FINM 32000 Practice Final Exam Solutions

University of Chicago

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(1a) $C(99.5) \approx C(100) + C'(100)(-0.5) + (1/2)C''(100) * (-0.5)^2$

$= 8 + 0.8 \times (F(100)/2 + p(100)/8) = 8.215$

(1b) $e^{-rT}F(101) \approx 0.8 \times (F(100) + 1 \times F'(100)) = 0.8 \times (0.5 + 0.15) = 0.52$

(1c) $\mathbb{P}(99 < S_T < 101) \approx (101 - 99)p(100) = 0.3$

(2a) Solution 1:

$\frac{1.05 - \mathtt{C}(15.0,0)}{0.1} + 0.75\frac{1.16 - 2\times 1.05 + 1.04}{0.5^2} = 0$ so $\mathtt{C}(15.0, 0) = 0.3\times 1.04 + 0.4\times 1.05 + 0.3\times 1.16 = 1.08$.

Solution 2: Use L3.10 with $\sigma^2/2 = 0.75$ and $\nu = r = 0$ to get $(q_u, q_m, q_d) = (0.3, 0.4, 0.3)$ and $\mathtt{C}(15.0, 0) = q_u \times 1.04 + q_m \times 1.05 + q_d \times 1.16 = 1.08$

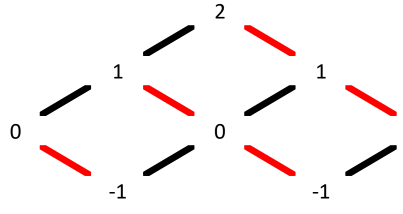
(2b) $\frac{1.01 - \mathtt{C}(15.0,0)}{0.2} + 0.75\frac{1.16 - 2\times 1.05 + 1.04}{0.5^2} = 0$

so $\mathtt{C}(15.0, 0) = 0.6 \times 1.04 - 1.2 \times 1.05 + 0.6 \times 1.16 + 1 \times 1.01 = 1.07$.

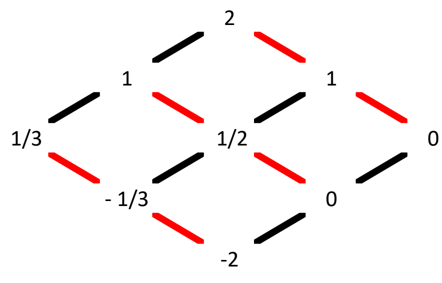
(2c) The standard explicit scheme (a) has the stability property because the weights $0.3, 0.4, 0.3$ are nonnegative, so their absolute values sum to 1.

The "leap-frog" scheme (b) does not have the stability property, because the weight -1.2 is negative, so the weights' absolute values sum to more than 1. Another way to see that the stability property fails, is to take specific values, for example change the 1.05 to 0.

(4) Problems 4a and 4b:



Optimized expectation if you play is 0.
Playing or not playing have the same expectation



Optimized expectation if you play is 1/3.
To maximize expectation you should play.

(5a) Using the implied volatility function from the homework, Impvol$(T_3) = \boxed{0.220}$. Now apply

$$\text{Impvol}^2(T) = \frac{1}{T}\int_0^T \sigma^2(t)\mathrm{d}t$$

repeatedly. Letting σ_1 and $\sigma_2 = 0.320$ and σ_3 denote the values of $\sigma(t)$ on the three disjoint time intervals ending at T_1, T_2, and T_3 respectively, we find $\sigma_1 = \text{Impvol}(T_1) = \boxed{0.25}$ and

$$\text{Impvol}^2(T_2) = \frac{1}{0.25}\Big(\sigma_1^2\times 0.1+\sigma_2^2\times 0.15\Big) \text{ and } \text{Impvol}^2(T_3) = \frac{1}{0.5}\Big(\sigma_1^2\times 0.1+\sigma_2^2\times 0.15+\sigma_3^2\times 0.25\Big)$$

which implies Impvol$(T_2) = \boxed{0.294}$ and $\sigma_3 = \boxed{0.102}$. Plug Impvol(T_1) and Impvol(T_2) into Black-Scholes to find $C(T_1) = \boxed{3.15}$ and $C(T_2) = \boxed{5.86}$.

(5b) We still have $\sigma_1 = \text{Impvol}(T_1) = 0.25$. Given Impvol$(T_3)$, the values of (σ_2, σ_3) which maximize Impvol(T_2) would be the extreme case that $\sigma_3 = \boxed{0}$, making all of the volatility between T_1 and T_3 attributable to σ_2 and none attributable to σ_3. Solving

$$\text{Impvol}^2(T_3) = \frac{1}{0.5}\Big(0.25^2 \times 0.1 + \sigma_2^2 \times 0.15 + 0^2 \times 0.25\Big)$$

produces $\sigma_2 = \boxed{0.625}$ and therefore Impvol$(T_2) = \sqrt{\frac{1}{0.25}(\sigma_1^2 \times 0.1 + \sigma_2^2 \times 0.15)} = \boxed{0.509}$