1. Indicator function (指示函数)
https://en.wikipedia.org/wiki/Indicator_function

指示函数 [编辑]

维基百科，自由的百科全书

建议将隶属函数并入本条目或章节。（讨论）

在集合论中，**指示函数**是定义在某集合*X*上的函数，表示其中有哪些元素属于某一子集*A*。

现在已经少用这一称呼。概率论有另一意思迥异的特征函数。

集*X*的子集*A*的指示函数是函数$1_A : X \to \{0,1\}$，定义为

$$1_A(x) = \begin{cases} 1 & 若\ x \in A \\ 0 & 若\ x \notin A \end{cases}$$

*A*的指示函数也记作$\chi_A(x)$或$I_A(x)$。

如上一例子所示，指示函数是组合数学一个有用记法。这记法也用在其他地方，例如在概率论：若*X*是概率空间，有概率测度*P*，*A*是可测集，那么1_A就是随机变量，其期望值等于*A*的概率。

$$E(1_A) = \int_X 1_A(x)\, dP = \int_A dP = P(A).$$

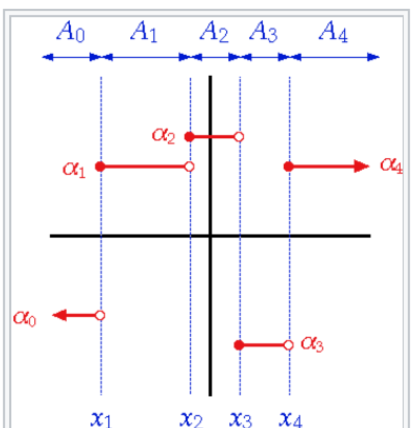
这等式用于马尔可夫不等式的一个简单证明里。

2. Step function (阶跃函数)
(a) https://zh.wikipedia.org/zh-cn/阶跃函数
“在数学中，**如果实数域上的某个函数可以用半开区间上的指示函数的有限次线性组合来表示，那么这个函数就是阶跃函数**，或者叫赫维赛德函数(Heaviside function)。换一种不太正式的说法就是，**阶跃函数是有限段分段常数函数的组合**。”

假设已知：

- 一个系数序列
  $\{\alpha_0, \ldots, \alpha_n\} \subset \mathbb{R},\ n \in \mathbb{N} \setminus \{0\}$
- 区间边界
  $\{x_1 < \cdots < x_{n-1}\} \subset \mathbb{R}$
- 区间序列
  $A_0 := (-\infty, x_1)$
  $A_i := [x_i, x_{i+1})$（对于$i = 1, \cdots, n-2$）
  $A_n := [x_{n-1}, \infty)$

（尽管这个例子中的区间下边界包含在内，而上边界不包含在内，但是这并不是定义所要求的。只要区间 A_n 互不相交，并且它们的组合是实数就可以了。）



n=4 的阶跃函数

定义： 函数 $f:\mathbb{R}\to\mathbb{R}$ 是 **阶跃函数**的条件是当且仅当它可以表示为

对于所有$x\in\mathbb{R}$ 有 $f(x)=\sum_{i=0}^{n}\alpha_i\cdot 1_{A_i}(x)$ 其中 1_A 是 A 的指示函数：

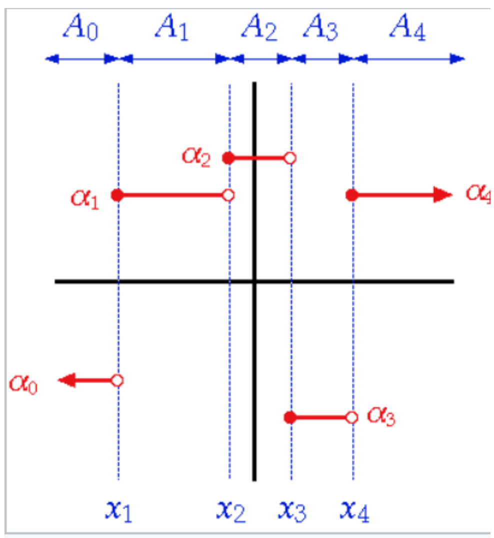
系数 α_i

$$1_A(x)=\begin{cases}1, & \text{if } x\in A\\ 0, & \text{otherwise}\end{cases}$$

注意： 对于所有的 $i=0,\cdots,n$ 及 $x\in A_i$ 满足： $f(x)=\alpha_i$.

(b) https://en.wikipedia.org/wiki/Step_function

"In mathematics, a function on the real numbers is called a step function (or staircase function) if it can be written as a **finite linear combination** of **indicator functions of intervals**. Informally speaking, a step function is a piecewise constant function having only finitely many pieces."



