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Imaging of multiple simultaneous sources in lossy and heterogeneous solids using an arbitrarily distributed virtual viscoelastic time reversal mirrors

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SUMMARY

The improved imaging, or localization, of seismic sources in a non-ideal solids with viscosities and velocity heterogeneities is a challenging and practical inverse problem. In this work, we validate the feasibility and evaluate the performance of multiple source imaging using an arbitrarily distributed virtual viscoelastic time reversal mirror, which back-play the recorded seismograms in a numerical lossy solid media to reconstruct the initial states. The closely located sources simultaneously emanating waves produce the interference pattern between each other. The intrinsic attenuation not only causes the wave dispersion associated with the propagation distance, but also breaks down the time reversal invariance of the attenuating wave equation. The velocity heterogeneities generate a massive codas from multiple scattering in volume, and the unattainable realization of the random velocity distributions can destroy the spatial reciprocity between the forward and backward propagation. Through numerical simulations, we illustrate that the image quality is some sensitive to the right selection of a reference velocity models but is tolerable to a rough estimation of the attenuation quality factors. The most important feature of a viscoelastic time reversal mirrors in source imaging in lossy and heterogeneous solids is that though the attenuating effect will blur the image, in return it can increase the stability thus provide an unbiased estimation of the source locations. We also illustrate that increasing the density of the arbitrarily distributed mirror can definitely improve the image quality.

1 INTRODUCTION

Modelling of seismic wave propagation in lossy and heterogeneous solid media

$$\rho(\partial_t^2 - \mathcal{L}_{\alpha,\beta} - \partial_t \mathcal{L}_{\eta_\alpha,\eta_\beta})\boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{f}(\boldsymbol{x},t)\mathbf{1}_{\{\boldsymbol{x}=\boldsymbol{x}_\mathrm{s}\}} \tag{1}$$

where α and β are the compressional and shear wave velocities, respectively, η_α and η_α are the viscosities of compressional and shear waves, respectively, and the isotropic elastic wave operator $\mathcal{L}$ can be expressed as

$$\mathcal{L}_{A,B}\boldsymbol{u} = A^2\nabla\nabla\cdot\boldsymbol{u} - B^2\nabla\times\nabla\times\boldsymbol{u} \tag{2}$$

Adopting a reference numerical model, the virtual time reversal wavefield can be numerically reproduced with different schemes

$$\rho_0(\partial_t^2 - \mathcal{L}_{\alpha_0,\beta_0} - \partial_t \mathcal{L}^{\mathrm{tr}}_{\eta_{\alpha 0},\eta_{\beta 0}})\boldsymbol{u}^{\mathrm{tr}}(\boldsymbol{x},t) = \boldsymbol{u}(\boldsymbol{x},-t)\mathbf{1}_{\{\boldsymbol{x}=\boldsymbol{x}_\mathrm{r}\}} \tag{3}$$

where the attenuating term of the time reversal wave operator can be chosen as

$$\partial_t \mathcal{L}^{\mathrm{tr}}_{\eta_{\alpha 0},\eta_{\beta 0}} = \begin{cases} 0 & \text{elastic} \\ \partial_t \mathcal{L}_{\eta_{\alpha 0},\eta_{\beta 0}} & \text{viscoelastic} \\ -\partial_t \mathcal{L}_{\eta_{\alpha 0},\eta_{\beta 0}} & \text{negative viscoelastic} \end{cases} \tag{4}$$

for different virtual time reversal mirrors.

The negative viscoelastic time reversal mirror, though can compensate the attenuating loss during the backward propagation thus improve the imaging resolution as well as in the elastic situation, is a mathematically ill-posed problem, which is unconditionally unstable that the high-frequency noise can be exponentially amplified that the numerical scheme quickly blows up. Such instability might be regularized in a trick ways by truncating the high-frequency noise in the frequency domain or wavenumber domain. Instead, here we are going to adopt the viscoelastic time reversal mirror in imaging sources in lossy and heterogeneous solids. Despite some loss of image sharpness, the backward propagation in viscoelastic virtual solid model will increase the stability of the image, which helps in source imaging in lossy and heterogeneous solids.

