유리함수의 적분법
무리함수의 적분법

5. 유리함수의 적분법

$$\int \frac{f(x)}{g(x)}\,dx$$

$$\int \frac{c}{ax+b}\,dx = \frac{c}{a}\ln|ax+b|$$

1. $\deg(f) \geq \deg(g)$ (차수)

(예제 5.5p 343)

$$\int \frac{x^2-1}{x+2}\,dx = \int \left(x-2+\frac{3}{x+2}\right)dx$$

$$= \frac{1}{2}x^2 - 2x + 3\ln|x+2| + C$$

$$\begin{array}{r|l} & x \quad -2 \\ \hline x+2 & x^2 \qquad -1 \\ & x^2+2x \\ \hline & -2x \quad -1 \\ & -2x-4 \\ \hline & 3 \end{array}$$

(예제) $\int \frac{x^3+3x-1}{x+1}dx = \int \left(x^2-x+4-\frac{5}{x+1}\right)dx$

$$
\begin{array}{r|l}
 & x^2 \quad -x \quad +4 \\
\hline
x+1 & x^3 + \qquad 3x \quad -1 \\
 & x^3+x^2 \\
\hline
 & \quad -x^2+3x \\
 & \quad -x^2-x \\
\hline
 & \qquad 4x-1 \\
 & \qquad 4x+4 \\
\hline
 & \qquad -5
\end{array}
$$

$$= \frac{1}{3}x^3 - \frac{1}{2}x^2 + 4x - 5\ln|x+1| + C$$

2. $\deg(f) < \deg(g)$ 부분 분수(partial fraction)분해법 이용

(1) $\dfrac{1}{x(x-1)(x+3)} = \dfrac{a}{x} + \dfrac{b}{x-1} + \dfrac{c}{x+3}$

(2) $\dfrac{1}{x(x-1)^2(x+3)} = \dfrac{a}{x} + \dfrac{b}{x-1} + \dfrac{c}{(x-1)^2} + \dfrac{d}{x+3}$ 분모의 인수가 1차식이다.

(3) $\dfrac{1}{x(x-1)\,(x^2+3)} = \dfrac{a}{x} + \dfrac{b}{x-1} + \dfrac{cx+d}{x^2+3}$ 분모보다 한 차수 낮게

a, b, c 를 열심히~^^!

(예제5.1 , p340) $\int \frac{x}{x^2-5x+6}\,dx =$

(1) 분모 인수분해 : $x^2-5x+6=(x-2)(x-3)$

(2) $\frac{x}{x^2-5x+6}=\frac{A}{x-2}+\frac{B}{x-3}$

(3) $x=A(x-3)+B(x-2)$ $\quad A=-2, B=3$

(4) $=\int\left(\frac{-2}{x-2}+\frac{3}{x-3}\right)dx$

(5) $=-2\ln|x-2|+3\ln|x-3|+C$

예제 $\int \frac{4x}{x^3-x^2-x+1}\,dx =$

(1) 분모 인수분해 : $x^3-x^2-x+1=(x-1)^2(x+1)$

(2) $\frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$ $\quad \frac{A}{x-1}+\frac{B}{x-1}+\frac{C}{x+1}$

(3) $4x = A(x-1)(x+1)+B(x+1)+C(x-1)^2$ $\quad A=1, B=2, C=-1$

(4) $= \int \left(\frac{1}{x-1}+\frac{2}{(x-1)^2}-\frac{1}{x+1}\right)dx$

(5) $= \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + C$

열씸열씸! 깡

(예제) $\int \frac{2x^2 - x + 4}{x^3 + 4x}\, dx =$

(1) 분모 인수분해 : $x^3 + 4x = x(x^2 + 4)$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

(2) $\frac{2x^2 - x + 4}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$

(3) $2x^2 - x + 4 = A(x^2 + 4) + (Bx + C)x$ $\quad A = 1, \quad B = 1, \quad C = -1$

(4) $= \int \left(\frac{1}{x} + \frac{x - 1}{x^2 + 4}\right) dx = \int \frac{1}{x}\, dx + \int \frac{x}{x^2 + 4}\, dx - \int \frac{1}{x^2 + 4}\, dx$

치환적분

피 땀 눈물! 2깡

(5) $= \ln|x| + \frac{1}{2}\ln(x^2 + 4) - \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

(예제) $\int \frac{1}{a^2 - x^2}\, dx$

(1) 분모 인수분해 : $a^2 - x^2 = (a - x)(a + x)$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

(2) $\frac{1}{(a-x)(a+x)} = \frac{A}{a-x} + \frac{B}{a+x}$

이 정도야~ 거뜬

$$\tanh^{-1}\frac{x}{a} = \frac{1}{2}\ln\left(\frac{1+\frac{x}{a}}{1-\frac{x}{a}}\right)$$

(3) $1 = A(a+x) + B(a-x)$ $\quad A = \frac{1}{2a}, B = \frac{1}{2a}$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

(4) $= \int \left(\frac{1}{2a}\frac{1}{a-x} + \frac{1}{2a}\frac{1}{a+x}\right) dx$

(5) $= \frac{1}{2a}(-\ln|a-x| + \ln|a+x|) + K = \frac{1}{2a}\left(\ln\left|\frac{a+x}{a-x}\right|\right) + K = \frac{1}{a}\tanh^{-1}\frac{x}{a} + K$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \qquad \int \frac{1}{a^2-x^2}\,dx = \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

(예제) (1) $\int \frac{1}{x^2+4x+11}\,dx = \int \frac{1}{x^2+4x+4+7}\,dx = \int \frac{1}{(x+2)^2+7}\,dx$

$x+2=u$로 치환, $dx=du$

$$= \int \frac{1}{u^2+7}\,du = \frac{1}{\sqrt{7}}\tan^{-1}\frac{u}{\sqrt{7}}$$

$$= \frac{1}{\sqrt{7}}\tan^{-1}\frac{x+2}{\sqrt{7}} + C$$

(2) $\int \frac{1}{15-4x-x^2}\,dx = \int \frac{1}{15+4-4-4x-x^2}\,dx = \int \frac{1}{19-(x+2)^2}\,dx$

$x+2=u$로 치환, $dx=du$

$$= \int \frac{1}{19-u^2}\,du = \frac{1}{\sqrt{19}}\tanh^{-1}\frac{u}{\sqrt{19}}$$

$$= \frac{1}{\sqrt{19}}\tan^{-1}\frac{x+2}{\sqrt{19}} + C$$