Notes for YouTube Video "Random Walks Tutorial: Fluctuation Dissipation Relationships"

Link: https://www.youtube.com/watch?v=gwoM-Hw8ztk

① An equation that involves the diffusion coeffvicient.

smoothing out of concentration fluctuations

(平滑)

$$\underset{\text{flux of particles}}{J} = -\frac{vl}{3}\frac{\partial c}{\partial x} = -D\frac{\partial c}{\partial x}$$

l : mean free path

concentration gradient

v : typical thermal velocity of molecules

------- Dissipation

This equation sometimes goes by the framework of dissipation because it's describing how a concentration gradient gradually dissipates because of molecular diffusion.

② Another Eqn:

$$\frac{\partial c}{\partial t} = \frac{(dx)^2}{2dt}\frac{d^2c}{dx^2} = D\frac{\partial^2 c}{\partial x^2}$$

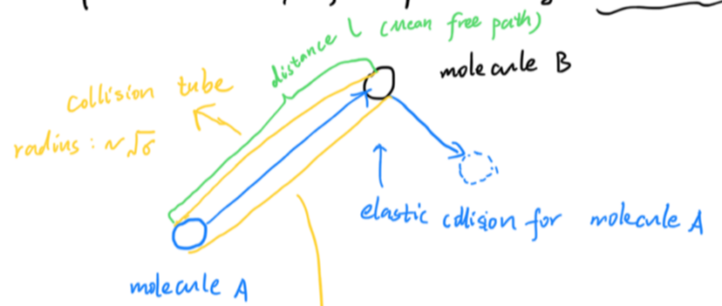
Fluctuation

These 2 Ds are identical since they are both related to the "Fluctuation - Dissipation Theorem".

Consider for gas molecules at Room Temperature

From the kinetic theory of gases

a gas molecular is moving freely with a velocity of $\sim$ 300 m/s, experiencing elastic collisions.



This molecule A has a cross-sectional area σ.

The volume of the collision tube : $l\sigma$

The # of molecules in the tube : $c\,l\sigma \approx 1$ (on the order of ~ 1)

c = concentration; defines collision

$$l \sim \frac{1}{c\sigma} \sim \frac{1}{10^{20}/cc \times 10^{-15}\ cm} = \sim 10^{-5}\ cm = \sim 100 \sim 1000 \text{ molecular diameters}$$

τ = relaxation time (time between collisions)

$$\tau \sim \frac{l}{v} \sim \frac{10^{-5}\ cm}{30000\ cm/s} \sim 10^{-9} \sim 10^{-10}\ sec$$

v = velocity

Random Walk (RW) Picture

Mean square displacement of random walk:

$$\langle x^2 \rangle = N l^2 = \frac{t}{\tau} l^2 = \frac{t}{l/v} l^2 = (vl)\,t$$

N = # of steps in the walk; t = total time elapsed; τ = relaxation time

(vl): This is the diffusion coefficient D we found in the above 2 eqns.