

# Economic growth Notebook

We will use a dataset that was used in the paper: I just ran two million regressions by Sala-I-Martin and Model uncertainty in cross country growth regression by Fernandez et. al. The dataset has 41 possible explanatory variables with 72 countries. The data consists of 43 columns with the Country, y (economic growth in per capita GDP) and 41 possible predictor variables.

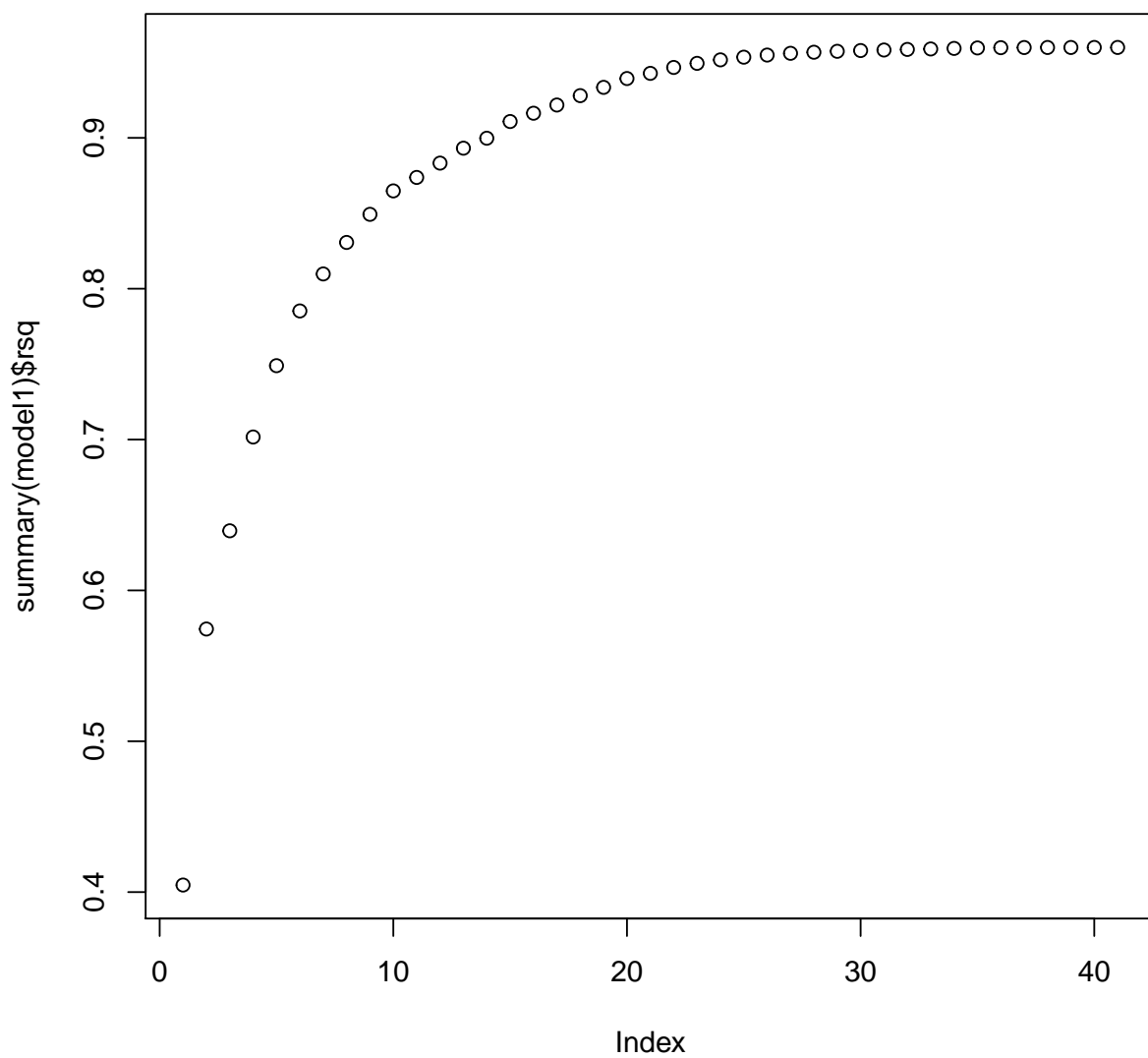
```
rm(list=ls()) # remove all objects

eg <- read.csv("economicgrowth.csv")
# str(eg)
eg1 <- subset(eg, select = -c(Country))
str(eg1)
```

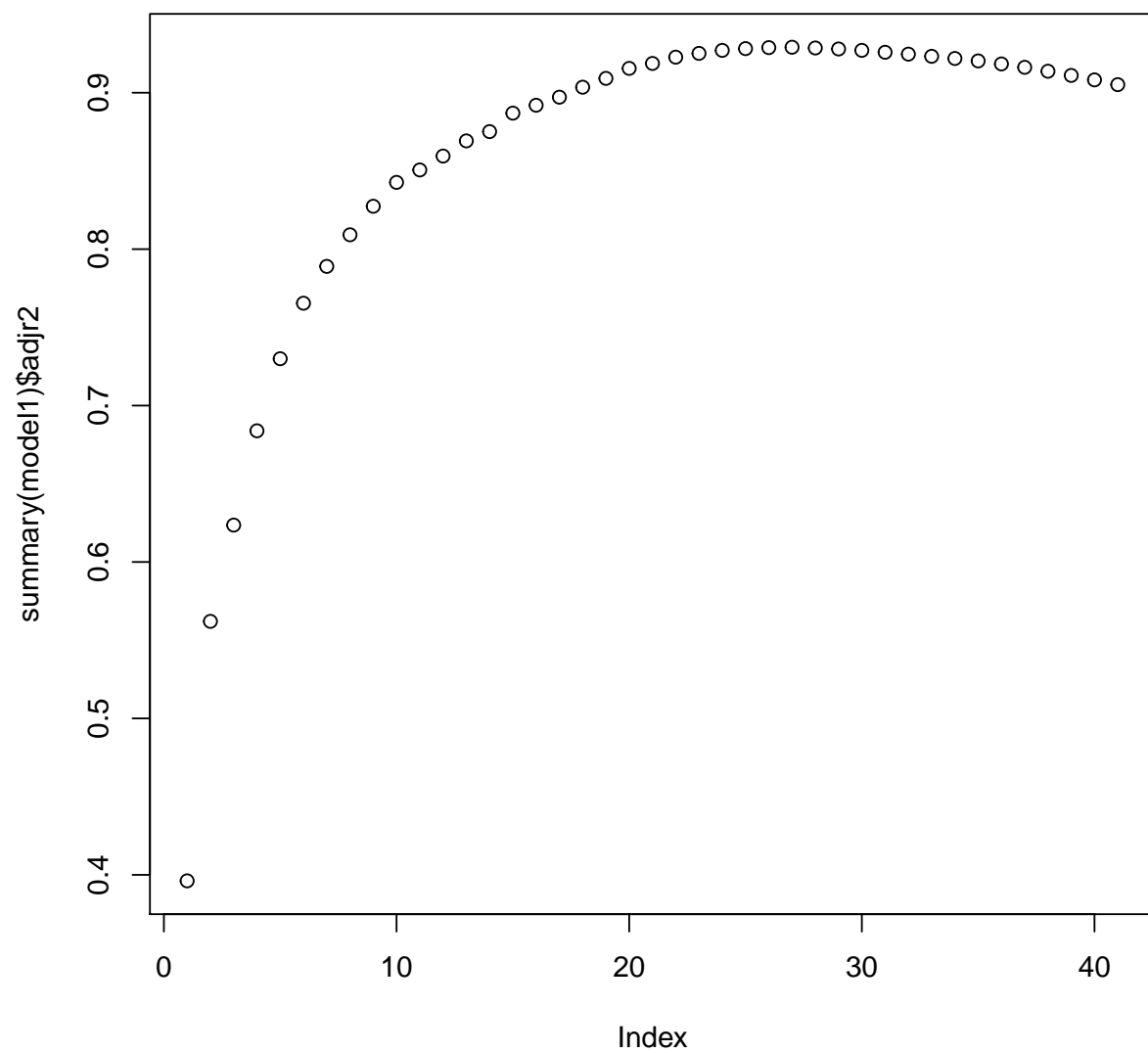
We now conduct model selection by exhaustive search. Note that we have  $2^{41}$  which is approximately 2 trillion possible regressions to run. The leaps package has some smart ways to search over this space by avoiding visiting parts of the space where the optimum cannot exist. It employs a branch and bound algorithm to search more efficiently. This would take a few minutes to run on a laptop. The model shows the bias-variance trade-off. We can also plot the variables identified using the plot command.

```
library(leaps)
model1 <- regsubsets(y ~ ., data = eg1, nvmax = 41)

plot(summary(model1)$rsq)
```



