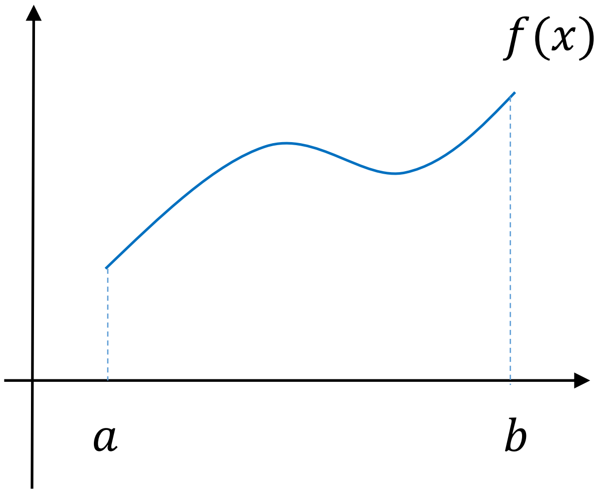
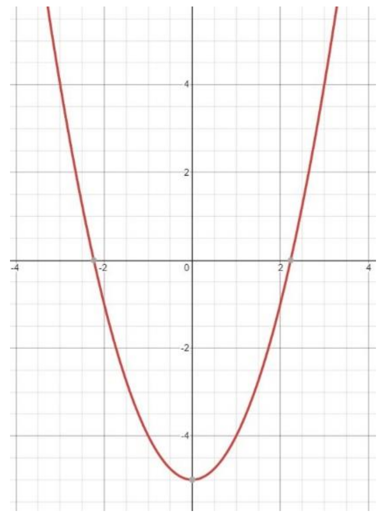
정적분의 응용1

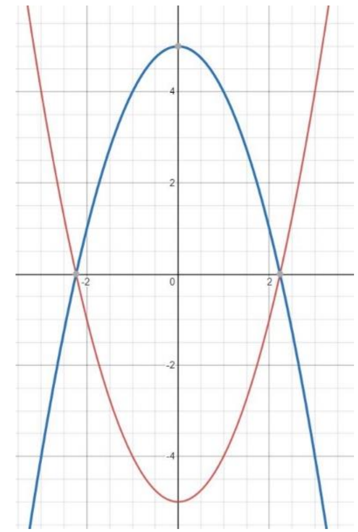
넓이와 부피

1 평면에서의 넓이



$$= \int_a^b f(x)\ dx$$





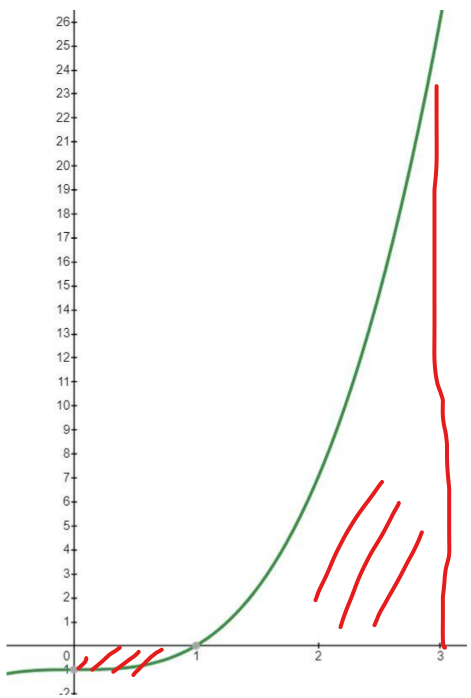
예제 1.1 , p.431 $y = x^2 - 5,$ x축, $x = -1,$ $x = 2$

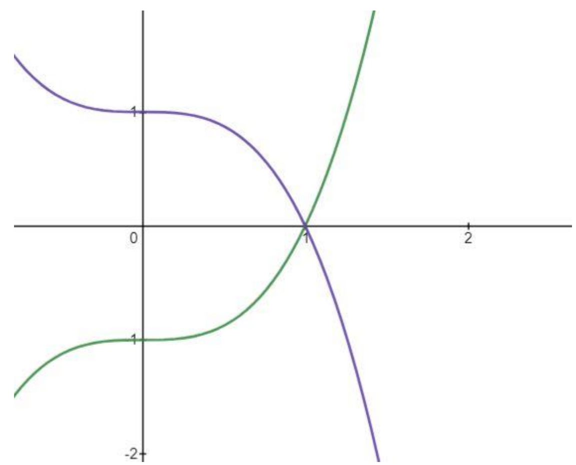
풀이 : $\int_{-1}^{2} |x^2 - 5|\, dx = \left[5x - \frac{1}{3}x^3\right]_{-1}^{2} =$ 12

예제 1.2 , p.432 $y = x^3 - 1,\quad x$축, $\quad x = 0, \qquad x = 3$

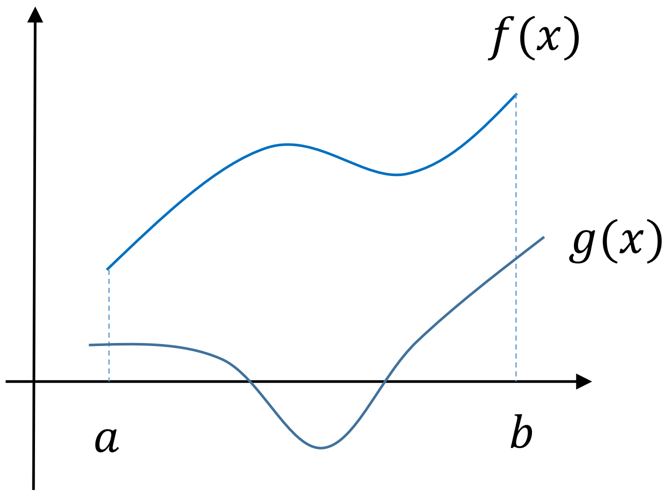
풀이 : $\int_0^3 |x^3 - 1|\, dx = -\int_0^1 (x^3 - 1)\, dx + \int_1^3 (x^3 - 1)\, dx =$

$$\left[x - \frac{1}{4}x^4\right]_0^1 + \left[\frac{1}{4}x^4 - x\right]_1^3 = \qquad \frac{75}{4}$$

