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Ad Hoc Black and Scholes Procedures with the Time-to-Maturity

Suk Joon Byun
College of Business,
Korea Advanced Institute of Science and Technology (KAIST),
85 Hoegi-ro, Dongdaemun-gu, Seoul, 02455, Korea
sjbyun@business.kaist.ac.kr

Sol Kim*
College of Business, Hankuk University of Foreign Studies
107, Imun-ro, Dongdaemun-gu, Seoul, 02450, Korea
solkim@hufs.ac.kr

Dong Woo Rhee
Credit Guarantee and Investment Facility (CGIF)
Asian Development Bank Building, 6 ADB Avenue,
Mandaluyong City, 1550 Metro Manila, Phlippines
drhee@cgif-abmi.org

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There are two *ad hoc* approaches to Black and Scholes model. The "relative smile" approach treats the implied volatility skew as a fixed function of moneyness, whereas the "absolute smile" approach treats it as a function of the strike price. Previous studies reveal that the "absolute smile" approach is superior to the "relative smile" approach as well as to other sophisticated models for pricing options. We find that the time-to-maturity factors improve the pricing and hedging performance of the *ad hoc* procedures and the superiority of the "absolute smile" approach still holds even after the time-to-maturity is considered.

Keywords: Options; relative approach; absolute approach; *ad hoc* Black and Scholes model; volatility smile.

JEL Classifications: G13, G14

*Corresponding author.

1. Introduction

Since Black and Scholes's (1973) seminal paper on option pricing, a number of empirical studies have investigated the Black–Scholes (BS) option pricing model. Because of its rather simple assumptions (e.g., a constant risk-free interest rate and constant volatility), many empirical studies have indicated that the BS model reveals some empirical discrepancies. The volatility "smile" phenomenon, which demonstrates that implied volatility is dependent on the strike price, is a fundamental shortcoming of the BS model. Various models have been developed to overcome this shortcoming, including a jump-diffusion model (Merton, 1976; Naik and Lee, 1990); stochastic volatility models (Hull and White, 1987; Johnson and Shanno, 1987; Scott, 1987; Wiggins, 1987; Melino and Turnbull, 1990; Stein and Stein, 1991; Heston, 1993; Duan, 1995; Heston and Nandi, 2000); and a regime-switching model (Naik, 1993).

Jackwerth and Rubinstein (2001) and Li and Pearson (2007) investigate the performance of a number of option pricing models and find them to be inferior to *ad hoc* BS (AHBS) models, which are frequently used by option traders. Dumas *et al.* (1998) examine the pricing and hedging performance of option valuation models, which consider the deterministic volatility function, and suggest that AHBS models, which just interpolate BS implied volatility across strike prices and time-to-maturity, are superior to deterministic volatility models in terms of pricing performance. By contrast, Kirgiz (2001) and Kim and Kim (2005) argue that AHBS models are not superior to other sophisticated option pricing models.

Kim (2009) examines these inconsistent results and finds that the differences arise from the way in which models treat the strike price as an independent variable. There are two AHBS approaches: "relative smile" and "absolute smile" approaches. The "relative smile" approach treats the implied volatility skew as a fixed function of moneyness (S/K) such that, even if the strike price (K) does not change, implied volatility is floated as the stock index (S) moves. This is known as the "sticky volatility" method. On the other hand, the "absolute smile" approach treats implied volatility as a fixed function of K such that, as long as K does not change, implied volatility is fixed regardless of the level of S. This is known as the "sticky delta" method. Both these models are referred to AHBS models. Dumas *et al.* (1998), Jackwerth and Rubinstein (2001), and Li and Pearson (2007) take the "absolute smile" approach and claim that AHBS models are superior to other models in terms of pricing performance. By contract, Kirgiz (2001) and

Kim and Kim (2005) take the "relative smile" approach and report that AHBS models are not superior to other models. These mixed results imply that the pricing performance of AHBS models depends on the type of AHBS model.

This study compares the pricing performance of the "absolute smile" approach with that of the "relative smile" approach with respect to KOSPI 200 index options. The KOSPI 200 index options market, one of the most liquid derivatives markets in the world, provides a unique opportunity for a statistically sound empirical analysis. AHBS models regard implied volatility as a function of the strike price (or moneyness) and the time-to-maturity. This study is the first to provide a comparison of pricing and hedging performance between different types of AHBS models by considering not only the strike price but also the time-to-maturity. We also examine what constitutes an efficient combination of independent variables. As the number of independent variables increases, pricing and hedging performance generally improves, but at the same time, there may be the over-fitting problem. Following Bakshi *et al.* (1997, 2000), we investigate one-day-ahead and one-week-ahead out-of-sample pricing and hedging performance as well as in-sample pricing performance. The results indicate that considering the time-to-maturity improves the pricing and hedging performance of AHBS models. In addition, the results are consistent with the findings of previous studies in that the "absolute smile" approach is found to be superior to the "relative smile" approach as well as to other models in terms of pricing and hedging performance.

The rest of this study is organized as follows. Section 2 considers AHBS models and the stochastic volatility model. Section 3 describes the KOSPI 200 options market and the data. Section 4 presents the empirical results, including those for in-sample and out-of-sample pricing performance, and Sec. 5 concludes.

2. Model

2.1. *AHBS models*

Despite its empirical deficiencies, the BS model remains popular among practitioners. However, market practitioners typically accept that the volatility parameter can vary according to the strike price and the time-to-maturity. According to Dumas *et al.* (1998), AHBS models, which fit implied volatility to the observed smile pattern, can mitigate some of the bias associated with the constant volatility assumption of the BS model.

We now introduce AHBS models, in which implied volatility is a function of the strike price and the time-to-maturity or a combination of both. Although some studies take a negative view of the time-to-maturity as an independent variable (e.g., Dumas *et al.*, 1998; Kim, 2009), we include it because market practitioners typically allow the volatility parameter to vary across not only strike prices but also option maturity dates. Also, the AHBS models with the time-to-maturity factor are consistent with the theoretical model suggested by Fouque *et al.* (2003). Fouque *et al.* (2003) proposed to use a combination of regular and singular perturbations to analyze parabolic PDEs that arise in the context of pricing options when the volatility is a stochastic process that varies on several characteristic time scales. Under the combination of regular and singular perturbations approach, the first factor driving the volatility is a fast mean reverting diffusion process and the second factor is a slowly varying diffusion process. The second factor dependent approximation for the term structure of implied volatility can be formulated as an affine function of log-moneyness to maturity Ratio, log-moneyness, and time-to-maturity. The implied volatility surface can be expressed in terms of these composite variables.