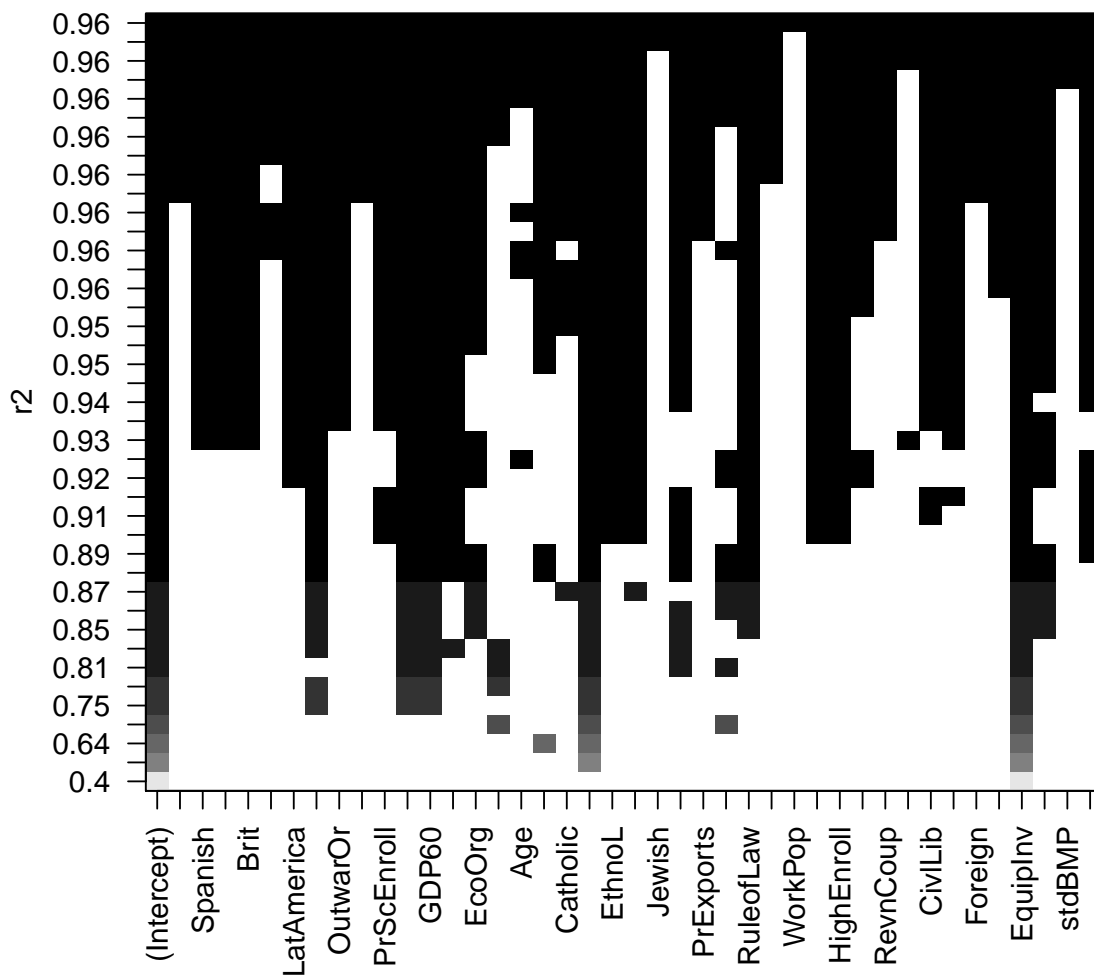
```
plot(summary(model1)$adjr2)
```



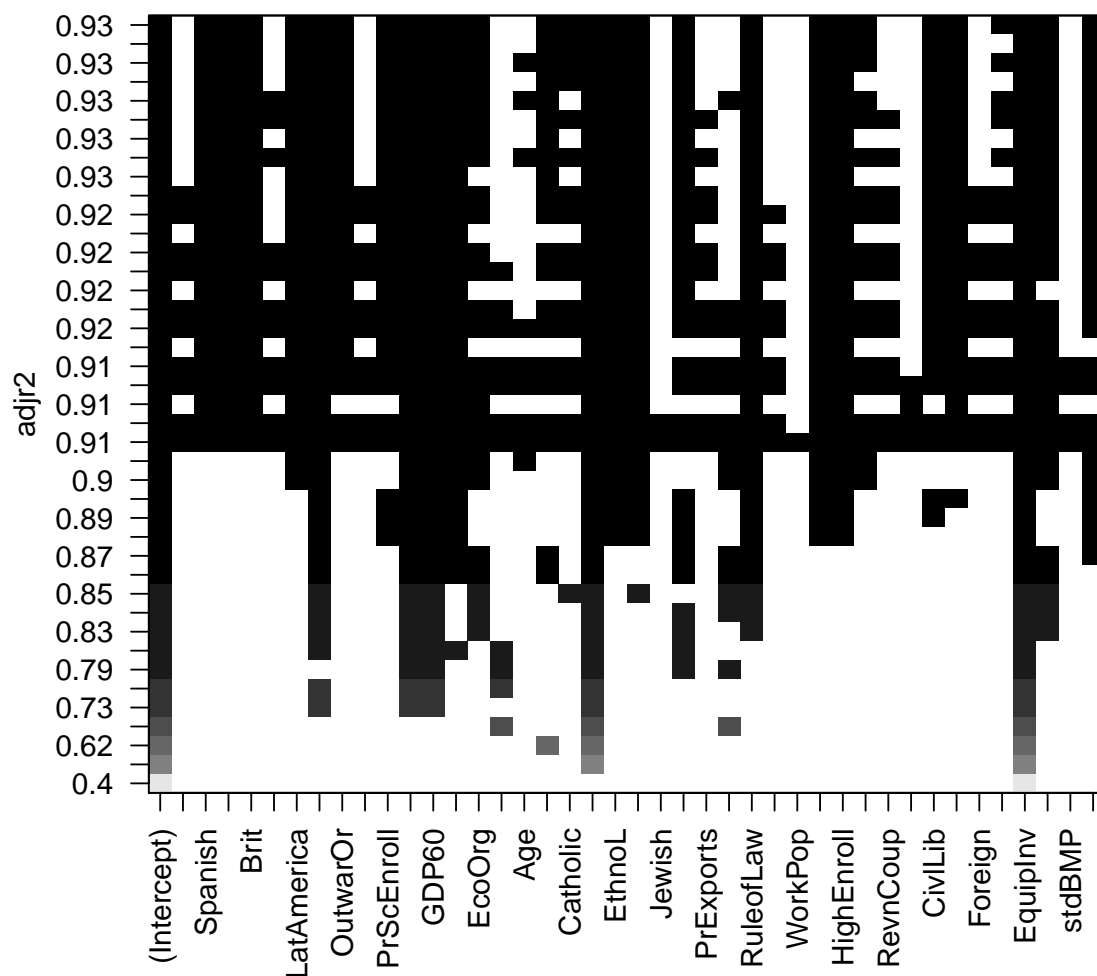
```
which.max(summary(model1)$adjr2)