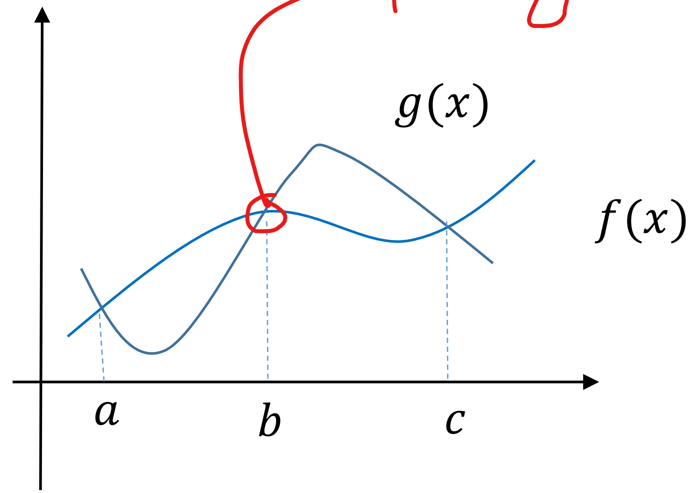




2. 두 곡선 사이의 넓이



f = g



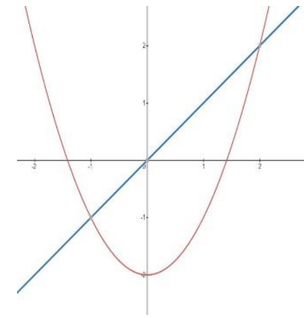
$$= \int_a^b (f(x) - g(x))\ dx$$

f > g

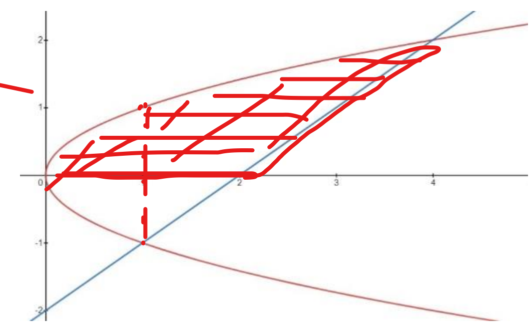
$$= \int_a^b (f(x) - g(x))\ dx + \int_b^c \big(g(x) - f(x)\big)\, dx$$

예제 1.6 , p.436 $\quad y = x^2 - 2, \qquad y = x$

풀이: $\displaystyle\int_{-1}^{2} \big((x) - (x^2 - 2)\big)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-1}^{2} = \frac{9}{2}$



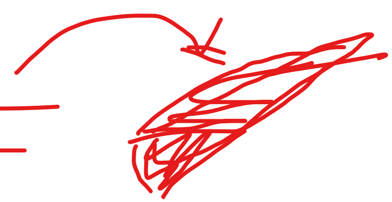
예제 1.7 , p.437 $\quad y^2 = x, \qquad y = x - 2, \qquad x$축



풀이: $\displaystyle\int_{0}^{2} \big((y + 2) - (y^2)\big)\, dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{0}^{2} = \frac{10}{3}$

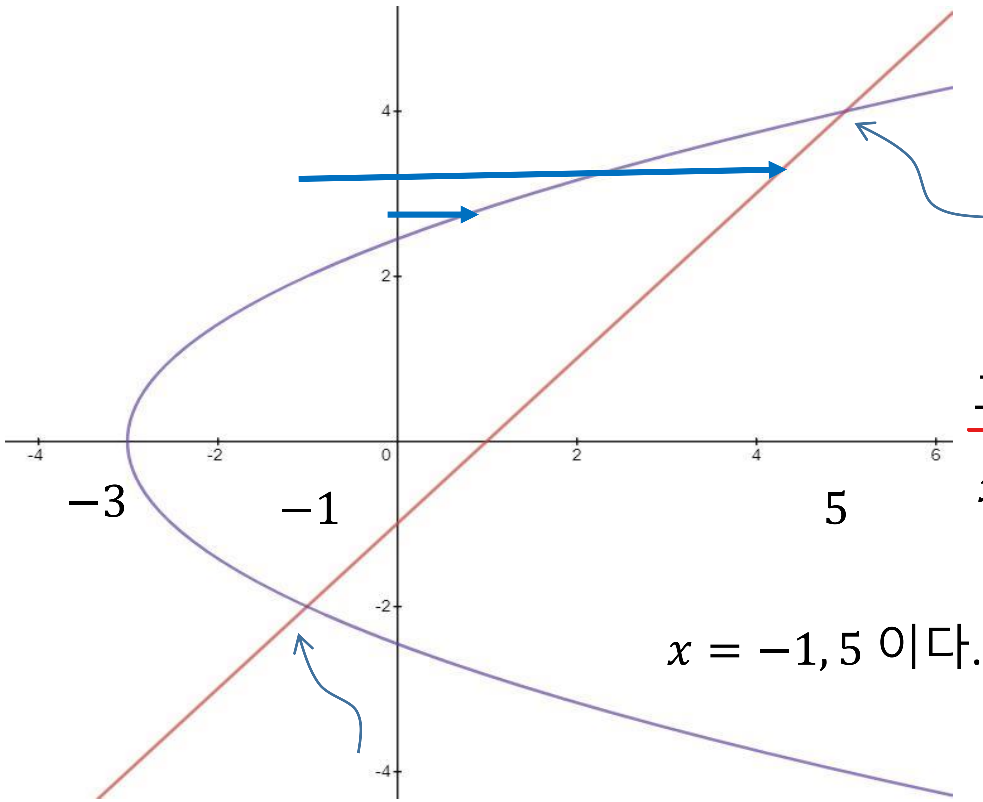
$$\cancel{\int_0^2 \sqrt{x}\, dx + \int_2^4 \left(\sqrt{x} - (x - 2)\right) dx = \frac{10}{3}}$$

