George Washington University
Department of Computer Science

Csci 6212 - Homework 1

Given: January 24, 2017
Due: 5PM, January 30, 2017 (submission through BB)

1. Asymptotic notations.

(a) Solve the recurrence: $T(n) = 1/n + T(n-1)$, where $T(0) = 0$.

(b) Prove or disprove:

(b1) $2^{n+1} = O(2^n)$

(b2) $2^{2n} = O(2^n)$

(c) Let P be a problem. The worst-case time complexity of P is $O(n^2)$. The worst-case time complexity of P is also $\Omega(n \log n)$. Let A be an algorithm that solves P. Which statements of the following statements are consistent with this information about the complexity of P?

(c1) A has worst-case time complexity $O(n^2)$.

(c2) A has worst-case time complexity $O(n^{3/2})$.

(c3) A has worst-case time complexity $O(n)$.

(c4) A has worst-case time complexity $\theta(n^2)$.

(c5) A has worst-case time complexity $\theta(n^3)$.

2. Let $f(n) = O(s(n))$ and $g(n) = O(t(n))$. Prove or disprove the following.

(a) $f(n) + g(n) = O(s(n) + t(n))$.

(b) $s(n) * t(n) = \Omega(f(n) * g(n))$.

(c) $f(n) - g(n) = O(s(n) - t(n))$.

(d) $s(n)/t(n) = \Omega(f(n)/g(n))$.

3. Let A and B be sets such that each has n positive integers in a non-decreasing order. We want to compute the set C such that $a \in C$ if and only if a appears either (i) in both A and B, or (ii) more than once in A or B. For example, if $A = \{1, 3, 5, 3, 7\}$ and $B == \{5, 2, 8, 4, 6\}$, then $C = \{3, 5\}$. Give an $O(n)$ time algorithm for the problem.