```

```
## [1] 27
```

```
plot(model1, scale = c("r2"))
```

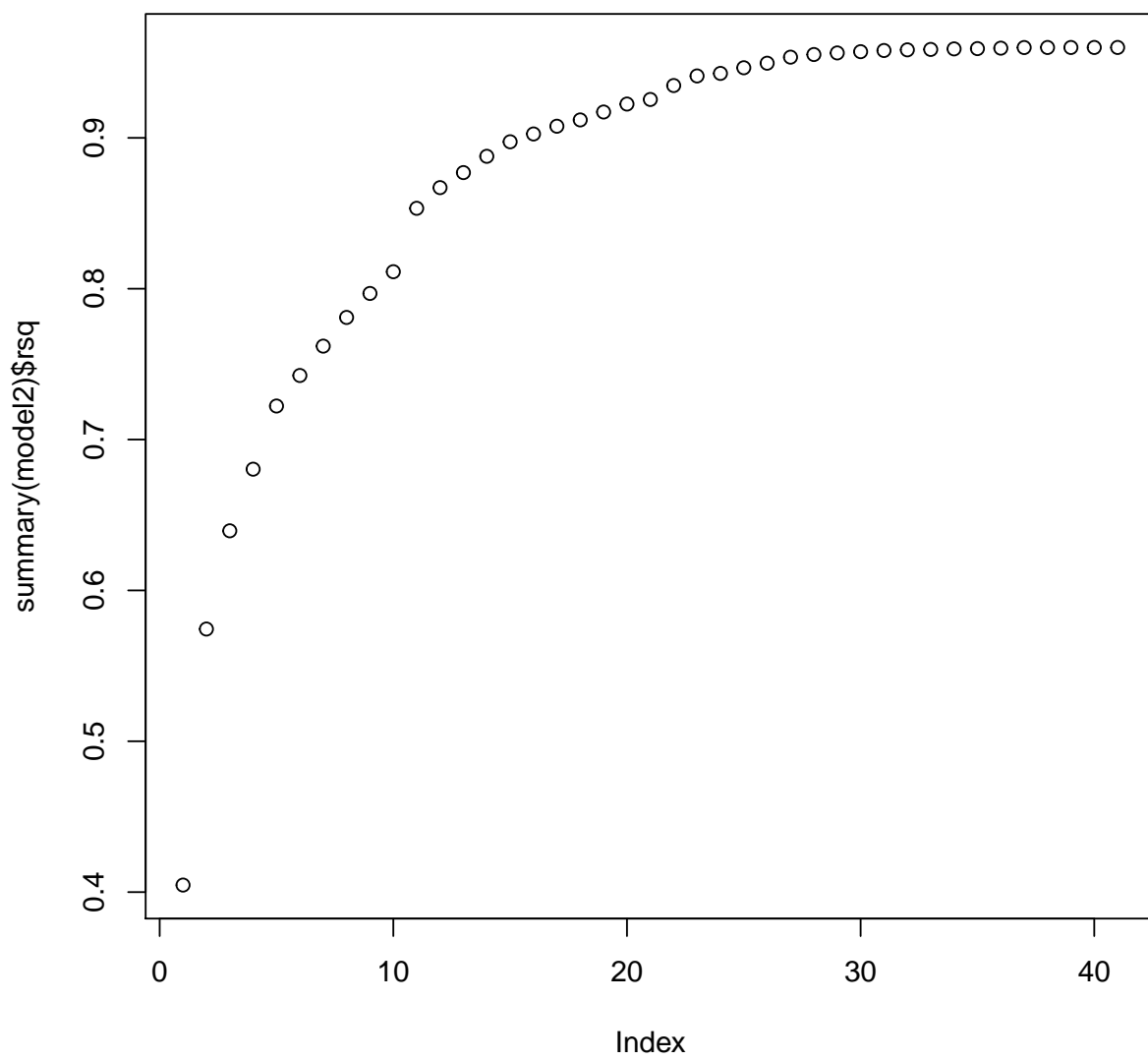


```
plot(model1, scale = c("adjr2"))
```

