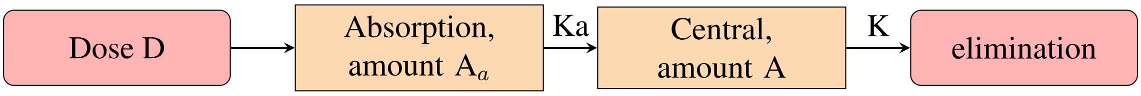
## First order PO, ODE derivation




First order, thus ODE:

$$\frac{\mathrm{d}A_a}{\mathrm{d}t} = -K_a A_a, \qquad \text{IC}: A_a = D \text{ when } t = 0;$$
$$\frac{\mathrm{d}A}{\mathrm{d}t} = K_a A_a - KA, \qquad \text{IC}: A = 0 \text{ when } t = 0;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} A_a \\ A \end{bmatrix} = \begin{bmatrix} -K_a & 0 \\ K_a & -K \end{bmatrix}\begin{bmatrix} A_a \\ A \end{bmatrix}$$

Solve for eigenvalues and eigenvectors and we obtain

$$\lambda_1 = -K_a, \qquad \mathbf{v}_1 = \begin{bmatrix} K - K_a \\ K_a \end{bmatrix}$$
$$\lambda_2 = -K, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Thus the equation is

$$\begin{bmatrix} A_a \\ A \end{bmatrix} = C_1 \mathbf{v}_1 \exp(-K_a t) + C_2 \mathbf{v}_2 \exp(-Kt)$$

in which $C_1$ and $C_2$ are constants. Given the initial conditions, we could obtain $C_1 = \frac{D}{K-K_a}$ and $C_2 = \frac{-K_a D}{K-Ka}$, thus the final equation will be:

$$\begin{bmatrix} A_a \\ A \end{bmatrix} = \frac{D}{K-K_a}\begin{bmatrix} K - K_a \\ K_a \end{bmatrix}\exp(-K_a t) + \frac{-K_a D}{K - Ka}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\exp(-Kt)$$

We are interested in central compartment which could represent the plasma drug amount/concentration

$$A = \frac{K_a D}{K_a - K}(\mathrm{e}^{-Kt} - \mathrm{e}^{-K_a t})$$
$$Conc = \frac{A}{V_d} = \frac{K_a D}{(K_a - K)V_d}(\mathrm{e}^{-Kt} - \mathrm{e}^{-K_a t})$$