As mentioned earlier, there are two AHBS approaches. In the "relative smile" approach, implied volatility is treated as a function of S/K, and thus, even if K is fixed, implied volatility is floated as the underlying asset while S moves. In the "absolute smile" approach, implied volatility is treated as a function of K such that implied volatility is not dependent on the level of the underlying asset as long as K remains unchanged. We introduce the following eight AHBS models:

$$\mathrm{AHBS_{A1}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_j, \tag{1}$$

$$\mathrm{AHBS_{A2}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_j, \tag{2}$$

$$\mathrm{AHBS_{A1.C}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_j + \beta_4 \cdot K_i \cdot \tau_j, \tag{3}$$

$$\mathrm{AHBS_{A2.C}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot (K_i)^2 + \beta_4 \cdot \tau_j + \beta_5 \cdot K_i \cdot \tau_j, \tag{4}$$

$$\mathrm{AHBS_{R1}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot \tau_j, \tag{5}$$

$$\mathrm{AHBS_{R2}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2 + \beta_4 \cdot \tau_j, \tag{6}$$

$$\mathrm{AHBS_{R1.C}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot \tau_j + \beta_4 \cdot (S/K_i) \cdot \tau_j, \tag{7}$$

$$\mathrm{AHBS_{R2.C}} : \sigma_{i,j} = \beta_1 + \beta_2 \cdot (S/K_i) + \beta_3 \cdot (S/K_i)^2 + \beta_4 \cdot \tau_j + \beta_5 \cdot (S/K_i) \cdot \tau_j, \tag{8}$$

where $\sigma_{i,j}$ is the implied volatility of the stock price, whose strike price is K_i and time-to-maturity is τ_j.

The first four models are "absolute smile" models, which use the strike price as the independent variable, whereas the remaining four are "relative smile" ones, which use moneyness as the independent variable. $\mathrm{AHBS_{A1}}$ is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; $\mathrm{AHBS_{A2}}$ considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\mathrm{AHBS_{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\mathrm{AHBS_{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\mathrm{AHBS_{R1}}$ considers the intercept, moneyness, and the time-to-maturity as independent variables; $\mathrm{AHBS_{R2}}$ considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\mathrm{AHBS_{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\mathrm{AHBS_{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables. Although more variables can be considered, such as the square of the time-to-maturity (or the strike price/moneyness) multiplied by the time-to-maturity, we do not consider those because such models can only marginally explain the role of additional variables.

To implement our model, we need to follow a four-step procedure. First, BS implied volatility has to be extracted from each option. Second, a regression model whose dependent variable is implied volatility and whose independent variables are the strike price (or moneyness) and the time-to-maturity is set. Third, the volatility "fitted" by the model for each option is calculated by using the parameter estimated from the second step and independent variables. Finally, the model option price is derived from the following BS formula, "whose volatility" is obtained from the third step:

$$C(K,\tau) = S \cdot N(d_1) - Ke^{-r\tau}N(d_2), \tag{9}$$

$$P(K,\tau) = Ke^{-r\tau}N(-d_2) - S \cdot N(-d_1), \tag{10}$$

$$d_1 = \frac{\ln(S/K) + (r+\sigma^2/2)\cdot\tau}{\sigma\sqrt{\tau}}, \quad d_2 = d_1 - \sigma\sqrt{\tau}, \tag{11}$$

where C, P, and r indicate the call option, the put option, and the risk-free rate, respectively, and $N(\cdot)$ is the cumulative standard normal density.

2.2. *Stochastic volatility model*

We use the continuous-time stochastic volatility (SV) model of Heston (1993), which models the square of the volatility process with mean-reverting dynamics, allowing for changes in the underlying asset price to be correlated contemporaneously with changes in the volatility process. We choose this model from various continuous-time stochastic models because it enables a correlation between asset returns and implied volatility and yields a closed-form solution. The actual diffusion processes for the underlying asset and its volatility are specified, respectively, as

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_{S,t}, \tag{12}$$

$$d\nu_t = \kappa(\theta - \nu_t)dt + \sigma_v \sqrt{\nu_t} dW_{\nu,t}, \tag{13}$$

where dW_S and dW_v have an arbitrary correlation ρ; ν_t is the instantaneous variance; κ is the speed of adjustment to the long-run mean θ; and σ_v the variation coefficient of variance.

Given the dynamics in (12) and (13), Heston (1993) shows that the closed-form pricing model of a European call option with τ periods to maturity is given by

$$C = SP_1 - Ke^{-r\tau} P_2, \tag{14}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi \ln[K]} f_j(x, v_t, \tau; \phi)}{i\phi}\right] d\phi \quad (j = 1, 2), \tag{15}$$

where $\mathrm{Re}[\cdot]$ denotes the real part of complex variables; i is the imaginary number; $f_j(x, v_t, \tau; \phi) = \exp[C(\tau; \phi) + D(\tau; \phi)v_t + i\phi x]$; and $C(\tau; \phi)$ and $D(\tau; \phi)$ are functions of θ, κ, ρ, σ, and v_t.

In applying option pricing models, one cannot observe spot volatility and structural parameters. As in standard practice, we estimate the parameters of each model for every sample day. Because closed-form solutions are available for each option price, a natural candidate for the estimation of parameters in the pricing and hedging formula is a nonlinear least squares procedure involving the minimization of the sum of squared errors between the model and market prices. Estimating parameters from asset returns can be an alternative method, but historical data reflect only what happened in the past. Furthermore, the procedure using historical data cannot identify risk premiums, which must then be estimated from options data conditional on the estimates of other parameters. An important advantage of using option prices to estimate parameters is that it allows one to use the forward-looking information contained in option prices.

