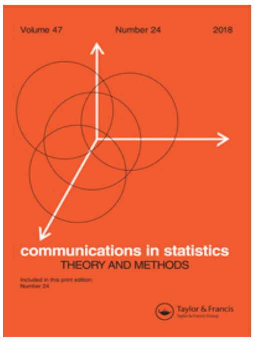

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Empirical analysis of SH50ETF and SH50ETF option prices under regime-switching jump-diffusion models

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ABSTRACT
In this paper, we use extensive empirical data sets from Shanghai 50ETF (SH50ETF) and SH50ETF options markets in China to study how regime-switching jump-diffusion models improve goodness of fit and option pricing performance. Firstly, the model parameters are estimated by using maximum likelihood estimation and the numerical analysis indicates that the regime-switching jump-diffusion models outperform a range of other models. Secondly, the analytical option pricing formulae are obtained via the fast Fourier transform and the empirical results using the proposed option pricing formulae are presented. Finally, we find that the calculated option prices are fairly consistent with the actual market prices.

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1. Introduction

The stock market index is a tool used by investors and financial managers to describe the market and compare returns on specific investments. In recent decades, there is an accelerating trend to create passively managed mutual funds based on market indices, such as index funds or exchange traded funds (ETFs). ETFs are characterized by small deviations from the underlying index, low costs, convenient transactions, low capital thresholds, and high liquidity. It has grown to be one of the most popular new-type of fund products in many countries in recent years. Designing ETFs and futures products around ETFs has become a new focus of competition for major exchanges. Many derivatives of stock indices have been developed, such as stock index futures and stock index options, which have also become important tools to hedge risks. On February 9, 2015, the Shanghai 50ETF (SH50ETF) option contract, the first stock option product in China's securities market, was traded on the Shanghai stock exchange, indicating that China's financial market has entered the option era. The launch of the SH50ETF option meets the needs of the development of China's securities capital market and promotes the innovation of the financial derivatives market. Therefore, how to establish a reasonable and effective option pricing model for SH50ETF option has important theoretical and practical contribution.

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/lsta.

Over the past decade or two, evidence has been found of financial returns exhibiting stochastic volatility and fatter tails than the standard normal model. Based on these features, many approaches have been proposed to capture the dynamics of financial returns, such as the jump-diffusion model and the regime-switching model. The jump-diffusion model introduced by Merton (1976) has shown increased prominence in option pricing. Merton (1976) introduces the jump diffusion model using a Poisson process for the jump timing and a log-normal process for the jump amplitudes to describe the market crashes and rallies. Kou (2002) proposes a new popular jump diffusion model by assuming that the jump size follows a double exponential distribution. Hanson, Westman, and Zhu (2004) and Synowiec (2008) estimate the parameters of the jump-diffusion model and perform a comparative analysis of different jump distributions.

Meanwhile, Markov regime-switching models introduced by Hamilton (1989) provide a natural and convenient way to describe structural changes in market interest rate, exchange rate, stock returns, and the like. Gray (1996) establishes a regime-switching model for the short-term interest rate process and conducts a numerical analysis. Filardo and Gordon (1998) use a regime-switching model to calculate the duration of the United States business cycle. Bollen, Gray, and Whaley (2000) use the regime-switching model to study the dynamic process of determining foreign exchange rates. Frömmel, MacDonald, and Menkhoff (2005) introduce the Markov regime switching into a monetary exchange rate model to develop a better description of the data. Chang and Feigenbaum (2008) examine the logarithmic periodicity of stock returns under the regime-switching model. Meanwhile, Chkili and Nguyen (2014) apply the regime-switching model on Brazil, Russia, India, China and South Africa to study the relationship between exchange rates and returns on the stock market. Moreover, the application of the regime switching model in pricing derivatives can be traced back to the work of Elliott, Chan, and Siu (2005), Siu, Yang, and Lau (2008), and Fan et al. (2014).

To clearly describe both the time-inhomogeneity and sudden shocks in the asset price dynamics, many studies have examined options pricing by incorporating jump-diffusion with Markov regime-switching models. Hsu et al. (2016) consider the application of the regime-switching jump-diffusion model with associated jump-scale risk in S&P500, DJIA and Nikkei225, revealing that the model is superior to others. Chen, Lin, and Li (2016) use the regime-switching jump-diffusion model to study the volatility accumulation characteristics of spot European Union quota returns and the beating caused by the carbon emission policy. In the pricing of derivatives, Elliott and Osakwe (2006) investigate option pricing for pure jump processes with Markov switching compensators. Bo, Wang, and Yang (2010) study the pricing of some currency options based on the Markov-modulated jump-diffusion models for spot foreign exchange rate.

In this paper, we make reference to the methodology in Chang, Fuh, and Lin (2013) and Costabile et al. (2014). Chang, Fuh, and Lin (2013) provide the explicit pricing formula for European options in a regime-switching jump-diffusion model and confirm the existence of jump intensity switching characteristics and aggregation in jump strength. Costabile et al. (2014) obtain a closed-form formula for European options under a two regime-switching economy with jumps when asset price process following a log-normal distribution. Comparing with Chang, Fuh, and Lin (2013) and Costabile et al. (2014), the differences between theirs and ours are also evident. First, Chang, Fuh, and Lin (2013) apply the regime-switching jump-diffusion model to the Dow Jones Industrial Average and perform empirical analysis. Costabile et al. (2014) establish a regime-switching jump-diffusion model for CAC40, DAX 30, FTSE 100, Nikkei 225 and S&P500 stock index returns using iterative maximum likelihood estimation (see Hamilton (1994) and Perlin (2015)). In fact, as far as we know, only few applications of regime-switching jump-diffusion models to China Stock Exchange Composite Index can be seen in the existing literatures. In this paper, we use the regime-switching jump-diffusion model to describe the SH50ETF price and expand the existing body of knowledge by examining whether regime-switching jump-diffusion models provide a more accurate valuation of SH50ETF option. Second, they obtain the explicit pricing formula for European options by using the probability density function of the occupation time of the states of the Markov chain. However, the method in our paper is different from theirs, we employ the Fourier transform method to get an explicit formula for SH50ETF option. Finally and the most important, we find that the calculated SH50ETF option prices under the regime-switching jump-diffusion model are well fitted with the actual market price. This empirical evidence also shows that it is very appropriate to apply the regime-switching jump-diffusion model to the SH50ETF price.

The rest of the paper is organized as follows. Section 2 presents the model dynamics and provides an empirical analysis of SH50ETF using the regime-switching jump-diffusion model. In Section 3, the SH50ETF option pricing formula is obtained by the FFT method and an empirical study is performed confirming the accuracy of the proposed model for SH50ETF options. Finally, Section 4 presents the conclusions.

