Basic Statistics Formulas

Population Measures

$$\text{Mean } \mu = \frac{1}{n}\sum x_i \tag{1}$$

$$\text{Variance } \sigma^2 = \frac{1}{n}\sum (x_i - \overline{x})^2 \tag{2}$$

$$\text{Standard Deviation } \sigma = \sqrt{\frac{1}{n}\sum (x_i - \overline{x})^2} \tag{3}$$

Sampling

$$\text{Sample mean } \overline{x} = \frac{1}{n}\sum x_i \tag{4}$$

$$\text{Sample variance } s_x^2 = \frac{1}{n-1}\sum (x_i - \overline{x})^2 \tag{5}$$

$$\text{Std. Deviation } s_x = \sqrt{\frac{1}{n-1}\sum (x_i - \overline{x})^2} \tag{6}$$

$$\text{z-score } z = \frac{x - \mu}{\sigma} \tag{7}$$

$$\text{Correlation } r =$$

$$\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{(x_i - \overline{x})}{s_x}\right)\left(\frac{(y_i - \overline{y})}{s_y}\right) \tag{8}$$

Linear Regression

$$\text{Line } \hat{y} = a + bx \tag{9}$$

$$b = r\frac{s_y}{s_x}, a = \overline{y} - b\overline{x} \tag{10}$$

$$s = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y})^2} \tag{11}$$

$$SE_b = \frac{s}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2}} \tag{12}$$

$$\text{To test } H_0 : b = 0, \text{ use } t = \frac{b}{SE_b} \tag{13}$$

$$CI = b \pm t^* SE_b \tag{14}$$

Probability

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \tag{15}$$

$$P(\text{not } A) = 1 - P(A) \tag{16}$$

$$P(A \text{ and } B) = P(A)P(B) \text{ (independent)} \tag{17}$$

$$P(B|A) = P(A \text{ and } B)/P(A) \tag{18}$$

$$0! = 1; n! = 1 \times 2 \times 3 \cdots \times (n-1) \times n \tag{19}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{20}$$

Binomial Distribution :

$$P(\mathcal{X} = k) = \binom{n}{k}p^k(1-p)^{n-k} \tag{21}$$

$$\mu = np, \ \sigma = \sqrt{np(1-p)} \tag{22}$$

One-Sample z-statistic

$$\text{To test } H_0 : \mu = \mu_0 \text{ use } z = \frac{\overline{z} - \mu_0}{\sigma/\sqrt{n}} \tag{23}$$

$$\text{Confidence Interval for } \mu = \overline{x} \pm z^*\frac{\sigma}{\sqrt{n}} \tag{24}$$

$$\text{Margin of Error } ME = z^*\frac{\sigma}{\sqrt{n}} \tag{25}$$

$$\text{Minimum sample size } n \geq \left[\frac{z^*\sigma}{ME}\right]^2 \tag{26}$$

One-Sample t-statistic

$$SEM = \frac{s_x}{\sqrt{n}}, \ t = \frac{\overline{x} - \mu}{s_x/\sqrt{n}} \tag{27}$$

$$\text{Confidence Interval} = \overline{x} \pm t^*\frac{s_x}{\sqrt{n}} \tag{28}$$

Two-Sample t-statistic

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \tag{29}$$

$$\text{Conf. Interval} = (\overline{x}_1 - \overline{x}_2) \pm t^*\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{30}$$

Sample Proportions

$$\mu_{\hat{p}} = p, \ \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{31}$$

$$\text{Conf. Int.} = \hat{p} \pm z^*(SE) \tag{32}$$

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \tag{33}$$

$$\text{sample size } n > \left[\frac{z^*}{ME}\right]^2 p^*(1-p^*) \tag{34}$$

$$\text{To test } H_0 : p = p_0, \text{ use } z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} \tag{35}$$

Two-Sample Proportions

$$SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \tag{36}$$

$$CI = (\hat{p}_1 - \hat{p}_2) \pm z^*(SE) \tag{37}$$

$$\text{To test } H_0 : p_1 = p_2, \text{ use} \tag{38}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}} \tag{39}$$

$$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}, \ X_i = \text{ successes} \tag{40}$$
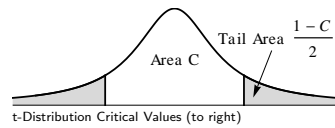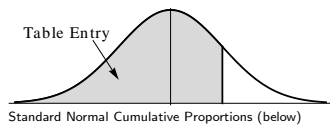
Chi-Square Statistic

$$\chi^2 = \sum_{i=1}^{n}\frac{(o_i - e_i)^2}{e_i} \tag{41}$$

$$o_i = \text{observed}, e_i = \text{expected}$$

Central Limit Theorem

$$s_{\overline{x}} \to \frac{\sigma}{\sqrt{n}} \text{ as } n \to \infty \tag{42}$$