$$\cancel{\int_{-1}^{2} \big((y + 2) - (y^2)\big)\, dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2} = \frac{9}{2}}$$



(예제 1.7) $\quad y = x - 1, \qquad y^2 = 2x + 6$

직 − 곡



교점을 구하면 $(x-1)^2 = 2x + 6$ 에서

$x^2 - 2x + 1 = 2x + 6 \qquad x^2 - 4x - 5 = 0$ 이므로

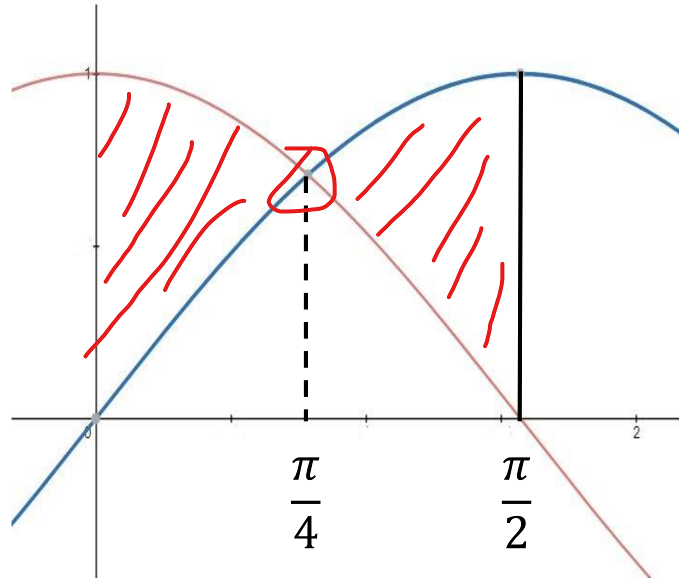
$x = -1, 5$ 이다. 이 때 $y = -2, 4$ 가 되어, 두 교점 $(-1,-2), (5,4)$

$$= \int_{-2}^{4} \left((y+1) - \left(\frac{1}{2} y^2 - 3 \right) \right) dy = 18$$

$$\cancel{= \int_{-3}^{-1} (\sqrt{2x+6} - (-\sqrt{2x+6}))\, dx + \int_{-1}^{5} (\sqrt{2x+6} - (x-1))\, dx}$$

(예제) $y = \cos x$, $\sin x$, $x = 0,$ $x = \frac{\pi}{2}$

$$\int_0^{\pi/2} (\cos x - \sin x)\, dx$$

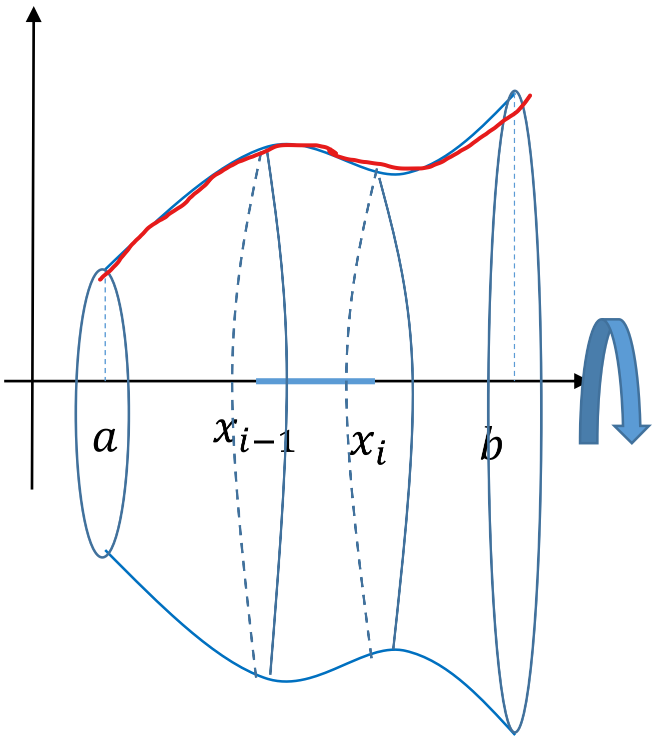


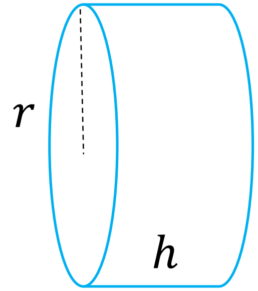
$$\int_0^{\pi/4} (\cos x - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$

$$= \left[\sin x + \cos x\right]_0^{\frac{\pi}{4}} + \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 2\sqrt{2} - 2$$

