

# Computer System Architecture

(THIRD EDITION)



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## Data Types

- ◆ Binary information is stored in **memory** or **processor registers**
- ◆ Registers contain either **data** or **control information**
  - Data** are numbers and other binary-coded information
  - Control information** is a bit or a group of bits used to specify the sequence of command signals
- ◆ Data types found in the registers of digital computers
  - Numbers** used in arithmetic computations
  - Letters** of the alphabet used in data processing
  - Other discrete symbols** used for specific purpose
    - » 위의 Number 와 Letter 이외 모두, 예) gray code, error detection code, ...
- ◆ The binary number system is the most natural system to use in a digital computer
- ◆ Number Systems
  - Base** or **Radix  $r$  system** : uses distinct symbols for  **$r$  digits**
  - Most common number system : Decimal, Binary, Octal, Hexadecimal
  - Positional-value(weight) System :  $r^2 \ r^1 r^0 . r^{-1} \ r^{-2} \ r^{-3}$ 
    - » Multiply each digit by an integer power of  $r$  and then form the sum of all weighted digits

## ◆ Decimal System/Base-10 System

Composed of 10 symbols or numerals(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0)

## ◆ Binary System/Base-2 System

Composed of 10 symbols or numerals(0, 1)

**Bit** = Binary digit

## ◆ Hexadecimal System/Base-16 System : Tab. 3-2

Composed of 16 symbols or numerals(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

## ◆ Binary-to-Decimal Conversions

$$\begin{aligned} 1011.101_2 &= (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) \\ &= 8_{10} + 0 + 2_{10} + 1_{10} + 0.5_{10} + 0 + 0.125_{10} \\ &= 11.625_{10} \end{aligned}$$

## ◆ Octal-to-Decimal Conversions

$$\begin{aligned} (736.4)_8 &= 7 \times 8^2 + 3 \times 8^1 + 6 \times 8^0 + 4 \times 8^{-1} \\ &= 7 \times 64 + 3 \times 8 + 6 \times 1 + 4/8 = (478.5)_{10} \end{aligned}$$

## ◆ Hexadecimal-to-Decimal Conversions

$$(F3)_{16} = F \times 16 + 3 = 15 \times 16 + 3 = (243)_{10}$$

# 3-1. Data Types

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## ◆ Conversion of decimal 41.6875 into binary

Repeated division(See p. 69, **Fig. 3-1**)

### Integer = 41

$41 / 2 = 20$  remainder 1 (binary number will end with 1) : **LSB**

$20 / 2 = 10$  remainder 0

$10 / 2 = 5$  remainder 0

$5 / 2 = 2$  remainder 1

$2 / 2 = 1$  remainder 0

$1 / 2 = 0$  remainder 1 (binary number will start with 1) : **MSB**

Read the result upward to give an answer of  $(41)_{10} = (101001)_2$

### Fraction = 0.6875

$0.6875 \times 2 = 1.3750$  integer 1 : **MSB**

$1.3750 \times 2 = 0.7500$  integer 0

$0.7500 \times 2 = 1.5000$  integer 1

$1.5000 \times 2 = 1.0000$  integer 1 : **LSB**

Read the result downward  $(0.6875)_{10} = (0.1011)_2$

$(41.6875)_{10} = (101001.1011)_2$

Handwritten notes for integer conversion:  
8 4 2 1  
0 0 0 1 (MSB)  
7 6 5 4 3 2 1  
↑  
1 0 1 0 0 1 (LSB)

Handwritten notes for fraction conversion:  
1 0 1 1 (MSB)  
↓  
1 0 1 1 (LSB)

곱한 결과 소수점 윗자리가  
2로 나오면 0으로 고치고,  
3이 나오면 1로 고치어 계산  
한다

# 3-1. Data Types

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## ◆ Hex-to-Decimal Conversion

$$\begin{aligned} 2AF_{16} &= (2 \times 16^2) + (10 \times 16^1) + (15 \times 16^0) \\ &= 512_{10} + 160_{10} + 15_{10} \\ &= 687_{10} \end{aligned}$$

## ◆ Decimal-to-Hex Conversion

$$\begin{aligned} 423_{10} / 16 &= 26 \text{ remainder } 7 \text{ (Hex number will end with 7) : **LSB**} \\ 26_{10} / 16 &= 1 \text{ remainder } 10 \\ 1_{10} / 16 &= 0 \text{ remainder } 1 \text{ (Hex number will start with 1) : **MSB**} \end{aligned}$$