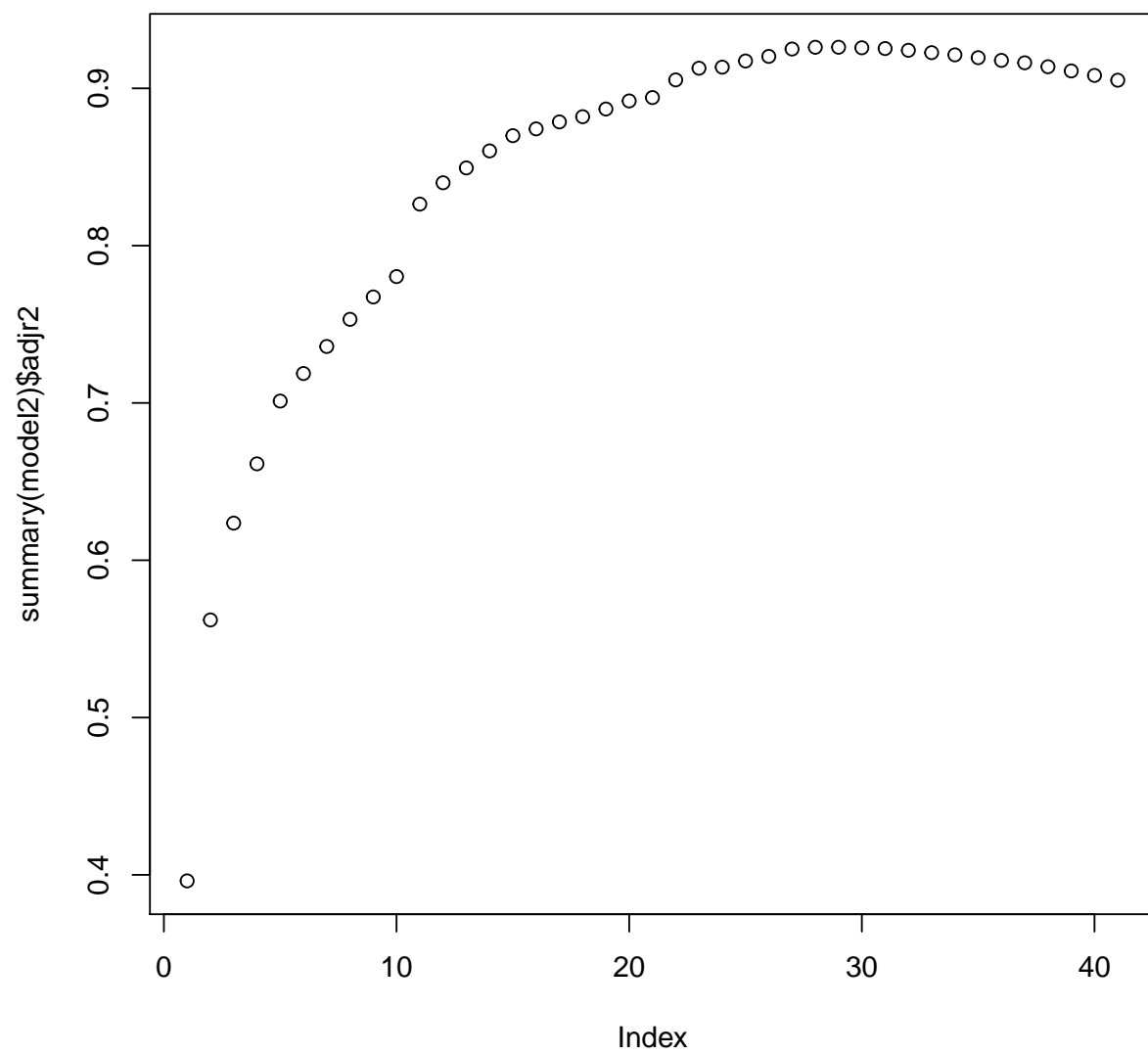


We next use the forward stepwise selection method which runs much faster as should be expected. Note that the results are not identical to what we obtained with the exhaustive selection approach.

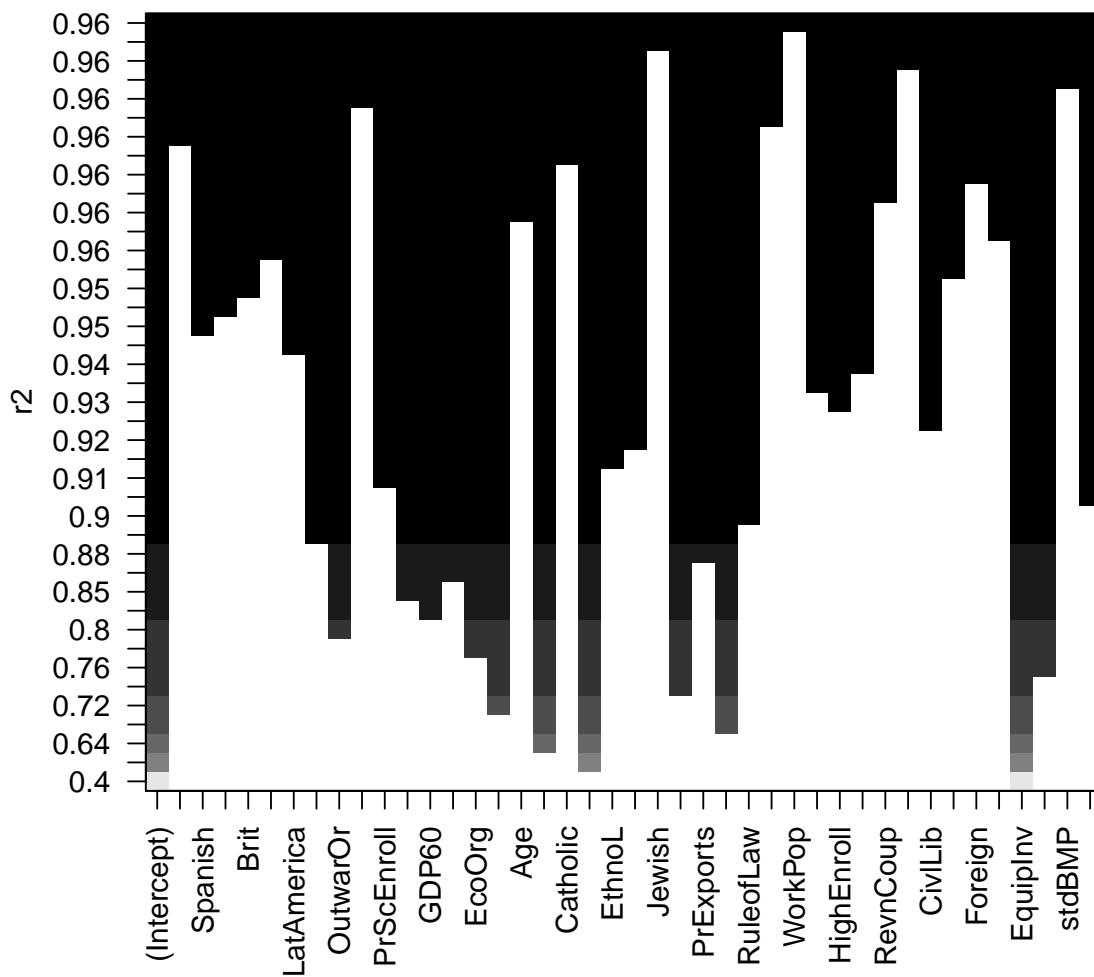
```
model2 <- regsubsets(y ~ ., data = eg1, nvmax = 41, method = "forward")
plot(summary(model2)$rsq)
```



```
plot(summary(model2)$adjr2)
```

