5.3 부분 적분법(Integration by parts)

$$\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g'(x) = \big(f(x)g(x)\big)' - f'(x)g(x)$$

양변 적분

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int udv = uv - \int vdu$$

$f(x) = u$ —미분→ $f'(x)dx = du$

$g'(x)dx = dv$ —적분→ $g(x) = v$

유형 1 .

$$\int udv = uv - \int vdu$$

$$\int x \sin x \ dx = -x \cos x - \int (-\cos x) dx$$

$x = u$ $\xrightarrow{\text{미분}}$ $dx = du$

$\sin x \, dx = dv$ $\xrightarrow{\text{적분}}$ $-\cos x = v$

$$= -x \cos x + \sin x + C$$

$$\int x \cos 5x \ dx =$$

$$\int t \ \sin 2t \ dt =$$

$$\int (x^2 + 2x) \ \cos x \, dx =$$

$$\int y \ \sin y \, dy =$$

유형 1-1 .

$$\int u dv = uv - \int v du$$

$$\int x\, e^x\, dx = xe^x - \int e^x dx$$

$x = u$ $\xrightarrow{\text{미분}}$ $dx = du$

$e^x dx = dv$ $\xrightarrow{\text{적분}}$ $e^x = v$

$$= xe^x - e^x + C$$

$$\int y\, e^{0.2y}\, dy =$$

$$\int t\, e^{t/2}\, dt =$$

$$\int z^3\, e^z dz =$$

$$\int (x^2 + 1)\, e^{-x}\, dx =$$

유형 2 .

$$\int u dv = uv - \int v du$$

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \left(\frac{1}{2}x^2\right)\left(\frac{1}{x}\right) dx = \frac{1}{2}x^2 \ln x - \int \left(\frac{1}{2}x\right) dx$$

$\ln x = u$ $\xrightarrow{\text{미분}}$ $\frac{1}{x}dx = du$

$xdx = dv$ $\xrightarrow{\text{적분}}$ $\frac{1}{2}x^2 = v$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

$\int x^4 \ln x \, dx =$

$\int w^2 \ln w \, dw =$

$\int \frac{\ln R}{R^2} dR =$

$\int \frac{\ln y}{\sqrt{y}} dy =$

유형3 .

$$\int u dv = uv - \int v du$$

$$\int \ln x \, dx = x \ln x - \int x \left(\frac{1}{x}\right) dx = x \ln x - \int dx$$

$\ln x = u$ —미분→ $\frac{1}{x} dx = du$

$dx = dv$ —적분→ $x = v$

$$= x \ln x - x + C$$

$$\int \tan^{-1} 2y \, dy =$$

$$\int \cos^{-1} x \, dx =$$

$$\int \cos(\ln x) \, dx =$$

u

$$\int (\arcsin x)^2 \, dx =$$

유형4.

$$\int udv = uv - \int vdu$$

$$\int e^x \sin x \; dx = -e^x \cos x - \int e^x(-\cos x)dx = -e^x \cos x + \left(e^x \sin x - \int e^x(\sin x)dx \right)$$

$e^x = u \xrightarrow{\text{미분}} e^x dx = du$

$\sin x \, dx = dv \xrightarrow[\text{적분}]{} -\cos x = v$

$e^x = u \xrightarrow{\text{미분}} e^x dx = du$

$\cos x \, dx = dv \xrightarrow[\text{적분}]{} \sin x = v$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2}(e^x \sin x - e^x \cos x) + C$$

~~Laplace 변환~~

혼자 해보기

1. $\int \cos(\ln x)\, dx$

$= \frac{1}{2} x \cos(\ln x) + \frac{1}{2} x \sin(\ln x) + C$

2. $\int e^{\sqrt{x}}\, dx$

$= 2\sqrt{x} e^{\sqrt{x}} - 2\, e^{\sqrt{x}} + C$

3. $\int e^{\cos x} \sin 2x \;\; dx$

hint : $\sin 2x \;\; = 2 \sin x \cos x$

연습 323P 1~6