1.3 함수의 극한

수열(Sequences)

$f: N \rightarrow R$

$f(1), f(2), \cdots, f(n), \cdots$

$a(1), a(2), \cdots, a(n), \cdots$ $\quad\quad$ $a_1, a_2, \cdots, a_n, \cdots$

$a_n = n = 1, 2, 3, \cdots$ → $\lim_{n\to\infty} n = \infty$ 발산: diverge

$a_n = \frac{1}{n} = 1, \quad \frac{1}{2}, \quad \frac{1}{3}, \cdots$ → $\lim_{n\to\infty} \frac{1}{n} = 0$ 수렴: converge

$a_n = (-1)^n = -1, 1, -1, \cdots$ → $\lim_{n\to\infty} (-1)^n = \infty$ 발산: diverge

$\lim_{x\to a} f(x) = L$ $\quad\quad$ $\lim_{x\to 3}(x+4) = 7,$ $\quad\quad$ $\lim_{x\to 3} 4 = 3$ ~~3~~ 4

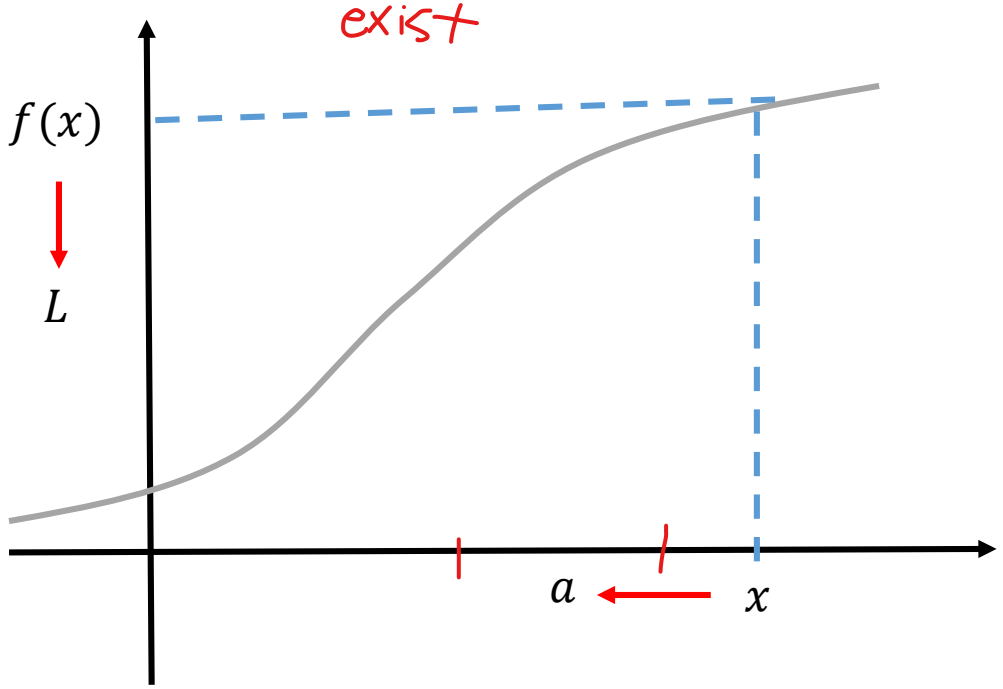
$$\lim_{x \to a} f(x) = L$$

$$\forall\ \varepsilon > 0, \quad \exists\ \delta > 0\ \ s.t. \quad 0 < |x - a| < \delta \ \Rightarrow\ |f(x) - L\,| < \varepsilon$$

for all

Here exist

조건



(1) $|x - a| < \delta \ \Leftrightarrow\ -\delta < x - a < \delta$

$$a - \delta < x < a + \delta$$

(2) $0 < |x - a|$

$$x \neq a$$

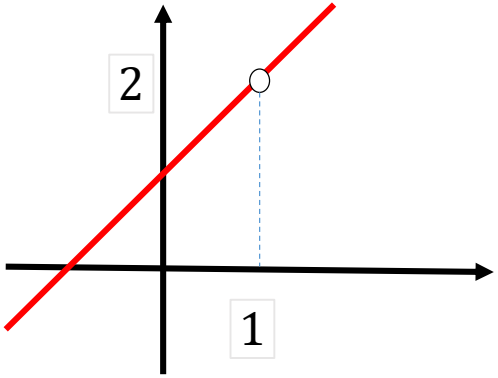
(3) $L - \varepsilon < f(x) < L + \varepsilon$