2. Model description and econometric analysis

2.1. Model description

We consider a continuous-time financial market with a finite time horizon $\mathcal{T} := [0, T]$, where $T < \infty$. Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space, where $\mathcal{P}$ is a physical probability measure. On the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, let $\{U_t\}_{t\in\mathcal{T}}$ be a continuous-time finite-state observable Markov chain, $\{W_t\}_{t\in\mathcal{T}}$ be a standard Brownian motion, $\{N_t\}_{t\in\mathcal{T}}$ be a Poisson process with intensity λ_t, and $\{Z_t\}_{t\in\mathcal{T}}$ be an independent and identically distributed random variable. Throughout this paper, we assume that $\{U_t\}_{t\in\mathcal{T}}, \{W_t\}_{t\in\mathcal{T}}$, $\{N_t\}_{t\in\mathcal{T}}$ and $\{Z_t\}_{t\in\mathcal{T}}$ are stochastically independent.

Let $N(t)$ denote the number of claims occurring within the time horizon $[0, t]$, and Z_i denote the size of the ith claim. The aggregate claims up to time t is represented by a compound Poisson process, i.e., $L_t = \sum_{i=1}^{N_t} Z_i$. We equip the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\mathcal{F} := \{\mathcal{F}_t | t \in \mathcal{T}\}$ generated by $\{W_t\}_{t\in\mathcal{T}}, \{L_t\}_{t\in\mathcal{T}}$ and $\{U_t\}_{t\in\mathcal{T}}$ satisfying the regular conditions (i.e., right continuity and $\mathcal{P}$-completeness). By convention, we assume that $\mathcal{F} = \mathcal{F}(\mathcal{T})$.

To describe the evolution of the state of an economy over time, we use the states of U to model the states of the economy and adopt the assumptions of Elliott (1994) that the states space of U is a limited collection of vectors $\{e_1, e_2, ..., e_N\}$, where $e_i = (0, ..., 1, ..., 0) \in \mathbf{R}^N$ with "1" in the ith component. Suppose that the time-invariant matrix $\mathbf{A}$ denote the generator or Q-matrix $(a_{ij})_{i,j=1,2,\dots,N}$ of U, where a_{ij} is the infinitesimal intensity of U from state e_j to state e_i with $a_{ii} = -\sum_{i\neq j,j=1}^{N} a_{ij}$. Then, based on the work of Elliott (1994), the semi-martingale decomposition of U is given by

$$dU_t = \mathbf{A}U_t dt + dM_t \tag{2.1}$$

where $M = \{M_t\}_{t\in\mathcal{T}}$ is an $\mathbf{R}^N$-valued martingale with respect to the filtration generated by $\{U_t\}_{t\in\mathcal{T}}$ under $\mathcal{P}$.

We assume that S_t is the SH50ETF price and follows a regime-switching jump-diffusion model under the physical probability measure $\mathcal{P}$,

$$\frac{dS_t}{S_{t-}} = \mu_t dt + \sigma_t dW_t + (e^{Z_t} - 1)dN_t \tag{2.2}$$

where μ_t is the instantaneous rate of SH50ETF return, σ_t is the volatility of asset. The jump amplitudes are controlled by Z_t which usually is assumed to follow a normal distribution or an asymmetric double exponential distribution. The parameters of the distributions also depend upon the regime observed at time t and consequently, they are defined for each state in the chain.

We assume that μ_t, σ_t and λ_t of the SH50ETF all depend on $\{U_t\}_{t\in\mathcal{T}}$, which are defined as

$$\mu_t = <\mu, U_t>, \sigma_t = <\sigma, U_t>, \lambda_t = <\lambda, U_t>$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_N) \in \mathbf{R}^N, \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \dots, \sigma_N) \in \mathbf{R}^N$ with $\sigma_i > 0$ for each $i = 1, 2, \dots, N$ and $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_N) \in \mathbf{R}^N$ with $\lambda_i > 0$ for each $i = 1, 2, \dots, N$. $<.,.>$ denotes the inner product in $\mathbf{R}^N$, for $i \neq j$, $<e_i, e_j> = 0$, otherwise, $<e_i, e_i> = 1$.

2.2. Econometric analysis of SH50ETF

For the sake of simplicity, we have limited the analysis to a Markov chain with only two states; however, its extension to a larger number of states remains straightforward.

The parameters of the asset dynamics may switch according to a continuous-time, homogeneous and stationary Markov process U_t, on the state space $\{e_1, e_2\}$ with generator $A \in R^{2\times 2}$ defined below.

$$A = \begin{pmatrix} -a_{1,2} & a_{1,2} \\ a_{2,1} & -a_{2,1} \end{pmatrix}$$

The generator A controls the transition probabilities of the process from the current state to the other. The transition probability matrix in the interval $[t, t+\Delta t]$ is given by

$$P = e^{A\Delta t} = \sum_{n=0}^{\infty} \frac{(A\Delta t)^n}{n!} = I + A\Delta t + o(\Delta t)$$

where I is the identity matrix. Thus, ignoring terms of order superior to Δt, if at time t we are in regime 1, then with probability $a_{1,2}\Delta t$, a switch to regime 2 occurs at time $t + \Delta t$, while the probability of remaining in regime 1 is $1 - a_{1,2}\Delta t$. Transition probabilities from regime 1 are constructed similarly.

Considering the following discretization of the regime-switching jump-diffusion model over $(t, t+\Delta t)$, we obtain

$$y_t = \Delta \ln S_t = \ln S_{(t+\Delta t)} - \ln S_t = \alpha_t \Delta t + \sigma_t \Delta W_t + Z_t \Delta N_t \tag{2.3}$$

where $\alpha_t = \mu_t - \frac{1}{2}\sigma_t^2, \Delta t$ is as small as necessary. In this paper, we assume that a year has 252 trading days, and $\Delta t = 1/252$. $\Delta W_t = W_{t+\Delta t} - W_t$ are i.i.d normal random variates with mean 0 and variance Δt, i.e $\Delta W_t \sim N(0, \Delta t)$. As we may observe, the distributions of the price process (2.2) and log-returns (2.3) depend on the distributions of $\Delta N_t = N_{t+\Delta t} - N_t$ and the log-return jump amplitude Z_t. We assume that $Z_t \sim N(\nu_t, \varpi_t^2)$, where $\nu_t = <\nu, U_t>, \varpi_t = <\varpi, U_t>$ with $\mathbf{v} = (\nu_1, \nu_2, ..., \nu_N) \in \mathbf{R}^N, \varpi = (\varpi_1, \varpi_2, ..., \varpi_N) \in \mathbf{R}^N$ and ΔN_t is a Poisson distribution with the parameter $\lambda_t \Delta t$, i.e $\Delta N_t \sim Poisson(\lambda_t \Delta t)$.

