# Solutions to Final-Mid term exam

2022/11/28

9:00a – 11:30a

1. (17%)

Evaluate the linear convolution ($h*f$) (8%) and 3×3 circular convolution ($h\otimes f$) (9%) of $h$ and $f$ below. The shaded grids indicate the origin (0,0) of the image spatial coordinate.

$h(x,y)$

| | | |
|---|---|---|
| **1** | **-1** | **0** |
| **-1** | **0** | **0** |
| **0** | **0** | **0** |

$f(x,y)$

| | | |
|---|---|---|
| **1** | **2** | **3** |
| **4** | **5** | **4** |
| **3** | **2** | **1** |

## *Solution*

$$g_{LC}(x,y) = h(x,y) * f(x,y) = \sum_{s=-\infty}^{\infty} \sum_{t=-\infty}^{\infty} h(s,t) f(x-s, y-t) = f(x,y) - f(x-1,y) - f(x,y-1)$$

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 4 |
| 3 | 2 | 1 |

−

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 2 | 3 |
| 4 | 5 | 4 |
| 3 | 2 | 1 |

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| | | | |
|---|---|---|---|
| 0 | 1 | 2 | 3 |
| 0 | 4 | 5 | 4 |
| 0 | 3 | 2 | 1 |

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| | | | |
|---|---|---|---|
| 1 | 2-1 | 3-2 | -3 |
| 4-1 | 5-2-4 | 4-3-5 | -4 |
| 3-4 | 2-5-3 | 1-4-2 | -1 |
| -3 | -2 | -1 | |

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| | | | |
|---|---|---|---|
| 1 | 1 | 1 | -3 |
| 3 | -1 | -4 | -4 |
| -1 | -6 | -5 | -1 |
| -3 | -2 | -1 | 0 |

8

**linear convolution (*h**f*)**

1. (18%) (continue)

Evaluate the linear convolution ($h*f$) (9%) and 3×3 circular convolution ($h\otimes f$) (9%) of $h$ and $f$ below. The shaded grids indicate the origin (0,0) of the image spatial coordinate.

$h(x,y)$

| | | |
|---|---|---|
| **1** | **-1** | **0** |
| **-1** | **0** | **0** |
| **0** | **0** | **0** |

$f(x,y)$

| | | |
|---|---|---|
| **1** | **2** | **3** |
| **4** | **5** | **4** |
| **3** | **2** | **1** |

## *Solution*

$$g_{CC}(x,y) = h(x,y) \otimes f(x,y) = \sum_{s=0}^{2}\sum_{t=0}^{2} h(s,t)\, f((x-s)_3, (y-t)_3)$$

$$= f((x)_3,(y)_3) - f((x-1)_3,(y)_3) - f((x)_3,(y-1)_3)$$

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 4 |
| 3 | 2 | 1 |

−

| | | |
|---|---|---|
| 3 | 2 | 1 |
| 1 | 2 | 3 |
| 4 | 5 | 4 |

−

| | | |
|---|---|---|
| 3 | 1 | 2 |
| 4 | 4 | 5 |
| 1 | 3 | 2 |

=

| | | |
|---|---|---|
| 1-3-3 | 2-2-1 | 3-1-2 |
| 4-1-4 | 5-2-4 | 4-3-5 |
| 3-4-1 | 2-5-3 | 1-4-2 |

=

| | | |
|---|---|---|
| **-5** | **-1** | **0** |
| **-1** | **-1** | **-4** |
| **-2** | **-6** | **-5** |

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**3×3 circular convolution ($h\otimes f$)**

2. (10%)

Consider the design of Wiener filter $H_W(u,v) = \hat{F}(u,v)/G(u,v)$. Assume the degradation function is a 2nd-order Butterworth lowpass filter ($\beta$=1) with cutoff frequency $D_0$. (a) Determine the SNR if the overall gain of Wiener filter is 1.2 at $D_0$ (5%). (b) What is the DC gain of the Wiener filter in (a) (5%)?