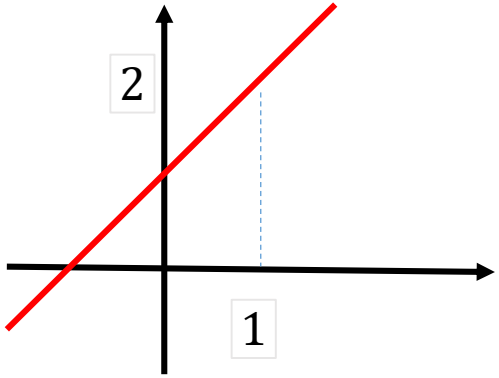
$$f(x) = L\ ?$$

$\frac{0}{0}$ 꼴 p. 65 예제1.5

$$\lim_{x\to 1}\frac{x^2-1}{x-1} \qquad \frac{x^2-1}{x-1}=\frac{(x+1)(x-1)}{x-1}=x+1$$

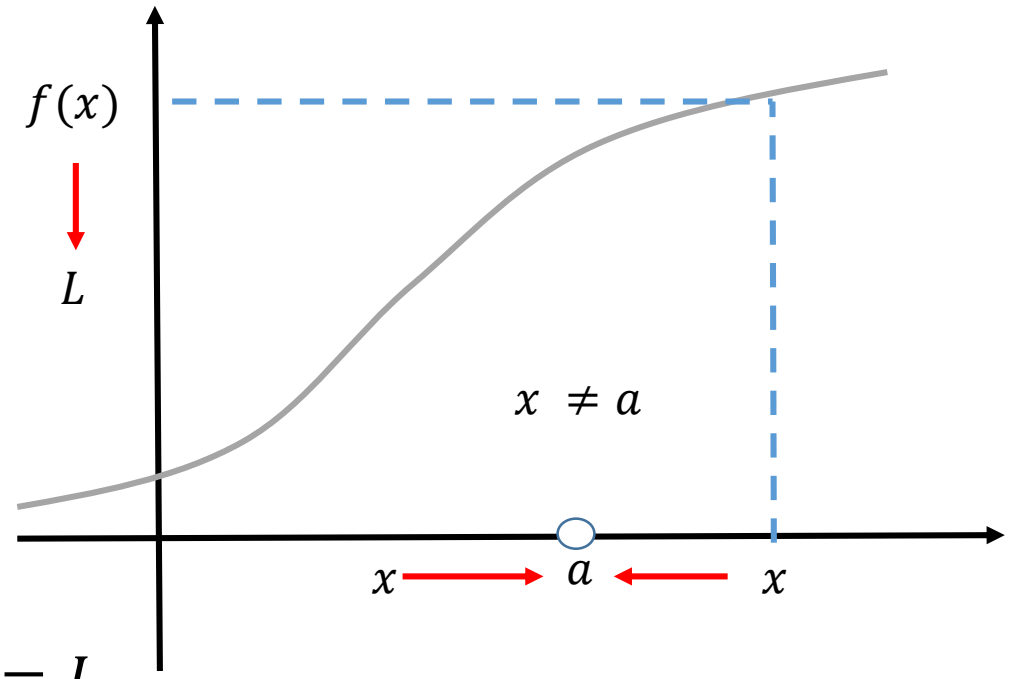
$$f(x)=\frac{x^2-1}{x-1} \quad \Rightarrow \quad f(x)=x+1$$





$$\lim_{x\to 1}\frac{x^2-1}{x-1}=\lim_{x\to 1}(x+1)=2$$

1.4 한쪽극한



$$(1)\ \lim_{x \to a+} f(x) = L$$

$$\forall\ \varepsilon > 0, \qquad \exists\ \delta > 0\ \ s.t. \quad a < x < a + \delta \ \Rightarrow\ |f(x) - L| < \varepsilon$$

