# 삼각함수의 적분법

유형1,2. $\int \sin^n x \; dx$ $\int \cos^n x \; dx$ $\int \sin^m x \; \cos^n x \; dx$

$n$ 과 $m$이 모두 짝수:

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$n$ 또는 $m$이 홀수:

$$\cos^2 x + \sin^2 x = 1$$

$$\begin{cases} \int \sin ax \, dx = -\frac{1}{a}\cos ax \\ \int \cos ax \, dx = \frac{1}{a}\sin ax \end{cases}$$

(예제1) $$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}dx = \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$$

(예제2) $$\int \sin^2 x \cos^2 x\,dx = \int (\frac{1-\cos 2x}{2}) \left(\frac{1+\cos 2x}{2}\right) dx = \frac{1}{4}\int (1-\cos^2 2x)dx$$

$$= \frac{1}{4}\int \sin^2 2x\,dx = \frac{1}{4}\int \frac{1-\cos 4x}{2}dx = \frac{1}{8}\left(x - \frac{1}{4}\sin 4\,x\right) + C$$

(예제3) $$\int \sin^3 x \cos^2 x\,dx = \int \underline{\sin^2 x} \cos^2 x \sin x\,dx \quad = \int \underline{(1-\cos^2 x)} \cos^2 x \sin x\,dx$$

$$\underline{\cos x = u \qquad -\sin x\,dx = du}$$

$$= \int -(1-u^2)u^2 du = \int -(u^2-u^4)\,du = -\frac{1}{3}u^3 + \frac{1}{5}u^5$$

$$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$$

유형3. $\int \sin ax \sin bx \, dx$ $\int \sin \overset{ax}{\cancel{a}} \cos bx \, dx$ $\int \cos ax \cos bx \, dx$

$$\sin x \cos y = \frac{1}{2}\left( \sin(x+y) + \sin(x-y) \right)$$

$$\cos x \cos y = \frac{1}{2}\left( \cos(x+y) + \cos(x-y) \right)$$

$$\sin x \, sin \; y = -\frac{1}{2}\left( \cos(x+y) - \cos(x-y) \right)$$

(예제 1) $\int \sin 4x \cos 3x\, dx \quad = \int \frac{1}{2}(\sin 7x + \sin x)\, dx \quad = \frac{1}{2}\left(-\frac{1}{7}\cos 7\mathrm{x} - \cos\mathrm{x}\right) + \mathrm{C}$

(예제 2) $\int_0^{2\pi} \sin mx \cos nx\, dx \quad = \frac{1}{2}\int_0^{2\pi} (\sin(m+n)x + \sin(m-n)x)dx$

$$= \frac{1}{2}\left(-\frac{1}{m+n}\cos(m+n)x - \frac{1}{m-n}\cos(m-n)x\right)_0^{2\pi} = 0$$

$$\int_{-\pi}^{\pi} \sin mx \cos nx\, dx = 0$$

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(예제 3) $\int \cos 4x \cos 3x \, dx \quad = \int \frac{1}{2} (\cos 7x + \cos x \,)\, dx \quad = \frac{1}{2}\left(\frac{1}{7} \sin 7\mathrm{x} \; + \sin \mathrm{x}\right) + \mathrm{C}$

(예제 4) $\int_0^{2\pi} \cos mx \cos nx \, dx \quad = \frac{1}{2}\int_0^{2\pi} (\, \cos(m+n)x \; + \cos(m-n)x) dx$

$m \neq n$

$$= \frac{1}{2}\left(\frac{1}{m+n} \sin(m+n)x + \frac{1}{m-n} \boldsymbol{\sin(m-n)x}\right)_0^{2\pi} \quad = 0$$

$m = n$

$$= \int_0^{2\pi} \cos^2 mx \, dx = \int_0^{2\pi} \frac{1 + \cos 2\overset{m}{x}}{2} dx \quad = \frac{1}{2}\left[x + \frac{1}{2} \sin 2\overset{m}{x}\right]_0^{2\pi} \quad = \pi$$

