EECS 70 Discrete Mathematics and Probability Theory
Spring 2014 Anant Sahai

Discussion 2A

1. Write the following statements using the notation covered in class. Use $\mathbb{N}$ to denote the set of natural numbers and $\mathbb{Z}$ to denote the set of integers. Also write $P(n)$ for the statement "n is odd".

   (a) For all natural numbers n, $2n$ is even.
   $\forall\, n \in \mathbb{N}, \neg P(2n)$

   (b) For all natural numbers n, n is odd if n^2 is odd.
   $\forall\, n \in \mathbb{N}, P(n^2) \implies P(n)$

   (c) There are no integer solutions to the equation $x^2 - y^2 = 10$.
   $\neg(\exists\, x, y \in \mathbb{Z}, x^2 - y^2 = 10)$

2. Which of the following statements are true? Let $Q(n)$ be the statement "n is divisible by 2." $\mathbb{N}$ denotes the set of natural numbers.

   (a) $\exists\, k \in \mathbb{N},\ Q(k) \wedge Q(k+1)$.

   (b) $\forall\, k \in \mathbb{N},\ Q(k) \implies Q(k^2)$.

   (c) $\exists\, x \in \mathbb{N}, \neg(\exists\, y \in \mathbb{N}, y < x)$.

   (a) false, (b) true, (c) true

3. Use truth tables to show that $\neg(A \vee B) \equiv \neg A \wedge \neg B$ and $\neg(A \wedge B) \equiv \neg A \vee \neg B$. These two equivalences are known as DeMorgan's Law.

A	B	$\neg(A \vee B)$	$\neg A \wedge \neg B$	$\neg(A \wedge B)$	$\neg A \vee \neg B$
0	0	1	1	1	1
0	1	0	0	1	1
1	0	0	0	1	1
1	1	0	0	0	0

4. You are on an island inhabited by two types of people: the Liars and the Truthtellers. Liars always make false statements, and Truthtellers always make true statements. In all other respects, the two types are indistinguishable. You meet an attractive local and ask him/her on a date. The local responds, "I will go on a date with you if and only if I am a Truthteller." Is this good news?

   Yes. Let T be the proposition that the local is a Truthteller, and D be the proposition that he/she will go on a date with you. If T is true, then the statement $D \iff T$ is true, so we can find the value of D in the last line of the truth table below. It is a 1, so D is true. If T is false, then the statement $T \iff D$ is false, so we can find the value of D in the third line of the truth table below. It is a 1 as well.

T	D	$D \iff T$
0	0	1
0	1	0
1	0	0
1	1	1

5. Prove that if you put $n+1$ apples into n boxes, any way you like, then at least one box must contain 2 apples. This is known as the *pigeon hole principle*.

   Suppose this is not the case. Then all the boxes would contain at most 1 apple. Then the maximum number of apples we could have would be n, but this is a contradiction since we have $n+1$ apples.

6. Prove that the length of the hypotenuse of a non-degenerate right triangle is strictly greater than the sum of the two remaining sides.

   (a) Write down the definition of a right triangle and the claim to be proven in mathematical notation.

   (b) Prove the statement by contradiction.

   (c) Prove the statement directly.

   Definition of a right triangle: $a^2+b^2=c^2$. Claim to be proven: $a+b>c$. We can prove this directly by adding $2ab$ (a positive number) to the LHS of the definition of a right triangle.

$$a^2+b^2=c^2 \implies a^2+2ab+b^2>c^2 \implies (a+b)^2>c^2 \implies a+b>c$$

   We can prove the claim with contradiction by assuming it is not true. This is basically the reverse of the previous proof:

$$a+b\leq c \implies (a+b)^2\leq c^2 \implies a^2+2ab+b^2\leq c^2 \implies a^2+b^2<c^2$$

$$\Longrightarrow\Longleftarrow$$