Let $O_i^*(t, \tau; K)$ denote the model price of option i on day t and $O_i(t, \tau; K)$ denote the market price of option i on day t. To estimate the parameters for each model, we minimize the sum of squared errors between the model and market prices:

$$\min_{\phi_t} \sum_{i=1}^{N} [O_i^*(t, \tau; K) - O_i(t, \tau; K)]^2 \quad (t = 1, \ldots, T), \tag{16}$$

where N denotes the number of options on day t and T denotes the number of days in the sample.

3. Data

In this empirical study, we consider data on KOSPI 200 index options. The KOSPI 200 index options market is the largest derivatives market in the world in terms of the number of contracts. According to the Futures Industry Association, its average daily trading volume in 2007 exceeded 10 million, which is approximately four times the number of Eurodollar futures contracts, the second largest derivative product in the world. In the KOSPI 200 options market, the contract months are three consecutive near-term delivery months and one additional month from the quarterly cycle (March, June, September, and December). In each contract month, expiration is the second Thursday. There are at least five strike prices in each options contract month, and this number increases as the index price moves. The transaction is fully automated, and KOSPI 200 options are European–style options, that is, they can be exercised only on the expiration date.

Because the trading volume of in-the-money (ITM) options is too thin to be reliable, out-of-the-money (OTM) and near-the-money (NTM) options for calls and puts are used. Minute-by-minute transaction prices of KOSPI 200 options, whose sample period is from January 4, 2000, to June 30, 2007, are obtained from the Korea Exchange. The 91-day CD (certificate of deposit) rate is used as the risk-free interest rate. To filter the data needed for the empirical test, we apply the following rules. The last traded price of each options contract before 2:50 p.m. is used. Because options whose maturity is less than seven days may lead to the liquidity bias because of low prices and wide bid-ask spreads, they are excluded from the sample. Because the liquidity of KOSPI 200 options contracts is concentrated in nearest and second nearest expiration contracts, we choose options with these maturity dates. To mitigate the impact of price discreteness, we exclude prices lower than 0.02. In addition, we exclude those prices that do not satisfy the arbitrage restriction.

Table 1 reports the average option price and the number of options by moneyness and option type. Because we investigate not only nearest maturity options but also second nearest maturity options, Table 1 is divided into two categories: short-term options and long-term options. Short-term options include 13,914 call options and 16,017 put options, whereas long-term options include 15,927 call options and 17,767 put options. Even through short-term options are more liquid than long-term ones, the number of long-term options in the sample is greater than that of short-term options because options whose maturity is less than seven days are excluded from the analysis. Deep OTM options (i.e., $S/K < 0.94$ for calls and $S/K > 1.06$ for puts) account for 52% of short-term call options, 57% of long-term call options, 63% of short-term put options, and 70% of long-term put options.

Table 2 shows the "volatility smile" phenomenon for 15 consecutive periods. There are six fixed intervals (based on the degree of moneyness). The numbers in Table 2 indicate the mean of implied volatility for each

Table 1. KOSPI 200 options data.

Call Options			Put Options		
Moneyness	Price	# of Options	Moneyness	Price	# of Options
Panel A: Short-term options					
$S/K < 0.94$	0.3796	7,242	$1.00 < S/K < 1.03$	2.3658	3,403
$0.94 < S/K < 0.96$	1.0112	2,937	$1.03 < S/K < 1.06$	1.2209	2,491
$0.96 < S/K < 1.00$	2.2701	3,735	$S/K > 1.06$	0.3383	10,123
Total	1.0204	13,914	Total	0.9063	16,017
Panel B: Long-term options					
$S/K < 0.94$	1.0290	9,064	$1.00 < S/K < 1.03$	3.8594	2,734
$0.94 < S/K < 0.96$	2.3134	3,284	$1.03 < S/K < 1.06$	2.6319	2,639
$0.96 < S/K < 1.00$	3.8364	3,579	$S/K > 1.06$	0.9235	12,394
Total	1.9247	15,927	Total	1.6290	17,767

Notes: This table reports the average option price and the number of options by moneyness and option type. The sample period is from January 4, 2000, to June 30, 2007. Daily data on final transaction prices (before 2:50 p.m.) for each options contract are used for summary statistics. The moneyness of an option is defined as S/K, where S denotes the spot price and K, the strike price. Short-term options refer to those options whose time-to-maturity is less than 40 days, whereas long-term options refer to those whose time-to-maturity exceeds 40 days.

Table 2. BS implied volatility.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06
Panel A: Short-term options						
2000 01-06	0.4275	0.4078	0.4086	0.4242	0.4158	0.4661
2000 07-12	0.5301	0.4811	0.4890	0.5163	0.5158	0.5280
2001 01-06	0.3883	0.3804	0.3816	0.3949	0.3919	0.4153
2001 07-12	0.3587	0.3240	0.3200	0.3720	0.3568	0.4501
2002 01-06	0.3867	0.3706	0.3652	0.3747	0.3756	0.4263
2002 07-12	0.3688	0.3469	0.3423	0.3904	0.3869	0.4150
2003 01-06	0.3328	0.3107	0.3131	0.3451	0.3575	0.3899
2003 07-12	0.2363	0.2268	0.2310	0.2547	0.2618	0.3194
2004 01-06	0.2652	0.2300	0.2430	0.2711	0.2740	0.3083
2004 07-12	0.2241	0.2152	0.2173	0.2782	0.2847	0.3201
2005 01-06	0.1804	0.1666	0.1679	0.1916	0.2038	0.2491
2005 07-12	0.1963	0.1794	0.1834	0.2200	0.2278	0.2711
2006 01-06	0.2094	0.1924	0.2007	0.2350	0.2457	0.2793
2006 07-12	0.4275	0.4078	0.4086	0.4242	0.4158	0.4661
2007 01-06	0.5301	0.4811	0.4890	0.5163	0.5158	0.5280
Panel B: Long-term options						
2000 01-06	0.4233	0.3942	0.3877	0.4246	0.4103	0.4430
2000 07-12	0.4893	0.4736	0.4767	0.5294	0.5204	0.5067
2001 01-06	0.3644	0.3477	0.3442	0.3675	0.3724	0.3922
2001 07-12	0.3179	0.2867	0.2875	0.3704	0.3514	0.3800
2002 01-06	0.3583	0.3446	0.3401	0.3745	0.3664	0.4015
2002 07-12	0.3430	0.3208	0.3197	0.3905	0.3902	0.3957
2003 01-06	0.3205	0.2963	0.3002	0.3394	0.3432	0.3593
2003 07-12	0.2203	0.2213	0.2270	0.2632	0.2713	0.2896
2004 01-06	0.2487	0.2228	0.2360	0.2613	0.2610	0.2815
2004 07-12	0.2090	0.1968	0.2001	0.2735	0.2770	0.2930
2005 01-06	0.1677	0.1621	0.1673	0.1939	0.2015	0.2280
2005 07-12	0.1783	0.1678	0.1677	0.2234	0.2319	0.2567
2006 01-06	0.1981	0.1892	0.1960	0.2367	0.2411	0.2594
2006 07-12	0.4233	0.3942	0.3877	0.4246	0.4103	0.4430
2007 01-06	0.4893	0.4736	0.4767	0.5294	0.5204	0.5067