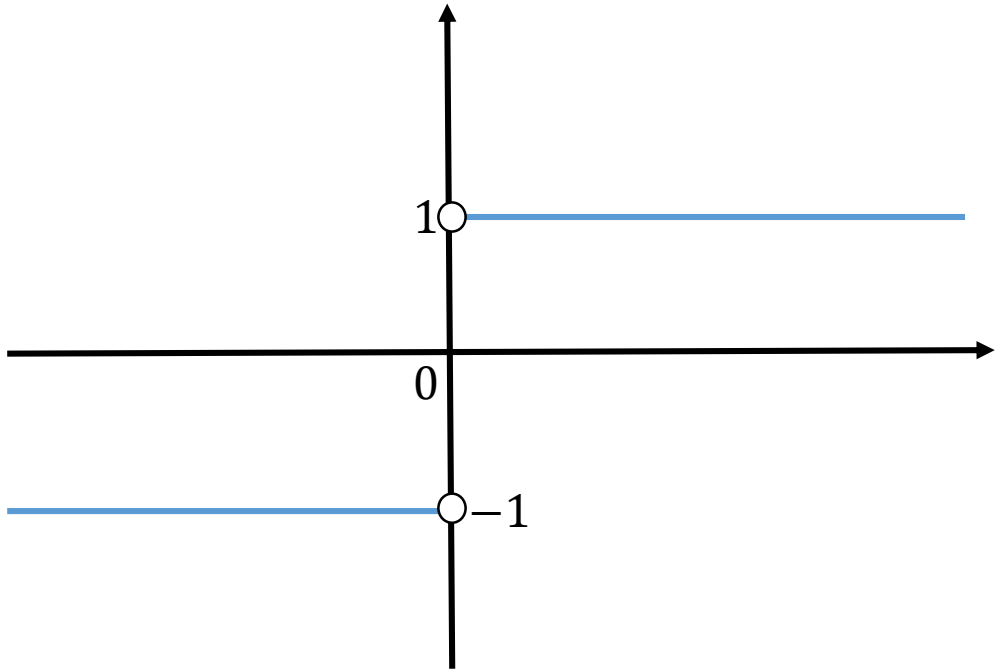
$$(2)\ \lim_{x \to a-} f(x) = L$$

$$\forall\ \varepsilon > 0, \qquad \exists\ \delta > 0\ \ s.t. \quad a - \delta < x < a \ \Rightarrow\ |f(x) - L| < \varepsilon$$

$$\lim_{x\to a} f(x) = L \Leftrightarrow \lim_{x\to a+} f(x) = \lim_{x\to a-} f(x)$$

