Notes for Video "Outline of Stochastic Calculus"

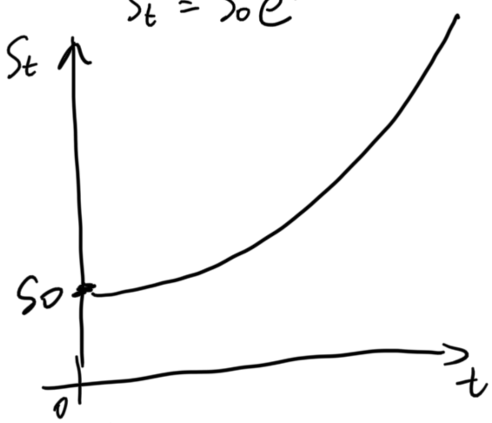
Video Link: https://www.youtube.com/watch?v=rvYfNz2H3Uk

Stochastic Calculus

$$\parallel$$

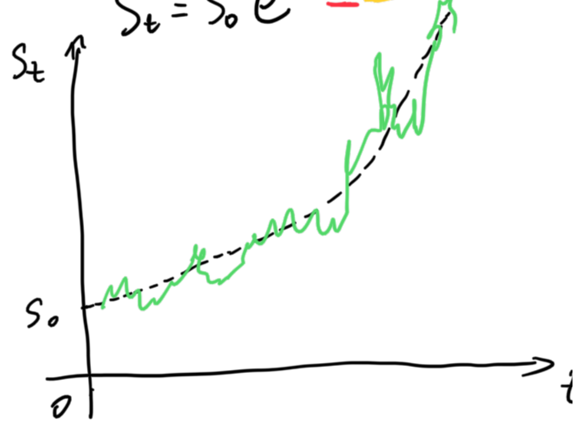
Ordinary Calculus + Randomness

$$S_t = S_0 e^{rt}$$



$$\frac{ds}{dt} = r \underbrace{S_0 e^{rt}} = r S_t$$

$$S_t = S_0 e^{rt + \beta B_t}$$



β : is a constant

B_t : Brownian Motion

It's hard to differentiate this Brownian Curve

Stochastic Calculus Comes in

To Accomodate the Randomness

Randomness does NOT mean there is no

order at all! In fact, randomness could have some distribution.

In this case, for the right curve,

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if we are going to calculate $\frac{dS}{dt}$ using the previous ordinary calculus method, what do we get? $\longrightarrow$ $\frac{dS}{dt} = \left[r + \beta \frac{dB_t}{dt}\right] S_0 e^{rt + \beta B_t}$

$$= \left[r + \beta \frac{dB_t}{dt}\right] S_t$$

$$= r S_t + \beta \underbrace{\frac{dB_t}{dt}}_{} S_t$$

Problem comes!
Does NOT work here!

Thus, we need to introduce new method: Differential Form.

(1) Rewrite $\underbrace{\frac{dS_t}{dt} = r S_t}_{\text{for left figure}}$ into $dS_t = r S_t \, dt$.

(2) $\int dS_t = \int r S_t \, dt$.

$\longrightarrow$ for the left figure (ordinary case, No Brownian motion.

For the right figure (with Brownian motion)

(1) Rewrite $\frac{dS_t}{dt} = rS_t + \beta S_t \frac{d\beta_t}{dt}$ into

$$dS_t = rS_t \, \underline{dt} + \beta S_t \, \underline{d\beta_t}$$

→ * No issues now!

Notes:
For ordinary function (no Brownian motion)

Differentiation $\xleftrightarrow{\text{Inverse to each other}}$ Integration

(2) Here, for our new case (with Brownian Motion)

We are gonna start with Integration first

SDE
↓
Integration

Differentiation ← Integration