회전체의 부피

$f(x)$



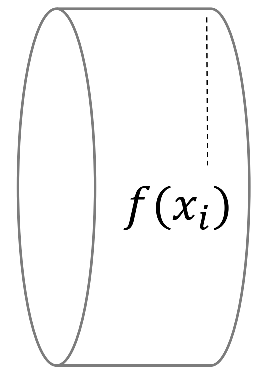


$= \pi r^2 h$

$$= \int_a^b \pi\, f(x)^2 \; dx$$

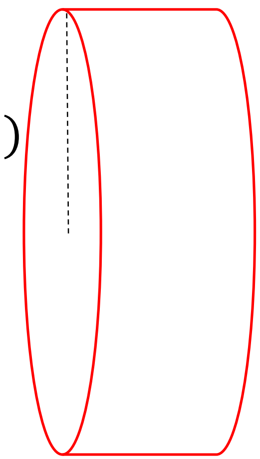
$\pi f(x_i)^2 \Delta x_i$

$$\sum_{i=1}^{n} \pi\, f(x_i)^2 \Delta x_i$$



Δx_i

$f(x_{i-1})$



$\boldsymbol{\pi f(x_{i-1})^2 \Delta x_i}$

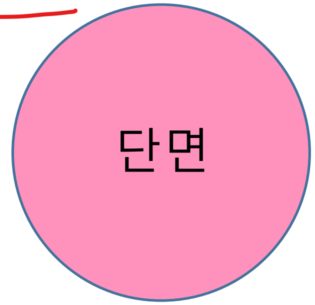
$$\boldsymbol{\sum_{i=1}^{n} \pi\, f(x_{i-1})^2 \Delta x_i}$$

$$\boldsymbol{\lim_{n\to\infty} \sum_{i=1}^{n} \pi\, f(x_{i-1})^2 \Delta x_i} = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \pi\, f(x_i)^2 \Delta x_i}$$

Δx_i

x 축 회전 : $\quad = \pi \int_a^b f(x)^2 \; dx$

y 축 회전 : $\quad = \pi \int_c^d f(y)^2 \; dy$



(예제2.4 p443)

$x = 0$ 부터 $x = 4$ 까지 $y = \sqrt{x}$, x 축 회전

$$= \pi \int_0^4 (\sqrt{x})^2 \; dx = \pi \int_0^4 x \, dx = 8\pi$$

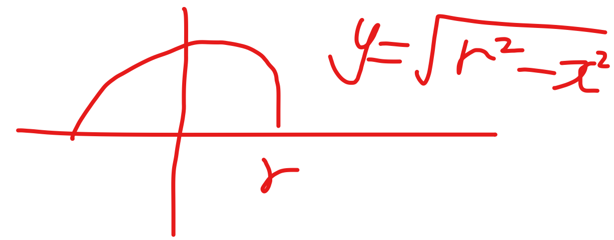
(예제2.5 p444)

$x = 0, \; y = 3, y = x^3$, y 축 회전

$$= \pi \int_0^3 (\sqrt[3]{y})^2 \; dy = \pi \int_0^3 y^{2/3} \, dy$$

$$= \frac{9\sqrt[3]{9}\pi}{5}$$

예제2.2 p.441 반지름이 r 인 구의 부피

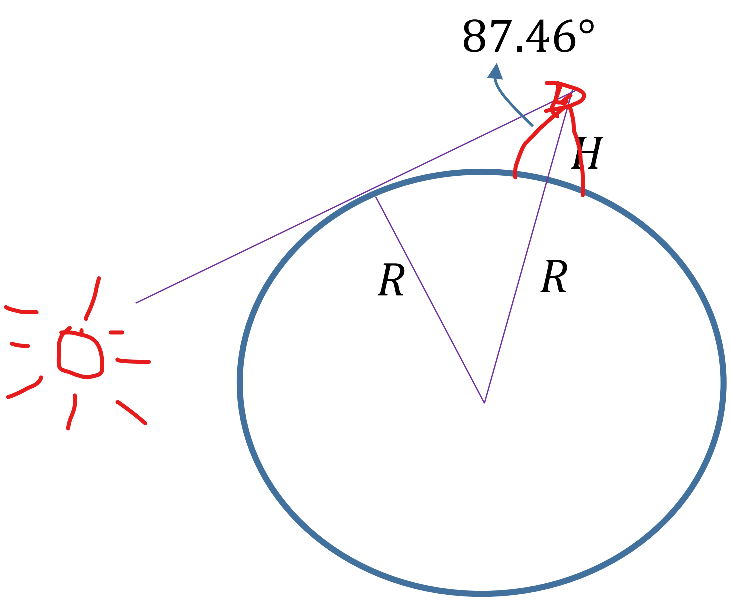


$$V = \int_{-r}^{r} \pi(r^2 - x^2)\,dx = 2\int_{0}^{r} \pi(r^2 - x^2)\,dx = \frac{4}{3}\pi r^3$$

지구의 반지름은 $6400km$ 이므로 지구의 부피는 $\frac{4}{3}\pi(6400)^3 \approx 1.08 \times 10^{12}(km^3)$: 1조$km^3$

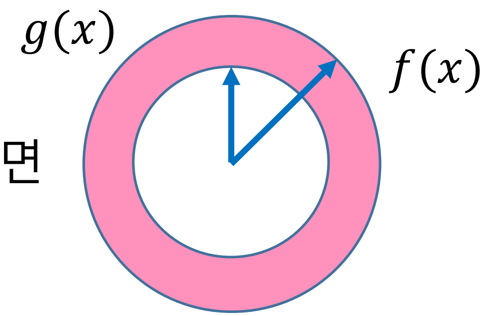
지구의 질량은 $5.98 \times 10^{24}kg$: 59조 8천억 톤