Due to the problem of "path dependence" observed in the Markov regime switching model, the form of likelihood function becomes complicated as sample size increases. Therefore, we calculate the likelihood function using the iterative method introduced by Hamilton (1994). Here, we follow the setup and notation of Hamilton to estimate the discrete time model (2.3) using a maximum likelihood procedure. Let θ be the vector of model parameters and Ω_t the observations up to time t as follows:

$$\theta = (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \nu_1, \nu_2, \varpi_1^2, \varpi_2^2, \lambda_1, \lambda_2, p_{11}, p_{22}), \quad \Omega_t = \{y_t, ..., y_0\}$$

Set

$$X_t = \alpha_t \Delta t + \sigma_t \Delta W_t, \quad Y_t = \sum_{i=1}^{\Delta N_t} Z_t(i), \quad V_n = \sum_{i=1}^{n} Z_t(i)$$

where $Z_t(i)$ denotes the ith jump amplitude in Δt time. Inference of model (2.3) requires defining the 2×1 vectors $\hat{\xi}_t$ with elements $p(U_t = l | \Omega_t, \theta), l = 0, 1$, and η_t with elements

$$\begin{aligned} f(y_t | U_t = l, \Omega_t; \theta) &= f_{X_l + Y_l}(x) \\ &= p_0(\lambda_l \Delta t) f_{X_l}(x) + \sum_{n=1}^{+\infty} p_n(\lambda_l \Delta t) f_{X_l + V_n}(x) \end{aligned} \tag{2.4}$$

where $P\{\Delta N_t = n\} = p_n(\lambda_t \Delta t) = \frac{(\lambda_t \Delta t)^n}{n!} e^{-\lambda_t \Delta t}$.

Considering the specific distribution information that W_t, N_t and Z_t are conditionally independent given U_t, the conditional density of y_t is represented as:

$$\begin{aligned} & f(y_t | U_t = l, \Omega_t; \theta) \\ &= \sum_{n=0}^{+\infty} \frac{(\lambda_l \Delta t)^n e^{-\lambda_l \Delta t}}{n!} \phi(\alpha_l \Delta t + n\nu_l, \sigma_l^2 \Delta t + n\varpi_l^2), l = 1, 2 \end{aligned} \tag{2.5}$$

where $\phi(\mu, \sigma^2)$ is the normal density function with mean μ and variance σ^2.

The logarithmic likelihood function for a sample of M observations is given by

$$l(\theta) = \ln L(\theta) = \sum_{t=1}^{M} \log f(y_t | \Omega; \theta)$$

where the densities $f(y_t | \Omega_t; \theta)$ are updated recursively as follows:

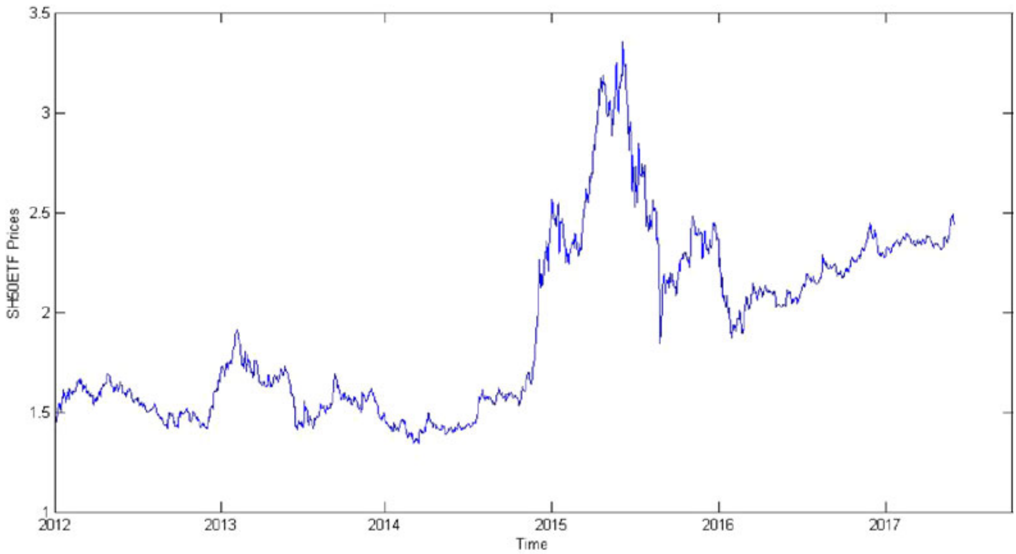

Figure 1. Time series dynamic of SH50ETF.

$$f(y_t|\Omega_t;\theta) = \iota'(P\hat{\xi}_{t-1} \odot \eta_t)$$

with

$$\iota = (1,1)', \quad P = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}, \quad \eta_t = \begin{pmatrix} f(y_t|U_t = 1, \Omega_t;\theta) \\ f(y_t|U_t = 2, \Omega_t;\theta) \end{pmatrix},$$

$$\hat{\xi}_t = \frac{P\hat{\xi}_{t-1} \odot \eta_t}{f(y_t|\Omega_t;\theta)}$$

Here $\odot$ is the Hadamard product. Typically, iterations are started by setting the vector $\hat{\xi}_0$ to the unconditional probabilities. The average duration of each regime can be obtained from the transition probability $p_{jj}(j = 1, 2)$ as follows:

$$d_j = \frac{1}{(1 - p_{jj})}$$

We fit model (2.3) to reflect SH50ETF daily returns. The sample period spans from January 4, 2012 to June 5, 2017, yielding a total of 1314 observations. Data are extracted from the Wind database and analyzed using Matlab software. We set S_t and y_t to represent the daily closing price and the return of SH50ETF at time t, respectively. Therefore the logarithmic return y_t of SH50ETF is calculated as follows:

$$y_t = \ln S_t - \ln S_{t-1}$$

The SH50ETF daily closing prices and logarithmic returns are shown in Figures 1 and 2. The prices of SH50ETF can be found to have significant fluctuations in early 2013 and 2015. In the latter year, the Chinese stock market went from soaring to plunging.

Table 1 presents a simple descriptive analysis and unit root tests for the SH50ETF return series. We find that all the return series are stationary at the 1% significance level.

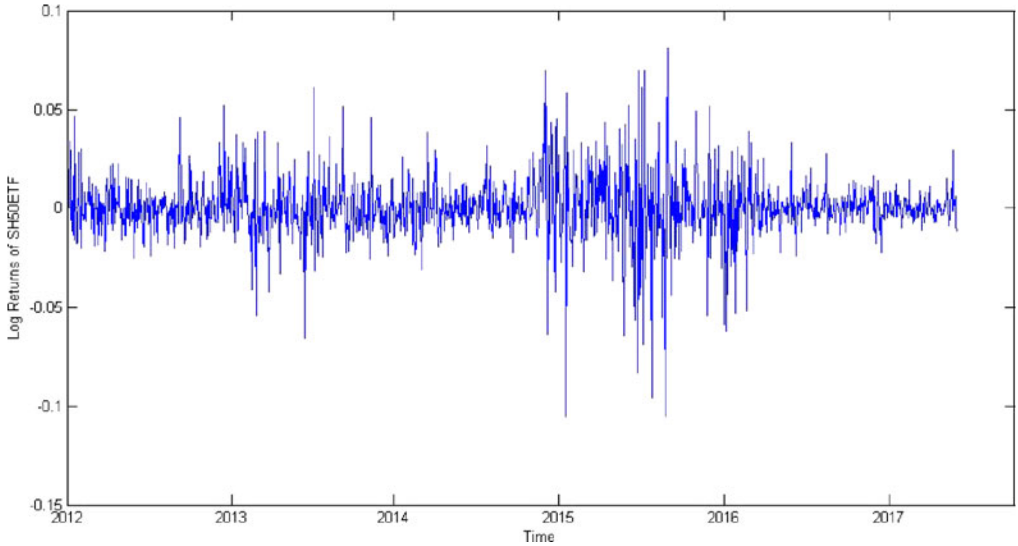

Figure 2. Time series dynamic of SH50ETF return.

