40.014 Engineering Systems Architecture
Term 5, 2020

Homework 2
Due date: April 19, 2019 11:59 PM

Note. *You must complete the homework on your own and submit it individually on eDimension (in a* ***single*** *PDF file). Put your name at the beginning of the file. Please use complete English sentences to explain the results.*

Problem 1. A company produces three products, P_1, P_2, and P_3. The table below reports the amount of profit, working hours, raw material needed, and pollution for each unit of P_1, P_2, and P_3. The company has an availability of 1,300 working hours and 1,000 kg of raw material. The goal is to maximize the profit and minimize the environmental impact (pollution). <u>Consider continuous decision variables for all tasks.</u>

	Profit ($/unit of product)	Working hours (h/unit of product)	Raw material (kg/unit of product)	Pollution (kg of CO_2/unit of product)
P_1	10	4	3	10
P_2	9	3	2	6
P_3	8	2	2	3

Task 1. Formulate the problem of maximizing the profit while satisfying all constraints (be sure to start by defining your variables; for example, "let P_1 denote …"). <u>In this formulation, disregard the objective of minimizing the environmental impact.</u>

Task 2. Use R to solve the problem formulated in Task 1. Report the optimal value of objective function and decision variables. What is the corresponding value of the environmental impact?

Task 3. Using the ε-constraint method, formulate a multi-objective optimization problem aimed at maximizing the profit while minimizing the environmental impact. <u>To work on this task, transform the environmental impact into a constraint.</u>

Task 4. The solution obtained by minimizing the environmental impact is obvious: set the production of all products to 0. Naturally, this makes profit and pollution equal to 0. Knowing this, what range of variability do you recommend for the parameter ε introduced in Task 3?

Task 5. Solve the problem defined in Task 3 and 4 using four different values of the parameter ε (in addition to the two extreme values defined at Task 4). Use R to solve the problem. Produce a 2D scatter plot to represent the objective space. Print and attach the scatter plot.

Task 6. Provide a comment on the relation between the value of the decision variables and objective functions found in Task 5.

Task 7. Formulate the problem using the weighting method. <u>Hint: remember that, when using the weighting method, it is advisable to normalize the objectives. For this task, it is not necessary to solve the problem.</u>

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Problem 2. You have been headhunted by the Excellent Widgets Corp. to optimize their business operations. The company produces two widgets: Wonder and Marvel. Relevant information for each product is shown in the table below.

	Wonder	Marvel
Labor required (hours)	4	2
Contribution to profit (SGD)	4	2

The company has a goal of 48 SGD in profits and incurs a 1 SGD penalty for each dollar it falls short of this goal. A total of 32 hours of labor are available. A 2 SGD penalty is incurred for each hour of overtime (labor over 32 hours) used, and a 1 SGD penalty is incurred for each hour of available labor that is unused. Marketing considerations require that at least 7 units of Wonder be produced and at least 10 units of Marvel be produced. For each unit (of either widget) by which production falls short of demand, a penalty of 5 SGD is incurred. The company has the following goals (in order of importance—lexicographic ordering):

- G1: Have a profit of at least 48 SGD;
- G2: Use the 32 hours of labor available;
- G3: Meet demand for Wonder;
- G4: Meet demand for Marvel.

Let:

- x_i = amount of product i produced, $i = 1,2$
- p_j = amount over the j-th goal, $j = 1, \dots, 4$
- n_j = amount under the j-th goal, $j = 1, \dots, 4$

The problem can be formulated as follows:

$$
\begin{aligned}
\min \quad & [n_1, (2p_2 + n_2), 5n_3, 5n_4] \\
s.t. \quad & 4x_1 + 2x_2 + n_1 - p_1 = 48 \\
& 4x_1 + 2x_2 + n_2 - p_2 = 32 \\
& x_1 + n_3 - p_3 = 7 \\
& x_2 + n_4 - p_4 = 10 \\
& x_i \geq 0, i = 1,2 \\
& p_j \geq 0, j = 1, \dots, 4 \\
& n_j \geq 0, j = 1, \dots, 4
\end{aligned}
$$

Task 1. Solve the problem with R (hint: use the *llgp* function, package *goalprog*). Report the value of the decision variables.

Task 2. Are all goals met? If not, report the deviation from the desired goals.

Task 3. Solve the problem using the following vector of priority levels: [(G1, G2), G3, G4] (this means that goal G1 and G2 have the same priority). Report the value of decision variables and deviations from the desired goals.

Task 4. Compare the value of the objective functions (for Task 1 and Task 3) using a radar chart. Hint: the term "objective function" denotes, for each goal, the (absolute value of the) difference between target, or goal, and attained goal.

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Problem 3. For this last question, we are going to work with the genetic algorithm implemented in the R package *GA*.

Task 1. Given the function $f(x, y) = -\cos(x)\cos(y)e^{-((x-\pi)^2+(y-\pi)^2)}$, defined over the interval $-100 \leq x, y \leq 100$, we would like to find the global minimum using the *ga* function. Solve this problem using an initial population (popSize) of 100 individuals and a maximum number of iterations equal to 100. Report the value of the decision variables (x, y) and the corresponding value of the function $f(x, y)$.

Task 2. Do the values of objective and fitness functions correspond? Explain your reasoning.

Task 3. Now, solve the problem outlined at Task 1 using the following values of the maximum number of iterations: 10, 25, 50, 75, 100, while always keeping the initial population (popSize) equal to 100. Report the value of the objective function attained by the genetic algorithm for these five different values of the maximum number of iterations. What can you conclude about the convergence of the algorithm?