High-dimensional Cost-constrained Regression via Non-convex Optimization

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Outline

- Research Problem
- Method
- Theoretical Properties
- Simulation Study
- Application to a diabetes study

High Dimensional Cost-constrained Regression

Consider the following linear regression model

$$y = x^T \beta^0 + \epsilon,$$

where

$$x = (x_1, x_2, \ldots, x_p)^T,\ \mathrm{E}(x) = 0, \mathrm{Cov}(x) = \mathbf{\Sigma},\ \mathrm{E}(\epsilon) = 0, \mathrm{Var}(\epsilon) = \sigma^2.$$

In practice, we need to spend money on collecting data. Suppose we need to spend c_j dollars on collecting the value of the j-th predictor x_j and our budget is C dollars. Assume that C is small and therefore our proposed model can only use a few predictors.

Question:

How to find a predictive linear model that satisfies the budget constraint and has good prediction performance?

High Dimensional Cost-constrained Regression

The regression coefficient vector of the best cost-constrained regression model is

$$
\begin{aligned}
\beta^* &\in \arg\min_{\beta} \mathrm{E}[(y - \sum_{j=1}^{p} x_j \beta_j)^2] \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \le C \\
&= \arg\min_{\beta} \mathrm{E}[(\sum_{j=1}^{p} x_j \beta_j^0 + \epsilon - \sum_{j=1}^{p} x_j \beta_j)^2] \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \le C \\
&= \arg\min_{\beta} (\beta - \beta^0)^T \mathbf{\Sigma} (\beta - \beta^0) \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \le C,
\end{aligned}
$$

where $\mathcal{I}(\beta_j)$ is an indicator function which equals to 1 if $\beta_j \neq 0$ and 0 otherwise.

High Dimensional Cost-constrained Regression

Given the training data $\{Y, \mathbf{X}\}$, it is natural to estimate β^* by solving the following sample-average approximation (SAA) problem

$$\min_{\beta} \frac{1}{2n} \|Y - \mathbf{X}\beta\|_2^2 \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C.$$

- This problem can be viewed as a generalized best subset selection problem (the case with $c_1 = c_2 = \cdots = c_p$).
- The constraint $\sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C$ makes this problem NP-hard.
- Even for the best subset selection problem, in order to find the global solution, most state-of-the-art algorithms do not scale to problems with more than 30 variables (Bertsimas et al. (2016)).

The difference between β^* and β^0

Since we aim to find the best cost-constrained regression model, the parameter of interest is β^* rather than β^0 in the true linear model.

$$\beta^* = \arg\min_{\beta} (\beta - \beta^0)^T \mathbf{\Sigma} (\beta - \beta^0) \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C,$$

Denote $S = \{j : \beta_j^0 \neq 0\}$.

- If $\sum_{j=1}^{p} c_j \mathcal{I}(\beta_j^0) = \sum_{j \in S} c_j \leq C$, then we know that β^0 is a feasible solution and $\beta^* = \beta^0$.
- If $\sum_{j \in S} c_j > C$ and $\mathrm{Cov}(x_S, x_{S^c}) = 0$, we can prove that $\beta^*_{S^c} = 0$ and

$$\beta^*_S = \arg\min_{\beta} (\beta_S - \beta^0_S)^T \mathbf{\Sigma}_{SS} (\beta_S - \beta^0_S)$$
$$\text{subject to } \sum_{j \in S} c_j \mathcal{I}(\beta_j) \leq C.$$

Could we use a two-step method?

For the above two cases, we have $\beta^*_{S^c} = \beta^0_{S^c} = 0$.

Therefore, if β^0 is sparse, we can use the following two-step method:

Step 1: Perform a screening to find a subset $\hat{S}$ which contains the true subset S with high probability and let $\hat{\beta}^*_{\hat{S}^c} = 0$;

Step 2: Solve the following optimization problem

$$\hat{\beta}^*_{\hat{S}} = \arg\min_{\beta_{\hat{S}}} \frac{1}{2n} \|Y - \mathbf{X}_{\hat{S}} \beta_{\hat{S}}\|_2^2$$

$$\text{subject to } \sum_{j \in \hat{S}} c_j \mathcal{I}(\beta_j) \leq C.$$

Could we use a two-step method?