Read the result upward to give an answer of  $423_{10} = 1A7_{16}$

Table 3-2

Hex	Binary	Decimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15
14	0001 0100	20
F8	1111 1000	248

## ◆ Hex-to-Binary Conversion

$$\begin{aligned} 9F2_{16} &= \begin{array}{ccc} 9 & F & 2 \\ \downarrow & \downarrow & \downarrow \\ & & \end{array} \\ &= 1001 \quad 1111 \quad 0010 \\ &= 100111110010_2 \end{aligned}$$

## ◆ Binary-to-Hex Conversion

$$\begin{aligned} 1110100110_2 &= \underbrace{0011}_{3} \underbrace{1010}_{A} \underbrace{0110}_{6} \\ &= 3A6_{16} \end{aligned}$$

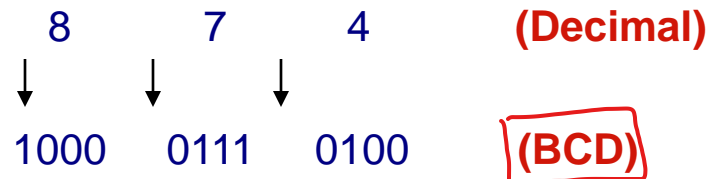
## ◆ Binary, octal, and hexadecimal Conversion

$$\begin{array}{ccccccccc} & 1 & 2 & & 7 & & 5 & & 4 & & 3 \\ & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} \\ A & & F & & 6 & & & & 3 & & & & & & & \end{array}$$

Octal 8진수  
Binary 2진수  
Hexadecimal 16진수

## ◆ Binary-Coded-Decimal Code

Each digit of a decimal number is represented by its binary equivalent



Only the four bit binary numbers from 0000 through 1001 are used

Comparison of BCD and Binary

$$137_{10} = 10001001_2 \quad (\text{Binary}) - \text{require only 8 bits}$$

$$137_{10} = 0001\ 0011\ 0111_{\text{BCD}} \quad (\text{BCD}) - \text{require 12 bits}$$

## ◆ Alphanumeric Representation

Alphanumeric character set (Tab. 3-4)

- » 10 decimal digits, 26 letters, special character (\$, +, =, ....)
- » A complete list of ASCII : p. 384, Tab. 11-1

ASCII (American Standard Code for Information Interchange)

- » Standard alphanumeric binary code uses seven bits to code 128 characters

### • Unicode

- 16bits code ( $2^{16} = 65536$ )

□ 코드는 다양한 정보를 2진수로 표현하기 위한 애진코드의 종류다

# 3-1. Data Types

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Character	Binary code	Character	Binary code	Character	Binary code	Character	Binary code
A	100 0001	U	101 0101	0	011 0000	/	010 1111
B	100 0010	V	101 0110	1	011 0001	,	010 1100
C	100 0011	W	101 0111	2	011 0010	=	011 1101
D	100 0100	X	101 1000	3	011 0011		
E	100 0101	Z	101 1010	4	011 0100		
F	100 0110			5	011 0101		
G	100 0111			6	011 0110		
H	100 1000			7	011 0111		
I	100 1001			8	011 1000		
J	100 1010			9	011 1001		
K	100 1011						
L	100 1100						
M	100 1101			space	010 0000		
N	100 1110			.	010 1110		
O	100 1111			(	010 1000		
P	101 0000			+	010 1011		
Q	101 0001			\$	010 0100		
R	101 0010			*	010 1010		
S	101 0011			)	010 1001		
T	101 0100			-	010 1101		

Table 3-4. ASCII

## 보수 3-2. Complements

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6에 대한 9의 보수는 3,  $(3+6=9)$

**Complements** are used in digital computers for simplifying the **subtraction operation** and for logical manipulation

### ◆ There are two types of complements for base r system

- 1) r's complement
- 2) (r-1)'s complement

- » Binary number : 2's or 1's complement
- » Decimal number : 10's or 9's complement

### ◆ (r-1)'s Complement

(r-1)'s Complement of N =  $(r^n - 1) - N$

- » 9's complement of N = **546700**

$$(10^6 - 1) - 546700 = (1000000 - 1) - 546700 = 999999 - 546700 = \underline{453299}$$

- » 1's complement of N = **101101**

$$(2^6 - 1) - 101101 = (1000000 - 1) - 101101 = 111111 - 101101 = \underline{010010}$$

