FINM 34000, Autumn 2023
Lecture 2

Reading: Notes, Section 3.

Exercise 1 *Suppose we change the probabilities in simple random walk so that*

$$\mathbb{P}\{X_j = 1\} = 1 - p, \qquad \mathbb{P}\{X_j = -1\} = p,$$

where $1/2 < p < 1$. *Let*

$$q_n = \mathbb{P}\{S_{2n} = 0\}$$

where we start at the origin.

- *Give an exact expression for* q_n.

- *Show that*

$$\sum_{n=1}^{\infty} q_n < \infty$$

  and conclude that the random walk does not return to the origin infinitely often.

Exercise 2 *Use the central limit theorem to find*

$$\lim_{n\to\infty} \mathbb{P}\{S_n < \frac{2}{3}\sqrt{n}\}.$$

Do this for both the symmetric simple random walk and the asymmetric random walk in Exercise 1.

Exercise 3 *Let us call* m *an upswing time for (symmetric) simple random walk if* $S_m = S_{m-5} + 5$, *that is, if we have had five consecutive* $+1$ *values. Find the expected number of steps until we have an upswing time. (Hint: a very similar problem was discussed in the August review and you should feel free to consult those notes.)*