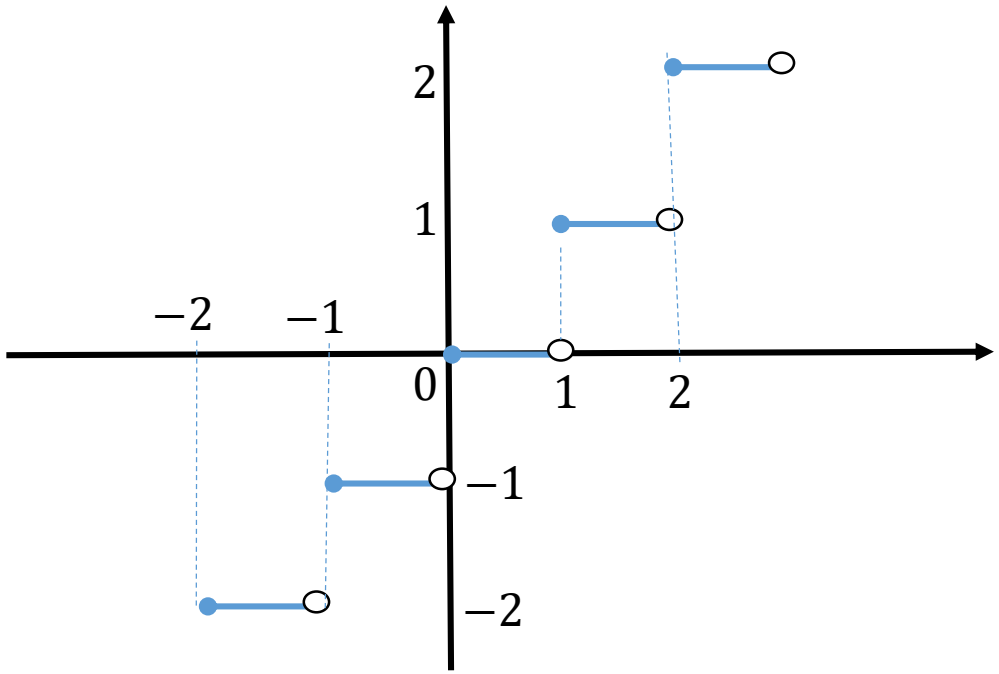
P67 예제1.9 $\lim_{x\to 0} \frac{|x|}{x}$

(1) $x > 0$, $\lim_{x\to 0+} \frac{|x|}{x} = \lim_{x\to 0+} \frac{x}{x} = 1$ (2) $x < 0$, $\lim_{x\to 0-} \frac{|x|}{x} = \lim_{x\to 0-} \frac{-x}{x} = -1$



(예제) 최대 정수 함수 $f(x) = [x]$ $[1.3] = 1$ $[1.99] = 1$

$[-1.3] = -2$ $[0] = 0$



$\lim_{x \to 2} [x]$

p.67극한의 기본성질

$\lim_{x\to a} f(x)$ $\lim_{x\to a} g(x)$

linearity 선형

(1) $\lim_{x\to a}\big(f(x)+g(x)\big)=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$

(2) $\lim_{x\to a}\big(f(x)-g(x)\big)=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$

(3) $\lim_{x\to a} cf(x)=c\lim_{x\to a} f(x)$

(4) $\lim_{x\to a}\big(f(x)*g(x)\big)=\lim_{x\to a} f(x)*\lim_{x\to a} g(x)$

(5) $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ $\lim_{x\to a} g(x)\neq 0$

p.69 예제

$$\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2} = \frac{1}{4}$$

$$\lim_{t\to 0}\frac{\sqrt{t^2+9}-3}{t^2}$$

$$= \lim_{t\to 0}\frac{(t^2+9)-9}{t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0}\frac{t^2}{t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0}\frac{1}{\sqrt{t^2+9}+3} = \frac{1}{6}$$

$$\lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}\frac{(x^2+2xh+h^2)-x^2}{h} = \lim_{h\to 0}\frac{2xh+h^2}{h} = 2x$$

$$\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \lim_{x\to a}\frac{\frac{a-x}{xa}}{x-a} = \lim_{x\to a}\frac{-1}{ax} = -\frac{1}{a^2}$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