However, in many cases, $\beta^*_{S^c}$ is not equal to 0 and therefore, we can not use the two-step method.

Let $p = 3, \beta = (\beta_1, \beta_2, \beta_3)^T$ and $\beta^0 = (t_0, t_0, 0)$ where $t_0 > 0$. Assume that the covariance matrix

$$\Sigma = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}$$

If $-\frac{1}{2} < \rho < -\frac{1}{3}$, we can show that $\beta^* = (0, 0, 2\rho t_0)$.

Therefore, $\{j : \beta^0_j \neq 0\} \cap \{j : \beta^*_j \neq 0\}$ is an empty set!

Method: Orthogonal Case

Assume that $n > p$ and $\mathbf{X}^T\mathbf{X} = n\mathbf{I}_n$. Under these assumptions, we can show that

$$\begin{aligned}\frac{1}{2n}\|Y - \mathbf{X}\beta\|_2^2 &= \frac{1}{2n}\|Y - \mathbf{X}\tilde{\beta}\|_2^2 + \frac{1}{2n}\|\mathbf{X}\beta - \mathbf{X}\tilde{\beta}\|_2^2 \\ &= \frac{1}{2n}\|Y - \mathbf{X}\tilde{\beta}\|_2^2 + \frac{1}{2}\|\beta - \tilde{\beta}\|_2^2,\end{aligned}$$

where $\tilde{\beta} = \mathbf{X}^T Y/n$ is the least squares estimate of the true regression coefficient β^0.

Therefore, the original problem is equivalent to the following optimization problem

$$\min_{\beta} \|\beta - \tilde{\beta}\|_2^2 \quad \text{subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C. \tag{1}$$

How to solve problem (1)?

If $\hat{\beta}$ is an optimal solution to the following problem

$$\min_{\beta} \|\beta - a\|_2^2 \quad \text{subject to} \ \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C,$$

then $\hat{\beta} = a \circ \hat{Z}$ where $\circ$ denotes the entrywise product of two vectors, and $\hat{Z} = (\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_p)$ is the solution to the following 0-1 knapsack problem

$$\max_{z_1, z_2, \ldots, z_p} \sum_{j=1}^{p} a_j^2 z_j \quad \text{subject to} \ \sum_{j=1}^{p} c_j z_j \leq C, \ \text{and} \ z_1, z_2, \ldots, z_p \in \{0, 1\}.$$

Therefore, to solve problem (1), we only need to solve a 0-1 knapsack problem.

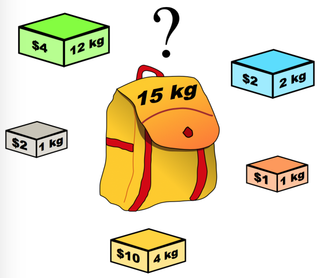
0-1 knapsack problem and dynamic programming

0-1 Knapsack Problem

$$\max_{z_1, z_2, \ldots, z_p} \sum_{j=1}^{p} v_j z_j$$

$$\text{subject to } \sum_{j=1}^{p} c_j z_j \leq C,$$

$$\text{and } z_1, z_2, \ldots, z_p \in \{0, 1\}.$$



Dynamic Programming

Denote $f[j, w]$ be the maximum value that can be attained with weight less than or equal to w using items up to j. We can define $f[j, w]$ recursively as follows:

- $f[0, w] = 0$;
- $f[j, w] = f[j-1, w]$ if $c_j > w$;
- $f[j, w] = \max(f[j-1, w], f[j-1, w-c_j] + v_j)$ if $c_j \leq w$.

The solution can then be found by calculating $f[p, C]$. The 0-1 knapsack problem can be solved in pseudo-polynomial time ($O(pC)$) using dynamic programming.

