George Washington University
Department of Computer Science

Csci 6212 - Homework 5

Given: February 28, 2017
No submission. Solution will be provided.

1. **Let T be a minimum spanning tree of G (where G is an edge-weighted graph). Suppose the cost of a certain edge e_0 in G has become changed. Give an example of G, edge weight $w(e)$ for each e in G, T, e_0, and the new weight $w'(e_0)$ such that T is no longer a minimum spanning tree of G after the weight of e_0 becomes $w'(e_0)$.**

2. **The prim's algorithm works correctly when an arbitrary vertex is chosen as a start vertex. Assume that there is only one edge say e_1 whose weight is smaller than any other edges. Prove that e_1 is always included in a tree generated by the Prim's algorithm regardless which vertex is chosen as a start vertex. .**

3. **The input to this problem consists of an ordered list of n words. The length of the ith word is w_i, that is the ith word takes up w_i spaces. (For simplicity assume that there are no spaces between words.) The goal is to break this ordered list of words into lines, this is called a layout. Note that you can not reorder the words. The length of a line is the sum of the lengths of the words on that line. The ideal line length is L. Assume that $w_i \leq L$ for all i. No line may be longer than L, although it may be shorter. The penalty for having a line of length K is $L - K$. Consider the following greedy algorithm.**

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For i = 1 to n
    Place the ith word on the current line if it fits
        else place the ith word on a new line
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(a) **The overall penalty is defined to be the sum of the line penalties. The problem is to find a layout that minimizes the overall penalty. Prove of disprove that the above greedy algorithm correctly solves this problem.**

(b) **The overall penalty is now defined to be the maximum of the line penalties. The problem is to find a layout that minimizes the overall penalty. Prove of disprove that the above greedy algorithm correctly solves this problem.**

Prove or disprove that if a path connecting two nodes (say s and t) is a shortest path using weight function w, it is also a shortest path connecting s and t when weight function w' is used.

4. Assume that edge weights are distinct in a given graph. Let T_K and T_P denote minimum spanning trees generated by the Kruskal's and the Prim's algorithm. Let $k_1 < k_2 < \cdots < k_{n-1}$ and $p_1 < p_2 < \cdots < p_{n-1}$, respectively, denote the weights in T_K and T_P. Prove that $k_i = p_i$ for each $1 \le i \le n-1$.

5. Let G be a connected edge-weighted graph. Let e be an edge in G. Denote by $T(e)$ a spanning tree of G that has minimum cost among all spanning trees of G that contain e. Design an algorithm to find $T(e)$ for a given $e \in E(G)$. Your algorithm should run in $O(n^2)$ time.

6. Suppose we assign n persons to n jobs, Let c_{ij} be the cost of assigning the ith person to the jth job. Use a greedy approach to write an algorithm that finds an assignment that minimizes the total cost of assigning all n persons to all n jobs. Is your algorithm optimal?