## Solution

$$H(u,v) = \frac{1}{1+\left[\dfrac{D^2(u,v)}{D_0^2}\right]^2} \Rightarrow H(D=D_0) = \frac{1}{2}$$

$$H_W(u,v) = \frac{\hat{F}(u,v)}{G(u,v)} = \frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K} = (2)\frac{1/4}{1/4+K} = 1.2 \Rightarrow K = \frac{1}{6}$$

(a) SNR = $1/K$ = 6

5

(b) $H(D=0)=1 \Rightarrow H_W(D=0) = \dfrac{1}{1+\frac{1}{6}} = \dfrac{6}{7}$

5

3. (14%)
(a) Determine the values of H, S and I in HSI color model for a color with normalized (R,G,B) = (0.6,0.7,0.5) (6%). (b) Determine the values of C, M, Y and K in CMYK model for this color (8%).

## Solution

(a)
$$H=\theta=\cos^{-1}\left\{\frac{\frac{1}{2}\left[(R-G)+(R-B)\right]}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right\}=\cos^{-1}\left\{\frac{\frac{1}{2}\left[(0.6-0.7)+(0.6-0.5)\right]}{\sqrt{(0.6-0.7)^2+(0.6-0.5)(0.7-0.5)}}\right\}$$

$$=\cos^{-1}(0)=90^\circ$$

$$G>R>B\Rightarrow H=120-60\frac{R-B}{G-B}=120-60\left(\frac{0.1}{0.2}\right)=90^\circ$$

2

$$S=1-\frac{3}{R+G+B}\left[\min(R,G,B)\right]=1-\frac{3}{1.8}(0.5)=\frac{1}{6}$$

2

$$I=\frac{1}{3}(R+G+B)=0.6$$

2

(b)

C, M, Y in CMY model:

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}-\begin{bmatrix} R \\ G \\ B \end{bmatrix}=\begin{bmatrix} 0.4 \\ 0.3 \\ 0.5 \end{bmatrix}$$

$$(C,M,Y)=(0.4,0.3,0.5)$$

CMYK model:

$$K=\min\{C,M,Y\}=0.3$$

$$\Rightarrow C=\frac{C-K}{1-K}=\frac{1}{7},\ M=\frac{M-K}{1-K}=0,\ Y=\frac{Y-K}{1-K}=\frac{2}{7}$$

$$(C,M,Y,K)=\left(\frac{1}{7},0,\frac{2}{7},0.3\right)$$

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4. (9%)
Prove the circular convolution property of 2D inverse DFT, that is, inverse DFT{$F(u,v)H(u,v)$} = $f(x,y) \otimes h(x,y)$, given $F(u,v)$ is $M\times N$ DFT of $f(x,y)$, $H(u,v)$ is $M\times N$ DFT of $h(x,y)$, and operator $\otimes$ indicates $M\times N$ circular convolution.

*Proof*

$$g(x,y) = DFT^{-1}\{F(u,v)H(u,v)\} = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\{F(u,v)H(u,v)\}W_M^{-ux}W_N^{-vy}$$

$$= \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left\{\left[\sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)W_M^{u\alpha}W_N^{v\beta}\right]H(u,v)\right\}W_M^{-ux}W_N^{-vy}$$

$$= \sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)\left[\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} H(u,v)W_M^{-u(x-\alpha)}W_N^{-v(y-\beta)}\right]$$

$$= \sum_{\alpha=0}^{M-1}\sum_{\beta=0}^{N-1} f(\alpha,\beta)h\big((x-\alpha)_M,(y-\beta)_N\big) = f(x,y)\otimes h(x,y)$$

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5. (30%) Briefly describe

(a) the differences of isopreference curves between crowd and Lena's face (5%),

(b) the assumption for the degradation function and the added noise in the degradation model (5%),

(c) the differences between the band filter and the notch filter (5%),

(d) the mechanism of ringing effect caused by ideal filter (5%).

(e) the scheme for determining the direction of motion from an image degraded by uniform-linear motion blurring (5%).

(f) Consider a real 4×4 function $f(x,y)$ and its 4×4 DFT $F(u,v)$. Determine $g(x,y)=\text{DFT}^{-1}\{F(u,v)(j)^{u}\}$, expressed in $f(x,y)$ (5%).