Notes: This table reports the implied volatility calculated by inverting the BS model separately for each moneyness category. The implied volatility of individual options is then averaged for each moneyness category and across the time-to-maturity. Moneyness is defined as S/K, where S denotes the spot price and K, the strike price. Short-term options refer to those options whose time-to-maturity is less than 40 days, whereas long-term options refer to those whose time-to-maturity exceeds 40 days.

category. Implied volatility is lowest for NTM options and increases as moneyness moves from NTM options to either ITM or OTM options regardless of the subperiod. Specifically, the implied volatility of OTM put options is generally higher than that of OTM call options, and thus, the implied volatility of KOSPI 200 options reflects a "volatility smirk". The volatility smirk is negatively skewed because a negative correlation between implied volatility and equity market returns is reflected in the KOSPI 200 market. As shown in Table 2, the implied volatility of short-term options is higher than that of long-term options, which is consistent with our intuition in that longer periods can contain mixed information on implied volatility such that the volatility per unit time generally decreases.

4. Empirical Results

This section discusses in-sample, out-of-sample pricing and hedging performance. To measure pricing and hedging performance, we apply the mean absolute percentage error (MAPE) and the mean squared error (MSE):

$$\mathrm{MAPE}_t = \sum_{i=1}^{N_t} \left(\frac{|\varepsilon_t|}{O_{i,t}} \right) \Big/ N_t (t = 1, \ldots, T), \tag{17}$$

$$\mathrm{MSE}_t = \sum_{i=1}^{N_t} \left(\frac{\varepsilon_t}{O_{i,t}} \right)^2 \Big/ N_t (t = 1, \ldots, T), \tag{18}$$

$$\varepsilon_t = O_{i,t} - O^*_{i,t} \quad \text{for the pricing errors,}$$
$$\varepsilon_t = \Delta O - \Delta O^* \quad \text{for the hedging errors,}$$

where O_i denotes the market price of option i; O^*_i denotes the model price of option i; N is the number of options on day t; and T is the number of days in the sample. The MAPE measures the magnitude of pricing and hedging errors, whereas the MSE measures the volatility of errors.

4.1. *In-sample pricing performance*

Panel A of Table 3 shows the mean and standard error of parameter estimates and R^2 for each AHBS model. Each parameter is estimated by using the ordinary least squares (OLS) method for each day. Panel B of Table 3 reports the mean and standard error of parameter estimates for BS and SV models.

Table 4 reports in-sample pricing performance, which is determined by comparing market prices with model prices computed using the parameter estimates for the current day. The results for in-sample pricing performance are consistent with R^2 results in Table 3. As expected, the models with more independent variables provide higher R^2 values and smaller in-sample

Table 3. Parameters.

	Constant	K (or S/K)	K^2 (or $(S/K)^2$)	T	$K \cdot T$ (or $(S/K) \cdot T$)	R^2
Panel A: AHBS models						
AHBS_{A1}	0.6133	−0.0027		−0.1413		0.6504
	(0.1638)	(0.0018)		(0.2649)		(0.2450)
AHBS_{A2}	1.4154	−0.0192	0.0001	−0.1619		0.7458
	(0.9282)	(0.0185)	(0.0001)	(0.2743)		(0.2009)
$\text{AHBS}_{\text{A1.C}}$	0.6699	−0.0031		−0.5649	0.0033	0.6928
	(0.3241)	(0.0036)		(2.3195)	(0.0251)	(0.2231)
$\text{AHBS}_{\text{A2.C}}$	1.4810	−0.0199	0.0001	−0.4924	0.0024	0.7806
	(0.9889)	(0.0192)	(0.0001)	(2.3758)	(0.0248)	(0.1797)
AHBS_{R1}	0.0338	0.2802		−0.1448		0.6749
	(0.2652)	(0.1837)		(0.2663)		(0.2450)
AHBS_{R2}	0.4663	−0.6034	0.4483	−0.1600		0.7426
	(1.0794)	(1.9952)	(0.9642)	(0.2832)		(0.1991)
$\text{AHBS}_{\text{R1.C}}$	−0.0290	0.3413		0.3256	−0.4631	0.7171
	(0.4080)	(0.3341)		(2.4627)	(2.2129)	(0.2229)
$\text{AHBS}_{\text{R2.C}}$	0.4303	−0.5867	0.4650	0.2549	−0.3943	0.7783
	(1.1200)	(2.0185)	(0.9674)	(2.6662)	(2.3629)	(0.1797)

Panel B: Other models					
	σ				
BS	0.2906				
	(0.0023)				
	κ	θ	σ_ν	ρ	ν_t
SV	6.4498	0.6701	1.3270	−0.4727	0.1087
	(0.3440)	(0.0455)	(0.0274)	(0.0074)	(0.0023)

Notes: The table reports the mean and standard error of parameter estimates for each model. The mean and standard deviation of R^2 values for each model are also reported. For AHBS models, each parameter is estimated by using the OLS method for each day. AHBS_{A1} is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; AHBS_{A2} considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\text{AHBS}_{\text{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; AHBS_{R1} considers the intercept, moneyness, and the time-to-maturity as independent variables; AHBS_{R2} considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\text{AHBS}_{\text{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables. BS is the Black-Scholes model, and SV is Heston's model. For the BS and SV models, each parameter is estimated by minimizing the sum of the squared difference between model and market option prices for each day.

