Navier-Stokes Equations 原式: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$. (More general) 因为在不可压缩流体的情况下，ρ不是时间或空间的函数，所以方程可以简化为: $\nabla\cdot\vec{u}=0$

Link: https://www.youtube.com/watch?v=ERBVFcutl3M

→ 此式仅在不可压缩流体的情况下成立.

$$\begin{cases} \nabla\cdot\vec{u} = 0 & \text{---- eqn (1)} \\ \rho\dfrac{D\vec{u}}{Dt} = \underbrace{-\nabla P + \mu\nabla^2\vec{u}}_{\text{Internal Forces}} + \rho\vec{F} & \text{---- eqn (2)} \end{cases}$$

eqn (1) Conservation of Mass (or the incompressibility equation)

P: Pressure

μ: viscosity

→ external force i.e. electrical force, gravity, etc

ρ: Density --- like mass

t: time

Internal Forces (See below for explanation)

eqn (2) Momentum Equation ⟺ Newton's 2nd Law

(They model every single fluid in the world).

注: $\frac{D}{Dt}(\ )$: 是随质导数，定义为算子(operator): $\frac{D}{Dt}(*) = \frac{\partial}{\partial t}(*) + (\vec{u}\cdot\nabla)(*)$

Eqn (1) just says that we have some blob (斑点) of fluid and it move around with some velocity and maybe change shape. We are not adding or removing anything, and we would want the same mass of fluid to still be there. --- Conservation of Mass.

$\vec{u}$ is the velocity vector, and could be written as:

$$\vec{u} = (u, v, w)$$

u → x-axis, v → y-axis, w → z-axis

∇ : gradient (named "Nabla").

$$\underbrace{\nabla\cdot\vec{u}}_{\text{Divergence of the velocity}} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

↑ Mass is conserved.

For Eqn (2), the <u>internal forces</u> are the force between all of those fluid particles hitting into each other, crashing, sliding, grinding past one another.

(i)

$$\nabla P = \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z}\right)$$

Gradient of pressure P is a vector representing the change of pressure. when there is a difference in pressure, the partiles tend to move from high pressure region to low pressure region.

(ii)

$$\mu \nabla^2 \vec{u}$$

$\uparrow$

viscosity : friction between the fluid particles (simply speaking) or could be considered as the friction between layers of fluids.

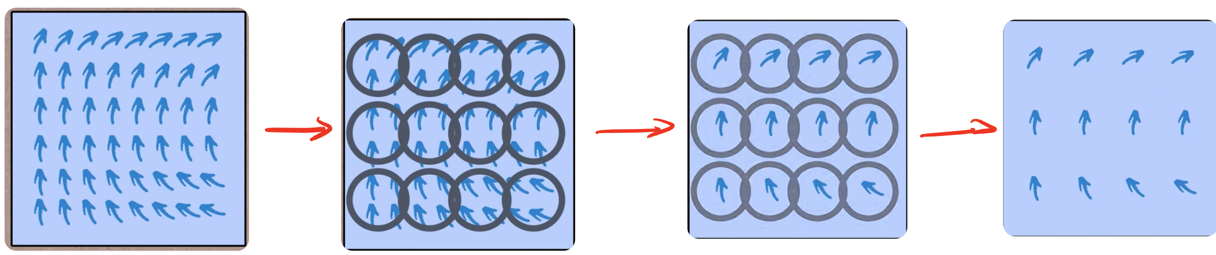
How strong is that friction is the viscosity.

We have now the Navier - Stokes equation, but the most important and tricky part is to solve this equation. However, sometimes we do not even know of such a solution exists.

ways to try to solve it :

① Simplification and assumptions (like initial and boundary condition)

② Averaging (I.e. Instead of having a velocity field defined everywhere, we could just select and combine into some big circle areas of fluid. Then we take the average velocity in the circles. And we want to know how that changes & behaves.



And this is called Reynolds Averaging of the N-S equation.

Now, we have the N-S equation.

For 2-D N-S equation, we have exact solutions

However, for 3-D, we could not obtain such an exact solution.