Special case: $c_1 = c_2 = \cdots = c_p = 1$

(**Bertsimas et al. (2016), Annals of Statistics**)

If $\hat{\beta}$ is an optimal solution to the following problem:

$$\hat{\beta} \in \arg\min_{\|\beta\|_0 \leq C} \|\beta - \tilde{\beta}\|_2^2,$$

then it can be computed as follows: if $|\tilde{\beta}_{(1)}| \geq |\tilde{\beta}_{(2)}| \geq \cdots \geq |\tilde{\beta}_{(p)}|$ denote the ordered values of the absolute values of the vector $\tilde{\beta}$, then

$$\hat{\beta}_j = \begin{cases} \tilde{\beta}_j, \text{ if } j \in \{(1), (2), \ldots, (C)\}, \\ 0, \text{ otherwise.} \end{cases}$$

Method: General Case

To solve the general high-dimensional cost-constrained regression problem, we use projected gradient descent methods.

- Denote $g(\beta) = \frac{1}{2n}\|Y - \mathbf{X}\beta\|_2^2$ and $L = \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$, we have

$$g(\eta) \leq Q_L(\eta, \beta) := g(\beta) + \frac{L}{2}\|\eta - \beta\|_2^2 + \langle \nabla g(\beta), \eta - \beta \rangle$$

  for all β, η with equality holding at $\beta = \eta$.

- Given a current solution $\beta^{(m)}$, we upper bound the function $g(\eta)$ by the function $Q_L(\eta, \beta^{(m)})$, and update the solution by

$$\beta^{(m+1)} = \arg\min_{\eta} Q_L(\eta, \beta^{(m)}) \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\eta_j) \leq C.$$

Method: General Case

We can show that

$$\begin{aligned}\beta^{(m+1)} &= \arg\min_{\eta} Q_L(\eta, \beta^{(m)}) \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\eta_j) \le C \\ &= \arg\min_{\eta} \|\eta - (\beta^{(m)} - \frac{1}{L}\nabla g(\beta^{(m)}))\|_2^2 \\ &\quad \text{subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\eta_j) \le C.\end{aligned}$$

Therefore, we can use the result shown in Theorem 1 to solve the above problem. As shown in our theoretical study, the sequence $g(\beta^{(m)}) = \frac{1}{2n}\|Y - X\beta^{(m)}\|_2^2$ is decreasing and the sequence $\{\beta^{(m)}\}$ converges to a near optimal solution.

High-dimensional Cost-constrained Regression (HCR)

Step 1: Choose $\delta > 0$, $L > \ell = \lambda_{max}(\frac{1}{n}\mathbf{X}^T\mathbf{X})$, and initialize $\beta^{(1)}$ such that $\sum_{j=1}^p c_j \mathcal{I}(\beta_j^{(1)}) \leq C$.

Step 2: For $m \geq 1$, denote $\mu^{(m)} = \beta^{(m)} + \frac{1}{nL}\mathbf{X}^T(Y - \mathbf{X}\beta^{(m)})$, apply dynamic programming to find

$$\beta^{(m+1)} \in \arg\min_{\eta} \|\eta - \mu^{(m)}\|_2^2 \text{ subject to } \sum_{j=1}^p c_j \mathcal{I}(\eta_j) \leq C$$

$$= \mu^{(m)} \circ Z^{(m)} = (\mu_1^{(m)} z_1^{(m)}, \mu_2^{(m)} z_2^{(m)}, \ldots, \mu_p^{(m)} z_p^{(m)}), \text{ where}$$

$$Z^{(m)} \in \arg \max_{z_1, z_2, \ldots, z_p} \sum_{j=1}^p (\mu_j^{(m)})^2 z_j \text{ subject to } \sum_{j=1}^p c_j z_j \leq C.$$

Step 3: Repeat **Step 2**, until $g(\beta^{(m)}) - g(\beta^{(m+1)}) \leq \delta$.

Extension 1: p is much larger than n and β^0 is sparse

We aim to develop a linear model that will

- satisfy the budget constraint;
- have good prediction performance.

Since some feasible models may have k $(k > n)$ variables, we need to use regularization techniques to solve the overfitting problem.

We propose to estimate the regression coefficient vector by

$$\hat{\beta} \in \arg\min_{\beta} \frac{1}{2n} \|Y - \mathbf{X}\beta\|_2^2 + \sum_{j=1}^{p} \lambda(\alpha|\beta_j| + \frac{1-\alpha}{2}\beta_j^2)$$

$$\text{subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C.$$

Extension 1: p is much larger than n and β^0 is sparse

How to solve the optimization problem?