Table 4. In-sample pricing errors.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel A: MAPE							
BS	0.4010	0.2658	0.1349	0.1199	0.2153	0.5501	0.3679
SV	0.1940	0.0912	0.0843	0.0780	0.0789	0.2451	0.1690
AHBS_{A1}	0.2230	0.1573	0.1356	0.0839	0.1014	0.1903	0.1717
AHBS_{A2}	0.1460	0.1263	0.1153	0.0876	0.0912	0.1678	0.1382
$\text{AHBS}_{\text{A1.C}}$	0.2079	0.1438	0.1270	0.0810	0.1019	0.1769	0.1606
$\text{AHBS}_{\text{A2.C}}$	0.1376	0.1115	0.1069	0.0848	0.0914	0.1518	0.1277
AHBS_{R1}	0.2028	0.1537	0.1287	0.0855	0.0999	0.1833	0.1629
AHBS_{R2}	0.1545	0.1295	0.1166	0.0871	0.0925	0.1652	0.1400
$\text{AHBS}_{\text{R1.C}}$	0.1900	0.1361	0.1190	0.0819	0.0992	0.1656	0.1502
$\text{AHBS}_{\text{R2.C}}$	0.1453	0.1150	0.1080	0.0835	0.0919	0.1474	0.1285
Panel B: MSE							
BS	0.0822	0.1905	0.2643	0.3048	0.2630	0.1066	0.1584
SV	0.0220	0.0338	0.1513	0.1878	0.0661	0.0224	0.0577
AHBS_{A1}	0.0388	0.0895	0.2524	0.1989	0.0955	0.0238	0.0830
AHBS_{A2}	0.0285	0.0617	0.2126	0.2092	0.0869	0.0223	0.0728
$\text{AHBS}_{\text{A1.C}}$	0.0327	0.0808	0.2416	0.1923	0.0920	0.0226	0.0780
$\text{AHBS}_{\text{A2.C}}$	0.0245	0.0545	0.2018	0.2024	0.0848	0.0208	0.0685
AHBS_{R1}	0.0348	0.0836	0.2342	0.2091	0.1041	0.0237	0.0810
AHBS_{R2}	0.0289	0.0629	0.2095	0.2061	0.0863	0.0225	0.0725
$\text{AHBS}_{\text{R1.C}}$	0.0294	0.0764	0.2235	0.2012	0.1002	0.0227	0.0762
$\text{AHBS}_{\text{R2.C}}$	0.0243	0.0563	0.2002	0.1983	0.0841	0.0210	0.0681

Notes: This table reports in-sample pricing errors for KOSPI 200 options in terms of moneyness. S/K is defined as moneyness, where S denotes the asset price, and K, the strike price. Each model is estimated on a daily basis, and in-sample pricing errors are computed using the parameter estimates for the current day. MAPE denotes the mean absolute percentage error; MSE, the mean squared error; BS, the Black–Scholes model; and SV, Heston's model. AHBS_{A1} is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; AHBS_{A2} considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\text{AHBS}_{\text{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; AHBS_{R1} considers the intercept, moneyness, and the time-to-maturity as independent variables; AHBS_{R2} considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\text{AHBS}_{\text{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables.

pricing errors. Although the SV model has five parameters, it is inferior to most AHBS models in terms of the MAPE. $\text{AHBS}_{\text{A2.C}}$ shows the best performance when the MAPE measure is applied. Among the AHBS models with four independent variables (including a constant), those with the square of moneyness (or the strike price) as an independent variable provide higher R^2 values and smaller in-sample pricing errors than those with moneyness (or the strike price) multiplied by the time-to-maturity as an independent variable. In-sample pricing performance is influenced by moneyness. When the MAPE measure is applied, the in-sample pricing error is smallest for NTM options and increases as moneyness moves from NTM options to either ITM or OTM options. Conversely, when the MSE measure is applied, the in-sample pricing error is largest for NTM options and decreases as moneyness moves from NTM options to either ITM or OTM options.

4.2. *Out-of-sample pricing performance*

In-sample pricing performance is positively related to the number of parameters even when some parameters lead to the over-fitting problem. In this regard, an out-of-sample pricing test is conducted to determine whether a particular model has the over-fitting problem. This test also evaluates the stability of model parameters over time. Further, out-of-sample pricing errors for the following day (week) are analyzed to control for the stability of parameters over subsequent periods. The structural parameters estimated from option prices for the current day are used to price options for the following day (week). Tables 5 and 6 report one-day-ahead and one-week-ahead out-of-sample pricing performance results, respectively.

Most of the AHBS models are superior to both BS and SV models regardless of the type of AHBS. This implies that including the time variable in the AHBS model improves its pricing performance. Previous studies have

Table 5. One-day-ahead out-of-sample pricing errors.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel A: MAPE							
BS	0.4243	0.2789	0.1431	0.1265	0.2171	0.5453	0.3751
SV	0.2458	0.1313	0.1048	0.0983	0.1173	0.2983	0.2124
AHBS_{A1}	0.2705	0.1811	0.1432	0.0956	0.1211	0.2228	0.2013
AHBS_{A2}	0.2299	0.1634	0.1264	0.1024	0.1194	0.2172	0.1858
$\text{AHBS}_{\text{A1.C}}$	0.2634	0.1715	0.1359	0.0940	0.1223	0.2173	0.1957
$\text{AHBS}_{\text{A2.C}}$	0.2308	0.1570	0.1208	0.1006	0.1209	0.2124	0.1830
AHBS_{R1}	0.2603	0.1832	0.1407	0.1011	0.1273	0.2286	0.2017
AHBS_{R2}	0.2255	0.1629	0.1291	0.1026	0.1208	0.2228	0.1870
$\text{AHBS}_{\text{R1.C}}$	0.2553	0.1701	0.1327	0.0995	0.1285	0.2180	0.1944
$\text{AHBS}_{\text{R2.C}}$	0.2219	0.1535	0.1230	0.1008	0.1236	0.2140	0.1815