The imaging functional is defined as the L^2-norm of the time reversal displacement wavefield at the refocusing time

$$\mathcal{I}(\boldsymbol{x}) = \|\boldsymbol{u}^{\mathrm{tr}}(\boldsymbol{x},0)\|_2 \tag{5}$$

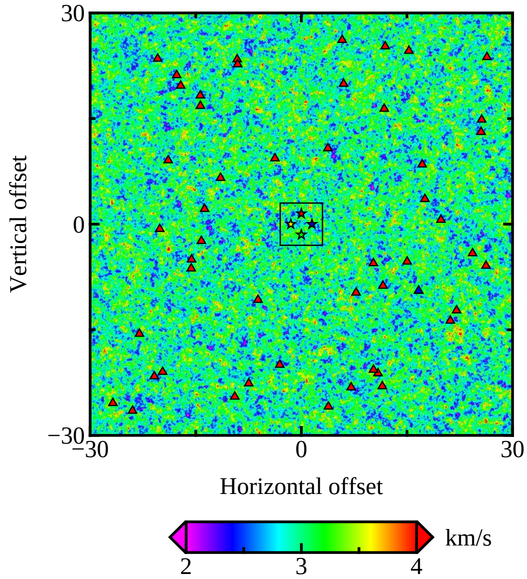

Figure 1. Realization of the random shear wave velocity, which is characterized by the von-Kármán autocorrelation function. The dimensionless horizontal and vertical offsets are normalized by a dominant shear wavelength. 4 single forces, denoted as the stars each with 1.5 offset from origin at 0, 3, 6, 9 o'clock directions, respectively, are pointing to the next in clockwise. 50 arbitrarily distributed transceivers are denoted as the triangles. A small solid-line rectangle is drawn to indicate the subregion.

- The first order time-derivative introduced by the viscosities breaks down the time reversal invariant
  - reduce the image sharpness
  - but increase the image stability
- The mismatch between the reference numerical model $\mathcal{M}_0 = (\rho_0, \alpha_0, \beta_0, \eta_{\alpha 0}, \eta_{\beta 0})$ and the real solid $\mathcal{M} = (\rho, \alpha, \beta, \eta_\alpha, \eta_\beta)$ breaks down the spatial reciprocity between the forward and backward propagation