Standard Normal Cumulative Proportions

Table Entry

Standard Normal Cumulative Proportions (below)

Tail Area $\frac{1-C}{2}$

Area C

t-Distribution Critical Values (to right)

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

t-Distribution Cumulative Proportions

					Confidence Level C					
df	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.8%
1	1	1.376	1.963	3.078	6.314	12.706	15.895	31.821	63.657	318.309
2	0.816	1.061	1.386	1.886	2.92	4.303	4.849	6.965	9.925	22.327
3	0.765	0.978	1.25	1.638	2.353	3.182	3.482	4.541	5.841	10.215
4	0.741	0.941	1.19	1.533	2.132	2.776	2.999	3.747	4.604	7.173
5	0.727	0.92	1.156	1.476	2.015	2.571	2.757	3.365	4.032	5.893
6	0.718	0.906	1.134	1.44	1.943	2.447	2.612	3.143	3.707	5.208
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.785
8	0.706	0.889	1.108	1.397	1.86	2.306	2.449	2.896	3.355	4.501
9	0.703	0.883	1.1	1.383	1.833	2.262	2.398	2.821	3.25	4.297
10	0.7	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	4.144
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	4.025
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.93
13	0.694	0.87	1.079	1.35	1.771	2.16	2.282	2.65	3.012	3.852
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.787
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.733
16	0.69	0.865	1.071	1.337	1.746	2.12	2.235	2.583	2.921	3.686
17	0.689	0.863	1.069	1.333	1.74	2.11	2.224	2.567	2.898	3.646
18	0.688	0.862	1.067	1.33	1.734	2.101	2.214	2.552	2.878	3.61
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.579
20	0.687	0.86	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.552
21	0.686	0.859	1.063	1.323	1.721	2.08	2.189	2.518	2.831	3.527
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.505
23	0.685	0.858	1.06	1.319	1.714	2.069	2.177	2.5	2.807	3.485
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.467
25	0.684	0.856	1.058	1.316	1.708	2.06	2.167	2.485	2.787	3.45
30	0.683	0.854	1.055	1.31	1.697	2.042	2.147	2.457	2.75	3.385
40	0.681	0.851	1.05	1.303	1.684	2.021	2.123	2.423	2.704	3.307
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	3.261
60	0.679	0.848	1.045	1.296	1.671	2	2.099	2.39	2.66	3.232
80	0.678	0.846	1.043	1.292	1.664	1.99	2.088	2.374	2.639	3.195
100	0.677	0.845	1.042	1.29	1.66	1.984	2.081	2.364	2.626	3.174
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.33	2.581	3.098
$z*$	0.674	0.842	1.036	1.282	1.645	1.960	2.054	2.326	2.576	3.090
1-Sided P	0.25	0.2	0.15	0.1	0.05	0.025	0.02	0.01	0.005	0.001
2-Sided P	0.5	0.4	0.3	0.2	0.1	0.05	0.04	0.02	0.01	0.002

Chi-Square Distribution Critical Values

Probability p

χ

					p					
df	0.25	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.0025	0.001
1	1.32	1.64	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83
2	2.77	3.22	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82
3	4.11	4.64	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27
4	5.39	5.99	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47
5	6.63	7.29	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.52
6	7.84	8.56	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46
7	9.04	9.80	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32
8	10.22	11.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12
9	11.39	12.24	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88
10	12.55	13.44	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59
11	13.70	14.63	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26
12	14.85	15.81	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91
13	15.98	16.98	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53
14	17.12	18.15	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12
15	18.25	19.31	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70
16	19.37	20.47	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25
17	20.49	21.61	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79
18	21.60	22.76	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31
19	22.72	23.90	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82
20	23.83	25.04	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31
21	24.93	26.17	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80
22	26.04	27.30	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27
23	27.14	28.43	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73
24	28.24	29.55	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18
25	29.34	30.68	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62
30	34.80	36.25	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70
40	45.62	47.27	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40
50	56.33	58.16	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66
60	66.98	68.97	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61
80	88.13	90.41	96.58	101.88	106.63	108.07	112.33	116.32	120.10	124.84
100	109.14	111.67	118.50	124.34	129.56	131.14	135.81	140.17	144.29	149.45