### ◆ r's Complement

r's Complement of N =  $r^n - N$

- » 10's complement of **2389** =  $7610 + 1 = \underline{7611}$

- » 2's complement of **1101100** =  $0010011 + 1 = \underline{0010100}$

간단하게 생각하면,

546700  
→ 453299

각자리 9에서  
빼면 9가 됨

N : given number  
r : base  
n : digit number

9의 보수,

$$546700(N) + 453299(9's \text{ com}) = 999999$$

$$101101(N) + 010010(1's \text{ com}) = 111111$$

1의 보수를 구하려면  
비트를 다 바꾼다

\* **r's Complement**

$$(r-1)'s \text{ Complement} + 1 = (r^n - 1) - N + 1 = r^n - N$$

10의 보수 = 9의 보수 + 1  
2의 보수 = 1의 보수 + 1



## 3-2. Complements

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### ◆ Subtraction of Unsigned Numbers (M-N), N≠0

- 1)  $M + (r^n - N)$
- 2)  $M \geq N$  : Discard end carry, Result =  $M - N$
- 3)  $M < N$  : No end carry, Result = - r's complement of  $(N - M)$

#### » Decimal Example)

$M \geq N$   $72532(M) - 13250(N) = 59282$

$$\begin{array}{r} 72532 \\ + 86750 \text{ (10's complement of 13250)} \\ \hline 159282 \end{array}$$

Discard End Carry → 159282  
Result = **59282**

$M < N$   $13250(M) - 72532(N) = -59282$

$$\begin{array}{r} 13250 \\ + 27468 \text{ (10's complement of 72532)} \\ \hline 040718 \end{array}$$

No End Carry → 040718  
Result = -(10's complement of 40718)  
= -(59281+1) = **-59282**

#### » Binary Example)

$X \geq Y$   $1010100(X) - 1000011(Y) = 0010001$

$$\begin{array}{r} 1010100 \\ + 0111101 \text{ (2's complement of 1000011)} \\ \hline 10010001 \end{array}$$

10010001  
Result = **0010001**

$X < Y$   $1000011(X) - 1010100(Y) = -0010001$

$$\begin{array}{r} 1000011 \\ + 0101100 \text{ (2's complement of 1010100)} \\ \hline 0110111 \end{array}$$

0110111  
Result = -(2's complement of 1101111)  
= -(0010000+1) = **-0010001**

## 3-2. Complements

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### 2진수의 뺄셈

#### [1's Complement]

10진법	9' Complement	보통방법	1' Complement
$\begin{array}{r} 6 \\ - 4 \\ \hline 2 \end{array}$	$\begin{array}{r} 6 \\ + 5 \\ \hline 11 \\ + 1 \text{ (carry를 아래로)} \\ \hline 12 \end{array}$	$\begin{array}{r} 0110 \\ - 0100 \\ \hline 0010 \end{array}$	$\begin{array}{r} 0110 \\ + 1011 \\ \hline 0001 \\ + 1 \\ \hline 0010 \end{array}$
			<p>← 0100에 대한 1' Complement</p> <p>← 자리 올림수를 더한다 <i>내린다</i></p> <p>← 연산결과</p>

$\begin{array}{r} 4 \\ - 6 \\ \hline -2 \end{array}$	$\begin{array}{r} 0100 \\ - 0110 \\ \hline -0010 \end{array}$	$\begin{array}{r} 0100 \\ + 1001 \\ \hline \boxed{1}101 \\ \downarrow \\ -0010 \end{array}$	<p>← 0110에 대한 1' Complement</p> <p>← 1의 보수로 변환 한다</p> <p>← 연산결과 (자리올림이 없으므로 '—'부호를 붙인다)</p>
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## 3-2. Complements

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### 2진수의 뺄셈

#### [2's Complement]

10진법	10' Complement	보통방법	2' Complement
$\begin{array}{r} 6 \\ - 4 \\ \hline 2 \end{array}$	$\begin{array}{r} 6 \\ + 6 \\ \hline 12 \\ \hline 2 \end{array}$	$\begin{array}{r} 0110 \\ - 0100 \\ \hline 0010 \end{array}$	$\begin{array}{r} 0110 \\ + 1100 \\ \hline 10010 \\ \hline 0010 \end{array}$
	<p>10의 보수</p> <p>올림수 삭제</p>		<p>0100에 대한 2' Complement</p> <p>올림수 무시</p> <p>연산결과</p>