(예제 5) $\int \sin 4x \sin 3x \, dx = \int -\frac{1}{2}(\cos 7x - \cos x)\, dx = -\frac{1}{2}\left(\frac{1}{7}\sin 7x - \sin x\right) + C$

(예제 6) $\int_0^{2\pi} \sin mx \sin nx \, dx = -\frac{1}{2}\int_0^{2\pi} (\cos(m+n)x - \cos(m-n)x) dx$

$m \neq n$ $\quad = -\frac{1}{2}\left(\frac{1}{m+n}\sin(m+n)x - \frac{1}{m-n}\sin(m-n)x\right)_0^{2\pi} = 0$

$m = n$ $\quad = \int_0^{2\pi} \sin^2 mx \, dx = \int_0^{2\pi} \frac{1-\cos 2mx}{2} dx = \frac{1}{2}\left[x - \frac{1}{2m}\sin 2mx\right]_0^{2\pi} = \pi$

유형4. $\int \tan^m x \sec^n x \, dx$

$m$ 이 홀수 : $\sec x = u$

$1 + \tan^2 x = \sec^2 x$

(예제 1)

$$\int \tan^3 x \sec^3 x \, dx = \int \tan^2 x \sec^2 x \, tanx \sec x \, dx = \int (\sec^2 x - 1) \sec^2 x \, tanx \sec x \, dx$$

$\sec x = u \quad \tan x \sec x \, dx = du$

$$= \int (u^2 - 1) u^2 du = \int (u^4 - u^2) \, du = \frac{1}{5} u^5 - \frac{1}{3} u^3$$

$$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

$n$ 이 짝수 :   $\tan x = u$

$$1 + \tan^2 x = \sec^2 x$$

(예제 2) $\displaystyle\int \tan^4 x \sec^4 x \, dx = \int \tan^4 x \sec^2 x \sec^2 x \, dx = \int \tan^4 x \, (1 + \tan^2 x) \sec^2 x \, dx$

$\tan x = u$   $\sec^2 x \, dx = du$

$$= \int u^4(1+u^2)du = \int (u^4 + u^6) \, du = \frac{1}{5}u^5 + \frac{1}{7}u^7$$

$$= \frac{1}{5}\tan^5 x + \frac{1}{7}tan^7 x + C$$

$\displaystyle\int \tan^5 x \sec^4 x \, dx$   $\cancel{\displaystyle\int \tan^4 x \sec^5 x \, dx}$

(예제 3) 알쓸 신부

1. $\int \tan x \, dx = -\ln|\cos x| \quad = \ln|\sec x|$

2. $\int \cot x \, dx = \ln|\sin x|$

3. $\int \sec x \; dx = \boxed{\ln|\sec x + \tan x|}$

미분

$$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \frac{\sec x\,(\tan x + \sec x)}{\sec x + \tan x}$$

4. $\int \csc x \; dx = -\ln|\csc x + \cot\ x|$

(예제) $\int \sec^3 x\, dx$

(풀이) $$\int \sec x \sec^2 x\, dx \; \boxed{= \sec x \tan x - \int \sec x \tan^2 x\, dx}$$

$$\sec x = u \xrightarrow{\text{미}} \sec x \tan x\, dx = du$$

$$\sec^2 x\, dx = dv \xrightarrow{\text{적}} \tan x = v$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx = \sec x \tan x - \int (\sec^3 x - \sec x)\, dx$$