Table 5. (*Continued*)

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel B: MSE							
BS	0.0952	0.2183	0.3084	0.3522	0.2909	0.1105	0.1777
SV	0.0474	0.0736	0.2245	0.2691	0.1183	0.0457	0.0968
AHBS_{A1}	0.0584	0.1179	0.2924	0.2481	0.1310	0.0368	0.1076
AHBS_{A2}	0.0519	0.0952	0.2603	0.2742	0.1333	0.0434	0.1051
$\text{AHBS}_{\text{A1.C}}$	0.1574	0.1082	0.2898	0.2482	0.1335	0.0425	0.1340
$\text{AHBS}_{\text{A2.C}}$	0.1682	0.0948	0.2666	0.2713	0.1481	0.0577	0.1416
AHBS_{R1}	0.0555	0.1160	0.2865	0.2678	0.1452	0.0403	0.1103
AHBS_{R2}	0.0520	0.0945	0.2624	0.2732	0.1314	0.0452	0.1057
$\text{AHBS}_{\text{R1.C}}$	0.1212	0.1084	0.2824	0.2702	0.1593	0.0488	0.1303
$\text{AHBS}_{\text{R2.C}}$	0.1384	0.0898	0.2624	0.2750	0.1561	0.0626	0.1357

Notes: This table reports one-day-ahead out-of-sample pricing errors for KOSPI 200 options in terms of moneyness. S/K is defined as moneyness, where S denotes the asset price, and K, the strike price. Each model is estimated on a daily basis, and one-day-ahead out-of-sample pricing errors are computed using the parameter estimates for the previous trading day. MAPE denotes the mean absolute percentage error; MSE, the mean squared error; BS, the Black–Scholes model; and SV, Heston's model. AHBS_{A1} is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; AHBS_{A2} considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\text{AHBS}_{\text{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; AHBS_{R1} considers the intercept, moneyness, and the time-to-maturity as independent variables; AHBS_{R2} considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\text{AHBS}_{\text{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables.

shown that some AHBS models outperform BS or SV models, whereas others do not.

In terms of one-day-ahead out-of-sample pricing performance, $\text{AHBS}_{\text{R2.C}}$ shows the best performance when the MAPE measure is applied. This is inconsistent with previous studies in that, all other factors being equal, the AHBS model with moneyness as an independent variable performs better than that with the strike price as an independent variable. In addition, this result is inconsistent with the findings of Kim (2009), who suggests that AHBS models with fewer parameters provide better out-of-sample pricing performance, in that $\text{AHBS}_{\text{R2.C}}$ has the highest number of independent variables (five). This provides evidence that an increase in the number of

Table 6. One-week-ahead out-of-sample pricing errors.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel A: MAPE							
BS	0.4918	0.3195	0.1685	0.1404	0.2269	0.5384	0.3990
SV	0.3334	0.1927	0.1401	0.1250	0.1679	0.4038	0.2889
AHBS_{A1}	0.3593	0.2270	0.1599	0.1168	0.1552	0.3031	0.2637
AHBS_{A2}	0.3392	0.2120	0.1454	0.1207	0.1544	0.3128	0.2592
$\text{AHBS}_{\text{A1.C}}$	0.3605	0.2269	0.1565	0.1175	0.1583	0.3092	0.2661
$\text{AHBS}_{\text{A2.C}}$	0.3493	0.2136	0.1424	0.1212	0.1566	0.3208	0.2646
AHBS_{R1}	0.3676	0.2436	0.1693	0.1257	0.1690	0.3303	0.2802
AHBS_{R2}	0.3424	0.2268	0.1599	0.1261	0.1647	0.3533	0.2788
$\text{AHBS}_{\text{R1.C}}$	0.3678	0.2382	0.1643	0.1255	0.1704	0.3354	0.2810
$\text{AHBS}_{\text{R2.C}}$	0.3415	0.2225	0.1561	0.1257	0.1664	0.3665	0.2825
Panel B: MSE							
BS	0.1237	0.2815	0.3929	0.4118	0.3308	0.1180	0.2125
SV	0.0770	0.1331	0.3130	0.3511	0.1849	0.0756	0.1442
AHBS_{A1}	0.0915	0.1702	0.3532	0.3220	0.1868	0.0784	0.1545
AHBS_{A2}	0.0851	0.1501	0.3139	0.3363	0.1832	0.0874	0.1507
$\text{AHBS}_{\text{A1.C}}$	0.1180	0.3532	0.3582	0.3381	0.2041	0.1330	0.2021
$\text{AHBS}_{\text{A2.C}}$	0.0964	0.3672	0.3296	0.3501	0.2011	0.1386	0.1975
AHBS_{R1}	0.0926	0.1789	0.3797	0.3443	0.2071	0.0661	0.1582
AHBS_{R2}	0.0885	0.1601	0.3533	0.3518	0.1985	0.1138	0.1691
$\text{AHBS}_{\text{R1.C}}$	0.0989	0.3250	0.3784	0.3562	0.2183	0.1423	0.2029
$\text{AHBS}_{\text{R2.C}}$	0.0959	0.3540	0.3646	0.3609	0.2107	0.1706	0.2133

Notes: This table reports one-week-ahead out-of-sample pricing errors for KOSPI 200 options in terms of moneyness. S/K is defined as moneyness, where S denotes the asset price, and K, the strike price. Each model is estimated on a daily basis, and one-week-ahead out-of-sample pricing errors are computed using the parameter estimates for the previous week. MAPE denotes the mean absolute percentage error; MSE, the mean squared error; BS, the Black–Scholes model; and SV, Heston's model. AHBS_{A1} is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; AHBS_{A2} considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\text{AHBS}_{\text{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; AHBS_{R1} considers the intercept, moneyness, and the time-to-maturity as independent variables; AHBS_{R2} considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\text{AHBS}_{\text{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables.

parameters can improve the structural fit. On the other hand, the effect of moneyness on out-of-sample pricing performance is similar to its effect on in-sample pricing performance. When the MAPE measure is applied, the out-of-sample pricing error is smallest for NTM options and increases as moneyness moves from NTM options to either ITM or OTM options. By contrast, when the MSE measure is applied, the out-of-sample pricing error is largest for NTM options and decreases as moneyness moves from NTM options to either ITM or OTM options.