$\begin{array}{r} 4 \\ - 6 \\ \hline -2 \end{array}$	$\begin{array}{r} 0100 \\ - 0110 \\ \hline -0010 \end{array}$	$\begin{array}{r} 0100 \\ + 1010 \\ \hline 1110 \\ \hline \square 1110 \\ \downarrow \\ -0010 \end{array}$	<p>0110에 대한 2' Complement</p> <p>자리 올림이 없으므로 2의 보수로 변환 하고 '—'부호를 붙인다)</p>
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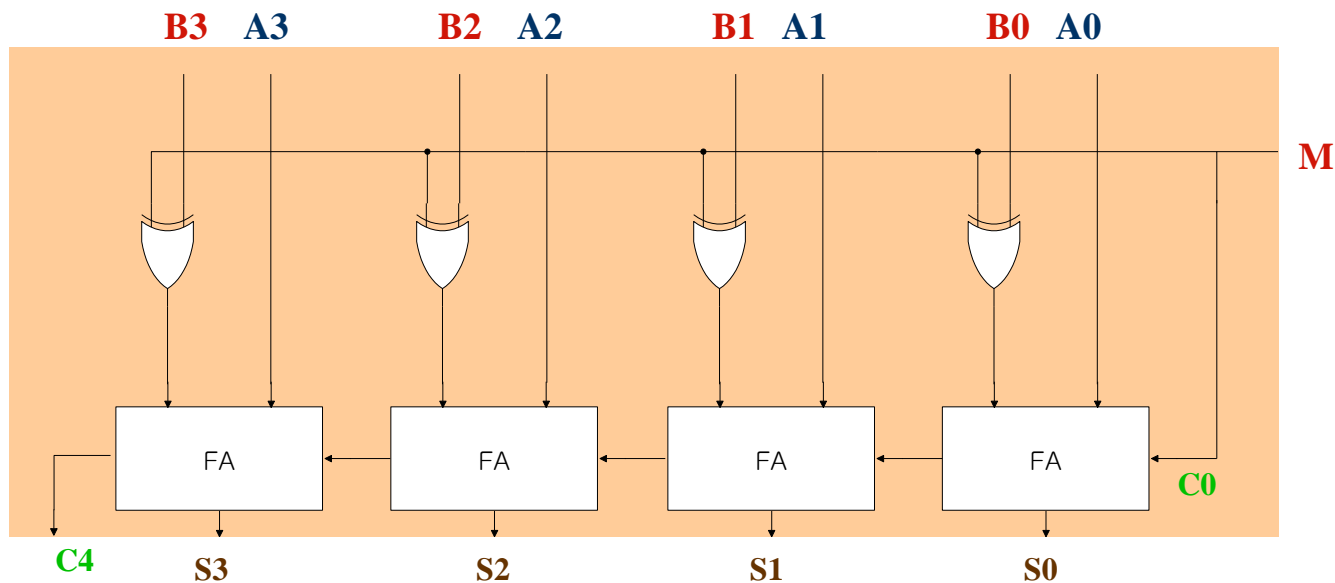
# 가감산기 4-bit Binary Adder-Subtractor

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## 4-bit Binary Adder-Subtractor : **Fig. 4-7**

- ◆ One common circuit by including an exclusive-OR gate with each full-adder
- ◆  $M = 0$  : Adder  $B \oplus M + C = B \oplus 0 + 0 = B, \therefore A + B$
- ◆  $M = 1$  : Subtractor  $B \oplus M + C = B \oplus 1 + 1 = B' + 1 = -B(2's \text{ comp}), \therefore A - B$

XOR : 달라야 1



# 3-3. Fixed-Point Representation

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## Fixed-Point Representation

\*Numeric Data

1) Fixed Point

2) Floating Point

고정소수점  
부동소수점

- ◆ Computers must represent everything with 1's and 0's, including the sign of a number and fixed/floating point number

- ◆ Binary/Decimal Point

\* 32.25

1) 0.25, 2) 32.0, 3) 32.25

The position of the binary/decimal point is needed to represent fractions, integers, or mixed integer-fraction number

- ◆ Two ways of specifying the position of the binary point in a register

1) Fixed Point : the binary point is always fixed in one position

» A binary point in the **extreme left** of the register(**Fraction** : 0.xxxxx)

» A binary point in the **extreme right** of the register(**Integer** : xxxxx.0)

*The binary point is not actually present, but the number stored in the register is treated as a fraction or as an integer*