Table 1. Descriptive statistics and unit root tests.

Mean	Median	Maximum	Minimum	Std. dev
0.000400	0.0000	0.080879	−0.105190	0.016318
Skewness	*Kurtosis*	*JB*	*ADF*	
−0.490240	10.22539	2908.708***	−35.3585***	

Note: JB is the Jarque-Bera test statistics for normality. ADF is the augmented Dickey-Fuller unit root tests. *** Null hypothesis is rejected at the 1% level. ** Null hypothesis is rejected at the 5% level. * Null hypothesis is rejected at the 10% level.

We compare a Black-Sholes model (BS), a lognormal jump diffusion model (LNJ), a double exponential jump diffusion model (DEJ), a regime-switching diffusion model, and two regime-switching jump-diffusion models (RSJ1 and RSJ2). In RSJ1 model, we assume that jump intensities and jump distribution parameters are not dependent on the states (i.e., $\lambda_1 = \lambda_2, \nu_1 = \nu_2, \varpi_1^2 = \varpi_2^2$). However, in the RSJ2 model, jump intensities and magnitudes are allowed to vary across regimes. The estimated results obtained using the maximum likelihood estimation method are listed in Tables 2 and 3. To study the goodness of fit of the models, we calculate the logarithmic likelihood, AIC and BIC for each model.

In Table 2, it has been shown that jump-diffusion models outperform the Black-Sholes model, while there is little difference between the lognormal jump-diffusion model and the double exponential jump-diffusion model. Estimation results, reported in Table 3, show that for daily data, using the regime-switching model is better than the Black-Sholes model and the jump diffusion models. Indeed, using only a regime-switching or only a jump diffusion model cannot sufficiently capture the return dynamics. According to information criteria used, models that incorporate both regime-switching features and jumps achieve the best fit. The numerical analysis indicates that SH50ETF returns evolve according to two different regimes: a low volatility regime and a high volatility regime. Based on the estimated parameters, high volatility regimes have

Table 2. Daily estimated parameters and standard errors for SH50ETF.

	BS model		LNJ model		DEJ model	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
α	0.000266**	0.0000104	−0.00046	0.0004	0.001403***	0.0004
σ^2	0.0004	0.000451	0.000012***	0.0000	0.000151***	0.0000
λ			1.7499***	0.1720	3.14748***	0.2619
ν			0.001114*	0.0007		
ϖ^2			0.000329***	0.0000		
p					0.23498***	0.2691
η_1					2.2738***	0.9031
η_2					21.6519***	4.4676
$\log L$	3541.03		3663.5286		3664.0119	
AIC	−7078.0682		−7317.0572		−7316.0237	
BIC	−7067.7080		−7291.1568		−7284.9433	

Table 3. Daily estimated parameters and standard errors for SH50ETF.

	RS model		RSJ1 model		RSJ2 model	
	Estimate	s.e.	Estimate	s.e.	Estimate	s.e.
α_1	−0.00063**	0.0003	0.000686	0.0025	0.000219	0.0004
α_2	0.002005**	0.0011	−0.001073**	0.0005	−0.000445	0.0019
σ_1^2	0.000049***	0.0000	0.000901***	0.0001	0.000046***	0.0000
σ_2^2	0.000602***	0.0001	0.000011*	0.0000	0.000052	0.0000
λ			1.736690***	0.4712		
λ_1					0.18567**	0.0780
λ_2					1.30147***	0.5184
ν			0.000665*	0.0004		
ν_1					0.000387	0.0029
ν_2					−0.000724	0.0016
ϖ^2			0.000051***	0.0000		
ϖ_1^2					0.000298*	0.0002
ϖ_2^2					0.000445***	0.0001
p_{11}	0.84***	0.2953	0.76***	0.1360	0.94***	0.2286
p_{22}	0.84**	0.3616	0.95***	0.1261	0.86***	0.2211
d_1	6.25		4.17		16.7	
d_2	6.25		20		7.14	
$\log L$	3792.8497		3828.3652		3830.9209	
AIC	−7573.6994		−7638.7304		−7637.8417	
BIC	−7542.6189		−7592.1098		−7575.6809	

associated larger positive means in the RS and RSJ1 models, but have associated smaller negative mean and jump magnitude mean but larger jump intensities and jump magnitude variance in the RSJ2 model. According to the logarithmic likelihood value, the RSJ2 model has the best goodness-of-fit, slightly higher than the RSJ1 model. However, if we consider the AIC and BIC criterias, the RSJ1 model is always the preferred model. Following a comprehensive comparison of complexity and accuracy of the models, we choose the RSJ1 model to analyze the actual market data of SH50ETF preferentially.

In the RSJ1 model, regime 1 is characterized by a high volatility level, whereas regime 2 exhibits a low volatility level. Table 3 also indicates that the probability of being in regime 1 is lower than the probability of staying in regime 2. The magnitude of these probabilities (p_{11} and p_{22}) suggests that the low volatility regime is more persistent than the high volatility one, such that the SH50ETF stays longer in Regime 2 than in Regime 1. This result is confirmed by the average duration of the dailies for each regime (d_1 and d_2). The high volatility regime lasts 4.17 dailies on average, whereas the average duration of the low volatility regime is 20 dailies.

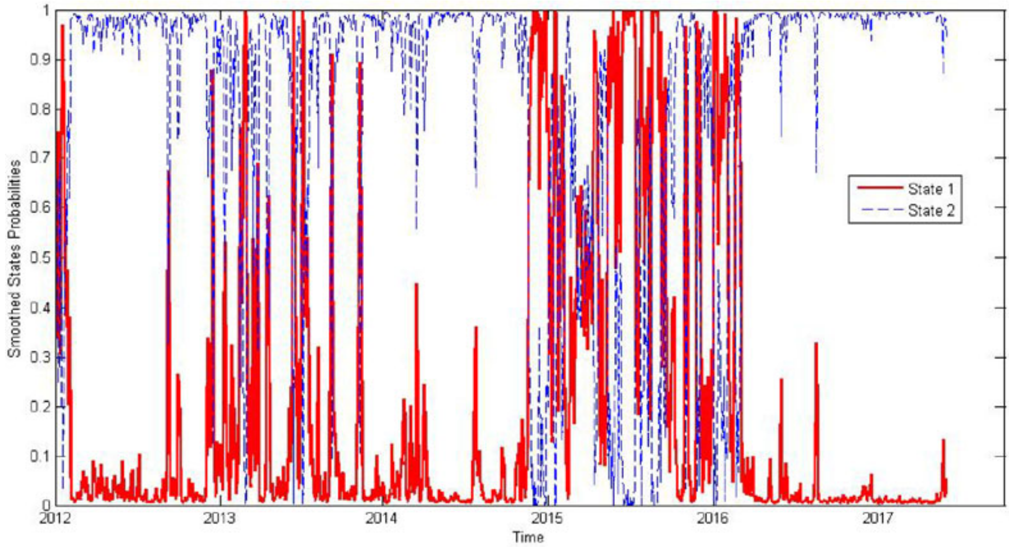

Figure 3. Smoothed states probabilities of SH50ETF return under RSJ1.