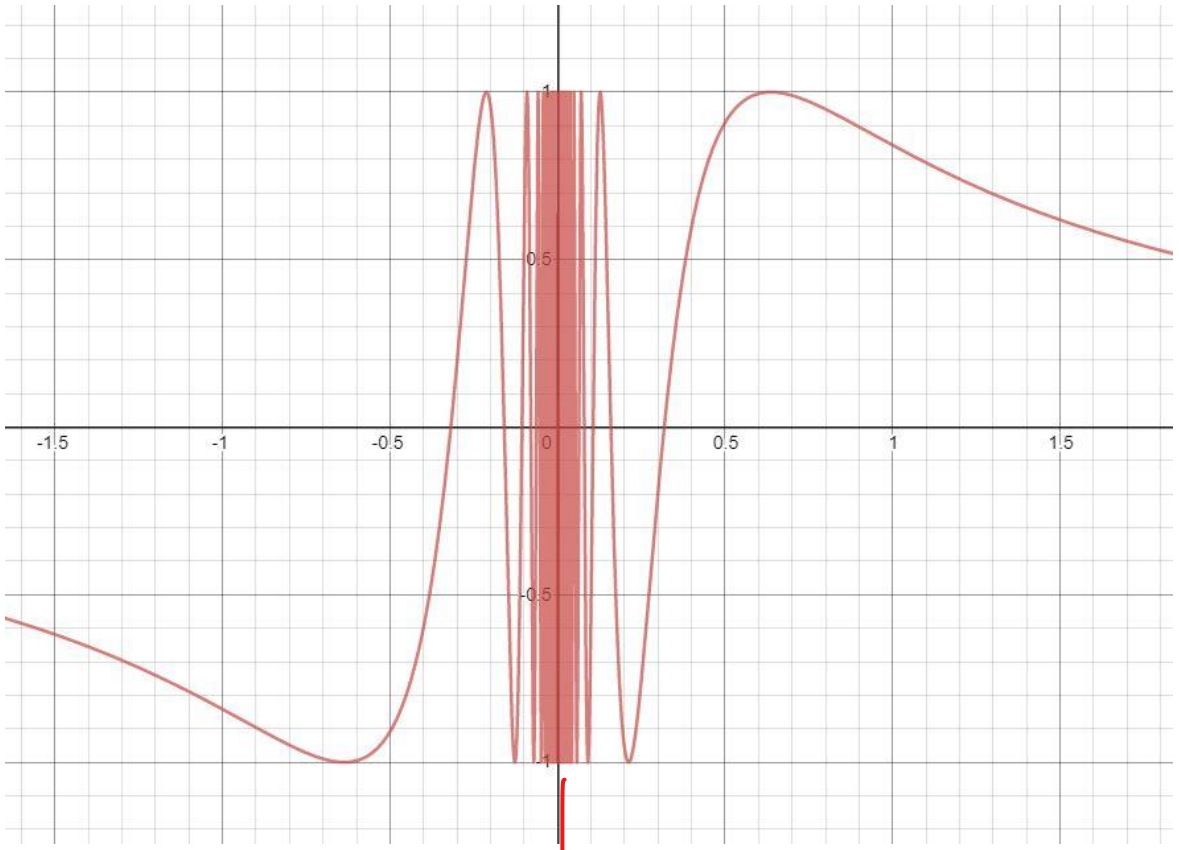
$$\lim_{x\to a} c = c$$

$$\lim_{x\to a} x = a \qquad \lim_{x\to a} x^n = a^n$$

$\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ n 은 양의 정수, n 이 짝수이면 $a > 0$

$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ n 은 양의 정수, n 이 짝수이면 $\lim_{x\to a} f(x) > 0$

$$\lim_{x \to 0} \sin \frac{1}{x}$$ 극한값 없음



└ ?

$$\lim_{x\to 0} x\sin\frac{1}{x} \neq \lim_{x\to 0} x \lim_{x\to 0}\sin\frac{1}{x} \neq 0 \qquad be\ careful\ \text{!!!}$$

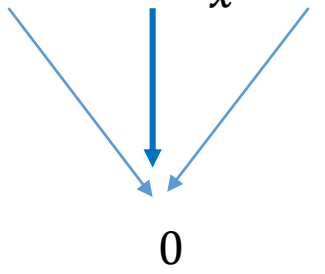
조임정리 (샌드위치, 압축)

$$f(x) \le g(x) \le h(x), \quad \lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L \ \Rightarrow\ \lim_{x\to a} g(x) = L$$

