

# Dynamic Prediction in Functional Concurrent Regression

2022-10-27

# Dynamic prediction

- ▶ With observations up to  $t_m$ , predict outcomes (or probabilities of outcome) after that time point
- ▶ Prediction updates with new observations

# Functional Concurrent Regression (FCR)

- ▶ For a subject  $i$ , we observe a function over  $t$

$$Y_i(t) = f_0(t) + b_i(t) + \epsilon_i(t)$$

- ▶ We usually observe  $Y_i$  on a series of discrete  $t_{ij}$

$$Y_{ij} = f_0(t_{ij}) + b_i(t_{ij}) + \epsilon_{ij}$$

where  $\epsilon_{ij} \sim N(0, \sigma_{\epsilon^2})$ .

- ▶ Subject-specific random effect

$$b_i(t) = \sum_{k=1}^c u_{ik} B_k(t)$$

where  $\mathbf{u}_i \sim N(0, \Gamma)$

## Dynamic prediction

- ▶ Use FPCA to estimate  $f_0$ ,  $\Gamma$  and  $\sigma_\epsilon$
- ▶ For a new subject with observations up to  $t_m$ , estimate its score:

$$\hat{\mathbf{u}} = E(\mathbf{u}|\mathbf{y}) = \hat{\mathbf{\Gamma}}\mathbf{B}^T(\mathbf{B}\hat{\mathbf{\Gamma}}\mathbf{B}^T + \hat{\sigma}_\epsilon^2\mathbf{I}_m)$$

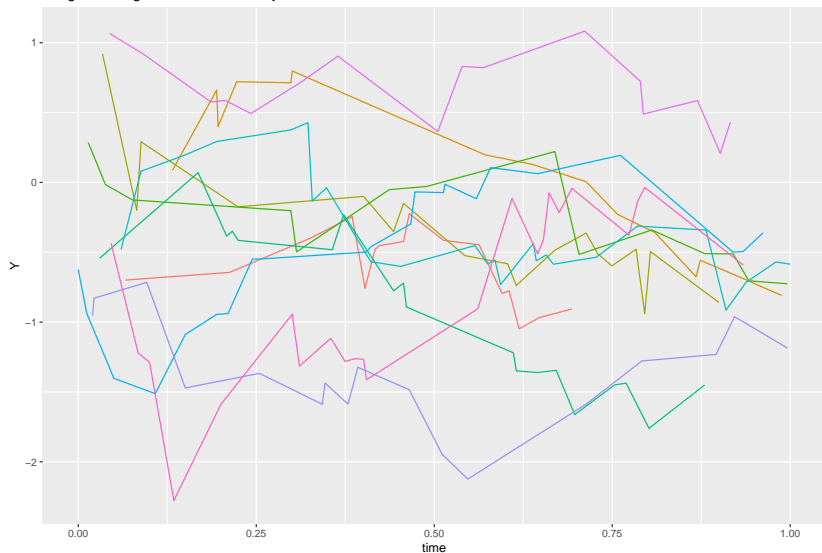
With the estimated score, we can predict its outcome in following time points

$$\hat{\mathbf{Y}} = \mathbf{B}^T \hat{\mathbf{u}}$$

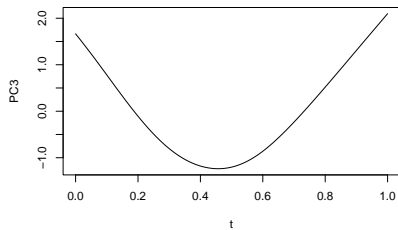
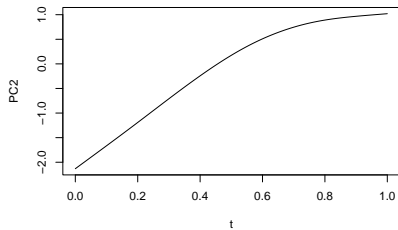
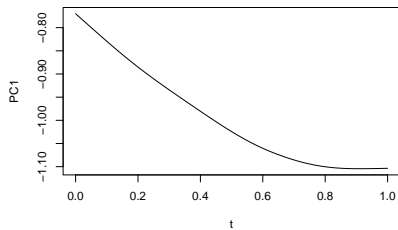
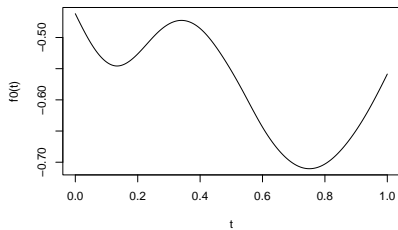
# Simulated child growth data