We have the so-called "weak solutions".

(Similar to the averaging solutions, i.e. when the initial velocity is small)

The N-S equation is essential to understand the turbulence. Turbulence is the chaotic random motion of fluid particles.

Reynolds Number : Re

Assuming a simple case — No external force $\vec{F}$.

Then

velocity

Pressure

velocity

$$\rho \frac{D\vec{u}}{Dt} = -\nabla P + \mu \nabla^2 \vec{u}$$

time

viscosity

length scale

density

Re is defined as $Re = \dfrac{\rho \cdot L \cdot \vec{u}}{\mu}$ → velocity

viscosity

Re number can help us simplify and then help us solve the N-S equation.

The Re number is what apportions (分配) different weightings to the different fact. (or situation).

(A) Low viscosity case $\Rightarrow$ indicates very Turbulent flow.

So if $Re \gg 1$, we could rewrite the N-S equation as :

$$\frac{D\vec{u}}{Dt} = -\nabla P + \frac{1}{Re}\mu\nabla^2\vec{u}$$

(By introducing the Re, we now Non-dimensionalize the system which means that we get rid of the units)

Re has no units, and it's just a number.

So if Re is really big,

$\frac{1}{Re}$ is really small,

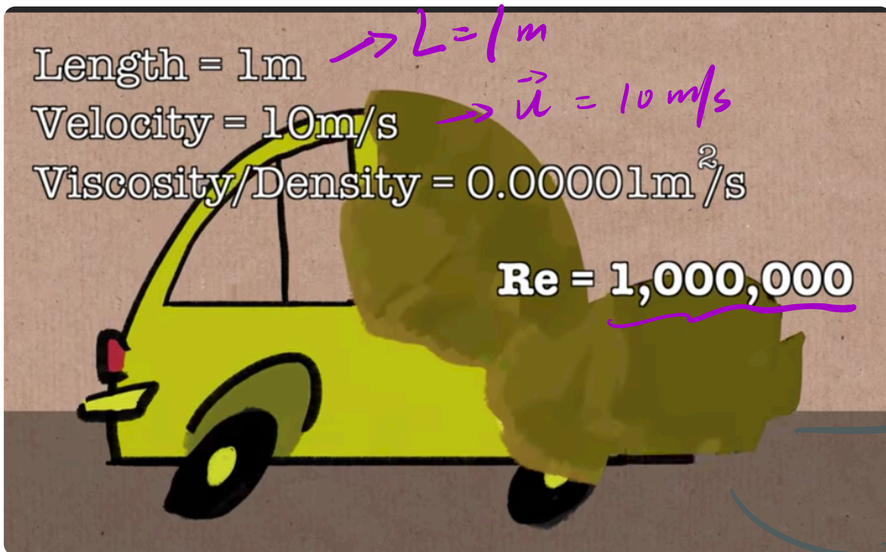
$\downarrow$

$$\frac{1}{Re} \to 0 \longrightarrow \cancel{\frac{1}{Re}\mu\nabla^2\vec{u}} \to 0$$

Thus, the above equation becomes

$$\frac{D\vec{u}}{Dt} = -\nabla P$$

Example: Car moving in air



→ $L = 1\,m$

→ $\vec{u} = 10\,m/s$

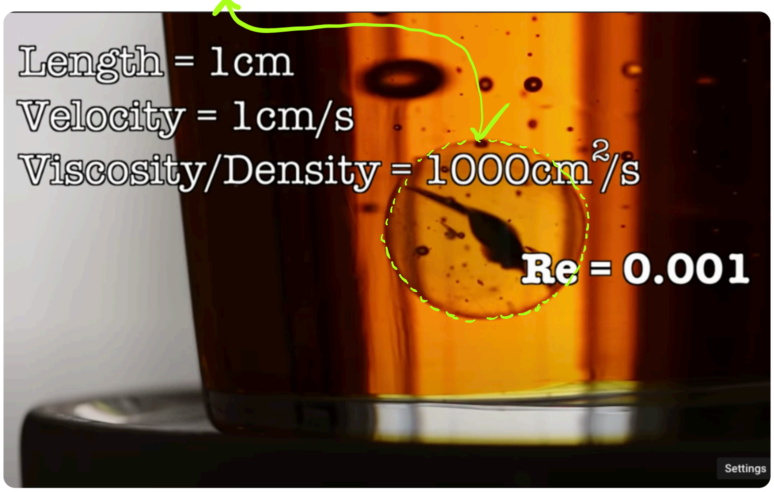
It's very hard for the air to clean the dirt on the car due to the large Re.

