

FINM 34000, Autumn 2023

Lecture 3

Reading: Notes, Section 4.1.

Note: you may use any program to do the matrix calculations. This is true for future problem sets as well.

Exercise 1 Suppose X_n is a Markov chain with state space $S = \{A, B, C\}$ and transition matrix

$$\mathbf{P} = \begin{array}{c} \begin{array}{ccc} & A & B & C \\ A & 1/2 & 1/4 & 1/4 \\ B & 1/3 & 1/3 & 1/3 \\ C & 1/4 & 1/2 & 1/4 \end{array} \end{array}.$$

1. Find the long range fraction of time that the chain spends in the three states A, B, C .
2. Suppose an investor gains \$10 when the chain is in state B , \$5 when the chain is in state C , and loses \$5 when the chain is in state A . Then the long range earnings after n steps is proportional to cn for some constant c . What is c ?
3. Suppose that $X_0 = A$. Find

$$\mathbb{P}\{X_4 = B \mid X_3 = C\}, \quad \mathbb{P}\{X_3 = C \mid X_4 = B\}.$$

Exercise 2 Consider the following Markov chain with state space $\{0, 1, 2, 3\}$.

- When the chain is in state j , then $3 - j$ fair dice are rolled. Let k be the number of these dice that come up either 5 or 6. Then we move to state k . In particular, if we are in state 3 we always move to state 0.

1. Write down the transition matrix for this chain.
2. Give the invariant probability for the chain.
3. Suppose we start at state 0. What is the probability that after three steps we are in state 3?
4. Suppose we start at state 0. What is the expected number of steps until we return to state 0?

Exercise 3 Suppose X_n is a finite irreducible Markov chain whose transition matrix is doubly stochastic, that is, both the rows and the columns add up to one. Show that the uniform distribution is the invariant probability distribution for the chain.