FINM 34000, Autumn 2023
Lectures 8 and 9

Reading: Notes, Sections 6.1 – 6.3 (we will not cover 6.4)

Exercise 1 *Suppose X_t is a Poisson process with parameter $\lambda = 3$. Find the following.*

1. $\mathbb{P}\{X_3 = 7\}$

2. $\mathbb{P}\{X_2 (X_5 - X_2) = 0\}$

3. *The expected amount of time until* $X_t = 6$

4. $\mathbb{P}\{X_1 = 2 \mid X_3 = 7\}$.

Exercise 2 *Suppose that the number of customers arriving at an automobile dealership in Chicago follows a Poisson process with $\lambda = 2$ (time is measured in hours).*

a. *What is the probability that at most two customers arrive in the first hour?*

b. *Suppose that exactly two customers arrive in the first hour. What is the probability that there will be exactly three customers in the second hour?*

c. *Suppose that exactly four customers arrived in the first two hours. What is the probability that exactly two customers arrived in the first hour?*

d. *An enthusiastic salesperson decides to wait until 10 customers have arrived before going to lunch. What is the expected amount of time she will have to wait?*

e. *Let N denote the number of customers that arrive in the first two hours. Find $\mathbb{E}[N^2]$.*

Exercise 3 *Suppose X_t is a continuous time Markov chain with state space $\{0, 1, 2, 3\}$ with rates*

$$\alpha(0,1) = 2, \quad \alpha(1,2) = 3, \quad \alpha(2,0) = 1, \quad \alpha(2,3) = 1, \quad \alpha(3,1) = 4,$$

with all other rates equal to zero.

1. *Write down the generator* $\mathbf{A}$.

2. *Is this chain irreducible?*

3. *What is the invariant probability?*

4. *Suppose we start with $X_0 = 0$. What is the expected amount of time until reaching state 3?*

5. *Suppose we start with $X_0 = 0$. What is the expected amount of time until the chain leaves state 0 for the first time?*

6. *Suppose we start with $X_0 = 0$. What is the expected amount of time for the chain to leave 0 and then return to 0 for the first time?*

Exercise 4 *Answer the same questions as in Exercise 3 with state space $\{0, 1, 2, 3, 4\}$ and rates*

$$\alpha(0,1) = \alpha(1,0) = \alpha(1,2) = \alpha(2,1) = \alpha(2,3) = \alpha(3,2) = \alpha(3,4) = \alpha(4,3) = 1.$$

This is a continuous time version of simple symmetric random walk with reflecting boundary.