- Predict length-for-age, observed with noise

Length-for-Age for the first 10 subjects

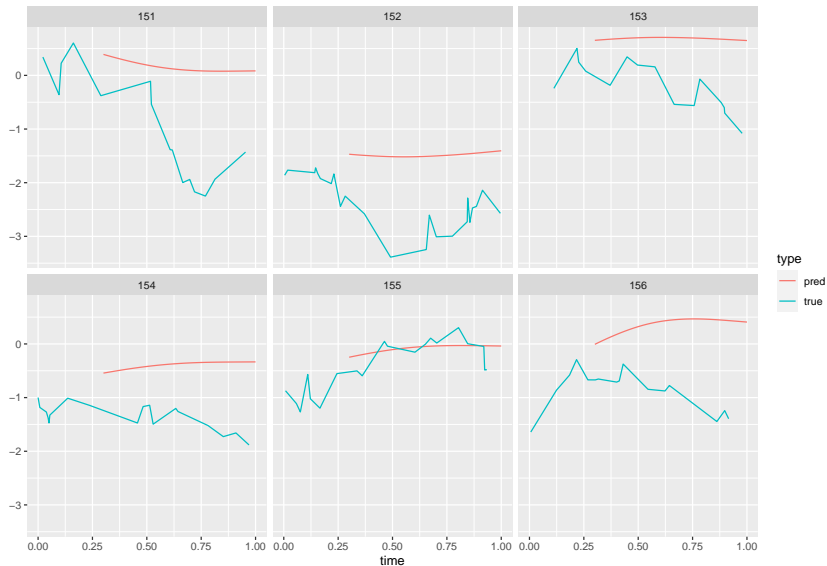


# FPCA on observed LAZ



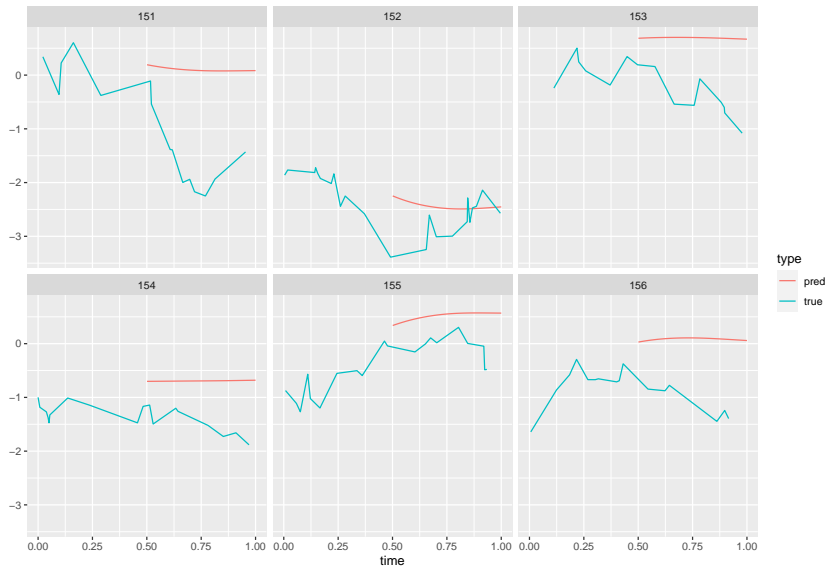
# Prediction of new partially observed sample

Prediction with observation up to  $t=0.3$



# Prediction of new partially observed sample

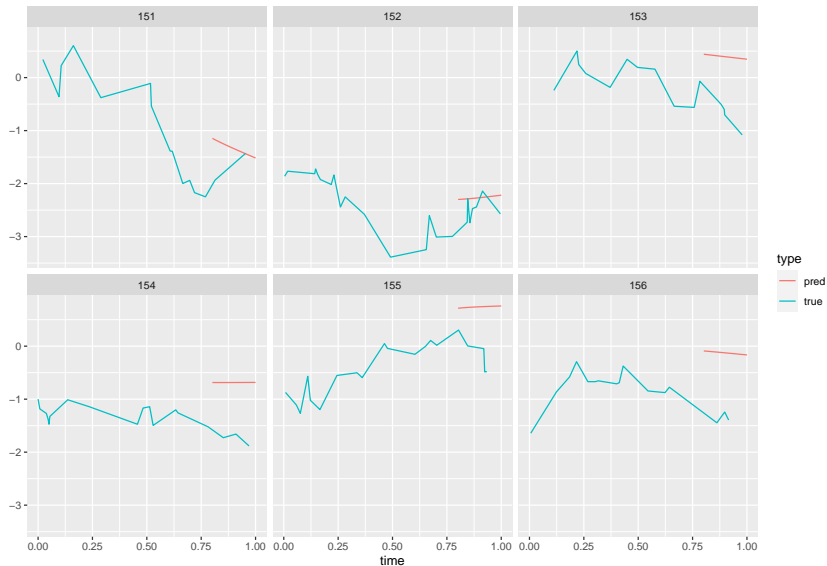
Prediction with observation up to  $t=0.5$





# Prediction of new partially observed sample

Prediction with observation up to  $t=0.8$



## Next steps

- ▶ Establish interval prediction
- ▶ ? fPCA on pooled prediction, instead of original functions observed with noise
- ▶ Extension to exponential family data