Figure 3 shows the smoothed probability of being in regime 1 and regime 2. If the smoothed probability of the state is greater than 0.5, the moment belongs to that state. As shown in Figure 3, the return of the SH50ETF is divided into two states based on the strength of volatility, where the volatility of state 1 is larger and that of state 2 is smaller. The smoothed probability graph allows us to clearly determine the persistence of each state. The smoothed probability of state 2 is relatively wide, indicating that the duration of state 2 is longer. The smoothed probability of staying in regime 1 (high volatility state) shows several high volatility periods which are very consistent with the actual market. The analysis of China's financial markets from January 4, 2012 to June 5, 2017 reveals large fluctuations in China's stock market in January 2012, from January to June 2013, from November 2014 to August 2015 and from January to March 2016. This finding reveals that the mean and volatility of SH50ETF returns are affected by market conditions and undergo a regime switching. Although the parameters of the jump also show the influence of regime switching, the empirical comparison results between RSJ1 and RSJ2 models suggest that the introduction of the regime switching does not significantly improve the fitting accuracy of the model.

The large fluctuations from actual financial market are examined in the following. The people's bank of China announced that from December 5, 2011, it would cut the RMB reserve requirement ratio of the deposit-type financial institutions by 0.5 percentage points which would stimulate the stock market to a certain extent in early 2012. From December 2012 to early February 2013, the Shanghai composite index soared and then decreased. After the "money shortage" in June 2013, the index bounced back but hit a new high, and decreased again. From November 2014 to August 2015, China's stock market experienced madness, overheating, panic and a slump. In 2014, the Shanghai-Hong Kong Stock Connect mechanism, the belt and road policy and the interest-rate cut became catalysts for the explosive growth of blue-chip stocks especially in the fourth quarter, when these policies became officially operational. An unprecedented surge brought China's stock market to stunning heights in June 2015. Since then, the index has tumbled, with the Shanghai index falling to 3847 points in June 30 and plunging more than 1300 points in 12 trading days. By closing time on July 7, more than 90% of the 2774 listed stocks in the Shanghai and Shenzhen stock markets had been suspended or dropped, and the stock market fell by a third in less than a month. The year 2016 began poorly with the circuit breaker mechanism being triggered twice and the CSI 300 index losing more than 7%. Although China's stock market is hot, some basic economic indicators did not grow at the same pace, as GDP growth, corporate profits, per capita disposable income of urban residents, and the like were well below the increase of the stock market. Thus, China's stock price has not been supported by fundamental growth.

3. Option valuation model and empirical analysis for SH50ETF options

3.1. Option pricing under a regime-switching jump-diffusion model

In this section, we present an explicit formula to compute the prices of SH50ETF options under the RSJ1 model which is optimal in terms of complexity and accuracy from the empirical results proposed in Section 2.

Due to the jumps and regime-switching in the option pricing model, the risk-neutral probability measure is not unique. We adopt the hypothesis of Merton (1976) and regard the jump risks as unsystematic risks that should not be priced. Thus, the parameters describing the jumps in the risky asset price processes will not be changed due to the measure transformation from physical measure $\mathcal{P}$ to risk-neutral measure $\mathcal{Q}$. Further, we assume that the regime risk is not priced in the market, hence the rate matrix $\mathbf{A}$ in (2.1) is the same under both the physical measure and the pricing measure. We can obtain the dynamics of the underlying asset value S_t following the regime-switching jump-diffusion process under the risk-neutral measure $\mathcal{Q}$.

$$S_t = S_0 \exp\left\{ \int_0^t \left(r_s - \frac{1}{2}\sigma_s^2 - \kappa\lambda \right) ds + \int_0^t \sigma_s dW_s + \int_0^t Z_s dN_s \right\} \tag{3.1}$$

where r_t denotes the interest rate and $r_t = <r, U_t>, r = (r_1, r_2, ..., r_N) \in \mathbf{R}^N$. For each $i = 1, 2, ..., N, r_i > 0$. The mean percentage jump size of the price is given by

$$\kappa = E(e^{Z_t} - 1)$$

where E denotes an expectation under the measure $\mathcal{P}$.

The payoff function of the call option is determined by

$$C(T) = (S_T - K)^+$$

where K is the strike price of SH50ETF option, and S_T is the price of the underlying asset at time T. Let $C(0, T, K)$ denote the value of the option at initial time. According to the no arbitrage pricing theory, the standard pricing formula is:

$$C(0, T, K) = E^{\mathcal{Q}}[e^{-\int_0^T r_t dt}(S_T - K)^+] \tag{3.2}$$

We obtain analytical pricing formulae via Fourier transform and discretize the pricing formulae via the FFT method introduced by Carr and Madan (1999) and Kwok, Leung, and Wong (2012). Let $X_t = \ln(S_t)$ and $k = \ln(K)$, denoting the logarithm of the SH50ETF price at time t and the strike value, respectively. Based on the work of Carr and Madan (1999), the modified SH50ETF call option price is defined by

$$c(0, T, k) = e^{ak} C(0, T, K) \tag{3.3}$$

where a is a predetermined positive constant such that $c(0, T, k)$ is square integrable in k over the entire real line. Then the Fourier transform of $c(0, T, k)$ is as follows:

$$\psi(0, T, u) = \int_{-\infty}^{\infty} e^{iuk} c(0, T, k) dk. \tag{3.4}$$

Applying the inverse Fourier transform to (3.3), we can derive the following equation:

$$\begin{aligned} C(0, T, K) &= e^{-ak} c(0, T, k) \\ &= e^{-ak} \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-iuk} \psi(0, T, u) du \\ &= e^{-ak} \frac{1}{\pi} \int_{0}^{+\infty} e^{-iuk} \psi(0, T, u) du \end{aligned}$$

The following proposition provides an integral expression for the price of the SH50ETF call option.

