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An Improved Approach to Computing Implied Volatility

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Abstract

A well-known problem in finance is the absence of a closed form solution for volatility in common option pricing models. Several approaches have been developed to provide closed form approximations to volatility. This paper examines Chance's (1993, 1996) model, Corrado and Miller's (1996) model and Bharadia, Christofides and Salkin's (1996) model for approximating implied volatility. We develop a simplified extension of Chance's model that has greater accuracy than previous models. Our tests indicate dramatically improved results.

Keywords: options, volatility

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1. Introduction

In common option pricing models, a closed form solution does not exist to implied volatility. Several approaches have been developed to provide closed form approximations to volatility. Recently, Chance (1996) developed and tested a model to approximate the implied volatility of an option using a closed form equation. Chance's model is relatively simple and rather accurate for near the money options.

One of the problems with Chance's model is that it requires the price of an at-the-money option. Thus, the user must input not only the standard option pricing variables (the strike price, the risk free rate, the time to expiration, the price of the underlying asset, and the option price), the user must also provide the price of

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another option (an at-the-money option) in order to provide a "starting point" for the model's estimation techniques.

Section 2 of this paper reviews Chance's model and provides an extension that is simpler and much more accurate. Corrado and Miller (1996) and Bharadia, Christofides and Salkin (1996) also derive models to approximate the implied volatility of an option. Unlike Chance's model, Corrado and Miller's model and Bharadia, Christofides and Salkin's model do not require the input of an "extra" option price and therefore rely only on the five standard variables (the strike price, the risk free rate, the time to expiration, the price of the underlying asset and the option price).

Both of these models, however, lack the precision of the Chance model. Additionally, in the Corrado and Miller model the formula for implied volatility is not defined over all reasonable ranges. Section 3 of this paper reviews Corrado and Miller's model and performs a minor modification so that the formula is defined for all reasonable ranges.

The Bharadia, Christofides and Salkin model is a highly simplified volatility approximation model. Section 4 reviews and tests that model. The final section summarizes the study and discusses conclusions.

2. The Chance model

Chance (1996) derived a model to estimate the implied volatility of an option price. This section reviews his model and develops an extension that dramatically simplifies his model and improves its accuracy. We will refer to the option whose volatility is being approximated as the target option.

The first step in approximating the implied volatility of this target option is to find a starting guess of the volatility. In Chance's model, in effect, the starting volatility value is derived from an at-the-money option price using the Brenner-Subrahmanyam (1988) approximation formula. The Brenner-Subrahmanyam formula is a very simple, closed form approximation of the volatility of an at-the-money option where at-the-money is defined as having an underlying asset price equal to the present value of the strike price.

Chance begins with an equation that illustrates that the true call price, c, is equal to the price of the at-the-money-call, c*, plus the difference in price caused by differences in their exercise prices and volatilities:

$$c = c^* + \Delta c^* \tag{1}$$

Chance then performs a second order Taylor Series expansion around the at-the-money option allowing both the volatility and the strike price of the target option to differ from the at-the-money option. The following equation is the first equation from the Appendix to Chance's paper:

$$\Delta c^* = \frac{\partial c^*}{\partial X^*}(\Delta X^*) + \frac{1}{2}\frac{\partial^2 c^*}{\partial X^{*2}}(\Delta X^*)^2 + \frac{\partial c^*}{\partial \sigma^*}(\Delta\sigma^*) + \frac{1}{2}\frac{\partial^2 c^*}{\partial \sigma^{*2}}(\Delta\sigma^*)^2 + \frac{\partial^2 c^*}{\partial \sigma^* \partial X^*}(\Delta\sigma^* \Delta X^*) \quad (2)$$

where X is the strike price, σ is the volatility and the superscript * refers to at-the-discounted-money values. The above second order equation is then solved using the quadratic equation to produce an estimate of the implied volatility of the target option. Chance demonstrates the solution in the form of the quadratic equation:[1]

$$\Delta\sigma^* = (-b + (b^2 - 4aq)^{1/2})/2a \quad (3)$$

where:

$a = \frac{1}{2} c^*_{\sigma^*\sigma^*}$

$b = c^*_{\sigma^*} + c^*_{\sigma^* X^*}(\Delta X^*)$

$q = c^* - c + c^*_{X^*}(\Delta X^*) + \frac{1}{2} c^*_{X^* X^*}(\Delta X^*)^2.$

The partial derivatives above are shown by Chance to depend on the values of the at-the-money option. Thus, Chance derives an estimate of volatility based on at-the-money values and the change in the exercise price. Chance demonstrates that this approximation produces reasonably accurate results for options that are reasonably close to being at-the-money.

