

1. What are the last digits of the following numbers?

- (a) 11^{2014}
- (b) 9^{10001}
- (c) $3^{987654321}$

2. Calculate the greatest common divisor (gcd) of the following pairs of numbers using the Euclidean algorithm.

[Hasty refresher: starting with a pair of input values, keep repeating the operation “Replace the larger value with its remainder modulo the smaller value” over and over, until one of the values becomes zero. At that point, the other value is the gcd of the original two inputs (as well as of every pair of values along the way).

In pseudocode: $\text{gcd}(x, y) = \text{if } y = 0 \text{ then return } x \text{ else return } \text{gcd}(y, x \bmod y)$].

- (a) 208 and 872
- (b) 1952 and 872
- (c) $1952 * n + 872$ and 1952

3. Solve the following system of equations:

$$5x \equiv 8y \pmod{13}$$

$$x \equiv 9y - 11 \pmod{13}$$

4. Prove that $\text{GCD}(7n + 4, 5n + 3) = 1$ for all $n \in \mathbb{N}$.

5. What is $(42^{42^{42}})! \bmod 300$?