ACKNOWLEDGMENTS

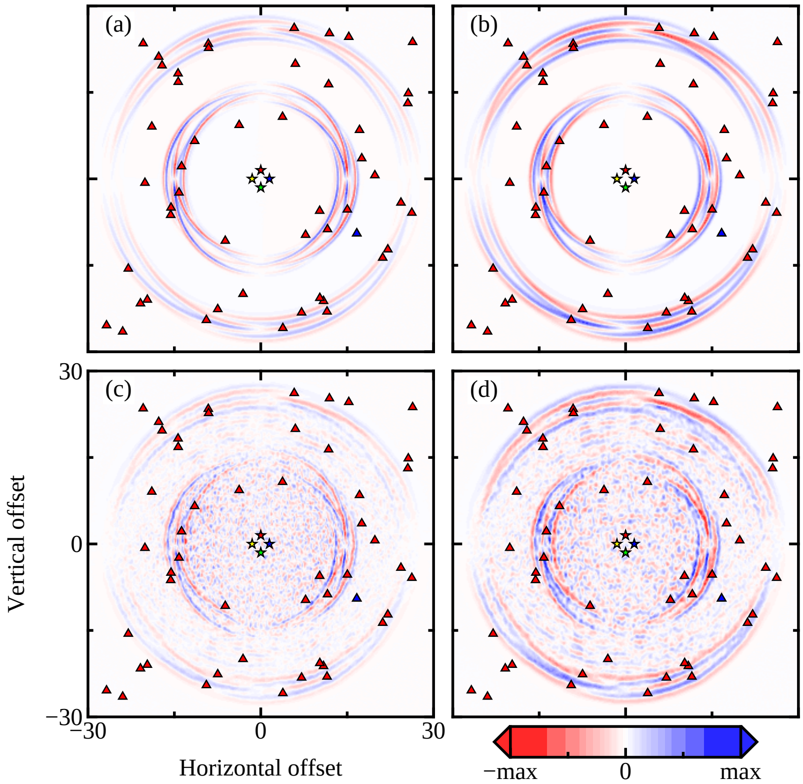

Figure 2. Vertical displacement snapshots at the time of 15th period taken in (a) homogeneous and lossless solid; (b) homogeneous and lossy solid; (c) heterogeneous and lossless solid, and (d) heterogeneous and lossy solid. The horizontal displacement snapshots are given in supplemental material.

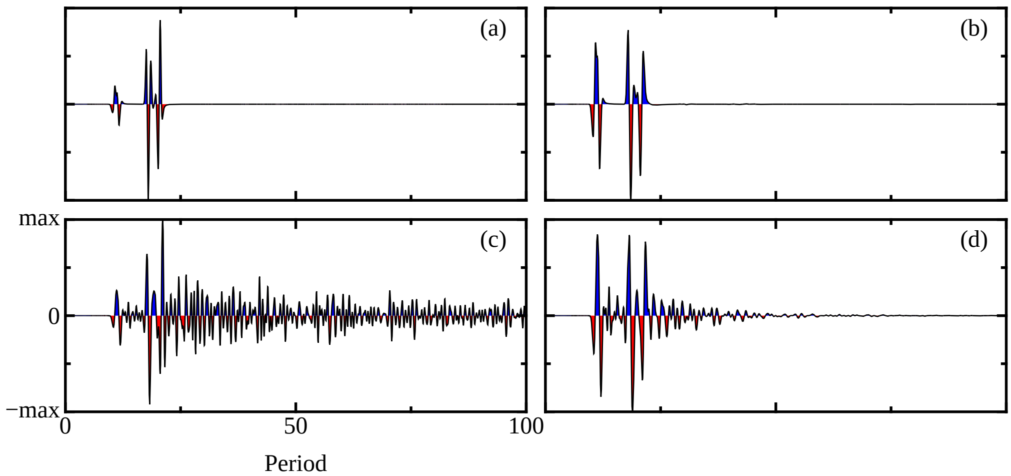

Figure 3. Vertical displacement traces recorded by the transceiver denoted as the blue triangle in (a) homogeneous and lossless solid; (b) homogeneous and lossy solid; (c) heterogeneous and lossless solid, and (d) heterogeneous and lossy solid. The horizontal displacement traces are given in supplemental material.