Table 1 of this paper is based on Table 1 of Chance's (1996) paper. Chance uses his model to estimate the implied volatility of 52 hypothetical options that differ by expiration date and strike price. The true volatility of each option is set equal to 0.41 and is compared with the model's volatility estimations to test the model's accuracy. Throughout this paper the underlying option pricing model is the Black-Scholes (1973) option pricing model for calls.

As discussed above, Chance's model requires the input of an option price (an at-the-money option) in addition to the price of the target option. For Table 1 of his paper, Chance assumes that the at-the-money option has a true volatility of 0.35 and approximates its volatility using the Brenner-Subrahmanyam (1988) approximation formula. Chance then uses the estimated implied volatility of the one year at-the-money option (0.348) and a second order Taylor Series expansion to approximate the volatilities of the 52 hypothetical options. If Chance's model were perfect, all of the estimated volatilities reported in his Table 1 would be 0.41. The accuracy of the approximation technique is measured by the closeness of the estimated volatilities to the true volatility of 0.41. Chance's Table 1 reports a variety of approximated volatilities in the area of 0.41.

[1] The formulas for the second derivative of the call price with respect to the volatility contain a typo in Chance's (1996) paper. The third and fourth equations on Page 865 need to have their right hand sides multiplied by d1.

Table 1

Approximation errors: Chance model

Each entry represents the estimation error: the difference between the true volatility (0.41) and the model's estimate of volatility. The starting point volatility for each entry in the Table is approximately 0.348. This is the implied volatility from the Brenner-Subrahmanyam formula using a one year at-the-discounted-money ($95.21) call option with a true volatility of 0.35, a strike price of $100, an interest rate of 5% and a price of $13.21. Volatility estimates are taken from Table 1 in Chance (1996).

	Time to Expiration				mean of
Strike Price	0.25	0.50	0.75	1.00	absolute values
$70	0.1115	0.0312	0.0143	0.0081	0.0413
$75	0.0636	0.0197	0.0100	0.0063	0.0249
$80	0.0342	0.0121	0.0068	0.0046	0.0144
$85	0.0170	0.0068	0.0040	0.0027	0.0076
$90	0.0070	0.0030	0.0017	0.0010	0.0032
$95	0.0015	0.0004	0.0000	−0.0003	0.0005
$100	−0.0004	−0.0006	−0.0007	−0.0007	0.0006
$105	0.0021	0.0007	0.0002	−0.0001	0.0008
$110	0.0108	0.0050	0.0031	0.0021	0.0052
$115	0.0282	0.0132	0.0085	0.0061	0.0140
$120	0.0566	0.0260	0.0168	0.0124	0.0280
$125	0.0977	0.0443	0.0285	0.0210	0.0479
$130	0.1529	0.0685	0.0438	0.0322	0.0744
mean of absolute values	0.0449	0.0178	0.0106	0.0075	0.0202

Equally weighted grand mean of absolute values = 0.0202
Vega weighted grand mean of absolute values = 0.0132

In order to develop an objective measure of accuracy, we computed the approximation error for each volatility in Table 1 of Chance's paper and computed the mean of their absolute values. This measure provides an indication of the average amount by which the approximations erred. In Table 1 we report the values of having subtracted the approximations from 0.41 to express the value of the errors directly. For example, the entry in the upper left hand corner of Table 1 indicates that in the case of an option with a strike price of $70 and 0.25 years to expiration, the Chance model underestimates the true volatility (0.41) by 0.1115. Table 1 of Chance's paper indicates a volatility approximation of 0.2985 for this option, an error of 0.1115 from the true volatility of 0.41.