## Solution

(a) The almost vertical isopreference curve of crowd indicates the subjective quality of crowd image is rather insensitive to the number of bits (number of gray levels); the isopreference curve of Lena's face bends to lower right indicating more pixels are needed to achieve the same subjective quality when using fewer gray levels. (5)

(b) Degradation function is assumed to be linear and shift invariant, noise is random variable, independent of spatial coordinates and uncorrelated with the image. (5)

(c) Band filter has a concentric-ring pass/stop band; notch filter has pairs of notches (holes) symmetrical to DC frequency (0,0). (5)

5. (30%) Briefly describe (continue)
(a) the differences of isopreference curves between crowd and Lena's face (5%),
(b) the assumption for the degradation function and the added noise in the degradation model (5%),
(c) the differences between the band filter and the notch filter (5%),
(d) the mechanism of ringing effect caused by ideal filter (5%).
(e) the scheme for determining the direction of motion from an image degraded by uniform-linear motion blurring (5%).
(f) Consider a real 4×4 function $f(x,y)$ and its 4×4 DFT $F(u,v)$. Determine $g(x,y) = \text{DFT}^{-1}\{F(u,v)(j)^u\}$, expressed in $f(x,y)$ (5%).

## Solution

(d) The linear convolution sum overlaps and sums up the scaled, shifted impulse response. Ideal filter's impulse response, a 2D sinc function, has significant sidelobes that cause ringing effect. (5)

(e) Compute DFT magnitude, from the direction of major (large magnitude) distribution, we may estimate the direction of motion as the direction perpendicular to the DFT major distribution. (5)

(f) According to spatial shift property, $\text{DFT}^{-1}\{F(u,v)(j)^u\} = \text{DFT}^{-1}\{F(u,v)(W_4^{-1})^u\} = f((x+1)_4,y) \rightarrow$
$g(x,y) = f((x+1)_4,y)$ (5)

6. (16%)

Consider a 3 bits/pixel image $f_r$ (size 10×10) with the gray levels $0 \le r_i \le 7$, $i = 0, \ldots, 7$. The number of pixels ($n_i$) with gray level $r_i$ is: $n_0$=10, $n_1$=30, $n_2$=25, $n_3$=20, $n_4$=15, $n_5$=$n_6$=$n_7$=0. (a) Determine the intensity transformation function $z = T(r)$ to produce the output image $f_z$ with the new histogram (probability density function) {0.2 0.2 0.1 0.1 0.1 0.1 0.1 0.1} (8%). (b) Determine the actual histogram of the output image (8%).

## *Solution*

| $r_k$ | $p_r(r_k)$ | $s_k$ | | $v_n$ | $p_z(z_k)$ | $z_k$ |
|---|---|---|---|---|---|---|
| 0 | 0.10 | 0.10 | | 0.2 | 0.2 | 0 |
| 1 | 0.30 | 0.40 | | 0.4 | 0.2 | 1 |
| 2 | 0.25 | 0.65 | | 0.5 | 0.1 | 2 |
| 3 | 0.20 | 0.85 | | 0.6 | 0.1 | 3 |
| 4 | 0.15 | 1 | | 0.7 | 0.1 | 4 |
| 5 | 0 | 1 | | 0.8 | 0.1 | 5 |
| 6 | 0 | 1 | | 0.9 | 0.1 | 6 |
| 7 | 0 | 1 | | 1 | 0.1 | 7 |

$z=T(r)$

| $r_k$ | $z_k$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 6 |
| 4 | 7 |
| 5 | 7 |
| 6 | 7 |
| 7 | 7 |

8

histogram

| $z_k$ | $p_z(z_k)$ |
|---|---|
| 0 | 0.10 |
| 1 | 0.30 |
| 2 | 0 |
| 3 | 0 |
| 4 | 0.25 |
| 5 | 0 |
| 6 | 0.20 |
| 7 | 0.15 |

8

7. (9%)

Consider the image in Fig 7A. Sketch the intensity transformation curves for generating the output images in Figs 7B – 7D.

Fig 7A

Fig 7B

Fig 7C

Fig 7D