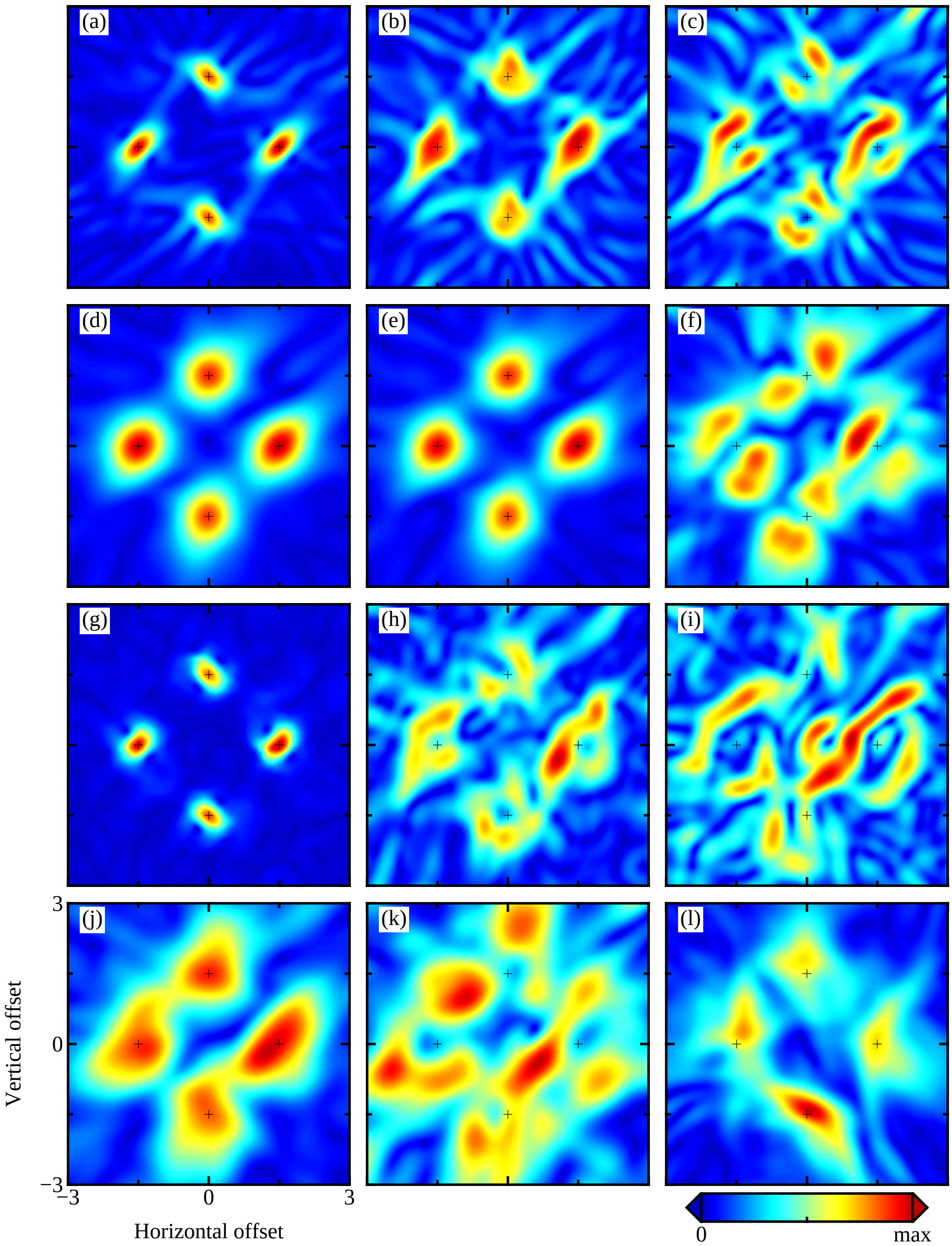

Figure 4. Images obtained under various conditions. In case of the homogeneous and lossless real solid, and the backward numerical model is selected as a homogeneous and lossless solid with a mismatch of shear wave velocity as (a) 0 m/s, (b) 30 m/s, and (c) 50 m/s; In case of the homogeneous and lossy real solid, and the backward numerical model is selected as a homogeneous and lossy solid with a mismatch of the shear quality factor as (d) 0, (e) 30, and (f) $+\infty$; In case of the heterogeneous and lossless real solid, and the backward numerical model is selected as (g) a heterogeneous and lossless numerical model exactly as the same as the real solid, a homogeneous and lossless solid with a mismatch of the mean shear wave velocity as (h) 0 m/s, and (i) 30 m/s; In case of the heterogeneous and lossy real solid, and the backward numerical model is selected as a homogeneous and lossy solid with a mismatches of the mean shear wave velocity and the shear quality factor as (j) 0 m/s and 0, (k) 30 m/s and 30, respectively, and (l) has the same numerical model as (k) except for increasing the transceiver number to 250.