2) Floating Point : the second register is used to designate the position of the binary point in the first register(**refer to 3-4**)

- ◆ Integer Representation

Signed-magnitude representation →

Signed-1's complement representation →

Signed-2's complement representation →

+14	-14
0 0001110	1 0001110
0 0001110	1 1110001
0 0001110	1 1110010

\* MSB for Sign  
"0" is plus +  
"1" is minus -

0 0001110  
MSB

Most  
Common

## ◆ Arithmetic Addition

Addition Rules of Ordinary Arithmetic

- » The signs are **same** : sign= **common** sign, result= **add**
- » The signs are **different** : sign= **larger** sign, result= **larger-smaller**

$$(-12) + (-13) = -25$$

$$(+12) + (+13) = +25$$

$$(+25) + (-37) = 37 - 25 = -12$$

Addition Rules of the signed 2's complement

- » Add the two numbers including their sign bits
- » Discard any carry out of the sign bit position →

**\*Addition Exam)**

+ 6	00000110	- 6	11111010
+ 13	00001101	+ 13	00001101
+ 19	00010011	+ 7	00000111

+ 6	00000110	- 6	11111010
- 13	11110011	- 13	11110011
- 7	11111001	- 19	11101101

## ◆ Arithmetic Subtraction

Subtraction is changed to an Addition

- »  $(\pm A) - (+ B) = (\pm A) + (- B)$
- »  $(\pm A) - (- B) = (\pm A) + (+ B)$

**\* Subtraction Exam)**  $(- 6) - (- 13) = +7$

$$11111010 - 11110011 = 11111010 + 2's \text{ comp of } 11110011$$

$$= 11111010 + 00001101$$

$$= 100000111 = +7$$

Discard  
End Carry

## ◆ Overflow

Two numbers of n digits each are added and the sum occupies n+1 digits  
n + 1 bit cannot be accommodated in a register with a standard length of n bits(*many computer detect the occurrence of an overflow, and a corresponding F/F is set*)

## ◆ Overflow 문제점

An overflow may occur if the two numbers added are both positive or both negative

BCD코드를 연산하려면 특별한 연산기가 필요하다

» When two unsigned numbers are added

an overflow is detected from the end carry out of the MSB position

» When two signed numbers are added

the MSB always represents the sign

- the sign bit is treated as part of the number

- the end carry does not indicate an overflow

### \* Overflow Exam)

out in		out in	
carries	0 1	carries	1 0
+ 70	0 1000110	- 70	1 0111010
+ 80	0 1010000	- 80	1 0110000
+ 150	1 0010110	- 150	0 1101010

## ◆ Overflow Detection

Detected by observing the **carry into** the sign bit position and the **carry out** of the sign bit position

If these two carries are not equal, an overflow condition is produced (**Exclusive-OR gate = 1**)

### \*Decimal Exam) (+375) + (-240)

375 + (10's comp of 240) = 375 + 760

0 375	(0000 0011 0111 0101)
+9 760	(1001 0111 0110 0000)
0 135	(0000 0001 0011 0101)

## ◆ Decimal Fixed-Point Representation

A 4 bit decimal code requires four F/Fs

for each decimal digit

The representation of 4385 in BCD requires 16 F/Fs (0100 0011 1000 0101)

The representation in decimal is **wasting a considerable amount of storage space** and the circuits required to perform decimal arithmetic are **more complex**

\* Advantage \*  
Computer I/O data are generated by people who use the decimal system

## 3-4 Floating-Point Representation

- The floating-point representation of a number has two parts

- 가수 -> 1) Mantissa : signed, fixed-point number
- 지수 -> 2) Exponent : position of binary(decimal) point

- Scientific notation :  $m \times r^e$  ( $+0.6132789 \times 10^{+4}$ )