If $\hat{\beta}$ is an optimal solution to the following problem

$$\min_{\beta} \frac{1}{2}\|\beta - a\|_2^2 + \sum_{j=1}^{p} \lambda(\alpha|\beta_j| + \frac{1-\alpha}{2}\beta_j^2) \quad \text{subject to} \quad \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C,$$

then $\hat{\beta} = \frac{1}{1+\lambda(1-\alpha)} \cdot \mathrm{sign}(a - \alpha\lambda) \circ (|a| - \alpha\lambda)_+ \circ \hat{Z}$,
where $\hat{Z} = (\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_p)$ is the solution to the following 0-1 knapsack problem

$$\max_{z_1, z_2, \ldots, z_p} \sum_{j=1}^{p} \frac{a_j^2 - 2\alpha\lambda|a_j| + \alpha^2\lambda^2}{2(1+\lambda(1-\alpha))} \cdot \frac{1 + \mathrm{sign}(|a_j| - \alpha\lambda)}{2} \cdot z_j$$

$$\text{subject to} \quad \sum_{j=1}^{p} c_j z_j \leq C, \quad \text{and } z_1, z_2, \ldots, z_p \in \{0, 1\}.$$

Extension 2: Convex differential loss functions with Lipschitz continuous gradient

Denote $f = \sum_{j=1}^{p} x_j\beta_j$ and let $\psi(y, f)$ be the loss function used to fit the model.

If the gradient of the convex differential loss function $\psi(y, f)$ satisfis the following Lipschitz condition

$$|\frac{\partial\psi}{\partial f}(y, f_1) - \frac{\partial\psi}{\partial f}(y, f_2)| \leq M_1|f_1 - f_2|,$$

for any y, f_1, f_2, and a positive constant M_1 (e.g., the squared hinge loss),

OR

$\frac{\partial\psi^2(y,f)}{\partial f^2}$ exists and $\frac{\partial\psi^2(y,f)}{\partial f^2} \leq M_2$ for any y and f, and a positive constant M_2 (e.g., the logistic regression loss),

if $L \geq 2M_1 \cdot \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$ or $L \geq M_2 \cdot \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$, we can also show that

$$g(\eta) \leq Q_L(\eta, \beta) = g(\beta) + \frac{L}{2}\|\eta - \beta\|_2^2 + \langle \nabla g(\beta), \eta - \beta \rangle,$$

for all β, η with equality holding at $\beta = \eta$.

Extension 3: Predictors are collected group-by-group

Suppose that we have G groups. For the g-th group, we need to spend $\tilde{c}_g$ dollars to simultaneously collect the values of all p_g variables in the g-th group $\mathcal{A}_g$.

We propose to estimate the regression coefficient vector by

$$\hat{\beta} \in \arg\min_{\beta} \frac{1}{2n}\|Y - \mathbf{X}\beta\|_2^2 + \sum_{j=1}^{p} \lambda(\alpha|\beta_j| + \frac{1-\alpha}{2}\beta_j^2)$$

$$\text{subject to } \sum_{g=1}^{G} \tilde{c}_g[1 - \prod_{j \in \mathcal{A}_g} (1 - \mathcal{I}(\beta_j))] \leq C.$$

Note that we assume that we always need to spend $\tilde{c}_g$ dollars if there is at least one variable in the g-th group $\mathcal{A}_g$ with a nonzero regression coefficient.