지구의 밀도(= 질량/부피)는 5.56 g/cm^3



$$\frac{R}{R+H} = 0.99924 = \sin(87.46°)$$

단면: 원환면



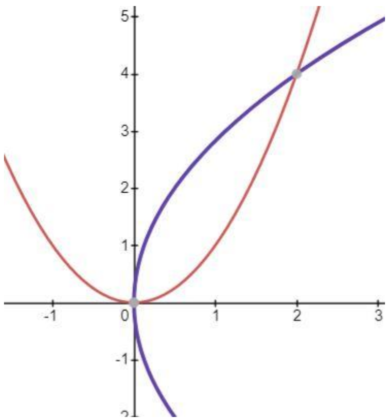
$$= \pi f(x)^2 - \pi g(x)^2$$

$$= \pi(f(x)^2 - g(x)^2)$$

x 축 회전 : $= \pi \int_a^b (f(x)^2 - g(x)^2)\ dx$

y 축 회전 : $= \pi \int_c^d (f(y)^2 - g(y)^2)\ dy$

(예제2.6, p445) $y = x^2, \qquad y^2 = 8x \qquad x$ 축 회전

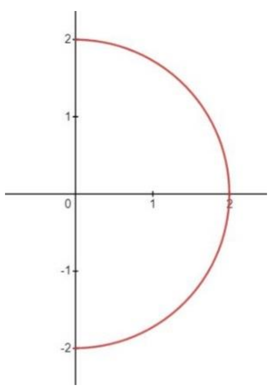


$$V = \pi \int_0^2 \left((\sqrt{8x})^2 - (x^2)^2 \right) dx$$

$$= \pi \int_0^2 (8x - x^4)\, dx$$

$$= \pi \left[4x^2 - \frac{1}{5} x^5 \right]_0^2 = \frac{48\pi}{5}$$

(예제2.7, p446) $x = \sqrt{4 - y^2} \qquad y$ 축, $\qquad x = -1$ 축 회전

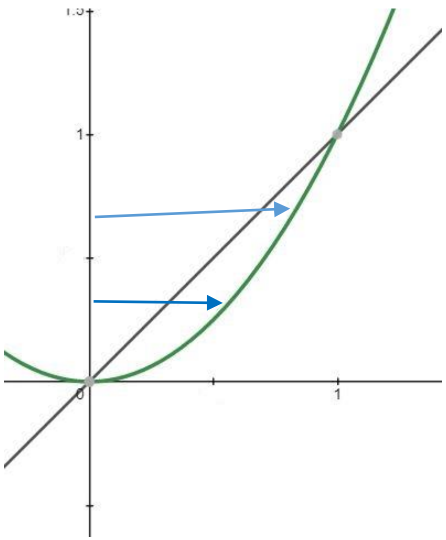


$$V = \pi \int_{-2}^2 \left((\sqrt{4 - y^2} + 1)^2 - (1)^2 \right) dy$$

$$= 2\pi \int_0^2 \left(2\sqrt{4 - y^2} + 4 - y^2 \right) dy$$

$$= 4\pi \int_0^2 \sqrt{4 - y^2}\, dy + 2\pi \int_0^2 (4 - y^2)\, dy = 4\pi^2 + \frac{32}{3}\pi$$

(예제) $y = x,\quad y = x^2\quad$ y 축 회전

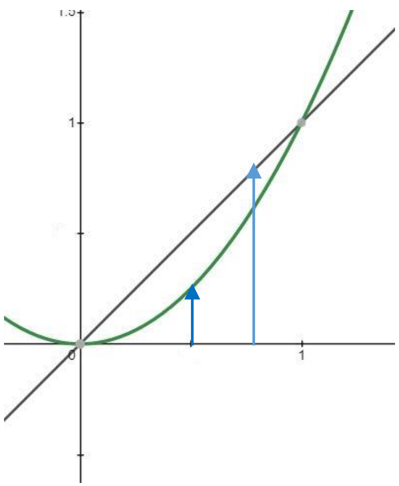


$$V = \pi \int_0^1 \left((\sqrt{y})^2 - (y)^2\right) dy$$

$$= \pi \int_0^1 (y - y^2)\, dy$$

$$= \pi \left[\frac{1}{2}\overset{y}{\cancel{x}}^{2} - \frac{1}{3}\overset{y}{\cancel{x}}^{3}\right]_0^1 = \frac{\pi}{6}$$

