Weibull: $S(t) = \exp[-(\lambda t)^p]$

$$\lambda = \lambda_0 \exp(x\beta) = \lambda_0 \exp(\eta)$$

$$S(t|\eta) = \exp[-(\lambda_0 \exp(\eta) t)^p]$$

$$S(t) = \int_\eta S(t|\eta) p(\eta) d\eta \quad , \ \eta \sim N(0, \Sigma\beta^2)$$

$$= \int_\eta \exp\{-[\lambda_0 \exp(\eta) t]^p\} p(\eta) d\eta$$

$$f(t) = -\frac{dS(t)}{dt} = -\frac{d}{dt}\int_\eta S(t|\eta) p(\eta) d\eta$$

$$= -\int_\eta p(\eta) \frac{dS(t|\eta)}{dt} d\eta$$

$$\frac{dS(t|\eta)}{dt} = -\exp[-(\lambda_0 \exp(\eta) t)^p] \, p[\lambda_0 \exp(\eta) t]^{p-1} \lambda_0 \exp(\eta)$$

$$= -p\lambda_0^p \exp(p\eta) t^{p-1} \exp[-(\lambda_0 \exp(\eta) t)^p]$$

$$f(t) = \int_\eta p\lambda_0^p t^{p-1} \exp[p\eta - (\lambda_0 \exp(\eta) t)^p] \, p(\eta) d\eta$$