Table 1 averages the absolute values of the approximation errors by row, by column and by matrix. These averaged absolute errors are a heuristic measure of the accuracy of the model. As seen in Table 1 of this paper and as noted by Chance, Chance's model is highly accurate for near-the-money options and has reduced effectiveness for options far from the money. The equally weighted grand mean of the absolute values in Table 1 is 0.0202 and reflects a crude estimate of the average error of the approximation.

In Table 1 the greatest approximation errors are for short term options that are far from the money and therefore have rather low vegas (the derivative or sensitivity of the call price with respect to volatility). The equally weighted mean of errors reported above is dominated by the large approximation errors that tend to be associated with the low vega observations. Therefore, Table 1 also reports a vega weighted grand mean of the absolute values of the approximation errors. The weight of each observation is directly proportional to the observation's vega. The vega was computed using the true volatility of each observation (0.41). The vega weighted mean is about one third lower: 0.0132.

Next, we extend Chance's model with one very important modification. Chance uses as his base case the at-the-money option. His Taylor Series expansion estimates the change in volatility that would explain the difference between the price of the option of interest and the at-the-money option. Given that differences in the strike price as well as the volatility can account for differences in these two option prices, his Taylor series expansion is in two variables, the strike price and the volatility. Our approach is simpler. We take for starters the implied volatility of the at-the-money option, using it to price our option and establish a base case option value. We then perform the Taylor series expansion with only the volatility. As we shall show, not only is this easier, but it gives better results.

Letting σ^* now represent the implied volatility of the at-the money option, obtainable from the Brenner-Subrahmanyam formula, and c^* be the price of our option obtained using the implied volatility of the at-the-money option, we have:

$$\Delta c^* = \frac{\partial c^*}{\partial \sigma^*}(\Delta\sigma^*) + \frac{1}{2}\frac{\partial^2 c^*}{\partial \sigma^{*2}}(\Delta\sigma^*)^2 \tag{4}$$

Here Δc^* is the actual option price minus the price obtained using the at-the-money implied volatility. This is the distance we need to travel by altering the volatility from the at-the-money volatility. This is a much simpler quadratic equation than the one used by Chance as it has three fewer terms. It essentially can be viewed as a second-order Newton-Raphson search that automatically stops after the first step. In comparison, Chance's method requires that we estimate the effect of the exercise price difference, when in fact, we can capture that effect precisely. We need only an at-the-money option and the use of the Brenner-Subrahmanyam formula:

$$\sigma_{atm} = \frac{c_{atm}\sqrt{2\pi}}{S\sqrt{T}}$$

where c_{atm} is the price of the at-the-money call. The formulas for the first and second derivatives with respect to volatility are shown in the Appendix.

As an example, consider a case used by Chance: $S = 95.12$, $X = 70$, $r = 0.05$, $T = 1$. The at-the-money option has $X=100$ (95.12 = present value of 100) and a price of 13.21. Its implied volatility using Brenner-Subrahmanyam's formula is

0.348114. We wish to find the implied volatility of a call that has an exercise price of 70 and is priced at 31.9521. The solution we seek is 0.41.

Using the at-the-money implied volatility, the call struck at 70 is priced at 30.715. Note that even at the wrong implied volatility, we are already quite close to the option price. The difference, which is our Δc^*, is 31.9521-30.715 = 1.236954. The vega is 18.503. The second order vega is 54.178. Thus, our equation is:

$$1.236954 = 18.503(\Delta\sigma^*) + \frac{1}{2}54.178(\Delta\sigma^*)^2$$

The solution is 0.06134, which means that our estimate of the implied volatility is 0.348114 + 0.06124 = 0.4095, a difference of only 0.41 − 0.4095 = 0.0005.

This extension produces a dramatic simplification and should produce improved accuracy since the estimation is taking place closer to the true values. We test the accuracy of this simplification by replicating Chance's test. We tested our extension in a manner analogous to Chance's test reported above. All of our one variable Taylor Series expansions are performed from the Brenner-Subrahmanyam starting point (in this case a volatility of 0.348). Our results are shown in Table 2 in a format analogous to Table 1. The equally weighted approximation error falls by over 98%, from 0.0202 to 0.0003. The vega weighted mean error also drops by over 98%, from 0.0132 to 0.0002.