(예제, p412) $y = x,\quad y = x^2\quad$ x 축 회전

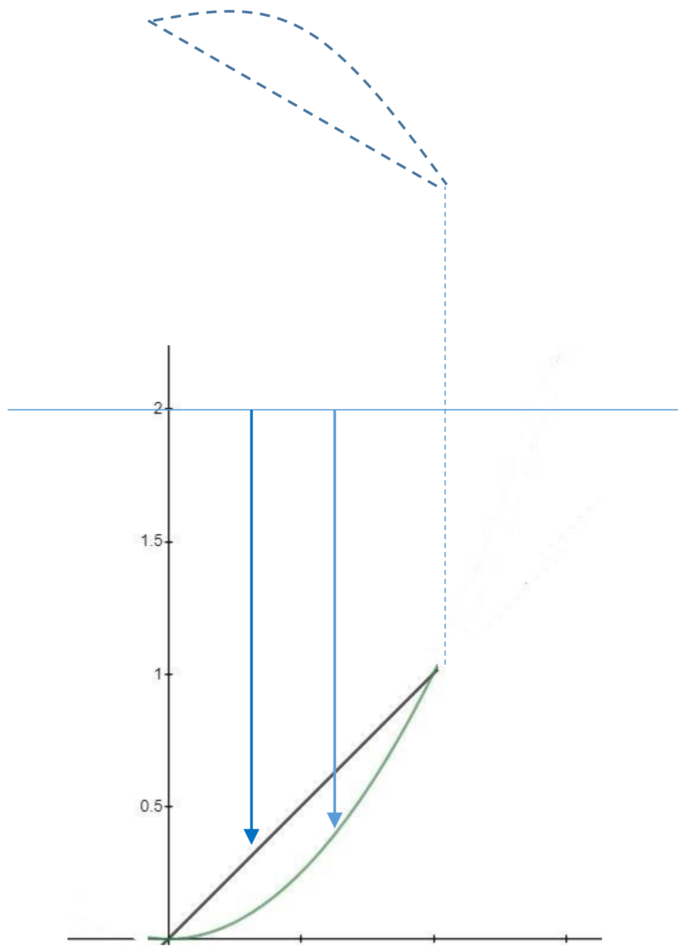


$$V = \pi \int_0^1 \left((x)^2 - (x^2)^2\right) dx$$

$$= \pi \int_0^1 (x^2 - x^4)\, dx$$

$$= \pi \left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1 = \frac{2\pi}{15}$$