$m$  : mantissa,  $r$  : radix,  $e$  : exponent

- Example :  $m \times 2^e = +(.1001110)_2 \times 2^{+4}$

\* Decimal + 6132.789  
**Fraction**      **Exponent**  
 +0.6132789      +4

~~$\rightarrow 0.6132789 \times 10^4$~~

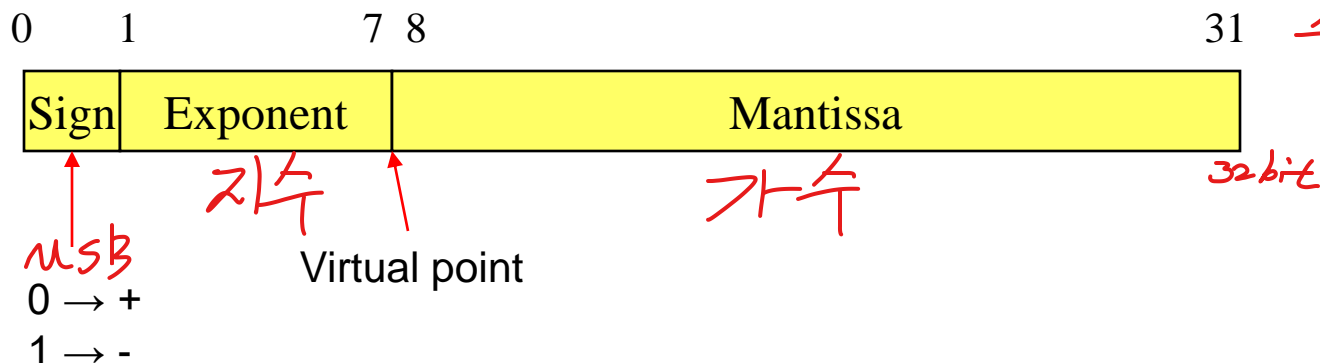
**Fraction**      **Exponent**  
 01001110      000100

가수 x 지수

- 정규화 Normalization - 가장 높은 정밀도 제공

Most significant digit of mantissa is **nonzero**

유효자리를 최대한 하기 위해 가수부분의 값이 0.1 ~ 1 사이에 있도록 조작 하는 것  
 $(0.0000145786)_{10}$ 를 정규화 하면  $0.145786 \times 10^{-4}$  1수정 아래 1번째 자리





## Gray Code

한 비트만 바뀜

Gray code **changes by only one bit** (Tab. 3-5 4-bit Gray Code )

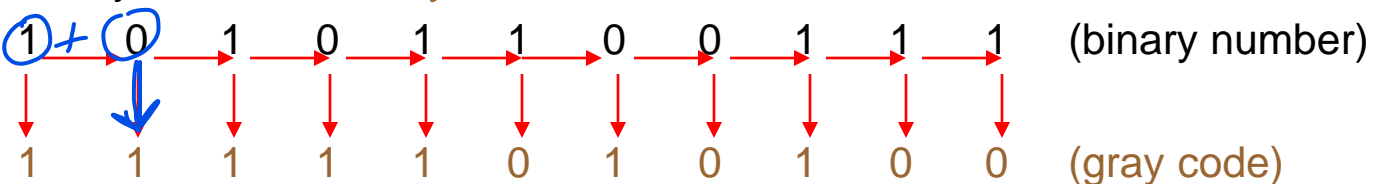
용도 :

- »The data must be converted into digital form before they can be used by a digital computer(**Analog to Digital Converter**)
- »The analog data are represented by the continuous change of a shaft position(**Rotary Encoder of Motor**)

BCD → Gray

Binary number → Gray code

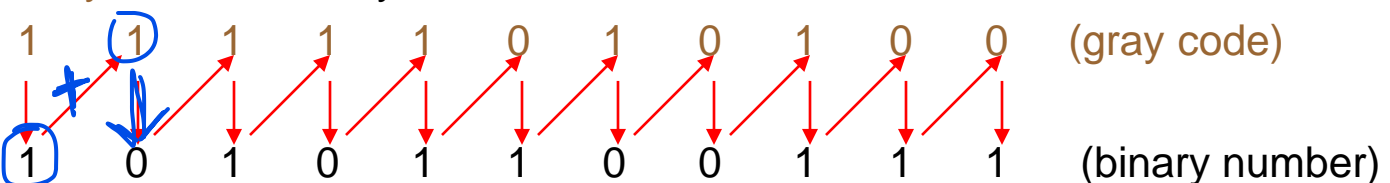
이웃해 있는 2진수의 합만을 다음의 그레이코드 비트로...



Gray → BCD

Gray code → Binary number

대각선으로 합해서 올림은 버리고 합만을 2진수로...



