through 33

    3.9100    4.0665    4.2214    4.3746    4.5260    4.6756    4.8233
4.9691    5.1130    5.2548    5.3946

  Columns 34 through 44

    5.5322    5.6676    5.8009    5.9319    6.0606    6.1869    6.3108
6.4324    6.5514    6.6679    6.7819

  Columns 45 through 55

    6.8933    7.0021    7.1082    7.2116    7.3123    7.4102    7.5053
7.5976    7.6871    7.7736    7.8573

  Columns 56 through 66

    7.9380    8.0158    8.0906    8.1624    8.2311    8.2968    8.3594
8.4190    8.4754    8.5287    8.5788

  Columns 67 through 77
```

8.6258 8.6696 8.7102 8.7476 8.7818 8.8128 8.8406
8.8651 8.8863 8.9043 8.9191

Columns 78 through 88

8.9306 8.9388 8.9437 8.9454 8.9438 8.9389 8.9307
8.9193 8.9046 8.8867 8.8655

Columns 89 through 99

8.8410 8.8133 8.7824 8.7482 8.7109 8.6703 8.6265
8.5796 8.5295 8.4763 8.4199

Columns 100 through 110

8.3604 8.2978 8.2322 8.1635 8.0917 8.0170 7.9393
7.8586 7.7749 7.6884 7.5990

Columns 111 through 121

7.5067 7.4116 7.3137 7.2131 7.1097 7.0036 6.8949
6.7835 6.6695 6.5530 6.4340

Columns 122 through 132

6.3125 6.1886 6.0622 5.9336 5.8026 5.6693 5.5339
5.3962 5.2565 5.1146 4.9708

Columns 133 through 143

4.8250 4.6772 4.5276 4.3762 4.2230 4.0680 3.9115
3.7533 3.5937 3.4325 3.2699

Columns 144 through 154

3.1060 2.9408 2.7743 2.6067 2.4380 2.2683 2.0976
1.9260 1.7536 1.5804 1.4065

Columns 155 through 161

1.2321 1.0571 0.8816 0.7057 0.5296 0.3532 0.1766

When **a =1.0e-5,** I got these improved Residuals when I run my Matlab code

IterativeRefinement(**1.0e-5**).

1.0e+15 *

Columns 1 through 11

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 12 through 22

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 23 through 33

0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001
0.0001 0.0001 0.0001 0.0001

Columns 34 through 44

0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001
0.0001 0.0001 0.0001 0.0001

Columns 45 through 55

0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001
0.0001 0.0001 0.0001 0.0001

Columns 56 through 66

0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001
0.0001 0.0001 0.0001 0.0001

Columns 67 through 77

0.0001 0.0002 0.0002 0.0002 0.0002 0.0002 0.0002
0.0002 0.0002 0.0002 0.0002

Columns 78 through 88

0.0002 0.0002 0.0002 0.0002 0.2215 0.4423 0.6624
0.8813 1.0987 1.3142 1.5274

Columns 89 through 99

1.7381 1.9458 2.1503 2.3512 2.5483 2.7412 2.9298
3.1136 3.2925 3.4662 3.6344

Columns 100 through 110

3.7970 3.9537 4.1043 4.2486 4.3864 4.5175 4.6418
4.7591 4.8693 4.9722 5.0676

Columns 111 through 121

5.1555 5.2358 5.3083 5.3730 5.4298 5.4786 5.5193
5.5520 5.5765 5.5928 5.6010

Columns 122 through 132

5.6009 5.5927 5.5763 5.5517 5.5190 5.4782 5.4293
5.3725 5.3077 5.2351 5.1548

Columns 133 through 143

5.0668 4.9713 4.8684 4.7582 4.6408 4.5165 4.3853
4.2474 4.1031 3.9525 3.7958

Columns 144 through 154

3.6332 3.4649 3.2912 3.1124 2.9285 2.7400 2.5471
2.3501 2.1492 1.9448 1.7371

Columns 155 through 161

1.5265 1.3134 1.0980 0.8807 0.6619 0.4419 0.2211

When **a =1.0e-15,** I got these improved Residuals when I run my Matlab code IterativeRefinement(**1.0e-15**).
1.0e+35 *

Columns 1 through 11

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 12 through 22

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 23 through 33

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 34 through 44

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 45 through 55

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 56 through 66

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 67 through 77

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000

Columns 78 through 88

0.0000 0.0000 0.0000 0.0000 0.2213 0.4421 0.6622
0.8811 1.0985 1.3139 1.5272

Columns 89 through 99

1.7378 1.9455 2.1500 2.3509 2.5480 2.7409 2.9294
3.1132 3.2921 3.4658 3.6341

Columns 100 through 110

3.7966 3.9533 4.1039 4.2482 4.3860 4.5171 4.6414
4.7588 4.8689 4.9718 5.0672

Columns 111 through 121

5.1552 5.2354 5.3080 5.3726 5.4294 5.4782 5.5190
5.5516 5.5761 5.5925 5.6006

Columns 122 through 132

5.6006 5.5924 5.5759 5.5514 5.5187 5.4778 5.4290
5.3722 5.3074 5.2348 5.1545

Columns 133 through 143

5.0665 4.9710 4.8681 4.7579 4.6406 4.5162 4.3850
4.2472 4.1029 3.9523 3.7956

Columns 144 through 154

3.6330 3.4648 3.2911 3.1122 2.9284 2.7399 2.5470
2.3500 2.1491 1.9447 1.7370

Columns 155 through 161

1.5264 1.3133 1.0979 0.8806 0.6618 0.4419 0.2211

Then, we can conclude that, these number has been improved by using Iterative refinement.