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50.007 Machine Learning
Homework 4

Q1

Case: All nodes take 2 different values

Node	Number of free parameters
A	$2-1=1$
B	$2-1=1$
C	$2-1=1$
D	$2-1=1$
E	$2\cdot 2\cdot 2\cdot (2-1)=8$
F	$2-1=1$
G	$2\cdot 2\cdot (2-1)=4$
H	$2\cdot 2\cdot (2-1)=4$
I	$2\cdot 2\cdot (2-1)=4$

$$Number\ of\ free\ parameters\ in\ this\ Bayesian\ network$$
$$= 1+1+1+1+8+1+4+4+4$$
$$= 25$$

Case: Nodes D and F take 4 different values, all other nodes take 2 different values

Node	Number of free parameters
A	$2-1=1$
B	$2-1=1$
C	$2-1=1$
D	$4-1=3$
E	$2\cdot 2\cdot 2\cdot (2-1)=8$
F	$4-1=3$
G	$4\cdot 2\cdot (2-1)=8$
H	$4\cdot 2\cdot (2-1)=8$
I	$2\cdot 2\cdot (2-1)=4$

$$Number\ of\ free\ parameters\ in\ this\ Bayesian\ network$$
$$= 1+1+1+3+8+3+8+8+4$$
$$= 37$$

Q2

Without knowing the actual value of any node, A and F are independent of each other. There are two possible paths to get from node A to node F.

- Looking at the path $A \rightarrow E \rightarrow H \rightarrow F$, there is a closed gate at node H (ie. $\rightarrow H \leftarrow$).
- Looking at the path $A \rightarrow E \rightarrow G \rightarrow I \rightarrow H \rightarrow F$, there is a closed gate at node I (ie. $\rightarrow I \leftarrow$).

If we know the value of node C and I, A and F are dependent of each other.

- Looking at the path $A \rightarrow E \rightarrow G \rightarrow I \rightarrow H \rightarrow F$, the gate at node I (ie. $\rightarrow I \leftarrow$) is now an open gate.

Q3

$$P(E = 1|C = 2) = \frac{P(E = 1, C = 2)}{P(C = 2)}$$

$$P(E, C) = \sum_{A,\mathrm{B}} P(E|A, B, C) \cdot P(C) \cdot P(A) \cdot P(B)$$

$$\begin{aligned}
&P(E = 1, C = 2) \\
&= \sum_{A,B} P(E = 1|A, B, C = 2) \cdot P(C = 2) \cdot P(A) \cdot P(B) \\
&= P(E = 1|A = 1, B = 1, C = 2) \cdot P(C = 2) \cdot P(A = 1) \cdot P(B = 1) \\
&+P(E = 1|A = 1, B = 2, C = 2) \cdot P(C = 2) \cdot P(A = 1) \cdot P(B = 2) \\
&+P(E = 1|A = 2, B = 1, C = 2) \cdot P(C = 2) \cdot P(A = 2) \cdot P(B = 1) \\
&+P(E = 1|A = 2, B = 2, C = 2) \cdot P(C = 2) \cdot P(A = 2) \cdot P(B = 2) \\
&= 0.3 \cdot 0.8 \cdot 0.2 \cdot 0.5 \\
&+ 0.0 \cdot 0.8 \cdot 0.2 \cdot 0.5 \\
&+0.6 \cdot 0.8 \cdot 0.8 \cdot 0.5 \\
&+0.5 \cdot 0.8 \cdot 0.8 \cdot 0.5 \\
&= 0.024 + 0.0 + 0.192 + 0.16 \\
&= 0.376
\end{aligned}$$

$$\begin{aligned}
&P(E = 1|C = 2) \\
&= \frac{P(E = 1, C = 2)}{P(C = 2)} \\
&= \frac{0.376}{0.8} \\
&= 0.47
\end{aligned}$$

Q4

Node A

1	**2**
$\frac{2}{3}$	$\frac{1}{3}$

Node B

1	**2**
$\frac{1}{2}$	$\frac{1}{2}$

Node C

1	**2**
$\frac{7}{12}$	$\frac{5}{12}$

Node D

1	**2**
$\frac{7}{12}$	$\frac{5}{12}$

Node E

A	**B**	**C**	**E**	
			1	**2**
1	1	1	0	1
1	1	2	$\frac{1}{2}$	$\frac{1}{2}$
1	2	1	$\frac{1}{3}$	$\frac{2}{3}$
1	2	2	0	1
2	1	1	1	0
2	1	2	1	0
2	2	1	1	0
2	2	2	0	1

Node F

1	**2**
$\frac{1}{2}$	$\frac{1}{2}$

Node G

D	E	G	
		1	**2**
1	1	0	1
1	2	$\frac{5}{6}$	$\frac{1}{6}$
2	1	$\frac{1}{4}$	$\frac{3}{4}$
2	2	0	1

Node H

E	F	H	
		1	**2**
1	1	1	0
1	2	$\frac{3}{4}$	$\frac{1}{4}$
2	1	$\frac{3}{5}$	$\frac{2}{5}$
2	2	$\frac{1}{2}$	$\frac{1}{2}$

Node I

G	H	I	
		1	**2**
1	1	$\frac{1}{2}$	$\frac{1}{2}$
1	2	$\frac{1}{2}$	$\frac{1}{2}$
2	1	$\frac{1}{2}$	$\frac{1}{2}$
2	2	$\frac{1}{2}$	$\frac{1}{2}$