처음 비트는 그대로 내려오고 두번째부터는 2개 더한 값

처음 비트는 그대로 내려오고

## 3-5. Other Binary Codes

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1비트만  
부인함

BCD code	Binary code	Decimal equivalent	Binary code	Decimal equivalent
0 0000	0000	0	1100	8
1 0001	0001	1	1101	9
2 0010	0011	2	1111	10
→ 3 0011	0010	3	1110	11
4 0100	0110	4	1010	12
5 0101	0111	5	1011	13
6 0110	0101	6	1001	14
7 0111	0100	7	1000	15

Table 3-5. 4-Bit Gray Code

## 3-5. Other Binary Codes

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### ◆ Other Decimal Codes *Gray code 2이든 더 있다*

Binary codes for decimal digits require four bits. A few possibilities are shown in **Tab. 3-6**

#### Excess-3 Gray Code

- » Gray code로 BCD 표현 시, 9 에서 0 으로 변하면 1101에서 0000으로 되어 3 비트가 동시에 변경되어 Tab. 3-5 에서 3 부터 12까지 사용하면 0010 에서 1010되어 1비트가 바뀜.

#### Self-Complementing : excess-3 code

- » 9's complement of a decimal number is easily obtained by 1's complement(=changing 1's to 0's and 0's to 1's)

#### Weighted Code : 2421 code *보수를 구하기 쉽다*

- » The bits are multiplied by the weights, and the sum of the weighted bits gives the decimal digit

#### \* Self-Complement Exam)

$$\begin{aligned} 4_{10} &= 0111 \text{ (3-excess)} \\ &= 1000 \text{ (1's comp)} \\ &= 5_{10} \text{ (3-excess in Tab. 3-6)} \\ &= 5_{10} \text{ (9's comp of 4)} \end{aligned}$$

### ◆ Other Alphanumeric Codes

*7bit* ASCII Code에서 Tab. 3-4 이외 : p. 384, Tab. 11-1

- » Format effector : Functional characters for controlling the layout of printing or display devices(carriage return-CR, line feed-LF, horizontal tab-HT,...)
- » Data communication flow control(acknowledge-ACK, escape-ESC, synchronous-SYN,...)

*8bit* EBCDIC(Extended BCD Interchange Code)

- » Used in IBM equipment(제어 문자만 약간 다름)

## 3-5. Other Binary Codes

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Decimal digit	BCD 8421	BCD 2421	Excess-3	Excess-3 gray
0	0000	0000	0011	0010
1	0001	0001	0100	0110
2	0010	0010	0101	0111
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1100
6	0110	1100	1001	1101
7	0111	1101	1010	1111
8	1000	1110	1011	1110
9	1001	1111	1100	1010
Unused bit combinations	1010	0101	0000	0000
	1011	0110	0001	0001
	1100	0111	0010	0011
	1101	1000	1101	1000
	1110	1001	1110	1001
	1111	1010	1111	1011

필요에 따라  
여러 코드  
만들 수 있다

Table 3-6. Four Different Binary Codes for the Decimal Digit

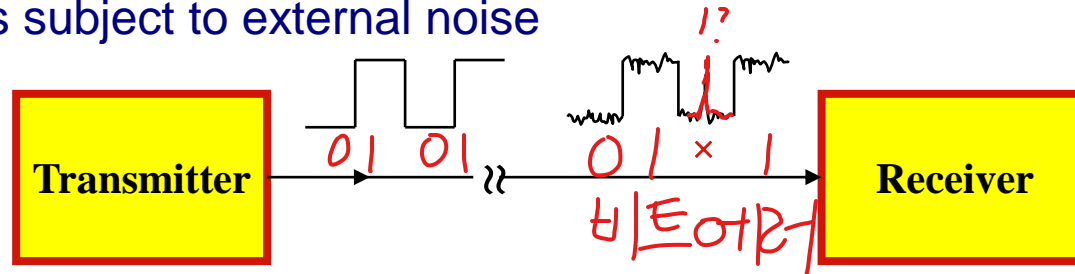
# 3-6. Error Detection Codes

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4 | 8 비트 방식 코드

## 3-6 Error Detection Codes

- Binary information transmitted through some form of communication medium is subject to external noise



- Parity Bit

An extra bit included with a binary message to make the total number of 1's either odd or even (Tab. 3-7) | 의 개수 세는 짝수/홀수인지 검사

- Even-parity method

The value of the parity bit is chosen so that the total number of 1s (including the parity bit) is an **even** number

1 1 0 0 0 0 1 1

Added parity bit

- Odd-parity method

Exactly the same way except that the total number of 1s is an **odd** number

1 1 0 0 0 0 0 1

Added parity bit

짝수로 맞추려, 나중에 다시 점검

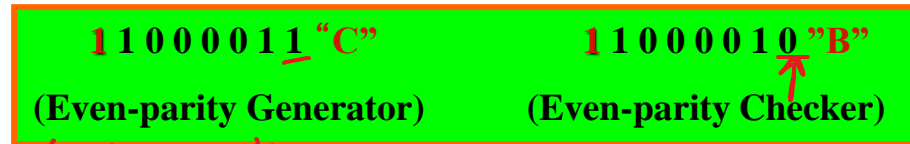
홀수로 보내기로 약속했는데

1이 짝수면 재전송 요청.