Example of a step function (the red graph). This particular step function is right-continuous.

Definition and first consequences [edit]

A function $f\colon \mathbb{R} \to \mathbb{R}$ is called a **step function** if it can be written as[citation needed]

$$f(x) = \sum_{i=0}^{n} \alpha_i \chi_{A_i}(x), \text{ for all real numbers } x$$

where $n \geq 0$, α_i are real numbers, A_i are intervals, and χ_A is the indicator function of A:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

In this definition, the intervals A_i can be assumed to have the following two properties:

1. The intervals are pairwise disjoint: $A_i \cap A_j = \emptyset$ for $i \neq j$
2. The union of the intervals is the entire real line: $\bigcup_{i=0}^{n} A_i = \mathbb{R}.$

Indeed, if that is not the case to start with, a different set of intervals can be picked for which these assumptions hold. For example, the step function

$$f = 4\chi_{[-5,1)} + 3\chi_{(0,6)}$$

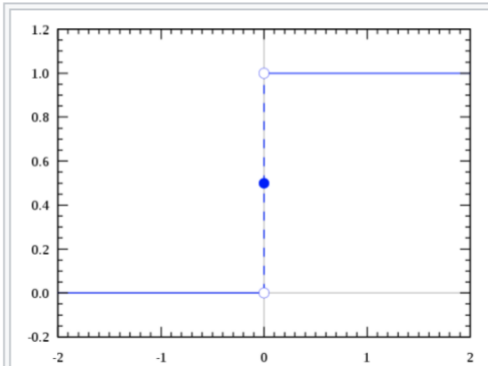
can be written as

$$f = 0\chi_{(-\infty,-5)} + 4\chi_{[-5,0]} + 7\chi_{(0,1)} + 3\chi_{[1,6)} + 0\chi_{[6,\infty)}.$$

Variations in the definition [edit]

Sometimes, the intervals are required to be right-open[1] or allowed to be singleton [2]. The condition that the collection of intervals must be finite is often dropped, especially in school mathematics[3][4][5], though it must still be locally finite, resulting in the definition of piecewise constant functions.

Examples [edit]



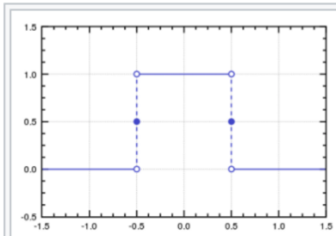
The Heaviside step function is an often-used step function.

- A constant function is a trivial example of a step function. Then there is only one interval, $A_0 = \mathbb{R}$.
- The sign function $\operatorname{sgn}(x)$, which is −1 for negative numbers and +1 for positive numbers, and is the simplest non-constant step function.
- The Heaviside function $H(x)$, which is 0 for negative numbers and 1 for positive numbers, is equivalent to the sign function, up to a shift and scale of range ($H = (\operatorname{sgn} + 1)/2$). It is the mathematical concept behind some test signals, such as those used to determine the step response of a dynamical system.
- The rectangular function, the normalized boxcar function, is used to model a unit pulse.

Non-examples [edit]

- The integer part function is not a step function according to the definition of this article, since it has an infinite number of intervals. However, some authors[6] also define step functions with an infinite number of intervals.[6]

Properties [edit]



The rectangular function, the next simplest step function.

- The sum and product of two step functions is again a step function. The product of a step function with a number is also a step function. As such, the step functions form an algebra over the real numbers.
- A step function takes only a finite number of values. If the intervals A_i, for $i = 0, 1, \dots, n$ in the above definition of the step function are disjoint and their union is the real line, then $f(x) = \alpha_i$ for all $x \in A_i$.
- The definite integral of a step function is a piecewise linear function.
- The Lebesgue integral of a step function $f = \sum_{i=0}^{n} \alpha_i \chi_{A_i}$ is $\int f\, dx = \sum_{i=0}^{n} \alpha_i \ell(A_i)$, where $\ell(A)$ is the length of the interval A, and it is assumed here that all intervals A_i have finite length. In fact, this equality (viewed as a definition) can be the first step in constructing the Lebesgue integral.[7]
- A discrete random variable is sometimes defined as a random variable whose cumulative distribution function is piecewise constant.[8] In this case, it is locally a step function (globally, it may have an infinite number of steps). Usually however, any random variable with only countably many possible values is called a discrete random variable, in this case their cumulative distribution function is not necessarily locally a step function, as infinitely many intervals can accumulate in a finite region.