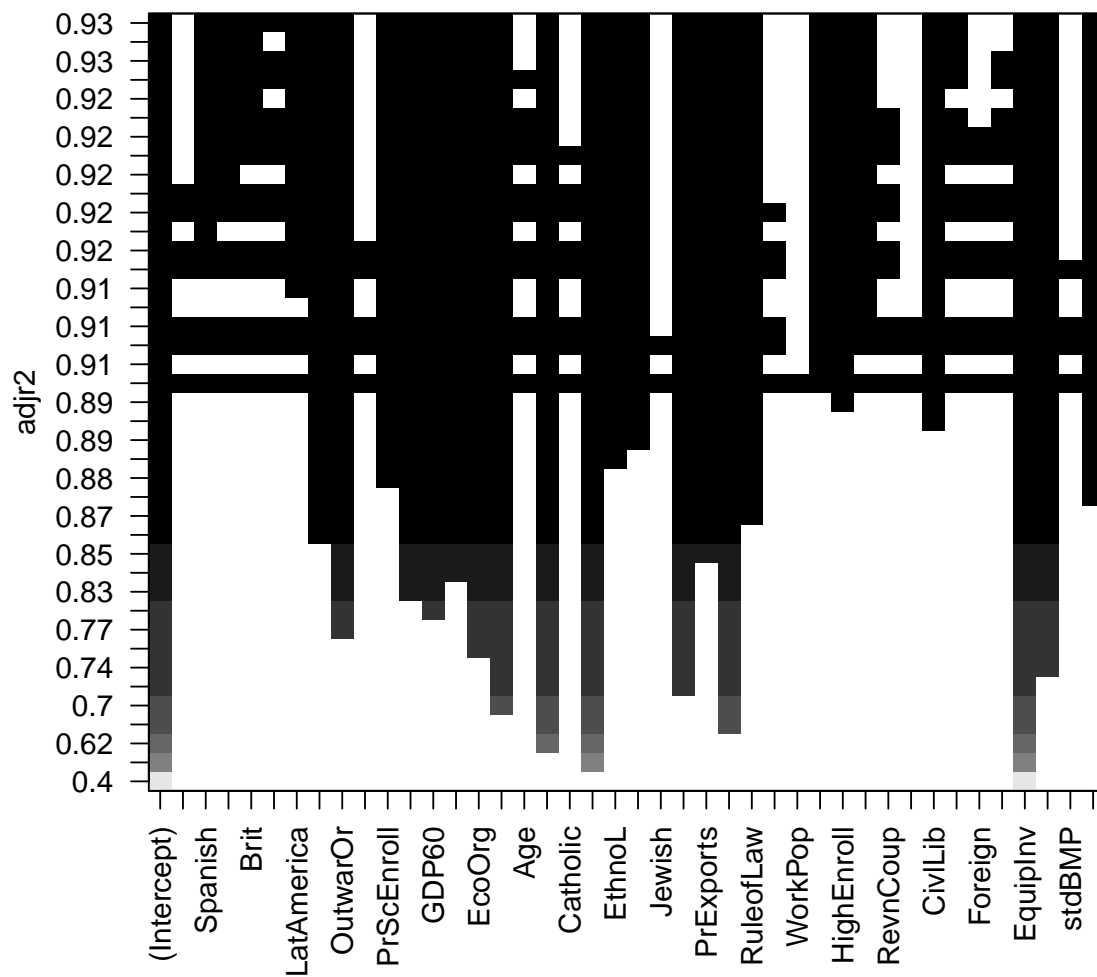


```
plot(model2, scale = c("r2"))
```



```
plot(model2, scale = c("adjr2"))
```





Which variables are selected:

```
summary(model1)$which[27,]
summary(model2)$which[27,]
```

The results indicate that with model 1, we have

- 1) EquipInv,
- 2) Confucian, EquipInv,
- 3) Buddha, Confucian, EquipInv,
- 4) YrsOpen, Confucian, Protestants, EquipInv

while for model 2, we have

- 1) EquipInv,
- 2) Confucian, EquipInv,

3) Buddha, Confucian, EquipInv,  
4) Buddha, Protestants, EquipInv, Confucian  
and so on. The results are different from the two models.

---

LASSO model: The results indicate that for variables such as EquipInv, YrsOpen and Confucian for many values of lambda, these occur while some other variables such as Abslat do not show up as often. Such results help illustrate the reliability of possible predictors for economic growth and can also cast doubts on the robustness of the results for certain variables which might be proposed as being correlated with growth.

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.0-2
```

```
x <- as.matrix(eg1[,c(2:42)])  
grid <- 10^seq(10, -2, length=100)  
model3 <- glmnet(x, eg1$y, lambda = grid)  
model3
```

```
##  
## Call:  glmnet(x = x, y = eg1$y, lambda = grid)  
##  
##      Df %Dev      Lambda  
## 1    0  0.0 1.000e+10  
## 2    0  0.0 7.565e+09  
## 3    0  0.0 5.722e+09  
## 4    0  0.0 4.329e+09  
## 5    0  0.0 3.275e+09  
## 6    0  0.0 2.477e+09  
## 7    0  0.0 1.874e+09  
## 8    0  0.0 1.417e+09  
## 9    0  0.0 1.072e+09  
## 10   0  0.0 8.111e+08  
## 11   0  0.0 6.136e+08  
## 12   0  0.0 4.642e+08  
## 13   0  0.0 3.511e+08  
## 14   0  0.0 2.656e+08  
## 15   0  0.0 2.009e+08  
## 16   0  0.0 1.520e+08  
## 17   0  0.0 1.150e+08  
## 18   0  0.0 8.697e+07  
## 19   0  0.0 6.579e+07  
## 20   0  0.0 4.977e+07  
## 21   0  0.0 3.765e+07  
## 22   0  0.0 2.848e+07  
## 23   0  0.0 2.154e+07  
## 24   0  0.0 1.630e+07  
## 25   0  0.0 1.233e+07  
## 26   0  0.0 9.326e+06
```