Extension 3: predictors are collected group-by-group

If $\hat{\beta}$ is an optimal solution to the following problem

$$\min_{\beta} \|\beta - a\|_2^2 \quad \text{subject to} \quad \sum_{g=1}^{G} \tilde{c}_g [1 - \prod_{j \in \mathcal{A}_g} (1 - \mathcal{I}(\beta_j))] \leq C,$$

then $\hat{\beta} = a \circ \hat{Z}$ where $\circ$ denotes the entrywise product of two vectors, $\hat{Z} = (\hat{z}_1 \mathbf{1}_{p_1}, \hat{z}_2 \mathbf{1}_{p_2}, \ldots, \hat{z}_g \mathbf{1}_{p_g})^T$, $\mathbf{1}_{p_g}$ is a row vector of p_g 1's, and $\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_g$ is the solution to the following 0-1 knapsack problem

$$\max_{z_1, z_2, \ldots, z_g} \sum_{g=1}^{G} (\sum_{j \in \mathcal{A}_g} a_j^2) z_g$$

$$\text{subject to} \quad \sum_{g=1}^{G} \tilde{c}_g z_g \leq C, \ \text{and} \ z_1, z_2, \ldots, z_g \in \{0, 1\}.$$

Extension 3: predictors are collected group-by-group

If $\hat{\beta}$ is an optimal solution to the following problem

$$\min_{\beta} \frac{1}{2}\|\beta - a\|_2^2 + \sum_{j=1}^{p} \lambda(\alpha|\beta_j| + \frac{1-\alpha}{2}\beta_j^2)$$

$$\text{subject to } \sum_{g=1}^{G} \tilde{c}_g[1 - \prod_{j \in \mathcal{A}_g} (1 - \mathcal{I}(\beta_j))] \leq C,$$

then $\hat{\beta} = \frac{1}{1+\lambda(1-\alpha)} \cdot \text{sign}(a - \alpha\lambda) \circ (|a| - \alpha\lambda)_+ \circ \hat{Z}$,
where $\hat{Z} = (\hat{z}_1 \mathbf{1}_{p_1}, \hat{z}_2 \mathbf{1}_{p_2}, \ldots, \hat{z}_g \mathbf{1}_{p_g})^T$ and $\hat{z}_1, \hat{z}_2, \ldots, \hat{z}_g$ is the solution to the following 0-1 knapsack problem

$$\max_{z_1, z_2, \ldots, z_g} \sum_{g=1}^{G} (\sum_{j \in \mathcal{A}_g} \frac{a_j^2 - 2\alpha\lambda|a_j| + \alpha^2\lambda^2}{2(1+\lambda(1-\alpha))} \cdot \frac{1 + \text{sign}(|a_j| - \alpha\lambda)}{2}) z_g$$

$$\text{subject to } \sum_{g=1}^{G} \tilde{c}_g z_g \leq C, \text{ and } z_1, z_2, \ldots, z_g \in \{0, 1\}.$$

Theoretical Properties

(a) For any $L > \ell = \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$, the sequence

$$g(\beta^{(m)}) = \frac{1}{2n}\|Y - \mathbf{X}\beta^{(m)}\|_2^2$$

is decreasing, converges and satisfies

$$g(\beta^{(m)}) - g(\beta^{(m+1)}) \geq \frac{L-\ell}{2}\|\beta^{(m+1)} - \beta^{(m)}\|_2^2.$$

Since the function $g(\beta)$ is nonnegative and convex, we know that the proposed algorithm will converge.

(b) If $L > \ell = \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$, then

$$\beta^{(m+1)} - \beta^{(m)} \to 0 \text{ as } m \to \infty.$$

Theoretical Properties

Definition 1: Given an $L \geq \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$, the vector $\eta \in R^p$ is said to be a first-order stationary point of our optimization problem

$$\min_{\beta} g(\beta) = \frac{1}{2n}\|Y - \mathbf{X}\beta\|_2^2 \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C$$

if

$$\sum_{j=1}^{p} c_j \mathcal{I}(\eta_j) \leq C$$

and

$$\eta = \arg\min_{\beta} \|\beta - (\eta - \frac{1}{L}\nabla g(\eta))\|_2^2 \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \leq C$$

Theoretical Properties

(c) If $\hat{\beta}$ is a global minimizer of the optimization problem

$$\min_{\beta} g(\beta) = \frac{1}{2n}\|Y - \mathbf{X}\beta\|_2^2 \text{ subject to } \sum_{j=1}^{p} c_j \mathcal{I}(\beta_j) \le C,$$

then it is a first-order stationary point.

(d) If η is a first-order stationary point and

$$(\max_{i \in A} c_i) + \sum_{i \in A^c} c_i \le C,$$

where $A = \{i : \eta_i = 0\}$, then η is a global minimizer.