Thus, the model derived in this paper provides a much simpler and much more accurate extension to Chance's model. However, this simplified model, like Chance's model, still requires the input of an ''extra'' option price -- the price of the at-the-money option to provide a starting point to the formula. In the next section, we review the model of Corrado and Miller that does not require an additional option price.

3. The Corrado-Miller model

As discussed above, a potential limitation of the Chance model is that it requires that the user know or be able to approximate reasonably the value of an at-the-money option. The purpose of the at-the-money option price is to provide an initial starting point for the search. Corrado and Miller (1996) reported a model that estimates implied volatility without requiring an option price other than the price of the target option. Corrado and Miller's model is an extension of the Brenner-Subrahmanyam formula. Corrado and Miller develop and test a formula for implied volatility given in their paper as Equation 10 and reproduced below:

$$\sigma\sqrt{T} = \left[\sqrt{2\pi}/(S + X)\right]\{C - [(S - X)/2] + \sqrt{\{C - [(S - X/2]\}^2 - [(S - X)^2/\pi]}\} \tag{5}$$

where X represents the discounted strike price.

Table 2

Approximation errors: single variable Taylor series expansion model (modified Chance model)

Each entry represents the estimation error: the difference between the true volatility (0.41) and the model's estimate of volatility. The starting point volatility for each entry in the Table is approximately 0.348. This is the implied volatility from the Brenner-Subrahmanyam formula using a one year at-the-discounted-money ($95.21) call option with a true volatility of 0.35, a strike price of $100, an interest rate of 5% and a price of $13.21.

	Time to Expiration				
Strike Price	0.25	0.50	0.75	1.00	mean of absolute values
$70	−0.0001	0.0006	0.0006	0.0005	0.0005
$75	0.0005	0.0005	0.0005	0.0004	0.0005
$80	0.0005	0.0004	0.0003	0.0003	0.0004
$85	0.0003	0.0002	0.0002	0.0002	0.0002
$90	0.0001	0.0001	0.0001	0.0001	0.0001
$95	0.0000	0.0000	0.0000	0.0000	0.0000
$100	0.0000	0.0000	0.0000	0.0000	0.0000
$105	0.0002	0.0001	0.0000	0.0000	0.0001
$110	0.0004	0.0002	0.0001	0.0001	0.0002
$115	0.0005	0.0003	0.0002	0.0001	0.0003
$120	0.0006	0.0004	0.0003	0.0002	0.0004
$125	0.0005	0.0005	0.0004	0.0003	0.0004
$130	0.0002	0.0006	0.0005	0.0004	0.0004
mean of absolute values	0.0003	0.0003	0.0002	0.0002	0.0003

Equally weighted grand mean of absolute values = 0.0003
Vega weighted grand mean of absolute values = 0.0002

Corrado and Miller then empirically demonstrate their model's accuracy for several actual option prices. One problem with Corrado and Miller's model is that it includes a square root term that in some cases does not have a real solution. Specifically, for short term options that are very substantially away from the money, the formula requires the square root of a negative value. This is a serious limitation since the user can not be sure if the formula will produce an error, although the problem is unlikely to occur for reasonable parameters.

The square root term on the right hand side of the above equation sometimes has a negative argument. We modified Corrado and Miller's formula by replacing the square root with a term that keeps the formula defined for all ranges. Specifically, when the formula called for the square root of a negative number, we substituted zero. Otherwise we used the square root from the formula.

We test the accuracy of the Corrado-Miller model, including our minor modification by replicating Chance's methodology. We used the modified Corrado-Miller model to estimate the same options as in Table 1 of the Chance paper. Our results are shown in Table 3 in a format analogous to Tables 1 and 2. The equally weighted error measure is 0.0241 compared to 0.0202 for Chance's model and 0.0003 for

Table 3

Approximation errors: modified Corrado-Miller model

Each entry represents the estimation error: the difference between the true volatility (0.41) and the model's estimate of volatility. The formula requires a call option price, a strike price and an underlying asset price. The call option price for each entry is the price of a call option with a volatility of 0.41, an interest rate of 5% and an underlying asset with a price of $95.21 (and other variables as indicated).