```
## 27 0 0.0 7.055e+06
## 28 0 0.0 5.337e+06
## 29 0 0.0 4.037e+06
## 30 0 0.0 3.054e+06
## 31 0 0.0 2.310e+06
## 32 0 0.0 1.748e+06
## 33 0 0.0 1.322e+06
## 34 0 0.0 1.000e+06
## 35 0 0.0 7.565e+05
## 36 0 0.0 5.722e+05
## 37 0 0.0 4.329e+05
## 38 0 0.0 3.275e+05
## 39 0 0.0 2.477e+05
## 40 0 0.0 1.874e+05
## 41 0 0.0 1.417e+05
## 42 0 0.0 1.072e+05
## 43 0 0.0 8.111e+04
## 44 0 0.0 6.136e+04
## 45 0 0.0 4.642e+04
## 46 0 0.0 3.511e+04
## 47 0 0.0 2.656e+04
## 48 0 0.0 2.009e+04
## 49 0 0.0 1.520e+04
## 50 0 0.0 1.150e+04
## 51 0 0.0 8.697e+03
## 52 0 0.0 6.579e+03
## 53 0 0.0 4.977e+03
## 54 0 0.0 3.765e+03
## 55 0 0.0 2.848e+03
## 56 0 0.0 2.154e+03
## 57 0 0.0 1.630e+03
## 58 0 0.0 1.233e+03
## 59 0 0.0 9.330e+02
## 60 0 0.0 7.060e+02
## 61 0 0.0 5.340e+02
## 62 0 0.0 4.040e+02
## 63 0 0.0 3.050e+02
## 64 0 0.0 2.310e+02
## 65 0 0.0 1.750e+02
## 66 0 0.0 1.320e+02
## 67 0 0.0 1.000e+02
## 68 0 0.0 7.600e+01
## 69 0 0.0 5.700e+01
## 70 0 0.0 4.300e+01
## 71 0 0.0 3.300e+01
## 72 0 0.0 2.500e+01
## 73 0 0.0 1.900e+01
## 74 0 0.0 1.400e+01
## 75 0 0.0 1.100e+01
## 76 0 0.0 8.000e+00
## 77 0 0.0 6.000e+00
## 78 0 0.0 5.000e+00
## 79 0 0.0 4.000e+00
## 80 0 0.0 3.000e+00
```

```
## 81  0  0.0 2.000e+00
## 82  0  0.0 2.000e+00
## 83  0  0.0 1.000e+00
## 84  0  0.0 1.000e+00
## 85  0  0.0 1.000e+00
## 86  0  0.0 0.000e+00
## 87  0  0.0 0.000e+00
## 88  0  0.0 0.000e+00
## 89  0  0.0 0.000e+00
## 90  0  0.0 0.000e+00
## 91  0  0.0 0.000e+00
## 92  0  0.0 0.000e+00
## 93  0  0.0 0.000e+00
## 94  0  0.0 0.000e+00
## 95  0  0.0 0.000e+00
## 96  0  0.0 0.000e+00
## 97  0  0.0 0.000e+00
## 98  0  0.0 0.000e+00
## 99  0  0.0 0.000e+00
## 100 2 10.1 0.000e+00
```

*# for a large portion, we are not explaining any part for the data for diff lambda values  
 # at the end, only explained 10% of the null deviance with only 2 variables at the end  
 # hence this does not seem to be a good grid to look for*

*# let the R system choose the grid here instead of specifying it*

*# model3 issues may be perhaps actual lambda values were too small and we were looking at much larger lambda values*

```
model14 <- glmnet(x, eg1$y)
model14
```

```
##
## Call:  glmnet(x = x, y = eg1$y)
##
##      Df  %Dev   Lambda
## 1    0  0.00 0.0115300
## 2    1  6.87 0.0105100
## 3    2 13.30 0.0095730
## 4    3 19.31 0.0087230
## 5    3 26.69 0.0079480
## 6    3 32.82 0.0072420
## 7    3 37.90 0.0065990
## 8    4 42.67 0.0060120
## 9    4 46.81 0.0054780
## 10   4 50.25 0.0049920
## 11   4 53.10 0.0045480
## 12   5 55.67 0.0041440
## 13   5 57.92 0.0037760
## 14   7 60.01 0.0034400
## 15   7 62.46 0.0031350
## 16   9 64.55 0.0028560
## 17  10 66.56 0.0026030
## 18  10 68.33 0.0023710
## 19  11 69.86 0.0021610
```

```
## 20 12 71.49 0.0019690
## 21 13 73.00 0.0017940
## 22 13 74.28 0.0016350
## 23 14 75.46 0.0014890
## 24 15 76.58 0.0013570
## 25 17 77.91 0.0012360
## 26 19 79.34 0.0011270
## 27 20 81.10 0.0010270
## 28 20 82.63 0.0009353
## 29 21 83.90 0.0008522
## 30 23 85.01 0.0007765
## 31 24 85.99 0.0007075
## 32 24 86.86 0.0006447
## 33 26 87.64 0.0005874
## 34 26 88.39 0.0005352
## 35 26 89.00 0.0004877
## 36 26 89.50 0.0004444
## 37 27 89.94 0.0004049
## 38 28 90.34 0.0003689
## 39 29 90.80 0.0003361
## 40 30 91.22 0.0003063
## 41 30 91.79 0.0002791
## 42 30 92.28 0.0002543
## 43 30 92.68 0.0002317
## 44 32 93.02 0.0002111
## 45 32 93.36 0.0001924
## 46 34 93.64 0.0001753
## 47 35 93.91 0.0001597
## 48 35 94.22 0.0001455
## 49 35 94.48 0.0001326
## 50 34 94.71 0.0001208
## 51 35 94.89 0.0001101
## 52 35 95.05 0.0001003
## 53 35 95.19 0.0000914
## 54 37 95.30 0.0000833
## 55 37 95.40 0.0000759
## 56 36 95.48 0.0000691
## 57 36 95.54 0.0000630
## 58 36 95.60 0.0000574
## 59 36 95.64 0.0000523
## 60 36 95.68 0.0000476
## 61 36 95.71 0.0000434
## 62 36 95.74 0.0000396
## 63 36 95.76 0.0000360
## 64 38 95.78 0.0000328
## 65 40 95.81 0.0000299
## 66 41 95.84 0.0000273
## 67 41 95.86 0.0000248
## 68 41 95.88 0.0000226
## 69 41 95.90 0.0000206
## 70 41 95.92 0.0000188
## 71 41 95.93 0.0000171
## 72 41 95.94 0.0000156
## 73 41 95.95 0.0000142
```

