George Washington University
Department of Computer Science

Csci 6212 - Homework 4

Given: February 14, 2017
Due: 6PM, February 21, 2017 (submission through BB)

1. Consider the following problem for n jobs, each one of which takes exactly one minute to complete. At any time $T = 1, 2, 3, \cdots$, we can execute exactly one job. Each job i earns a profit of p_i dollars if and only if it is executed no later than time d_i, where d_i is given as an input. Assume that d_i is an integer value. The problem is to schedule the jobs to maximize the profit.

   Consider the following greedy strategy: For all the jobs with deadline at 1 minute, schedule the job with the maximum profit. Next, for all the jobs with deadline at 2 minutes or less, pick the job with maximum profit from the remaining unscheduled jobs. And so on. For example, consider $n = 4$, profits $P = (50, 10, 15, 300)$ and deadlines $D = (2, 1, 2, 1)$. The greedy strategy will yield the following solution: job 4 and job 1 to be scheduled for a total of profit of 80 dollars.

   Give a counter example to establish that this greedy strategy does not always work.

2. Consider the "coin change problem" we discussed in class: Consider a currency system with coins worth $a_1, a_2, \cdots, a_k$ cents where $a_1 = 1$. Assume that you are given an unlimited numbers of coins of each type. The input to the problem is an integer M and the objective is to determine the number of coins of each type to make up M cents using the minimum number of coins. Consider a greedy algorithm that takes as many coins as possible from the highest denomination, and repeat this with the next highest one, etc.

   Prove that this greedy algorithm correctly solve the coin change problem for the case when $a_1 = 1$, $a_2 = 5$, $a_3 = 10$, $a_4 = 25$, and $a_5 = 50$.

3. A k-coloring og a graph $G = (V, E)$ is a mapping $f : V \to \{1, 2, \cdots, k\}$ such that adjacent vertices are mapped out different colors, i.e., no two neighbors in G receive the same color (i.e., same integer).

For $i = 1, \cdots, n$ do
    Color vertex v_i using the smallest available color in $\{1, 2, \cdots, \Delta(G) + 1\}$.

(a) Prove that the following GREEDY algorithm colors the given graph G with at most $\Delta(G) + 1$ colors where $\Delta(G)$ denote the maximum degree (number of adjacent vertices) of any node $v \in V$.

(b) Show an example of a graph G that requires $\Delta(G)$ colors.

4. Consider the knapsack problem discussed in class. Now we have one additional constraint that $x_\in\{0, 1\}$, i.e., you are not allowed to put a fraction of any object in the knapsack. This problem is called the 0-1 knapsack problem.

(a) Show that the greedy algorithm that considers ratio of p_i/w_i is not an optimal algorithm for this case.

(b) Suppose that the order of the items when sorted by increasing weight is the same as their order when sorted by decreasing profit. The greedy algorithm then finds an optimal solution. Prove the optimality.

5. A source node of a data communication network has n communication lines connected to its destination node. Each line i has a transmission rate r_i representing the number of bits that can be transmitted per second. A data needs to be transmitted with transmission rate at least M bits per second from the source node to its destination node. If a fraction x_i ($0 \leq x_i \leq 1$) of line i is used (for example, a fraction x_i of the full bandwidths of line i is used), the transmission rate through line i becomes $x_i \cdot r_i$ and a cost $c_i \cdot x_i$ is incurred. Assume that the cost function c_i ($1 \leq i \leq n$) is given. The objective of the problem is to compute x_i, for $1 \leq i \leq n$, such that $\sum_{1\leq i\leq n} r_i x_i \geq M$ and $\sum_{1\leq i\leq n} c_i x_i$ is minimized.

(a) Describe an outline of a greedy algorithm to solve the problem.

(b) Prove that your algorithm in part (a) always produces an optimum solution. You should give all the details of your proof.