Proposition 1. *Under the Markovian regime-switching jump-diffusion model, the price of SH50ETF option is given by the following integral formula*

$$C(0, T, K) = \frac{e^{-ak}}{\pi} \int_{0}^{\infty} e^{-iuk} \psi(0, T, u) du \tag{3.5}$$

where

$$\begin{aligned} \psi(0, T, u) = &\frac{\exp\left\{i(u - i(a_F + 1))(X_0 - \kappa\lambda T) + \lambda T\left(e^{i(u-i(a+1))\nu - \frac{1}{2}(u-i(a+1))^2 \varpi^2} - 1\right)\right\}}{a^2 + a + i(2a + 1)u - u^2} \\ &\times \left\langle U_0 \exp\left\{\int_0^T diag(\mathbf{g}(t, u))dt + \mathbf{A}T\right\}, 1\right\rangle \end{aligned}$$

where $1 = \{1, 1, \ldots, 1\} \in \mathbf{R}^N$.

Define

$$\mathbf{g}(t, u) := \left(g_1(t, u), g_2(t, u), \ldots, g_N(t, u)\right) \in \mathfrak{C}^N$$

where $\mathfrak{C}$ *is the complex space and* $\mathfrak{C}^N$ *is the N-fold product of* $\mathfrak{C}$. *For each* $j = 1, 2, \ldots, N$,

$$g_j(t, u) := -r_j + i(u - i(a + 1))\left(r_j - \frac{1}{2}\sigma_j^2\right) - \frac{1}{2}(u - i(a + 1))^2 \sigma_j^2 \tag{3.6}$$

Proof. Let $f^{\mathcal{Q}}_{X_T|\mathcal{F}^U_T}(x)$ and $\phi^{\mathcal{Q}}_{X_T|\mathcal{F}^U_T}(u)$ denote the conditional distribution function and the conditional characteristic function of X_T given $\mathcal{F}^U_T$ under $\mathcal{Q}$, respectively. Then by direct calculation we have

$$\begin{aligned}
\psi(0,T,u) &= \int_{-\infty}^{+\infty} e^{iuk} c(0,T,k)dk \\
&= \int_{-\infty}^{+\infty} e^{iuk} e^{ak} E^{\mathcal{Q}}[e^{-\int_0^T r_t dt}(S_T - K)^+]dk \\
&= E^{\mathcal{Q}}\left[\int_{-\infty}^{+\infty} e^{iuk} e^{ak} E^{\mathcal{Q}}[e^{-\int_0^T r_t dt}(e^{X_T} - e^k)^+ | \mathcal{F}_T^U]dk\right] \\
&= E^{\mathcal{Q}}\left[\int_{-\infty}^{+\infty} e^{iuk} e^{ak} e^{-\int_0^T r_t dt} E^{\mathcal{Q}}[(e^{X_T} - e^k)^+ | \mathcal{F}_T^U]dk\right] \\
&= E^{\mathcal{Q}}\left[\int_{-\infty}^{+\infty} e^{iuk} e^{ak} e^{-\int_0^T r_t dt} \int_k^{+\infty} (e^x - e^k) f^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(x)dxdk\right] \\
&= E^{\mathcal{Q}}\left[\int_{-\infty}^{+\infty} e^{-\int_0^T r_t dt} \int_{-\infty}^{x} (e^{x+(a+iu)k} - e^{(1+a+iu)k})dk f^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(x)dx\right] \\
&= E^{\mathcal{Q}}\left[\int_{-\infty}^{+\infty} e^{-\int_0^T r_t dt} \left(\frac{e^{(1+a+iu)x}}{a^2 + a + i(2a+1)u - u^2}\right) f^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(x)dx\right] \\
&= \frac{E^{\mathcal{Q}}\left[e^{-\int_0^T r_t dt} \phi^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(u - i(a+1))\right]}{a^2 + a + i(2a+1)u - u^2}
\end{aligned} \tag{3.7}$$

The conditional characteristic function of X_T given $\mathcal{F}_T^U$ under $\mathcal{Q}$ is calculated as follows:

$$\begin{aligned}
&\phi^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(u) \\
&= E^{\mathcal{Q}}[e^{iuX_T} | \mathcal{F}_T^U] \\
&= \exp\left\{ iu\left(X_0 - \kappa\lambda T + \int_0^T \left(r_t - \frac{1}{2}\sigma_t^2\right)dt\right) - \frac{1}{2}u^2 \int_0^T \sigma_t^2 dt + \lambda T\left(e^{iu\nu - \frac{1}{2}u^2\varpi^2} - 1\right)\right\}
\end{aligned}$$

Then we have

$$\begin{aligned}
&E^{\mathcal{Q}}\left[e^{-\int_0^T r_t dt} \phi^{\mathcal{Q}}_{X_T|\mathcal{F}_T^U}(u - i(a+1))\right] \\
&= \exp\left\{ i(u - i(a+1))(X_0 - \kappa\lambda T) + \lambda T\left(e^{i(u-i(a+1))\nu - \frac{1}{2}(u-i(a+1))^2\varpi^2} - 1\right)\right\} \\
&\times E^{\mathcal{Q}}\left[\exp\left(\int_0^T < \mathbf{g}(t,u), U_t > dt\right)\right]
\end{aligned} \tag{3.8}$$

Referring to Lemma 3.3 in Fan et al. (2014), we know that

$$E^{\mathcal{Q}}\left[\exp\left(\int_0^T < \mathbf{g}(t,u), U_t > dt\right)\right] = \left\langle U_0 \exp\left\{\int_0^T diag(\mathbf{g}(t,u))dt + \mathbf{A}T\right\}, 1\right\rangle \tag{3.9}$$

Finally, by (3.8) and (3.9), the conclusion is established.

3.2. Empirical analysis of SH50ETF option

In this section, we provide the empirical analysis of SH50ETF option prices using the explicit formula in Proposition 1. For simplicity, the analysis is limited to a Markov chain with only two states. Similar to other options, the SH50ETF option is also divided into call and put options. For each type of SH50ETF call and put option, contracts can be divided, in accordance with the expiration month of the option contract, into the current month option, next month option and next quarterly month option. For example, on February 3, 2017, the options on the trade (whether the call or put option) may expire in February making them current month options; may expire in March making them next month options; or may expire in June or September making them next quarterly month options. Strike price is another item of the option contract when considering the call or put and delivery month; thus the third division can be made according to the strike price. In general, quarterly month options have more strike prices than non-quarterly month options as the duration of quarterly month options is longer in the market and the strike price increases according to the underlying asset price changes. For example, the option contract that expired in September 2017 has 30 strike prices.

It is known that the strike price of an at-the-money option is extremely close to the market price of the underlying asset. Generally speaking, the trading of at-the-money options is usually the most active and the direction of converting into in-the-money options or out-of-the-money options is difficult to determine. In terms of in-the-money options, the buyer profits, whereas in out-of-the-money options, the seller profits. Thus, at-the-money options are the most speculative type and have the greatest time value. As the actual virtual value degree is different, the implied volatility is also considerably different. The implied volatility of at-the-money options does not change significantly; thus, considering the volatility smile is unnecessary. Therefore, it is of practical importance to choose at-the-money options for analysis.

In this paper, the prices of at-the-money options that expired in September 2017 (the expiry date is September 27, 2017) were analyzed. The sample period spans from April 18 to September 1, 2017, yielding a total of 96 observations. First, using the proposed RSJ1 model, we considered two states to model the underlying asset SH50ETF and then estimated the parameters. The change trend of the data is shown in Figure 4, and the logarithmic return is shown in Figure 5.