```
## 74 41 95.95 0.0000130
## 75 41 95.96 0.0000118
## 76 41 95.96 0.0000108
## 77 41 95.97 0.0000098
## 78 40 95.97 0.0000089
## 79 40 95.97 0.0000081
## 80 40 95.98 0.0000074
## 81 40 95.98 0.0000068
## 82 40 95.98 0.0000062
## 83 40 95.98 0.0000056
## 84 40 95.98 0.0000051
## 85 40 95.98 0.0000047
## 86 40 95.99 0.0000042
```

```
model4$df
```

```
## [1] 0 1 2 3 3 3 3 4 4 4 4 5 5 7 7 9 10 10 11 12 13 13 14 15 17
## [26] 19 20 20 21 23 24 24 26 26 26 26 27 28 29 30 30 30 30 32 32 34 35 35 35 34
## [51] 35 35 35 37 37 36 36 36 36 36 36 36 36 38 40 41 41 41 41 41 41 41 41 41 41
## [76] 41 41 40 40 40 40 40 40 40 40 40
```

```
model4$beta["EquipInv",]
```

```
##          s0          s1          s2          s3          s4          s5          s6
## 0.00000000 0.02989896 0.05202483 0.06760673 0.08042867 0.09212352 0.10277946
##          s7          s8          s9          s10          s11          s12          s13
## 0.11258867 0.12154191 0.12971174 0.13715581 0.14406270 0.15040049 0.15383321
##          s14          s15          s16          s17          s18          s19          s20
## 0.15867593 0.16258346 0.16444915 0.16555791 0.16661977 0.16734255 0.16832148
##          s21          s22          s23          s24          s25          s26          s27
## 0.16948025 0.17126770 0.17293008 0.17351524 0.17220683 0.16702669 0.16130824
##          s28          s29          s30          s31          s32          s33          s34
## 0.15651196 0.15187566 0.14705527 0.14234527 0.13738774 0.13193100 0.12706968
##          s35          s36          s37          s38          s39          s40          s41
## 0.12274067 0.11929709 0.11510558 0.11298039 0.11149575 0.11069796 0.10979416
##          s42          s43          s44          s45          s46          s47          s48
## 0.10902316 0.10822285 0.10826583 0.10805193 0.10978025 0.11122019 0.11265440
##          s49          s50          s51          s52          s53          s54          s55
## 0.11364832 0.11454541 0.11517577 0.11571790 0.11637044 0.11769020 0.11837771
##          s56          s57          s58          s59          s60          s61          s62
## 0.11911269 0.11981383 0.12046601 0.12107996 0.12163600 0.12214121 0.12259898
##          s63          s64          s65          s66          s67          s68          s69
## 0.12310262 0.12363149 0.12418720 0.12446285 0.12470910 0.12491215 0.12509626
##          s70          s71          s72          s73          s74          s75          s76
## 0.12525557 0.12540204 0.12553559 0.12566579 0.12577917 0.12588798 0.12597718
##          s77          s78          s79          s80          s81          s82          s83
## 0.12607745 0.12615294 0.12619169 0.12622857 0.12626553 0.12629465 0.12632607
##          s84          s85
## 0.12635177 0.12637656
```

```
model4$beta["YrsOpen",]
```

```
##          s0          s1          s2          s3          s4
## 0.000000e+00 0.000000e+00 8.411755e-04 2.348983e-03 3.459478e-03
##          s5          s6          s7          s8          s9
## 4.469387e-03 5.389575e-03 6.018196e-03 6.519700e-03 6.974666e-03
##          s10         s11         s12         s13         s14
## 7.389210e-03 7.501796e-03 7.459716e-03 7.359030e-03 7.580682e-03
##          s15         s16         s17         s18         s19
## 7.776759e-03 7.780396e-03 7.624393e-03 7.531362e-03 7.530964e-03
##          s20         s21         s22         s23         s24
## 7.433922e-03 7.333904e-03 7.238579e-03 7.105824e-03 7.186242e-03
##          s25         s26         s27         s28         s29
## 6.986698e-03 6.950097e-03 7.060531e-03 7.172980e-03 7.283439e-03
##          s30         s31         s32         s33         s34
## 7.355836e-03 7.181306e-03 7.000231e-03 6.575529e-03 6.222930e-03
##          s35         s36         s37         s38         s39
## 5.895569e-03 5.633759e-03 5.344985e-03 4.899859e-03 4.378597e-03
##          s40         s41         s42         s43         s44
## 3.952915e-03 3.557960e-03 3.207090e-03 2.878671e-03 2.436668e-03
##          s45         s46         s47         s48         s49
## 1.985109e-03 1.678964e-03 1.473317e-03 1.286642e-03 1.069770e-03
##          s50         s51         s52         s53         s54
## 8.549662e-04 6.260712e-04 4.303385e-04 2.377157e-04 3.966491e-05
##          s55         s56         s57         s58         s59
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s60         s61         s62         s63         s64
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 -3.471530e-05
##          s65         s66         s67         s68         s69
## -2.031494e-04 -4.074152e-04 -5.981505e-04 -7.729290e-04 -9.349521e-04
##          s70         s71         s72         s73         s74
## -1.081157e-03 -1.216336e-03 -1.332769e-03 -1.445648e-03 -1.548900e-03
##          s75         s76         s77         s78         s79
## -1.638620e-03 -1.721495e-03 -1.802933e-03 -1.871868e-03 -1.934607e-03
##          s80         s81         s82         s83         s84
## -1.986890e-03 -2.049210e-03 -2.087723e-03 -2.138088e-03 -2.175417e-03
##          s85
## -2.212684e-03
```