In terms of one-week-ahead out-of-sample pricing performance, $\mathrm{AHBS_{A2}}$ shows the best performance when both the MAPE and MSE measures are applied. Because the interval between the date when parameters are estimated and the date when parameter estimates are used to price options is increased, $\mathrm{AHBS_{A2}}$ performs better than the other AHBS models. This result is partially consistent with Jackwerth and Rubinstein (2001) and Li and Pearson (2007) in that the "absolute smile" model performs better than the "relative smile" model. However, it is inconsistent with Kim (2009), who suggests that the "absolute smile" model with the lowest number of independent variables shows the best performance. As shown in Table 6, most of the "absolute smile" models perform better than not only "relative smile" models but also the BS or SV models. This result is noteworthy in that including the time variable in the AHBS model does not change pricing performance rankings, which is inconsistent with the findings of previous studies.

Finally, the difference in the pricing error between in-sample and out-of-sample pricing performances does not exceed that in Kim (2009), who does not include the time variable in the AHBS model. The average difference in the pricing error between in-sample and out-of-sample pricing performances in Kim (2009) is approximately 0.4823, whereas that in the present study is only 0.1245. This implies that including the time variable in the AHBS model reduces the over-fitting problem. Thus, the results do not provide support for Dumas *et al.* (1998) argument that the time variable is an important cause of the over-fitting problem.

4.3. *Hedging performance*

Hedging performance is an important tool for gauging the forecasting power of the volatility of the underlying assets. In practice, option traders usually focus on the risk due to the underlying asset price volatility alone, and carry out a delta-neutral hedge, employing only the underlying asset as the

Table 7. One-day-ahead hedging errors.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel A: MAPE							
BS	0.2831	0.1225	0.0578	0.0612	0.0876	0.1772	0.1487
SV	0.1948	0.1086	0.0679	0.0737	0.0993	0.1446	0.1206
$\mathrm{AHBS}_{\mathrm{A1}}$	0.1929	0.1001	0.0571	0.0596	0.0807	0.1362	0.1129
$\mathrm{AHBS}_{\mathrm{A2}}$	0.2002	0.0991	0.0567	0.0596	0.0796	0.1354	0.1140
$\mathrm{AHBS}_{\mathrm{A1.C}}$	0.1881	0.0938	0.0567	0.0596	0.0809	0.1377	0.1118
$\mathrm{AHBS}_{\mathrm{A2.C}}$	0.1943	0.0919	0.0563	0.0596	0.0797	0.1367	0.1125
$\mathrm{AHBS}_{\mathrm{R1}}$	0.2292	0.1272	0.0661	0.0697	0.0952	0.1451	0.1290
$\mathrm{AHBS}_{\mathrm{R2}}$	0.2135	0.1226	0.0657	0.0706	0.0976	0.1491	0.1268
$\mathrm{AHBS}_{\mathrm{R1.C}}$	0.2210	0.1184	0.0654	0.0696	0.0947	0.1448	0.1263
$\mathrm{AHBS}_{\mathrm{R2.C}}$	0.2067	0.1147	0.0654	0.0707	0.0972	0.1484	0.1243
Panel B: MSE							
BS	0.1190	0.0953	0.1462	0.2352	0.1362	0.0605	0.0989
SV	0.1399	0.1508	0.2063	0.3367	0.2025	0.0807	0.1347
$\mathrm{AHBS}_{\mathrm{A1}}$	0.0980	0.0922	0.1472	0.2239	0.1215	0.0443	0.0867
$\mathrm{AHBS}_{\mathrm{A2}}$	0.0917	0.0903	0.1465	0.2242	0.1212	0.0426	0.0846
$\mathrm{AHBS}_{\mathrm{A1.C}}$	0.0982	0.0923	0.1477	0.2242	0.1221	0.0445	0.0869
$\mathrm{AHBS}_{\mathrm{A2.C}}$	0.0920	0.0905	0.1471	0.2243	0.1213	0.0427	0.0847
$\mathrm{AHBS}_{\mathrm{R1}}$	0.1598	0.1431	0.1875	0.3078	0.1858	0.0777	0.1317
$\mathrm{AHBS}_{\mathrm{R2}}$	0.1454	0.1439	0.1933	0.3275	0.2079	0.1098	0.1433
$\mathrm{AHBS}_{\mathrm{R1.C}}$	0.1572	0.1392	0.1827	0.3014	0.1791	0.0747	0.1283
$\mathrm{AHBS}_{\mathrm{R2.C}}$	0.1430	0.1418	0.1914	0.3233	0.2035	0.1060	0.1404

Notes: This table reports one-day-ahead out-of-sample pricing errors for KOSPI 200 options in terms of moneyness. S/K is defined as moneyness, where S denotes the asset price, and K, the strike price. For each option, its hedging error is the difference between the change in the reported market price and the change in the model's theoretical price from day t until day $t+1$. MAPE denotes the mean absolute percentage error; MSE, the mean squared error; BS, the Black–Scholes model; and SV, Heston's model. $\mathrm{AHBS}_{\mathrm{A1}}$ is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{A2}}$ considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{R1}}$ considers the intercept, moneyness, and the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{R2}}$ considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\mathrm{AHBS}_{\mathrm{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\mathrm{AHBS}_{\mathrm{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables.

Table 8. One-week-ahead hedging errors.