	Time to Expiration				mean of
Strike Price	0.25	0.50	0.75	1.00	absolute values
$70	0.1807	0.1483	0.0344	0.0224	0.0964
$75	0.1645	0.0226	0.0132	0.0110	0.0528
$80	0.0206	0.0075	0.0062	0.0063	0.0101
$85	0.0041	0.0030	0.0035	0.0042	0.0037
$90	0.0011	0.0018	0.0025	0.0033	0.0022
$95	0.0007	0.0015	0.0022	0.0029	0.0018
$100	0.0008	0.0015	0.0021	0.0029	0.0018
$105	0.0016	0.0017	0.0023	0.0029	0.0021
$110	0.0050	0.0025	0.0027	0.0032	0.0033
$115	0.0164	0.0044	0.0036	0.0038	0.0070
$120	0.0668	0.0087	0.0052	0.0047	0.0213
$125	0.1714	0.0174	0.0082	0.0063	0.0508
$130	0.1812	0.0368	0.0131	0.0087	0.0600
mean of absolute values	0.0627	0.0198	0.0076	0.0064	0.0241

Equally weighted grand mean of absolute values = 0.0241
Vega weighted grand mean of absolute values = 0.0122

our extension of Chance's model. The vega weighted error measure is 0.0122 compared to 0.0132 for Chance's model and 0.0002 for our extension of Chance's model.

Corrado and Miller's model is highly accurate for options near the money but deteriorates for options very far out of the money. The largest approximation errors in the results of Table 3 were the same cases for which the original Corrado-Miller Model fails to produce real solutions (short term options very far from the money). However, it should be noted that the results in Table 2 (the modified Chance model) are far more accurate than the results in Table 3 (the Corrado-Miller model) even if the extreme cases are removed that required our modification to the Corrado-Miller model. For all strike prices and for all expiration dates reported in Tables 2 and 3, the modified Chance model is substantially more accurate than the Corrado-Miller model.

Thus, Corrado and Miller's model offers a reduced information requirement but generally is far less accurate than our extension of Chance's model. Perhaps more importantly, the Corrado-Miller formula only has imaginary solutions in extreme cases.

Table 4

Approximation errors: Bharadia-Christofides-Salkin model

Each entry represents the estimation error: the difference between the true volatility (0.41) and the model's estimate of volatility. The formula requires a call option price, a strike price and an underlying asset price. The call option price for each entry is the price of a call option with a volatility of 0.41, an interest rate of 5% and an underlying asset with a price of $95.21 (and other variables as indicated).

	Time to Expiration				mean of
Strike Price	0.25	0.50	0.75	1.00	absolute values
$70	−0.4092	−0.2358	−0.1717	−0.1383	0.2388
$75	−0.2685	−0.1534	−0.1124	−0.0913	0.1564
$80	−0.1554	−0.0894	−0.0665	−0.0549	0.0915
$85	−0.0727	−0.0433	−0.0333	−0.0282	0.0444
$90	−0.0214	−0.0140	−0.0116	−0.0103	0.0143
$95	−0.0002	−0.0002	−0.0003	−0.0003	0.0002
$100	−0.0061	−0.0001	0.0019	0.0029	0.0027
$105	−0.0349	−0.0116	−0.0039	−0.0000	0.0126
$110	−0.0819	−0.0330	−0.0163	−0.0079	0.0348
$115	−0.1425	−0.0622	−0.0343	−0.0202	0.0648
$120	−0.2126	−0.0977	−0.0569	−0.0361	0.1008
$125	−0.2888	−0.1379	−0.0831	−0.0549	0.1412
$130	−0.3684	−0.1815	−0.1122	−0.0761	0.1846
mean of absolute values	0.1587	0.0816	0.0542	0.0401	0.0836

Equally weighted grand mean of absolute values = 0.0836
Vega weighted grand mean of absolute values = 0.0565

4. The Bharadia, Christopher, and Salkin model

Bharadia, Christofides and Salkin (1996) derive a highly simplified volatility approximation model as shown:

$$\sigma = \sqrt{(2\pi / T)}\ [(c - \delta) / (S - \delta)] \tag{6}$$

Where $\delta = (S - X)/2$, c is the call price, S is the stock price and X is the discounted strike price. Bharadia, Christofides and Salkin demonstrate the accuracy of their formula relative to the formula of Manaster and Koehler's (1982) technique and the formula of Brenner and Subrahmanyam (1988).

