FINM 33000 Midterm

November 2, 2022

Closed book, closed notes, no calculators, no devices

Your Name: ______________________________

Scores

1	2	3	4	5	Total

General Directions

Results from lecture/homework may be used without justification. Unless otherwise directed, assume frictionless markets and no arbitrage, and assume also the following:

The share price (the price of 1 unit) of any stock is always nonnegative.

All options are European-style (so no early exercise). Calls and puts are standard plain vanilla, unless otherwise noted. All strike prices are positive. All assets pay no dividends/coupons. A binary call with strike K pays 1 if the underlying is $\geq K$ at expiration, otherwise it pays 0. A binary put with strike K pays 1 if the underlying is $< K$ at expiration, otherwise it pays 0.

Let T, T_1, T_2, T^*, and K, K_1, K_2, K^* denote positive constants. "Positive" means > 0.

Let P denote the physical probability measure, and let E denote expectation with respect to P.

Let $\mathbb{P}$ denote martingale measure (with respect to numeraire B), and let $\mathbb{E}$ denote expectation with respect to $\mathbb{P}$. Let the time-0 filtration be trivial; thus $\mathbb{E}$ and $\mathbb{E}_0$ are the same thing.

Let $\mathbb{R}$ denote the real numbers.

For events F and G, recall that $\mathbb{P}(F|G)$ denotes conditional probability .

Let N denote the standard normal CDF, and N' denote its derivative $N'(x) = (1/\sqrt{2\pi})\exp(-x^2/2)$.

Problem 1

In each part, arbitrage exists.

Find a static arbitrage portfolio, by specifying how many units of each asset to hold. If you do not mention an asset, we will assume that your portfolio holds zero units of that asset. Either type 1 or type 2 arbitrage is acceptable. Do not propose more than one arbitrage portfolio.

The assets listed inside the curly braces { } are the *only* assets available for you to trade.

In all parts of this problem, S is the share price of a non-dividend-paying stock.

In every part, the expiry of every option is time $T = 1.0$.

Aside from the conditions on page 1, and any other conditions explicitly stated in each part of the problem, we make no further assumptions about the distribution of the underlying S_T; your arbitrage must be valid irrespective of the distribution of S_T.

Assumptions in the introduction (the lines above here) apply to every part of the problem.

Assumptions in individual parts of the problem (a,b,c,d) are specific to that part of the problem, and do not carry over into other parts of the problem.

(a) Asset Z is the bond, with time-0 price $Z_0 = 0.9$, and time-T price $Z_T = 1$.

Asset C is a 8-strike call on S with time-0 price $C_0 = 7.0$.

Asset P is a 8-strike put on S with time-0 price $P_0 = 5.0$.

Assume $S_0 = 10$.

Find an arbitrage, using some or all of the assets $\{C, P, S, Z\}$.

(b) Asset Z is the bond, with time-0 price $Z_0 = 0.9$, and time-T price $Z_T = 1$.

Asset P is a 9-strike put on S with time-0 price $P_0 = 4.0$.

Asset Q is a 7-strike put on S with time-0 price $Q_0 = 1.0$.

Assume $S_0 = 10$.

Find an arbitrage, using some or all of the assets $\{P, Q, S, Z\}$.

(c) Asset Z is the bond, with time-0 price $Z_0 = 0.9$, and time-T price $Z_T = 1$.

Asset C is a 15-strike call on S with time-0 price 1.0.

Asset X is a contract with time-T payoff $\min(4S_T, 60)$ and time-0 price $X_0 = 38.0$.

Assume $S_0 = 10$.

Find an arbitrage, using some or all of the assets $\{C, S, X, Z\}$

(d) Asset Z is the bond, with time-0 price $Z_0 = 0.9$, and time-T price $Z_T = 1$.

Asset V is a 12-strike straddle on S, with time-T payoff $|S_T - 12|$ and time-0 price $V_0 = 6.0$.

Asset C is a 12-strike call on S with time-0 price $C_0 = 3.0$.

Assume $S_0 = 10$.

Find an arbitrage, using some or all of the assets $\{C, S, V, Z\}$.

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Problem 2

Assume that discount bonds maturing at T have time-0 price 1 per unit, so the interest rate is zero.

Assume that T-expiry European **binary** calls, all on the same underlying random variable S_T are available at the following strikes K and time-0 prices $C_0(K)$.