(예제) $y = x, \quad y = x^2 \quad y = 2$ 축 회전

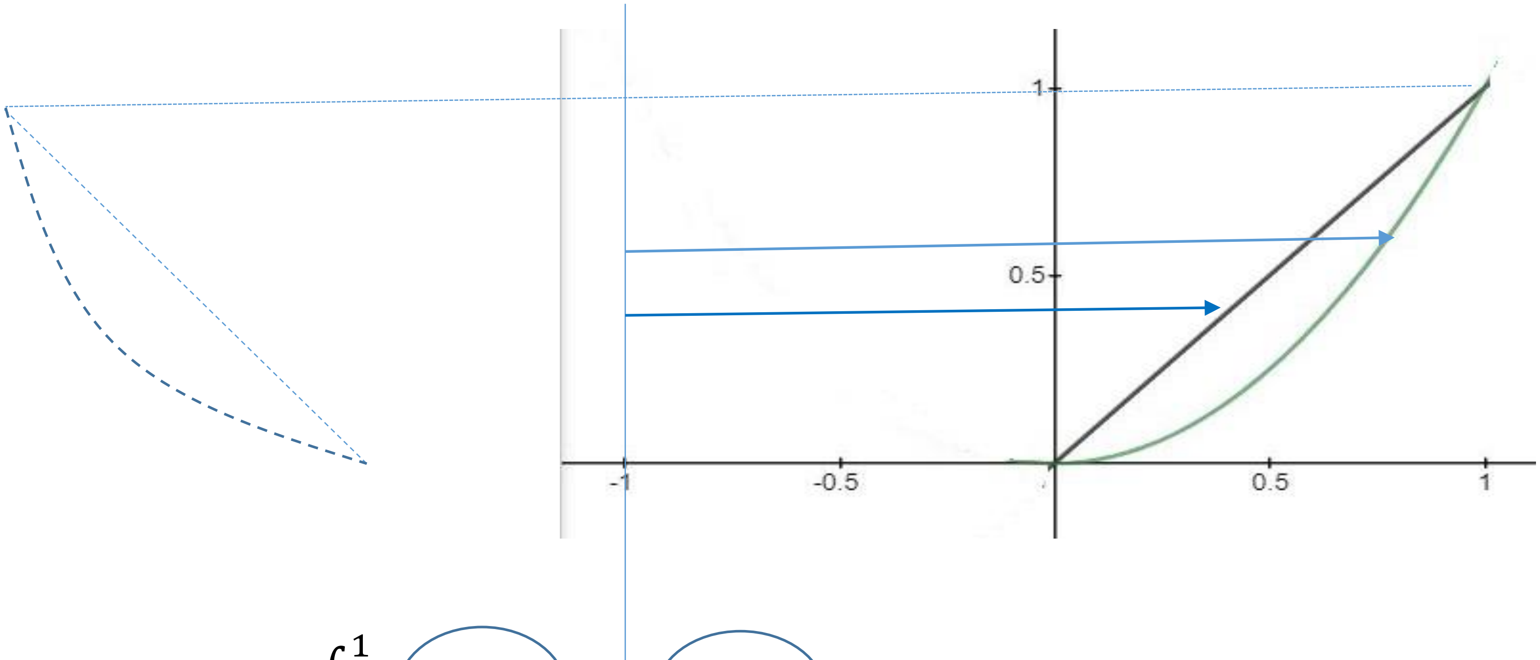


$$V = \pi \int_0^1 \left((2 - x^2)^2 - (2 - x)^2 \right) dx$$

$$= \pi \int_0^1 (x^4 - 5x^2 + 4x)\, dx$$

$$= \frac{8\pi}{15}$$

(예제) $y = x, \qquad y = x^2 \qquad x = -1$ 축 회전



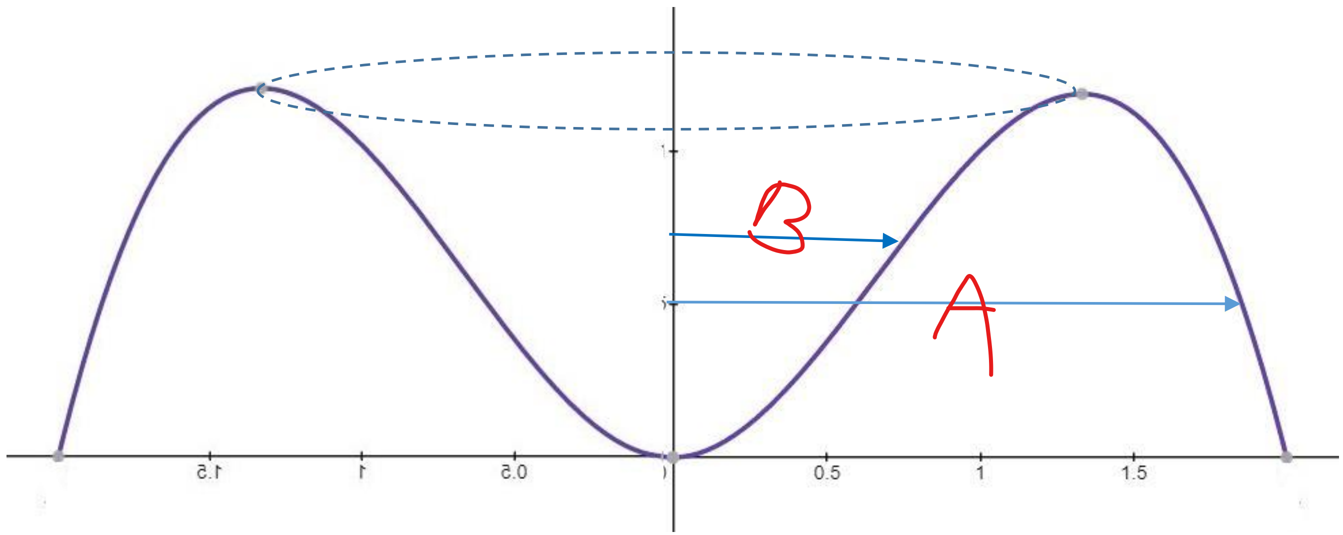
$$V = \pi \int_0^1 \left((1+\sqrt{y})^2 - (1+y)^2 \right) dy$$

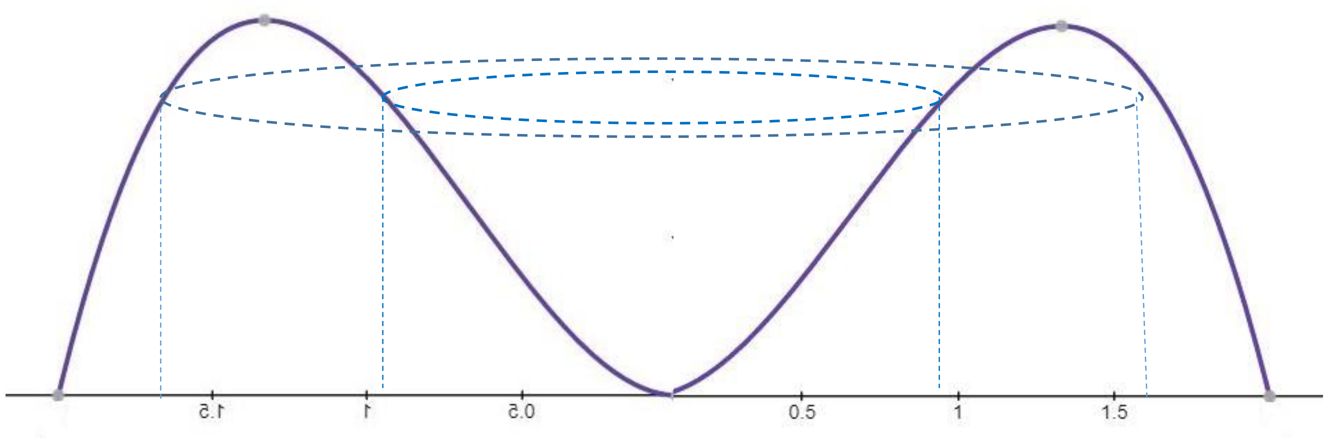
$$= \pi \int_0^1 (2\sqrt{y} - y - y^2)\, dy$$

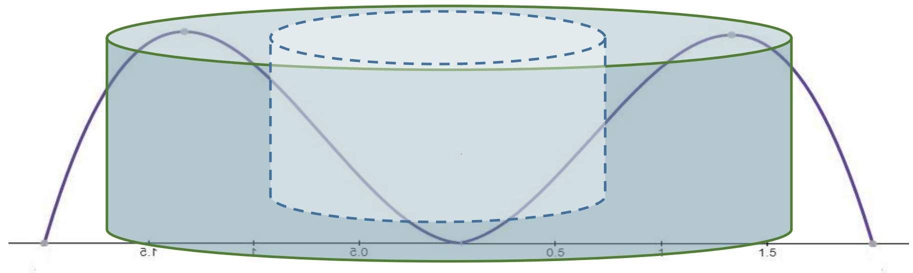
$$= \frac{\pi}{2}$$

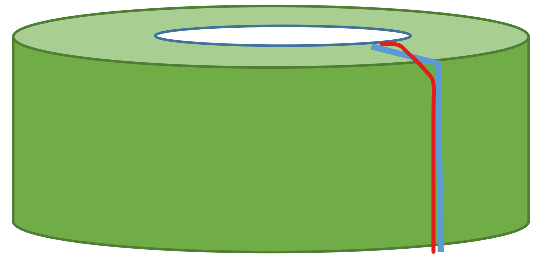
원기둥 껍질법 (p447)