### ◆ Parity Generator/Checker

At the sending end, the message is applied to a parity generator

At the receiving end, all the incoming bits are applied to a parity checker



보낼 때 1 붙임

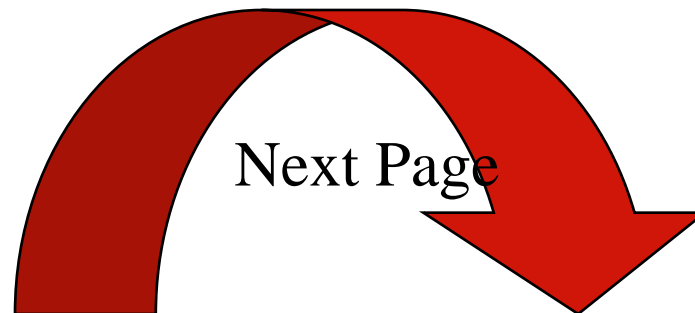
받을 때 검사

Can not tell which bit in error

Can detect only single bit error(odd number of errors)

3 bit data line example : **Fig. 3-3**

### ◆ 4 bit data line example :



## Odd Parity Generator/Checker

### ◆ Truth Table

A	B	C	D	E	O
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	0	1

### ◆ K-Map(Odd Parity)

		C	
		0	1
		3	2
		4	5
		7	6
A	{	12	13
		15	14
		8	9
		11	10
		D	

XOR는 1의 개수가  
홀수면 1, 짝수면 0

### ◆ Expression

$$\begin{aligned}
 & \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\overline{C}\overline{D} + ABC\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} \\
 &= \overline{A}\overline{B}(\overline{C}\overline{D} + CD) + \overline{A}B(\overline{C}\overline{D} + CD) + AB(\overline{C}\overline{D} + CD) + \overline{A}\overline{B}(\overline{C}\overline{D} + CD) \\
 &= \overline{A}\overline{B}(C \otimes D) + \overline{A}B(C \oplus D) + AB(C \otimes D) + \overline{A}\overline{B}(C \oplus D) \quad \oplus: XOR \\
 &= (C \otimes D)(\overline{A}\overline{B} + AB) + (C \oplus D)(\overline{A}B + A\overline{B}) \quad \otimes: XNOR \\
 &= (C \otimes D)(A \otimes B) + (C \oplus D)(A \oplus B) \\
 &= \overline{(C \oplus D)(A \oplus B)} + (C \oplus D)(A \oplus B) \\
 &= \overline{x}\overline{y} + xy \\
 &= x \otimes y \\
 &= \overline{x \oplus y} \\
 &= \overline{(C \oplus D) \oplus (A \oplus B)} \\
 &= C \oplus D \oplus A \oplus B
 \end{aligned}$$

$$\begin{aligned}
 x &= C \oplus D \\
 y &= A \oplus B
 \end{aligned}$$

## Parity check

Error 검출용 비트를 하나 더 추가 시켜서 언제나 전체 부호 속에 포함 되어 있는 1의 수가 홀수 또는 짝수개가 되도록 하는 것

10진수	$2^3$	$2^2$	$2^1$	$2^0$	패리티비트	1의 합계
0	0	0	0	0	1	1
1	0	0	0	1	0	1
2	0	0	1	0	0	1
3	0	0	1	1	1	3
4	0	1	0	0	0	1
5	0	1	0	1	1	3
6	0	1	1	0	1	3
7	0	1	1	1	0	3
8	1	0	0	0	0	1
9	1	0	0	1	1	3

패리티비트는  
단순하지만  
기능은 강하지  
않아서,  
한 비트 에러만  
알아낼 수 있지만  
어느 자리에서  
에러났는 지는 X,  
같은 홀, 짝일 때도 X,



# 3-6. Error Detection Codes

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☞ Word parity 패리티비트는 일괄하여, 상황에 따라 좋은 게 다름,

코드를 한 묶음 단위로 하는 Block단위, 에러 검출은 물론 정정 까지도 가능 하다