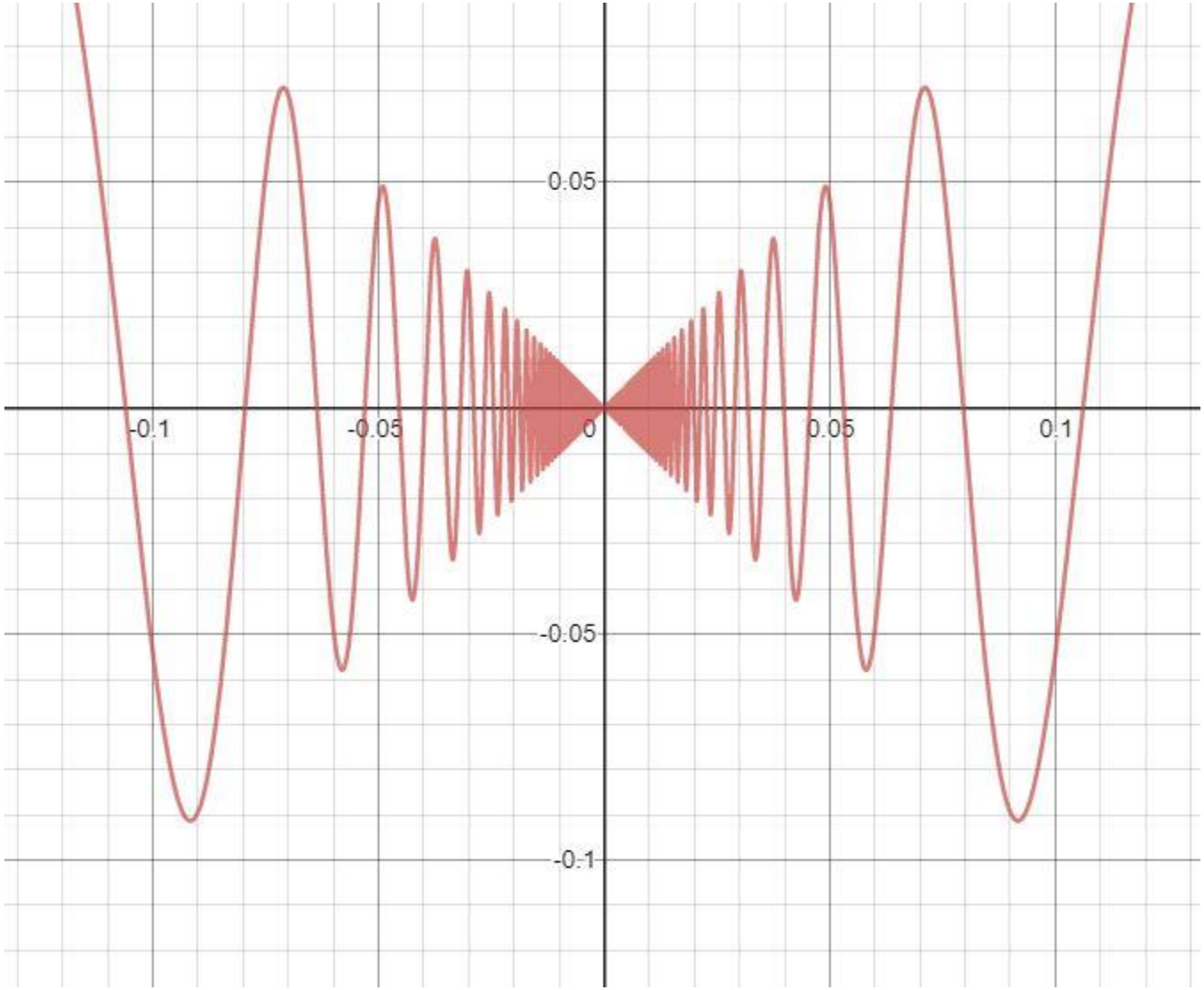
p. 71예제1.15 $\lim_{x\to 0} x\sin\frac{1}{x}$

