FINM 33000 Practice Final Solutions

December 2023

1(a,b) As in HW1.2 (Trump+Biden = 1 because *exactly* 1 *winner*) or HW2.1 (binary call + same-strike binary put = 1 because exactly 1 of those contracts pays a dollar), here the sum of all 4 contract values should be 2 because exactly 2 teams advance (and interest rate is 0).

1a: Long 1 unit of each team, short 2 units of bank B. Time-0 value -0.04, time-T value 0.

1b: Short 1 unit of each team, long 2 units of bank B. Time-0 value -0.02, time-T value 0.

1(c) To find a lower bound, *subreplicate* the contract (see HW2, "Discussion of Problem 1").

$(+1X, +1Y, -1B)$ subreplicates the parlay contract, because the portfolio pays 1 if both X, Y advance, and otherwise either 0 or -1. So lower bound $0.92 + 0.60 - 1 = \boxed{0.52}$.

(How to obtain that subreplicating portfolio? Ask yourself, is the parlay contract bullish or bearish team X? Is it bullish or bearish team Y? Answer: Intuitively, bullish X, and bullish Y. So imagine a portfolio, longing $+1$ unit of each. But that pays 0/1/2 dollars depending on how many of those two teams advance, dominating the parlay contract which pays 0/0/1. So subtract 1 unit of B from the portfolio, creating a payoff $-1/0/1$, which subreplicates.)

2(a) Put delta -0.495, call delta 0.505. HW6.3c.

2(b) Put gamma 0.035, call gamma 0.035. HW6.3d.

2(c) Call theta -9.71, put theta $-9.71 + rKe^{-r(T-t)} = -9.71 + 0.02(40)e^{-0.02(0.25)}$ using a similar put-call parity type of calculation as in HW6.3.

2(d) $1 - 0.387 = 0.613$ because put is ITM if and only if call is OTM, same logic as in HW2.1b.

2(e) $38 \times 0.505 - 40e^{-0.005} \times 0.387$

2(f) As strike increases, the deltas of calls and puts decrease (see the IB delta tables!) Intuition: Higher $K \Rightarrow$ call more OTM $\Rightarrow$ smaller Δ. Higher $K \Rightarrow$ put more ITM $\Rightarrow$ Δ more negative. Mathematical reason: if K increases, then d_1 decreases, so both $N(d_1)$ and $N(d_1)-1$ decrease.

The call deltas are positive, while the put deltas are negative, so $\boxed{F < D < B < E < C < A}$

3(a) Mean is $X_0 + 3 \times 0.36 = 2.08$. Variance is $2^2 \times 0.36 = 1.44$.

3(b) By the same type of calculation as done to obtain the first term of the HW7.1a result,

$e^{-rT}\mathbb{E}(X-\mu)^+ = \frac{e^{-rT}}{\sqrt{2\pi\sigma^2 T}}\int_\mu^\infty (x-\mu)e^{-(x-\mu)^2/(2\sigma^2 T)}\mathrm{d}x = \frac{\sigma\sqrt{T}}{\sqrt{2\pi}}e^{-rT} = \frac{1.2}{\sqrt{2\pi}}e^{-0.0036}$

3(c) Same type of calculation as done to obtain the second term of the HW7.1a result:

$e^{-rT}\mathbb{P}(X_T \geq 2) = e^{-rT}\mathbb{P}(\frac{X_T-2.08}{\sqrt{1.44}} \geq \frac{2-2.08}{\sqrt{1.44}}) = e^{-0.0036}N(0.08/1.2)$

4. Payoff decomposition: Long 8 shares, short 8 calls at strike 25, long 5 calls at strike 40.

How to see this? For this payoff, try drawing pictures (rather than doing algebraically):

Solution 1: Picture drawn in L1.35 showed how to build piecewise linear payoff from calls.

Solution 2: Imagine that the payoff for $S_T > 40$ were to be flat at level 200 (rather than upward-sloping); then the payoff would be 8 covered call combinations (long stock, short 25-strike call) as in HW1.1c. To get the actual upward-sloping payoff for $S_T > 40$, need to additionally include 5 calls at strike 40.

(Note: this is a simplified version of the payoff of a "mandatory convertible")

4(a) Apply Black-Schoes with $\sigma\sqrt{T} = 0.3$ and $d_{1,2} = \log(32e^{0.0144}/K)/0.3 \pm 0.3/2$.

(Note that there are two different values of d_1, and two different values of d_2, depending on whether we are calculating the 25 or 40 strike).

$$\begin{aligned}
\text{Contract value} &= 8S_0 - 8\times(\text{price of 25-strike call}) + 5\times(\text{price of 40-strike call}) \\
&= 8\times 32 \\
&\quad - 8\times\left(32N\Big(\frac{\log(32e^{0.0144}/25)}{0.3}+\frac{0.3}{2}\Big) - 25e^{-0.0144}N\Big(\frac{\log(32e^{0.0144}/25)}{0.3}-\frac{0.3}{2}\Big)\right) \\
&\quad + 5\times\left(32N\Big(\frac{\log(32e^{0.0144}/40)}{0.3}+\frac{0.3}{2}\Big) - 40e^{-0.0144}N\Big(\frac{\log(32e^{0.0144}/40)}{0.3}-\frac{0.3}{2}\Big)\right)
\end{aligned}$$

4(b) Number of shares to hold is given by the Delta:

$$8 - 8\times N\Big(\frac{\log(32e^{0.0144}/25)}{0.3}+\frac{0.3}{2}\Big) + 5\times N\Big(\frac{\log(32e^{0.0144}/40)}{0.3}+\frac{0.3}{2}\Big)$$

Similar to HW6.1c (but here you are asked to *replicate* the risk, rather than to hedge/neutralize/cancel that risk).

5(a) By L6.14, the risk-neutral drift and volatility of S are r and σ respectively.

By HW6.2b, the physical drift and volatility of L are $2\mu - r$ and 2σ respectively. Under risk-neutral measure the μ becomes r, so the risk-neutral drift and volatility of L are $2r - r = r$ and 2σ respectively. (Alternative solution for the risk-neutral drift: self-financing portfolios of tradeable assets can be regarded as tradeable, so the risk-neutral drift of L must be r.)

5(b) Use the HW6.3 put price formula: $d_{1,2} = 0.03/0.50 \pm 0.50/2$ so the put price is

$Ke^{-rT}N(-d_2) - L_0N(-d_1) = 10e^{-0.03}N(-0.03/0.50+0.50/2) - 10N(-0.03/0.50-0.50/2)$.