```
model4$beta["Confucian",]
```

```
##          s0          s1          s2          s3          s4          s5
## 0.0000000000 0.0000000000 0.0000000000 0.0001621431 0.0073470601 0.0138931647
##          s6          s7          s8          s9          s10         s11
## 0.0198577304 0.0247184313 0.0289443053 0.0327943465 0.0363023597 0.0394825701
##          s12         s13         s14         s15         s16         s17
## 0.0423720755 0.0451703608 0.0473677815 0.0494006357 0.0513695192 0.0532800898
##          s18         s19         s20         s21         s22         s23
## 0.0550396318 0.0568168456 0.0580045782 0.0590249207 0.0597445690 0.0602435836
##          s24         s25         s26         s27         s28         s29
## 0.0602429141 0.0596761196 0.0592215257 0.0587600550 0.0583418572 0.0577364260
##          s30         s31         s32         s33         s34         s35
## 0.0568774528 0.0562989043 0.0556491892 0.0554179214 0.0552196992 0.0550505400
##          s36         s37         s38         s39         s40         s41
## 0.0547964120 0.0546579435 0.0549502195 0.0555138745 0.0567310638 0.0577869720
##          s42         s43         s44         s45         s46         s47
```

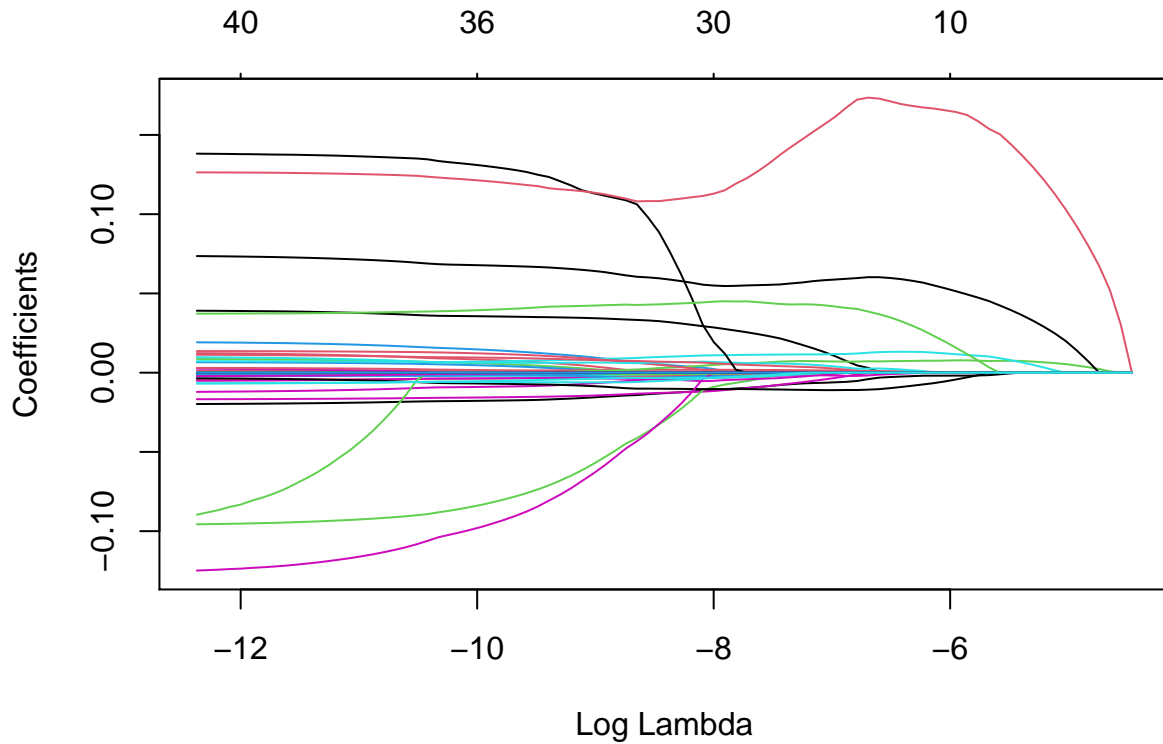
```
## 0.0587342490 0.0595534109 0.0601290032 0.0605148323 0.0613936418 0.0624443098
##          s48          s49          s50          s51          s52          s53
## 0.0635110808 0.0642446690 0.0649079907 0.0654469803 0.0659337479 0.0663722431
##          s54          s55          s56          s57          s58          s59
## 0.0666905089 0.0669304933 0.0671519438 0.0673599910 0.0675530982 0.0677332176
##          s60          s61          s62          s63          s64          s65
## 0.0678979832 0.0680485327 0.0681856232 0.0684322068 0.0688534260 0.0693838201
##          s66          s67          s68          s69          s70          s71
## 0.0698945900 0.0703152378 0.0706821885 0.0710215911 0.0713174773 0.0715927904
##          s72          s73          s74          s75          s76          s77
## 0.0718321509 0.0720705210 0.0722813913 0.0724716965 0.0726395166 0.0728104598
##          s78          s79          s80          s81          s82          s83
## 0.0729514088 0.0730629130 0.0731568200 0.0732688430 0.0733399001 0.0734313972
##          s84          s85
## 0.0735005287 0.0735691984
```

```
model4$beta["Abslat",]
```

```
##          s0          s1          s2          s3          s4
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s5          s6          s7          s8          s9
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s10         s11         s12         s13         s14
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s15         s16         s17         s18         s19
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s20         s21         s22         s23         s24
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s25         s26         s27         s28         s29
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s30         s31         s32         s33         s34
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s35         s36         s37         s38         s39
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s40         s41         s42         s43         s44
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s45         s46         s47         s48         s49
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s50         s51         s52         s53         s54
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s55         s56         s57         s58         s59
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
##          s60         s61         s62         s63         s64
## 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00 -1.622437e-06
##          s65         s66         s67         s68         s69
## -7.518398e-06 -1.431802e-05 -2.042576e-05 -2.591297e-05 -3.102535e-05
##          s70         s71         s72         s73         s74
## -3.556897e-05 -3.979984e-05 -4.337126e-05 -4.690859e-05 -5.016168e-05
##          s75         s76         s77         s78         s79
## -5.296114e-05 -5.55386e-05 -5.806661e-05 -6.007978e-05 -6.195865e-05
##          s80         s81         s82         s83         s84
## -6.347919e-05 -6.536241e-05 -6.648517e-05 -6.799466e-05 -6.910986e-05
##          s85
## -7.022663e-05
```



```
plot(model4, xvar = "lambda")
```



```
#model4$beta
```

```
# find the set beta coefficient for all lambdas  
# whenever a beta is non zero, it gives 1  
model4$beta !=0
```

```
## 41 x 86 sparse Matrix of class "lgCMatrix"
```

```
##      [[ suppressing 86 column names 's0', 's1', 's2' ... ]]
```

```
##  
## Abslat      . . . . .  
## Spanish    . . . . .  
## French     . . . . .  
## Brit       . . . . .  
## WarDummy   . . . . .  
## LatAmerica . . . . .  
## SubSahara  . . . . .  
## OutwarOr   . . . . .  
## Area       . . . . .  
## PrScEnroll . . . . .  
## LifeExp    . . . . .
```

[illegible]

[illegible]

## Foreign																			
## RFEXDist																			
## EquipInv																			
## NequipInv																			
## stdBMP																			
## BlMktPm																			