Table 4 shows the parametric estimations of SH50ETF index returns from April 18 to September 1, 2017 under the RSJ1 model.

As shown in Figure 6, the return of the SH50ETF is divided into two states based on the strength of volatility. The volatility of state 1 is larger than that of state 2. Using the smoothed probability graph, we determine the persistence and transformation of each state precisely. To examine the option pricing performance, the estimated model is applied to price the at-the-money SH50ETF options expired in September 2017.

For comparative analysis, we first adopted the BS formula to reflect the price of the at-the-money SH50ETF option that expired in September 2017, using risk-free interest rates $r = 0.03$ and the volatility calculated by the latest 60-day closing price data. The prices of the BS formula are compared with the actual market prices shown in Figure 7. It can be seen that the overall fitting situation of the price of the BS formula is not satisfactory, and that the mechanism transformation and jump change cannot be tracked in time.

We adopt the fast Fourier algorithm to perform a numerical analysis of the SH50ETF option prices under the regime-switching jump-diffusion model. The damping

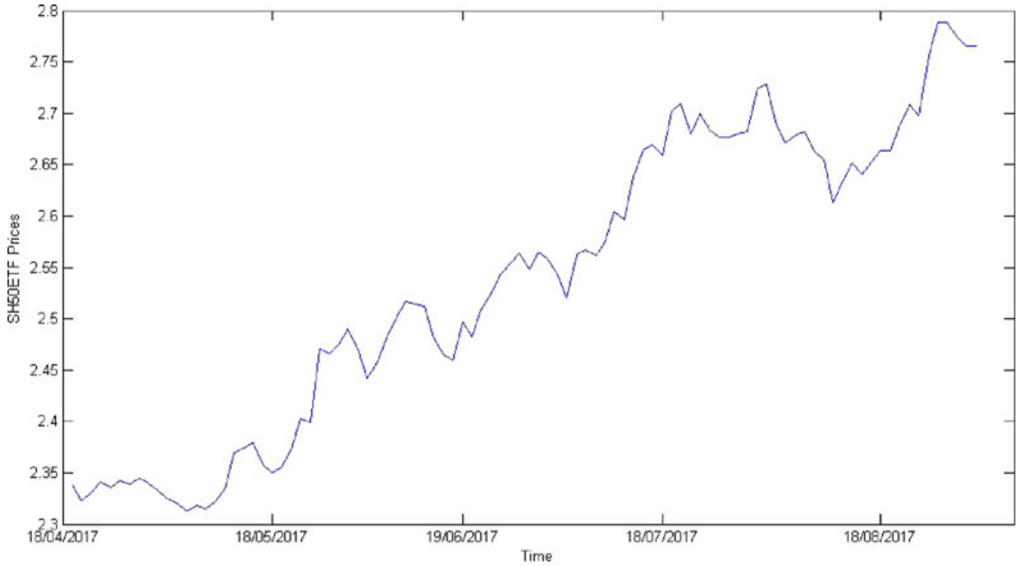

Figure 4. Time series dynamic of SH50ETF.

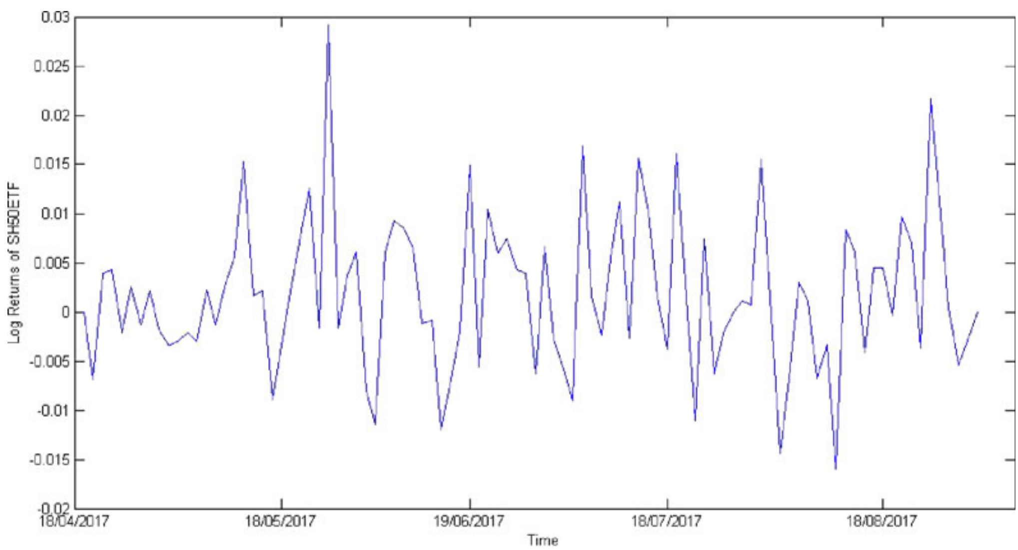

Figure 5. Time series dynamic of SH50ETF return.

coefficient is set at $a = 1.5$ which is consistent with that in Carr and Madan (1999) and Wong and Lo (2009). Without loss of generality, we assume the risk-free interest rate to be $r_1 = 0.04$ and $r_2 = 0.02$. The generator of the Markov chain is obtained through the estimated parameters presented in Table 4 as follows:

$$A = \begin{pmatrix} -0.16 & 0.16 \\ 0.16 & -0.16 \end{pmatrix}$$

The prices of the at-the-money options that expired in September 2017 are calculated in state 1 and state 2. As shown in Figure 8, the option prices in state 1 are higher than

Table 4. Parametric estimations under RSJ1 model.

α_1	α_2	σ_1^2	σ_2^2	λ	ν	ϖ^2
−0.004706	−0.002499	0.00003	0.000001	1.750003	0.003180	0.000015
(0.0039)	(0.0005)	(0.0000)	(0.0000)	(1.0815)	(0.0024)	(0.0000)
p_{11}	p_{22}	$\log L$	AIC	BIC		
0.84	0.84	327.7246	−637.4491	−614.4643		