Theoretical Properties

(e) If $L > \lambda_{max}(\frac{\mathbf{X}^T\mathbf{X}}{n})$ and

$$\lim_{m\to\infty} \inf_{1\leq j\leq p} \min \{\max\{|\beta_j^{(m)}|, 0\}\} > 0,$$

then

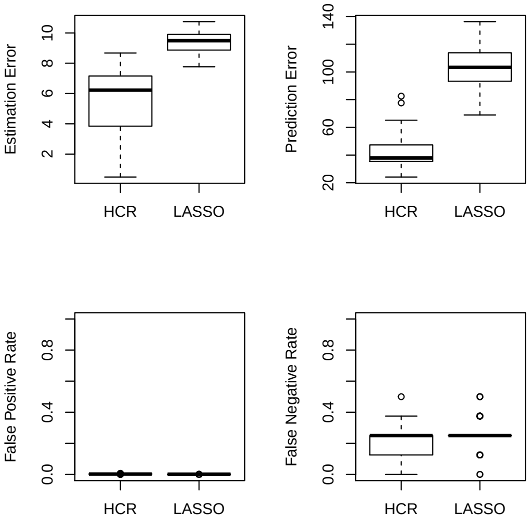
- The sparsity pattern sequence $Z^{(m)}$ converges after finitely many steps, that is, there exists an iteration index M^* such that $Z^{(m)} = Z^{(m+1)}$ for all $m \geq M^*$.
- The sequence $\beta^{(m)}$ is bounded and converges to a first-order stationary point.

The above theoretical properties about our HCR algorithm are similar to the theoretical results shown in Bertsimas et al. (2016) for the best subset selection.

Simulation Study: Example 1

- 200 training samples, 10000 testing samples;
- The dimension $p = 1000$ and $(x_1, x_2, \ldots, x_p)^T \sim N(0, \mathbf{I}_p)$;
- The model parameter β^0: the first 10 elements are randomly generated from $N(4, 1)$ and the other elements are 0;
- The costs of different variables are randomly selected from $\{1, 2, 3\}$. They are used for all the 100 experiments.
- The budget is $C = 12$ and the variance $\sigma^2 = 0.25$. Since the generated costs satisfy $\sum_{j=1}^{10} c_j = 18 > C$, the parameter of interest β^* is different from β^0.

Simulation Study: Example 1



Simulation Study: Example 2

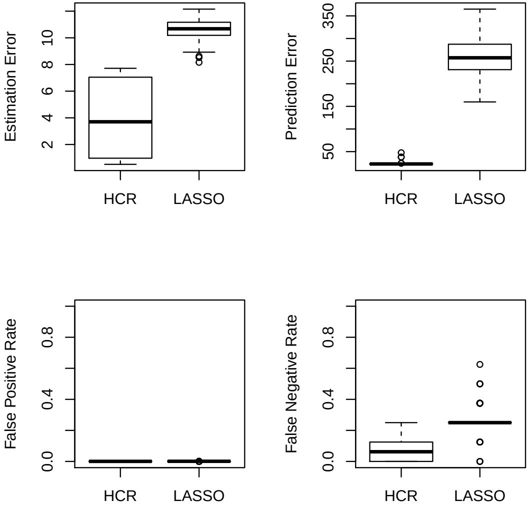
- The predictors $(x_1, x_2, \ldots, x_{10})^T \sim N(0, \mathbf{A})$ where $a_{jt} = 0.2$ if $j \neq t$ and 1 otherwise. The other $p - 10$ predictors are generated from $N(0, \mathbf{B})$ where $b_{jt} = 0.2$ if $j \neq t$ and 1 otherwise.

- The model parameter β^0: the first 10 elements are randomly generated from $N(4, 1)$ and the other elements are 0;

- The costs of different variables are randomly selected from $\{1, 2, 3\}$. They are used for all the 100 experiments.

- The budget is $C = 12$ and the variance $\sigma^2 = 0.25$. Since the generated costs satisfy $\sum_{j=1}^{10} c_j = 18 > C$, the parameter of interest β^* is different from β^0.