so that viscosity does not matter

B. Re << 1 (Re is always > 0)

Example: Marble (大理石) ball falling in treacle (糖蜜)

High viscosity case



Then the N-S equation becomes:

$$Re \frac{D\vec{u}}{Dt} = -\nabla P + \nabla^2 \vec{u}$$

Since Re is very small.

$$0 = -\nabla P + \nabla^2 \vec{u}$$

Time doesn not matter! → (No time dependence in this case!)

雷诺数 [编辑]

维基百科，自由的百科全书

在流体力学中，**雷诺数**（Reynolds number）是流体的惯性力$\frac{\rho v^2}{L}$与黏性力$\frac{\mu v}{L^2}$比值的量度，它是一个无量纲量。

雷诺数较小时，黏滞力对流场的影响大于惯性力，流场中流速的扰动会因黏滞力而衰减，流体流动稳定，为层流；反之，若雷诺数较大时，惯性力对流场的影响大于黏滞力，流体流动较不稳定，流速的微小变化容易发展、增强，形成紊乱、不规则的紊流流场。

定义 [编辑]

对于不同的流场，雷诺数可以有很多表达方式。这些表达方式一般都包括流体性质（密度、黏度）再加上流体速度和一个特征长度或者特征尺寸。这个尺寸一般是根据习惯定义的。比如说半径和直径对于球型和圆形并没有本质不同，但是习惯上只用其中一个。对于管内流动和在流场中的球体，通常使用直径作为特征尺寸。对于表面流动，通常使用长度。

管内流场 [编辑]

对于在管内的流动，雷诺数定义为：

$$\mathrm{Re} = \frac{\rho \mathrm{V} D}{\mu} = \frac{\mathrm{V} D}{\nu} = \frac{\mathrm{Q} D}{\nu A}$$

式中：

- V是平均流速（国际单位：m/s）
- D管直径（一般为特征长度）（m）
- μ流体动力黏度（Pa·s或N·s/m²）
- ν 运动黏度（$\nu = \mu/\rho$）（m²/s）
- ρ流体密度（kg/m³）
- Q体积流量（m³/s）
- A横截面积（m²）

假如雷诺数的体积流速固定，则雷诺数与密度（ρ）、速度的开方（$\sqrt{u}$）成正比；与管径（D）和黏度（u）成反比

假如雷诺数的质量流速（即是可以稳定流动）固定，则雷诺数与管径（D）、黏度（u）成反比；与√速度（$\sqrt{u}$）成正比；与密度（ρ）无关

2

平板流 [编辑]

对于在两个宽板(板宽远大于两板之间距离)之间的流动,特征长度为两倍的两板之间距离

3

流体中的物体 [编辑]

对于流体中的物体的雷诺数,经常用**Re_p**表示。用雷诺数可以研究物体周围的流动情况，是否有旋涡分离，还可以研究沉降速度。

流体中的球 [编辑]

对于在流体中的球，特征长度就是这个球的直径，特征速度是这个球相对于远处流体的速度，密度和黏度都是流体的性质。在这种情况下，层流只存在于Re=10或者以下。 在小雷诺数情况下，力和运动速度的关系遵从斯托克斯定律。

球在流体中的雷诺数可以用下式计算，其中v_f为流体速度，v_s为球速度，d_s为球直径，ρ_f为流体密度，μ_f为流体粘度[1]。

$$Re = \frac{|v_f - v_s| d_s \rho_f}{\mu_f}$$