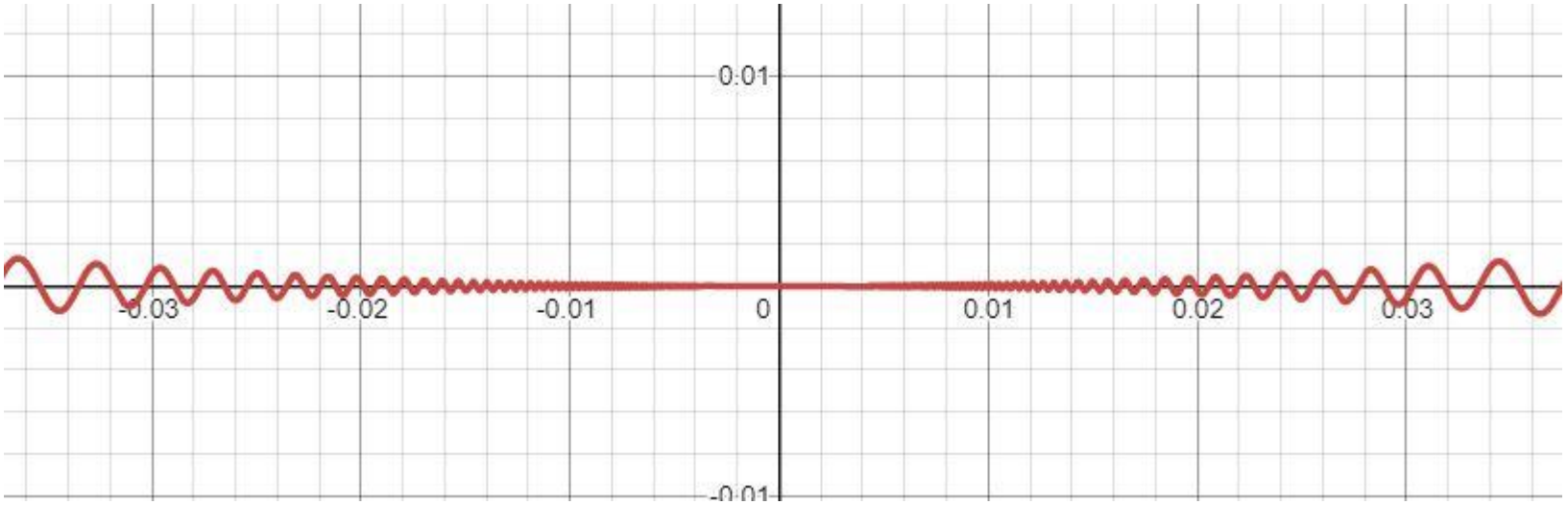
$$-1 \le \sin\frac{1}{x} \le 1$$

$$-x \le x\sin\frac{1}{x} \le x$$

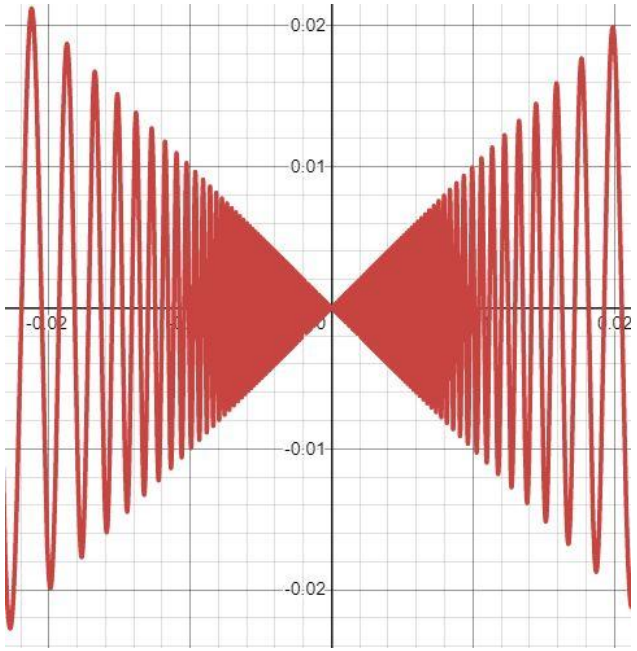


$$\lim_{x \to 0} x \sin \frac{1}{x} = 0$$





$$f(x) = x^2 \sin\frac{1}{x}$$



$$f(x) = x\,cos\,\frac{1}{x}$$

혼자 해보기

(1) $f(x)=\begin{cases} x^2, & x\text{: 유리수} \\ 0, & x\text{: 무리수} \end{cases}$ 일 때 $\lim_{x\to 0} f(x)$ 를 구하시오.

(2) $\lim_{x\to\frac{\pi}{2}}[\cos x\,]$ 은 존재하는가? (단 $-\pi \le x \le \pi$)

(3) $\lim_{x\to 0+}\left(\sqrt{x}\, e^{\sin\left(\frac{3\pi}{x}\right)}\right)$ 를 구하시오.