Table 4 repeats this paper's empirical test analogous to Tables 1 through 3 using Bharadia, Christofides and Salkin's model. As would be expected from such a simplified model, the approximation errors are generally much larger than from the more complex models reported in Tables 1 through 3. The equally weighted approximation error in Table 4 is 0.0836 compared with 0.0003 from Table 2. The vega weighted mean error in Table 4 is 0.0565 compared with 0.0002 in Table 2.

5. Conclusion

Implied volatility can be easily calculated using an iterative searching technique. However, to simplify some applications such as spreadsheets, it may be useful to have a closed form approximation solution if that solution is reasonably simple, accurate and valid for a wide range of cases.

Chance's model, especially as extended in this study, is relatively simple and accurate for most cases. Corrado and Miller's model offers the potential advantage of not requiring additional information (for example, the price of an at-the-money option). However, this requirement is not very demanding and would nearly always be met. In fact, the only true requirement of Chance's model and our extension of the Chance model is that there be a reasonable estimate of volatility to serve as a starting point to the approximation.

The Corrado-Miller model's performance is substantially diminished for nearly all cases and tremendously diminished in extreme cases (i.e., short term and far out-of-the-money). The Corrado-Miller model's most potentially serious difficulty is that its solution is imaginary in extreme cases. Bharadia, Christofides and Salkin's model for approximating implied volatility is highly simplified but far less accurate.

Iterative techniques to solve for implied volatility include the approach of Manaster and Koehler (1982). However, the cost and inconvenience of iterating motivate the search for closed form approximations. Therefore, for options relatively near the money, the extended Chance model developed in this study offers a relatively simple and accurate approximation.

Appendix

This appendix provides the first and second derivatives of the call price with respect to volatility.

The first derivative is:

$$c^*_\sigma = [S\ T^{0.5}\ exp\ (-d_1{}^{*2}/2)]/(2\pi)^{0.5} \tag{A1}$$

For an at-the-money (present valued) call, this gives:

$$c^*_\sigma\ (at\ the\ money) = [S\ T^{0.5}\ exp\ (-\sigma^{*2}\ T/8)]/(2\pi)^{0.5} \tag{A2}$$

The second derivative is:

$$c^*_{\sigma\sigma} = [S\ T^{0.5}\ n(d_1)\ d_1\ d_2\ ]/\sigma \tag{A3}$$

where n(.) is the normal density function.
For an at-the-money (present valued) call, this gives:

$$c^*_{\sigma\sigma}\ (at\ the\ money) = -S\ T^{1.5}\ \sigma\ exp[-\sigma^2\ T\ /\ 8\ ]/[4\ (2\pi)^{0.5}] \tag{A4}$$

The formulas for the first derivative are from Chance's (1996) paper. The formulas for the second derivative of the call price with respect to the volatility are factored differently from Chance's equations and correct for a typo in Chance's (1996) paper. Specifically, the third and fourth equations on Page 865 of Chance's (1996) paper need to have their right hand sides multiplied by d1.

References

Black, F. and M. Scholes, 1973. The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637-659.

Bharadia, M. A., N. Christofides and G. R. Salkin, 1996. Computing the Black Scholes Implied Volatility, *Advances in Futures and Options Research, Vol. 8*, 15-29, (JAI Press, London).

Brenner, Menachem and Marti G. Subrahmanyam, 1988. A Simple Formula to Compute Implied Standard Deviation, *Financial Analyst Journal 5*, 80-83.

Chance, Don M., 1996. A Generalized Simple Formula to Compute the Implied Volatility, *The Financial Review 4*, 859-867.

Chance, Don M., 1993. Leap Into the Unknown, *Risk 6*, 60-66.

Corrado, Charles J. and Thomas W. Miller, Jr., 1996. A Note on a Simple, Accurate Formula to Compute Implied Standard Deviations, *Journal of Banking & Finance, 20*, 595-603.

Corrado, Charles J. and Thomas W. Miller, Jr., 1996. Volatility without Tears, *Risk 7*, 49-52.

Manaster, S. and G. Koehler, 1982. The Calculation of Implied Variances from the Black-Scholes Model, *Journal of Finance 38*, 227-230.