

1. Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.
2. Which of the following statements are true? Let $Q(n)$ be the statement “ n is divisible by 2.” \mathbb{N} denotes the set of natural numbers.
 - (a) $\exists k \in \mathbb{N}, Q(k) \wedge Q(k+1)$.
 - (b) $\forall k \in \mathbb{N}, Q(k) \implies Q(k^2)$.
 - (c) $\exists x \in \mathbb{N}, \neg(\exists y \in \mathbb{N}, y < x)$.
3. Write the following statements using the notation covered in class. Use \mathbb{N} to denote the set of natural numbers and \mathbb{Z} to denote the set of integers. Also write $P(n)$ for the statement “ n is odd”.
 - (a) For all natural numbers n , $2n$ is even.
 - (b) For all natural numbers n , n is odd if n^2 is odd.
 - (c) There are no integer solutions to the equation $x^2 - y^2 = 10$.
4. You are on an island inhabited by two types of people: the Liars and the Truthtellers. Liars always lie, and Truthtellers always tell the truth. In all other respects, the two types are indistinguishable. You meet a very attractive local and ask him/her on a date. The local responds, “I will go on a date with you if and only if I am a Truthteller.” Is this good news?
5. Prove that if you put $n+1$ apples into n boxes, any way you like, then at least one box must contain 2 apples. This is known as the *pigeon hole principle*.
6. Prove that the length of hypotenuse of a right triangle is strictly greater than the sum of the two remaining sides.
 - (a) Write down the definition of a right triangle and the claim to be proven in mathematical notation.
 - (b) Prove the statement by contradiction.
 - (c) Prove the statement directly.