FINM 33000 Practice Midterm Solutions

November 2023

Scoring P1: $6+6+6+6$, P2: $5+5+5+5$, P3: $6+8+6$, P4: $10+6$, P5: $10+5+5$.

Problem 1

(a) Put call parity $C_T = P_T + S_T - KZ_0$ but $7 < 5 + 10 - 0.9 \times 8$.

Type 2 arb $(+1C, -1P, -1S, +8Z)$

(b) Put spread should not be priced higher than $2Z_0$. (Similar to call spread in HW1.1d)

Type-2 arb $(-1P, +1Q, +3Z)$

(c) $\min(4S_T, 60) = 4\min(S_T, 15) = 4S_T - 4\max(S_T - 15, 0)$, but $40 - 4 < 38$.

Type 2 arb: $(-1X, +4S, -4C)$. The contract is 4 covered call combinations (HW1.1c).

(d) Build a straddle from a call, using put-call parity:

$C_T + P_T = 2C_T - S_T + KZ_0$ but $6 < 6 - 10 + 12 \times 0.9$.

Type-2 arb: $(+1V, -2C, +1S, -12Z)$.

Problem 2

(a) Long the bond, short the 70-strike binary call, to replicate the binary put (HW2.1b).

Price $1 - 0.16 = 0.84$

(b) The question should have had only one strict inequality: payoff zero for $50 \leq S_T < 60$.

Replicating portfolio: Long 1 bond, short a 50-strike binary call, long a 60-strike binary call:

(which is equivalent to being long a binary 50-put and a binary 60-call).

Price $1 - 0.64 + 0.44 = 0.80$.

(c) Replicating portfolio: (2 units of the 40-strike call, -1 units of the bond).

Price $2 \times 0.83 - 1 = 0.66$

(d) Subreplicating portfolio: (10 units of the 60-strike call, 10 units of the 70-strike call).

Lower bound on time-0 price $10 \times 0.44 + 10 \times 0.16 = 6.0$

Problem 3

(a) See HW3.2. Solve

$$\begin{pmatrix} 248 & 152 & 176 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_A \\ p_R \\ p_C \end{pmatrix} = \begin{pmatrix} 200 \\ 1 \end{pmatrix} \quad \Longrightarrow \quad \begin{pmatrix} p_A \\ p_R \\ p_C \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \\ 0 \end{pmatrix} + \begin{pmatrix} -1/4 \\ -3/4 \\ 1 \end{pmatrix} c$$

for any $0 < c < 2/3$.

For example $c = 0.4$ gives $(p_A, r_R, p_C) = (0.4, 0.2, 0.4)$.

(b) See HW3.1. Hold α units of the bank account and β units of stock respectively, where

$$\alpha(1, 1, 1) + \beta(248, 152, 176) = (60, 100, 90)$$

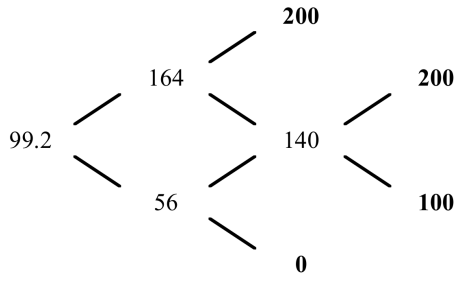
Solution: $\alpha = 490/3$ and $\beta = -5/12$

Value of the replicating portfolio is $(490/3) \times 1 - (5/12) \times 200 = 80$

(c) $0.4 \times 60 + 0.2 \times 100 + 0.4 \times 90 = 80$. See HW3.1.

Problem 4

(a) See HW3.3. Risk-neutral probability p of Simona winning set n satisfies $0.4 = p \times 1 + (1-p) \times 0$, so $p = 0.4$.



Time-0 value of the "total sets won contract" is 99.2

(b) You should hold $\frac{164-56}{1-0} = 108$ contracts on Simona winning set 1. See HW3.3.

Problem 5

(a) See HW5.1 solution. Let $X_t = at + 6W_t$. Then $\mathrm{d}X_t = a\mathrm{d}t + 6\mathrm{d}W_t$ and $\mathrm{d}Z_t = e^{X_t}\mathrm{d}X_t + \frac{1}{2}e^{X_t}(\mathrm{d}X_t)^2 = (a + 18)e^{X_t}\mathrm{d}t + 6e^{X_t}\mathrm{d}t$ by Ito.

(b) Martingale condition is that drift vanishes (L4.12 or HW5.1c) which occurs for $a = -18$.

(c) By L3.24 definition of martingale, $\mathbb{E}Z_5 = Z_0 = e^0 = 1$.