S/K	< 0.94	0.94–0.96	0.96–1.00	1.00–1.03	1.03–1.06	> 1.06	Total
Panel A: MAPE							
BS	0.7520	0.2425	0.0967	0.1058	0.1722	0.5547	0.3556
SV	0.4476	0.2054	0.1191	0.1290	0.1974	0.3484	0.2371
AHBS_{A1}	0.4644	0.1931	0.0958	0.1021	0.1609	0.3274	0.2271
AHBS_{A2}	0.4409	0.1880	0.0952	0.1024	0.1568	0.3134	0.2176
$\text{AHBS}_{\text{A1.C}}$	0.4538	0.1829	0.0960	0.1019	0.1597	0.3278	0.2243
$\text{AHBS}_{\text{A2.C}}$	0.4244	0.1753	0.0952	0.1020	0.1555	0.3141	0.2136
AHBS_{R1}	0.5238	0.2455	0.1180	0.1227	0.1917	0.3339	0.2493
AHBS_{R2}	0.4788	0.2325	0.1168	0.1248	0.1950	0.3425	0.2427
$\text{AHBS}_{\text{R1.C}}$	0.5059	0.2293	0.1156	0.1223	0.1888	0.3342	0.2444
$\text{AHBS}_{\text{R2.C}}$	0.4631	0.2181	0.1150	0.1244	0.1922	0.3418	0.2381
Panel B: MSE							
BS	0.3700	0.2236	0.3226	0.4968	0.3042	0.1809	0.2177
SV	0.3740	0.3420	0.5014	0.7883	0.4683	0.2357	0.2868
AHBS_{A1}	0.2719	0.2207	0.3301	0.4419	0.2627	0.1258	0.1768
AHBS_{A2}	0.2516	0.2153	0.3271	0.4445	0.2653	0.1206	0.1711
$\text{AHBS}_{\text{A1.C}}$	0.2747	0.2254	0.3377	0.4427	0.2627	0.1273	0.1788
$\text{AHBS}_{\text{A2.C}}$	0.2577	0.2203	0.3341	0.4436	0.2641	0.1211	0.1732
AHBS_{R1}	0.4628	0.3594	0.4699	0.7182	0.4433	0.2249	0.2933
AHBS_{R2}	0.3967	0.3413	0.4757	0.7780	0.4743	0.3596	0.3266
$\text{AHBS}_{\text{R1.C}}$	0.4563	0.3453	0.4509	0.6968	0.4146	0.2155	0.2834
$\text{AHBS}_{\text{R2.C}}$	0.3939	0.3358	0.4653	0.7633	0.4495	0.3472	0.3186

Notes: This table reports one-week-ahead hedging errors for KOSPI 200 options in terms of moneyness. S/K is defined as moneyness, where S denotes the asset price, and K, the strike price. For each option, its hedging error is the difference between the change in the reported market price and the change in the model's theoretical price from day t until day $t+7$. MAPE denotes the mean absolute percentage error; MSE, the mean squared error; BS, the Black–Scholes model; and SV, Heston's model. AHBS_{A1} is an AHBS model that considers the intercept, the strike price, and the time-to-maturity as independent variables; AHBS_{A2} considers the intercept, the strike price, the square of the strike price, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{A1.C}}$ considers the intercept, the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; $\text{AHBS}_{\text{A2.C}}$ considers the intercept, the strike price, the square of the strike price, the time-to-maturity, and the strike price multiplied by the time-to-maturity as independent variables; AHBS_{R1} considers the intercept, moneyness, and the time-to-maturity as independent variables; AHBS_{R2} considers the intercept, moneyness, the square of moneyness, and the time-to-maturity as independent variables; $\text{AHBS}_{\text{R1.C}}$ considers the intercept, moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables; and $\text{AHBS}_{\text{R2.C}}$ considers the intercept, moneyness, the square of moneyness, the time-to-maturity, and moneyness multiplied by the time-to-maturity as independent variables.

hedging instrument. While this seems plausible for the BS and AHBS-type models, the SV model lead to incomplete markets. It is well known that a simple delta hedging strategy is sub-optimal in this setting.

Because there are several risk factors in the proposed SV model, the need for a perfect hedge may arise in situations where not only is the underlying price risk present, but also is volatility risk present. To implement this hedging practice, we should recognize that a perfect hedge is not practically feasible in the presence of stochastic volatility. So, in line with the measure of hedging performances in Dumas *et al.* (1998) and Gemmill and Saflekos (2000), we define the hedge portfolio error as follows.

$$\varepsilon_t = \Delta O - \Delta O^*, \tag{19}$$

where ΔO is the change in the reported market price from day t until day $t+1$ or $t+7$ and ΔO^* is the change in the model's theoretical price.

Tables 7 and 8 present one day and one week hedging errors over alternative moneyness categories, respectively. First, the AHBS_{A2} model has the best hedging performance for one day and one week ahead MSE errors. The $\text{AHBS}_{\text{A1.C}}$ and $\text{AHBS}_{\text{A2.C}}$ models show better hedging performance than other models for one day and one week ahead MAPE errors. The "absolute smile" approaches show better performance than "relative smile" approaches, the BS and SV models. For MSE errors, even the performances of the SV model are less than that of the BS model. As the term of hedging becomes longer, the difference between the worst model and the best model becomes smaller. As a result, these results are also consistent with those from pricing performance. The superiority of the model for pricing options remains unchanged in hedging performance. Similar to the results of the pricing performance, the "absolute smile" approaches can also decrease the hedging errors, but is not drastic.

5. Conclusion

Although the "absolute smile" approach is known to be superior to the "relative smile" approach, this study is the first to provide an empirical comparison of pricing and hedging performance between these two approaches by taking the time-to-maturity into account. The results for in-sample pricing performance are partially consistent with the findings of previous studies demonstrating the superiority of the "absolute smile" approach in that one "absolute smile" model shows better in-sample pricing performance than all the other models. The results for one-day-ahead out-of-sample pricing performance do not provide support for the superiority of the "absolute smile" approach because the "relative smile" approach shows the smallest pricing error. However, the results for one-week-ahead out-of-sample pricing performance are largely consistent with the findings of previous studies. Most of the "absolute smile" models show better one-week-ahead out-of-sample pricing performance than not only "relative smile" models but also the BS and SV models. Irrespective of the forecasting term, for hedging performance, the "absolute smile" approaches show better performance than "relative smile" approaches, the BS and SV models. Overall, the time-to-maturity factors improve the pricing performance of the *ad hoc* Black and Scholes procedures and the superiority of the "absolute smile" approach in terms of pricing performance still holds even after the time variable is taken into account in AHBS modeling.

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