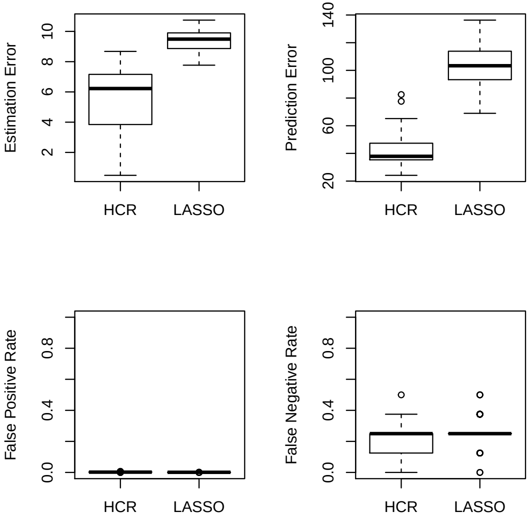
Simulation Study: Example 2



Simulation Study: Example 3

- The predictors $(x_{i1}, x_{i2}, \ldots, x_{ip})^T \sim N(0, \mathbf{\Sigma})$ where $\sigma_{jt} = 0.5^{|j-t|}$.
- The model parameter β^0: the first 10 elements are randomly generated from $N(4, 1)$ and the other elements are 0;
- The costs of different variables are randomly selected from $\{1, 2, 3, \ldots, 10\}$. They are used for all the 100 experiments.
- The budget is $C = 100$ and the variance $\sigma^2 = 0.25$. As C is equal to the cost of collecting all important variables, for this example, the true parameter of interest β^* is the same as β^0.

Simulation Study: Example 3



Application to a diabetes study

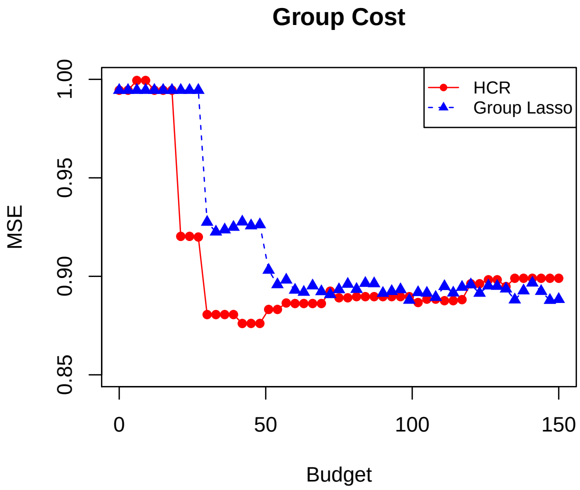
- The glycated hemoglobin (HbA1c) is now recommended as a standard of care (SOC) for testing and monitoring diabetes, specifically the type 2 diabetes.
- In general, the higher the HbA1c, the greater the risk of developing diabetes-related complications.
- We are interested in developing a cost-effective model to predict the change in HbA1c using some demographical information and predictors from several clinical tests.

Application to a diabetes study

- Our data are collected from 181 diabetes patients;
- The efficacy endpoint is the change in HbA1c from baseline to week 52;
- There are 20 predictors collected from 10 groups;
- We develop cost-constrained models for a sequence of budgets.

Costs	Predictors
$200	HDL
	LDL
	Total Cholesterol
	Triglycerides
$50	Creatinine
$20	Fasting BG
$30	HbA1c
$100	ALT
	AST
$100	GGT
	C.Peptide
$50	Fasting insulin test
$20	Age
	Weight
	BMI
	Waist
$5	Duraton of diabetes
$10	BP diastolic
	BP Systolic
	Pulse

Application to a diabetes study



Summary

- The cardinality budget constraint makes the high-dimensional cost-constrained regression problem NP-hard.
- We propose a new discrete extension of the first-order continuous optimization methods to deliver a near optimal solution.
- Our HCR algorithm generates a series of estimates of the regression coefficient vector by solving a sequence of 0-1 knapsack problems that can be efficiently addressed by the dynamic programming.
- The proposed HCR method can be extended to the other statistical learning problems and problems with more complicated constraints.

Thank you!