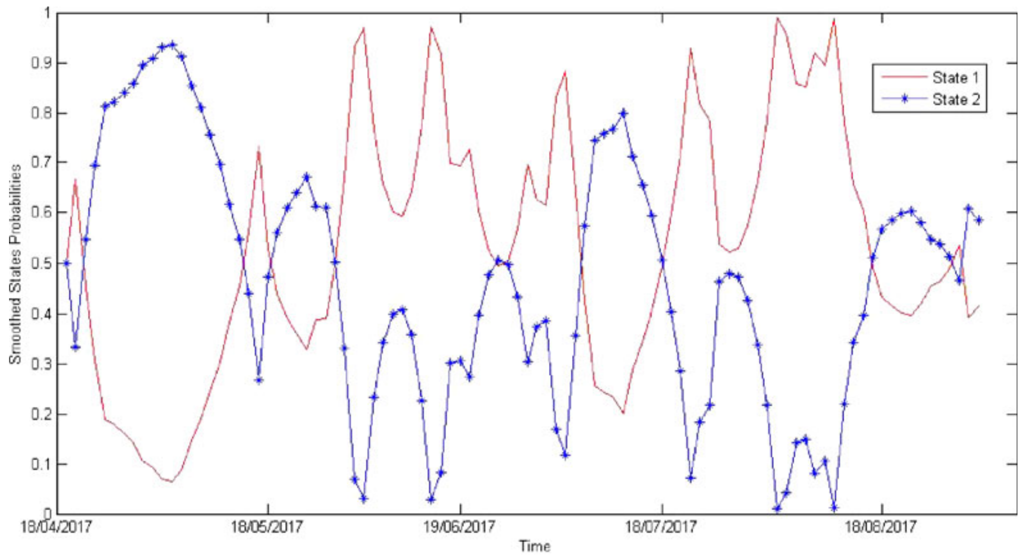

Figure 6. Smoothed states probabilities of SH50ETF return under RSJ1.

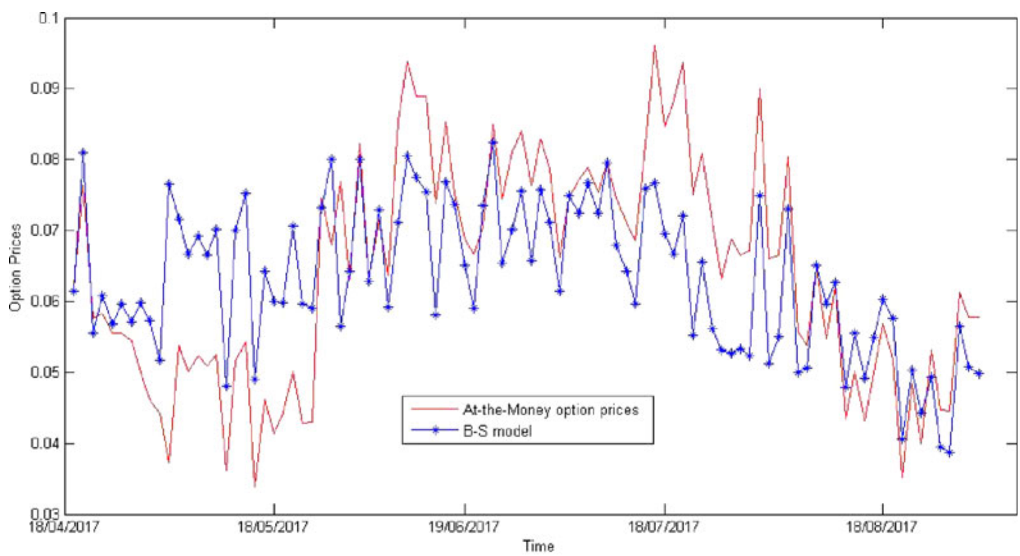

Figure 7. Actual market prices and SH50ETF option prices calculated by BS model.

those in state 2, and the difference of the option prices in the two states decreases as the maturity date approaches. By comparing these figures with the smoothed state probabilities presented in Figure 6, we find that the option prices under the regime-

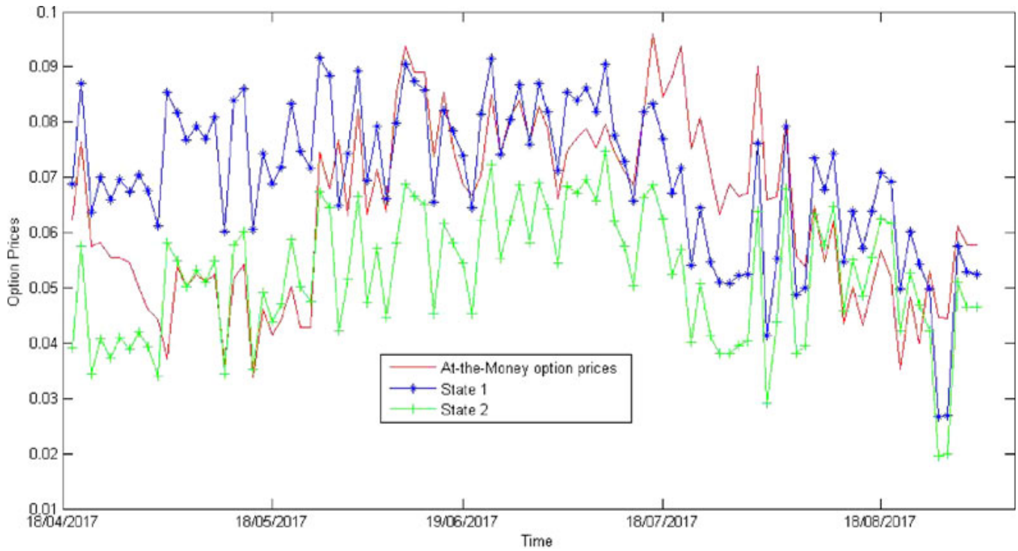

Figure 8. Actual market prices and SH50ETF option prices calculated under regime-switching jump-diffusion model in state 1 and state 2.

Table 5. RMSE and MAPE of BS model and RSJ1 model.

Errors	BS model	RSJ1 model
RMSE	0.0117	0.0096
MAPE	15.99%	12.59%

switching jump-diffusion model can fit the actual market prices better than those in the BS formula as well as track the market characteristics and jumping mechanism following the transformation of the two states.

Furthermore, Table 5 reports the selection criteria of root mean square error (RMSE) and mean absolute percentage error (MAPE) for the fitting of the BS and RSJ1 models. As indicated in Table 5, the RSJ1 model has the lower RMSEs and MAPEs. From the perspective of an investor, using the RSJ1 model for the valuation of SH50ETF options can provide more accurate option prices and simultaneously reduce model risks.

4. Conclusion

This paper investigates the ability of the regime-switching jump-diffusion model to capture the time series properties of SH50ETF and option pricing performance. The results indicate that the regime-switching jump-diffusion model is more effective than competing models in describing the time series of SH50ETF. Moreover, the states transfer situation is in satisfactory agreement with the fluctuation of the actual financial market environment. Thus, establishing the regime-switching jump-diffusion model has practical meaning for the pricing of SH50ETF options. We obtain analytical pricing formulae for SH50ETF call options via Fourier transform and discretize the pricing formulae via the FFT method under the regime-switching jump-diffusion model. The at-the-money option market prices of the SH50ETF that expired in September 2017 have been selected for the empirical analysis and the prices calculated under the regime-switching jump-diffusion model perform better than those of the BS formula. The results provide theoretical basis and technical support for further practical work.

This paper adopts the most commonly used two state regime-switching model, but whether it is appropriate to be divided into several states, as well as the question of the number of states of the regime-switching model must be explored. In addition, although this paper focuses on the comparison of the fitting of different models, no further analysis of the predictive ability of each model was performed due to space limitations. This task should be conducted in future studies.

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