$$= \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$$

$$\int \sec^3 x\, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$

반각 치환 : 피적분함수가 $\sin x$ 나 $\cos x$ 에 관한 유리 함수인 경우

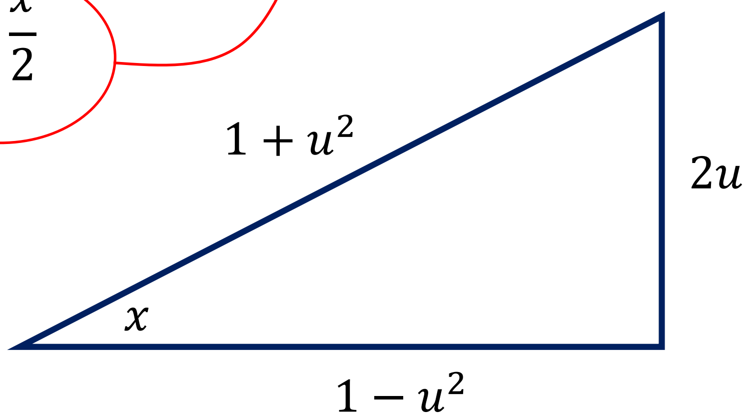
$$u = \tan\frac{x}{2}$$

양변 미분 $\frac{du}{dx} = \frac{1}{2}\sec^2\frac{x}{2} = \frac{1}{2}\left(1 + \tan^2\frac{x}{2}\right) = \frac{1}{2}(1+u^2)$

$$dx = \frac{2}{1+u^2}du$$

한편 $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\theta = \frac{x}{2}$




$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \frac{2u}{1-u^2}$$

$$\sin x = \frac{2u}{1+u^2}$$

$$\cos x = \frac{1-u^2}{1+u^2}$$

(예제1) $$\int \frac{1}{4\sin x + 3\cos x}\,dx$$

$\tan\frac{x}{2} = u$ $\quad \sin x = \frac{2u}{1+u^2} \quad \cos x = \frac{1-u^2}{1+u^2} \quad dx = \frac{2}{1+u^2}du$

$$= \int \frac{\frac{2}{1+u^2}}{4\frac{2u}{1+u^2} + 3\frac{1-u^2}{1+u^2}}du = -2\int \frac{1}{3u^2 - 8u - 3}du = -2\int \frac{1}{(3u+1)(u-3)}du$$

유리함수 의 적분

$$= -2\int\left(-\frac{3}{10}\cdot\frac{1}{3u+1} + \frac{1}{10}\cdot\frac{1}{u-3}\right)du = \frac{3}{5}\int\frac{1}{3u+1}du - \frac{1}{5}\int\frac{1}{u-3}du$$

$$= \frac{1}{5}\ln|3u+1| - \frac{1}{5}\ln|u-3| + C$$

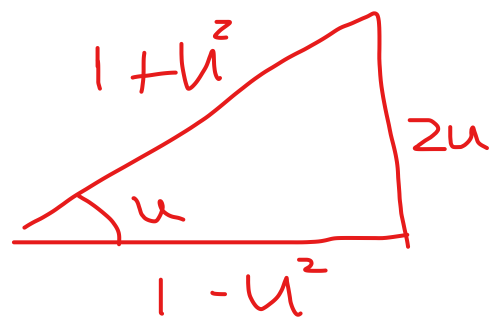
$$\int \frac{c}{ax+b}\,dx = \frac{c}{a}\ln|ax+b|$$

(예제2) $\int \sec x \, dx = \ln(\sec x + \tan x) + C$

$$= \int \frac{1}{\cos x} dx = \int \frac{\cancel{1+u^2}}{1-u^2} \cdot \frac{2}{\cancel{1+u^2}} du = \int \frac{2}{1-u^2} du = \int \left( \frac{1}{1+u} + \frac{1}{1-u} \right) du$$

$$= \ln(1+u) - \ln(1-u)$$

$$= \ln\left(\frac{1+u}{1-u}\right) = \ln\left(\frac{(1+u)^2}{(1-u)(1+u)}\right) = \ln\left(\frac{1+2u+u^2}{1-u^2}\right) = \ln\left(\underbrace{\frac{1+u^2}{1-u^2}}_{\sec x} + \underbrace{\frac{2u}{1-u^2}}_{\tan x}\right)$$




+연습문제

5.4(1~5)