

George Washington University
Department of Computer Science

Csci 6212 - Homework 3

Given: February 9, 2017

Due: 7PM, February 14, 2017 (submission through BB)

1. Given positive integers B, n and M as inputs, design an algorithm to compute the value of $(B^n \bmod M)$ in $O(\log n)$ time in the worst case.
Note that $B^{pq} \bmod M = ((B^p \bmod M) \times (B^q \bmod M)) \bmod M$.
2. Suppose an array A consists of n elements, each of which is red, white, or blue. We seek to rearrange the elements so that all the reds come before all the whites, which come before all the blues. Your algorithm should run in $O(n)$ time in the worst case using $O(1)$ space.
3. Given a list of n element A , outline an $O(n)$ time in-space algorithm by modifying *Partition* algorithm discussed in class such that upon completion of the algorithm, there exists two index variables i and j ($1 \leq i < j \leq n$) such that $A(i+1) = A(i+2) = \dots = A(j)$; $A(k) < A(i+1)$ for any k , $1 \leq k \leq i$; and $A(k) > A(i+1)$ for any k , $j+1 \leq k \leq n$
4. We are given an array A with n elements and a number p . Assume that the sum of the elements in A is larger than p . We would like to compute a smallest subset A' of A such that the sum of the elements in A' is at least p . (For example, if $A = \{8, 3, 9, 2, 7, 1, 5\}$ and $p = 18$, then the answer is $A' = \{8, 9, 1\}$.) Give an $O(n)$ time algorithm for this problem. (HINT: use the linear-time SELECT algorithm)
5. Let $X = \{x_1, x_2, \dots, x_n\}$ be a sequence of arbitrary numbers (positive or negative). Give an $O(n)$ time algorithm to find the subsequence of consecutive elements x_i, x_{i+1}, \dots, x_j whose sum is maximum over all consecutive subsequences. For example, for $X = \{2, 5, -10, 3, 12, -2, 10, -7, 5\}$, $\{3, 12, -2, 10\}$ is a solution
6. Consider the $O(n)$ time Select algorithm discussed in class and modify it in such a way that the group size is 9 instead of 5. Prove that the time complexity of this algorithm is also $O(n)$.