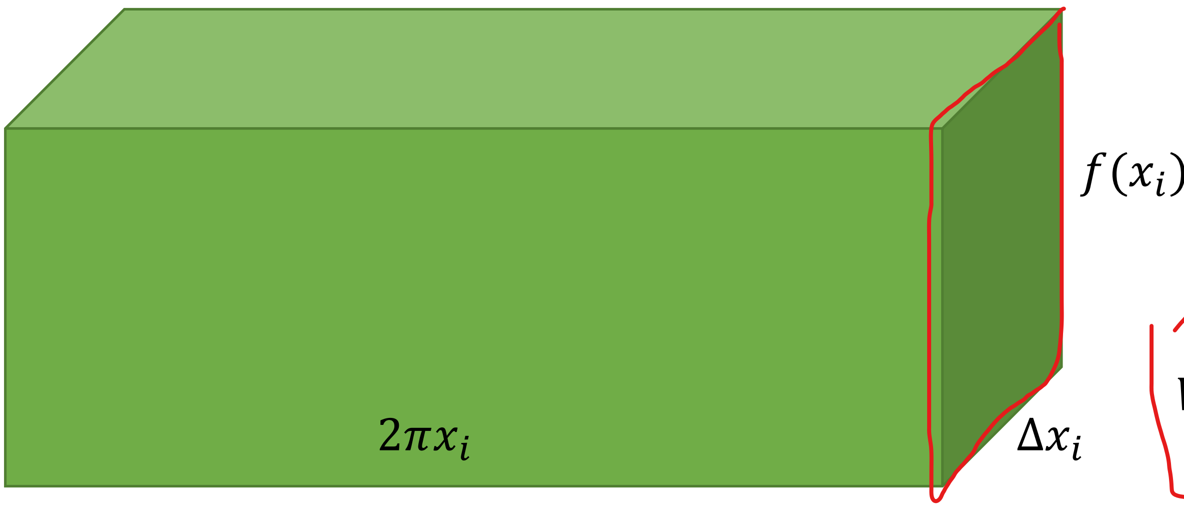
$y = 2x^2 - x^3$ 과 $y = 0$ 으로 둘러싸인 영역을 y 축 회전 $\quad V = \pi \int_c^d (A^2 - B^2)\, dy$











$$2\pi x_i f(x_i)\Delta x_i$$

$$\sum_{i=1}^{n} 2\pi x_i f(x_i)\Delta x_i$$

$$V = \lim_{n\to\infty} \sum_{i=1}^{\infty} 2\pi x_i f(x_i)\Delta x_i$$

y 축 회전 : $V = 2\pi \int_a^b x\, f(x)\, dx$

(예제) $y = 2x^2 - x^3$ 과 $y = 0$ 으로 둘러싸인 영역을 y 축 회전

(풀이) $V = 2\pi \int_0^2 x\,(2x^2 - x^3)\; dx \quad = 2\pi \left[\frac{1}{2}x^4 - \frac{1}{5}x^5\right]_0^2 = \frac{16\pi}{5}$

(예제) $y = \sin(x^2)$ 과 $y = 0$ 으로 둘러싸인 영역을 y 축 회전

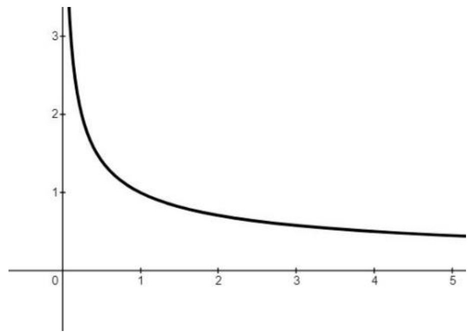
(풀이) $V = 2\pi \int_0^{\sqrt{\pi}} x \sin(x^2)\; dx$ 따라서 $V = 2\pi \cdot \left(-\frac{1}{2}\right) \cdot [\cos(x^2)]_0^{\sqrt{\pi}} = 2\pi$

$\int x \sin(x^2)\, dx$ $x^2 = u$ 로 치환 $= \frac{1}{2}\int \sin u\, du \;= -\frac{1}{2}\cos u$

$2xdx = du$ 가 되어

예제2.8 p.448 $y = \frac{1}{\sqrt{x}}$, $x = 1$, $x = 4$ y축 회전

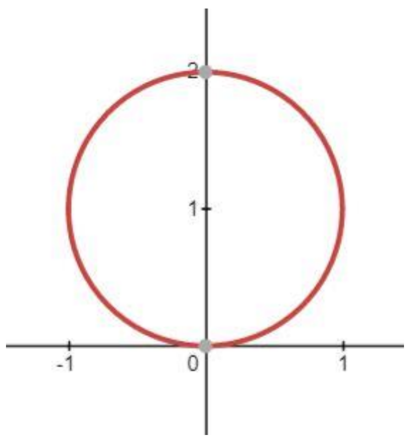
풀이:



$$V = 2\pi \int_1^4 x \times \frac{1}{\sqrt{x}}\, dx = 2\pi \int_1^4 \sqrt{x}\, dx = \frac{28}{3}\pi$$

y 축 회전 : $V = 2\pi \int_c^d y\, f(y)\, dy$

예제2.9 p.448 $x^2 + (y-1)^2 = 1$, $x \geq 0$ x축 회전



$$V = 2\pi \int_0^2 y \times \sqrt{1-(y-1)^2}\, dy = 2\pi \int_{-1}^{1} (t+1)\sqrt{1-t^2}\, dt = \pi^2$$

치환