K	$C_0(K)$
40	0.83
50	0.64
60	0.44
70	0.16

The underlying random variable S_T (which will be revealed at time T) is not available to be traded, and does not have a time-0 price. The only available assets are the binary calls listed above, and the bond.

Find the time-0 prices of contracts with the following payoffs:

(a) Payoff: 1 if $S_T < 70$, otherwise 0

(b) Payoff: 0 if $50 < S_T < 60$, otherwise 1

(c) Payoff: 1 if $S_T \geq 40$, otherwise -1 (negative 1).

Now consider a contract which pays $(S_T - 50)^+$. Its time-0 price cannot be exactly determined given only the information in the table above. However a lower bound can be found.

(d) Find a lower bound on the time-0 price of the contract with payoff $(S_T - 50)^+$. Your bound must be valid irrespective of the distribution of S_T.

You do not need to show that your bound is tight (meaning, that it cannot be increased without making further assumptions), but bounds which are not tight will not get full credit.

In particular, only a small amount of credit will be awarded for giving the trivial lower bound that comes from the fact that the payoff is guaranteed to be nonnegative.

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Problem 3

Interest rates are zero; units of the bank account have constant price 1.

BigPharma drug company has a time-0 stock price of $S_0 = 200$.

BigPharma has applied to the FDA (the US Food and Drug Administration) for approval of a new drug that BigPharma is proposing to sell. The FDA could announce any one of three possible actions at time $T > 0$:

approval, rejection, CRL

(CRL="Complete Response Letter", which is typically a request that the company provide further data on the drug).

In the cases of approval, or rejection or CRL, the price S_T of BigPharma stock would become

248, or 152, or 176, respectively.

Exactly one of the 3 scenarios will happen; there are no other cases.

(a) Let p_A, p_R, p_C denote risk-neutral probabilities of acceptance, rejection, and CRL respectively. There is more than one martingale measure with positive probabilities consistent with the prices of the bank account and the stock.

Find any one of those measures, by giving the three numbers (p_A, p_R, p_C).

You are not required to report *all* combinations of (p_A, p_R, p_C) that work, just *one* combination of those three numbers.

(b) By replication using a static portfolio of the bank account and BigPharma stock, find the time-0 price of a contract that pays 100 or 90 or 60 at time T, in the case of rejection (100), or CRL (90), or acceptance (60), respectively.

(c) Find the time-0 price of the contract in (b), using the probabilities from your (a) answer, rather than replication.

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Problem 4

Emma and Simona play a tennis match, which consists of "sets": a first set, then a second set, and then possibly a third set.

The possible outcomes of each set are that either Emma wins the set or Simona wins the set.

If a player wins both the first set and the second set, then that player wins the match, which ends without playing a third set.

If the players split the first two sets (each player winning one set), then a third set is played, and whoever wins the third set wins the match.

Interest rates are zero; units of the bank account have constant price 1 dollar.

Immediately before nth set (where $n = 1, 2$, and, if the third set is played, 3), you can buy or sell, at a price of **0.4** dollars per unit, arbitrary quantities of "Simona wins set n" contracts, which pay you +1 dollar if Simona wins set n but pay you 0 if Emma wins set n. (Note that the payoff if Simona loses is zero, not a negative payoff.)

Assume that Simona has a physical probability of 50% of winning each game, independently of all other games.

(a) Find the no-arbitrage time-0 price of a "total sets won" contract which pays $n \times 100$ dollars if Simona wins exactly n sets in the match. Thus the possible payout amounts are $0, 100, 200$, depending on whether Simona wins zero sets, 1 set, or 2 sets in the match.

(b) In order to perfectly replicate the "total sets won" contract of part (a), how many units of "Simona wins set 1" contracts should you hold (immediately before set 1)?

If the number of units is negative, you must either say "short" or use a negative sign, otherwise we will assume you intend to hold a positive number of units of the "Simona wins set 1" contract.

You do not need to report how many units of the bank account your replicating portfolio holds.

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Problem 5

Let $Z_t = \exp(6W_t + at) = e^{6W_t + at}$ where a is a constant.

(a) Write Z_t explicitly in terms of drift and diffusion components. You may use either differential notation

$$\mathrm{d}Z_t = \_\_\_\_\mathrm{d}t + \_\_\_\_\mathrm{d}W_t,$$

or integral notation. Your answer may depend on a.

(b) For which value of a is Z a martingale?

(c) For the value of a that solves part (b), find the expectation of Z_5.

Hint: This can be solved without using the answers to parts (a,b).

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